Tomograms and data analysis applied on reflectometry signals

9th International Reflectometry Workshop

Tomograms applied on reflectometry signals

- Collaboration
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  1. Filtering, Fourier Transform
  2. Time-Frequency representation : Spectrogram, Wigner-Ville, Wavelet, Tomogram

Contents

- **Tomogram**
- **Some results on simulated data**
- **Some results on reflectometry data**

Tomogram

- Time-Frequency operator

$$\hat{x} = \mu\hat{t} + \nu\hat{\omega} == \mu t + i\nu\frac{d}{dt}$$

Eigen vectors $\{\psi^X_{\mu,\nu}\}$

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- Tomogram : Time-frequency distibution of the analytical signal s

$$M_s\,(X, \mu, \nu) = \left| < s, \psi^{X}_{\mu,\nu} > \right|^2$$

- with normalisation $\int M_s(X, \mu, \nu)dX = 1$
- $\mu = 1$, $\nu = 0$, distribution in the time domain $M_s(t, 1, 0) = |s(t)|^2$
- $\mu = 0$, $\nu = 1$, distribution in frequency domain $M_s(\omega, 0, 1) = |\tilde{s}(\omega)|^2$

Tomogram Symplectic

- single parameter θ

$$\hat{x} = \cos\theta \hat{t} + \sin\theta \hat{\omega}$$

$$M_s(X,\theta) = \left| \int \psi_\theta^X(t)\, s(t)\, dt \right|^2$$

$$\psi_\theta^X(t) = \frac{1}{2\pi\,|\sin\theta|} \exp\left(\frac{i\cos\theta}{2\sin\theta}\, t^2 - \frac{iX}{\sin\theta}\, t \right)$$

SD # 1 : Component decomposition

- Sinusoidal signal

$$y(t) = y_1(t) + y_2(t) + y_3(t) + b(t)$$

$$\begin{aligned} y_1(t) &= \exp(i\omega_0 t), t \in [0, T] \\ y_2(t) &= \exp(i\omega_1 t), t \in \left[0, \frac{T}{4}\right] \\ y_3(t) &= \exp(i\omega_1 t), t \in \left[\frac{T}{2}, T\right] \end{aligned}$$

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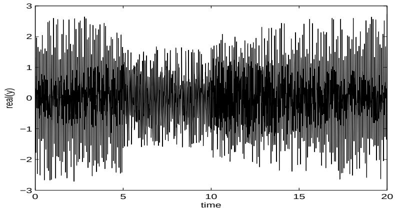
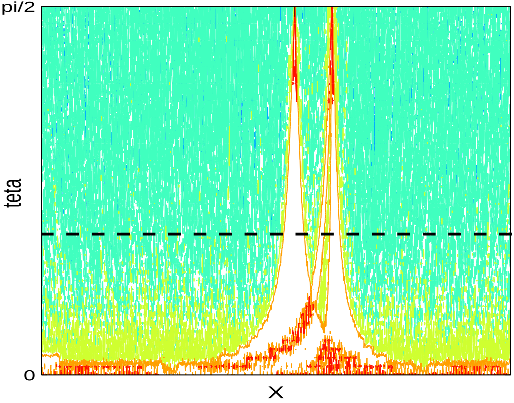
- Real part of the time signal (SNR = 10 dB)



SD # 1 : Component decomposition

- Tomogram (contour plot)



SD # 1 : Component decomposition

Projection, in the case of finite time $t \in [0, T]$

- orthonormal basis $\left\{\psi^{T}_{\theta,X}\right\}$

$$< \psi^{T}_{\theta,X_n} \psi^{T}_{\theta,X_m} >= \delta_{m,n}$$

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$$c_n =< s, \psi^T_{\theta,X_n} >$$

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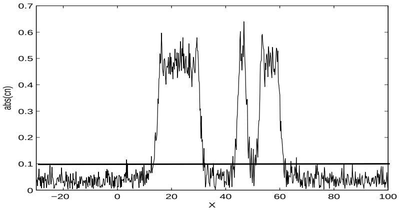
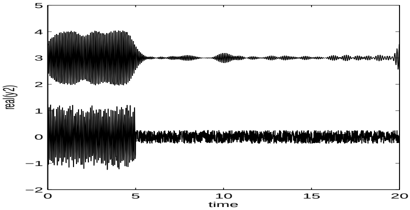
- Representation of the projections c_n of the signal $y(t)$ for $\theta = \frac{\pi}{5}$



SD # 1 : Component decomposition

- Component $\tilde{y}_2(t)$



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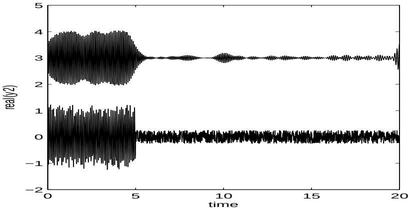
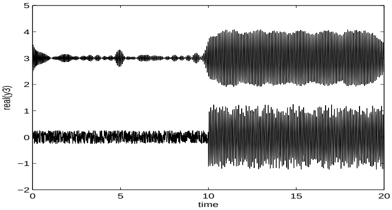


- Component $\tilde{y}_3(t)$ components



SD # 2 : Component decomposition and phase derivative

- Chirps : $y(t) = e^{i\Phi_1(t)} + e^{i\Phi_2(t)} + b(t)$ SNR=10dB

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  - with $\frac{\partial}{\partial t}\Phi_1(t) = a_1 t^2 + b_1 t + c_1$
    and $\frac{\partial}{\partial t}\Phi_2(t) = b_2 t + d_2\sqrt{t} + c_2$.

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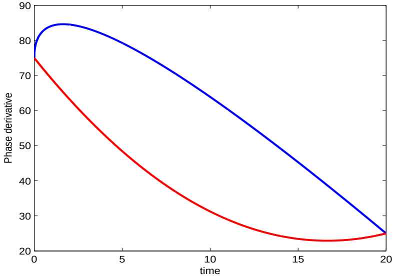
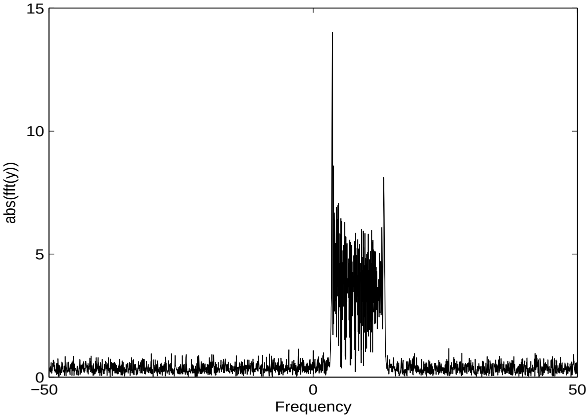
    and $\frac{\partial}{\partial t}\Phi_2(t) = b_2 t + d_2\sqrt{t} + c_2$.
- Representation of $\frac{d}{dt}\Phi_1(t)$ and $\frac{d}{dt}\Phi_2(t)$ as a function of time.



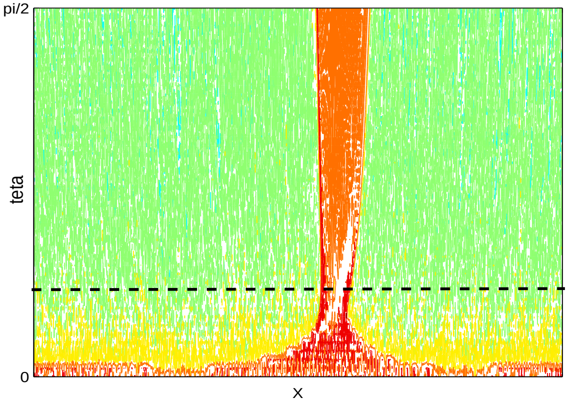
SD # 2 : Component decomposition and phase derivative

- Frequency representation



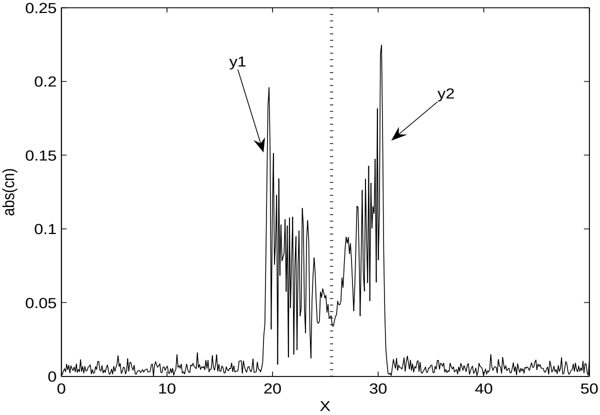
SD # 2 : Component decomposition and phase derivative

- Tomogram of the chirps signal



SD # 2 : Component decomposition and phase derivative

- Projection $\theta = atan(\frac{\Delta T}{\Delta F}) \approx \frac{\pi}{8}$



SD # 2 : Component decomposition and phase derivative

Phase derivative

- The phase derivative can be estimated from the projections $\{c_n\}_n$

$$\frac{\partial}{\partial t}\phi(t) = Im\left(\frac{\Gamma(t)}{\tilde{y}(t)}\right) \tag{1}$$

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  - with

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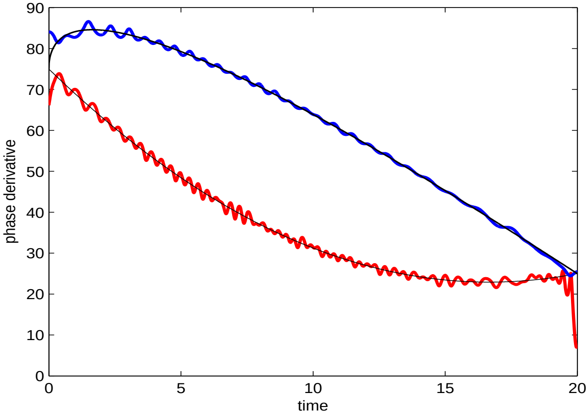
$$\tilde{y}(t) = \sum_n c_n \psi_{x_n}^{\theta,T}(t) \tag{3}$$

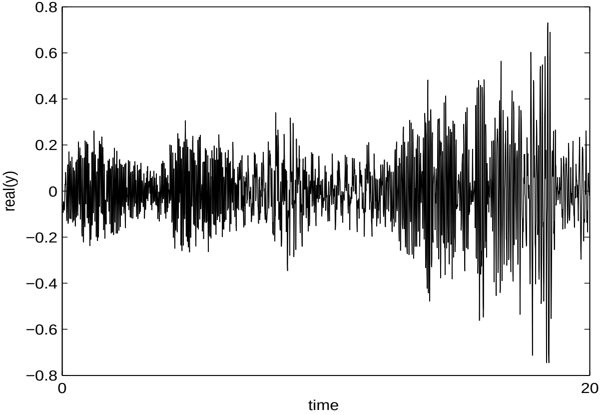
SD # 2 : Component decomposition and phase derivative

- Phase derivative $\theta \approx \frac{\pi}{8}$



Reflectometry data : choc 42824

- Time representation of the signal
  Fast-sweep X-mode reflectometer on Tore Supra, V band



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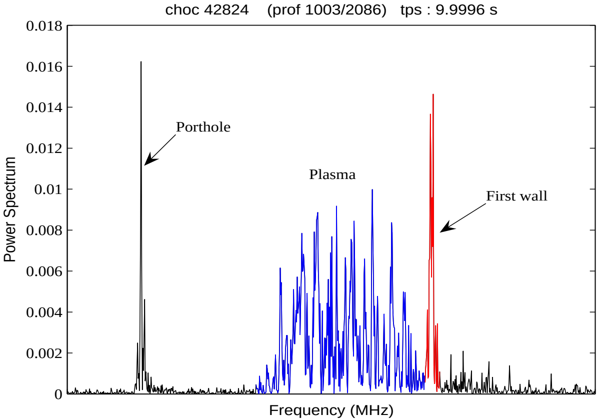
- Frequency representation of the signal



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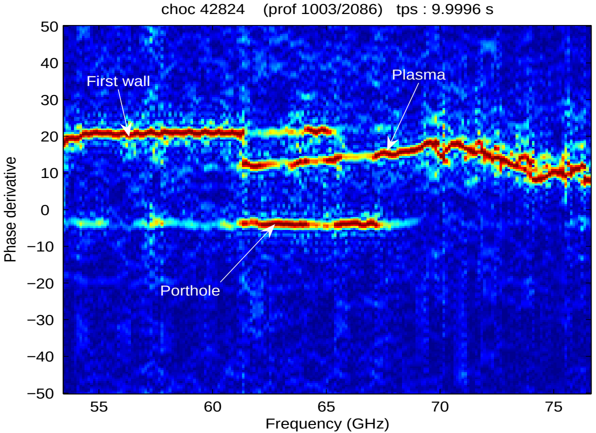
- Spectrogram of the signal



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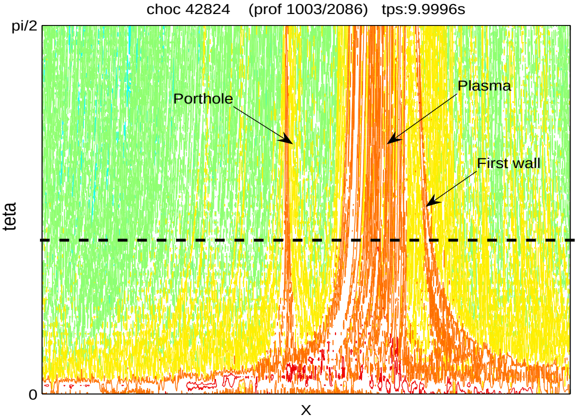
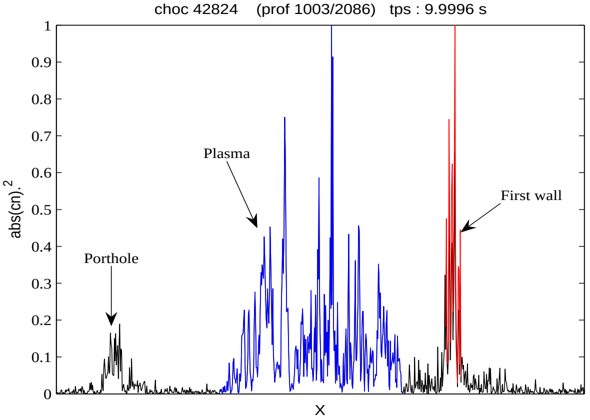
- Tomogram of the signal

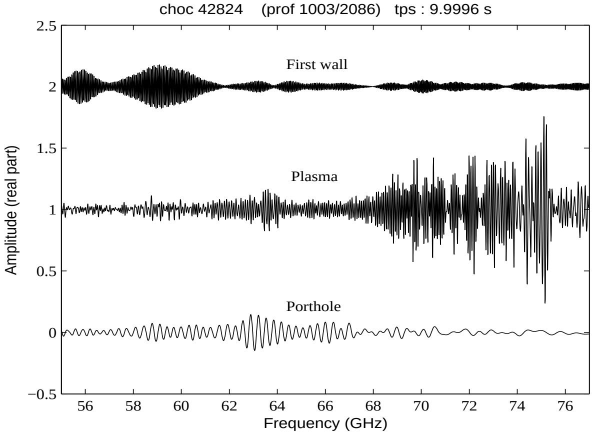


Reflectometry data : choc 42824

- Spectrum $c^{\theta}_{x_n}$ of the reflectometry signal $y(t)$ for $\theta = \pi - \frac{\pi}{5}$



- Components of the reflectometry signal : $\theta = \pi - \frac{\pi}{5}$



Reflectometry data : choc 42824

- Separation of the components in the reflectometry signal

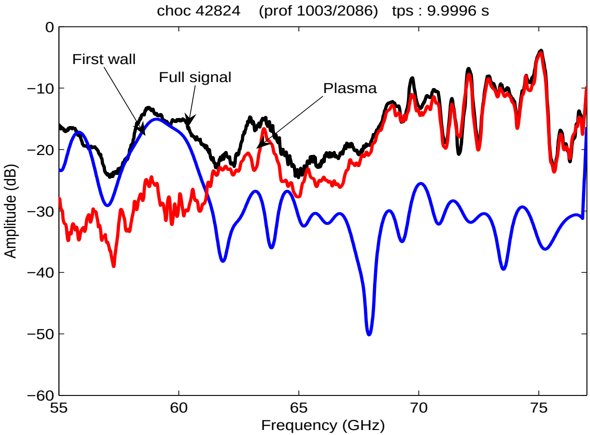
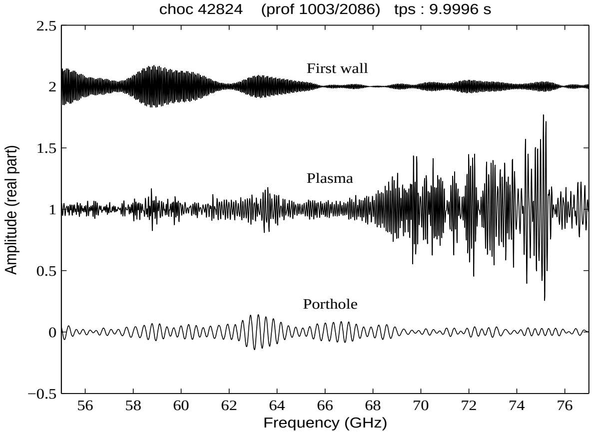
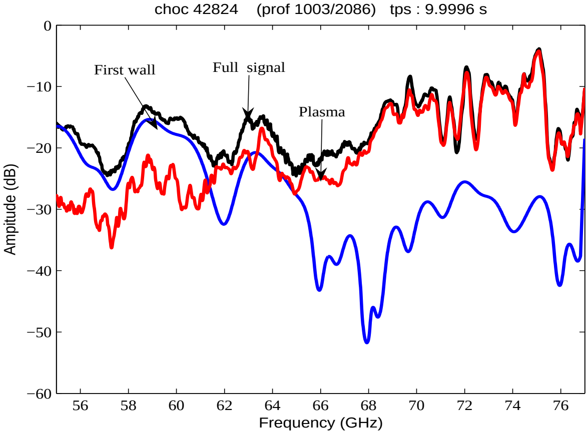


Figure: Full signal and reflexions on the wall and on the plasma (dB)

- Components of the reflectometry signal : $\theta = \frac{\pi}{2}$

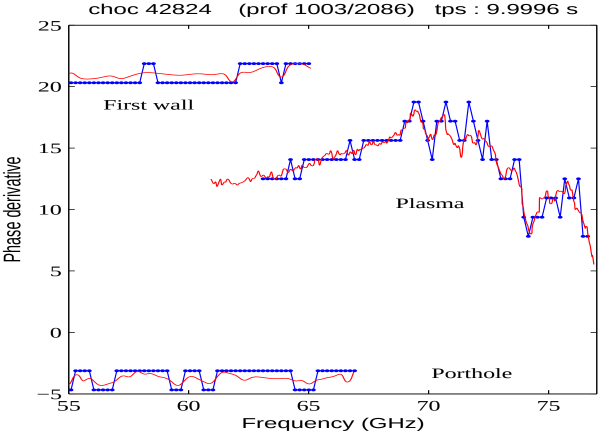


- Components in the reflectometry signal : $\theta = \frac{\pi}{2}$

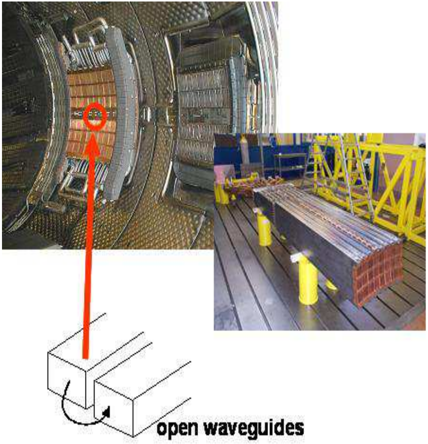


Reflectometry data : choc 42824

- Phase derivative of the three components ($\theta = \pi - \frac{\pi}{5}$)



Reflectometry data : LH antenna choc 29286 tps: 1.43536



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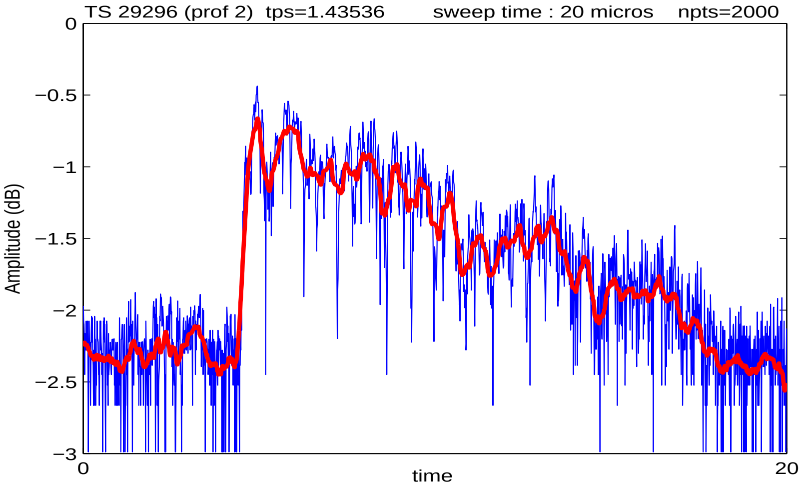
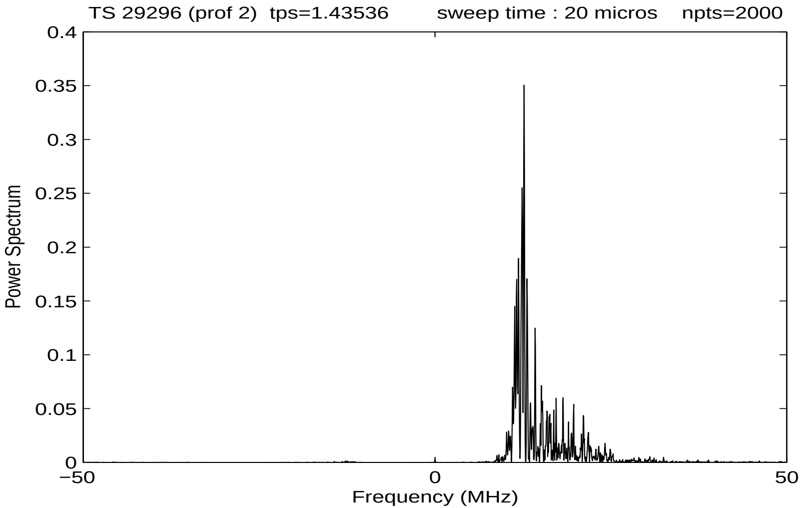
- Time representation of the data (red line filtered data)



- Frequency representation of the data



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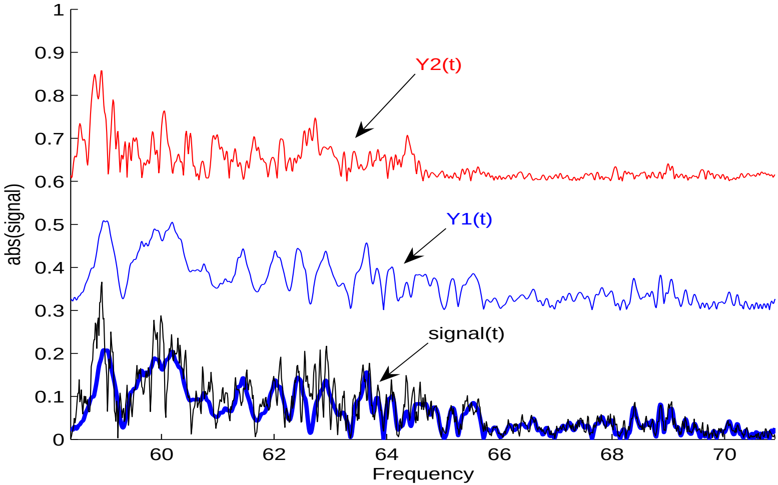
- Extraction of the two components



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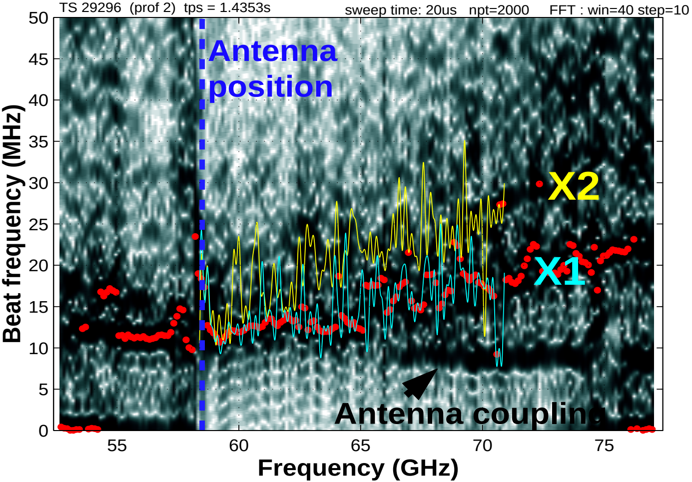

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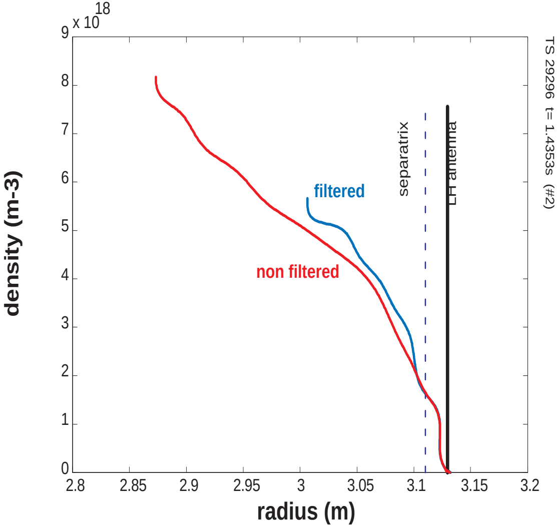

CONCLUSIONS

- Tomogram gives promissing results for reflectometry data
  - separation of components
  - phase derivative estimation
- Applications
  - First frequency cutoff determination
  - Separation of close multicomponents (reflectometers in Heating Antennas ICRH, LH)

Tomograms associated to the conformal group

- Time-frequency tomogram

$$x_1 = \mu \hat{t} + \nu \hat{\omega} = \mu t + i\nu \frac{d}{dt}$$

- Time-scale

$$x_2 = \mu t + i\nu \left(t\frac{d}{dt} + \frac{1}{2} \right)$$

- Frequency-scale

$$x_3 = i\mu \frac{d}{dt} + i\nu \left(t\frac{d}{dt} + \frac{1}{2} \right)$$

- Time-conformal

$$x_4 = \mu t + i\nu \left(t^2\frac{d}{dt} + t \right)$$