Biases in the Cosmological Distance-Redshift Relation: Implications for cosmic flows, supernovae and the CMB

Nick Kaiser
Institute for Astronomy, U. Hawaii

Quy Nhon talk 2016/07/05
Introduction

• Several papers in recent years doing relativistic perturbation theory analysis of cosmological distance-redshift relation $D(z)$

• Three areas:
  • “Doppler lensing” as a new probe of structure
    • related to SN Ia error analysis
  • Bias in $H_0$ at low-$z$
    • 2nd order effect
  • Bias in distance to the “cosmic photosphere”
    • claimed big effect on CMB cosmology
    • would also impact SN cosmology

• But things are not quite as they might appear…..
I) Doppler-lensing and anti-lensing
Cosmology with Doppler lensing

David J. Bacon$^{1}$, Sambatra Andrianomena$^{2}$, Chris Clarkson$^{2}$, Krzysztof Bolejko$^{3}$, Roy Maartens$^{1,4}$

$^{1}$Institute of Cosmology & Gravitation, University of Portsmouth, Dennis Sciama Building, Portsmouth, PO1 3FX
$^{2}$Astrophysics, Cosmology Gravity Centre, and Department of Mathematics and Applied Mathematics, University of Cape Town, Cape Town, 7701, South Africa.
$^{3}$Sydney Institute for Astronomy, The University of Sydney, NSW 2006, Australia
$^{4}$Physics Department, University of the Western Cape, Cape Town 7535, South Africa

Accepted —-. Received —-; in original form —-.

ABSTRACT

Doppler lensing is the apparent change in object size and magnitude due to peculiar velocities. Objects falling into an overdensity appear larger on its near side, and smaller on its far side, than typical objects at the same redshifts. This effect dominates over the usual gravitational lensing magnification at low redshift. Doppler lensing is a promising new probe of cosmology, and we explore in detail how to utilize the effect with forthcoming surveys. We present cosmological simulations of the Doppler and gravitational lensing effects based on the Millennium simulation. We show that Doppler lensing can be detected around stacked voids or unvirialised over-densities. New power spectra and correlation functions are proposed which are designed to be sensitive to Doppler lensing. We consider the impact of gravitational lensing and intrinsic size correlations on these quantities. We compute the correlation functions and forecast the errors for realistic forthcoming surveys, providing predictions for constraints on cosmological parameters. Finally, we demonstrate how we can make 3-D potential maps of large volumes of the Universe using Doppler lensing.

Key words: Cosmology: theory; cosmology: observations; gravitational lensing: weak
Cosmology with Doppler lensing

David J. Bacon\textsuperscript{1}, Sambatra Andrianomena\textsuperscript{2}, Chris Clarkson\textsuperscript{2}, Krzysztof Bolejko\textsuperscript{3}, Roy Maartens\textsuperscript{1,4}

1 INTRODUCTION

Light rays from distant sources are focused by overdensities (or defocused by underdensities) along the line of sight, leading to apparent magnification (or demagnification) of images. But besides this \textit{gravitational lensing}, there is a further effect which appears to magnify or demagnify the images of objects in the Universe. This \textit{Doppler lensing} effect arises from the peculiar velocity of the source, and was first highlighted and investigated in general by Bonvin (2008) (see also Bonvin et al. (2006)). Bolejko et al. (2013) then showed that the effect can dominate over gravitational lensing, and even reverse its effect, leading to an ‘anti-lensing’ phenomenon. Doppler lensing gives a new window into the peculiar velocity field in addition to the usual redshift space distortion measurements.

The effect is a consequence of the distortion introduced by mapping from redshift-space to real space, as illustrated in Figure 1. Imagine we have three spherical galaxies with the same physical size, and (as an extreme case) the same measured redshift. Finally, galaxy C has a peculiar velocity away from us so that its density while B and C are falling towards the centre.

\textbf{Figure 1.} Three spherical galaxies of the same physical size and same observed redshift. A is at the centre of a spherical overdensity while B and C are falling towards the centre.
Antilensing: The Bright Side of Voids

Krzysztof Bolejko,\textsuperscript{1} Chris Clarkson,\textsuperscript{2} Roy Maartens,\textsuperscript{3,4} David Bacon,\textsuperscript{4} Nikolai Meures,\textsuperscript{4} and Emma Beynon\textsuperscript{4}

\textsuperscript{1}Sydney Institute for Astronomy, The University of Sydney, Sydney, New South Wales 2006, Australia
\textsuperscript{2}Centre for Astrophysics, Cosmology and Gravitation and, Department of Mathematics and Applied Mathematics, University of Cape Town, Cape Town 7701, South Africa
\textsuperscript{3}Physics Department, University of the Western Cape, Cape Town 7535, South Africa
\textsuperscript{4}Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth PO1 3FX, United Kingdom

(Received 1 October 2012; published 10 January 2013)

More than half of the volume of our Universe is occupied by cosmic voids. The lensing magnification effect from those underdense regions is generally thought to give a small dimming contribution: objects on the far side of a void are supposed to be observed as slightly smaller than if the void were not there, which together with conservation of surface brightness implies net reduction in photons received. This is predicted by the usual weak lensing integral of the density contrast along the line of sight. We show that this standard effect is swamped at low redshifts by a relativistic Doppler term that is typically neglected. Contrary to the usual expectation, objects on the far side of a void are brighter than they would be otherwise. Thus the local dynamics of matter in and near the void is crucial and is only captured by the full relativistic lensing convergence. There are also significant nonlinear corrections to the relativistic linear theory, which we show actually underpredicts the effect. We use exact solutions to estimate that these can be more than 20\% for deep voids. This remains an important source of systematic errors for weak lensing density reconstruction in galaxy surveys and for supernovae observations, and may be the cause of the reported extra scatter of field supernovae located on the edge of voids compared to those in clusters.

DOI: 10.1103/PhysRevLett.110.021302 PACS numbers: 98.62.Sb
We study the effect of peculiar motion in weak gravitational lensing. We derive a fully relativistic formula for the cosmic shear and the convergence in a perturbed Friedmann universe. We find a new contribution related to galaxies’ peculiar velocities. This contribution does not affect cosmic shear in a measurable way, since it is of second order in the velocity. However, its effect on the convergence (and consequently on the magnification, which is a measurable quantity) is important, especially for redshifts $z \leq 1$. As a consequence, peculiar motion modifies also the relation between the shear and the convergence.

FIG. 1. A light beam emitted by a galaxy at spacetime position $S$ and received by an observer at $O$. At the observer position, the plane normal to the observer four-velocity is indicated.
Fluctuations of the luminosity distance

Camille Bonvin,* Ruth Durrer,† and M. Alice Gasparini‡

Département de Physique Théorique, Université de Genève, 24 quai Ernest Ansermet, CH-1211 Genève 4, Switzerland

(Received 7 November 2005; published 27 January 2006)

We derive an expression for the luminosity distance in a perturbed Friedmann universe. We define the correlation function and the power spectrum of the luminosity distance fluctuations and express them in terms of the initial spectrum of the Bardeen potential. We present semianalytical results for the case of a pure CDM (cold dark matter) universe. We argue that the luminosity distance power spectrum represents a new observational tool which can be used to determine cosmological parameters. In addition, our results shed some light into the debate whether second order small scale fluctuations can mimic an accelerating universe.

\begin{align}
\tilde{d}_L(z_s, \mathbf{n}) &= (1 + z_s)(\eta_0 - \eta_s) \left\{ 1 - \frac{1}{(\eta_0 - \eta_s)\mathcal{H}_s} \mathbf{v}_o \cdot \mathbf{n} - \left(1 - \frac{1}{(\eta_0 - \eta_s)\mathcal{H}_s}\right) \mathbf{v}_s \cdot \mathbf{n} \\
&- \left(2 - \frac{1}{(\eta_0 - \eta_s)\mathcal{H}_s}\right) \Psi_s + \left(1 - \frac{1}{(\eta_0 - \eta_s)\mathcal{H}_s}\right) \Psi_o \\
&+ \frac{2}{(\eta_0 - \eta_s)} \int_{\eta_s}^{\eta_o} d\eta \Psi + \frac{2}{(\eta_0 - \eta_s)\mathcal{H}_s} \int_{\eta_s}^{\eta_o} d\eta \dot{\Psi} - 2 \int_{\eta_s}^{\eta_o} d\eta \frac{(\eta - \eta_s)}{(\eta_0 - \eta_s)} \dot{\Psi} + \int_{\eta_s}^{\eta_o} d\eta \frac{(\eta - \eta_s)(\eta_0 - \eta)}{(\eta_0 - \eta_s)} \dddot{\Psi} \\
&- \int_{\eta_s}^{\eta_o} d\eta \frac{(\eta - \eta_s)(\eta_0 - \eta)}{(\eta_0 - \eta_s)} \nabla^2 \Psi \right\}. \tag{59}
\end{align}

\[\mathcal{H} \equiv \dot{a}/a = a^{-1} \frac{da}{d\eta} \equiv H a\]

We now consider a Friedmann universe with scalar perturbations. In longitudinal (or Newtonian) gauge the metric is given by

\[\tilde{g}_{\mu\nu} dx^\mu dx^\nu = a^2\left[-(1 + 2\Psi) d\eta^2 + (1 - 2\Phi) \gamma_{ij} dx^i dx^j\right].\]
What’s new in Doppler (or anti) lensing?
Motion of the Galaxy and the Local Group determined from the velocity
anisotropy of distant Sc I galaxies. II. The analysis for the motion

Vera C. Rubin,*† Norbert Thonnard,‡ and W. Kent Ford Jr.†‡

Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, DC 20015

Morton S. Roberts

National Radio Astronomy Observatory, Green Bank, West Virginia
(Received 18 May 1976; revised 28 June 1976)

For an all-sky sample of 96 Sc I–Sc II galaxies, \(3500 < V_c < 6500\) km sec\(^{-1}\), for which radial velocities and
magnitudes have been obtained, the quantity \(HM = \log V_c - 0.2m_c = \log H - 0.2M_0 - 5\) varies across the
sky. The form of the variation is consistent with a motion of the Sun of \(V_\odot = 600 \pm 125\) km sec\(^{-1}\) toward
\(\alpha = 32^\circ \pm 20^\circ, \delta = +53^\circ \pm 11^\circ, (l = 135^\circ, b = -8^\circ)\), corresponding to a motion of the Galaxy and the Local Group
of galaxies of \(V_{GM} = 454 \pm 125\) km sec\(^{-1}\) toward \(l = 163^\circ, b = -11^\circ\). The mean error arises from the scatter
of the data and does not take possible systematic errors into account. The Galaxy is moving almost edge-on;
the leading edge is in the anticenter direction. Alternative explanations which might account for the observed
anisotropy are examined: (1) that apex galaxies are intrinsically fainter than antapex galaxies; (2) that apex
(anticenter) galaxies are more obscured; (3) that the Hubble constant varies by 20% across the sky. Each of
these explanations is shown to be less likely than a motion of the observer. It is also demonstrated that a
Malmquist bias does not produce the observed anisotropy. Additionally, undetected systematic errors in the
magnitude system are probably no larger than \(0.1\), so can account for no more than one-fourth of the observed
effect. Moreover, 22 nearer galaxies, \(1600 < V_c < 3500\) km sec\(^{-1}\) exhibit a more pronounced anisotropy in HM
than the sample \(3500 < V_c < 6500\) km sec\(^{-1}\). Of the explanations considered above, only a motion of our
Galaxy is consistent with the variation in HM observed at both distances. Support for this explanation comes
also from a sample of E and S0 galaxies, \(3500 < V_c < 6500\) km sec\(^{-1}\) (Sandage 1975). After correction for
the motion of the observer, the random motions of these Sc galaxies are small, \(\sigma(\Delta V)_{\text{radial}} < 200\) km sec\(^{-1}\),
and the Hubble flow is uniform, \(\sigma(\Delta H / H) < 0.04\).
THE VELOCITY FIELD OF BRIGHT NEARBY GALAXIES. II. LUMINOSITY FUNCTIONS FOR VARIOUS HUBBLE TYPES AND LUMINOSITY CLASSES: THE PECULIAR MOTION OF THE LOCAL GROUP RELATIVE TO THE VIRGO CLUSTER

G. A. TAMMANN, Amos Yahil, and Allan Sandage

Received 1979 April 30; accepted 1979 June 25

ABSTRACT

Galaxies of all morphological classes in the Revised Shapley-Ames (RSA) catalog show the same type of correlation between absolute magnitude and redshift as was found for E and S0 galaxies in Paper I of this series. Mean absolute magnitudes become brighter with increasing redshift due to the bias caused by a broad luminosity function for objects in a magnitude limited sample. Using the methods developed in Paper I, we have computed luminosity and catalog completeness functions for five separate luminosity classes \( L_C \) of Sc galaxies, two of Sb, and one of Sa. These functions have been obtained with and without applying corrections for internal absorption caused by different inclinations and, together with the E and S0 functions determined in Paper I, cover all classes of the RSA.

The correlations of \( \langle M \rangle \) on log \( v_0 \) for the different classes are used to calculate the photometrically expected velocity \( \langle v \rangle_p \) of the Virgo cluster. The method uses only the observed distribution of apparent magnitudes of cluster members of given morphological types and of \( L_C \) classes. Comparison of \( \langle v \rangle_p \) with the directly observed systemic velocity \( \langle v \rangle_0 \) gives a peculiar motion of Virgo (relative to the velocity frame of the RSA) of \( 60 \pm 132 \) km s\(^{-1}\). Such a small peculiar motion, although not particularly well determined by this new method, nevertheless agrees with previous determinations by Sandage and Tammann, Kormendy, and others using different methods. Assuming that there is no uniformly distributed unseen mass that would significantly reduce the observed density contrast toward Virgo, this result requires that \( q_0 < 0.1 \) by a wide margin.

An estimate of the global mass-to-luminosity ratio follows by summing the luminosity density for each morphological type and luminosity class. The total volume emissivity contributed by all galaxies of all types and luminosities (averaged over the volume spanned by the RSA) is \( \mathcal{L} = 11 \times 10^7 \) solar B luminosities per Mpc\(^3\), as found by integrating the present individual luminosity functions and summing. Correction to the lower global value by a factor of 1.5 to account for local density enhancement gives the global ratio of total mass to visible B luminosity (in solar units) of \( \langle M \rangle / \langle L \rangle = 1900q_0 \). An open universe with \( q_0 = 0.02 \) (Paper IV) would yield \( \langle M \rangle / \langle L \rangle = 40 \), in agreement with recent estimates in galaxies, and would imply that most of the matter in the Universe is in galaxies. By comparison, an \( \langle M \rangle / \langle L \rangle \) of \( \sim 1000 \) is required to close the Universe.
What’s new in Doppler (or anti) lensing?

• Long history of observations
  • Rubin-Ford effect (1976)
  • …… Tully-Fisher … Faber-Jackson … Dn-sigma …
  • Cosmic flows II; 6df survey…
• What’s new in theory?
The magnitude–redshift relation in a perturbed Friedmann universe

Misao Sasaki Research Institute for Theoretical Physics, Hiroshima University, Takehara, Hiroshima 725, Japan

Accepted 1987 April 30. Received 1987 April 29; in original form 1987 March 2

Summary. A general formula for the magnitude–redshift relation in a linearly perturbed Friedmann universe is derived. The formula does not assume any specific gauge condition, but the gauge-invariance of it is explicitly shown. Then the application of the formula to the spatially flat background model is considered and the implications are discussed.

\[ ds^2 = a(\eta)^2 \quad ds^2 = a(\eta)^2 g_{\mu\nu} \quad dx^\mu \quad dx^\nu \]

\[ g_{\mu\nu} = g_{\mu\nu}^{(b)} + \delta g_{\mu\nu} \]

\[
\frac{\Delta d_L(z, \Psi S)}{d_L(z)} = \frac{1}{\lambda_s} \int_0^{\lambda_s} d\lambda \left[ (\lambda - \lambda_s) \lambda^3 \Delta \Psi + 2(\Psi - \Psi_0) \right] + \frac{1}{2} \left( \frac{\eta_0}{\lambda_s} - 3 \right) (\Psi_s - \Psi_0)
\]

\[ + \frac{1}{6} \left( \frac{\eta_0}{\lambda_s} - 3 \right) \left[ \eta_s (\psi_{\eta i} \gamma^i)_s - \eta_0 (\psi_{\eta i} \gamma^i)_0 \right] - \frac{1}{3} \eta_0 (\psi_{\eta i} \gamma^i)_0 + \Psi_0 + \frac{3}{\eta_0} \delta_s \eta_0, \]
What’s new in Doppler (or anti) lensing?

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- What’s new in theory?
  - classic paper by Sasaki et al ’87
- Wasn’t this all thrashed out in relation to SN1a cosmology?
Correlated fluctuations in luminosity distance and the importance of peculiar motion in supernova surveys

Lam Hui\textsuperscript{1,2,*} and Patrick B. Greene\textsuperscript{1,3,†}

\textsuperscript{1}Institute for Strings, Cosmology and Astro-particle Physics (ISCAP)  
\textsuperscript{2}Department of Physics, Columbia University, New York, New York 10027, USA  
\textsuperscript{3}Department of Physics and Astronomy, University of Texas at San Antonio, Texas 78249, USA

(Received 7 December 2005; published 23 June 2006)

Large scale structure introduces two different kinds of errors in the luminosity distance estimates from standardizable candles such as supernovae Ia (SNe)—a Poissonian scatter for each SN and a coherent component due to correlated fluctuations between different SNe. Increasing the number of SNe helps reduce the first type of error but not the second. The coherent component has been largely ignored in forecasts of dark energy parameter estimation from upcoming SN surveys. For instance it is commonly thought, based on Poissonian considerations, that peculiar motion is unimportant, even for a low redshift SN survey such as the Nearby Supernova Factory (SNfactory; \(z = 0.03–0.08\)), which provides a useful anchor for future high redshift surveys by determining the SN zero point. We show that ignoring coherent peculiar motion leads to an underestimate of the zero-point error by about a factor of 2, despite the fact that SNfactory covers almost half of the sky. More generally, there are four types of fluctuations: peculiar motion, gravitational lensing, gravitational redshift and what is akin to the integrated Sachs-Wolfe effect. Peculiar motion and lensing dominates at low and high redshifts, respectively. Taking into account all significant luminosity distance fluctuations due to large scale structure leads to a degradation of up to 60\% in the determination of the dark energy equation of state from upcoming high redshift SN surveys, when used in conjunction with a low redshift anchor such as the SNfactory. The most relevant fluctuations are the coherent ones due to peculiar motion and the Poissonian ones due to lensing, with peculiar motion playing the dominant role. We also discuss to what extent the noise here can be viewed as a useful signal, and whether corrections can be made to reduce the degradation.

DOI: 10.1103/PhysRevD.73.123526  
PACS numbers: 98.80.–k, 95.30.Sf, 98.80.Es, 98.80.Jk
Hui & Greene 2006

In summary, the total peculiar motion and lensing contributions to $\delta_{dL}$ are

$$\delta_{dL}(z, n) = v_e \cdot n - \frac{1}{X_e} \left[ a \right] a' \left( v_e \cdot n - v_0 \cdot n \right)$$

$$- \int_{0}^{X_e} d\chi \frac{(X_e - \chi) \chi}{X_e} \nabla^2 \phi(\chi).$$

(18)

To reiterate: $v_e$ and $v_0$ are the peculiar velocities of the emitter and observer, and $n$ is the line-of-sight unit vector pointing away from the observer ($n$ here plays the role of $\theta$ in Eq. (7)); the comoving distance to emitter $X_e$, the scale factor at emission $a_e$ and its derivative with respect to conformal time $a'_e$ are evaluated at redshift $z$. One can see from above that for small $X_e$ or at a low redshift, the peculiar motion term proportional to $1/X_e$ becomes important, while at a large redshift, the lensing term (second line) is more important. A more rigorous derivation of $\delta_{dL}$,

$$\left( \sigma_{\text{Poiss., vel.}}^i \right)^2 \equiv \left[ \frac{5}{\ln 10} \right]^2 \left[ 1 - \frac{a_i}{a'_i X_i} \right]^2 (D_i')^2$$

$$\times \int \frac{d^3 k}{(2\pi)^3} \frac{k_z^2}{k^4} P(k, a = 1),$$
Large-scale bulk motions complicate the Hubble diagram

Asantha Cooray

1Center for Cosmology, Department of Physics and Astronomy, University of California, Irvine, California 92697, USA

Robert R. Caldwell

2Department of Physics and Astronomy, Dartmouth College, 6127 Wilder Laboratory, Hanover, New Hampshire 03755, USA

We investigate the extent to which correlated distortions of the luminosity distance-redshift relation due to large-scale bulk flows limit the precision with which cosmological parameters can be measured. In particular, peculiar velocities of type 1a supernovae at low redshifts, \( z < 0.2 \), may prevent a sufficient calibration of the Hubble diagram necessary to measure the dark energy equation of state to better than 10\%, and diminish the resolution of the equation of state time-derivative projected for planned surveys. We consider similar distortions of the angular-diameter distance, as well as the Hubble constant. We show that the measurement of correlations in the large-scale bulk flow at low redshifts using these distance indicators may be possible with a cumulative signal-to-noise ratio of order 7 in a survey of 300 type 1a supernovae spread over 20,000 square degrees.

\[
C_{HH}(z_i, z_j, \theta_{ij}) = \sum_{\text{even } \ell} \frac{2l+1}{4\pi} \cos\theta_{ij} \frac{2}{\pi} F_l \times \int dkP_{mm}(k, z_i, z_j) j_l(k[X_i - X_j \cos\theta_{ij}]) \times j_l(kX_j \sin\theta_{ij}) \left[ D'(z_i) \left( 1 - \frac{a}{a'X_i} \right) \right] \left[ D'(z_j) \left( 1 - \frac{a}{a'X_j} \right) \right] \times \left( 1 - \frac{a}{a'X_i} \right) + X_j \left[ D'(z_j) \left( 1 - \frac{a}{a'X_j} \right) \right],
\]

\[
\text{Combination, we obtain (see, Ref. [28] for details including their equation 3.15; also [29,31]):}
\]

\[
\frac{\delta d_L}{d_L} = \hat{\mathbf{n}} \cdot \left[ \mathbf{v}_{\text{SN}} - \frac{a}{a'} (\mathbf{v}_{\text{SN}} - \mathbf{v}_{\text{obs}}) \right],
\]

where \( \hat{\mathbf{n}} \) is the unit vector along the line-of-sight, \( \mathbf{v}_{\text{SN}} \) is the SN velocity, \( \mathbf{v}_{\text{obs}} \) is the velocity of the observer, \( \chi \) is the comoving radial distance to the SN, and the prime denotes the derivative with respect to the conformal time. Unless otherwise stated, here and throughout, we take a unit system in which \( c = 1 \). The covariance matrix of errors in luminosity distance is

\[
\text{Cov}_{ij} \approx \sigma_{\text{int}}^2(z_i) \delta_{ij} + C^{uv}(z_i, z_j, \theta_{ij}).
\]
THE EFFECT OF PECULIAR VELOCITIES ON SUPERNova COSMOLOGY

Tamara M. Davis, Lam Hui, Joshua A. Frieman, Troels Haugbølle, Richard Kessler, Benjamin Sinclair, Jesper Sollerman, Bruce Bassett, John Marriner, Edvard Mörtsell, Robert C. Nichol, Michael W. Richmond, Masao Sako, Donald P. Schneider, and Mathew Smith

We analyze the effect that peculiar velocities have on the cosmological inferences we make using luminosity distance indicators, such as Type Ia supernovae. In particular we study the corrections required to account for (1) our own motion, (2) correlations in galaxy motions, and (3) a possible local under- or overdensity. For all of these effects we present a case study showing the impact on the cosmology derived by the Sloan Digital Sky Survey-II Supernova Survey (SDSS-II SN Survey). Correcting supernova (SN) redshifts for the cosmic microwave background (CMB) dipole slightly overcorrects nearby SNe that share some of our local motion. We show that while neglecting the CMB dipole would cause a shift in the derived equation of state of $\Delta w \sim 0.04$ (at fixed $\Omega_m$), the additional local-motion correction is currently negligible ($\Delta w \lesssim 0.01$). We then demonstrate a covariance-matrix approach to statistically account for correlated peculiar velocities. This down-weights nearby SNe and effectively acts as a graduated version of the usual sharp low-redshift cut. Neglecting coherent velocities in the current sample causes a systematic shift of $\Delta w \sim 0.02$. This will therefore have to be considered carefully when future surveys aim for percent-level accuracy and we recommend our statistical approach to down-weighting peculiar velocities as a more robust option than a sharp low-redshift cut.

The peculiar-motion-induced magnitude covariance is related to the velocity correlation function $\xi_{ij}^{vel}$ by

$$C_{ij}^{vel} = \left[ \frac{5}{c \ln 10} \right]^2 \left[ 1 - \frac{a_i}{\tilde{c}_i} \right] \left[ 1 - \frac{a_j}{\tilde{c}_j} \right] \xi_{ij}^{vel} , \quad (26)$$

where $c$ is the speed of light, $\tilde{c} \equiv R_0 \tilde{x}$ is the radial comoving distance, $a = R/R_0$ is the normalized scale factor, and the prime denotes the conformal time derivative. All quantities with
What’s new in Doppler (or anti) lensing?

- Long history of observations
  - Rubin-Ford effect (1976)
  - Tammann, Sandage & Yahil (1979)
  - Tully-Fisher … Faber-Jackson … Dn-sigma …
  - Cosmic flows II; 6df survey…

- What’s new in theory?
  - long history back to Zel’dovich ’64
  - classic paper by Sasaki et al ’87

- Wasn’t this all thrashed out in relation to SN Ia cosmology?
  - Hui & Greene ’06; Cooray & Caldwell ’06; Davis et al 2011

- So it’s “not even wrong”?
  - not quite… lowest order effect is traditional pec. vel.
  - but next order (finite z) effect depends on absolute motion
  - violates Equivalence Principle!
• Perturbation to the distance (at fixed $z$) from velocities (alone)
  \[
  (\delta d/d)_z = - (a/a')\chi(v_S\cdot n - v_O\cdot n) + v_S\cdot n
  \]
• $a$=scale factor, $\chi$=conformal distance ($d\chi=dz/H$), $n$=LOS
• $S$=source, $O$=observer, $'= d/d\eta$, $\eta$=conformal time
• At low $z$, $a'\chi/a=z$, so first term dominates
• it depends only on relative velocity
• But for finite $z$ 2nd term becomes important
  • this depends on \textit{absolute} peculiar motion of sources
  • what if observer and sources share a common motion?
    • perhaps caused by the attraction of a distant mass excess
  • would we see a dipole in $(\delta d/d)_z = v_S\cdot n$?
• this would conflict with the Equivalence Principle
  • our observations 'transform gravity away' locally since both sources and observer are in free fall so we should only be able to detect the tidal influence of distant matter
Observed region \xrightarrow{g} \text{Distant attractor}
Whence \((\delta d/d)_z = -(a/a'\chi)(v_s.n - v_O.n) + v_s.n\) ?

- Size of source of diameter \(l\) at distance \(\chi\) is \(\theta = \theta_0(\chi) = l/a\chi\)
- and \(d\theta/dz = (l/\chi)(1 - a/a'\chi)\)
- so at fixed \(z\), \(\delta\theta = \theta_0(\chi) - \theta_0(\chi(z+\delta z)) = -(d\theta/dz)\delta z\)
- But a moving observer will suffer relativistic aberration
  - angular sizes modulated by factor \((1 - n.v_O)\)
  - so \((\delta\theta/\theta)_z = -(d\theta/dz)(\delta z/\theta) - n.v_O\)
  - or \((\delta\theta/\theta)_z = (a/a'\chi - 1)\delta z/(1+z) - n.v_O\)
  - which, with \(\delta z/(1+z) = n.(v_s-v_O)\) is equivalent to above
    - \(-((\delta\theta/\theta)_z = (\delta d_A/d_A)_z = (\delta d/d)_z \) (since \((\delta \Sigma/\Sigma)_z = 0)\)
    - note - asymmetry caused by motion of the observer
Q: So is \((\delta d/d)_z\) an absolute speedometer?

- A: No. Velocities in the formula are velocities on the past light cone of the observer. And velocities change with time.

- Case 1: Unsupported motions: these decay as \(v \sim 1/a = (1+z)\)
  - can rewrite \((\delta \theta/\theta)_z = (a/a'\chi)\left[(1-a'\chi/a)(1+z)n.v_s - n.v_o\right]\)
  - where all velocities are at the same (present) time \(\eta_0\)
  - but \(a'\chi/a \sim z\) at small \(z\), so
  - \((\delta \theta/\theta)_z = (a/a'\chi)n.(v_s - v_o)\)
    - plus corrections that are smaller by factor \(\sim z^2\)
    - only depends on relative velocity
    - no effect from distant attractors
Q: So is \((\delta d/d)_z\) an absolute speedometer?

- Case 2: More interesting: velocities associated with structure.
  - \(dv/dt = -Hv + g\)
  - \(g\) is the peculiar gravity

- Expressing \((\delta \theta/\theta)_z\) in terms of velocity at present epoch does not banish dependence on absolute velocity (or gravity)

- But to be consistent, one needs to include the gravitational redshift (Sachs & Wolfe effect) in \(\delta z/(1+z)\)

- Result is \((\delta \theta/\theta)_z = (a/a')n.(v_S - v_O) - n.g_S/H + z^{-1} \int dX.g(X)\)

- no observable effect from distant attractors

- independent of choice of "background"

- gives consistent approach to calculating e.g. SNIa covariance
2) Bias in $H_0$ from 2nd order perturbation theory
Scale dependence of cosmological backreaction

Nan Li* and Dominik J. Schwarz+

Fakultät für Physik, Universität Bielefeld, Universitätsstraße 25, D-33615 Bielefeld, Germany

(Received 2 November 2007; published 23 October 2008)

Because of the noncommutation of spatial averaging and temporal evolution, inhomogeneities and anisotropies (cosmic structures) influence the evolution of the averaged Universe via the cosmological backreaction mechanism. We study the backreaction effect as a function of averaging scale in a perturbative approach up to higher orders. We calculate the hierarchy of the critical scales, at which 10% effects show up from averaging at different orders. The dominant contribution comes from the averaged spatial curvature, observable up to scales of $\sim 200$ Mpc. The cosmic variance of the local Hubble rate is 10% (5%) for spherical regions of radius 40 (60) Mpc. We compare our result to the one from Newtonian cosmology and Hubble Space Telescope Key Project data.

FIG. 2 (color online). Relative fluctuation of the Hubble rate from cosmological backreaction and its cosmic variance band (thick lines) compared to the empirical mean and variance of $\delta_H$ obtained from the HST Key Project data [5] as a function of averaging radius. The thin line shows the ensemble mean of $\delta_H$. The band enclosed by the thick lines indicates the effect of the inhomogeneities ($\propto 1/r^2$), and the dashed lines are the effect from sampling with given measurement errors in an otherwise perfectly homogeneous Universe.
We investigate the effect that the average backreaction of structure formation has on the dynamics of the cosmological expansion, within the concordance model. Our approach in the Poisson gauge is fully consistent up to second order in a perturbative expansion about a flat Friedmann background, including a cosmological constant. We discuss the key length scales which are inherent in any averaging procedure of this kind. We identify an intrinsic homogeneity scale that arises from the averaging procedure, beyond which a residual offset remains in the expansion rate and deceleration parameter. In the case of the deceleration parameter, this can lead to a quite large increase in the value, and may therefore have important ramifications for dark energy measurements, even if the underlying nature of dark energy is a cosmological constant. We give the intrinsic variance that affects the value of the effective Hubble rate and deceleration parameter. These considerations serve to add extra intrinsic errors to our determination of the cosmological parameters, and, in particular, may render attempts to measure the Hubble constant to percent precision overly optimistic.
The Hubble rate in averaged cosmology

Obinna Umeh, Julien Larena and Chris Clarkson

Astrophysics, Cosmology and Gravity Center and Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7701, South Africa

E-mail: umebinna@gmail.com, julien.larena@gmail.com, chris.clarkson@uct.ac.za

Received November 23, 2010
Revised January 27, 2011
Accepted March 3, 2011
Published March 21, 2011

Abstract. The calculation of the averaged Hubble expansion rate in an averaged perturbed Friedmann-Lemaître-Robertson-Walker cosmology leads to small corrections to the background value of the expansion rate, which could be important for measuring the Hubble constant from local observations. It also predicts an intrinsic variance associated with the finite scale of any measurement of $H_0$, the Hubble rate today. Both the mean Hubble rate and its variance depend on both the definition of the Hubble rate and the spatial surface on which the average is performed. We quantitatively study different definitions of the averaged Hubble rate encountered in the literature by consistently calculating the backreaction effect at second order in perturbation theory, and compare the results. We employ for the first time a recently developed gauge-invariant definition of an averaged scalar. We also discuss the variance of the Hubble rate for the different definitions.

Keywords: cosmic flows, cosmological perturbation theory, dark energy theory
The second-order luminosity-redshift relation in a generic inhomogeneous cosmology

Ido Ben-Dayan, Giovanni Marozzi, Fabien Nugier and Gabriele Veneziano

Published November 22, 2012

Abstract. After recalling a general non-perturbative expression for the luminosity-redshift relation holding in a recently proposed “geodesic light-cone” gauge, we show how it can be transformed to phenomenologically more convenient gauges in which cosmological perturbation theory is better understood. We present, in particular, the complete result on the luminosity-redshift relation in the Poisson gauge up to second order for a fairly generic perturbed cosmology, assuming that appreciable vector and tensor perturbations are only generated at second order. This relation provides a basic ingredient for the computation of the effects of stochastic inhomogeneities on precision dark-energy cosmology whose results we have anticipated in a recent letter. More generally, it can be used in connection with any physical information carried by light-like signals traveling along our past light-cone.
Backreaction on the luminosity-redshift relation from gauge invariant light-cone averaging

I. Ben-Dayan$^{a,b}$ M. Gasperini$^{c,d}$ G. Marozzi$^e$ F. Nugier$^f$ and G. Veneziano$^{e,g}$

Figure 4. The distance-modulus difference of eq. (6.3) is plotted for a pure CDM model (thin line), for a CDM model including the contribution of IBR$_2$ (dashed blue line) plus/minus the dispersion (coloured region), and for a $\Lambda$CDM model with $\Omega_\Lambda = 0.73$ (thick line) and $\Omega_\Lambda = 0.1$ (dashed-dot thick line). We have used for all backreaction integrals the cut-off $k = 1\,\text{Mpc}^{-1}$. 
Average and dispersion of the luminosity-redshift relation in the concordance model

I. Ben-Dayan, M. Gasperini, G. Marozzi, F. Nugier and G. Veneziano

Figure 6. The fractional correction to the flux ($f_\Phi$, thin curves) and to the luminosity distance ($f_d$, thick curves), for a perturbed $\Lambda$CDM model with $\Omega_{\Lambda 0} = 0.73$. Unlike in figure 3, we have taken into account the non-linear contributions to the power spectrum given by the HaloFit model of [17] (including baryons), and we have used the following cutoff values: $k_{UV} = 10h\text{Mpc}^{-1}$ (dashed curves) and $k_{UV} = 30h\text{Mpc}^{-1}$ (solid curves).
Figure 7. The averaged distance modulus $\langle \mu \rangle - \mu^M$ of eq. (3.6) (thick solid curve), and its dispersion of eq. (3.9) (shaded region), for a perturbed $\Lambda$CDM model with $\Omega_{\Lambda 0} = 0.73$. Unlike figure 4, we have taken into account the non-linear contributions to the power spectrum given by the HaloFit model of [17] (including baryons), and used the cut-off $k_{UV} = 30h \text{Mpc}^{-1}$. The averaged results are compared with the homogeneous values of $\mu$ predicted by unperturbed $\Lambda$CDM models with (from bottom to top) $\Omega_{\Lambda 0} = 0.68, 0.69, 0.71, 0.73, 0.75, 0.77, 0.78$ (dashed curves). The right panel simply provides a zoom of the same curves, plotted in the smaller redshift range $0.5 \leq z \leq 2$. 
Do Stochastic Inhomogeneities Affect Dark-Energy Precision Measurements?

I. Ben-Dayan, M. Gasperini, G. Marozzi, F. Nugier, and G. Veneziano

The effect of a stochastic background of cosmological perturbations on the luminosity-redshift relation is computed to second order through a recently proposed covariant and gauge-invariant light-cone averaging procedure. The resulting expressions are free from both ultraviolet and infrared divergences, implying that such perturbations cannot mimic a sizable fraction of dark energy. Different averages are estimated and depend on the particular function of the luminosity distance being averaged. The energy flux being minimally affected by perturbations at large \( z \) is proposed as the best choice for precision estimates of dark-energy parameters. Nonetheless, its irreducible (stochastic) variance induces statistical errors on \( \Omega_\Lambda(z) \) typically lying in the few-percent range.

FIG. 2. The fractional correction to the flux \( f_d \) of Eq. (7) (thin curves) is compared with the fractional correction to the luminosity distance \( f_d \) of Eq. (13) (thick curves) for a \( \Lambda \)CDM model with \( \Omega_\Lambda = 0.73 \). We have used two different cutoff values: \( k_{UV} = 0.1 \) Mpc\(^{-1} \) (dashed curves) and \( k_{UV} = 1 \) Mpc\(^{-1} \) (solid curves). The spectrum is the same as that of Fig. 1 adapted to \( \Lambda \)CDM.

FIG. 3. The averaged distance modulus \( \langle \mu \rangle - \mu^M \) (thick solid curve) and its dispersion of Eq. (15) (shaded region) are computed for \( \Omega_\Lambda = 0.73 \) and compared with the homogeneous value for the unperturbed \( \Lambda \)CDM models with \( \Omega_\Lambda = 0.69, 0.71, 0.73, 0.75, 0.77 \) (dashed curves). We have used \( k_{UV} = 1 \) Mpc\(^{-1} \) and the same spectrum as in Fig. 2.
Value of $H_0$ in the Inhomogeneous Universe

Ido Ben-Dayan,$^1$ Ruth Durrer,$^2$ Giovanni Marozzi,$^2$ and Dominik J. Schwarz$^3$

$^1$Deutsches Elektronen-Synchrotron DESY, Theory Group, D-22603 Hamburg, Germany
$^2$Université de Genève, Département de Physique Théorique and CAP, 24 quai Ernest-Ansermet, CH-1211 Genève 4, Switzerland
$^3$Fakultät für Physik, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany

(Received 5 February 2014; published 6 June 2014)

Local measurements of the Hubble expansion rate are affected by structures like galaxy clusters or voids. Here we present a fully relativistic treatment of this effect, studying how clustering modifies the mean distance- (modulus-)redshift relation and its dispersion in a standard cold dark matter universe with a cosmological constant. The best estimates of the local expansion rate stem from supernova observations at small redshifts ($0.01 < z < 0.1$). It is interesting to compare these local measurements with global fits to data from cosmic microwave background anisotropies. In particular, we argue that cosmic variance (i.e., the effects of the local structure) is of the same order of magnitude as the current observational errors and must be taken into account in local measurements of the Hubble expansion rate.

$$\langle d_L^{-2}(z) \rangle = (d_L^{FL})^{-2}[1 + f_\Phi(z)], \quad (4)$$

where for $z \ll 1$,

$$f_\Phi(z) \approx -\frac{1}{\mathcal{H}(z)\Delta \eta} \frac{2}{\langle (\vec{v}_s \cdot \vec{n})^2 \rangle}. \quad (5)$$

would nearly double the effect in Eq. (5). The dominant peculiar velocity contribution at low redshift gives

$$f_\Phi(z) \approx -\frac{1}{\mathcal{H}(z)\Delta \eta} \frac{2}{3} \int_{H_0}^{k_{UV}} dk k^2 \mathcal{P}_\psi(k), \quad (6)$$

The brightness of supernovae is typically expressed in terms of the distance modulus $\mu$. Because of the nonlinear function relating $\mu$ and $\Phi$, one obtains different second order contributions,

$$\langle \mu \rangle - \mu^{FL} = -\frac{2.5}{\ln(10)} \left[ f_\Phi - \frac{1}{2} \langle (\Phi_1/\Phi_0)^2 \rangle \right], \quad (7)$$

where at $z \ll 1$, we also find

$$\langle (\Phi_1/\Phi_0)^2 \rangle \approx -4f_\Phi. \quad (8)$$
Bias in $H_0$ from 2nd order pert$^n$ theory

- Backreaction causes systematic bias in $H$ measurement
  - very large effects on DM??
  - interesting bias in flux density, distance etc at low-z
Bias in $H_0$ from 2nd order perturbative theory

- Backreaction causes systematic bias in $H$ measurement
  - very large effects on DM??
  - interesting bias in flux density, distance etc at low-z
- But isn’t this just the residual “homogeneous Malmquist bias” in “inverse + type II” method?
Malmquist bias?

- Objects in a region of estimated distance space will have a distance that is biased
  - because of (large) uncertainty in distance
- But “Schechter’s method” largely avoids that
  - don't measure velocity as a function of distance
  - do it the other way round
  - small scatter in distance for objects same redshift
- but not completely free from bias
  - analysed by Lynden-Bell ’92 and Willick & Strauss ‘97
Eddington–Malmquist Bias, Streaming Motions, and the Distribution of Galaxies

D. Lynden-Bell

ABSTRACT Schechter’s method of eliminating Malmquist bias is reviewed and presented in the context of the $D_n - \sigma$ relationship for elliptical galaxies. A Malmquist-like correction occurs which is dependent on the dispersion in the velocity field of galaxies; however, this correction does not increase with distance so it is much less important than the normal Malmquist bias that this method eliminates. The method is applied to a bulk flow model of the ellipticals and gives almost identical results to those found using the other reduction method which employs the Malmquist corrections. Ways of using the method to model the density and velocity fields out to 10,000 km/sec are briefly indicated.

is already small.

Solving for $R$, we obtain the value $R_m$ at which the maximum occurs

$$R_m = \frac{1}{2} \left\{ w + \sqrt{w^2 + 4\sigma_v^2 \left[ 3 + \frac{d \ln (n/\sigma_v)}{d \ln r} \right] [1 + u'(v)]^{-2}} \right\}.$$

Equations (9.16) and (9.15) constitute our solution for $R_m$. Notice that when $w >> \sigma_v$, then

$$R_m = w \left\{ 1 + \frac{\sigma_v^2}{w^2} \left[ 3 + \frac{d \ln (n/\sigma_v)}{d \ln r} \right] [1 + u'(v)]^{-2} \right\}.$$

(9.17)
2.2.2. Further discussion of the velMOD likelihood

The physical meaning of the velMOD likelihood expressions is clarified by considering them in a suitable limit. If we take $\sigma_v$ to be “small,” in a sense to be made precise below, the integrals in Eqs. (11) and (12) may be approximated using standard techniques. If in addition we neglect sample selection ($S = 1$) and density variations ($n(r) = \text{constant}$), and assume that the redshift-distance relation is single-valued, we find for the forward relation:

$$P(m|\eta, cz) \simeq \frac{1}{\sqrt{2\pi}\sigma_e} \exp \left\{ -\frac{1}{2\sigma_e^2} \left[ m - \left[ M(\eta) + 5 \log w + \frac{10}{\ln{10}} \Delta_v^2 \right] \right]^2 \right\},$$  \hspace{1cm} (15)

$$P(m|\eta, cz) \simeq \frac{1}{\sqrt{2\pi}\sigma_e} \exp \left[ -\frac{1}{2\sigma_e^2} \left[ m - \left[ M(\eta) + 5 \log w + 3 \times \frac{5}{\ln{10}} \Delta_v^2 \right] \right]^2 \right],$$  \hspace{1cm} (15)

We thank Marc Davis, Carlos Frenk, and Amos Yahil for extensive discussions of various aspects of this project, as well as the support of the entire Mark III team: David Burstein, Stéphane Courteau, and Sandra Faber. We also thank the referee, Alan Dressler, for an insightful report that improved the quality of the paper. J. A. W. and M. A. S. are grateful for the

- **KH15:** The “3” here comes from the standard formula for HMB.
- **The right answer is 1.5**
  - as found by the relativistic backreaction folks!
Kinematic Bias in Cosmological Distance Measurement

Nick Kaiser & Michael J. Hudson

1Institute for Astronomy, University of Hawaii
2Department of Physics and Astronomy, University of Waterloo

ABSTRACT
Recent calculations using non-linear relativistic cosmological perturbation theory show biases in the mean luminosity distance and distance modulus at low redshift. We show that these effects may be understood very simply as a non-relativistic, and purely kinematic, Malmquist-like bias, and we describe how the effect changes if one averages over sources that are limited by apparent magnitude. This effect is essentially identical to the distance bias from small-scale random velocities that has previously been considered by astronomers, though we find that the standard formula overestimates the homogeneous bias by a factor 2.

Figure 1. Dotted lines are lines of longitude and latitude on the surface of constant redshift. On this surface, peculiar velocities are equally likely to be positive as negative. The cone illustrates how a section of this sphere maps to real space for the case of a negative peculiar velocity. The section is pushed out radially away from the observer – who resides at the centre of the sphere – and consequently is expanded in area. Similarly, for a positive peculiar velocity the section would be compressed. The result of this is that the average of the distance, when weighted by real-space area, is positive. This is the cause of the bias found in the relativistic perturbation theory analyses. More relevant to real observations is the bias in distance averaged over the sources that lie in a shell of given redshift. We consider this in §2.2. There we find that there are some relatively minor differences that arise from the clustering of sources and from the Jacobian involved in transforming volumes from redshift to real space, but the main difference is that the generalisations of (8) have different numerical pre-factors when the sources are subject to selection based on flux density.
Conclusions

• There have been a number of recent papers computing perturbations to the cosmological distance and claiming either

  • new probes of structure (Doppler- or anti-lensing) that rely on previously ignored relativistic effects

  • subtle relativistic effects (backreaction) that bias cosmological parameters

• The “lensing” effects are mostly well known and studied pec. vel. effect

  • but to the extent that they go beyond the lowest order effect are wrong

• The bias in local measurement of H is just kinematic Malmquist bias

  • though astronomers got this wrong
21 This assumes the observer has already corrected both for the motion of the Earth around the Sun, which contributes up to 30 km s\(^{-1}\) depending on the time of observation and for atmospheric refraction, which contributes up to 90 km s\(^{-1}\) (the index of refraction of air is 1.0003, so \(\Delta z = 0.0003\) and \(c\Delta z = 90\) km s\(^{-1}\)). Usually this is done as a standard step in wavelength calibration.