

The results given below are mentioned and used in the article :

[“ The Horn Problem for Real Symmetric and Quaternionic Self-Dual Matrices”](#)

by R. Coquereaux and J.-B. Zuber (September 2018)

The results given below can be obtained by using the Mathematica package SymPol\$Package (the file itself is called SymPol\$Package.wl in the same directory).

See also the companion file SymPol\$Examples.nb (usage and examples).

See also the file HistogramsUsingSymPol.nb that contains some extra code related to the specific example studied below in the first section (square of {2,1,0} and of its multiples); in particular this file contains commands for visualization.

The present file only contains results. Most cells are non editable and non evaluatable.

Structure constants of zonal polynomials and SU(n) reduction

Structure constants of zonal polynomials : the square of {2*s, 1*s, 0} -- extended partition -- for s=2,...,6

Consider the following 3-variables zonal polynomial ZP[{2,1,0}], using the P normalization (hence ZP) :

(M9bis) In[]:= ZonalPolP[{2, 1, 0}] // TraditionalForm

(M9bis) Out[]//TraditionalForm=

$$\frac{1}{4} (4 x_2 x_1^2 + 4 x_3 x_1^2 + 4 x_2^2 x_1 + 4 x_3^2 x_1 + 6 x_2 x_3 x_1 + 4 x_2 x_3^2 + 4 x_2^2 x_3)$$

We now consider its square, and decompose on zonal polynomials (same normalization). We get :

$$\begin{aligned} \text{ZP}[\{2, 1, 0\}]^2 == & \frac{25}{12} \text{ZP}[\{2, 2, 2\}] + \frac{12}{5} \text{ZP}[\{3, 2, 1\}] + \\ & \frac{4}{3} \text{ZP}[\{3, 3, 0\}] + \frac{4}{3} \text{ZP}[\{4, 1, 1\}] + \text{ZP}[\{4, 2, 0\}] \end{aligned}$$

Arguments, above and below (in this subsection), denote extended partitions.

More generally we consider the square of the 3-variables zonal polynomials ZP[{2s,s,0}] and obtain :

toZonalPProdPair[{2, 1, 0}, {2, 1, 0}] =

$$\begin{aligned} & \frac{25}{12} \text{ZP}[\{2, 2, 2\}] + \frac{12}{5} \text{ZP}[\{3, 2, 1\}] + \\ & \frac{4}{3} \text{ZP}[\{3, 3, 0\}] + \frac{4}{3} \text{ZP}[\{4, 1, 1\}] + \text{ZP}[\{4, 2, 0\}]; \end{aligned}$$

toZonalPProdPair[2 {2, 1, 0}, 2 {2, 1, 0}] = $\frac{49}{20} \text{ZP}[\{4, 4, 4\}] +$

$$\begin{aligned} & \frac{252}{125} \text{ZP}[\{5, 4, 3\}] + \frac{132}{175} \text{ZP}[\{5, 5, 2\}] + \frac{132}{175} \text{ZP}[\{6, 3, 3\}] + \frac{8531 \text{ZP}[\{6, 4, 2\}]}{3087} + \\ & \frac{4128 \text{ZP}[\{6, 5, 1\}]}{2695} + \frac{64}{45} \text{ZP}[\{6, 6, 0\}] + \frac{4128 \text{ZP}[\{7, 3, 2\}]}{2695} + \frac{32}{27} \text{ZP}[\{7, 4, 1\}] + \\ & \frac{16}{21} \text{ZP}[\{7, 5, 0\}] + \frac{64}{45} \text{ZP}[\{8, 2, 2\}] + \frac{16}{21} \text{ZP}[\{8, 3, 1\}] + \text{ZP}[\{8, 4, 0\}]; \end{aligned}$$

$$\begin{aligned}
& \text{toZonalPProdPair}[3 \{2, 1, 0\}, 3 \{2, 1, 0\}] = \\
& \frac{184\,041 \text{ ZP}[\{6, 6, 6\}]}{70\,000} + \frac{1\,104\,246 \text{ ZP}[\{7, 6, 5\}]}{546\,875} + \frac{531\,674 \text{ ZP}[\{7, 7, 4\}]}{765\,625} + \\
& \frac{531\,674 \text{ ZP}[\{8, 5, 5\}]}{765\,625} + \frac{27\,805\,063 \text{ ZP}[\{8, 6, 4\}]}{13\,781\,250} + \frac{228\,488 \text{ ZP}[\{8, 7, 3\}]}{240\,625} + \\
& \frac{22\,984 \text{ ZP}[\{8, 8, 2\}]}{39\,375} + \frac{228\,488 \text{ ZP}[\{9, 5, 4\}]}{240\,625} + \frac{2\,336\,792 \text{ ZP}[\{9, 6, 3\}]}{804\,375} + \\
& \frac{11\,268 \text{ ZP}[\{9, 7, 2\}]}{6875} + \frac{102\,336 \text{ ZP}[\{9, 8, 1\}]}{74\,375} + \frac{256}{175} \text{ ZP}[\{9, 9, 0\}] + \\
& \frac{22\,984 \text{ ZP}[\{10, 4, 4\}]}{39\,375} + \frac{11\,268 \text{ ZP}[\{10, 5, 3\}]}{6875} + \frac{2\,201\,913 \text{ ZP}[\{10, 6, 2\}]}{1\,730\,300} + \\
& \frac{12\,896 \text{ ZP}[\{10, 7, 1\}]}{15\,125} + \frac{128}{175} \text{ ZP}[\{10, 8, 0\}] + \frac{102\,336 \text{ ZP}[\{11, 4, 3\}]}{74\,375} + \\
& \frac{12\,896 \text{ ZP}[\{11, 5, 2\}]}{15\,125} + \frac{324}{325} \text{ ZP}[\{11, 6, 1\}] + \frac{36}{55} \text{ ZP}[\{11, 7, 0\}] + \\
& \frac{256}{175} \text{ ZP}[\{12, 3, 3\}] + \frac{128}{175} \text{ ZP}[\{12, 4, 2\}] + \frac{36}{55} \text{ ZP}[\{12, 5, 1\}] + \text{ZP}[\{12, 6, 0\}];
\end{aligned}$$

$$\begin{aligned}
& \text{toZonalPProdPair}[4 \{2, 1, 0\}, 4 \{2, 1, 0\}] = \\
& \frac{5\,909\,761 \text{ ZP}[\{8, 8, 8\}]}{2\,160\,900} + \frac{4\,298\,008 \text{ ZP}[\{9, 8, 7\}]}{2\,100\,875} + \frac{126\,205\,144 \text{ ZP}[\{9, 9, 6\}]}{185\,297\,175} + \\
& \frac{126\,205\,144 \text{ ZP}[\{10, 7, 7\}]}{185\,297\,175} + \frac{2\,525\,883\,698 \text{ ZP}[\{10, 8, 6\}]}{1\,331\,669\,031} + \\
& \frac{32\,502\,678\,624 \text{ ZP}[\{10, 9, 5\}]}{38\,364\,750\,655} + \frac{2\,177\,598\,816 \text{ ZP}[\{10, 10, 4\}]}{4\,626\,546\,925} + \\
& \frac{32\,502\,678\,624 \text{ ZP}[\{11, 7, 6\}]}{38\,364\,750\,655} + \frac{111\,080\,141\,728 \text{ ZP}[\{11, 8, 5\}]}{54\,963\,377\,469} + \\
& \frac{263\,971\,874\,032 \text{ ZP}[\{11, 9, 4\}]}{252\,609\,462\,105} + \frac{488\,934\,144 \text{ ZP}[\{11, 10, 3\}]}{660\,935\,275} + \\
& \frac{6\,806\,528 \text{ ZP}[\{11, 11, 2\}]}{13\,073\,445} + \frac{2\,177\,598\,816 \text{ ZP}[\{12, 6, 6\}]}{4\,626\,546\,925} + \\
& \frac{263\,971\,874\,032 \text{ ZP}[\{12, 7, 5\}]}{252\,609\,462\,105} + \frac{86\,964\,163\,635\,033 \text{ ZP}[\{12, 8, 4\}]}{29\,242\,552\,494\,155} + \\
& \frac{166\,677\,160\,576 \text{ ZP}[\{12, 9, 3\}]}{97\,950\,607\,755} + \frac{43\,567\,872 \text{ ZP}[\{12, 10, 2\}]}{31\,244\,213} + \\
& \frac{5\,292\,032 \text{ ZP}[\{12, 11, 1\}]}{4\,029\,025} + \frac{16\,384 \text{ ZP}[\{12, 12, 0\}]}{11\,025} + \frac{488\,934\,144 \text{ ZP}[\{13, 6, 5\}]}{660\,935\,275} + \\
& \frac{166\,677\,160\,576 \text{ ZP}[\{13, 7, 4\}]}{97\,950\,607\,755} + \frac{2\,956\,771\,344 \text{ ZP}[\{13, 8, 3\}]}{2\,241\,873\,725} + \\
& \frac{77\,219\,568 \text{ ZP}[\{13, 9, 2\}]}{84\,805\,721} + \frac{11\,889\,664 \text{ ZP}[\{13, 10, 1\}]}{15\,450\,435} + \frac{8192 \text{ ZP}[\{13, 11, 0\}]}{11\,319} + \\
& \frac{6\,806\,528 \text{ ZP}[\{14, 5, 5\}]}{13\,073\,445} + \frac{43\,567\,872 \text{ ZP}[\{14, 6, 4\}]}{31\,244\,213} + \frac{77\,219\,568 \text{ ZP}[\{14, 7, 3\}]}{84\,805\,721} + \\
& \frac{740\,556 \text{ ZP}[\{14, 8, 2\}]}{728\,875} + \frac{211\,072 \text{ ZP}[\{14, 9, 1\}]}{302\,575} + \frac{20\,736 \text{ ZP}[\{14, 10, 0\}]}{35\,035} + \\
& \frac{5\,292\,032 \text{ ZP}[\{15, 5, 4\}]}{4\,029\,025} + \frac{11\,889\,664 \text{ ZP}[\{15, 6, 3\}]}{15\,450\,435} + \frac{211\,072 \text{ ZP}[\{15, 7, 2\}]}{302\,575} + \\
& \frac{768}{833} \text{ ZP}[\{15, 8, 1\}] + \frac{64}{105} \text{ ZP}[\{15, 9, 0\}] + \frac{16\,384 \text{ ZP}[\{16, 4, 4\}]}{11\,025} + \\
& \frac{8192 \text{ ZP}[\{16, 5, 3\}]}{11\,319} + \frac{20\,736 \text{ ZP}[\{16, 6, 2\}]}{35\,035} + \frac{64}{105} \text{ ZP}[\{16, 7, 1\}] + \text{ZP}[\{16, 8, 0\}];
\end{aligned}$$

$$\begin{aligned}
& \text{toZonalPProdPair}[5\{2, 1, 0\}, 5\{2, 1, 0\}] = \\
& \frac{17\,631\,601 \text{ ZP}[\{10, 10, 10\}]}{6\,286\,896} + \frac{1\,356\,277 \text{ ZP}[\{11, 10, 9\}]}{654\,885} + \frac{2\,399\,567 \text{ ZP}[\{11, 11, 8\}]}{3\,536\,379} + \\
& \frac{2\,399\,567 \text{ ZP}[\{12, 9, 9\}]}{3\,536\,379} + \frac{21\,551\,763\,175 \text{ ZP}[\{12, 10, 8\}]}{11\,585\,177\,604} + \\
& \frac{4\,511\,185\,960 \text{ ZP}[\{12, 11, 7\}]}{5\,562\,724\,167} + \frac{3\,647\,341\,840 \text{ ZP}[\{12, 12, 6\}]}{8\,452\,451\,007} + \\
& \frac{4\,511\,185\,960 \text{ ZP}[\{13, 9, 8\}]}{5\,562\,724\,167} + \frac{317\,760\,358\,600 \text{ ZP}[\{13, 10, 7\}]}{172\,671\,499\,143} + \\
& \frac{11\,213\,600\,044 \text{ ZP}[\{13, 11, 6\}]}{12\,209\,095\,899} + \frac{22\,817\,972\,480 \text{ ZP}[\{13, 12, 5\}]}{37\,253\,395\,179} + \\
& \frac{850\,227\,200 \text{ ZP}[\{13, 13, 4\}]}{2\,191\,376\,187} + \frac{3\,647\,341\,840 \text{ ZP}[\{14, 8, 8\}]}{8\,452\,451\,007} + \\
& \frac{11\,213\,600\,044 \text{ ZP}[\{14, 9, 7\}]}{12\,209\,095\,899} + \frac{293\,103\,454\,115 \text{ ZP}[\{14, 10, 6\}]}{144\,958\,789\,404} + \\
& \frac{47\,733\,500\,800 \text{ ZP}[\{14, 11, 5\}]}{43\,335\,582\,147} + \frac{8\,335\,016\,000 \text{ ZP}[\{14, 12, 4\}]}{10\,160\,016\,867} + \\
& \frac{110\,510\,080 \text{ ZP}[\{14, 13, 3\}]}{169\,702\,533} + \frac{118\,784 \text{ ZP}[\{14, 14, 2\}]}{243\,243} + \\
& \frac{22\,817\,972\,480 \text{ ZP}[\{15, 8, 7\}]}{37\,253\,395\,179} + \frac{47\,733\,500\,800 \text{ ZP}[\{15, 9, 6\}]}{43\,335\,582\,147} + \\
& \frac{10\,905\,744\,368 \text{ ZP}[\{15, 10, 5\}]}{3\,621\,246\,993} + \frac{2\,086\,257\,595\,600 \text{ ZP}[\{15, 11, 4\}]}{1\,198\,075\,639\,761} + \\
& \frac{25\,448\,219\,392 \text{ ZP}[\{15, 12, 3\}]}{17\,975\,414\,457} + \frac{4\,693\,504 \text{ ZP}[\{15, 13, 2\}]}{3\,648\,645} + \\
& \frac{143\,360 \text{ ZP}[\{15, 14, 1\}]}{111\,969} + \frac{65\,536 \text{ ZP}[\{15, 15, 0\}]}{43\,659} + \frac{850\,227\,200 \text{ ZP}[\{16, 7, 7\}]}{2\,191\,376\,187} + \\
& \frac{8\,335\,016\,000 \text{ ZP}[\{16, 8, 6\}]}{10\,160\,016\,867} + \frac{2\,086\,257\,595\,600 \text{ ZP}[\{16, 9, 5\}]}{1\,198\,075\,639\,761} + \\
& \frac{15\,179\,395\,24 \text{ ZP}[\{16, 10, 4\}]}{1\,128\,196\,755} + \frac{7\,957\,060\,192 \text{ ZP}[\{16, 11, 3\}]}{8\,378\,151\,327} + \\
& \frac{20\,569\,904 \text{ ZP}[\{16, 12, 2\}]}{25\,664\,067} + \frac{3\,891\,200 \text{ ZP}[\{16, 13, 1\}]}{5\,316\,597} + \frac{409\,600 \text{ ZP}[\{16, 14, 0\}]}{567\,567} + \\
& \frac{110\,510\,080 \text{ ZP}[\{17, 7, 6\}]}{169\,702\,533} + \frac{25\,448\,219\,392 \text{ ZP}[\{17, 8, 5\}]}{17\,975\,414\,457} + \\
& \frac{7\,957\,060\,192 \text{ ZP}[\{17, 9, 4\}]}{8\,378\,151\,327} + \frac{382\,136\,000 \text{ ZP}[\{17, 10, 3\}]}{370\,494\,243} + \\
& \frac{67\,208\,560 \text{ ZP}[\{17, 11, 2\}]}{92\,067\,597} + \frac{55\,552 \text{ ZP}[\{17, 12, 1\}]}{89\,505} + \frac{5120 \text{ ZP}[\{17, 13, 0\}]}{9009} + \\
& \frac{118\,784 \text{ ZP}[\{18, 6, 6\}]}{243\,243} + \frac{4\,693\,504 \text{ ZP}[\{18, 7, 5\}]}{3\,648\,645} + \frac{20\,569\,904 \text{ ZP}[\{18, 8, 4\}]}{25\,664\,067} + \\
& \frac{67\,208\,560 \text{ ZP}[\{18, 9, 3\}]}{92\,067\,597} + \frac{98\,899\,025 \text{ ZP}[\{18, 10, 2\}]}{108\,893\,484} + \frac{265\,600 \text{ ZP}[\{18, 11, 1\}]}{423\,453} + \\
& \frac{5120 \text{ ZP}[\{18, 12, 0\}]}{9639} + \frac{143\,360 \text{ ZP}[\{19, 6, 5\}]}{111\,969} + \frac{3\,891\,200 \text{ ZP}[\{19, 7, 4\}]}{5\,316\,597} + \\
& \frac{55\,552 \text{ ZP}[\{19, 8, 3\}]}{89\,505} + \frac{265\,600 \text{ ZP}[\{19, 9, 2\}]}{423\,453} + \frac{500}{567} \text{ ZP}[\{19, 10, 1\}] + \\
& \frac{100}{171} \text{ ZP}[\{19, 11, 0\}] + \frac{65\,536 \text{ ZP}[\{20, 5, 5\}]}{43\,659} + \frac{409\,600 \text{ ZP}[\{20, 6, 4\}]}{567\,567} +
\end{aligned}$$

$$\frac{5120 \text{ ZP}[\{20, 7, 3\}]}{9009} + \frac{5120 \text{ ZP}[\{20, 8, 2\}]}{9639} + \frac{100}{171} \text{ ZP}[\{20, 9, 1\}] + \text{ZP}[\{20, 10, 0\}];$$

Structure constants of zonal polynomials reduced to SU(3) : the square of [s,s] -- Dynkin basis -- for s=2,...,8

The SU(3) zonal-characters $\chi_Z([1,1])$ is obtained by SU(3) reduction from the 3-variables zonal polynomial $\text{ZP}[\{2,1,0\}]$.

`ZonalPToSUnCharPol[[1, 1]] // TraditionalForm (* This is $\chi_Z([1,1])$ *)`

(M9bis) Out[] // TraditionalForm=

$$\frac{1}{2} \left(\frac{2y_1^2}{y_2} + 2y_2y_1 + \frac{2y_1}{y_2^2} + \frac{2y_2^2}{y_1} + \frac{2}{y_2y_1} + \frac{2y_2}{y_1^2} + 3 \right)$$

The notation [1,1] denotes the components, in the Dynkin basis (basis of fundamental weights), of the highest weight of the adjoint representation of SU(3).

The corresponding integer partition (Young diagram) is {2,1} and the corresponding extended partition (three components, with one trailing 0) is {2,1,0}.

We now consider the square of $\chi_Z([1,1])$ and decompose on the the $\chi_Z([v1,v2])$.

One gets

$$\chi_Z([1, 1])^2 ==$$

$$\frac{25}{12} \chi_Z([0, 0]) + \frac{12}{5} \chi_Z([1, 1]) + \frac{4}{3} \chi_Z([3, 0]) + \frac{4}{3} \chi_Z([0, 3]) + \chi_Z([2, 2])$$

It is obtained by SU(3) reduction from the square of the zonal polynomial $\text{ZP}[\{2,1,0\}]$, using the P normalization (hence ZP).

We consider the square of [s,s] for s = 1, 2, 3, ...

Arguments { , } below (in this subsection) denote highest weights [v1,v2] in the SU(3) Dynkin basis.

The last entry m of each pair {{v1,v2},m} is the structure constant for the triple : [s,s] * [s,s] --> [v1,v2].

`ZonalPproductCoeffsDynkLab[[1, 1], {1, 1}, 3] =`

$$\{ \{ \{0, 0\}, \frac{25}{12} \}, \{ \{1, 1\}, \frac{12}{5} \}, \{ \{3, 0\}, \frac{4}{3} \}, \{ \{0, 3\}, \frac{4}{3} \}, \{ \{2, 2\}, 1 \} \};$$

`ZonalPproductCoeffsDynkLab[2 {1, 1}, 2 {1, 1}, 3] =`

$$\begin{aligned} & \{ \{ \{0, 0\}, \frac{49}{20} \}, \{ \{1, 1\}, \frac{252}{125} \}, \{ \{3, 0\}, \frac{132}{175} \}, \{ \{0, 3\}, \frac{132}{175} \}, \\ & \{ \{2, 2\}, \frac{8531}{3087} \}, \{ \{4, 1\}, \frac{4128}{2695} \}, \{ \{1, 4\}, \frac{4128}{2695} \}, \{ \{6, 0\}, \frac{64}{45} \}, \\ & \{ \{3, 3\}, \frac{32}{27} \}, \{ \{0, 6\}, \frac{64}{45} \}, \{ \{5, 2\}, \frac{16}{21} \}, \{ \{2, 5\}, \frac{16}{21} \}, \{ \{4, 4\}, 1 \} \}; \end{aligned}$$

$$\text{ZonalPproductCoeffsDynkLab}[3 \{1, 1\}, 3 \{1, 1\}, 3] =$$

$$\left\{ \left\{ \{0, 0\}, \frac{184\,041}{70\,000} \right\}, \left\{ \{1, 1\}, \frac{1\,104\,246}{546\,875} \right\}, \left\{ \{3, 0\}, \frac{531\,674}{765\,625} \right\}, \left\{ \{0, 3\}, \frac{531\,674}{765\,625} \right\}, \right.$$

$$\left. \left\{ \{2, 2\}, \frac{27\,805\,063}{13\,781\,250} \right\}, \left\{ \{4, 1\}, \frac{228\,488}{240\,625} \right\}, \left\{ \{1, 4\}, \frac{228\,488}{240\,625} \right\}, \left\{ \{6, 0\}, \frac{22\,984}{39\,375} \right\}, \right.$$

$$\left. \left\{ \{3, 3\}, \frac{2\,336\,792}{804\,375} \right\}, \left\{ \{0, 6\}, \frac{22\,984}{39\,375} \right\}, \left\{ \{5, 2\}, \frac{11\,268}{6875} \right\}, \left\{ \{2, 5\}, \frac{11\,268}{6875} \right\}, \right.$$

$$\left. \left\{ \{7, 1\}, \frac{102\,336}{74\,375} \right\}, \left\{ \{4, 4\}, \frac{2\,201\,913}{1\,730\,300} \right\}, \left\{ \{1, 7\}, \frac{102\,336}{74\,375} \right\}, \left\{ \{9, 0\}, \frac{256}{175} \right\}, \right.$$

$$\left. \left\{ \{6, 3\}, \frac{12\,896}{15\,125} \right\}, \left\{ \{3, 6\}, \frac{12\,896}{15\,125} \right\}, \left\{ \{0, 9\}, \frac{256}{175} \right\}, \left\{ \{8, 2\}, \frac{128}{175} \right\}, \right.$$

$$\left. \left\{ \{5, 5\}, \frac{324}{325} \right\}, \left\{ \{2, 8\}, \frac{128}{175} \right\}, \left\{ \{7, 4\}, \frac{36}{55} \right\}, \left\{ \{4, 7\}, \frac{36}{55} \right\}, \left\{ \{6, 6\}, 1 \right\} \right\};$$

$$\text{ZonalPproductCoeffsDynkLab}[4 \{1, 1\}, 4 \{1, 1\}, 3] =$$

$$\left\{ \left\{ \{0, 0\}, \frac{5\,909\,761}{2\,160\,900} \right\}, \left\{ \{1, 1\}, \frac{4\,298\,008}{2\,100\,875} \right\}, \left\{ \{3, 0\}, \frac{126\,205\,144}{185\,297\,175} \right\}, \right.$$

$$\left. \left\{ \{0, 3\}, \frac{126\,205\,144}{185\,297\,175} \right\}, \left\{ \{2, 2\}, \frac{2\,525\,883\,698}{1\,331\,669\,031} \right\}, \left\{ \{4, 1\}, \frac{32\,502\,678\,624}{38\,364\,750\,655} \right\}, \right.$$

$$\left. \left\{ \{1, 4\}, \frac{32\,502\,678\,624}{38\,364\,750\,655} \right\}, \left\{ \{6, 0\}, \frac{2\,177\,598\,816}{4\,626\,546\,925} \right\}, \left\{ \{3, 3\}, \frac{111\,080\,141\,728}{54\,963\,377\,469} \right\}, \right.$$

$$\left. \left\{ \{0, 6\}, \frac{2\,177\,598\,816}{4\,626\,546\,925} \right\}, \left\{ \{5, 2\}, \frac{263\,971\,874\,032}{252\,609\,462\,105} \right\}, \left\{ \{2, 5\}, \frac{263\,971\,874\,032}{252\,609\,462\,105} \right\}, \right.$$

$$\left. \left\{ \{7, 1\}, \frac{488\,934\,144}{660\,935\,275} \right\}, \left\{ \{4, 4\}, \frac{86\,964\,163\,635\,033}{29\,242\,552\,494\,155} \right\}, \left\{ \{1, 7\}, \frac{488\,934\,144}{660\,935\,275} \right\}, \right.$$

$$\left. \left\{ \{9, 0\}, \frac{6\,806\,528}{13\,073\,445} \right\}, \left\{ \{6, 3\}, \frac{166\,677\,160\,576}{97\,950\,607\,755} \right\}, \left\{ \{3, 6\}, \frac{166\,677\,160\,576}{97\,950\,607\,755} \right\}, \right.$$

$$\left. \left\{ \{0, 9\}, \frac{6\,806\,528}{13\,073\,445} \right\}, \left\{ \{8, 2\}, \frac{43\,567\,872}{31\,244\,213} \right\}, \left\{ \{5, 5\}, \frac{2\,956\,771\,344}{2\,241\,873\,725} \right\}, \right.$$

$$\left. \left\{ \{2, 8\}, \frac{43\,567\,872}{31\,244\,213} \right\}, \left\{ \{10, 1\}, \frac{5\,292\,032}{4\,029\,025} \right\}, \left\{ \{7, 4\}, \frac{77\,219\,568}{84\,805\,721} \right\}, \right.$$

$$\left. \left\{ \{4, 7\}, \frac{77\,219\,568}{84\,805\,721} \right\}, \left\{ \{1, 10\}, \frac{5\,292\,032}{4\,029\,025} \right\}, \left\{ \{12, 0\}, \frac{16\,384}{11\,025} \right\}, \right.$$

$$\left. \left\{ \{9, 3\}, \frac{11\,889\,664}{15\,450\,435} \right\}, \left\{ \{6, 6\}, \frac{740\,556}{728\,875} \right\}, \left\{ \{3, 9\}, \frac{11\,889\,664}{15\,450\,435} \right\}, \right.$$

$$\left. \left\{ \{0, 12\}, \frac{16\,384}{11\,025} \right\}, \left\{ \{11, 2\}, \frac{8192}{11\,319} \right\}, \left\{ \{8, 5\}, \frac{211\,072}{302\,575} \right\}, \right.$$

$$\left. \left\{ \{5, 8\}, \frac{211\,072}{302\,575} \right\}, \left\{ \{2, 11\}, \frac{8192}{11\,319} \right\}, \left\{ \{10, 4\}, \frac{20\,736}{35\,035} \right\}, \left\{ \{7, 7\}, \frac{768}{833} \right\}, \right.$$

$$\left. \left\{ \{4, 10\}, \frac{20\,736}{35\,035} \right\}, \left\{ \{9, 6\}, \frac{64}{105} \right\}, \left\{ \{6, 9\}, \frac{64}{105} \right\}, \left\{ \{8, 8\}, 1 \right\} \right\};$$

$$\begin{aligned}
& \text{ZonalPproductCoeffsDynkLab}[5 \{1, 1\}, 5 \{1, 1\}, 3] = \\
& \left\{ \left\{ \{0, 0\}, \frac{17\,631\,601}{6\,286\,896} \right\}, \left\{ \{1, 1\}, \frac{1\,356\,277}{654\,885} \right\}, \left\{ \{3, 0\}, \frac{2\,399\,567}{3\,536\,379} \right\}, \right. \\
& \left\{ \{0, 3\}, \frac{2\,399\,567}{3\,536\,379} \right\}, \left\{ \{2, 2\}, \frac{21\,551\,763\,175}{11\,585\,177\,604} \right\}, \left\{ \{4, 1\}, \frac{4\,511\,185\,960}{5\,562\,724\,167} \right\}, \\
& \left\{ \{1, 4\}, \frac{4\,511\,185\,960}{5\,562\,724\,167} \right\}, \left\{ \{6, 0\}, \frac{3\,647\,341\,840}{8\,452\,451\,007} \right\}, \left\{ \{3, 3\}, \frac{317\,760\,358\,600}{172\,671\,499\,143} \right\}, \\
& \left\{ \{0, 6\}, \frac{3\,647\,341\,840}{8\,452\,451\,007} \right\}, \left\{ \{5, 2\}, \frac{11\,213\,600\,044}{12\,209\,095\,899} \right\}, \left\{ \{2, 5\}, \frac{11\,213\,600\,044}{12\,209\,095\,899} \right\}, \\
& \left\{ \{7, 1\}, \frac{22\,817\,972\,480}{37\,253\,395\,179} \right\}, \left\{ \{4, 4\}, \frac{293\,103\,454\,115}{144\,958\,789\,404} \right\}, \left\{ \{1, 7\}, \frac{22\,817\,972\,480}{37\,253\,395\,179} \right\}, \\
& \left\{ \{9, 0\}, \frac{850\,227\,200}{2\,191\,376\,187} \right\}, \left\{ \{6, 3\}, \frac{47\,733\,500\,800}{43\,335\,582\,147} \right\}, \left\{ \{3, 6\}, \frac{47\,733\,500\,800}{43\,335\,582\,147} \right\}, \\
& \left\{ \{0, 9\}, \frac{850\,227\,200}{2\,191\,376\,187} \right\}, \left\{ \{8, 2\}, \frac{8\,335\,016\,000}{10\,160\,016\,867} \right\}, \left\{ \{5, 5\}, \frac{10\,905\,744\,368}{3\,621\,246\,993} \right\}, \\
& \left\{ \{2, 8\}, \frac{8\,335\,016\,000}{10\,160\,016\,867} \right\}, \left\{ \{10, 1\}, \frac{110\,510\,080}{169\,702\,533} \right\}, \left\{ \{7, 4\}, \frac{2\,086\,257\,595\,600}{1\,198\,075\,639\,761} \right\}, \\
& \left\{ \{4, 7\}, \frac{2\,086\,257\,595\,600}{1\,198\,075\,639\,761} \right\}, \left\{ \{1, 10\}, \frac{110\,510\,080}{169\,702\,533} \right\}, \left\{ \{12, 0\}, \frac{118\,784}{243\,243} \right\}, \\
& \left\{ \{9, 3\}, \frac{25\,448\,219\,392}{17\,975\,414\,457} \right\}, \left\{ \{6, 6\}, \frac{1\,517\,939\,524}{1\,128\,196\,755} \right\}, \left\{ \{3, 9\}, \frac{25\,448\,219\,392}{17\,975\,414\,457} \right\}, \\
& \left\{ \{0, 12\}, \frac{118\,784}{243\,243} \right\}, \left\{ \{11, 2\}, \frac{4\,693\,504}{3\,648\,645} \right\}, \left\{ \{8, 5\}, \frac{7\,957\,060\,192}{8\,378\,151\,327} \right\}, \\
& \left\{ \{5, 8\}, \frac{7\,957\,060\,192}{8\,378\,151\,327} \right\}, \left\{ \{2, 11\}, \frac{4\,693\,504}{3\,648\,645} \right\}, \left\{ \{13, 1\}, \frac{143\,360}{111\,969} \right\}, \\
& \left\{ \{10, 4\}, \frac{20\,569\,904}{25\,664\,067} \right\}, \left\{ \{7, 7\}, \frac{382\,136\,000}{370\,494\,243} \right\}, \left\{ \{4, 10\}, \frac{20\,569\,904}{25\,664\,067} \right\}, \\
& \left\{ \{1, 13\}, \frac{143\,360}{111\,969} \right\}, \left\{ \{15, 0\}, \frac{65\,536}{43\,659} \right\}, \left\{ \{12, 3\}, \frac{3\,891\,200}{5\,316\,597} \right\}, \\
& \left\{ \{9, 6\}, \frac{67\,208\,560}{92\,067\,597} \right\}, \left\{ \{6, 9\}, \frac{67\,208\,560}{92\,067\,597} \right\}, \left\{ \{3, 12\}, \frac{3\,891\,200}{5\,316\,597} \right\}, \\
& \left\{ \{0, 15\}, \frac{65\,536}{43\,659} \right\}, \left\{ \{14, 2\}, \frac{409\,600}{567\,567} \right\}, \left\{ \{11, 5\}, \frac{55\,552}{89\,505} \right\}, \left\{ \{8, 8\}, \frac{98\,899\,025}{108\,893\,484} \right\}, \\
& \left\{ \{5, 11\}, \frac{55\,552}{89\,505} \right\}, \left\{ \{2, 14\}, \frac{409\,600}{567\,567} \right\}, \left\{ \{13, 4\}, \frac{5120}{9009} \right\}, \left\{ \{10, 7\}, \frac{265\,600}{423\,453} \right\}, \\
& \left\{ \{7, 10\}, \frac{265\,600}{423\,453} \right\}, \left\{ \{4, 13\}, \frac{5120}{9009} \right\}, \left\{ \{12, 6\}, \frac{5120}{9639} \right\}, \left\{ \{9, 9\}, \frac{500}{567} \right\}, \\
& \left\{ \{6, 12\}, \frac{5120}{9639} \right\}, \left\{ \{11, 8\}, \frac{100}{171} \right\}, \left\{ \{8, 11\}, \frac{100}{171} \right\}, \left\{ \{10, 10\}, 1 \right\};
\end{aligned}$$

$$\begin{aligned}
& \text{ZonalPproductCoeffsDynkLab}[6 \{1, 1\}, 6 \{1, 1\}, 3] = \\
& \left\{ \left\{ \{0, 0\}, \frac{34\,493\,775\,625}{12\,086\,906\,832} \right\}, \left\{ \{1, 1\}, \frac{275\,950\,205}{131\,900\,769} \right\}, \right. \\
& \left\{ \{3, 0\}, \frac{6\,898\,755\,125}{10\,156\,359\,213} \right\}, \left\{ \{0, 3\}, \frac{6\,898\,755\,125}{10\,156\,359\,213} \right\}, \left\{ \{2, 2\}, \frac{468\,266\,314\,648\,675}{253\,380\,849\,645\,924} \right\}, \\
& \left\{ \{4, 1\}, \frac{10\,745\,825\,630\,000}{13\,518\,114\,112\,503} \right\}, \left\{ \{1, 4\}, \frac{10\,745\,825\,630\,000}{13\,518\,114\,112\,503} \right\}, \\
& \left\{ \{6, 0\}, \frac{25\,475\,264\,600\,000}{61\,621\,533\,162\,189} \right\}, \left\{ \{3, 3\}, \frac{20\,042\,492\,350\,930\,000}{11\,329\,559\,025\,676\,749} \right\}, \\
& \left\{ \{0, 6\}, \frac{25\,475\,264\,600\,000}{61\,621\,533\,162\,189} \right\}, \left\{ \{5, 2\}, \frac{231\,416\,988\,025\,000}{267\,026\,643\,702\,819} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \{2, 5\}, \frac{231\,416\,988\,025\,000}{267\,026\,643\,702\,819} \right\}, \left\{ \{7, 1\}, \frac{123\,353\,912\,800\,000}{219\,859\,544\,245\,341} \right\}, \\
& \left\{ \{4, 4\}, \frac{7\,451\,208\,629\,814\,303\,125}{4\,123\,120\,258\,751\,842\,062} \right\}, \left\{ \{1, 7\}, \frac{123\,353\,912\,800\,000}{219\,859\,544\,245\,341} \right\}, \\
& \left\{ \{9, 0\}, \frac{1\,750\,101\,280\,000\,000}{5\,160\,232\,832\,581\,827} \right\}, \left\{ \{6, 3\}, \frac{263\,080\,302\,598\,000\,000}{273\,913\,583\,623\,374\,123} \right\}, \\
& \left\{ \{3, 6\}, \frac{263\,080\,302\,598\,000\,000}{273\,913\,583\,623\,374\,123} \right\}, \left\{ \{0, 9\}, \frac{1\,750\,101\,280\,000\,000}{5\,160\,232\,832\,581\,827} \right\}, \\
& \left\{ \{8, 2\}, \frac{2\,186\,908\,564\,495\,000\,000}{3\,181\,987\,209\,400\,230\,231} \right\}, \left\{ \{5, 5\}, \frac{19\,825\,685\,983\,990\,000}{9\,809\,577\,374\,941\,773} \right\}, \\
& \left\{ \{2, 8\}, \frac{2\,186\,908\,564\,495\,000\,000}{3\,181\,987\,209\,400\,230\,231} \right\}, \left\{ \{10, 1\}, \frac{1\,363\,027\,840\,000\,000}{2\,658\,301\,762\,239\,123} \right\}, \\
& \left\{ \{7, 4\}, \frac{7\,494\,109\,787\,091\,250\,000}{6\,582\,841\,263\,891\,481\,589} \right\}, \left\{ \{4, 7\}, \frac{7\,494\,109\,787\,091\,250\,000}{6\,582\,841\,263\,891\,481\,589} \right\}, \\
& \left\{ \{1, 10\}, \frac{1\,363\,027\,840\,000\,000}{2\,658\,301\,762\,239\,123} \right\}, \left\{ \{12, 0\}, \frac{3\,043\,654\,400\,000}{8\,805\,563\,437\,671} \right\}, \\
& \left\{ \{9, 3\}, \frac{1\,602\,787\,715\,680\,000}{1\,840\,362\,758\,473\,239} \right\}, \left\{ \{6, 6\}, \frac{59\,046\,627\,043\,228\,744\,564}{19\,463\,215\,064\,030\,181\,675} \right\}, \\
& \left\{ \{3, 9\}, \frac{1\,602\,787\,715\,680\,000}{1\,840\,362\,758\,473\,239} \right\}, \left\{ \{0, 12\}, \frac{3\,043\,654\,400\,000}{8\,805\,563\,437\,671} \right\}, \\
& \left\{ \{11, 2\}, \frac{566\,507\,453\,504\,000}{788\,726\,896\,488\,531} \right\}, \left\{ \{8, 5\}, \frac{116\,954\,188\,085\,349\,152}{66\,149\,489\,106\,507\,807} \right\}, \\
& \left\{ \{5, 8\}, \frac{116\,954\,188\,085\,349\,152}{66\,149\,489\,106\,507\,807} \right\}, \left\{ \{2, 11\}, \frac{566\,507\,453\,504\,000}{788\,726\,896\,488\,531} \right\}, \\
& \left\{ \{13, 1\}, \frac{666\,227\,200\,000}{1\,105\,460\,344\,989} \right\}, \left\{ \{10, 4\}, \frac{3\,296\,155\,484\,066\,000}{2\,299\,410\,158\,545\,929} \right\}, \\
& \left\{ \{7, 7\}, \frac{1\,373\,942\,937\,165\,099\,584}{1\,009\,203\,744\,060\,824\,235} \right\}, \left\{ \{4, 10\}, \frac{3\,296\,155\,484\,066\,000}{2\,299\,410\,158\,545\,929} \right\}, \\
& \left\{ \{1, 13\}, \frac{666\,227\,200\,000}{1\,105\,460\,344\,989} \right\}, \left\{ \{15, 0\}, \frac{40\,960\,000}{87\,362\,847} \right\}, \\
& \left\{ \{12, 3\}, \frac{2\,019\,900\,918\,656\,000\,000}{1\,574\,677\,884\,029\,680\,287} \right\}, \left\{ \{9, 6\}, \frac{336\,587\,170\,854\,640}{344\,329\,324\,441\,047} \right\}, \\
& \left\{ \{6, 9\}, \frac{336\,587\,170\,854\,640}{344\,329\,324\,441\,047} \right\}, \left\{ \{3, 12\}, \frac{2\,019\,900\,918\,656\,000\,000}{1\,574\,677\,884\,029\,680\,287} \right\}, \\
& \left\{ \{0, 15\}, \frac{40\,960\,000}{87\,362\,847} \right\}, \left\{ \{14, 2\}, \frac{33\,990\,707\,200\,000}{27\,735\,170\,399\,937} \right\}, \left\{ \{11, 5\}, \frac{24\,952\,742\,393\,600}{30\,181\,854\,293\,019} \right\}, \\
& \left\{ \{8, 8\}, \frac{73\,402\,029\,394\,115}{70\,427\,989\,466\,604} \right\}, \left\{ \{5, 11\}, \frac{24\,952\,742\,393\,600}{30\,181\,854\,293\,019} \right\}, \\
& \left\{ \{2, 14\}, \frac{33\,990\,707\,200\,000}{27\,735\,170\,399\,937} \right\}, \left\{ \{16, 1\}, \frac{142\,082\,048}{112\,763\,651} \right\}, \\
& \left\{ \{13, 4\}, \frac{60\,390\,048\,640\,000}{80\,857\,560\,266\,483} \right\}, \left\{ \{10, 7\}, \frac{7\,563\,017\,340\,800}{10\,025\,036\,778\,141} \right\}, \\
& \left\{ \{7, 10\}, \frac{7\,563\,017\,340\,800}{10\,025\,036\,778\,141} \right\}, \left\{ \{4, 13\}, \frac{60\,390\,048\,640\,000}{80\,857\,560\,266\,483} \right\}, \\
& \left\{ \{1, 16\}, \frac{142\,082\,048}{112\,763\,651} \right\}, \left\{ \{18, 0\}, \frac{1\,048\,576}{693\,693} \right\}, \left\{ \{15, 3\}, \frac{62\,095\,360}{87\,362\,847} \right\}, \\
& \left\{ \{12, 6\}, \frac{7\,156\,640\,000}{11\,108\,522\,007} \right\}, \left\{ \{9, 9\}, \frac{941\,995\,100}{1\,039\,525\,641} \right\}, \left\{ \{6, 12\}, \frac{7\,156\,640\,000}{11\,108\,522\,007} \right\}, \\
& \left\{ \{3, 15\}, \frac{62\,095\,360}{87\,362\,847} \right\}, \left\{ \{0, 18\}, \frac{1\,048\,576}{693\,693} \right\}, \left\{ \{17, 2\}, \frac{262\,144}{363\,363} \right\},
\end{aligned}$$

$$\begin{aligned} & \left\{ \{14, 5\}, \frac{14\,888\,960\,000}{25\,468\,476\,033} \right\}, \left\{ \{11, 8\}, \frac{1\,818\,807\,500}{2\,821\,569\,597} \right\}, \left\{ \{8, 11\}, \frac{1\,818\,807\,500}{2\,821\,569\,597} \right\}, \\ & \left\{ \{5, 14\}, \frac{14\,888\,960\,000}{25\,468\,476\,033} \right\}, \left\{ \{2, 17\}, \frac{262\,144}{363\,363} \right\}, \left\{ \{16, 4\}, \frac{163\,840}{294\,151} \right\}, \\ & \left\{ \{13, 7\}, \frac{48\,000\,000}{87\,272\,339} \right\}, \left\{ \{10, 10\}, \frac{13\,743\,185}{16\,194\,277} \right\}, \left\{ \{7, 13\}, \frac{48\,000\,000}{87\,272\,339} \right\}, \\ & \left\{ \{4, 16\}, \frac{163\,840}{294\,151} \right\}, \left\{ \{15, 6\}, \frac{409\,600}{820\,743} \right\}, \left\{ \{12, 9\}, \frac{7\,100\,000}{12\,097\,701} \right\}, \\ & \left\{ \{9, 12\}, \frac{7\,100\,000}{12\,097\,701} \right\}, \left\{ \{6, 15\}, \frac{409\,600}{820\,743} \right\}, \left\{ \{14, 8\}, \frac{8\,000}{16\,093} \right\}, \left\{ \{11, 11\}, \frac{2\,592}{3\,025} \right\}, \\ & \left\{ \{8, 14\}, \frac{8\,000}{16\,093} \right\}, \left\{ \{13, 10\}, \frac{144}{253} \right\}, \left\{ \{10, 13\}, \frac{144}{253} \right\}, \left\{ \{12, 12\}, 1 \right\}; \end{aligned}$$

ZonalProductCoeffsDynkLab[7 {1, 1}, 7 {1, 1}, 3] =

$$\begin{aligned} & \left\{ \{0, 0\}, \frac{232\,074\,122\,405}{80\,287\,027\,392} \right\}, \left\{ \{1, 1\}, \frac{57\,335\,959\,653}{27\,180\,504\,065} \right\}, \left\{ \{3, 0\}, \frac{144\,383\,831\,171}{212\,007\,931\,707} \right\}, \\ & \left\{ \{0, 3\}, \frac{144\,383\,831\,171}{212\,007\,931\,707} \right\}, \left\{ \{2, 2\}, \frac{8\,874\,392\,930\,459\,075}{4\,810\,318\,631\,810\,692} \right\}, \\ & \left\{ \{4, 1\}, \frac{16\,360\,815\,833\,396\,820}{20\,787\,448\,373\,181\,919} \right\}, \left\{ \{1, 4\}, \frac{16\,360\,815\,833\,396\,820}{20\,787\,448\,373\,181\,919} \right\}, \\ & \left\{ \{6, 0\}, \frac{9\,919\,121\,206\,822\,380}{24\,566\,984\,441\,033\,177} \right\}, \left\{ \{3, 3\}, \frac{54\,837\,696\,089\,307\,851\,300}{31\,617\,708\,975\,609\,698\,799} \right\}, \\ & \left\{ \{0, 6\}, \frac{9\,919\,121\,206\,822\,380}{24\,566\,984\,441\,033\,177} \right\}, \left\{ \{5, 2\}, \frac{2\,951\,914\,809\,704\,187\,950}{3\,513\,078\,775\,067\,744\,311} \right\}, \\ & \left\{ \{2, 5\}, \frac{2\,951\,914\,809\,704\,187\,950}{3\,513\,078\,775\,067\,744\,311} \right\}, \left\{ \{7, 1\}, \frac{3\,796\,206\,881\,623\,380\,000}{7\,099\,858\,503\,458\,588\,153} \right\}, \\ & \left\{ \{4, 4\}, \frac{32\,011\,351\,215\,580\,773\,090\,153\,225}{18\,673\,025\,456\,172\,280\,523\,326\,568} \right\}, \left\{ \{1, 7\}, \frac{3\,796\,206\,881\,623\,380\,000}{7\,099\,858\,503\,458\,588\,153} \right\}, \\ & \left\{ \{9, 0\}, \frac{2\,499\,720\,068\,750\,000\,000}{7\,935\,135\,974\,453\,716\,171} \right\}, \left\{ \{6, 3\}, \frac{5\,291\,769\,863\,786\,738\,051\,650\,000}{5\,892\,449\,466\,501\,917\,193\,112\,667} \right\}, \\ & \left\{ \{3, 6\}, \frac{5\,291\,769\,863\,786\,738\,051\,650\,000}{5\,892\,449\,466\,501\,917\,193\,112\,667} \right\}, \left\{ \{0, 9\}, \frac{2\,499\,720\,068\,750\,000\,000}{7\,935\,135\,974\,453\,716\,171} \right\}, \\ & \left\{ \{8, 2\}, \frac{1\,800\,991\,676\,745\,965\,250\,000}{2\,864\,584\,086\,777\,791\,537\,731} \right\}, \left\{ \{5, 5\}, \frac{1\,129\,924\,433\,830\,578\,361\,912\,566}{633\,073\,083\,177\,891\,929\,838\,551} \right\}, \\ & \left\{ \{2, 8\}, \frac{1\,800\,991\,676\,745\,965\,250\,000}{2\,864\,584\,086\,777\,791\,537\,731} \right\}, \left\{ \{10, 1\}, \frac{143\,235\,377\,550\,000\,000\,000}{315\,241\,310\,985\,115\,815\,157} \right\}, \\ & \left\{ \{7, 4\}, \frac{12\,174\,470\,759\,121\,447\,280\,943\,250}{12\,320\,576\,157\,231\,281\,403\,781\,031} \right\}, \\ & \left\{ \{4, 7\}, \frac{12\,174\,470\,759\,121\,447\,280\,943\,250}{12\,320\,576\,157\,231\,281\,403\,781\,031} \right\}, \left\{ \{1, 10\}, \frac{143\,235\,377\,550\,000\,000\,000}{315\,241\,310\,985\,115\,815\,157} \right\}, \\ & \left\{ \{12, 0\}, \frac{27\,974\,183\,000\,000\,000}{95\,847\,160\,530\,591\,613} \right\}, \left\{ \{9, 3\}, \frac{89\,057\,557\,885\,319\,570\,368}{121\,246\,658\,071\,198\,390\,445} \right\}, \\ & \left\{ \{6, 6\}, \frac{16\,725\,426\,741\,387\,148\,626\,748\,221}{8\,284\,411\,079\,364\,766\,947\,136\,250} \right\}, \left\{ \{3, 9\}, \frac{89\,057\,557\,885\,319\,570\,368}{121\,246\,658\,071\,198\,390\,445} \right\}, \\ & \left\{ \{0, 12\}, \frac{27\,974\,183\,000\,000\,000}{95\,847\,160\,530\,591\,613} \right\}, \left\{ \{11, 2\}, \frac{14\,091\,547\,116\,257\,721\,600}{24\,249\,331\,614\,239\,678\,089} \right\}, \\ & \left\{ \{8, 5\}, \frac{26\,234\,985\,402\,909\,724\,930\,664}{22\,536\,063\,619\,755\,353\,006\,625} \right\}, \left\{ \{5, 8\}, \frac{26\,234\,985\,402\,909\,724\,930\,664}{22\,536\,063\,619\,755\,353\,006\,625} \right\}, \\ & \left\{ \{2, 11\}, \frac{14\,091\,547\,116\,257\,721\,600}{24\,249\,331\,614\,239\,678\,089} \right\}, \left\{ \{13, 1\}, \frac{43\,888\,255\,971\,840\,000}{95\,847\,160\,530\,591\,613} \right\}, \end{aligned}$$

$$\begin{aligned}
& \left\{ \{10, 4\}, \frac{505\,152\,498\,768\,444\,831\,528}{557\,734\,627\,127\,512\,596\,047} \right\}, \left\{ \{7, 7\}, \frac{72\,605\,087\,324\,733\,320\,207\,728}{23\,826\,240\,129\,082\,139\,649\,615} \right\}, \\
& \left\{ \{4, 10\}, \frac{505\,152\,498\,768\,444\,831\,528}{557\,734\,627\,127\,512\,596\,047} \right\}, \left\{ \{1, 13\}, \frac{43\,888\,255\,971\,840\,000}{95\,847\,160\,530\,591\,613} \right\}, \\
& \left\{ \{15, 0\}, \frac{11\,800\,931\,063\,169\,024}{36\,864\,292\,511\,766\,005} \right\}, \left\{ \{12, 3\}, \frac{4\,212\,282\,681\,043\,404\,800\,000}{5\,535\,461\,062\,123\,257\,425\,589} \right\}, \\
& \left\{ \{9, 6\}, \frac{145\,822\,929\,832\,957\,596\,712\,444}{81\,612\,566\,941\,560\,846\,659\,325} \right\}, \left\{ \{6, 9\}, \frac{145\,822\,929\,832\,957\,596\,712\,444}{81\,612\,566\,941\,560\,846\,659\,325} \right\}, \\
& \left\{ \{3, 12\}, \frac{4\,212\,282\,681\,043\,404\,800\,000}{5\,535\,461\,062\,123\,257\,425\,589} \right\}, \left\{ \{0, 15\}, \frac{11\,800\,931\,063\,169\,024}{36\,864\,292\,511\,766\,005} \right\}, \\
& \left\{ \{14, 2\}, \frac{345\,963\,627\,790\,745\,600}{524\,340\,348\,785\,001\,177} \right\}, \left\{ \{11, 5\}, \frac{1\,309\,287\,136\,953\,054\,485\,632}{904\,487\,101\,654\,127\,030\,325} \right\}, \\
& \left\{ \{8, 8\}, \frac{168\,850\,729\,070\,302\,835\,114\,839}{123\,141\,845\,991\,787\,846\,526\,160} \right\}, \left\{ \{5, 11\}, \frac{1\,309\,287\,136\,953\,054\,485\,632}{904\,487\,101\,654\,127\,030\,325} \right\}, \\
& \left\{ \{2, 14\}, \frac{345\,963\,627\,790\,745\,600}{524\,340\,348\,785\,001\,177} \right\}, \left\{ \{16, 1\}, \frac{1\,518\,352\,766\,337\,024}{2\,653\,743\,804\,837\,125} \right\}, \\
& \left\{ \{13, 4\}, \frac{56\,841\,663\,078\,888\,371\,712}{44\,219\,369\,414\,201\,765\,927} \right\}, \left\{ \{10, 7\}, \frac{74\,199\,800\,424\,410\,425\,152}{74\,354\,883\,725\,723\,884\,855} \right\}, \\
& \left\{ \{7, 10\}, \frac{74\,199\,800\,424\,410\,425\,152}{74\,354\,883\,725\,723\,884\,855} \right\}, \left\{ \{4, 13\}, \frac{56\,841\,663\,078\,888\,371\,712}{44\,219\,369\,414\,201\,765\,927} \right\}, \\
& \left\{ \{1, 16\}, \frac{1\,518\,352\,766\,337\,024}{2\,653\,743\,804\,837\,125} \right\}, \left\{ \{18, 0\}, \frac{18\,077\,974\,528}{39\,656\,234\,475} \right\}, \\
& \left\{ \{15, 3\}, \frac{839\,852\,131\,378\,798\,592}{696\,769\,367\,753\,348\,175} \right\}, \left\{ \{12, 6\}, \frac{2\,852\,323\,980\,881\,220\,160}{3\,368\,939\,811\,829\,137\,117} \right\}, \\
& \left\{ \{9, 9\}, \frac{6\,744\,914\,415\,157\,874\,378}{6\,426\,473\,758\,335\,625\,995} \right\}, \left\{ \{6, 12\}, \frac{2\,852\,323\,980\,881\,220\,160}{3\,368\,939\,811\,829\,137\,117} \right\}, \\
& \left\{ \{3, 15\}, \frac{839\,852\,131\,378\,798\,592}{696\,769\,367\,753\,348\,175} \right\}, \left\{ \{0, 18\}, \frac{18\,077\,974\,528}{39\,656\,234\,475} \right\}, \\
& \left\{ \{17, 2\}, \frac{77\,090\,555\,625\,472}{64\,966\,840\,816\,163} \right\}, \left\{ \{14, 5\}, \frac{285\,102\,608\,837\,283\,840}{374\,326\,645\,758\,793\,013} \right\}, \\
& \left\{ \{11, 8\}, \frac{4\,642\,879\,949\,730\,070}{6\,005\,240\,306\,290\,797} \right\}, \left\{ \{8, 11\}, \frac{4\,642\,879\,949\,730\,070}{6\,005\,240\,306\,290\,797} \right\}, \\
& \left\{ \{5, 14\}, \frac{285\,102\,608\,837\,283\,840}{374\,326\,645\,758\,793\,013} \right\}, \left\{ \{2, 17\}, \frac{77\,090\,555\,625\,472}{64\,966\,840\,816\,163} \right\}, \\
& \left\{ \{19, 1\}, \frac{38\,102\,106\,112}{30\,572\,584\,185} \right\}, \left\{ \{16, 4\}, \frac{5\,334\,795\,855\,470\,592}{7\,471\,186\,693\,858\,745} \right\}, \\
& \left\{ \{13, 7\}, \frac{19\,101\,738\,533\,472\,000}{28\,794\,357\,366\,061\,001} \right\}, \left\{ \{10, 10\}, \frac{23\,126\,556\,776\,480\,625\,181}{25\,501\,879\,993\,395\,341\,250} \right\}, \\
& \left\{ \{7, 13\}, \frac{19\,101\,738\,533\,472\,000}{28\,794\,357\,366\,061\,001} \right\}, \left\{ \{4, 16\}, \frac{5\,334\,795\,855\,470\,592}{7\,471\,186\,693\,858\,745} \right\}, \\
& \left\{ \{1, 19\}, \frac{38\,102\,106\,112}{30\,572\,584\,185} \right\}, \left\{ \{21, 0\}, \frac{4\,194\,304}{2\,760\,615} \right\}, \left\{ \{18, 3\}, \frac{5\,317\,904\,564\,224}{7\,625\,100\,764\,853} \right\}, \\
& \left\{ \{15, 6\}, \frac{19\,490\,550\,381\,568}{32\,477\,281\,035\,485} \right\}, \left\{ \{12, 9\}, \frac{39\,354\,732\,631\,256}{59\,676\,661\,122\,507} \right\}, \\
& \left\{ \{9, 12\}, \frac{39\,354\,732\,631\,256}{59\,676\,661\,122\,507} \right\}, \left\{ \{6, 15\}, \frac{19\,490\,550\,381\,568}{32\,477\,281\,035\,485} \right\}, \\
& \left\{ \{3, 18\}, \frac{5\,317\,904\,564\,224}{7\,625\,100\,764\,853} \right\}, \left\{ \{0, 21\}, \frac{4\,194\,304}{2\,760\,615} \right\}, \left\{ \{20, 2\}, \frac{29\,360\,128}{40\,673\,061} \right\}, \\
& \left\{ \{17, 5\}, \frac{69\,868\,273\,664}{123\,903\,701\,493} \right\}, \left\{ \{14, 8\}, \frac{21\,877\,068\,103\,576}{38\,623\,876\,423\,275} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \{11, 11\}, \frac{35\,317\,334\,986\,696}{42\,280\,544\,109\,375} \right\}, \left\{ \{8, 14\}, \frac{21\,877\,068\,103\,576}{38\,623\,876\,423\,275} \right\}, \\
& \left\{ \{5, 17\}, \frac{69\,868\,273\,664}{123\,903\,701\,493} \right\}, \left\{ \{2, 20\}, \frac{29\,360\,128}{40\,673\,061} \right\}, \left\{ \{19, 4\}, \frac{520\,224\,768}{944\,518\,861} \right\}, \\
& \left\{ \{16, 7\}, \frac{1\,013\,227\,520}{1\,974\,903\,073} \right\}, \left\{ \{13, 10\}, \frac{83\,220\,751\,908}{140\,002\,451\,875} \right\}, \\
& \left\{ \{10, 13\}, \frac{83\,220\,751\,908}{140\,002\,451\,875} \right\}, \left\{ \{7, 16\}, \frac{1\,013\,227\,520}{1\,974\,903\,073} \right\}, \\
& \left\{ \{4, 19\}, \frac{520\,224\,768}{944\,518\,861} \right\}, \left\{ \{18, 6\}, \frac{28\,672\,000}{59\,445\,243} \right\}, \left\{ \{15, 9\}, \frac{27\,971\,776}{55\,028\,259} \right\}, \\
& \left\{ \{12, 12\}, \frac{54\,681\,333\,077}{67\,440\,720\,204} \right\}, \left\{ \{9, 15\}, \frac{27\,971\,776}{55\,028\,259} \right\}, \left\{ \{6, 18\}, \frac{28\,672\,000}{59\,445\,243} \right\}, \\
& \left\{ \{17, 8\}, \frac{1\,120\,000}{2\,437\,149} \right\}, \left\{ \{14, 11\}, \frac{65\,434\,208}{116\,698\,725} \right\}, \left\{ \{11, 14\}, \frac{65\,434\,208}{116\,698\,725} \right\}, \\
& \left\{ \{8, 17\}, \frac{1\,120\,000}{2\,437\,149} \right\}, \left\{ \{16, 10\}, \frac{508\,032}{1\,068\,925} \right\}, \left\{ \{13, 13\}, \frac{4116}{4901} \right\}, \\
& \left\{ \{10, 16\}, \frac{508\,032}{1\,068\,925} \right\}, \left\{ \{15, 12\}, \frac{196}{351} \right\}, \left\{ \{12, 15\}, \frac{196}{351} \right\}, \left\{ \{14, 14\}, 1 \right\};
\end{aligned}$$

ZonalPproductCoeffsDynkLab[8 {1, 1}, 8 {1, 1}, 3] =

$$\begin{aligned}
& \left\{ \{0, 0\}, \frac{154\,341\,336\,769}{52\,874\,979\,300} \right\}, \left\{ \{1, 1\}, \frac{389\,914\,956\,048}{183\,593\,678\,125} \right\}, \left\{ \{3, 0\}, \frac{6\,840\,613\,264}{10\,014\,200\,625} \right\}, \\
& \left\{ \{0, 3\}, \frac{6\,840\,613\,264}{10\,014\,200\,625} \right\}, \left\{ \{2, 2\}, \frac{1\,054\,430\,940\,199\,436}{571\,380\,245\,060\,625} \right\}, \\
& \left\{ \{4, 1\}, \frac{15\,623\,960\,694\,976}{19\,952\,960\,938\,625} \right\}, \left\{ \{1, 4\}, \frac{15\,623\,960\,694\,976}{19\,952\,960\,938\,625} \right\}, \\
& \left\{ \{6, 0\}, \frac{219\,693\,135\,586\,624}{551\,790\,065\,229\,975} \right\}, \left\{ \{3, 3\}, \frac{31\,241\,286\,589\,921\,856}{18\,209\,072\,152\,589\,175} \right\}, \\
& \left\{ \{0, 6\}, \frac{219\,693\,135\,586\,624}{551\,790\,065\,229\,975} \right\}, \left\{ \{5, 2\}, \frac{1\,973\,170\,137\,313\,312}{2\,391\,090\,282\,663\,225} \right\}, \\
& \left\{ \{2, 5\}, \frac{1\,973\,170\,137\,313\,312}{2\,391\,090\,282\,663\,225} \right\}, \left\{ \{7, 1\}, \frac{9\,202\,549\,729\,168\,896}{17\,718\,592\,094\,606\,975} \right\}, \\
& \left\{ \{4, 4\}, \frac{10\,966\,978\,990\,642\,028\,634}{6\,587\,772\,540\,774\,873\,305} \right\}, \left\{ \{1, 7\}, \frac{9\,202\,549\,729\,168\,896}{17\,718\,592\,094\,606\,975} \right\}, \\
& \left\{ \{9, 0\}, \frac{55\,056\,772\,839\,424}{182\,798\,144\,686\,443} \right\}, \left\{ \{6, 3\}, \frac{43\,111\,391\,298\,475\,281\,152}{49\,892\,011\,619\,994\,320\,205} \right\}, \\
& \left\{ \{3, 6\}, \frac{43\,111\,391\,298\,475\,281\,152}{49\,892\,011\,619\,994\,320\,205} \right\}, \left\{ \{0, 9\}, \frac{55\,056\,772\,839\,424}{182\,798\,144\,686\,443} \right\}, \\
& \left\{ \{8, 2\}, \frac{10\,614\,803\,415\,235\,328}{17\,786\,813\,411\,762\,681} \right\}, \left\{ \{5, 5\}, \frac{3\,943\,542\,320\,446\,187\,424}{2\,351\,008\,232\,057\,148\,625} \right\}, \\
& \left\{ \{2, 8\}, \frac{10\,614\,803\,415\,235\,328}{17\,786\,813\,411\,762\,681} \right\}, \left\{ \{10, 1\}, \frac{80\,603\,115\,436\,916\,736}{190\,470\,127\,425\,555\,835} \right\}, \\
& \left\{ \{7, 4\}, \frac{734\,481\,611\,138\,762\,075\,232}{799\,784\,065\,059\,908\,951\,165} \right\}, \left\{ \{4, 7\}, \frac{734\,481\,611\,138\,762\,075\,232}{799\,784\,065\,059\,908\,951\,165} \right\}, \\
& \left\{ \{1, 10\}, \frac{80\,603\,115\,436\,916\,736}{190\,470\,127\,425\,555\,835} \right\}, \left\{ \{12, 0\}, \frac{63\,205\,175\,219\,658\,752}{238\,883\,939\,078\,874\,375} \right\}, \\
& \left\{ \{9, 3\}, \frac{143\,823\,785\,341\,110\,969\,344}{214\,278\,893\,353\,750\,314\,375} \right\}, \left\{ \{6, 6\}, \frac{1\,366\,943\,406\,384\,038\,971\,420\,968}{773\,056\,477\,624\,811\,133\,203\,125} \right\}, \\
& \left\{ \{3, 9\}, \frac{143\,823\,785\,341\,110\,969\,344}{214\,278\,893\,353\,750\,314\,375} \right\}, \left\{ \{0, 12\}, \frac{63\,205\,175\,219\,658\,752}{238\,883\,939\,078\,874\,375} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \{11, 2\}, \frac{2\,760\,519\,171\,895\,845\,326\,848}{5\,321\,139\,742\,981\,926\,703\,125} \right\}, \left\{ \{8, 5\}, \frac{357\,410\,362\,128\,277\,999\,257\,472}{354\,696\,501\,498\,442\,755\,234\,375} \right\}, \\
& \left\{ \{5, 8\}, \frac{357\,410\,362\,128\,277\,999\,257\,472}{354\,696\,501\,498\,442\,755\,234\,375} \right\}, \left\{ \{2, 11\}, \frac{2\,760\,519\,171\,895\,845\,326\,848}{5\,321\,139\,742\,981\,926\,703\,125} \right\}, \\
& \left\{ \{13, 1\}, \frac{4\,208\,668\,028\,895\,232}{10\,638\,555\,991\,366\,875} \right\}, \left\{ \{10, 4\}, \frac{328\,986\,744\,969\,527\,135\,808\,128}{428\,772\,065\,600\,854\,379\,064\,375} \right\}, \\
& \left\{ \{7, 7\}, \frac{9\,039\,263\,067\,056\,029\,101\,576\,448}{4\,483\,727\,570\,223\,904\,572\,578\,125} \right\}, \\
& \left\{ \{4, 10\}, \frac{328\,986\,744\,969\,527\,135\,808\,128}{428\,772\,065\,600\,854\,379\,064\,375} \right\}, \left\{ \{1, 13\}, \frac{4\,208\,668\,028\,895\,232}{10\,638\,555\,991\,366\,875} \right\}, \\
& \left\{ \{15, 0\}, \frac{110\,561\,348\,930\,240\,512}{419\,813\,786\,428\,554\,375} \right\}, \left\{ \{12, 3\}, \frac{1\,427\,751\,805\,691\,101\,184}{2\,282\,260\,402\,584\,322\,875} \right\}, \\
& \left\{ \{9, 6\}, \frac{4\,947\,372\,456\,680\,013\,487\,706\,432}{4\,182\,776\,444\,148\,194\,816\,746\,875} \right\}, \\
& \left\{ \{6, 9\}, \frac{4\,947\,372\,456\,680\,013\,487\,706\,432}{4\,182\,776\,444\,148\,194\,816\,746\,875} \right\}, \left\{ \{3, 12\}, \frac{1\,427\,751\,805\,691\,101\,184}{2\,282\,260\,402\,584\,322\,875} \right\}, \\
& \left\{ \{0, 15\}, \frac{110\,561\,348\,930\,240\,512}{419\,813\,786\,428\,554\,375} \right\}, \left\{ \{14, 2\}, \frac{3\,413\,124\,396\,089\,344}{6\,563\,363\,249\,262\,105} \right\}, \\
& \left\{ \{11, 5\}, \frac{130\,905\,685\,925\,150\,947\,328}{140\,590\,224\,146\,125\,771\,875} \right\}, \left\{ \{8, 8\}, \frac{1\,643\,205\,994\,104\,086\,903\,573}{537\,762\,569\,366\,842\,117\,155} \right\}, \\
& \left\{ \{5, 11\}, \frac{130\,905\,685\,925\,150\,947\,328}{140\,590\,224\,146\,125\,771\,875} \right\}, \left\{ \{2, 14\}, \frac{3\,413\,124\,396\,089\,344}{6\,563\,363\,249\,262\,105} \right\}, \\
& \left\{ \{16, 1\}, \frac{1\,113\,238\,409\,918\,283\,776}{2\,629\,552\,583\,838\,984\,375} \right\}, \left\{ \{13, 4\}, \frac{43\,266\,729\,705\,070\,592}{54\,694\,693\,743\,850\,875} \right\}, \\
& \left\{ \{10, 7\}, \frac{2\,103\,382\,330\,606\,988\,772\,352}{1\,168\,308\,039\,815\,707\,171\,875} \right\}, \left\{ \{7, 10\}, \frac{2\,103\,382\,330\,606\,988\,772\,352}{1\,168\,308\,039\,815\,707\,171\,875} \right\}, \\
& \left\{ \{4, 13\}, \frac{43\,266\,729\,705\,070\,592}{54\,694\,693\,743\,850\,875} \right\}, \left\{ \{1, 16\}, \frac{1\,113\,238\,409\,918\,283\,776}{2\,629\,552\,583\,838\,984\,375} \right\}, \\
& \left\{ \{18, 0\}, \frac{99\,486\,382\,686\,208}{328\,215\,425\,484\,375} \right\}, \left\{ \{15, 3\}, \frac{4\,704\,943\,175\,400\,030\,208}{6\,759\,334\,115\,510\,315\,625} \right\}, \\
& \left\{ \{12, 6\}, \frac{1\,382\,345\,077\,849\,673\,491\,456}{947\,703\,332\,806\,880\,203\,125} \right\}, \left\{ \{9, 9\}, \frac{5\,031\,239\,250\,509\,151\,136}{3\,653\,369\,721\,778\,651\,875} \right\}, \\
& \left\{ \{6, 12\}, \frac{1\,382\,345\,077\,849\,673\,491\,456}{947\,703\,332\,806\,880\,203\,125} \right\}, \left\{ \{3, 15\}, \frac{4\,704\,943\,175\,400\,030\,208}{6\,759\,334\,115\,510\,315\,625} \right\}, \\
& \left\{ \{0, 18\}, \frac{99\,486\,382\,686\,208}{328\,215\,425\,484\,375} \right\}, \left\{ \{17, 2\}, \frac{1\,911\,005\,672\,262\,074\,368}{3\,072\,424\,597\,959\,234\,375} \right\}, \\
& \left\{ \{14, 5\}, \frac{52\,839\,083\,684\,724\,736}{40\,965\,661\,306\,123\,125} \right\}, \left\{ \{11, 8\}, \frac{3\,640\,704\,977\,262\,251\,936}{3\,592\,858\,422\,039\,046\,875} \right\}, \\
& \left\{ \{8, 11\}, \frac{3\,640\,704\,977\,262\,251\,936}{3\,592\,858\,422\,039\,046\,875} \right\}, \left\{ \{5, 14\}, \frac{52\,839\,083\,684\,724\,736}{40\,965\,661\,306\,123\,125} \right\}, \\
& \left\{ \{2, 17\}, \frac{1\,911\,005\,672\,262\,074\,368}{3\,072\,424\,597\,959\,234\,375} \right\}, \left\{ \{19, 1\}, \frac{12\,680\,404\,402\,176}{23\,003\,132\,384\,375} \right\}, \\
& \left\{ \{16, 4\}, \frac{158\,437\,751\,260\,250\,112}{132\,331\,499\,855\,537\,125} \right\}, \left\{ \{13, 7\}, \frac{78\,821\,591\,726\,108\,672}{91\,384\,936\,759\,813\,125} \right\}, \\
& \left\{ \{10, 10\}, \frac{4\,585\,830\,448\,773\,460\,376}{4\,349\,249\,668\,784\,109\,375} \right\}, \left\{ \{7, 13\}, \frac{78\,821\,591\,726\,108\,672}{91\,384\,936\,759\,813\,125} \right\}, \\
& \left\{ \{4, 16\}, \frac{158\,437\,751\,260\,250\,112}{132\,331\,499\,855\,537\,125} \right\}, \left\{ \{1, 19\}, \frac{12\,680\,404\,402\,176}{23\,003\,132\,384\,375} \right\}, \\
& \left\{ \{21, 0\}, \frac{3\,154\,116\,608}{7\,062\,193\,125} \right\}, \left\{ \{18, 3\}, \frac{1\,106\,423\,767\,585\,783\,808}{958\,234\,124\,733\,546\,375} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \{15, 6\}, \frac{472\,154\,338\,410\,496}{609\,406\,531\,826\,625} \right\}, \left\{ \{12, 9\}, \frac{4\,125\,140\,218\,059\,392}{5\,236\,554\,785\,158\,125} \right\}, \\
& \left\{ \{9, 12\}, \frac{4\,125\,140\,218\,059\,392}{5\,236\,554\,785\,158\,125} \right\}, \left\{ \{6, 15\}, \frac{472\,154\,338\,410\,496}{609\,406\,531\,826\,625} \right\}, \\
& \left\{ \{3, 18\}, \frac{1\,106\,423\,767\,585\,783\,808}{958\,234\,124\,733\,546\,375} \right\}, \left\{ \{0, 21\}, \frac{3\,154\,116\,608}{7\,062\,193\,125} \right\}, \\
& \left\{ \{20, 2\}, \frac{1\,791\,320\,129\,536}{1\,544\,777\,983\,125} \right\}, \left\{ \{17, 5\}, \frac{4\,466\,304\,403\,111\,936}{6\,194\,006\,130\,121\,875} \right\}, \\
& \left\{ \{14, 8\}, \frac{1\,687\,944\,916\,820\,608}{2\,485\,942\,246\,198\,125} \right\}, \left\{ \{11, 11\}, \frac{4\,634\,402\,544\,815\,838\,592}{5\,104\,156\,070\,126\,953\,125} \right\}, \\
& \left\{ \{8, 14\}, \frac{1\,687\,944\,916\,820\,608}{2\,485\,942\,246\,198\,125} \right\}, \left\{ \{5, 17\}, \frac{4\,466\,304\,403\,111\,936}{6\,194\,006\,130\,121\,875} \right\}, \\
& \left\{ \{2, 20\}, \frac{1\,791\,320\,129\,536}{1\,544\,777\,983\,125} \right\}, \left\{ \{22, 1\}, \frac{981\,400\,027\,136}{793\,729\,096\,875} \right\}, \\
& \left\{ \{19, 4\}, \frac{356\,044\,091\,949\,056}{514\,350\,040\,114\,625} \right\}, \left\{ \{16, 7\}, \frac{677\,735\,220\,772\,864}{1\,103\,997\,340\,265\,625} \right\}, \\
& \left\{ \{13, 10\}, \frac{2\,154\,095\,729\,438\,912}{3\,206\,396\,928\,515\,625} \right\}, \left\{ \{10, 13\}, \frac{2\,154\,095\,729\,438\,912}{3\,206\,396\,928\,515\,625} \right\}, \\
& \left\{ \{7, 16\}, \frac{677\,735\,220\,772\,864}{1\,103\,997\,340\,265\,625} \right\}, \left\{ \{4, 19\}, \frac{356\,044\,091\,949\,056}{514\,350\,040\,114\,625} \right\}, \\
& \left\{ \{1, 22\}, \frac{981\,400\,027\,136}{793\,729\,096\,875} \right\}, \left\{ \{24, 0\}, \frac{1\,073\,741\,824}{703\,956\,825} \right\}, \left\{ \{21, 3\}, \frac{64\,290\,291\,712}{93\,405\,180\,375} \right\}, \\
& \left\{ \{18, 6\}, \frac{757\,129\,277\,341\,696}{1\,321\,216\,276\,404\,375} \right\}, \left\{ \{15, 9\}, \frac{14\,337\,472\,850\,944}{24\,703\,879\,516\,875} \right\}, \\
& \left\{ \{12, 12\}, \frac{17\,078\,696\,876\,824\,906\,148}{20\,604\,457\,223\,888\,484\,375} \right\}, \left\{ \{9, 15\}, \frac{14\,337\,472\,850\,944}{24\,703\,879\,516\,875} \right\}, \\
& \left\{ \{6, 18\}, \frac{757\,129\,277\,341\,696}{1\,321\,216\,276\,404\,375} \right\}, \left\{ \{3, 21\}, \frac{64\,290\,291\,712}{93\,405\,180\,375} \right\}, \\
& \left\{ \{0, 24\}, \frac{1\,073\,741\,824}{703\,956\,825} \right\}, \left\{ \{23, 2\}, \frac{1\,073\,741\,824}{1\,486\,131\,075} \right\}, \left\{ \{20, 5\}, \frac{260\,835\,377\,152}{473\,731\,914\,825} \right\}, \\
& \left\{ \{17, 8\}, \frac{63\,042\,670\,592}{119\,891\,113\,875} \right\}, \left\{ \{14, 11\}, \frac{8\,530\,479\,630\,848}{14\,155\,330\,921\,875} \right\}, \\
& \left\{ \{11, 14\}, \frac{8\,530\,479\,630\,848}{14\,155\,330\,921\,875} \right\}, \left\{ \{8, 17\}, \frac{63\,042\,670\,592}{119\,891\,113\,875} \right\}, \\
& \left\{ \{5, 20\}, \frac{260\,835\,377\,152}{473\,731\,914\,825} \right\}, \left\{ \{2, 23\}, \frac{1\,073\,741\,824}{1\,486\,131\,075} \right\}, \left\{ \{22, 4\}, \frac{234\,881\,024}{429\,326\,755} \right\}, \\
& \left\{ \{19, 7\}, \frac{1\,897\,398\,272}{3\,860\,665\,575} \right\}, \left\{ \{16, 10\}, \frac{353\,271\,559\,168}{680\,353\,756\,875} \right\}, \left\{ \{13, 13\}, \frac{7\,629\,326\,565\,056}{9\,662\,906\,556\,875} \right\}, \\
& \left\{ \{10, 16\}, \frac{353\,271\,559\,168}{680\,353\,756\,875} \right\}, \left\{ \{7, 19\}, \frac{1\,897\,398\,272}{3\,860\,665\,575} \right\}, \left\{ \{4, 22\}, \frac{234\,881\,024}{429\,326\,755} \right\}, \\
& \left\{ \{21, 6\}, \frac{58\,720\,256}{124\,294\,599} \right\}, \left\{ \{18, 9\}, \frac{1\,395\,064\,832}{2\,967\,432\,975} \right\}, \left\{ \{15, 12\}, \frac{105\,939\,184\,064}{188\,737\,745\,625} \right\}, \\
& \left\{ \{12, 15\}, \frac{105\,939\,184\,064}{188\,737\,745\,625} \right\}, \left\{ \{9, 18\}, \frac{1\,395\,064\,832}{2\,967\,432\,975} \right\}, \\
& \left\{ \{6, 21\}, \frac{58\,720\,256}{124\,294\,599} \right\}, \left\{ \{20, 8\}, \frac{3\,211\,264}{7\,311\,447} \right\}, \left\{ \{17, 11\}, \frac{565\,784\,576}{1\,176\,946\,875} \right\}, \\
& \left\{ \{14, 14\}, \frac{651\,808\,816}{830\,722\,035} \right\}, \left\{ \{11, 17\}, \frac{565\,784\,576}{1\,176\,946\,875} \right\}, \left\{ \{8, 20\}, \frac{3\,211\,264}{7\,311\,447} \right\}, \\
& \left\{ \{19, 10\}, \frac{1\,605\,632}{3\,700\,125} \right\}, \left\{ \{16, 13\}, \frac{24\,561\,152}{45\,287\,125} \right\}, \left\{ \{13, 16\}, \frac{24\,561\,152}{45\,287\,125} \right\},
\end{aligned}$$

$$\left\{ \{10, 19\}, \frac{1\,605\,632}{3\,700\,125} \right\}, \left\{ \{18, 12\}, \frac{351\,232}{763\,425} \right\}, \left\{ \{15, 15\}, \frac{2048}{2475} \right\},$$

$$\left\{ \{12, 18\}, \frac{351\,232}{763\,425} \right\}, \left\{ \{17, 14\}, \frac{256}{465} \right\}, \left\{ \{14, 17\}, \frac{256}{465} \right\}, \left\{ \{16, 16\}, 1 \right\};$$

History and citation

The package SymPol\$Package was written by R.C. during the academic year 2017/2018.

It was made public on the R.C. web site in September 2018.

If you use the SymPol\$Package or the results obtained with it (for instance the results given above) in a scientific publication or talk, please give proper academic credit to the author.

Thank you.