# Microdynamics in stationary complex networks

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**Many complex systems, including networks, are not static but can display strong fluctuations at various time scales. Characterizing the dynamics in complex networks is thus of the utmost importance in the understanding of these networks and of the dynamical processes taking place on them. In this article, we study the example of the US airport network in the time period 1990–2000. We show that even if the statistical distributions of most indicators are stationary, an intense activity takes place at the local (''microscopic'') level, with many disappearing/appearing connections (links) between airports. We find that connections have a very broad distribution of lifetimes, and we introduce a set of metrics to characterize the links' dynamics. We observe in particular that the links that disappear have essentially the same properties as the ones that appear, and that links that connect airports with very different traffic are very volatile. Motivated by this empirical study, we propose a model of dynamical networks, inspired from previous studies on firm growth, which reproduces most of the empirical observations both for the stationary statistical distributions and for the dynamical properties.**

critical infrastructures | dynamics of infrastructures | nonequilibrium processes

Despite the presence of stable statistical regularities at the global level, many systems exhibit an intense activity at the level of individual components, i.e., at the "microscopic" level. An important illustration of this fact was recently put forward by Batty (1) in the case of city populations. Indeed, even if the population Zipf plots display negligible changes in time, the same city can have very different ranks in the course of history. Similarly, many other systems, in particular occurring in human dynamics studies, present simultaneously stationary statistical distributions and strong time fluctuations at the microscopic level, with activity bursts separated by very heterogeneous time intervals (2–5). These systems thus challenge us with the fundamental puzzle that consists in reconciling an important dynamical activity occurring at the local level on many time scales and the emergence of stable distributions at a macroscopic level that can be maintained even when the external conditions are highly nonstationary (6). For instance, the dynamics of the rank is not consistent with processes, such as preferential attachment (7), where the rank is essentially constant in time.

These issues naturally apply to the case where complex systems are structured under the form of large networks. In most recent studies, these networks have been considered as static objects with a fixed topology. However, their structure may in principle evolve, links may appear and disappear. Such topological fluctuations have important consequences: Many dynamical processes take place on complex networks (8–11), and a nontrivial interplay can occur between the evolutions of the topology and of these dynamical processes. The structure of the network strongly influences the characteristics of the dynamical processes (11), and the topology of the network can simultaneously be modified as a consequence of the process itself. In this framework, recent studies have been devoted to simple models of coevolution and adaptive networks (12–16). Another illustration of the importance of taking into account the dynamics of the network is given by concurrency effects in epidemiology (17). Indeed, although a contact network is usually measured at a certain instant or aggregated over a certain period, the actual spread of epidemics depends on the instantaneous contacts. In such contexts, it is thus crucial to gain insights into the dynamics of the network, possibly by putting forward convenient new measures and to propose possible models for it.

These considerations emphasize the need for empirical observations and models for the dynamics of complex networks, which are up to now quite scarce. In this article, we study the case of the US airport network (USAN) where nodes are airports and links represent direct connections between them. It is indeed possible to gather data on the time evolution of this network (www.bts.gov) (see also ref. 18 for a study of the yearly evolution of the Brazilian airport network), which represents an important indicator of human activity and economy. Moreover, air transportation has a crucial impact on the spread of infectious diseases (19, 20), and it would be interesting to include its dynamical variations in large-scale epidemiological modeling. We first present empirical measures on the dynamics of the USAN. In particular, we provide evidence for the large-scale statistical regularity of many indicators, and we also define convenient metrics that enable us to characterize the small-scale dynamical activity. We then propose a model, based on simple but realistic mechanisms, which reproduces most empirical observations.

## Empirical Observations: Stable Statistical Distributions in a Fluctuating System

We analyze data available from the Bureau of Transportation Statistics (www.bts.gov). These data give the number of passengers per month on every direct connection between the US airports in the period 1990–2007. We limit ourselves to the period 1990–2000 during which the data collection technique is consistent. We obtain $12 \times 11 = 132$ weighted, undirected* networks where the nodes are airports, the links are direct connections, and the weights represent the number of passengers on a given link during a given month. We denote by $w$ the weight of a link, by $k$ the degree (number of neighbors) of a node, and by $s$ its strength, equal to the sum of the weights of the issuing links (21) (in the air-transportation case, the strength gives thus the total traffic handled by each airport).

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*From a rigorous point of view, the weights are not fully symmetric, but the deviations are small so we will use the symmetric undirected graph and avoid the complication deriving from flow imbalances.

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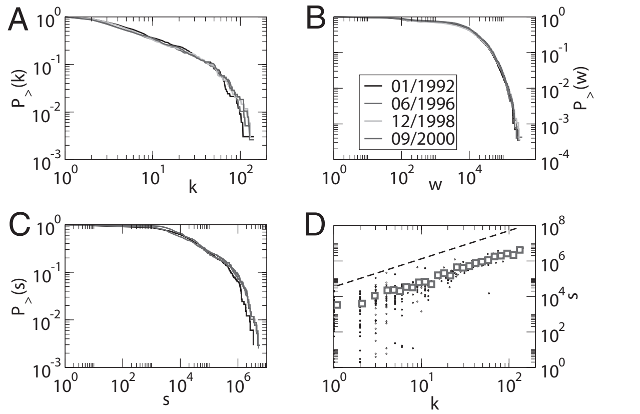


**Fig. 1.** Characteristics of the US airport network measured at 4 different dates: January 1992, June 1996, December 1998, and September 2000. (*A–C*) Cumulative distributions of degrees (*A*), weights (*B*), and strengths (*C*). (*D*) Strengths versus degrees for the year 2000 (circles, raw data; squares, average strength for each degree value). The dashed line is a power law with exponent 1.6. These results are quantitatively similar with published studies on static snapshots of the World Airport Network and the North American Airport Network (21–23).

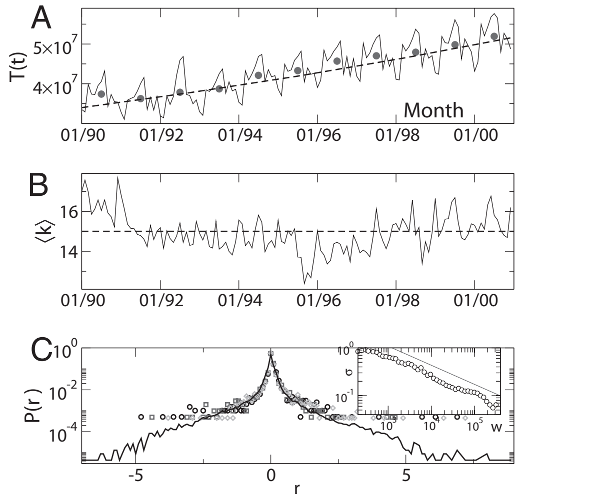


**Fig. 2.** Dynamics of the U.S. airport. (*A*) Total traffic on the US airport network versus time. The symbols represent the annual traffic and the dashed line is an exponential fit with time scale $\delta \approx 312$ months. (*B*) Average degree $\langle k \rangle$, approximately constant and of order 15 (dashed line). (*C*) Distribution of the monthly growth rates $r = \log(w(t+1)/w(t))$. The full line corresponds to the distribution obtained over the 11 years under study. Symbols correspond to 3 single months (May 1993, May 1995, and May 1997). The *Inset* shows the standard deviation $\sigma$ of the conditional distributions $P(r|w)$ as a function of $w$, showing that the distribution of the growth rates become narrower for larger weights. The line represents a power law $w^{-0.4}$.

Fig. 1 displays the cumulative distributions of degrees, weights, and strengths at 4 different times. These distributions are broad, as already shown in previous studies (21–25) highlighting the strong heterogeneities present in the air transportation network for both the topology and the traffic, leading to interesting behavior for various quantities such as the optimal paths (26). Fig. 1*D* moreover shows the dependence of the strength $s$ of an airport on its number of connections $k$, with a clear nonlinear behavior denoting a strong correlation between weights and topology (21, 23). Interestingly, Fig. 1 clearly shows that the distributions of degrees, weights, and strengths measured at different times are identical (we have obtained the same distributions at other dates). These distributions are therefore stationary even if, as we will show later, nontrivial dynamics occur continuously.

The first and simplest evidence for the presence of a dynamical evolution in the network is displayed in Fig. 2*A*, which shows the total traffic $T(t)$ (equal to the sum of the weights of all links) as a function of time. When the seasonal effects are averaged out, the data can be fitted, as often assumed in economics, by an exponential growth $T(t) = T(0)\exp(t/\delta)$ with $\delta \approx 312$ months ($\delta \approx 25$ years). Note that the data can also be fitted linearly, because of the large value of $\delta$. We also observe similar growth with seasonal fluctuations of the total number $N(t)$ of connected airports, and of the total number $L(t)$ of links (Fig. S1). The fits give the same growth rate for $N(t)$ and $L(t)$ (approximately half the growth rate of $T(t)$), and the average degree $\langle k \rangle = 2L(t)/N(t)$ has small fluctuations ($\pm 3$) around a constant value ($\approx 15$), as shown in Fig. 2*B*, over the 10 years period under study, whereas the average weight $\langle w \rangle$ grows exponentially with a typical time of order $2\delta$ (Fig. S1).

## Dynamics at the Microscopic Level

We now study in detail the dynamics at the microscopic level, i.e., the evolution of single links. We denote by $w_{ij}(t)$ the weight of the link between nodes $i$ and $j$ at time $t$ (in months) and by

$$r_{ij}(t) = \log\left(\frac{w_{ij}(t+1)}{w_{ij}(t)}\right) \quad [1]$$

the growth rate from one month to the next. Fig. 2*C* shows the distributions of $r_{ij}(t)$ for all links present in the network both at $t$ and $t+1$, for all months in the 11 years dataset under study (period January 1990 through December 2000), and for 3 single months ($t$ = May 1993, May 1995, and May 1997). The fact that the distributions can be superimposed leads to the conclusion that the distribution of $r_{ij}$ is independent of time. This distribution, however, depends on the link's properties. As shown in Fig. 2*C Inset*, the standard deviation $\sigma$ of the conditional distributions $P(r|w)$ as a function of $w$ decreases as $\sigma \sim w^{-0.4}$, a behavior similar to the one observed for firm growth (27). The weights' evolution from one month to the next can thus be modeled by

$$w_{ij}(t+1) = w_{ij}(t)(1+\eta_{ij}) \quad [2]$$

where the multiplicative noise $\eta = e^r - 1$ is a random variable whose distribution does not depend on time but does depend on the link weight. The distribution of $\eta$ is broad (Fig. S2), which indicates that most of the increments are small but that sudden and large variations of the weights can be observed with a small but nonnegligible probability.

Links can be created or suppressed between airports, and in fact the number of link creation events is $4 \times 10^4$ for the 11 years period under study, for a total number of links in the 132 networks close to $3 \times 10^5$. This result immediately raises the question of the lifetime $\tau$ of links. As shown in Fig. 3*A*, the distribution of $\tau$ is very broad, with a power law behavior $P(\tau) \sim \tau^{-\alpha}$ with $\alpha = 2.0 \pm 0.1$. Some comments are in order. First, we consider in this distribution only the links that appear and disappear during the period under study. This is necessary because we cannot know the real lifetime of a link that is already present at the start of the period or still present at the end. Second, although the most probable value for $\tau$ is small, which implies that new links are the most fragile, the distribution extends over all available time scales: links of an arbitrary age may disappear. This indicates a nontrivial dynamics with appearance/disappearance of both "young" and "old" links. This strong heterogeneity of lifetimes is in line with other results about human activity (3), where it has been shown to have a strong impact on dynamical processes (28). It is therefore

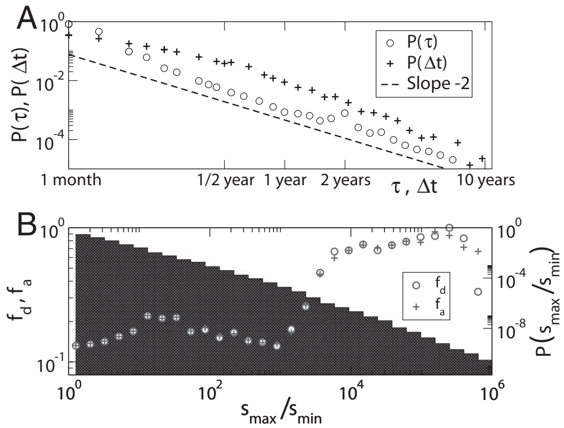


**Fig. 3.** Links' dynamical properties. (*A*) Lifetime distribution of links (in months). Links that existed at the beginning of the measure (January 1990) and still present at the end (December 2000) are discarded. The full line is a power law fit with an exponent $-2.0 \pm 0.1$. We also show the distribution of time $\Delta t$ during which a given link is absent from the system. (*B*) Fraction $f_d$ (open circles) of disappearing links and $f_a$ (pluses) of appearing links as a function of the ratio $s_{max}/s_{min}$ of the strengths of their extremities. The scale is on the left-hand *y* axis. We also show the logarithmically binned reference distribution $P(s_{max}/s_{min})$ (line above the shaded area, scale on the right-hand *y* axis).

important to characterize and incorporate it into models of dynamically evolving complex networks (29).

Most links appearing at a certain point disappear and reappear again at later times: A substantial part of the dynamics is due to links appearing and disappearing several times, rather than to uncorrelated phenomena of appearance and disappearance. We have thus measured the number of times each link appears and disappears, and the durations $\Delta t$ of links absence. Fig. 3*A* displays the distribution of these absence durations, showing a large heterogeneity of the absence periods. The number of times a given link appears and disappears is exponentially distributed (Fig. S3): Most links appear and disappear less than 5 times during the 11 years period of study.

These results show that, behind the stability of the statistical characteristics of the USAN, incessant microscopic rearrangements occur. We now propose a systematic way to characterize the corresponding fluctuating connections, whose importance stem from the fact that they induce changes in the topology of the network. Each link $(i, j)$ can be characterized by a certain number of quantities such as its weight $w_{ij}$, the strengths of its extremities $s_i$ and $s_j$, etc. It is usual to consider the distributions of these quantities over the whole network, and we will consider these distributions as reference (see Fig. 1). In addition, we propose to focus at each time $t$ on the links that appear (or disappear), to study the distributions of these links' characteristics, and to compare them with the reference distributions. For instance, if $N_t(w)$ is the number of links with weight $w$ at time $t$, and $N_t^d(w)$ is the number of such links that disappear between $t$ and $t + 1$, we measure the fraction of links of weight $w$ that disappear at time $t$,

$$f_d(w) = \frac{N_t^d(w)}{N_t(w)}. \qquad [3]$$

We also define the number $N_t^a(w)$ and fraction $f_a(w)$ of links of weight $w$ that appear at $t$. Similarly, $f_d$ and $f_a$ can be measured for other links characteristics as we will investigate. A priori, all these quantities depend on the measurement time $t$. We have already seen that the reference distributions are stationary (Fig. 1). Strikingly, we observe that the fractions $f_d$ and $f_a$ display as well a stationary behavior (see Fig. S4 for an example), even if they clearly highlight a strong dynamical evolution. In the following, we will therefore drop any $t$ index and measure $f_d$ and $f_a$ averaged over the whole period under study. In all cases, we also observe a strong similarity between $f_d$ and $f_a$, partly because most disappearing links had appeared some time before, and vice versa, and to the large number of links with lifetime of order a few months, during which no strong evolution occurs (see the above discussion on the lifetime and the absence intervals).

The measure of $f_d(w)$ and $f_a(w)$ (Fig. S5) indicate that most links have a small weight just before they disappear or just after their birth, which is not a surprise. However, $f_d$ and $f_a$ are broad, extending on several orders of magnitude of $w$ values: Appearing and disappearing connections occur with nonnegligible probabilities even at strong weights. A more detailed analysis shows the presence of 2 regimes in $f_{d(a)}$: for links with $w < 10^2$ passengers per month, $f_d$ and $f_a$ present rather large values close to 0.8. For $w > 10^2$, these fractions decrease slowly: Also, links with large weights can appear or disappear. We also observe that for $w > 10^3$ there are more links that appear than that disappear, an effect that is consistent with the increase of the total traffic.

As previously mentioned, a similar analysis can be carried out for various links' characteristics; particularly relevant quantities include the traffic of the airports located at both ends of the link. In the following we denote by $s_{max}(l) = \max_{l=(i,j)}(s_i, s_j)$ and $s_{min}(l) = \min_{l=(i,j)}(s_i, s_j)$ the larger and smaller traffic of the extremities of a link $l = (i, j)$. A measure of the importance of the link for $i$ and $j$ is given by $w/s_{min}$ and $w/s_{max}$. For instance, if $w/s_{min}$ is small, the link carries only a small fraction of $i$'s and $j$'s traffic; on the contrary, a large $w/s_{max}$ indicates that the link is important for both its extremities. The study of $f_{d(a)}(w/s_{min})$ and $f_{d(a)}(w/s_{max})$ (Fig. S6) shows that most links that disappear/appear display small values of these ratios, of order $w/s_{min} < 10^{-3}$ and $w/s_{max} < 10^{-4}$. This means that most of these links have a small importance for the airports to which they are attached. For larger values of $w/s_{min(max)}$, the ratios $f_d$ and $f_a$ decrease, from $\approx 0.7$ to $\approx 10^{-2}$, and surprisingly increase again (from $\approx 10^{-2}$ to $\approx 10^{-1}$) for $w/s_{min} > 10^{-1}$ and $w/s_{max} > 10^{-2}$. This phenomenon corresponds to links that are very important for some airports, the extreme case being airports with a single connection (these airports have thus usually a small strength).

Finally, we also consider the ratio $s_{max}/s_{min}$ of the traffic of the links extremities. This quantity indicates indeed how similar the airports connected by the link are, in terms of traffic. We plot in Fig. 3*B* the fractions $f_{d(a)}(s_{max}/s_{min})$ of links that disappear (appear) as a function of $s_{max}/s_{min}$. On this figure we also show the reference probability distribution $P(s_{max}/s_{min})$ that displays a broad behavior: Most links connect airports of similar importance, but the ratio $s_{max}/s_{min}$ varies over 6 orders of magnitude, and a nonnegligible fraction of links connect very different airports. Interestingly, $f_{d(a)}(s_{max}/s_{min})$ displays 2 different regimes. For $s_{max}/s_{min} < 10^3$, small values of $f_d$ are obtained: Links that connect airports of similar, or not too dissimilar, sizes are rather stable. In the opposite case when $s_{max}/s_{min} > 10^3$, the fraction $f_d$ increases rapidly to reach another plateau, at values of order 0.7–0.8. This last regime corresponds to links connecting airports with very different traffic, which turn out to be the most fragile and to have a short lifetime.

We can now summarize the results of our empirical observations, obtained through the analysis of the tools introduced in Eq. **3** (*i*) The links that disappear have essentially the same properties as the ones that appear. (*ii*) The disappearing/appearing links have a weight that is low on average but broadly distributed: large weights links may appear or disappear with a nonnegligible probability. (*iii*) Most disappearing links have small weights with respect to the traffic of their extremities, but links appear or disappear in the whole range of *w*/*s*. (*iv*) Links that connect airports with very different traffic are very volatile.

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($v$) The lifetime of links is broadly distributed and covers all available time scales.

The set of measures we have presented, although not exhaustive, is able to give a clear characterization of the dynamics of the network under study†. They are also easily applicable to any network undergoing topological changes, and can be generalized to include other links characteristics.

The results of the empirical analysis may moreover serve as guidelines in the elaboration of a model for dynamically fluctuating networks. In particular and in contrast with most models found in the literature, topological modifications of the network result here from the stochastic evolution of weights.

## Model for Dynamical Networks

Using the results of the empirical analysis of the airport network as guidelines, we now propose a model for dynamically fluctuating networks able to reproduce the main features observed for the USAN, and that highlights important features of dynamical networks modeling. We consider simple ingredients that can easily be extended with more detailed rules, and can therefore serve as a modeling basis in many other fields where the dynamics of weights and links is essential. In this model, topological modifications of the network result from the stochastic evolution of weights.

We start from ideas developed in refs. 27, 31, and 32 to model firm growth through a process based on multiplicative growth of subunits together with fusion/creation rules. In our framework, we consider airports (nodes) and connections (links) instead of firms and subunits. The equivalent of a firm's size is then given by the traffic of the airport as measured by its strength, and the subunits sizes correspond to the traffic on each link. An essential difference distinguishes our model from the firm growth model where the various firms undergo independent evolutions: Here, each node is connected to many others by links whose weights evolve randomly, so that the evolution of the airports sizes are correlated.

Let us present the details of the modeling framework. We start (at time $t = 0$) from an initial network composed of $N_0$ and $L_0$ links of unit weights, with $L_0 \approx N_0$ (we have checked that the initial conditions do not influence the results). At each time step $t$, we first compute for each link $(i, j)$ with weight $w_{ij}(t)$ a random increment

$$\delta w_{ij}(t) = w_{ij}(t)\eta, \qquad [4]$$

where we chose for simplicity $\eta$ as a random variable drawn from a distribution independent from time and from the pair $(i, j)$, and that may a priori take values in $]-1, +\infty[$. For $\langle\eta\rangle > 0$, the total traffic will on average grow exponentially. For the sake of simplicity we will choose for $\eta$ a Gaussian distribution (truncated at $-1$), with variance $\sigma_\eta^2$ (We have also run simulations with broad distributions of $\eta$ for $\eta > 0$, with the same results.). The weights' increments govern the evolution of the network's topology: Depending on the values of $\delta w_{ij}$, the nodes $i$ and $j$ can either update the weight of $(i, j)$, delete it or create new links toward other nodes. More precisely, each airport $i$ has a threshold value $s_s(i)$, which sets a criterium of viability for a connection: If a link's weights drops below this threshold, the airport $i$ does not consider the link anymore as interesting and removes it. For simplicity, we take thresholds independent from time and uniform: $s_s(i) = 1.0$ for all $i$. The detailed evolution rules are as follows:

1. If $\delta w_{ij}(t) < 0$, $i$ and $j$ test each the viability of the connection $(i, j)$. If $w_{ij}(t) + \delta w_{ij}(t) < \max(s_s(i), s_s(j))$, the link disappears and its weight is uniformly redistributed over the other connections of $i$ and $j$. In the opposite case, $w_{ij}(t) + \delta w_{ij}(t) > \max(s_s(i), s_s(j))$, the link's weight is simply updated: $w_{ij}(t + 1) = w_{ij}(t) + \delta w_{ij}(t)$.
2. If the weight increment $\delta w_{ij}(t)$ is positive, we assume that $i$ and $j$ have contributed equally to it and can decide each on how half of it should be used: If $\delta w_{ij}(t) > s_s(i)$, with probability $p_f$ node $i$ will use its part $\delta w_{ij}(t)/2$ of the increment to create a new link $(i,\ell)$ with weight $w_{i\ell} = \delta w_{ij}(t)/2$. With probability $1 - p_d$, $\ell$ is an existing airport chosen at random, and with probability $p_d$ it is a new node. $p_d$ therefore governs the rate of growth of the number of nodes. With probability $1 - p_f$, node $i$ simply increases the weight $w_{ij}$ of $\delta w_{ij}(t)/2$. Node $j$ then chooses independently either to create a new link $(j,k)$, or to increase the weight $w_{ij}$ by an amount equal to $\delta w_{ij}(t)/2$.
3. If $0 < \delta w_{ij}(t) < s_s(i)$, node $i$ increases the weight of $(i, j)$ of $\delta w_{ij}(t)/2$. The same procedure is applied to node $j$.

Rules 1–3 express the concept that the evolution of the traffic governs the topological modifications of the network. If a weight becomes too small, the corresponding connection will be stopped. However, if it grows too fast, new connections can be created. The quantity $p_f$ determines the rate of new connections. If $p_f$ is close to 1, as soon as an increment $\delta w$ is large enough a new link will be created, which in turn will limit the growth of weights because they are used to create new connections. In the opposite case of small $p_f$, the number of links will grow very slowly but the weights will reach more easily large values. At each time step, the total traffic $T(t)$ is multiplied on average by $1 + \langle\eta\rangle$ leading to an exponential growth $T(t) \propto \exp(t/d)$ with $d = 1/\langle\ln(1 + \eta)\rangle$. The number of nodes and links also grow in time, and their simultaneous growth, controlled by $p_f$ and $p_d$, results in an average degree $\langle k\rangle$, which fluctuates around a constant value, function of the parameters $p_f$, $\sigma_\eta$, $\langle\eta\rangle$, and $p_d$ (see Fig. S7). For instance, for larger $p_d$, $N(t)$ grows faster and $\langle k\rangle$ is smaller.

The model rules can easily be modified to incorporate other elements, such as preferential attachment mechanisms or random distributions of the threshold values $s_s(i)$. Although we will focus here on the simplest version as described above, we have also considered variants ($i$) in which the link's relevance is tested if $\delta w_{ij} < \max(s_s(i), s_s(j))$ (instead of the condition $\delta w_{ij} < 0$), or ($ii$) where the weight of deleted links is redistributed at random, or ($iii$) only 1 new link can be created, either from $i$ or $j$. The conclusion is that the qualitative features are not modified, showing that the simulation results presented below are robust with respect to such changes. We have also simulated the case $p_d = 0$ in which no new nodes are inserted, $N(t) = N_0$. In this case, the global increase of traffic leads at large time to a fully connected network, but during a long time, it remains sparse ($\langle k\rangle \ll N(t)$) and the same results are again obtained in this regime.

Figs. 4 and 5 summarize some results of our numerical simulations of the dynamical network model. Although the network evolves with many links creations and deletions, the distributions of degrees, weights, and strengths display a remarkable stability, as shown in Fig. 4 for $N(t)$ growing from $10^4$ to $10^5$. All these distributions are broad, consistently with empirical observations, and the nonlinear behavior of the strength versus degree is reproduced as well. Interestingly, this behavior (a power law with an exponent of the order 1.4; see Fig. 4$D$) emerges here as a result of the stochastic dynamics without any reference to preferential attachment mechanisms combined with spatial constraints (23) or with link additions between nodes (33, 34).

Although many network models are able to produce broad degree and strength distributions, the focus of this article lies in the small-scale dynamical aspects. We show in Fig. 5$A$ that the

†Note that a straightforward other set of measures concerns the degrees of the links' extremities, see also (30) for the definition of interesting measures combining weights and degrees. We have focused here on traffic properties, which are particularly relevant in the airport network case.

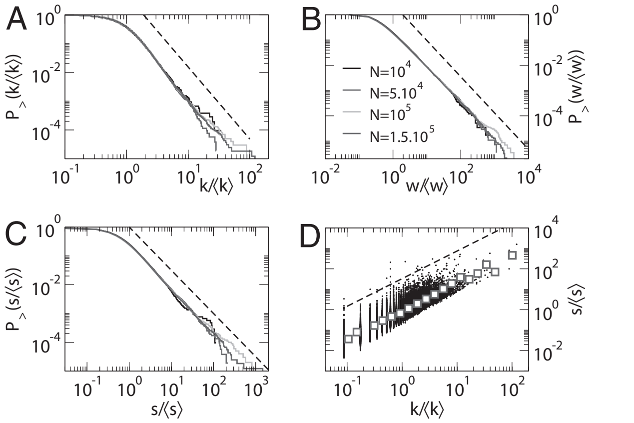


**Fig. 4.** Model simulation obtained with the parameters $p_d = 0.01$, $p_f = 0.1$, $\sigma_\eta = 0.0225$ and $\langle\eta\rangle = 10^{-3}$. Cumulative distributions obtained at different times of the network growth ($N = 10^4$, $5.10^4$, $10^5$, and $1.5.10^5$) of normalized degrees $P(k/\langle k\rangle)$ (the dashed line represents a power law with exponent $-2.5$) (*A*); normalized weights $P(w/\langle w\rangle)$ (the dashed line represents a power law of exponent $-1.5$) (*B*); and normalized strength $P(s/\langle s\rangle)$ (the dashed line represents a power law with exponent $-1.5$) (*C*). (*D*) Strength versus degree. Circles represent $s(k)$ for each node; full squares represent the same data, averaged for each $k$, and binned logarithmically. This measure is done for $N = 10^5$ and the dashed line represents a power law with exponent 1.4.

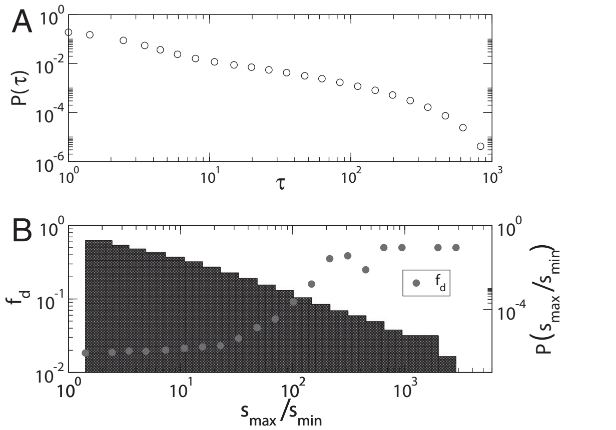


**Fig. 5.** Model simulation with the same parameters as Fig. 4. (*A*) Lifetime distribution $P(\tau)$ displaying a broad behavior. A power law fit gives an exponent of order $-1.1 \pm 0.1$. (*B*) Fraction of disappearing links $f_d$ versus $s_{max}/s_{min}$ (circles). We also represent the logarithmically binned reference distribution of $s_{max}/s_{min}$ (line above shaded ares).

lifetime distribution of the links is broad, as in the USAN case, and we report in Fig. 5*B* the behavior of $f_d(s_{max}/s_{min})$. Strikingly, our model reproduces the empirical behavior shown in Fig. 3*B*, with 2 different plateaus at small and large $s_{max}/s_{min}$. Other properties of the appearing or disappearing links coincide in the model with the empirical results, for example, most disappearing links have a small weight, or the nontrivial shape of $f_{d(a)}(w/s_{min(max)})$, with a decreasing $f_d$ for $w/s > 0.01$, and an increase at $w/s > 0.1$ (Fig. S8).

In summary, the simple assumptions on which our model is based yields stationary nontrivial emergent properties such as broad distributions and nonlinearities, together with an active local dynamics of links occuring on all time scales, and whose characteristics reproduce the empirical findings concerning the USAN's microscopic dynamics.

## Discussion

The question of the dynamical evolution of networks is crucial in the study of many dynamical processes and complex systems. If the time scales governing the dynamics of the network and of the process taking place on it are comparable, one can indeed expect a highly nontrivial behavior, which in principle could be very different from the static network case. In this article, we have used as a case study the US airline network, and we have shown that it exhibits stationary distributions despite the incessant creation and deletion of connections on broadly distributed time scales. We have introduced a set of measures in a systematic way to characterize this dynamics. Finally, we have proposed a model based on simple assumptions that reproduces the main empirical features, both for stationary and local dynamical properties.

The coexistence of stationary distributions and strong microscopic activity taking place at very different time scales occurs in many different systems and our model can provide a framework that can easily be extended and serve as a basis for further and more detailed modeling. For instance, we have observed that a bimodal distribution of the thresholds $s_s(i)$ for the deletion of a link results in the following picture: Nodes with small $s_s$ typically have a large degree, but are connected to weak links, whereas nodes with large $s_s$ reach a smaller number of stronger connections. This behavior does not correspond to infrastructure networks such as the USAN but could describe social behavior where individuals with many connections do not have intense (i.e., with large weight) relations. In these perspectives, the present work should stimulate further studies on the coexistence of dynamics at different scales and on the impact of network dynamics on different processes.

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