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Black Holes, Quantum Mechanics, and the Topology of Space and Time

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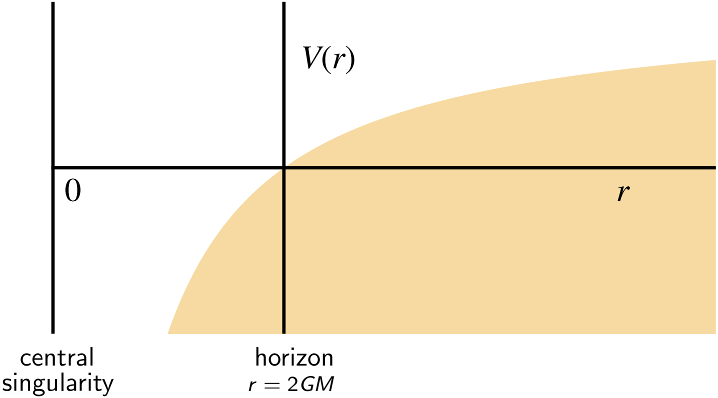
Schwarzschild metric:

$$\mathrm{d}s^2 = \quad -\mathrm{d}t^2\left(1 - \frac{2GM}{r}\right) + \frac{\mathrm{d}r^2}{1 - 2GM/r} + r^2\mathrm{d}\Omega^2 \ ;$$

$$\mathrm{d}\Omega^2 \equiv \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2 \ .$$



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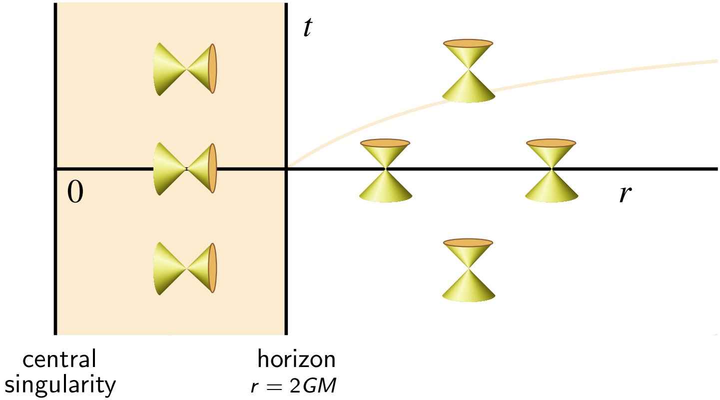
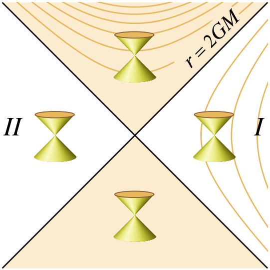


The tortoise coordinates

Kruskal-Szekeres coordinates x, y, defined by

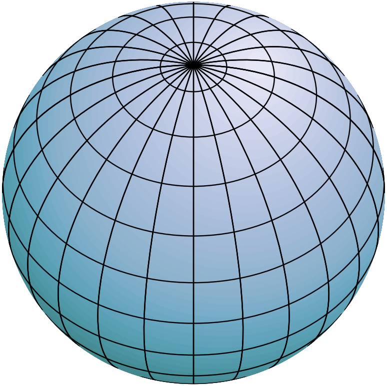
$$\begin{aligned} x\,y &= \left(\frac{r}{2GM}-1\right)e^{r/2GM} \\ y/x &= e^{t/2GM} \end{aligned}$$

In these coordinates, the metric stays regular at the horizon ($r \to 2GM$). x and y are light cone coordinates.

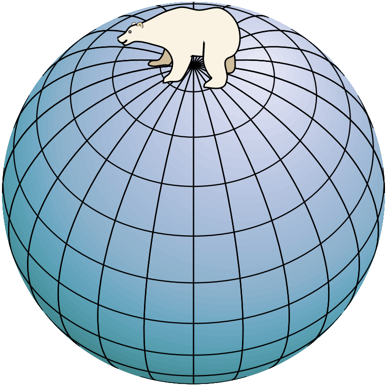


For the outside observer, *time stands still* at the horizon (the origin of this diagram)

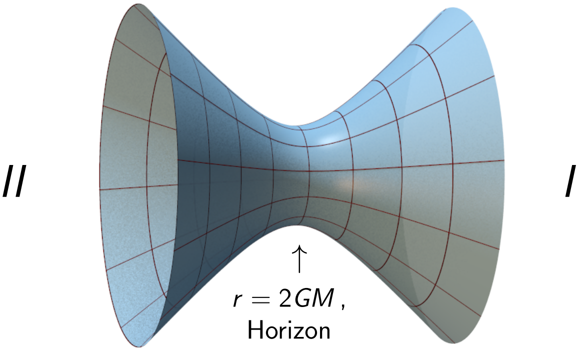
At the horizon, the outside observer has a coordinate-singularity. It is not physical. Compare polar coordinates on planet Earth.

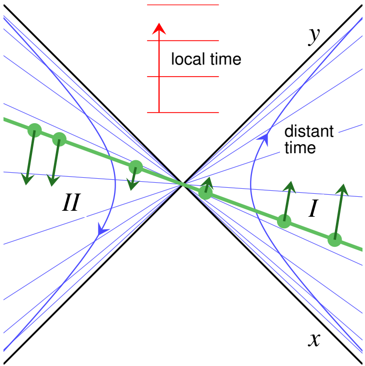


A polar bear walking on the North Pole would not notice anything unusual there.



The Einstein - Rosen bridge, at *fixed* time



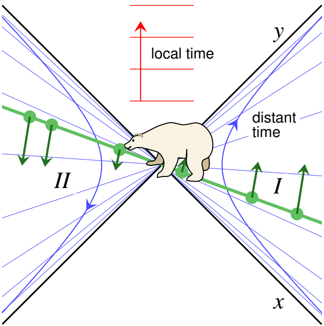


$$x(\tau) = x(0)e^{-\tau}$$
$$y(\tau) = y(0)e^{\tau}$$

As time $\tau = t/4GM$ goes forwards, x approaches the horizon asymptotically;

as time goes backwards, y approaches the past horizon asymptotically (tortoises).

If the outside observer makes a time boost, the local observer makes a Lorentz transformation.



Hartle-Hawking vacuum:

$$|\, HH \,\rangle \;=\; C \sum_{E,n} e^{-\frac{1}{2}\beta E}\, |E,n\rangle_I\, |E,n\rangle_{II}$$

Time boost for distant observer = Lorentz boost for local observer.

Usual interpretation:
I = outside
II = other universe, or inside [?]

→ quantum entanglement becomes entropy:
→ a thermal state . . .

In- and out-going particles: energies E for distant observer stay small.

But for the local observer, energies of in-particles in distant past, as well as the out-particles in distant future, rapidly tend to infinity.

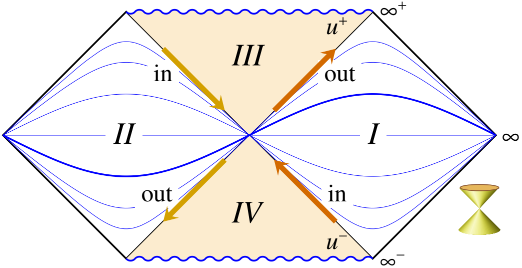
The black hole with surrounding universe: the Penrose diagram

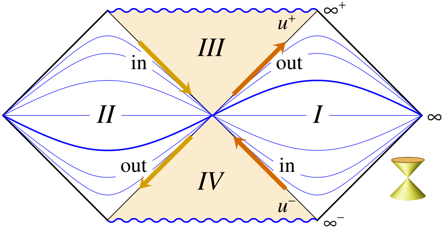
Hawking particles are seen as being formed by a *local vacuum state.* There, they have no effect on the metric at all.

By time reversal invariance, we must now also first regard the states of the in coming particles as having no effect on the metric as well.

Thus, consider first the black hole metric without the effects of matter – the *eternal black hole*.

The Penrose diagram is a conformally compressed picture of all of space-time:





Problems:

1) If we allow large time translations, infinitely many infinitely energetic Hawking particles will crowd both the future and the past horizon.

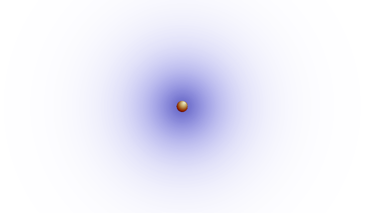
2) The particles going in do not seem to affect particles going out: no unitarity in the evolution process.

The black hole does not seem to respond to our messages,

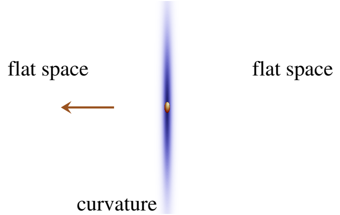
and it houses infinitely many particles, with no bounds on their energy-momentum.

The *gravitational effect* of a fast, massless particle is easy to understand:

Schwarzschild metric of a particle with tiny rest mass $m \ll M_{\text{Planck}}$:



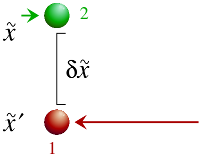
And now apply a strong *Lorentz boost*, so that $E/c^2 \gg M_{\text{Planck}}$:

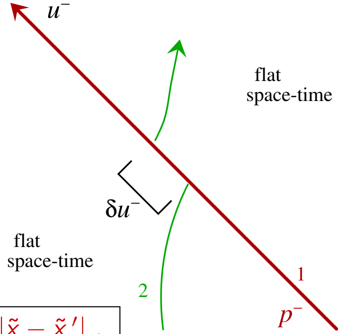


This gives us the

gravitational backreaction:

Lorentz boosting the light (or massless) particle gives the *Shapiro time delay* caused by its grav. field:





$$\boxed{\delta u^-(\tilde{x}) = -4G\, p^-(\tilde{x}\,')\, \log|\tilde{x} - \tilde{x}\,'|\ .}$$

P.C. Aichelburg and R.U. Sexl, J. Gen. Rel. Grav. **2** (1971) 303,
W.B. Bonnor, *Commun. Math. Phys.* **13** (1969) 163,
T. Dray and G. 't Hooft, *Nucl. Phys.* **B253** (1985) 173.

Hard and soft particles

The particles populating this space-time: their energies may go way beyond M_{Planck}.

If so, we call them **hard particles**:
they cause shifts in the orbits of the soft particles

This effect is strong and chaotic: no observer trying to cross such a curtain of particles can survive: **firewalls**

Almheiri, Marolf, Polchinski, Sully (2013)

Particles whose energies, *in a given Lorentz frame*, are small compared to M_{Planck} will be called **soft particles**.

Their effects on curvature are small compared to L_{Planck}, and will be ignored (or taken care of in *perturbative* Qu.Gravity).

During its entire history, a black hole has in-going matter (grav. implosion) and out-going matter (Hawking).
If we want to express these in terms of pure quantum states, we must expect firewalls both on the future and past event horizon.

(The pure quantum theory must be CPT symmetric)

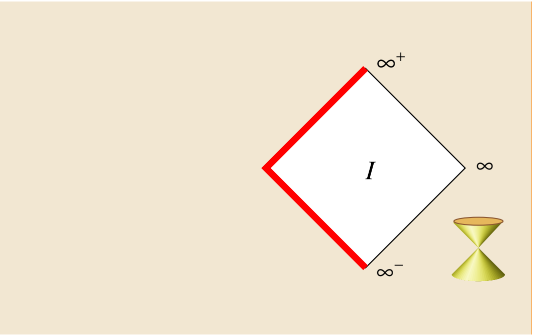
Such firewalls would form a natural boundary surrounding region I

That can't be right

Derivation of Hawking radiation asks for analytic extension to region *III*

Time reversal symmetry then asks for analytic extension to region *IV*

In combination, you then also get region *II*



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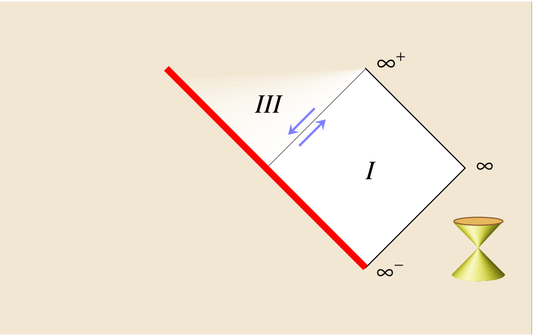

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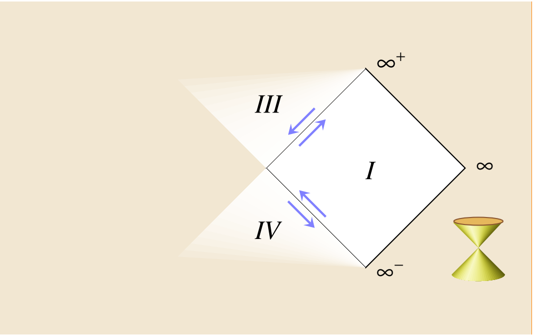

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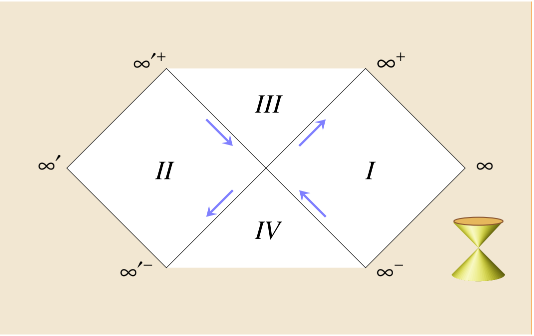


All this suggests that firewalls can be switched on and off:

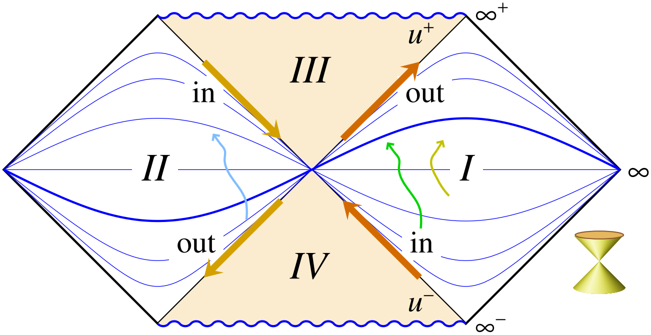
the firewall transformation.

(1.) Note: Hawking's wave function seems to form a single quantum state *(if we assume both regions I and II of the Penrose diagram to be physical! – see later)*. A firewall would form infinitely many quantum states. What kind of mapping do we have? Aren't we dealing with an information problem here??

(2.) Region *II* would have its own asymptotic regions: ∞', ∞'^{+}, and ∞'^{-}. What is their physical significance?

Wait and see ...

We start with only soft particles populating space-time in the Penrose diagram. Claim: this is the most generic wave function of a Schwarzschild(-like) black hole:



We now first wish to understand the evolution operator for short time intervals only. Firewalls have no time to develop.

The evolution law for the soft particles is fully dictated by QFT on curved space-time.

At $|\tau| = \mathcal{O}(1)$, particles going in, and Hawking particles going out, are soft.

However, during our short time interval, some soft particles might pass the borderline between soft and hard: they now interact with the out-particles.
The interaction through QFT forces stay weak, but the *gravitational forces* make that (early) in-particles interact strongly with (late) out-particles.

Effect of gravitational force between them easy to calculate . . .

Calculate Shapiro shift,

Every in-particle with momentum p^- at solid angle $\Omega = (\theta, \varphi)$ causes a shift δu^- of *all* out-particles at solid angles $\Omega' = (\theta', \varphi')$:

$$\delta u^-(\Omega') = 8\pi G\, f(\Omega', \Omega) p^- \ ; \qquad (1 - \Delta_W) f(\Omega', \Omega) = \delta^2(\Omega', \Omega)\ .$$

Many particles: $p^-(\Omega) = \sum_i p_i^- \delta^2(\Omega, \Omega_i) \ \to$

$$\delta u^-(\Omega') = 8\pi G \int \mathrm{d}^2\Omega\, f(\Omega', \Omega) p^-(\Omega)\ .$$

"Small modification": replace $\delta u^-_{\mathrm{out}}$ by u^-_{out} , then:

$$\boxed{u^-_{\mathrm{out}}(\Omega) = 8\pi G \int \mathrm{d}^2\Omega' f(\Omega, \Omega') p^-_{\mathrm{in}}(\Omega')}$$

adding an in-going particle with momentum p^-_{in}, corresponds to *displacing* all out-going particles by u^-_{out} as given by our equation.

All u^-_{out} are generated by *all* p^-_{in} .

A mapping of the momenta $p^-_{\rm in}$ of the in-particles onto the positions $u^-_{\rm out}$ of the out-particles. Agrees with time evolution:

$$p^-_{\rm in} \to p^-_{\rm in}(0)e^{\tau} \ , \quad u^+_{\rm in} \to u^+_{\rm in}(0)e^{-\tau} \ ;$$
$$p^+_{\rm out} \to p^+_{\rm out}(0)e^{-\tau} \ , \quad u^-_{\rm out} \to u^-_{\rm out}(0)e^{\tau} \ .$$

What we calculated is the *footprint* of in-particles onto the out-particles, caused by gravity.

And then: $\delta u^- \to u^-$ implies that now the in-particles are to be *replaced* by the out-particles. *The out-particles are their footprints!*

This makes sense: u^- is the particle p^+ in position space – just Fourier transform the quantum state!

Footprints promoted to the status of particles themselves.
Avoids double counting: only describe the in-particle *or* its footprint (the out-particle), not both, as in the older equations.

Note: *hard* in-particles generate *soft* out-particles and *vice versa*.

This way, replace all hard particles by soft ones.
This removes the firewalls: the *firewall transformation*.

Authors of older papers, when encountering "firewalls", did not take into account that neither in-going nor out-going particles should be followed for time intervals $\delta\tau$ with $|\delta\tau| \gg \mathcal{O}(1)$.

This evolution law involves soft particles only. Is it unitary ?

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Yes, provided only the variables $p^{\pm}(\Omega)$ and $u^{\pm}(\Omega)$ are involved.
No quantum numbers like baryon, lepton ...

$p^{\pm}(\Omega)$ are like vertex insertions in string theories.
Postulating that this respects unitarity makes sense ...

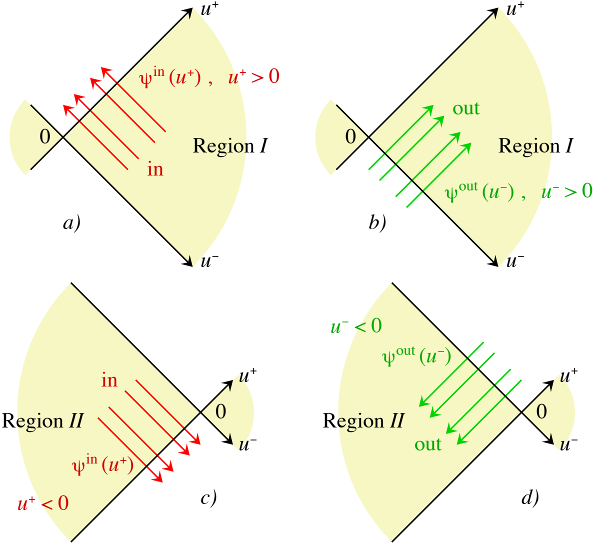
First amendment on Nature's Constitution:

A particle may be replaced by its gravitational footprint: At a horizon, out-particles are the Fourier transforms of in-particles.

Second problem:

What is the relation between regions I and II ?
Both have asymptotic domains: two universes !

a) Wave functions $\psi(u^+)$ of the in-particles live in region I, therefore $u^+ > 0$.
b) Out-particles in region I have $\psi(u^-)$ with $u^- > 0$.



The physical picture

c, d) In region II, the in-particles have $u^+ < 0$ and the out-particles $u^- < 0$.

Expand in $\boxed{\text{Spherical harmonics}}$:

$$u^{\pm}(\Omega) = \sum_{\ell,m} u_{\ell m} Y_{\ell m}(\Omega) \ , \qquad p^{\pm}(\Omega) = \sum_{\ell,m} p^{\pm}_{\ell m} Y_{\ell m}(\Omega) \ ;$$

$$[u^{\pm}(\Omega),\, p^{\mp}(\Omega')] = i\delta^2(\Omega,\, \Omega') \ , \qquad [u^{\pm}_{\ell m},\, p^{\mp}_{\ell' m'}] = i\delta_{\ell\ell'}\delta_{mm'} \ ;$$

$$u^{-}_{\mathrm{out}} = \frac{8\pi G}{\ell^2+\ell+1} p^{-}_{\mathrm{in}} \ , \qquad u^{+}_{\mathrm{in}} = -\frac{8\pi G}{\ell^2+\ell+1} p^{+}_{\mathrm{out}} \ ,$$

$$p^{\pm}_{\ell m} \ = \ \text{total momentum in of } {}^{\mathrm{out}}_{\mathrm{in}}\text{-particles in } (\ell, m)\text{-wave} \ ,$$

$$u^{\pm}_{\ell m} \ = \ (\ell, m)\text{-component of c.m. position of } {}^{\mathrm{in}}_{\mathrm{out}}\text{-particles} \ .$$

Because we have linear equations, all different ℓ, m waves decouple, and for one (ℓ, m)-mode we have just the variables $u^{\pm}$ and $p^{\pm}$. They represent only one independent coordinate u^{+}, with $p^{-} = -i\partial/\partial u^{+}$.

Great advantage of the expansion in spherical harmonics:

At every (ℓ, m), we get a single wave function in 1 space- and 1 time coordinate, and we can see that it evolves in a unitary way.

All we need to do is regard the positions u and the momenta p as canonical operators as always in QM. As soon as we replace the momenta p of the hard particles, by the shifted positions u of the soft particles, we get rid of the firewalls, and we see unitary evolution.

The in-particles can now be replaced by their *footprints* in the out-particles. The Fourier transform is unitary !

The in-particles never get the opportunity to become truly hard particles.

Like a "soft wall"-boundary condition near the origin of the Penrose diagram. Wave functions going in reflect as wave functions going out. Soft in-particles emerge as soft out-particles.

No firewall, ever.

The total of the in-particles in regions *I* and *II* are transformed (basically just a Fourier transform) into out-particles in the same two regions.

Regions *III* and *IV* are best to be seen as lying somewhere on the time-line *where time t is somewhere beyond infinity*

The antipodal identification

Sanchez(1986), Whiting(1987)

Regions I and II of the Penrose diagram are exact copies of one another.
Does region II represent the "inside" of the black hole?
NO! There are asymptotic regions. Region I is carbon copy of region II.

We must assume that region II describes the same black hole as region I.
It represents some other part of the same black hole. Which other part?
The local geometry stays the same, while the square of this $O(3)$ operator must be the identity.

Search for $A \in O(3)$ such that : $\quad A^2 = \mathbb{I}\,,$ and
$Ax = x$ has no real solutions for x.

$\Rightarrow$ All eigenvalues of A must be -1. Therefore: $A = -\mathbb{I}$:

the antipodal mapping.

We stumbled upon a new restriction for all general coordinate transformations:

Amendment # 2 for Nature's Constitution"

For a curved space-time background to be used to describe a region in the universe, one must demand that every point on our space-time region represent exactly one point in a single universe

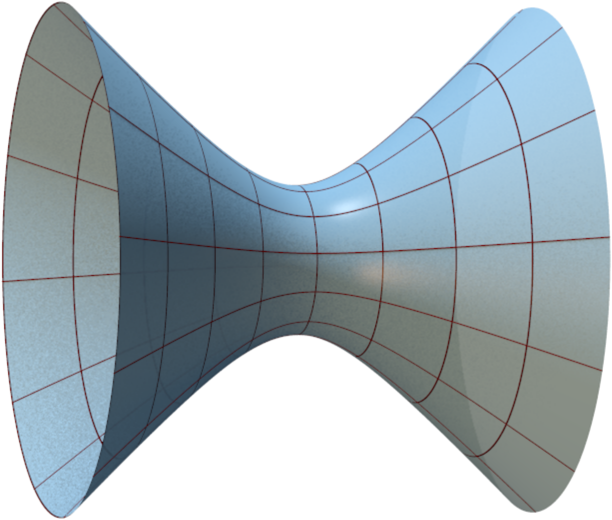
(not two, as in analytically extended Schwarzschild metric)

The emergence of a non-trivial topology needs not be completely absurd, as long as no signals can be sent around. We think that this is the case at hand here.

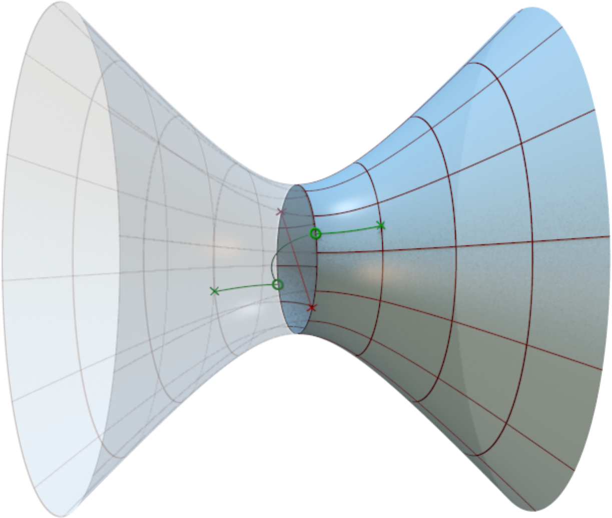
It is the absence of singularities in the physical domain of space-time that we must demand.

Note that, now, ℓ has to be odd!

II



I



I

The basic, explicit, calculation of the total scattering process.

Our algebra generates the scattering matrix, by giving us the *boundary condition* that replaces $|\mathrm{in}\rangle$-states by $|\mathrm{out}\rangle$-states.

NOT a model, theory, or assumption . . .

Apart from the most basic assumption of unitary evolution, this is nothing more than applying GR and quantum mechanics !

Spherical harmonics diagonalises S-matrix into 1 dimensional partial diff. equations !

Commutator equation for u and p:

$$[u,\, p] = i \;, \qquad \text{so that} \qquad \langle u|p\rangle = \tfrac{1}{\sqrt{2\pi}} e^{ipu} \;.$$

Tortoise coordinates, and split u and p in a positive part and a negative part:

$$u \equiv \sigma_u\, e^{\varrho_u} \;, \quad p = \sigma_p\, e^{\varrho_p} \;; \quad \sigma_u = \pm 1 \;, \quad \sigma_p = \pm 1 \;,$$

write: $\tilde\psi_{\sigma_u}(\varrho_u) \equiv e^{\frac12 \varrho_u}\, \psi(\sigma_u\, e^{\varrho_u}) \;, \quad \tilde{\hat\psi}_{\sigma_p}(\varrho_p) \equiv e^{\frac12 \varrho_p}\, \hat\psi(\sigma_p\, e^{\varrho_p}) \;;$

normalized: $|\psi|^2 = \sum_{\sigma_u=\pm} \int_{-\infty}^{\infty} \mathrm{d}\varrho_u |\tilde\psi_{\sigma_u}(\varrho_u)|^2 = \sum_{\sigma_p=\pm} \int_{-\infty}^{\infty} \mathrm{d}\varrho_p |\tilde{\hat\psi}_{\sigma_p}(\varrho_p)|^2 \;.$

Then $\tilde{\hat\psi}_{\sigma_p}(\varrho_p) = \sum_{\sigma_u=\pm 1} \int_{-\infty}^{\infty} \mathrm{d}\varrho\, K_{\sigma_u\sigma_p}(\varrho)\, \tilde\psi_{\sigma_u}(\varrho - \varrho_p) \;,$

with $K_\sigma(\varrho) \equiv \tfrac{1}{\sqrt{2\pi}} e^{\frac12 \varrho}\, e^{-i\sigma\, e^{\varrho}} \;.$

If $\varrho_p \to \varrho_p + \lambda$, then simply $u \to u\, e^{-\lambda}$, $p \to p\, e^{\lambda}$, a symmetry of the Fourier transform

Use this symmetry to write plane waves:

$$\tilde{\psi}_{\sigma_u}(\varrho_u) \equiv \breve{\psi}_{\sigma_u}(\kappa)\, e^{-i\kappa\varrho_u} \quad \text{and} \quad \tilde{\hat{\psi}}_{\sigma_p}(\varrho_p) \equiv \breve{\hat{\psi}}_{\sigma_p}(\kappa)\, e^{i\kappa\varrho_p} \quad \text{with}$$

$$\breve{\hat{\psi}}_{\sigma_p}(\kappa) = \sum_{\sigma_p=\pm 1} F_{\sigma_u\sigma_p}(\kappa)\breve{\psi}_{\sigma_u}(\kappa)\ ; \quad F_\sigma(\kappa) \equiv \int_{-\infty}^{\infty} K_\sigma(\varrho) e^{-i\kappa\varrho} \mathrm{d}\varrho\ .$$

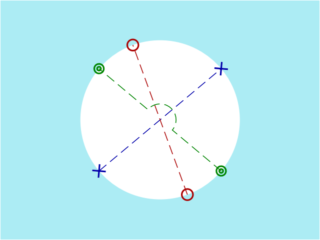
Thus, we see left-going waves produce right-going waves. On finds (just do the integral):

$$F_\sigma(\kappa) = \int_0^\infty \frac{\mathrm{d}y}{y} y^{\frac{1}{2}-i\kappa}\, e^{-i\sigma y} \;=\; \Gamma(\tfrac{1}{2} - i\kappa)\, e^{-\frac{i\sigma\pi}{4} - \frac{\pi}{2}\kappa\sigma}\ .$$

Matrix $\begin{pmatrix} F_+ & F_- \\ F_- & F_+ \end{pmatrix}$ is unitary: $F_+F_-^* = -F_-F_+^*$ and $|F_+|^2 + |F_-|^2 = 1$.

Black emptiness: blue regions are the accessible part of space-time; dotted lines indicate identification.

The white sphere within is *not* part of space-time. Call it a 'vacuole'.



At given time t, the black hole is a 3-dimensional vacuole. The entire life cycle of a black hole is a vacuole in 4-d Minkowski space-time: *an instanton*

N.Gaddam, O.Papadoulaki, P.Betzios (Utrecht PhD students)

Space coordinates change sign at the identified points

– *and also time changes sign*

(Note: time stands still at the horizon itself).

Virtual black holes and space-time foam (Summary)

Virtual black holes must be everywhere in space and time. Due to vacuum fluctuations, amounts of matter that can contract to become black holes, must occur frequently. They also evaporate frequently, since they are very small. This produces small vacuoles in the space-time fabric.
How to describe multiple vacuoles is not evident. The emerging picture could be that of "space-time foam":

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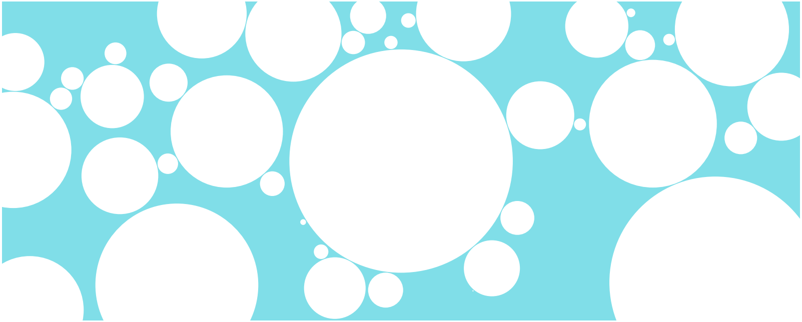

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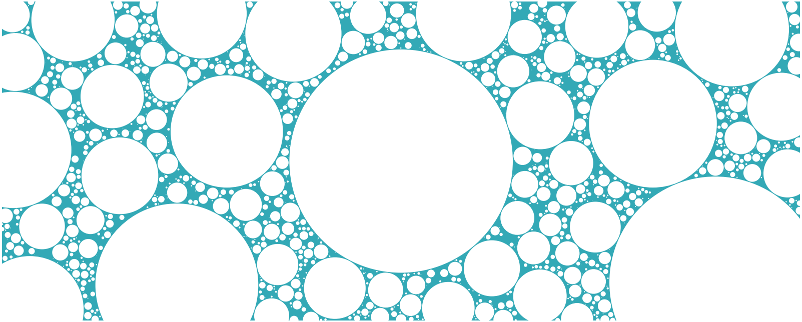
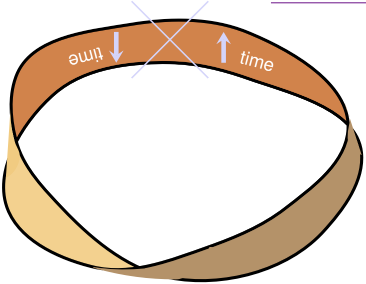


A timelike Möbius strip



Draw a spacelike closed curve:

Begin on the horizon at a point

$r_0 = 2GM\ ,\ \ t_0 = 0\ ,\ \ (\theta_0, \varphi_0)\ \ .$

Move to larger r values, then travel to the antipode:

$r_0 = 2GM\ ,\ \ t_0 = 0\,,\ \ (\pi - \theta_0, \varphi_0 + \pi)\ .$

You arrived at the same point, so the (space-like) curve is closed.

Now look at the environment $\{\mathrm{d}x\}$ of this curve. Continuously transport $\mathrm{d}x$ around the curve. The identification at the horizon demands

$$\mathrm{d}x \leftrightarrow -\mathrm{d}x\ , \quad \mathrm{d}t \leftrightarrow -\mathrm{d}t\ ,.$$

So this is a Möbius strip, in particular in the time direction.
Note that it makes a *CPT* inversion when going around the loop.

There are no direct contradictions, but take in mind that
the Hamiltonian switches sign as well.
Demanding that the external observer chooses the point where the Hamilton density switches sign as being on the horizon, gives us a good practical definition for the entire Hamiltonian.

Note that the soft particles near the horizon adopted the dilaton operator as their Hamiltonian. That operator leaves regions *I* and *II* invariant. Also, the boundary condition, our "scattering matrix", leaves this Hamiltonian invariant. Therefore, indeed, breaking the Hamiltonian open exactly at the horizon still leaves the total Hamiltonian conserved. So indeed, there are no direct contradictions.

However, this is a peculiarity that we have to take into consideration.

More to be done. Searching for like-minded colleagues.

See: G. 't Hooft, arxiv:1612.08640 [gr-qc] + references there;
http://www.phys.uu.nl/~thooft/lectures/GtHBlackHole_2017.pdf

P. Betzios, N. Gaddam and O. Papadoulaki, The Black Hole S-Matrix from Quantum Mechanics, JHEP 1611, 131 (2016), arxiv:1607.07885.

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