Detection and characterization of Levy flights in chaotic advection phenomena

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Motivation : Transport of advected passive particles in two dimensional flows with coherent structures (vortex) is anomalous when it contains Levy flights. We suggest a method for detecting these Levy flights in the signals, allowing to characterize the transport (diffusive or anomalous). We use time-frequency techniques such as Fractional Fourier transform and matching pursuit in order to be robust to noise.

Length of some particles trajectories in a chaotic advection

The erratic motion of particles finds its origin in chaotic advection. The regular part of the motion are due to stickiness : trajectory are trapped around islands of regular motion.

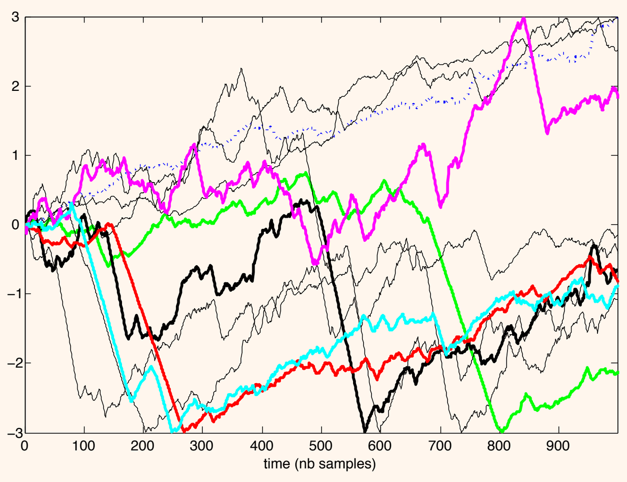

Figure 1: Length of some particles trajectories

Levy flights are coherent parts in the random signal (particle trapped in a vortex during some time). Some examples of length of trajectories, as a function of time are shown in figure1. Coherent parts are straight lines appearing in the signal.

Problems: The Levy flights are composed of a regular part with the addition of noise; they are not perfectly linear, even if the amplitude of noise is small. They occur randomly, and each time with a different length and slope. How to detect and describe these coherent behaviors?

Length trajectory as a phase derivative of a new signal S(t)

1. Time-frequency transformation.

The signal s is transformed into a path in the time-frequency plane.

$$s(t) \longrightarrow S(t) = e^{i\int_0^t s(\tau)d\tau}$$

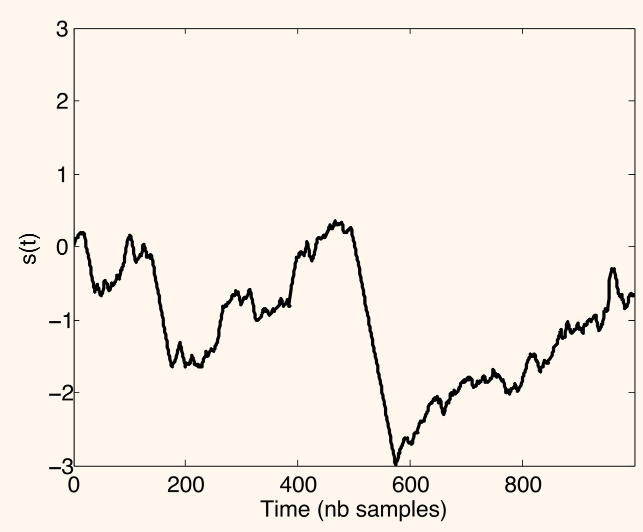

Figure 2: Length of a single particle

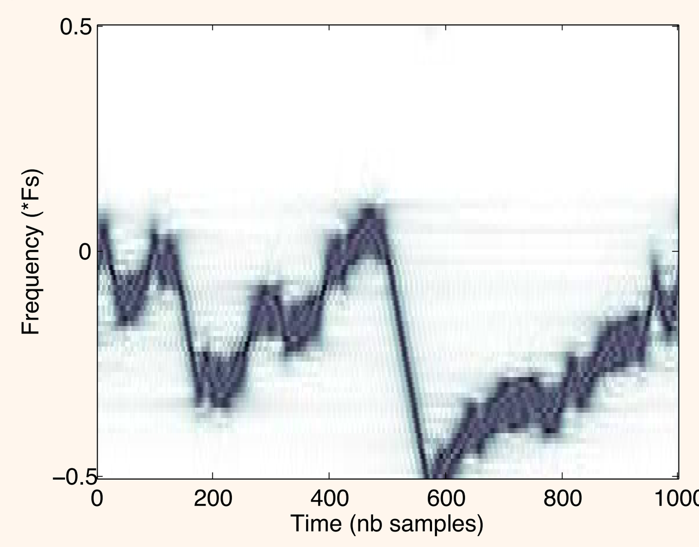

Figure 3: Short-time Fourier transform of S.

Because of the uncertainty principle, brownian fluctuations become diffuse stains in figure 3.

⇒ Pure random behavior is blurred, linear parts remain sharp.

Search for lines in the time-frequency 'picture'

2. Projection on a basis of chirps.

$$C(\theta,\mu) = \int S(t)\overline{\psi_{\theta,\mu}(t)}dt \quad \text{with} \quad \psi_{\theta,\mu} = e^{i(\frac{1}{2\tan\theta}t^2 + \frac{\mu}{\sin\theta}t)}$$

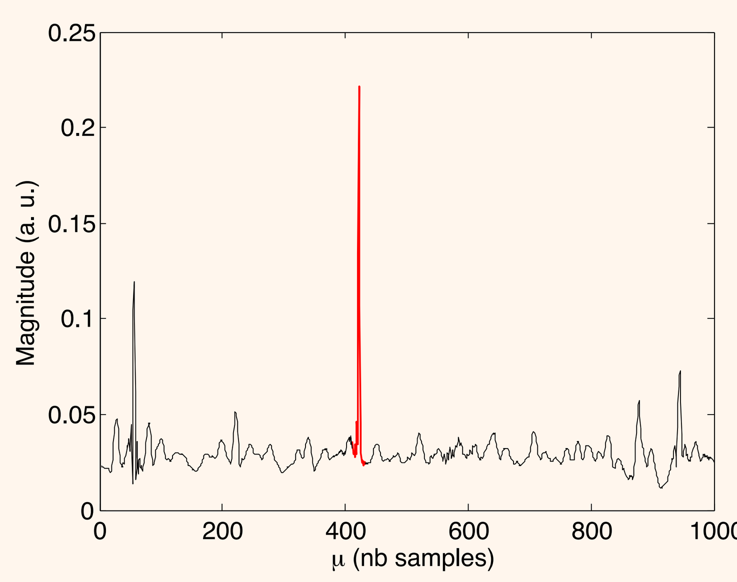

Figure 4: For θ_M, signal projections $|C(\theta_M,\mu)|$.

This Process detects linear parts in the time-frequency plane. For this particular choice of ψ, it is the Fractional Fourier transform. Several θ are chosen for the projection giving the matrix $C(\theta,\mu)$. For $\theta_M = -1.54rd$, the result of the projection of the signal on the chirps are presented on figure 4. The sharp peak give evidence that there is a Levy flight with a particular slope $\frac{1}{\tan\theta_M} \sim -0.085$ around $\mu = 420$.

3. Extraction of the maximum : $M(\theta_M,\mu_M) = \max_{\theta,\mu}|C(\theta,\mu)|$.

Levy flight removing and matching poursuit

4. Re-synthesis of S without the Levy flight:

$$S_1 = S - C(\theta_M,\mu_M)\psi_{\theta_M,\mu_M}$$

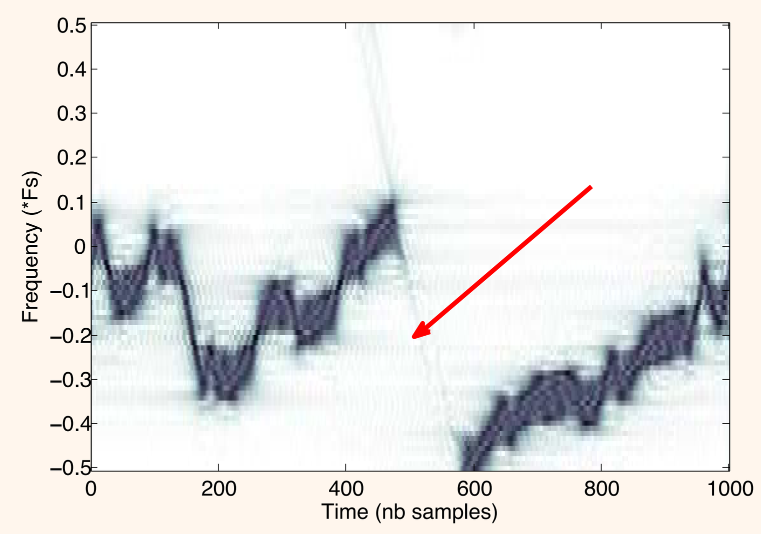

Figure 5: Short-time Fourier transform of S_1.

S_1 is now the original signal where the linear part of the phase derivative has been 'removed'. The short-time Fourier transform of S_1 is presented on figure 5.

Since there can be several Levy flights in the signal, this process has to be iterated to find the next ones.

5. Iteration (matching pursuit). Stopping condition: $|C_n| < \varepsilon$

Characterization of Levy flights: some results

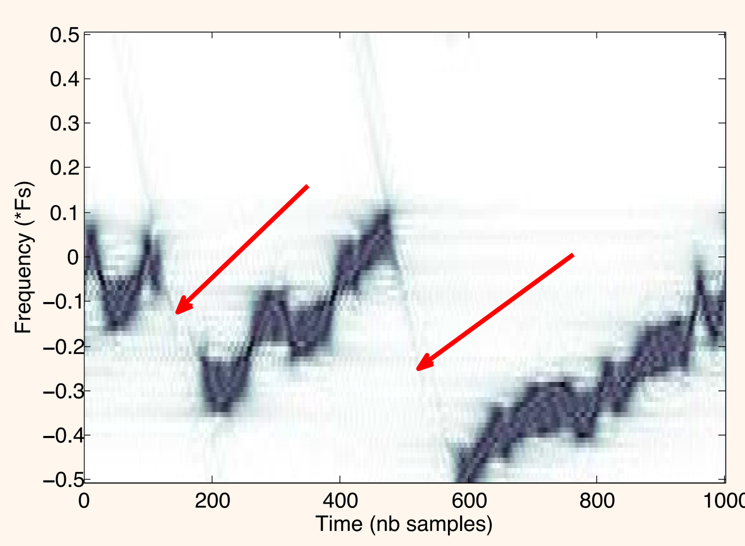

Figure 6: Short-time Fourier transform of S_2.

In this example, the process is iterated twice. As shown in figure 6, largest Levy flights have been removed.

The method has been applied to several signals in order to make a statistical study. The characteristic values of a simulation with N particles are the number of Levy fights, their mean length and their mean slope.

Results obtained with the length of some particles trajectories (fig 1)

$\theta(rd)$	−1.54	−1.44	−1.48	−1.49	−1.50	−1.54	−1.52	−1.49
length	0.2245	0.1832	0.2112	0.1574	0.2307	0.2365	0.1710	0.1435

Conclusions

The first step of the method help emphasing the straight lines over random fluctuations.

The second step consists in the detection of straight lines in the time-frequency image. The Fractional Fourier transform applied to a one-variable signal is similar to a Radon transform or Hough transform of a standard image.

Open Questions

- How to analyse other coherent shapes in the signal (more complex than linear)?
- Can we analyse the remaining random signal S_n and recover brownian motion?
- This method can detect noisy flights, what is the maximal level of noise admitted?
- What is the minimal length of a Levy flight?
- Is it possible to quantify anomalous transport with this technique?

References :

1 X. Leoncini, L. Kuznetsov, and G. M. Zaslavsky, *Chaotic advection near 3-vortex collapse*, Phys. Rev.E, 63(036224), 2001.

2 B. Ricaud, F. Briolle, F. Clairet, *Analysis and separation of time-frequency components in signals with chaotic behavior*, submitted.

3 F. Briolle, R. Lima, and R. Vilela Mendes, *A tomographic analysis of reflectometry data II : phase derivative*, Meas. Sci. Technol. **20**, No 10 105502, 2009.