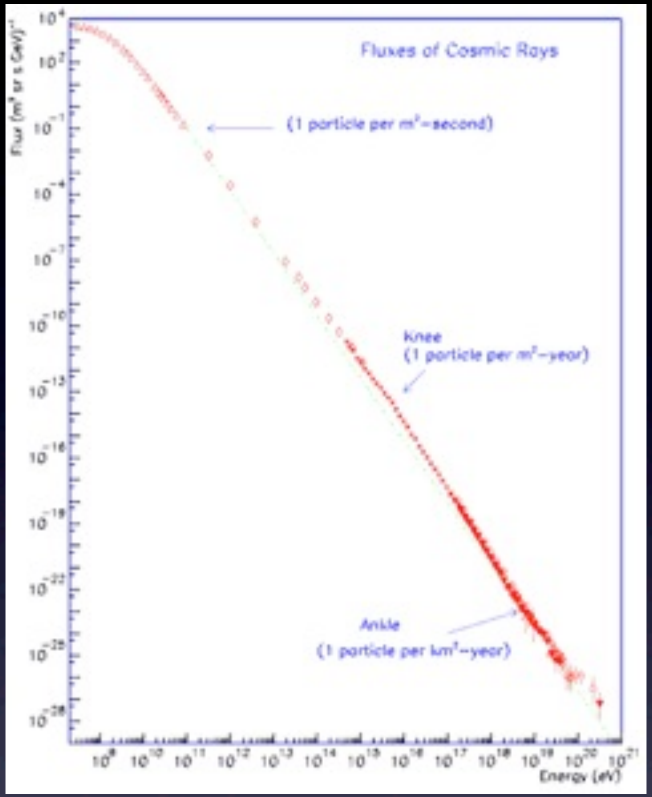
Back to basis: Leptonic Processes in High Energy Sources

École de physique des astroparticles:
Observatoire de Haute Provence
23-28 Mai 2011

R. Belmont

High Energy Astrophysics



- Observed High- and Very High Energy
  - photons (> 10 TeV)
  - particles (>10^{20} eV)
- Goal: understand the physics of HE particles:
  - How are they created/accelerated
  - How do they propagate/evolve ?
- HE photons are tracers of the emitting HE particles
- Photons interact with matter
  - Change the emitting matter properties
  - Can be altered after their emission
- => Complex system matter+radiation

Outline

- ✓ Goals of this course:
  - ✓ Present the main leptonic processes
    - ✓ Assumptions, approximations, properties
  - ✓ Sum up their role in HE sources
    - ✓ individual, coupled
  - ✓ Present some methods

- ✓ Main books/reviews:

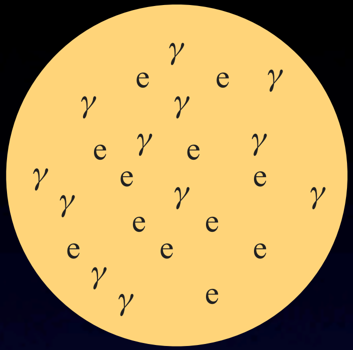
    - ✓ Rybicki G.B. & Lightman A.P., 1979, Radiative processes in astrophysics, New York, Wiley-Interscience
    - ✓ Jauch J.M. & Rohrlich F., 1980, The theory of photons and electrons (2nd edition), Berlin, Springer
    - ✓ Aharonian F. A., 2004, Very High energy cosmic gamma radiation, World scientific publishing
    - ✓ Heitler W., 1954, Quantum theory of radiation, International Series of Monographs on Physics, Oxford: Clarendon
    - ✓ Blumenthal G.R. & Gould R.J., 1970, Reviews of Modern Physics

Contents

Photon/Particles

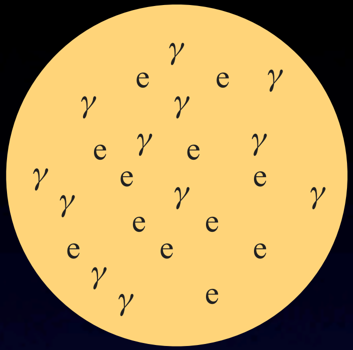
- Photons:
  - energy: $E = h\nu = \omega\, mc^2$
  - momentum: $\mathbf{p}=h\nu/c\ \mathbf{n}$
- leptons:
  - energy $E = \gamma\, mc^2$
  - velocity: $\beta=\mathbf{v}/c$, momentum $p = \sqrt{\gamma^2 - 1} = \beta\gamma$
- Distributions: $N(\vec{p}) = \partial^3 N/\partial\vec{p}$
  - Isotropic distribution: N(p) or N(γ)
  - At high energy: collisions inefficient => non-thermal distributions

Total Cross Section



- Source with 2 interacting species:
- The simplest case:
  - one species at rest, with number density n_1
  - one species with one single velocity v_2, number density n_2
  - otherwise: change of frame...

Total Cross Section



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$$\frac{dn}{dt} = \sigma n_1 (v_2 n_2)$$

total cross section   target density   incoming flux

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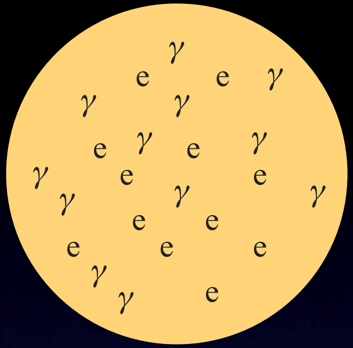

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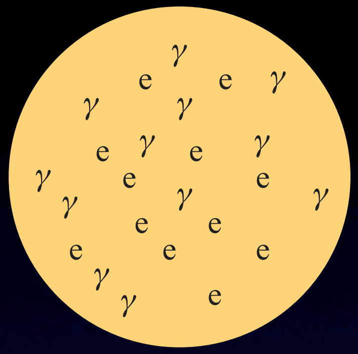
- ✓ $\sigma=\sigma(v_2)$
  - ✓ Hard spheres: σ=cst
  - ✓ Coulomb collisions: $\sigma=1/v$

Total Cross Section



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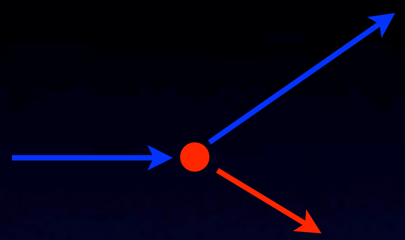
- $\sigma = \sigma(v_2)$
  - Hard spheres: σ=cst
  - Coulomb collisions: $\sigma = 1/v$

$$\sigma_T = \frac{8\pi}{3} r_e^2 = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2$$

Differential cross sections

- ✓ Unique interaction event:

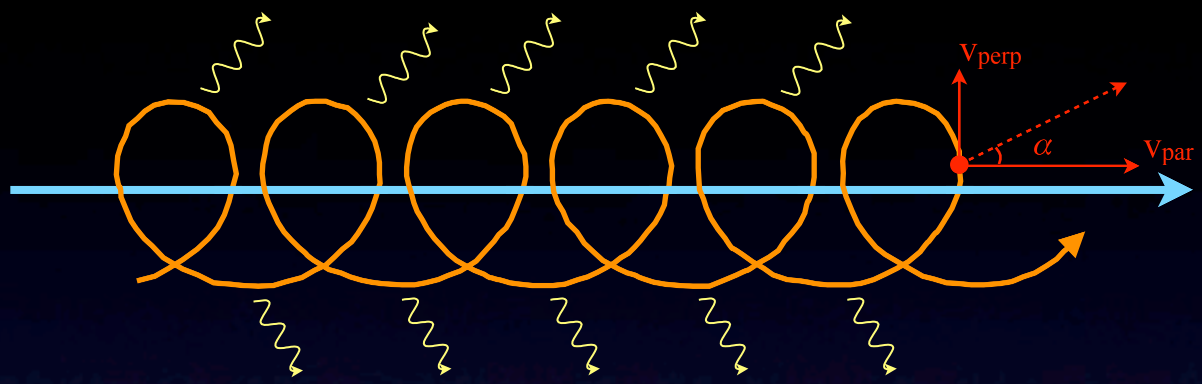


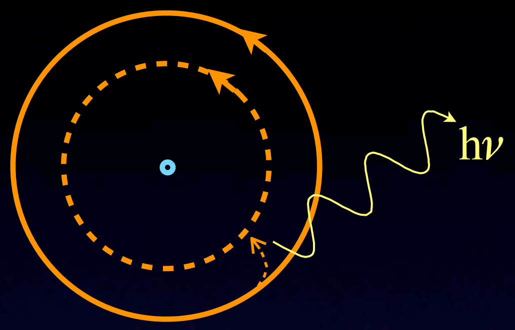
- ✓ Result = direction and energy of the outgoing species
- ✓ Conservation rules (E and p) not sufficient to tell the answer
- ✓ Most of time, only one undefined quantity
- ✓ Quantum mechanics gives the outcome as probabilities

$$\frac{\partial\sigma}{\partial E_1} \quad \frac{\partial\sigma}{\partial E_2} \quad \frac{\partial\sigma}{\partial\Omega_2} \quad \frac{\partial\sigma}{\partial\Omega_1}$$

- ✓ These are the differential cross sections
- ✓ Integration yields the total cross section: $\sigma = \int \frac{\partial\sigma}{\partial\Omega} d\Omega = \int \frac{\partial\sigma}{\partial E} dE$

Cyclo-synchrotron radiation





✓ Radiation from particles gyrating the magnetic field lines

$$\nu_L = \frac{qB}{2\pi mc} \qquad \nu_B = \frac{1}{\gamma}\frac{qB}{2\pi mc} = 2.80 \times 10^6 \,\frac{B(\mathrm{G})}{\gamma}\ \mathrm{Hz}$$

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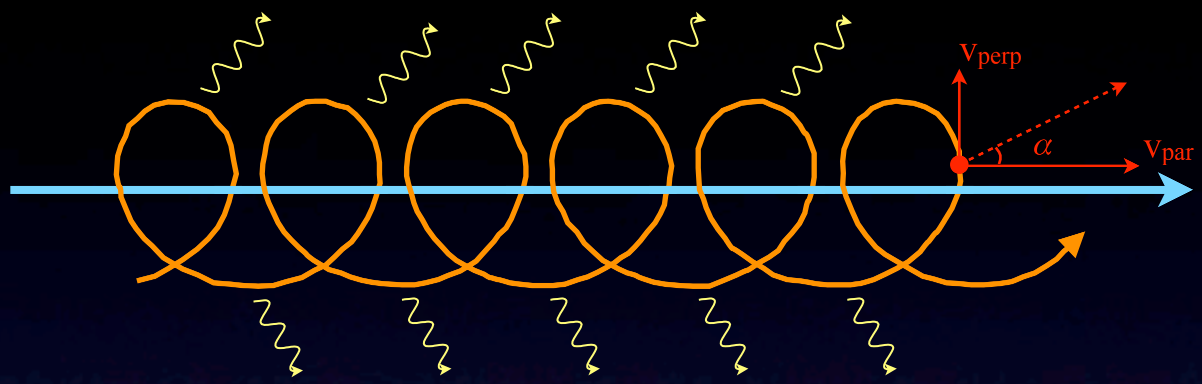

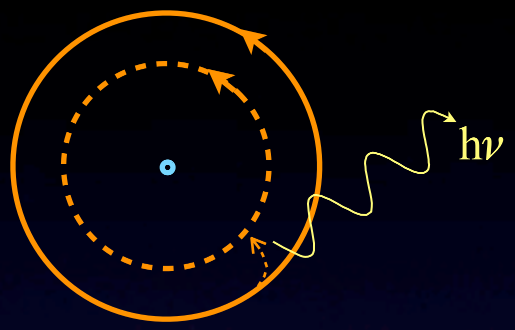

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- Assumptions:
  - B uniform at the Larmor scale (parallel and perp)
    - ! Strong B curvature (pulsar and rapidly rotating neutron stars)
    - ! Small scale turbulence (at the Larmor scale)

$$r_L = p_\perp \frac{mc^2}{qB} = 1.70 \times 10^3 \frac{p_\perp}{B(\mathrm{G})}\ \mathrm{cm}$$

  - Small losses ($t_{cool} >> 1/\nu_B$)
  - Classical limit:
    - Otherwise: quantization of energies, Larmor radii...
    - Observable cyclotron lines in accreting neutron stars...

$$B < B_c = \frac{m^2 c^3}{\hbar q} = 4.4 \times 10^{13}\ \mathrm{G}$$

- Emission/Absorption

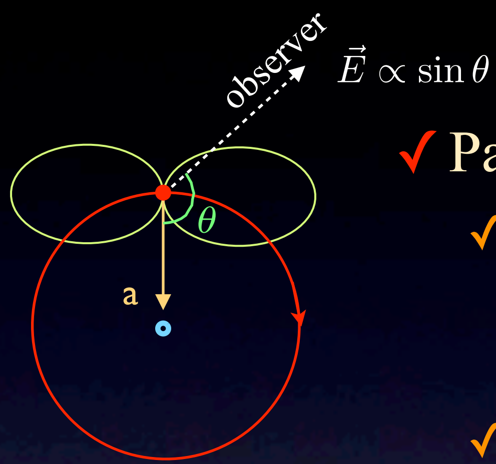
Synchrotron Emitted Power

- Emission of an accelerated particle (erg/s):
  - Non-relativistic: $P = \frac{2q^2}{3c^3} a^2$
  - Relativistic: $P = \frac{2q^2}{3c^3}\gamma^4 \left(a_\perp^2 + \gamma^2 a_\parallel^2\right)$
- Circular motion: $a = a_\perp = \frac{\nu_B}{2\pi} v_\perp$ $\qquad P = 2c\sigma_T U_B p_\perp^2$
- Isotropic distribution of pitch angles: $p_\perp = p \sin\alpha$

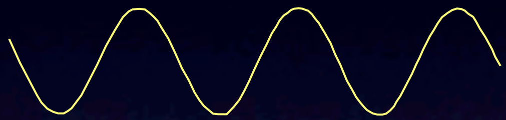
$$\boxed{P = \frac{4}{3} c\sigma_T U_B p^2}$$

- Maximal loss limit: $\gamma mc^2/P > 1/\nu_B$ => $\gamma^2 B < \frac{2q}{r_0^2} = 1.2 \times 10^{16}$ G

Synchrotron Emission Spectrum

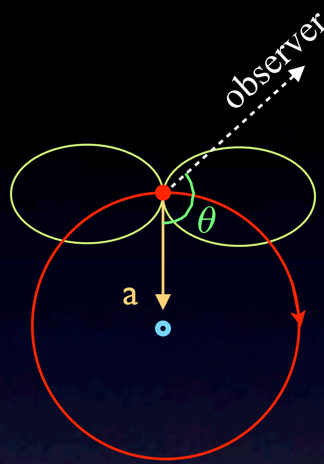


- ✓ Particles nearly at rest:
  - ✓ Simple modulation of the electric field at ν_B: $E(t) = \sin(2\pi\nu_B t)$



  - ✓ Spectrum = one line at ν_B

Synchrotron Emission Spectrum



$\vec{E} \propto \sin\theta$

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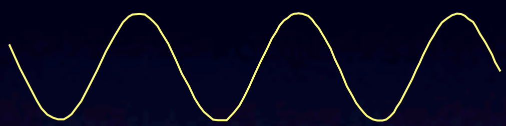

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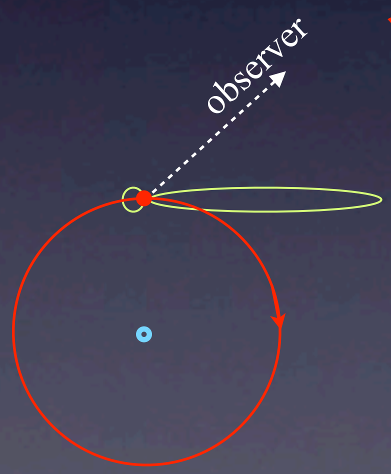
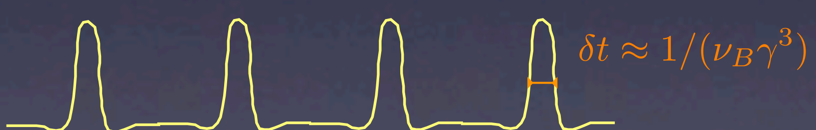
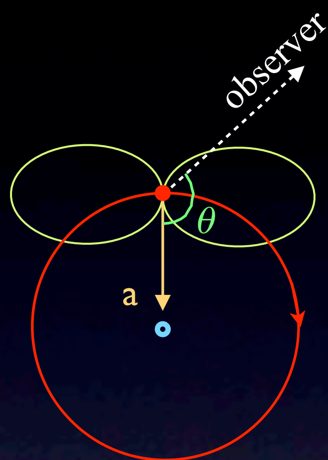


- Relativistic particles:
  - Relativistic beaming: $\delta\theta = 1/\gamma$
  - Pulsed modulation of the electric field at ν_B:



  - Multiple harmonics of ν_B up to a critical frequency:
    $$\nu_c = \frac{3}{2}\gamma^3 \nu_B \sin\alpha$$
  - Spectrum = many lines at $k\nu_B$
  - For $\gamma >> 1$: continuum

Synchrotron Emission Spectrum



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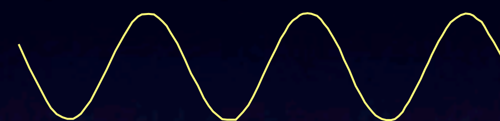

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- Relativistic particles:
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$\delta t \approx 1/(\nu_B \gamma^3)$

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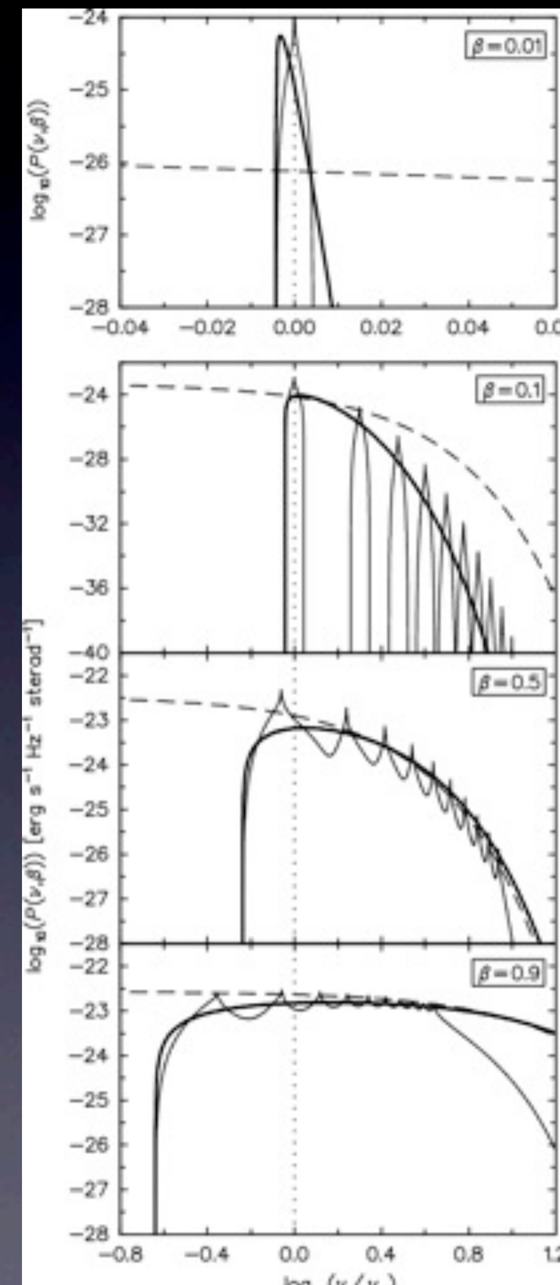

Synchrotron Emission Spectrum

✓ Exact spectrum of mono-energetic particles of any energy (erg/s/Hz):

$$\frac{\partial P}{\partial \nu}(\alpha, p, \nu) = \frac{2\pi q^2 \nu^2}{c} \int \sum_{n=1}^{\infty} \left[\frac{(\cos\theta - \beta_{\parallel})^2}{\sin^2\theta} J_n^2(x) + \beta_{\perp}^2 J_n'^2(x) \right] \delta\left(\nu(1 - \beta_{\parallel}\cos\theta) - n\nu_B\right) d\Omega$$

$$x = (\nu/\nu_B)\beta_{\perp}\sin\theta$$

$$\tan\alpha = \beta_{\perp}/\beta_{\parallel}$$

Marcowith&Malzac03

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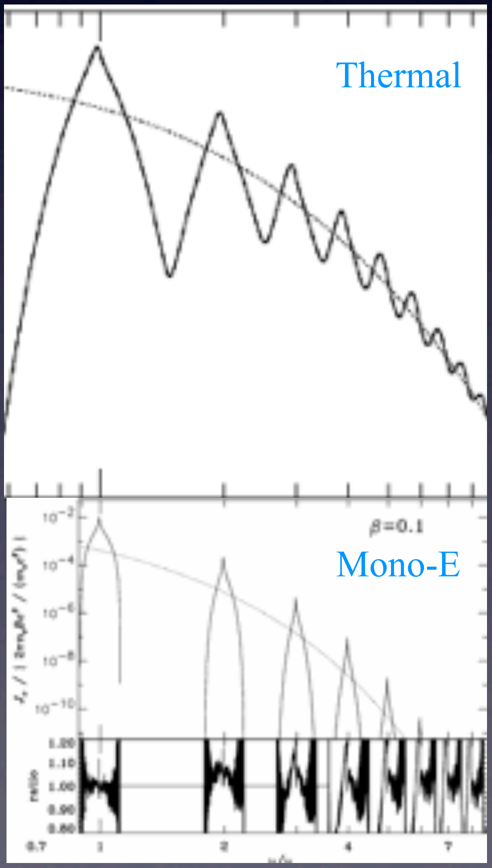


Marcowith&Malzac03

- ✓ Real systems have particle distributions in
  - ✓ Pitch angle
  - ✓ Energy

Convolution => Line broadening

Spectrum of Relativistic Particles

- ✓ Relativistic particles have a continuous spectrum (erg/s/Hz)
  - ✓ Pitch-angle dependent spectrum:

$$\frac{\partial P}{\partial \nu}(\alpha, p, \nu) = \frac{\sqrt{3} q^3 B}{mc^2} \sin\alpha F(\nu/\nu_c)$$

$$F(x) = x \int_x^\infty K_{5/3}(z)dz \qquad \nu_c = \frac{3}{2}\nu_B \gamma^3 \sin\alpha$$

  - ✓ Pitch-angle averaged spectrum:

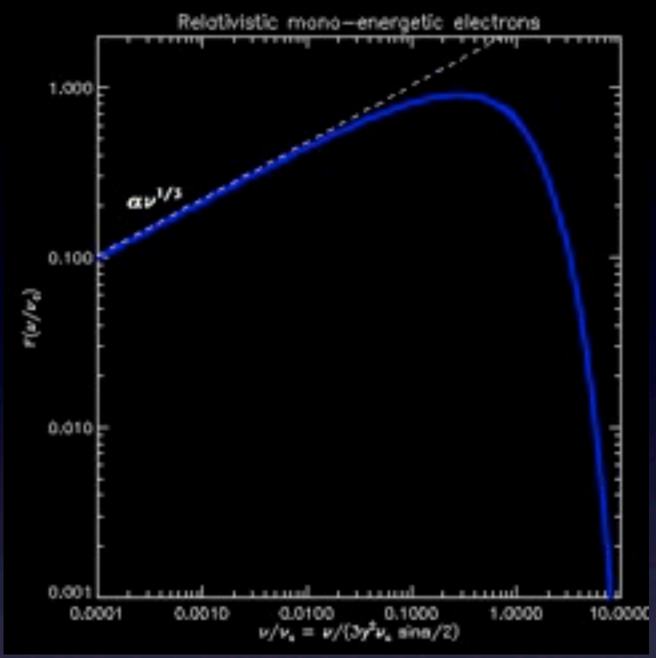
$$\frac{\partial P}{\partial \nu}(\gamma, \nu) = \frac{12\sqrt{3}\sigma_T c U_B}{\nu_L} G\left(\frac{\nu}{2\nu_c^*}\right)$$

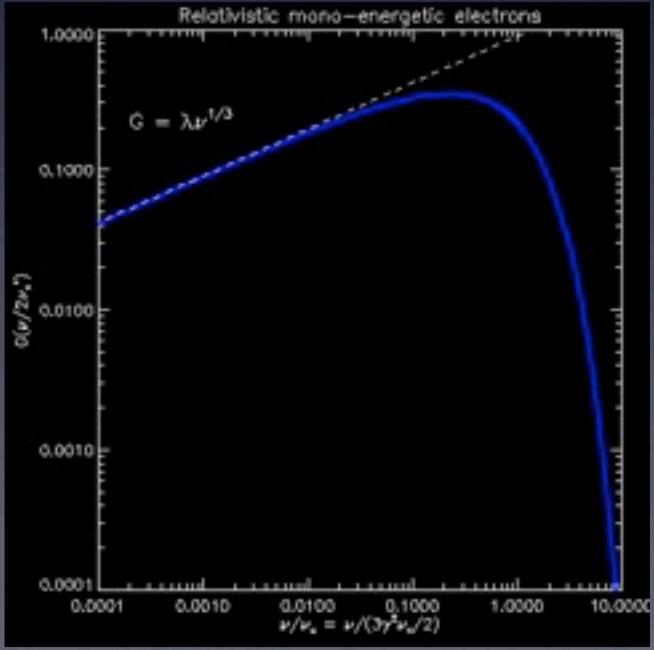
$$G(x) = x^2 \left[K_{4/3}(x)K_{1/3}(x) - \frac{3x}{5}\left(K_{4/3}^2(x) - K_{1/3}^2(x)\right)\right]$$

- ✓ Emission peaked at: $\boxed{\nu \approx \nu_c^* = \frac{3}{2}\nu_L \gamma^2}$ $\nu \propto B\gamma^2$

- ✓ Maximal energy

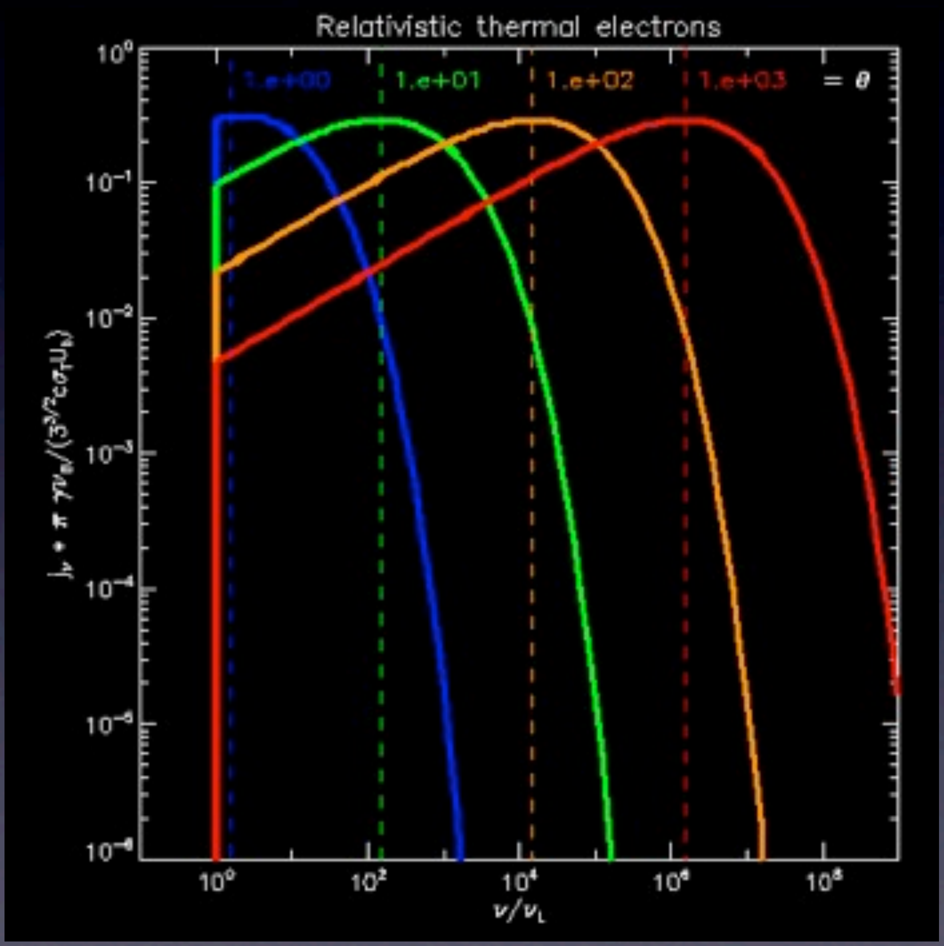
$$\gamma^2 B < \frac{2q}{r_0^2} \qquad h\nu = 3\ mc^2/\alpha_f = 70\ \text{MeV}$$





Spectrum of Relativistic Particles

$N_\gamma \propto \gamma^2 e^{-\gamma/\theta}$ for $\theta = k_B T / m_e c^2 > 1$



Spectrum peaking at $h\nu = h\nu_c^*(\theta) = 3/2\ h\nu_L\ \theta^2$

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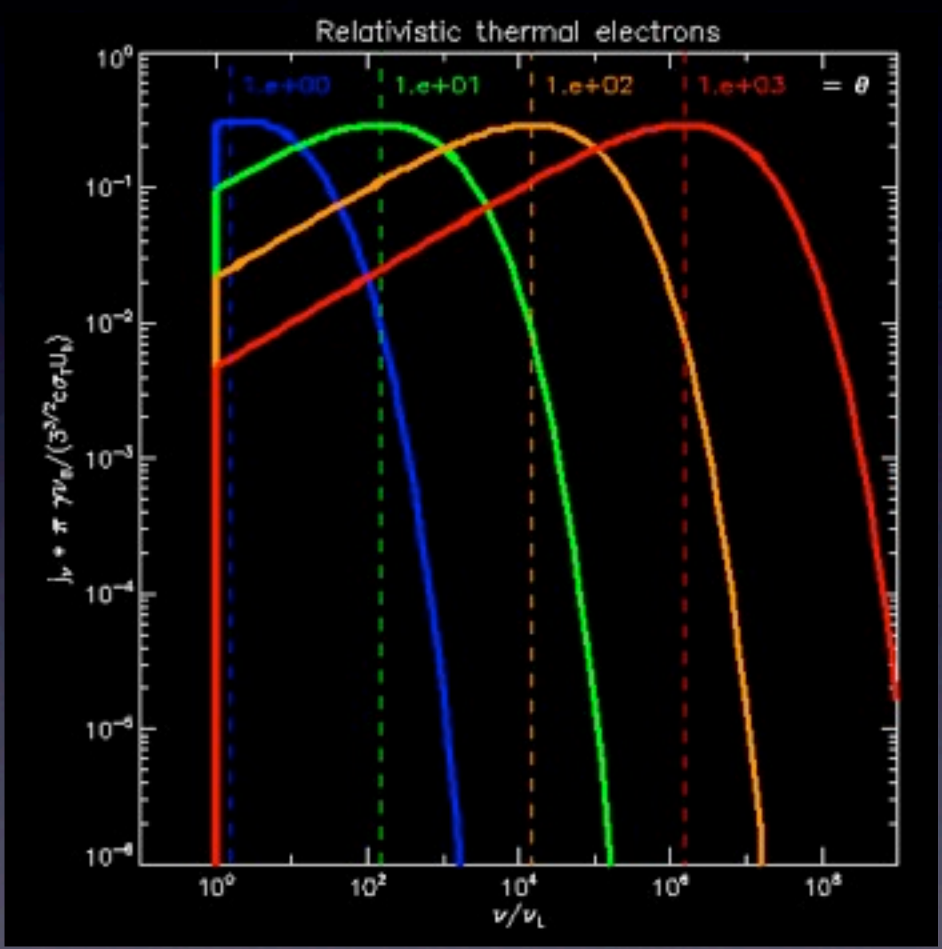
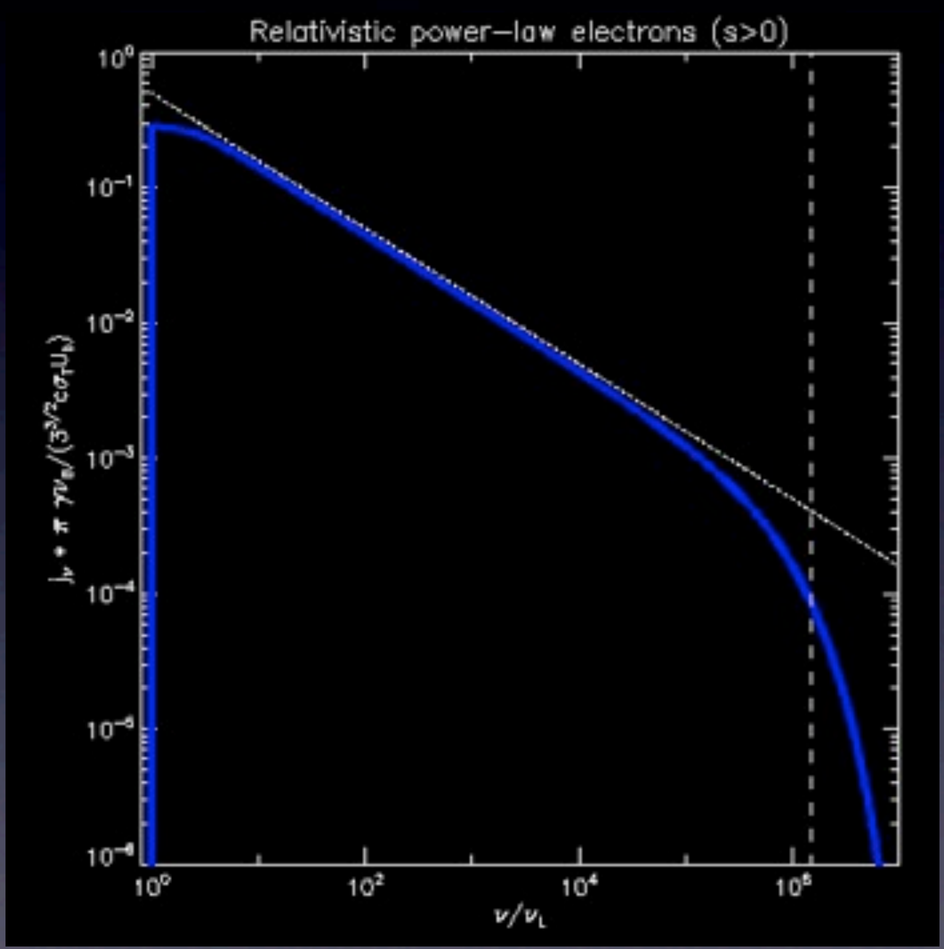


$N_\gamma \propto \gamma^{-s}$ for $1 < \gamma_{\min} < \gamma < \gamma_{\max}$

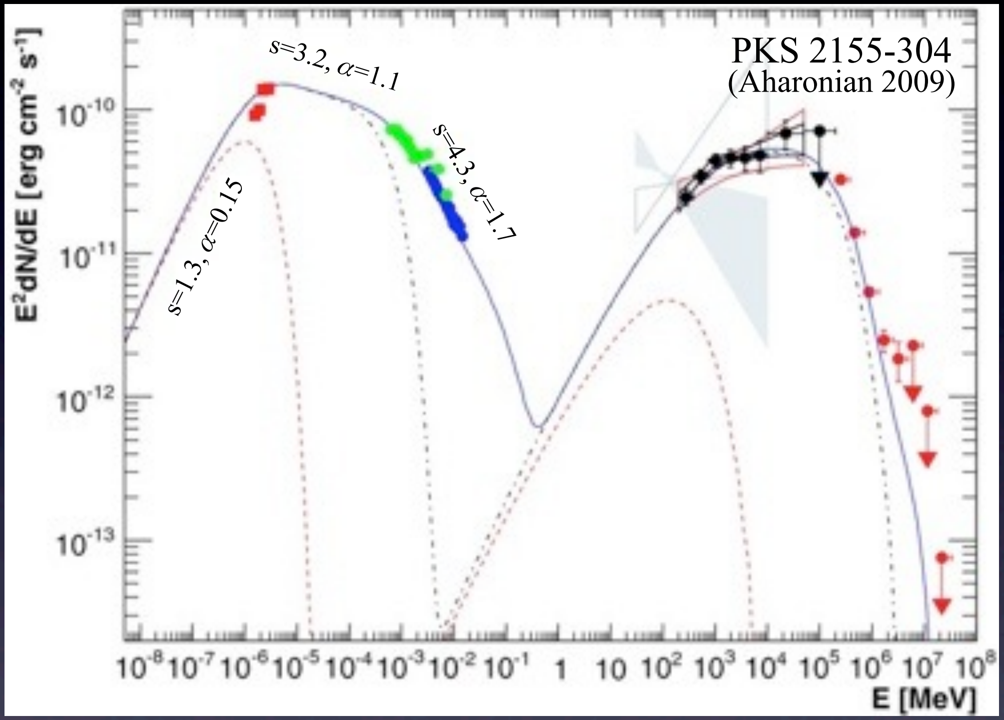


Spectrum peaking at $h\nu = h\nu_c^*(\theta) = 3/2\ h\nu_L\ \theta^2$

Power-law spectrum $P = \nu^{-\alpha}$
with slope: $\alpha=(s-1)/2$
cutoff: $h\nu = h\nu_c^*(\gamma_{max}) = 3/2\ h\nu_L\ \gamma_{max}^2$

Some Examples

Blazars:



A model:

One-zone

Broken power law distributions

Doppler factor

Emission in Optical-X-ray: need for TeV electrons (B=10 mG)

Seed photons for comptonization ?

X-ray binaries:

Some Examples

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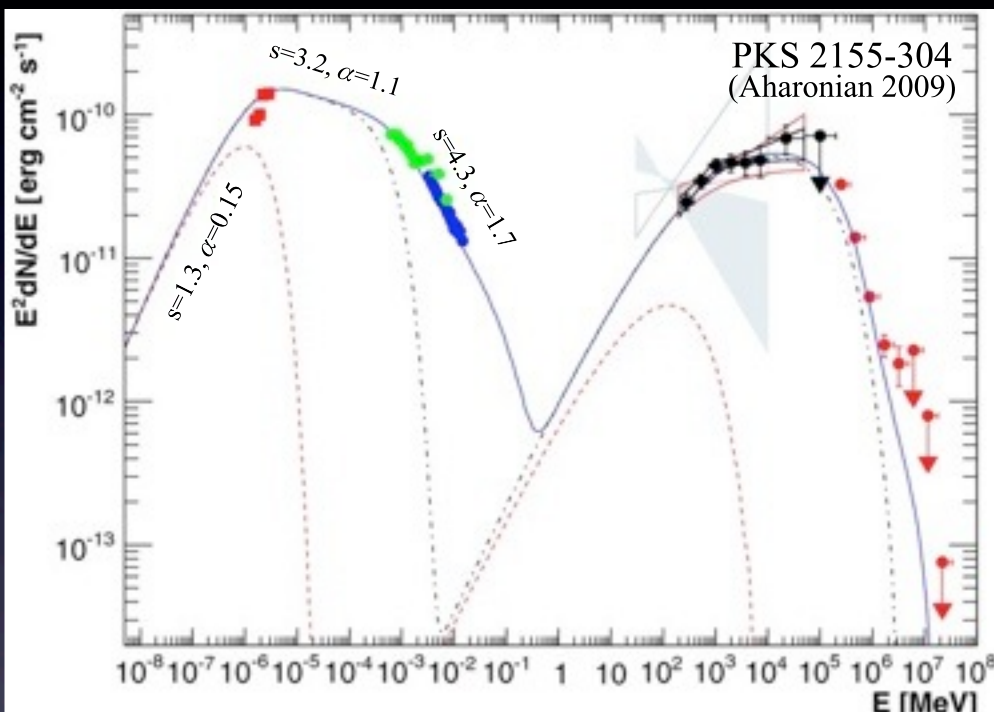
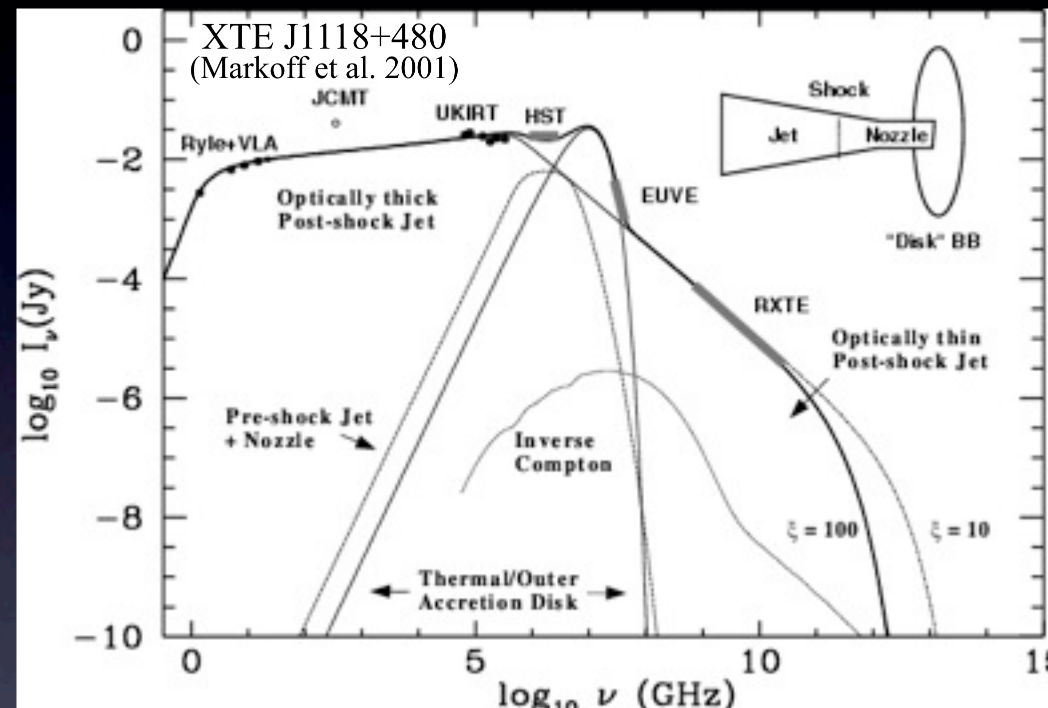


X-ray binaries:



A model:

- One-zone
- Broken power law distributions
- Doppler factor

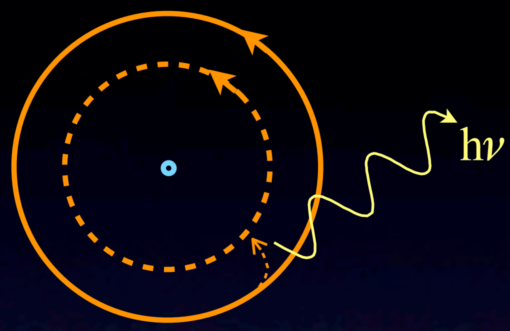
Emission in Optical-X-ray: need for TeV electrons (B=10 mG)

Seed photons for comptonization ?

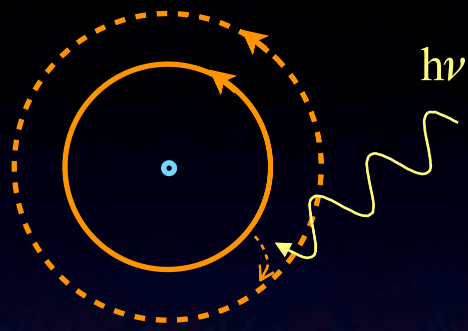
Model:

- Sum of the emission from various regions of the jet

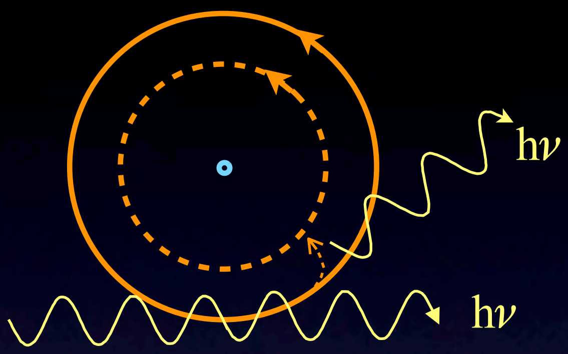
Synchrotron self-absorption



Spontaneous emission

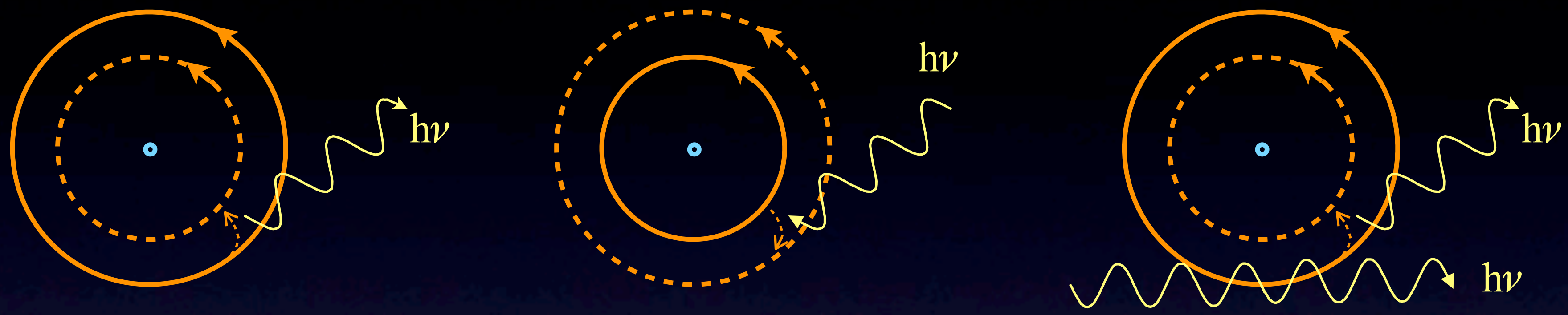


True absorption



Stimulated emission
(negative absorption)

Synchrotron self-absorption



Spontaneous emission

True absorption

Stimulated emission
(negative absorption)

- ✓ Transition probabilities described by Einstein coefficients
- ✓ Absorption ($\mathrm{cm^{-1}Hz^{-1}}$) can be expressed as a function of the emissivity:

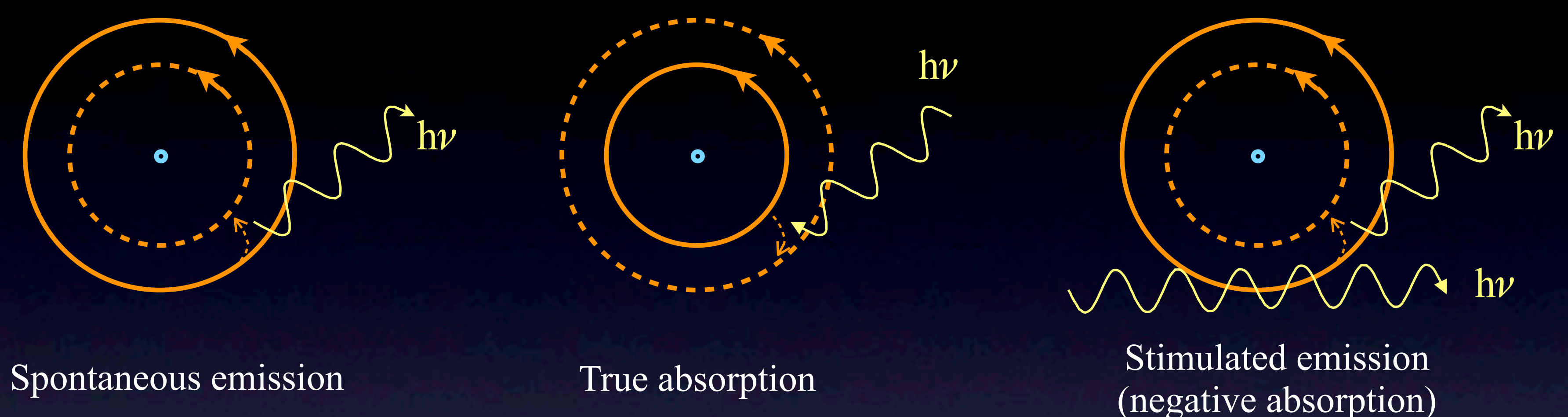
True absorption

$$\alpha_\nu(p,\nu) = \frac{c^2}{2mh\nu^3}\frac{1}{p\gamma}\left[\gamma p j_\nu\right]_\gamma^{\gamma+h\nu/mc^2}$$

Stimulated emission
(negative absorption)

$$\alpha_\nu(p,\nu) \approx \frac{1}{2m\nu^2}\frac{1}{p\gamma}\partial_\gamma\left(\gamma p j_\nu\right)$$

Synchrotron self-absorption



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Stimulated emission
(negative absorption)

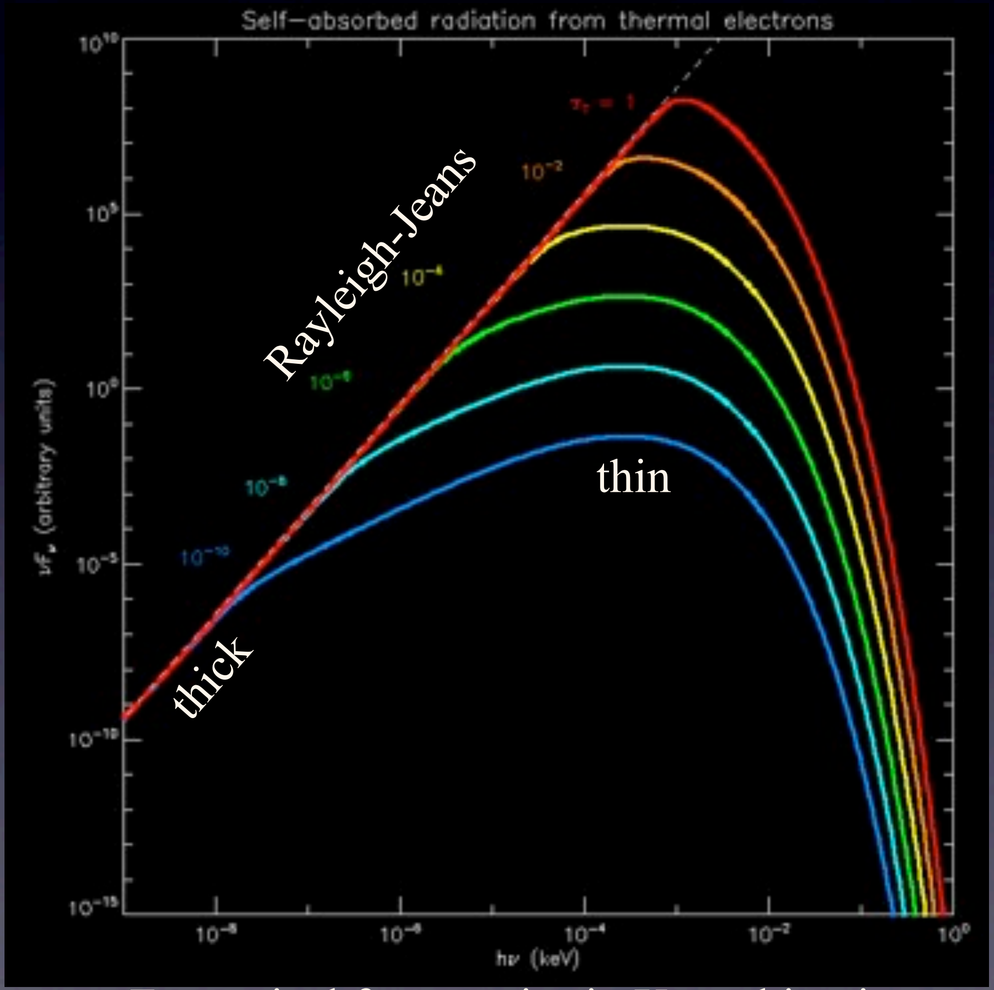
$$\alpha_\nu(p,\nu) \approx \frac{1}{2m\nu^2}\frac{1}{p\gamma}\partial_\gamma\left(\gamma p j_\nu\right)$$

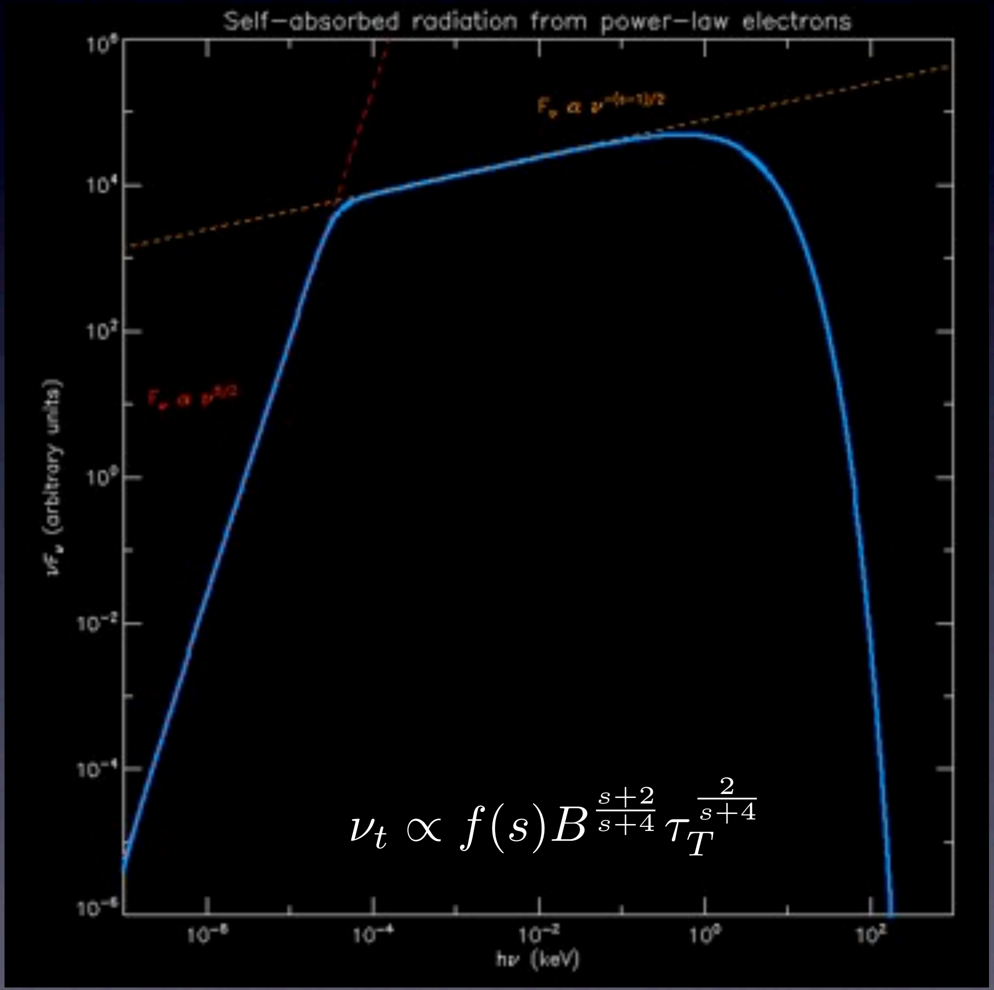
- ✓ Radiative transfer problem: $I_\nu \approx \frac{j_\nu}{\alpha_\nu}\left(1-e^{-\alpha_\nu L}\right)$
- ✓ Transition thick/thin at the turnover frequency defined by: $\alpha_\nu(\nu_t)L \approx 1$

Self-absorbed Spectra

✓ Thermal distribution

✓ Power-law distribution





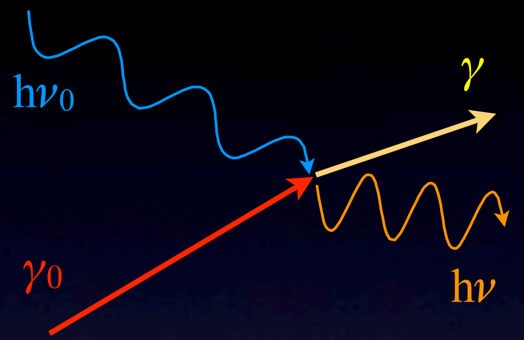
$$\nu_t \propto f(s) B^{\frac{s+2}{s+4}} \tau_T^{\frac{2}{s+4}}$$

For typical frequencies in X-ray binaries:

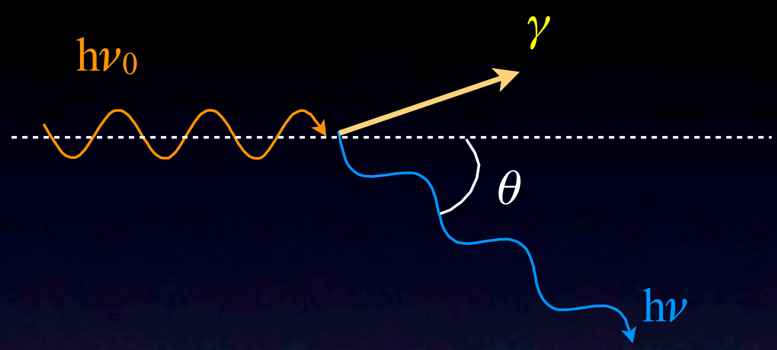
Wardzinsky&Zdziarsky00

Compton Scattering

In the lab frame:



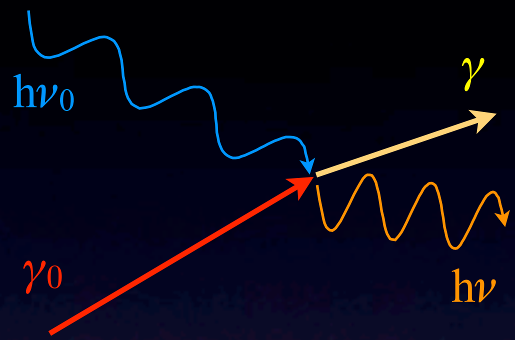
In the electron rest frame:



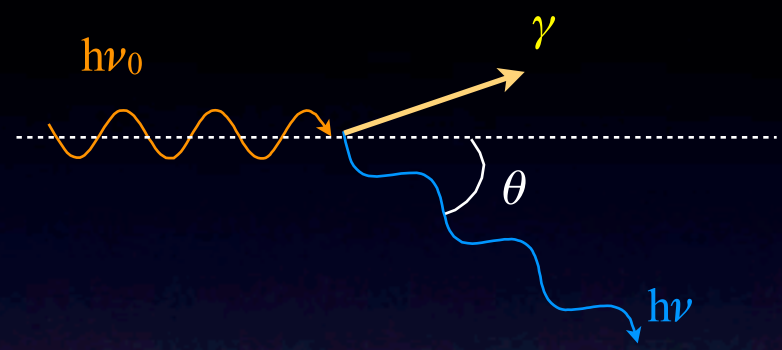
✓ Energy and momentum conservation: $$\frac{h\nu}{h\nu_0} = \frac{1}{1 + \frac{h\nu_0}{m_e c^2}(1 - \cos\theta)}$$

Compton Scattering

In the lab frame:



In the electron rest frame:

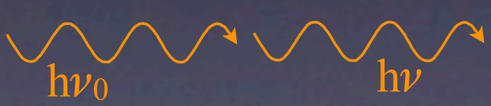


✓ Energy and momentum conservation: $$\frac{h\nu}{h\nu_0} = \frac{1}{1 + \frac{h\nu_0}{m_e c^2}(1 - \cos\theta)}$$

$$\frac{h\nu_0}{1 + 2h\nu_0/m_e c^2} \quad \leq \quad h\nu \quad \leq \quad h\nu_0$$

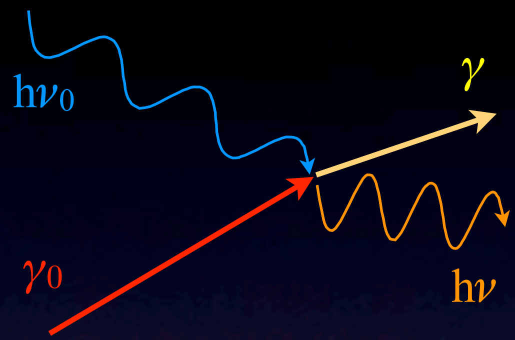
backward scattering (θ=π)

forward scattering (θ=0)

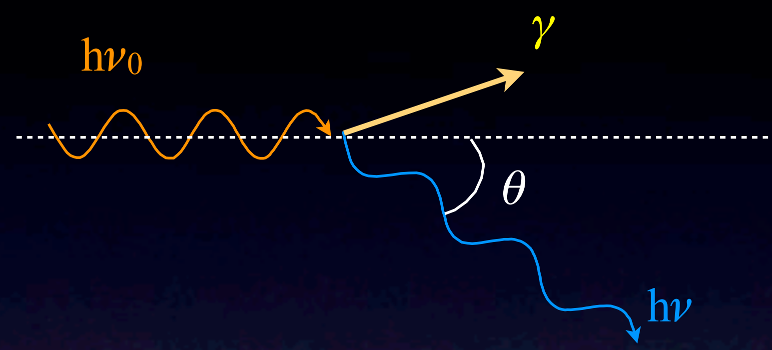


Compton Scattering

In the lab frame:



In the electron rest frame:



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$$\frac{h\nu_0}{1+2h\nu_0/m_e c^2} \leq h\nu \leq h\nu_0$$

backward scattering (θ=π)

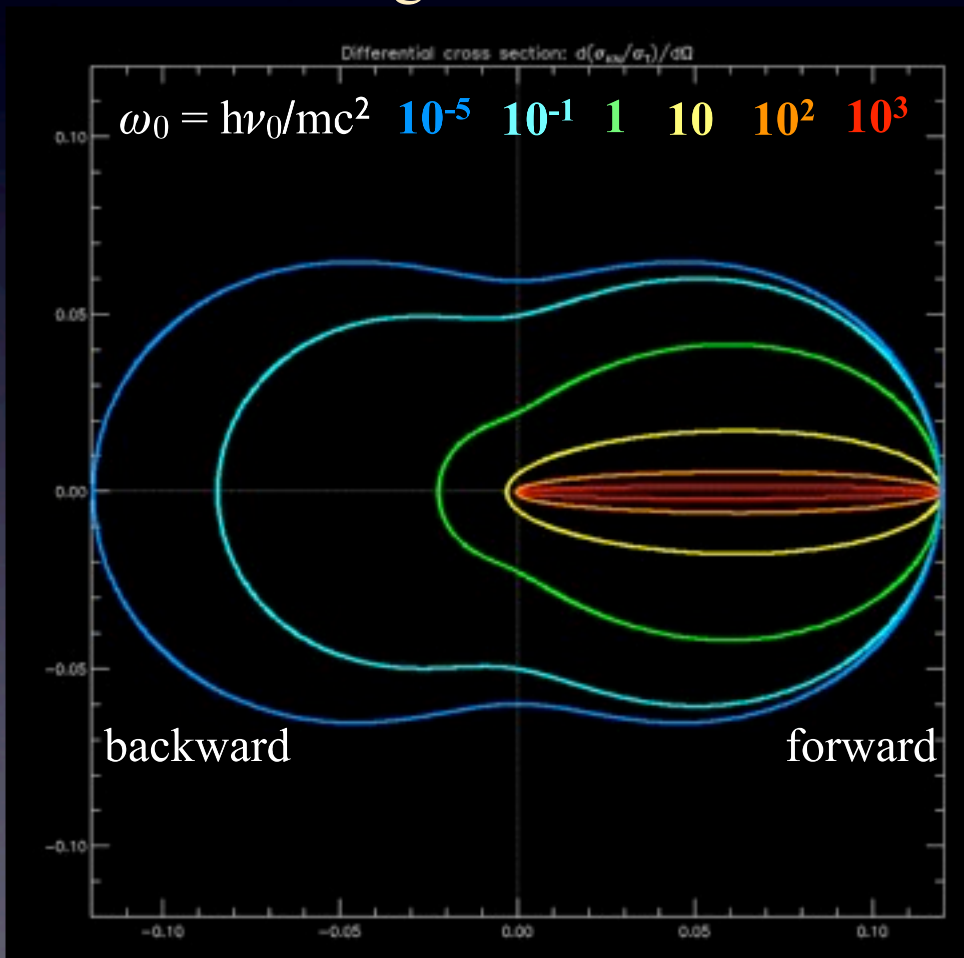
forward scattering (θ=0)

✓ For low energy photons ($h\nu_0 << mc^2$): Coherent scattering $h\nu_0 = h\nu$

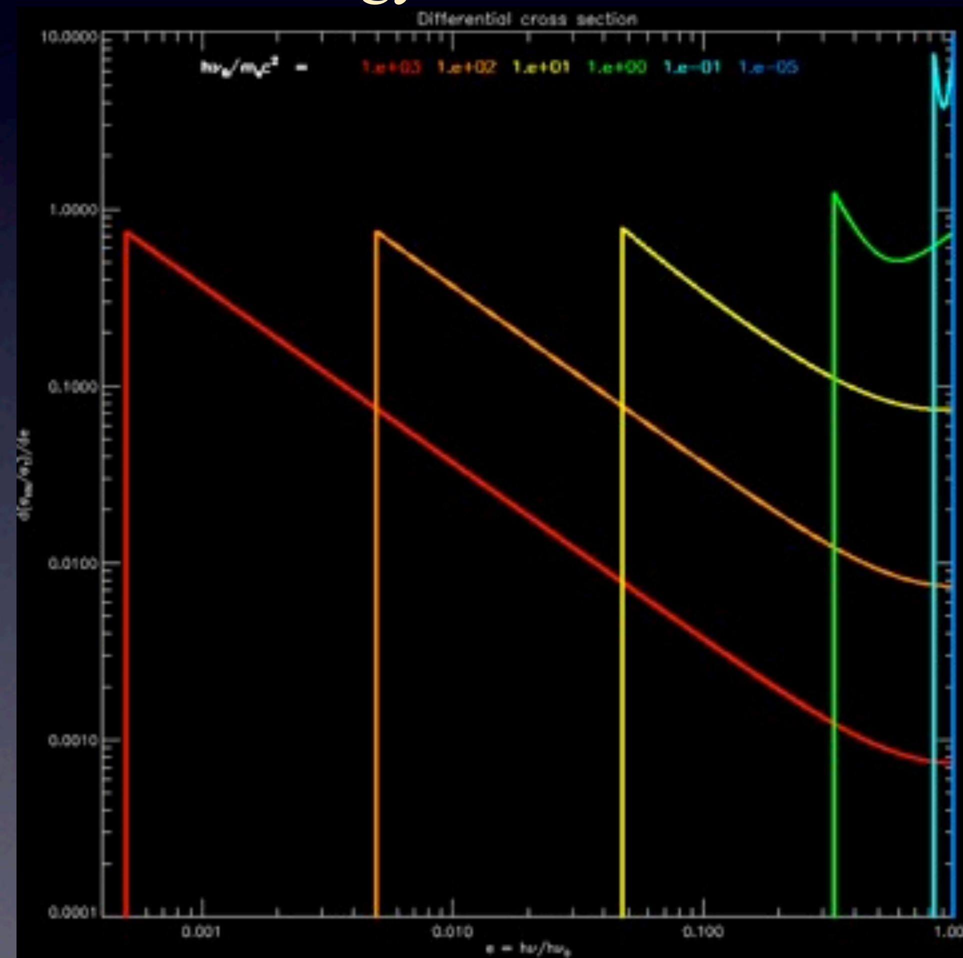
Compton differential cross section

$$\frac{d\sigma}{d\Omega} = \sigma_T \frac{3}{16\pi}\left(\frac{h\nu}{h\nu_0}\right)^2\left(\frac{h\nu_0}{h\nu} + \frac{h\nu}{h\nu_0} - \sin^2\theta\right)$$

Solid angle distribution



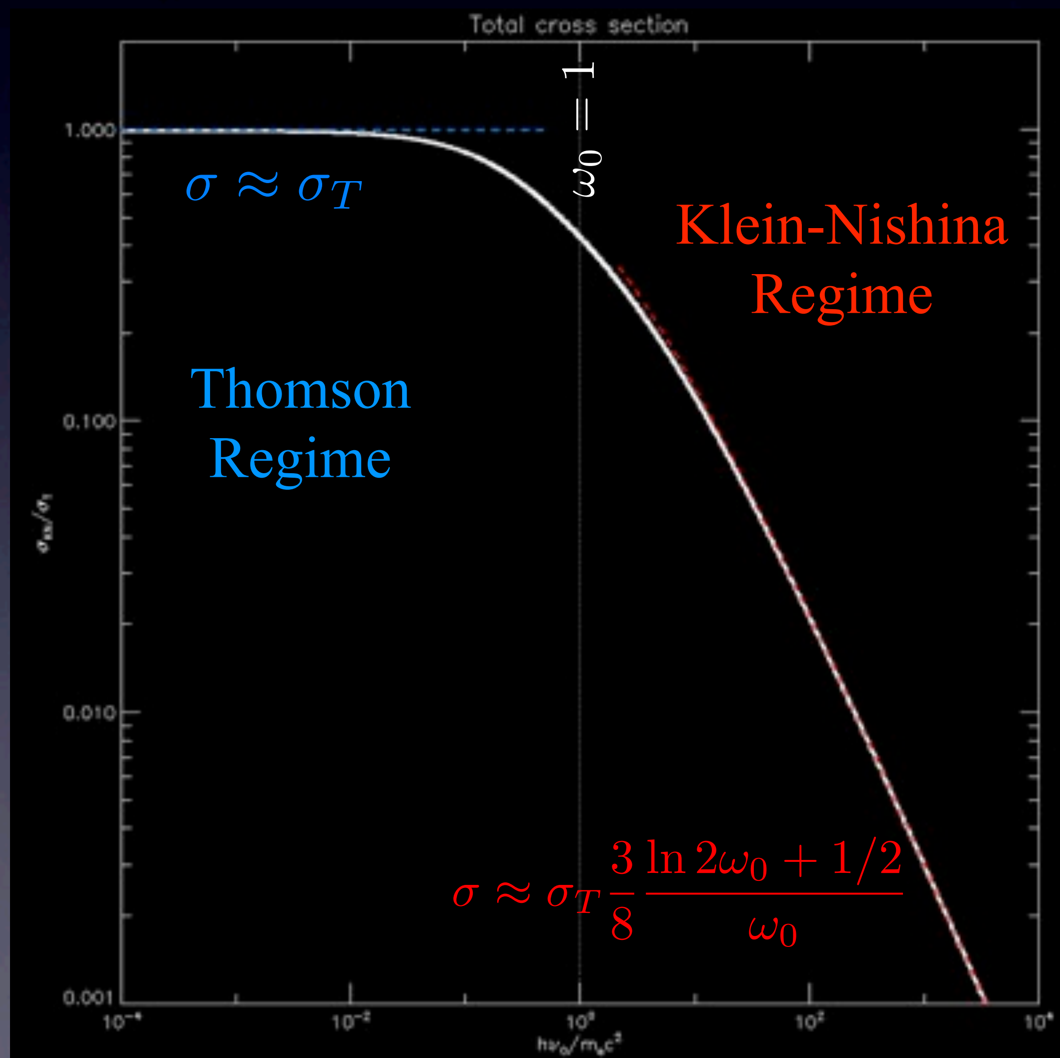
Energy distribution



(in the particle rest frame)

Compton Total cross section

$$\sigma = \sigma_T \frac{3}{4}\left[\frac{1+\omega_0}{\omega_0^3}\left(\frac{2\omega_0(1+\omega_0)}{1+2\omega_0} - \ln(1+2\omega_0)\right) + \frac{\ln(1+2\omega_0)}{2\omega_0} - \frac{1+3\omega_0}{(1+2\omega_0)^2}\right]$$

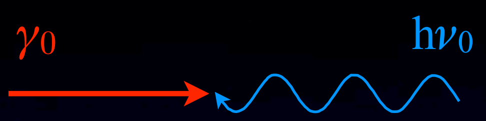


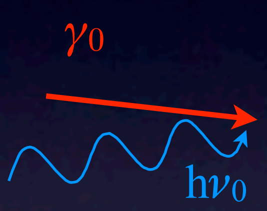
$$\omega_0 = \frac{h\nu_0}{m_e c^2}$$

(in the particle rest frame)

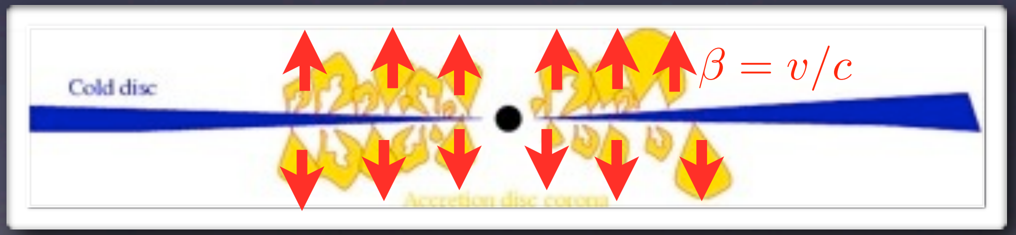
In the lab frame

- For one particle: doppler shift
  - Head-on collisions:
    - higher energy photon in the particle rest frame
    - larger energy change, scattering more anisotropic
  - Trailing collisions:
    - low energy photon in the particle rest frame
    - smaller energy change, scattering more isotropic





- For an angle distribution: integration over solid angles
  - highly anisotropic problems (e.g. outflowing corona...)



(Beloborodov 1999; Malzac et al. 2001)

- Isotropic medium (at least in a given frame):
    - Complicated formula (Jones 68, Nirganer&Poutanen 94, Belmont 08)

The Total cross section

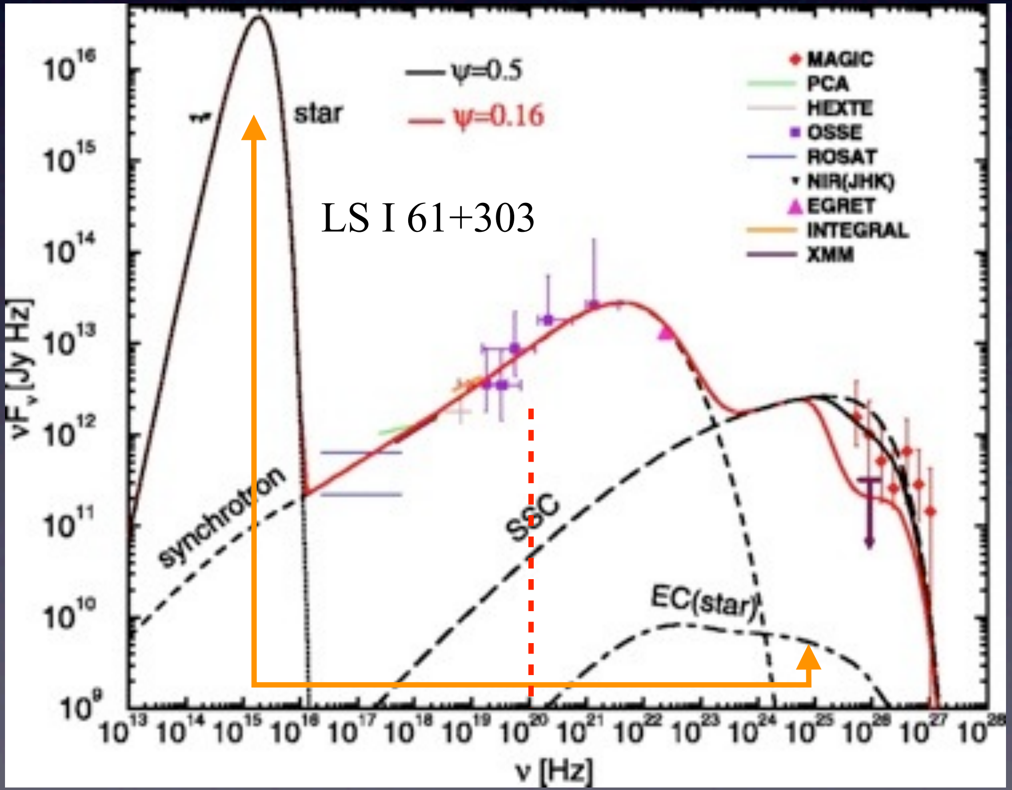
(in the plasma frame)

$$\sigma(\omega_0, p_0) \approx \sigma_T \frac{3}{4}\left[\frac{1+x}{x^3}\left(\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right) + \frac{\ln(1+2x)}{2x} - \frac{1+3x}{(1+2x)^2}\right]$$

$$x = \gamma_0 \omega_0$$

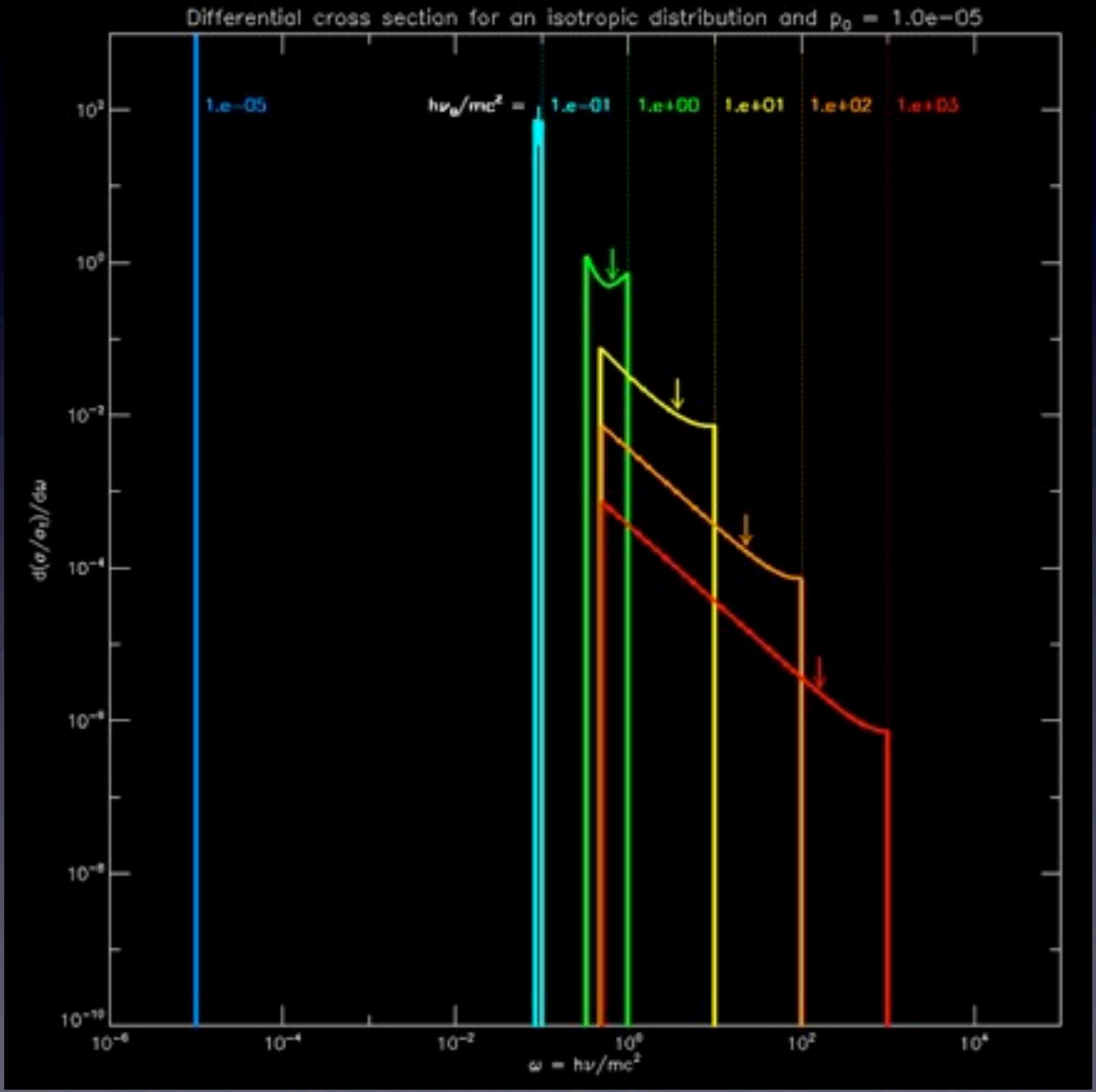
Thomson-KN transition at x=1:

	$h\nu_0$	$E_0=\gamma mc^2$
CMB	1 K	PeV
Radio jet of blazars	10^{13} Hz	10 TeV
Star	10 000 K	100 GeV
AGN accretion disk	10 eV	10 GeV
NS/SMBH accretion disk	1 keV	100 MeV

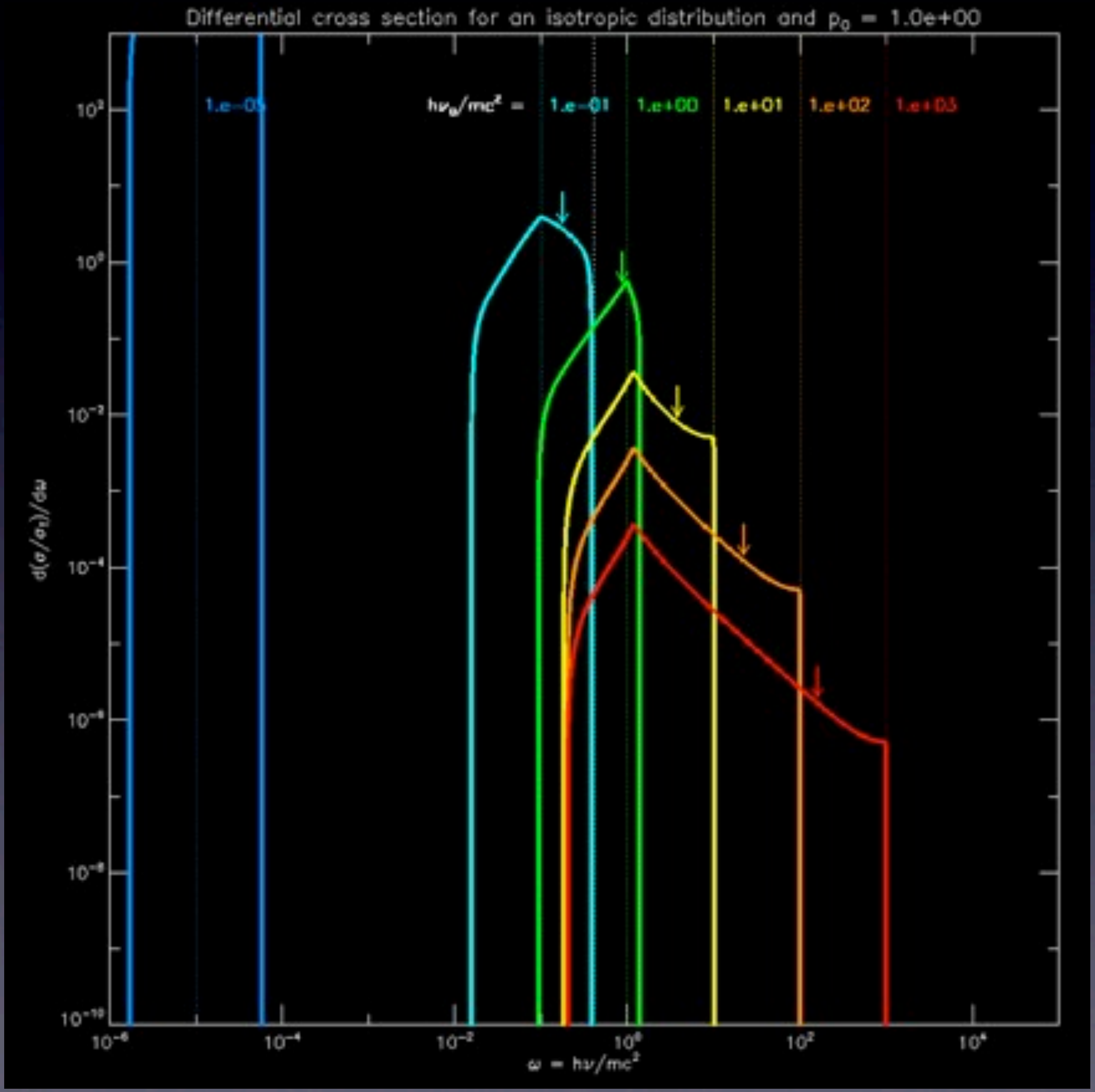


Boettcher 2006

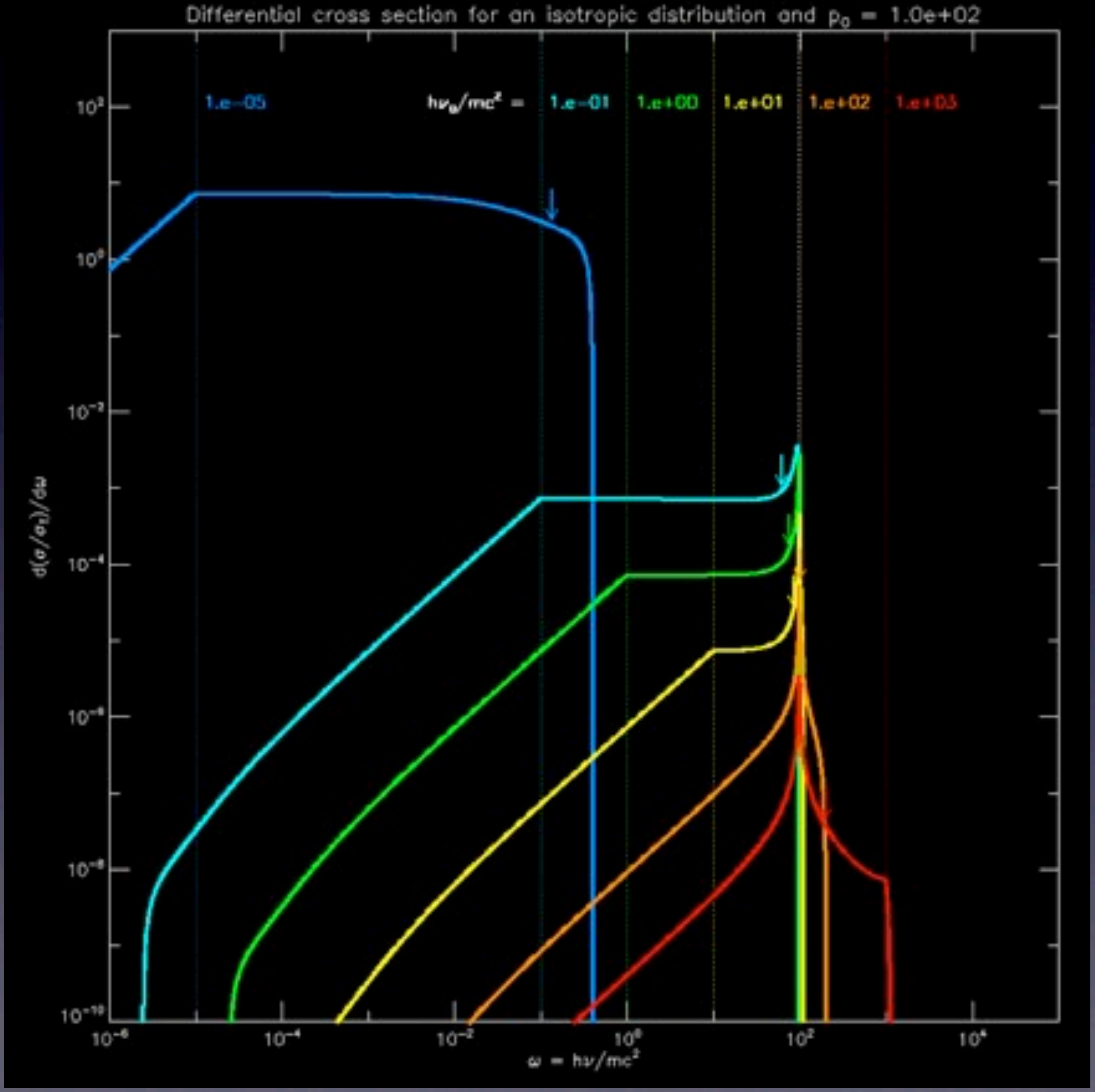
Spectrum of a single scattering



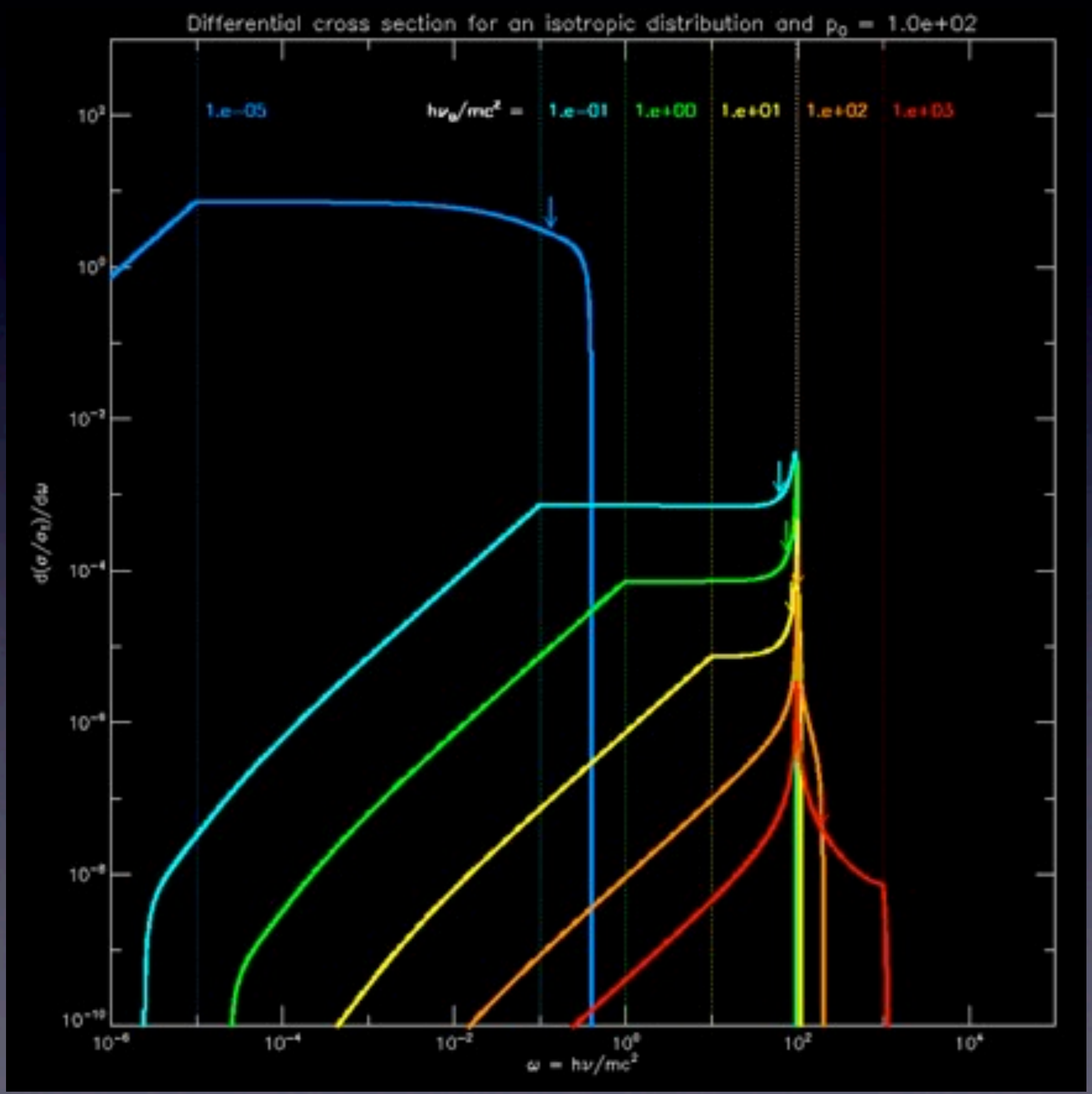
Spectrum of a single scattering



Spectrum of a single scattering



Spectrum of a single scattering



✓ γ^2-1 << ω: down-scattering

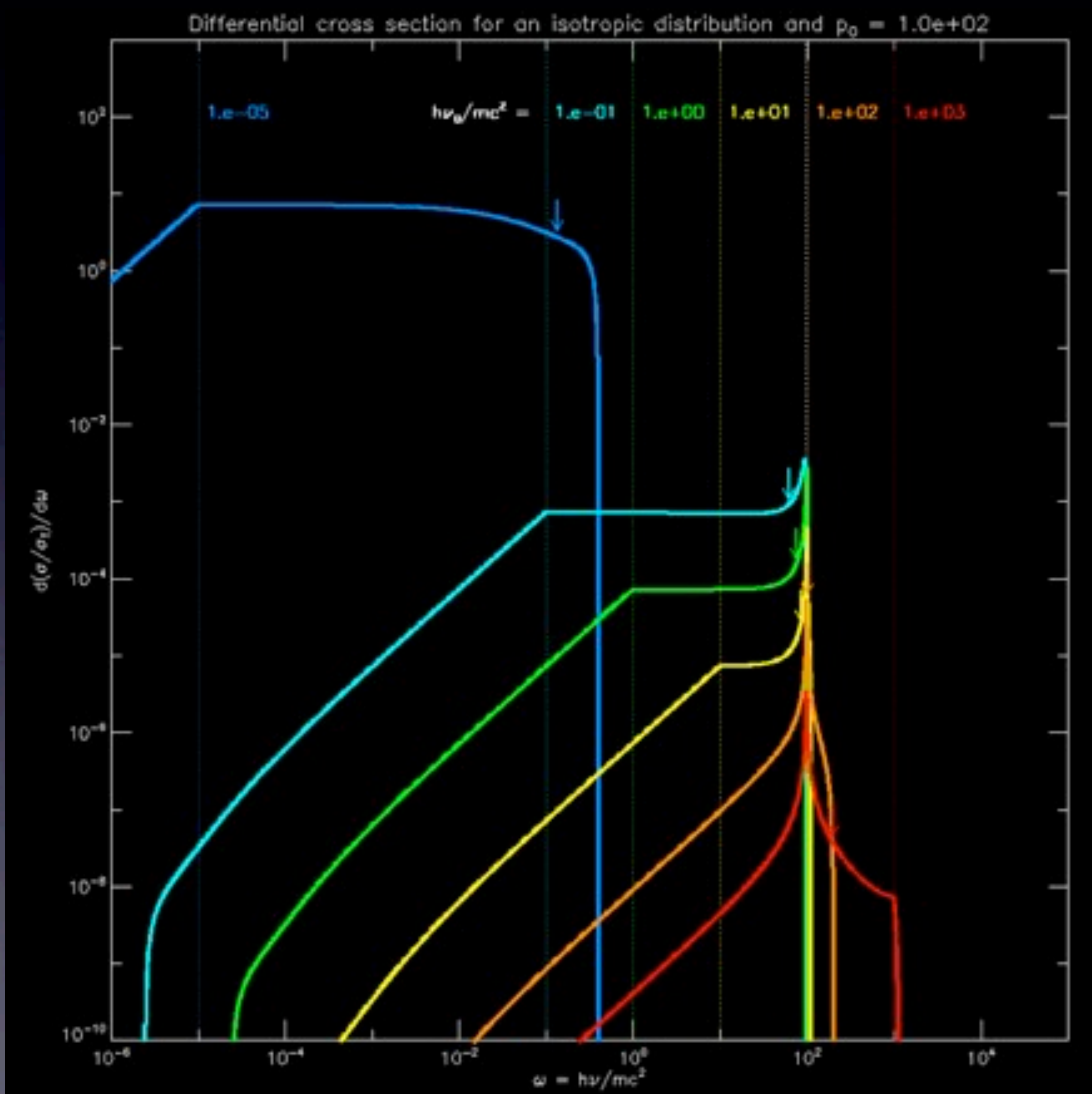
✓ γ^2-1 >> ω: up-scattering

- Amplification factor:

$$A = \frac{< \omega >}{\omega_0}$$

- In the Thomson regime ($\gamma\omega$<<1):

Spectrum of a single scattering



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✓ γ^2-1 >> ω: up-scattering

- Amplification factor:

$$A = \frac{<\omega>}{\omega_0}$$

- In the Thomson regime ($\gamma\omega$<<1):

$$A \approx 1 + \frac{4}{3}p^2$$

$$A \approx \frac{4}{3}p^2 \quad \text{for} \quad p >> 1$$

Effect of energy distributions

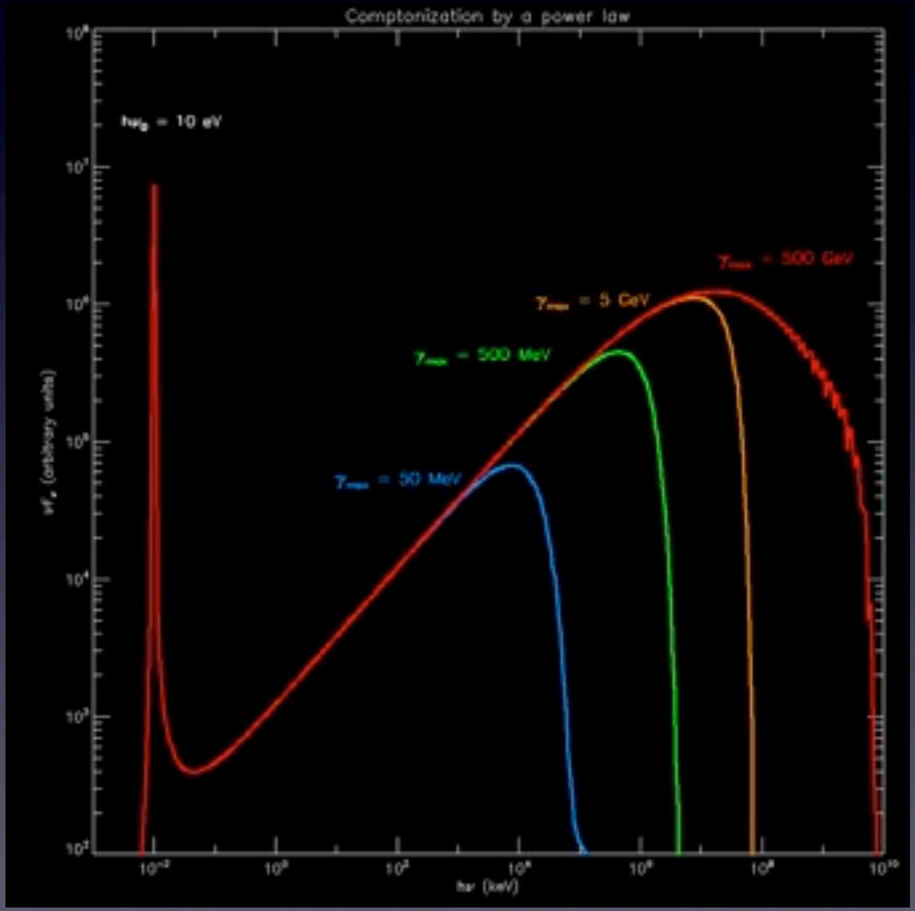
- Each photon can scatter off each electron => integration over both distribution

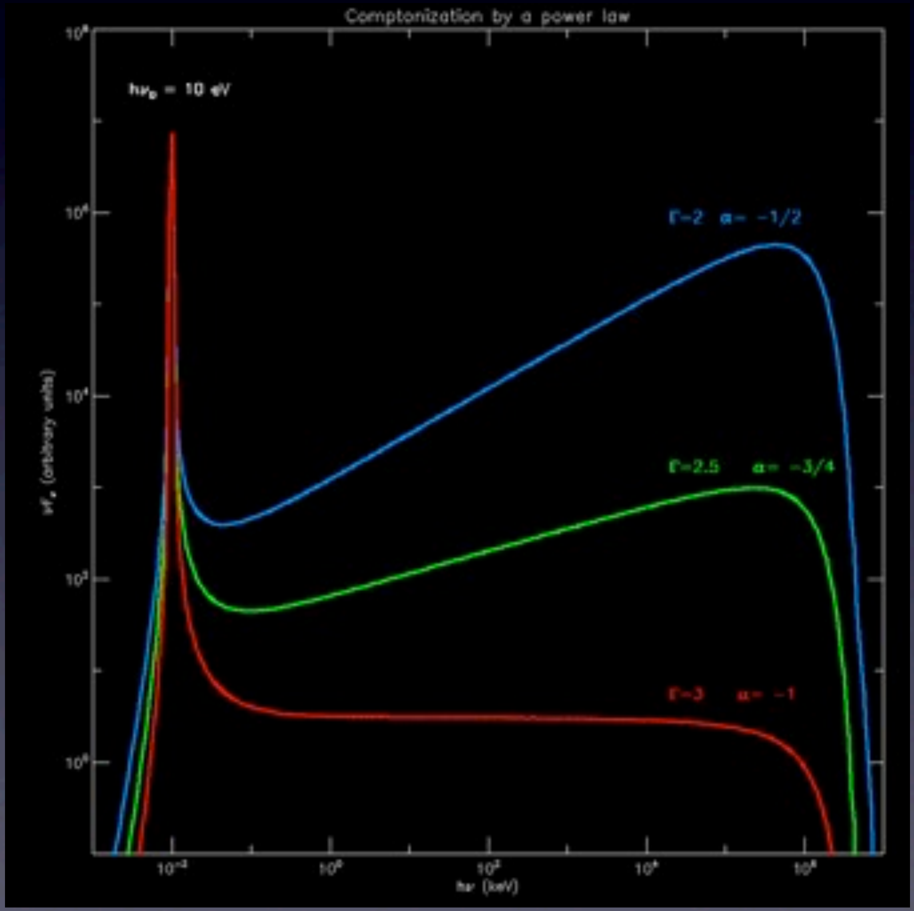
$$\frac{\partial N(\omega)}{\partial t} = \iint c \frac{\partial \sigma}{\partial \omega}(\omega_0, p_0) dN_p(p_0) dN_\omega(\omega_0)$$

- Up-scattering by relativistic particles in the Thomson regime:
  - Amplification: $A = 4/3 * p^2$
  - Energy exchange rate: $\frac{\partial E}{\partial t} = c\sigma_T \frac{4}{3}\omega p^2$
  - Total particle cooling: $\frac{\partial E_p}{\partial t} = \frac{4}{3} c\sigma_T p^2 U_{ph}$
- Similar to synchrotron losses

Single scattering off a power-law

- ✓ Power-law particle distribution $N(\gamma) = \gamma^{-s}$ for $\gamma_{min} < \gamma < \gamma_{max}$
- ✓ Spectrum: power-law: $F_\nu = \nu^{-\alpha}$





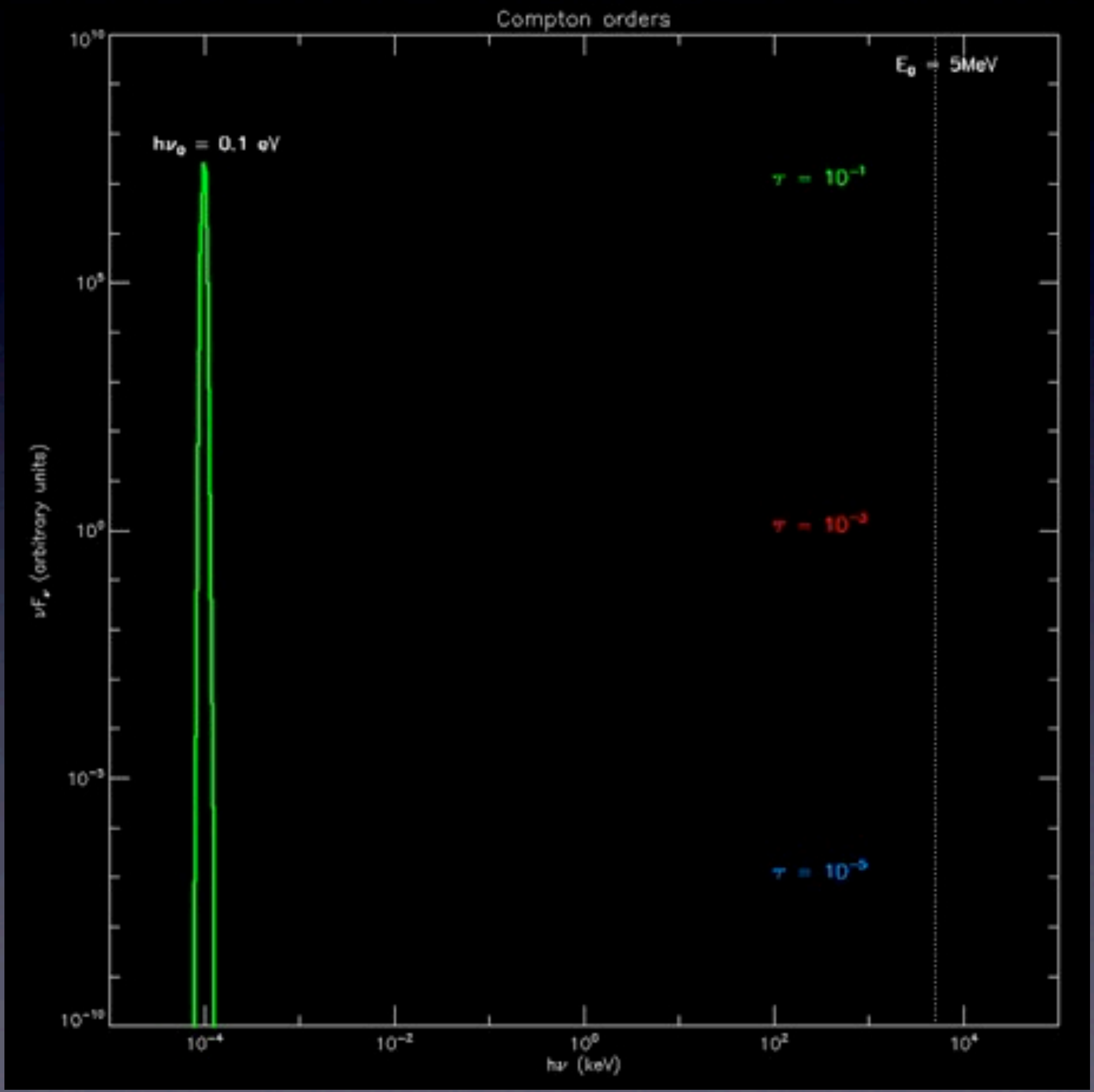
$h\nu_{max} = \gamma_{max} mc^2$

$\alpha=(s-1)/2$ (like synchrotron)

Multiple Scatterings

- Photons can undergo successive scattering
- Medium of finite size L: Thomson optical depth: $\tau = \sigma_T N_e L$
- Competition scattering/escape/absorption:
  - τ = Mean number of scattering before escape (or τ^2)
    - small τ: inefficient Compton scattering
    - large τ: efficient Compton scattering
- y parameter = <photon energy change> before escape
  - y= <Energy change per scattering> * <number scattering>
  - For thermal: $y=4\tau\theta(1+4\theta)$
- Absorption processes add to escape => effective optical depth

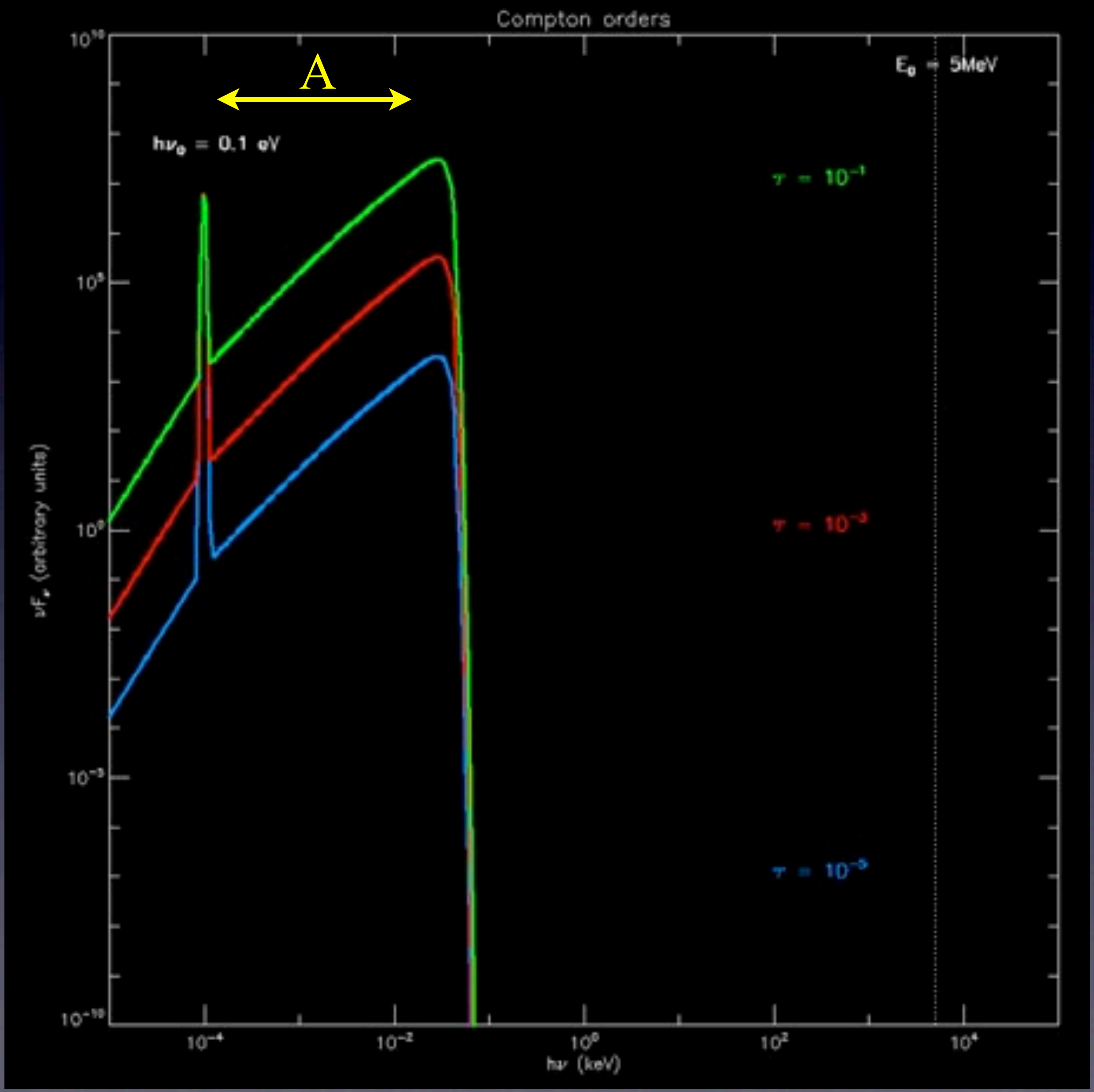
Compton orders



$\omega_0 = 10^{-7}$, $p_0 = 10$ $(\gamma_0 \omega_0 << 1)$

$A = 4/3\ p^2 = 100$

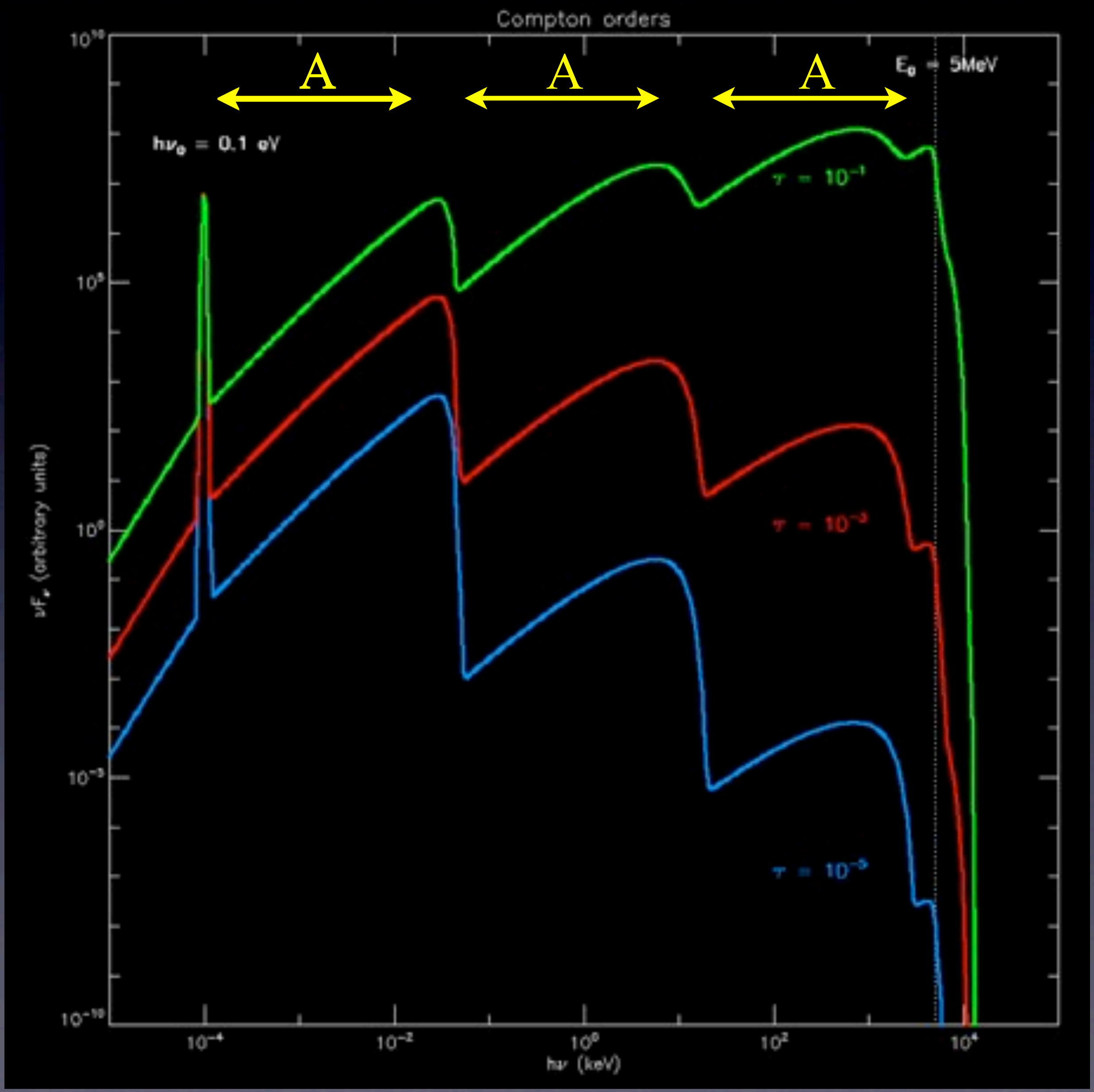
Compton orders



ω_0=10^{-7}, p_0=10 ($\gamma_0\omega_0$<<1)

A = 4/3 p^2 = 100

Compton orders



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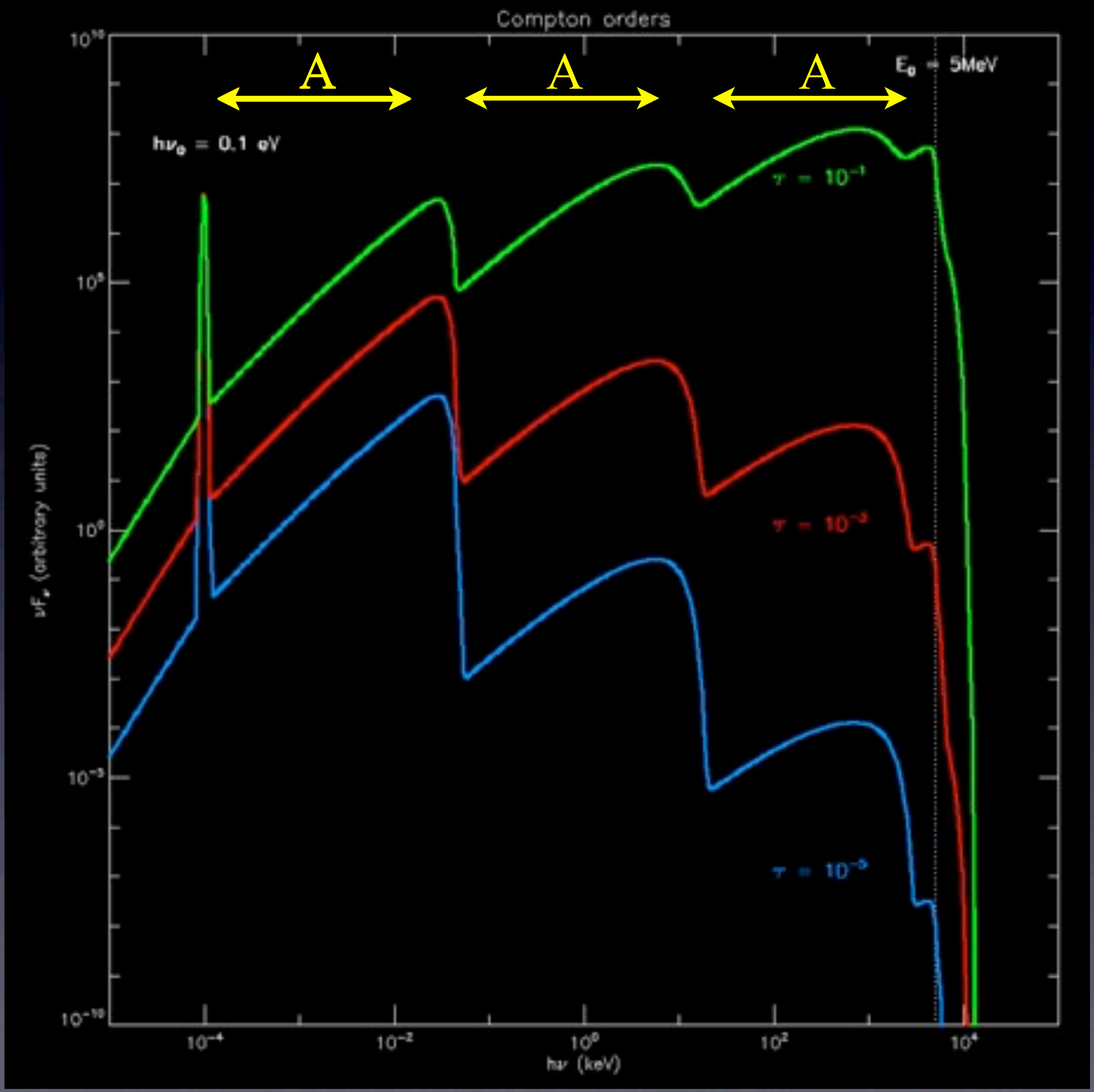
Compton orders



$\omega_0 = 10^{-7}$, $p_0 = 10$ ($\gamma_0 \omega_0 << 1$)
$A = 4/3\ p^2 = 100$

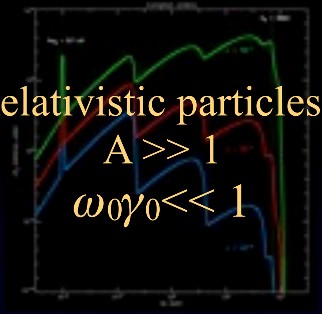
bumpy spectrum

cutoff at the particle energy

τ => spectrum hardness

Compton regimes

Sub-relativistic particles: $\omega_0 < p < 1$
Inefficient scattering: A = 1

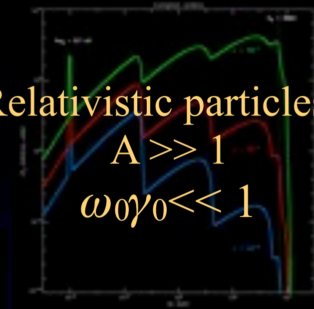


Relativistic particles...
A >> 1
$\omega_0 \gamma_0 << 1$

Ultra-relativistic particles: $\omega_0 \gamma_0 >> 1$
A >> 1
$A\,\omega_0 > \gamma_0$

Compton regimes

Sub-relativistic particles: ω_0<p<1
Inefficient scattering: A = 1



Relativistic particles...
A >> 1
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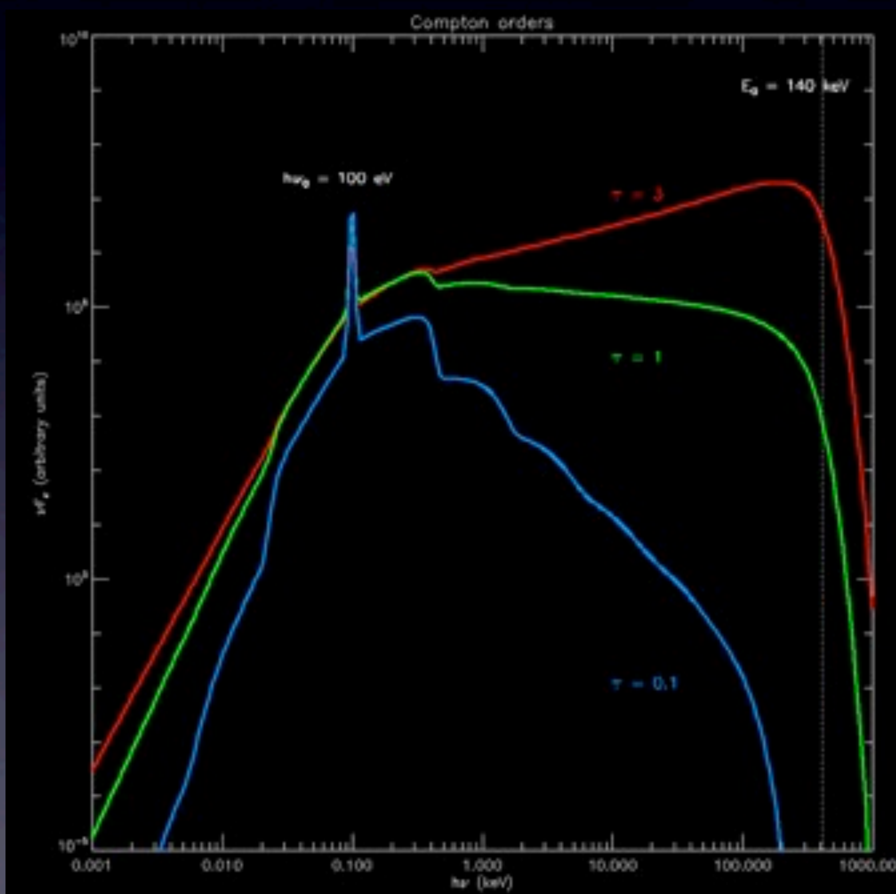
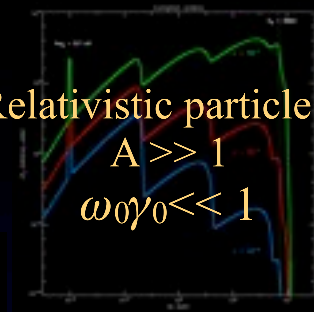
A >> 1
A ω_0 > γ_0



Power-law spectrum
- cutoff at the particle energy
- Slope = ln(τ)/ln(A) (ref)

Compton regimes

Sub-relativistic particles: $\omega_0 < p < 1$
Inefficient scattering: $A = 1$



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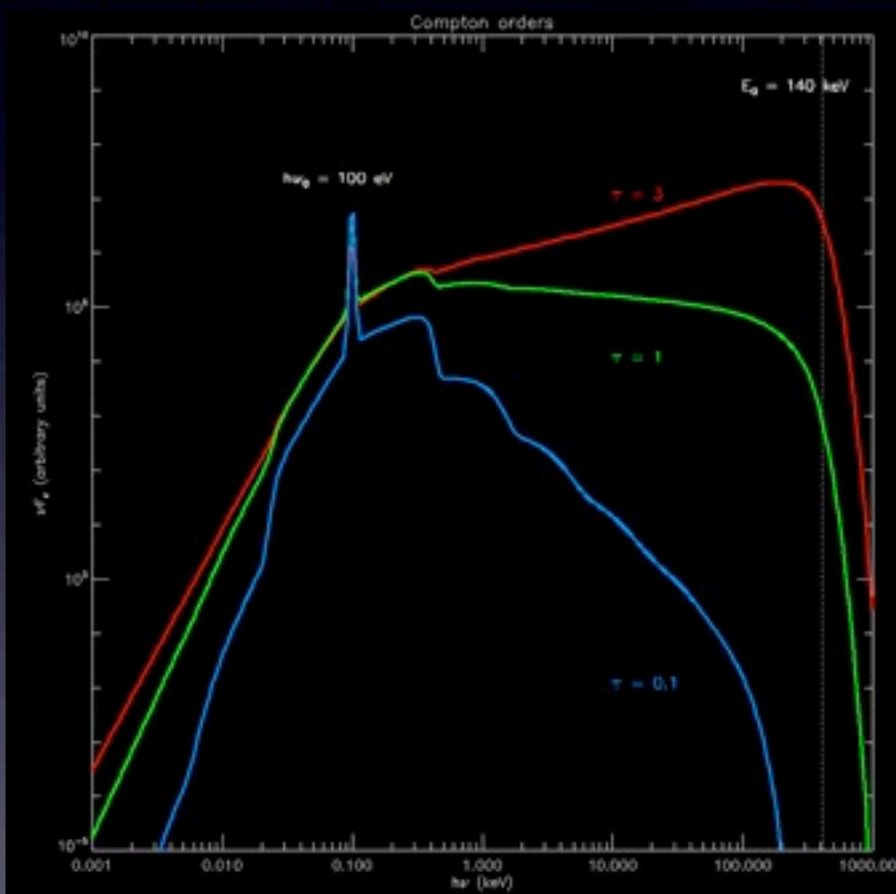

Power-law spectrum
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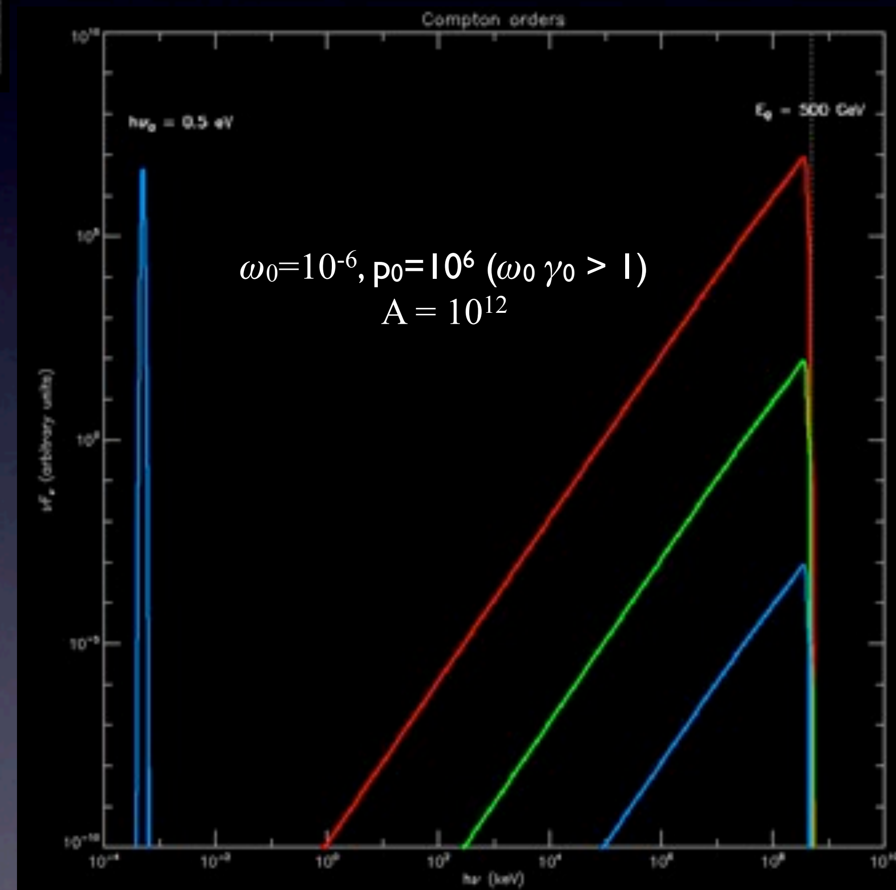
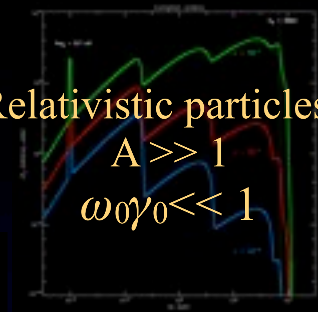


one-bump spectrum
= single scattering !

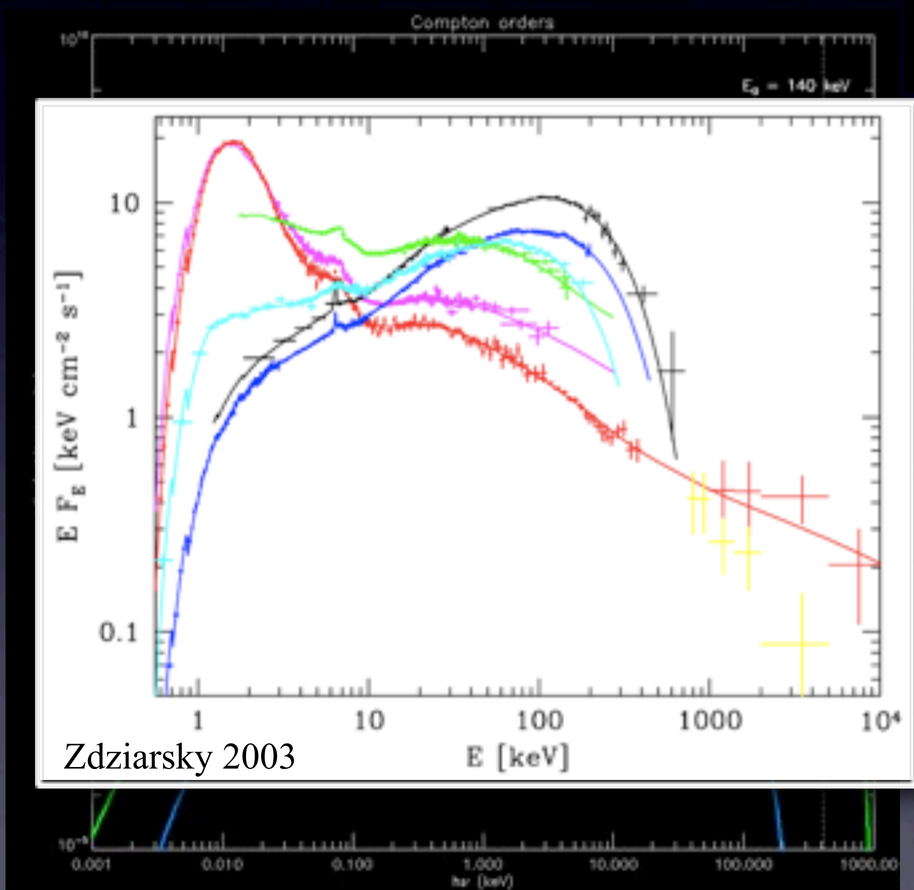
Compton regimes

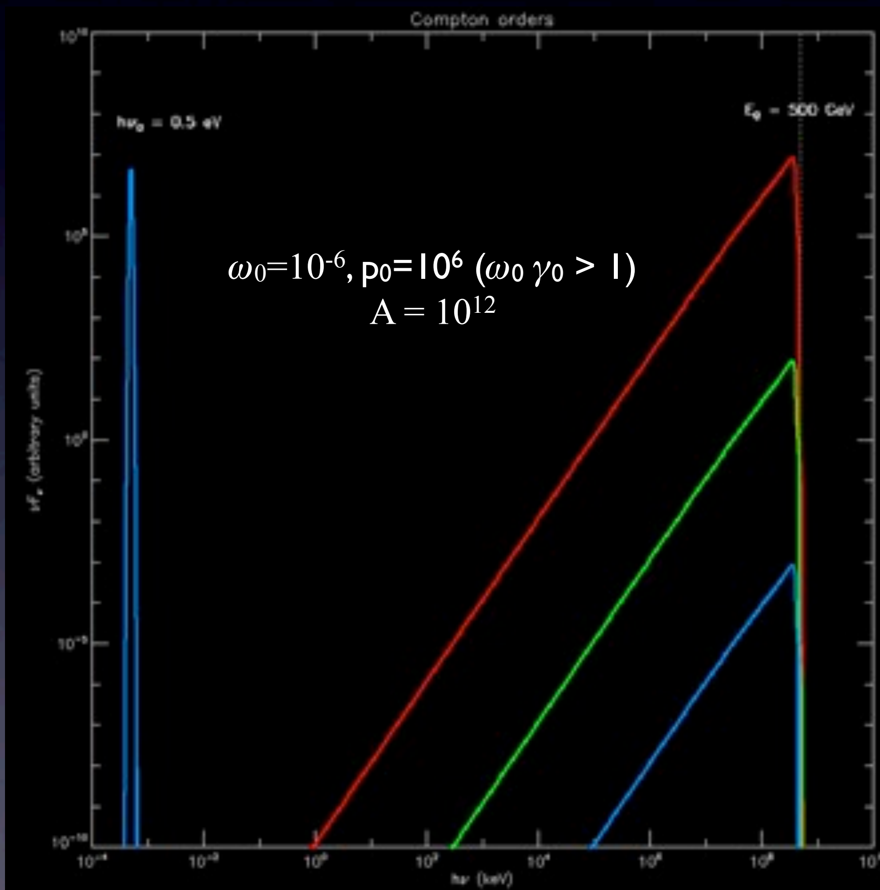
Sub-relativistic particles: ω_0<p<1
Inefficient scattering: A = 1



Relativistic particles...
A >> 1
$\omega_0\gamma_0$<< 1

Ultra-relativistic particles: $\omega_0\gamma_0$>> 1
A >> 1
A ω_0 > γ_0





Power-law spectrum
- cutoff at the particle energy
- Slope = ln(τ)/ln(A) (ref)

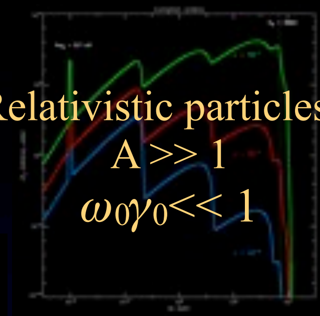
=> X-ray binaries

one-bump spectrum
= single scattering !

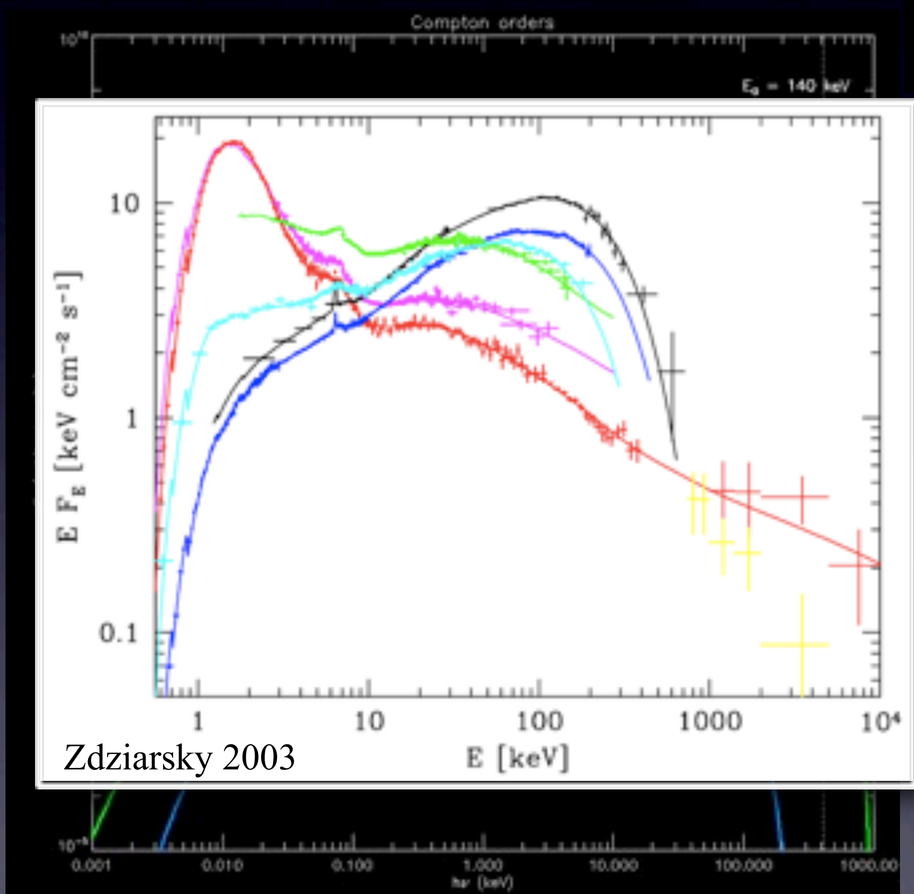
Compton regimes

Sub-relativistic particles: $\omega_0 < p < 1$
Inefficient scattering: $A = 1$

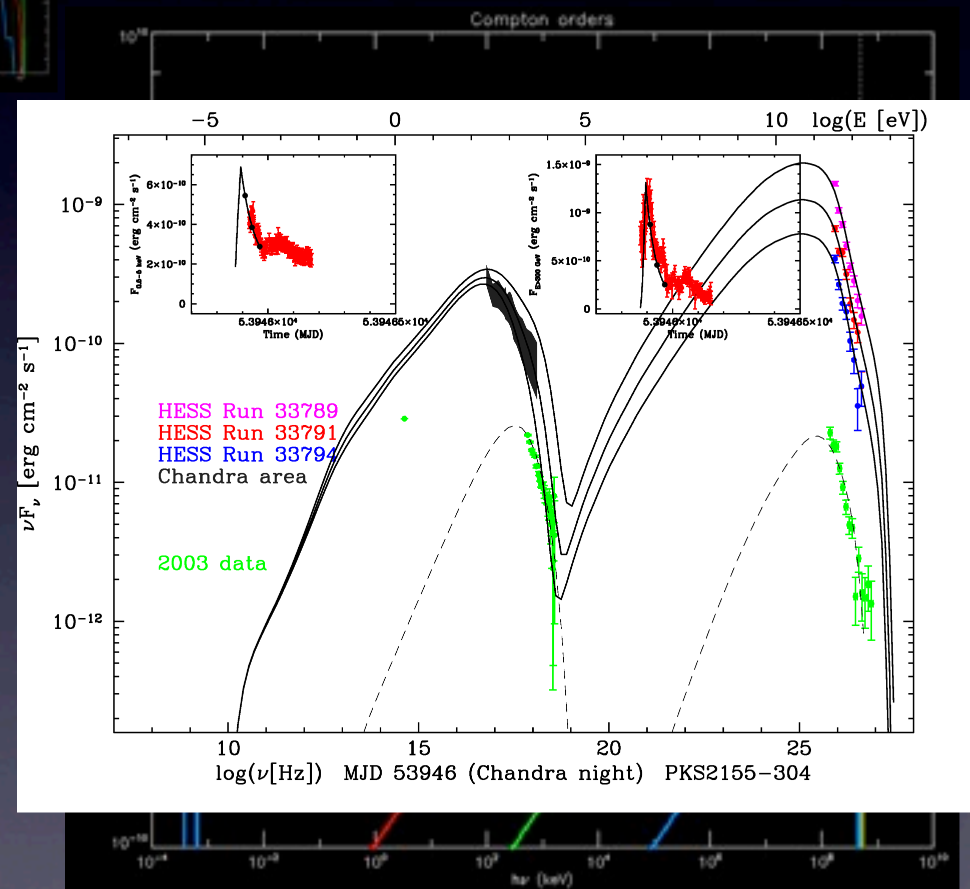
Relativistic particles...
$A >> 1$
$\omega_0 \gamma_0 << 1$



Ultra-relativistic particles: $\omega_0 \gamma_0 >> 1$
$A >> 1$
$A\,\omega_0 > \gamma_0$



Zdziarsky 2003



Power-law spectrum
- cutoff at the particle energy
- Slope = ln(τ)/ln(A) (ref)

=> X-ray binaries

one-bump spectrum
= single scattering !

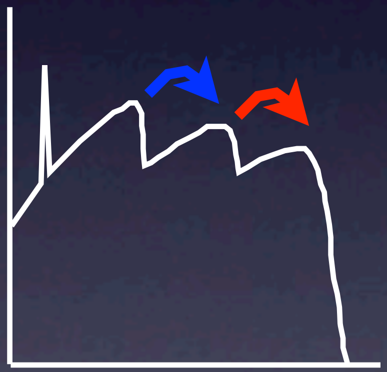
=> AGN/Blazars

General Methods

- Multiple scatterings:
  - Contribution to a kinetic Boltzmann equation:

$$\frac{dN_\omega(\omega)}{dt} = c\iint\left(\frac{d\sigma(p_0,\omega_0\to\omega)}{d\omega}N_\omega(\omega_0) - \frac{d\sigma(p_0,\omega\to\omega_0)}{d\omega_0}N_\omega(\omega)\right)N_p(p_0)dp_0d\omega_0$$

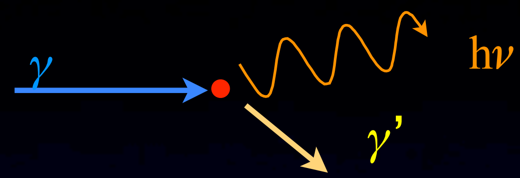
  - Complex problem (Belmont 09):
    - Integral equation, Numerical cancelation issues
    - Kompaneets equation for non-relativistic, thermal distributions (Kompaneets, Barbosa 82)



- Single scattering (τ<<1, γ>>1, KN regime)
  - Simpler problem

$$\frac{dN_\omega(\omega)}{dt} = c\iint\frac{d\sigma(p_0,\omega_0\to\omega)}{d\omega}N_\omega(\omega_0)N_p(p_0)dp_0d\omega_0$$

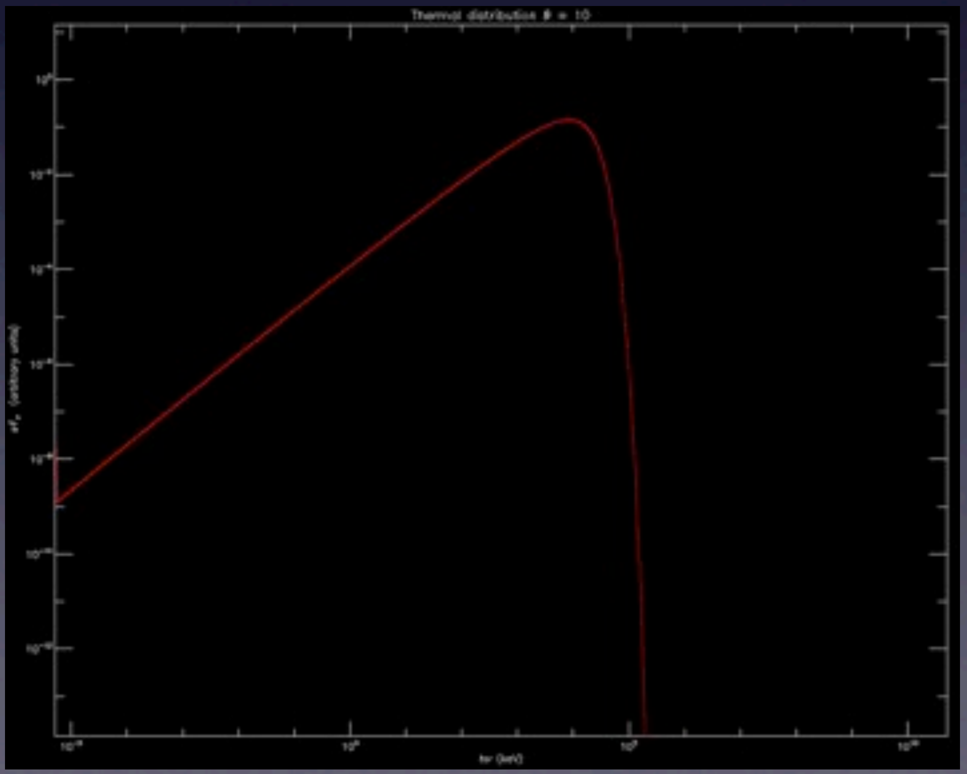
e-p Bremsstrahlung



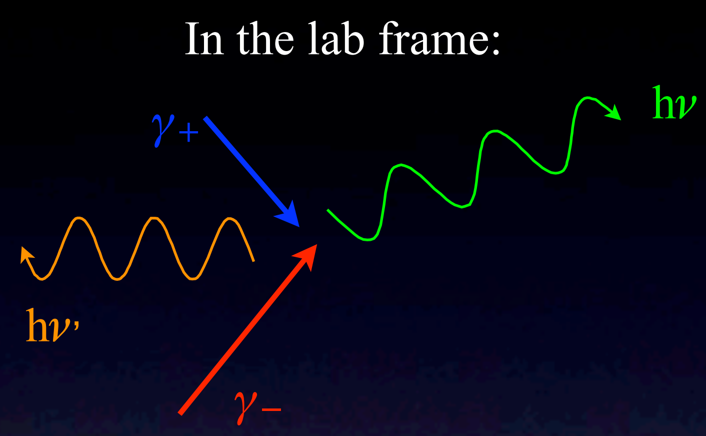
- ✓ protons at rest
- ✓ Relativist particles:

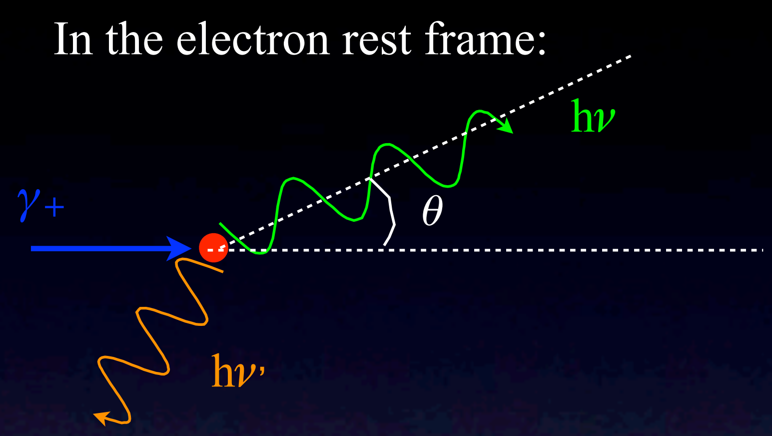
$$\frac{\partial \sigma}{\partial \omega}(\omega, p) \approx \alpha_f \sigma_T \frac{3}{2}\frac{1}{\omega}\left(1 + \frac{\gamma'}{\gamma} - \frac{2}{3}\frac{\gamma'}{\gamma}\right)\left(\ln\left(2\gamma\gamma'/\omega\right) - \frac{1}{2}\right)$$

- ✓ Spectrum: roughly $F_\nu \propto \nu$ up to $h\nu = \gamma mc^2$
- ✓ Radiated energy: $\partial_t E \propto N_e N_p \gamma$
- ✓ Cooling rate = constant
- ✓ e-e bremsstrahlung = complicated...
- ✓ Self-absorption



Pair annihilation / Photon production





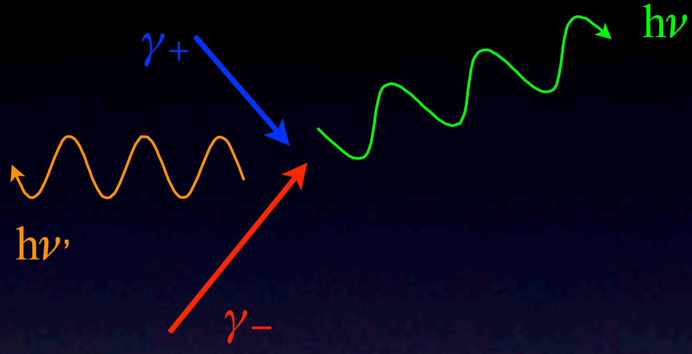
✓ Energy and momentum conservation:

$$\omega + \omega' = \gamma_+ + 1$$

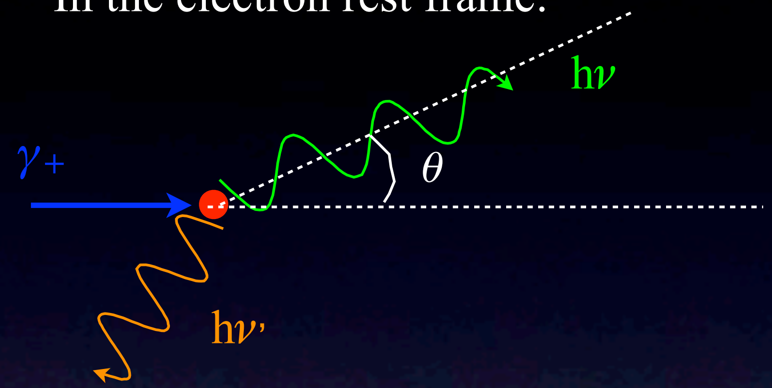
$$\omega = \frac{h\nu}{mc^2} = \frac{1+\gamma_+}{1+\gamma_+ - p_+ \cos\theta}$$

Pair annihilation / Photon production

In the lab frame:



In the electron rest frame:



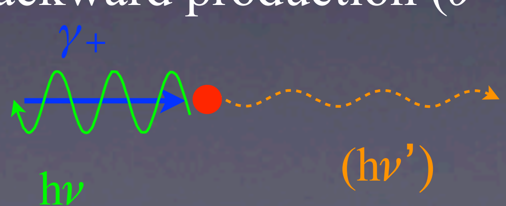
✓ Energy and momentum conservation:

$$\omega + \omega' = \gamma_+ + 1$$

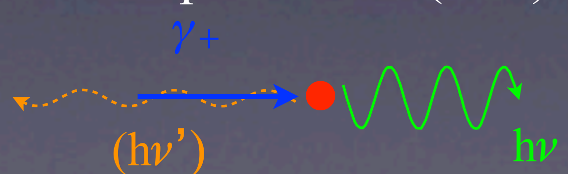
$$\omega = \frac{h\nu}{mc^2} = \frac{1+\gamma_+}{1+\gamma_+ - p_+\cos\theta}$$

$$\frac{1+\gamma_+}{1+\gamma_+ + p_+} \leq \omega \leq \frac{1+\gamma_+}{1+\gamma_+ - p_+}$$

backward production (θ=0)



Forward production (θ=π)



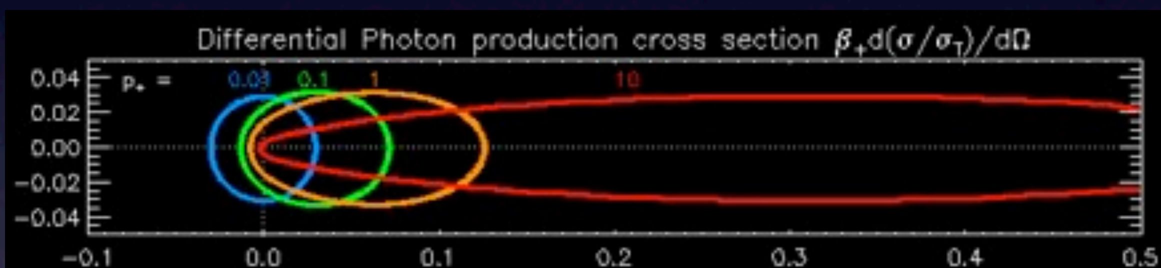
Differential cross section

In the electron rest frame

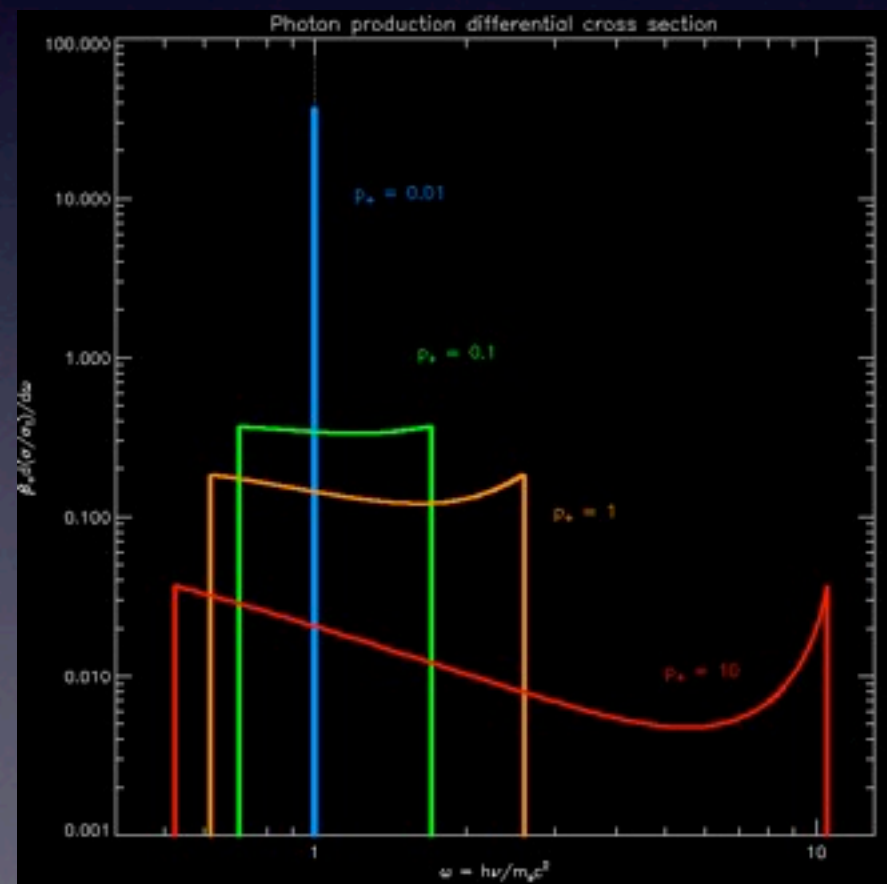
$$\frac{\partial \sigma_{e_+ e_-}}{\partial \Omega} = \sigma_T \frac{3}{16\pi} \frac{1}{p} \frac{\omega}{1 + \gamma_+ - \omega} \left(1 + \gamma_+ + \frac{1 + C^2}{1 - C} \right)$$

$$C = 1 - \frac{1}{\omega} - \frac{1}{\omega'}$$

Solid angle distribution



Photon spectrum



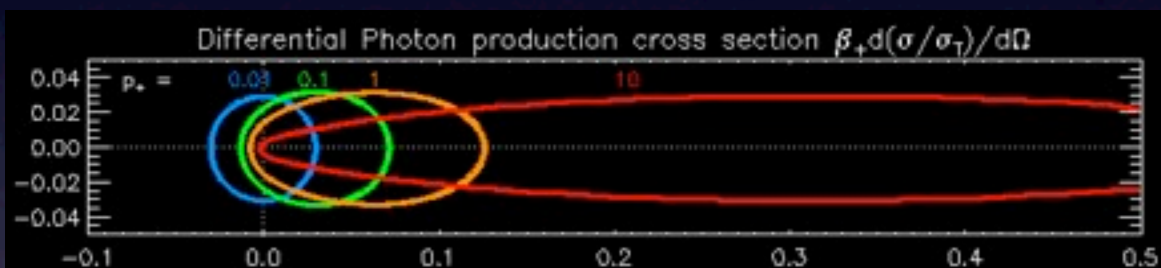
Differential cross section

In the electron rest frame

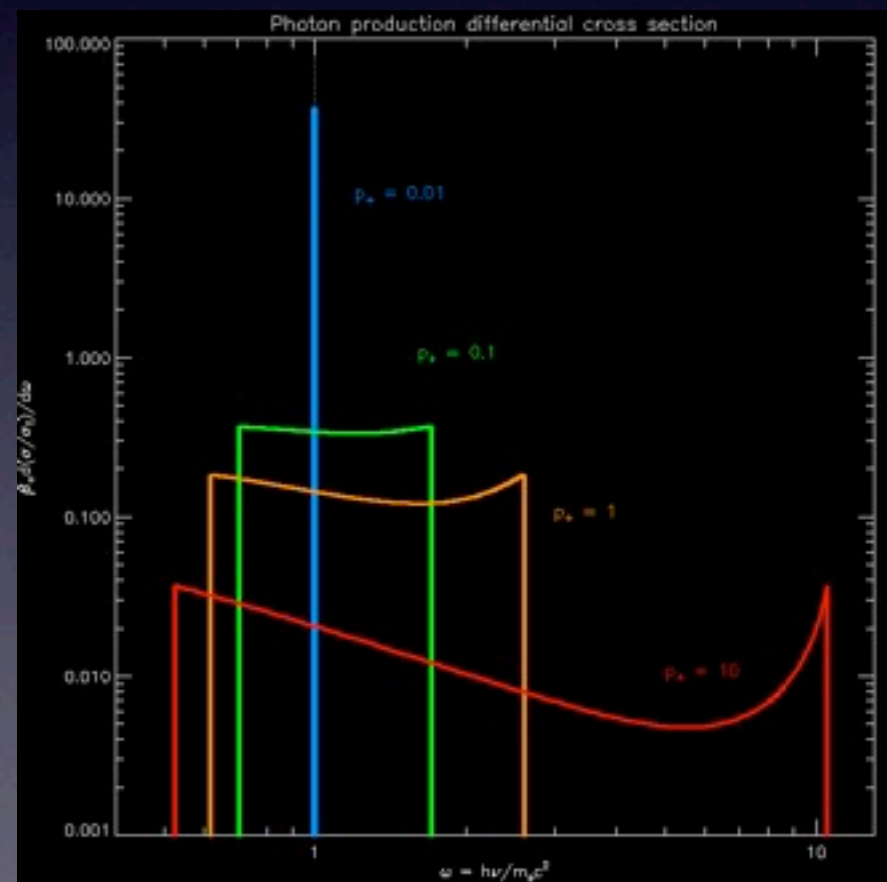
$$\frac{\partial\sigma_{e_+e_-}}{\partial\Omega} = \sigma_T \frac{3}{16\pi}\frac{1}{p}\frac{\omega}{1+\gamma_+-\omega}\left(1+\gamma_+ + \frac{1+C^2}{1-C}\right)$$

$$C = 1 - \frac{1}{\omega} - \frac{1}{\omega'}$$

Solid angle distribution



Photon spectrum

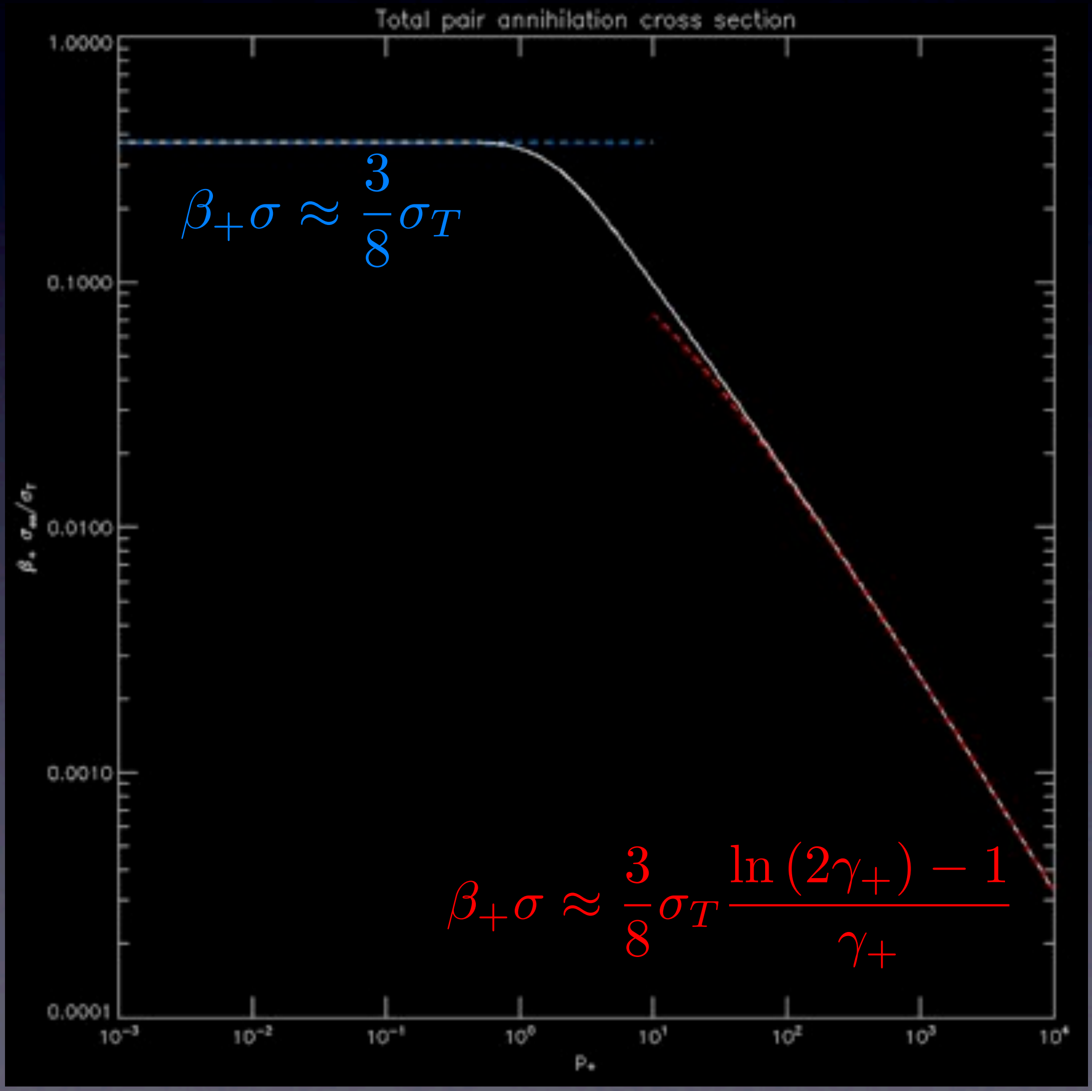


- ✓ Case of Cosmic ray positrons + electrons at rest:
  - ✓ Low energy positrons => sharp line
  - ✓ high energy positrons => broad line
  - ✓ Very high energy positrons =>
    - ✓ 1 high energy photon ($\omega = \gamma$)
    - ✓ 1 low energy photon

Total cross section

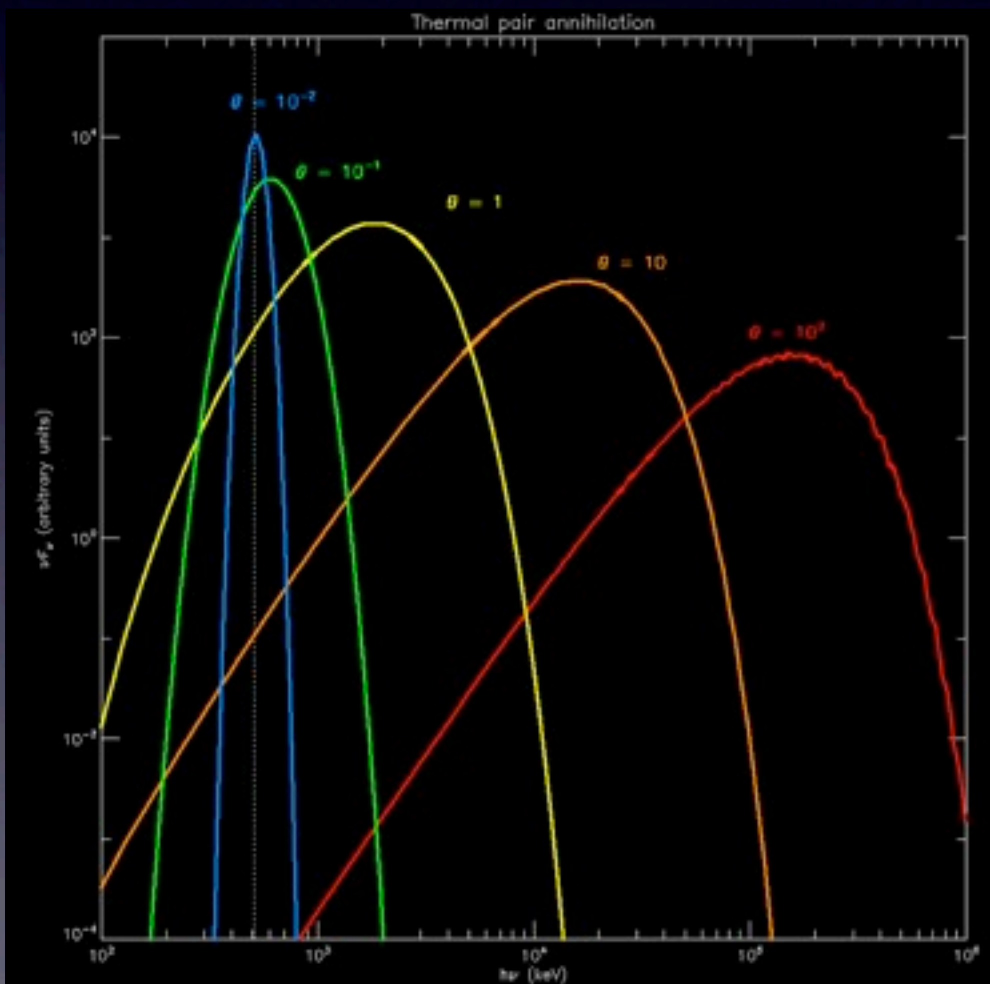
In the electron rest frame

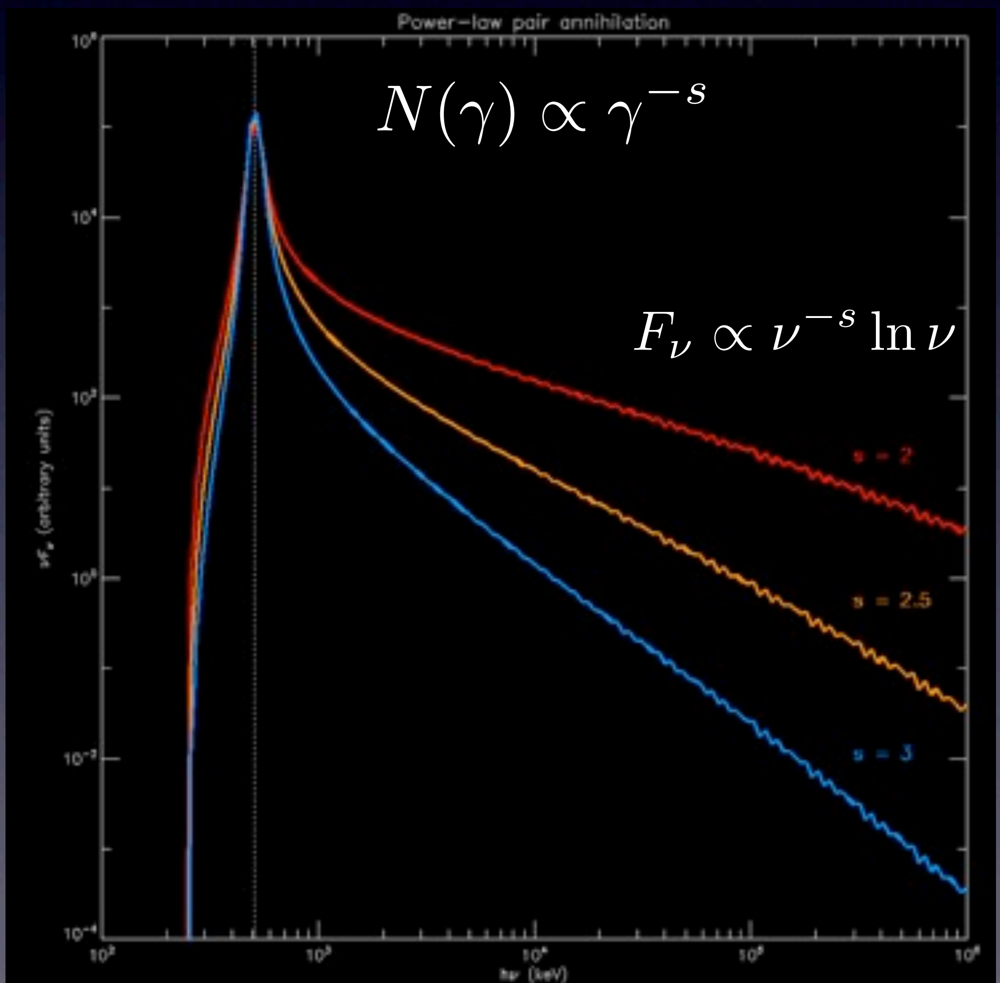
$$\beta_+ \sigma = \sigma_T \frac{3}{8} \frac{1}{\gamma_+(\gamma_+ + 1)} \left[(\gamma_+^2 + 4\gamma_+ + 1) \frac{\ln(p_+ + \gamma_+)}{p_+} - 3 - \gamma_+ \right]$$



In the lab frame for Isotropic particles

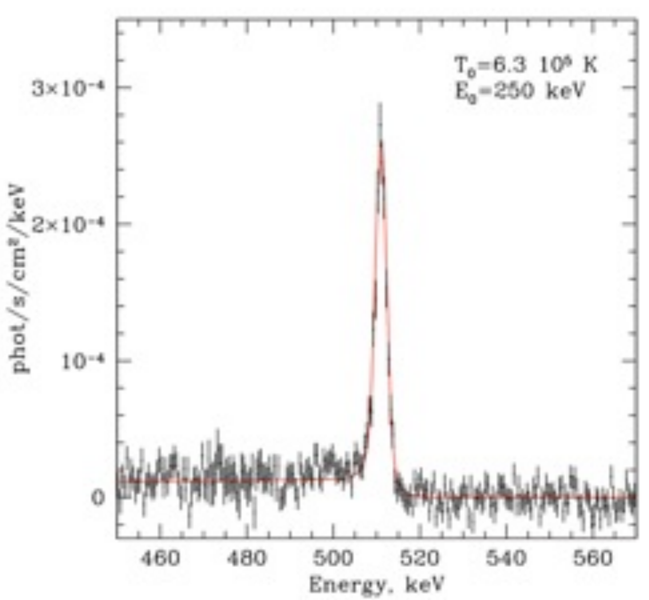
- ✓ Total cross section: cutoff for: $\gamma_{+}\gamma_{-} > 1$ (similar to KN Compton regime)
- ✓ Production spectrum... Svensson 82
- ✓ Spectrum of thermal/non-thermal leptons



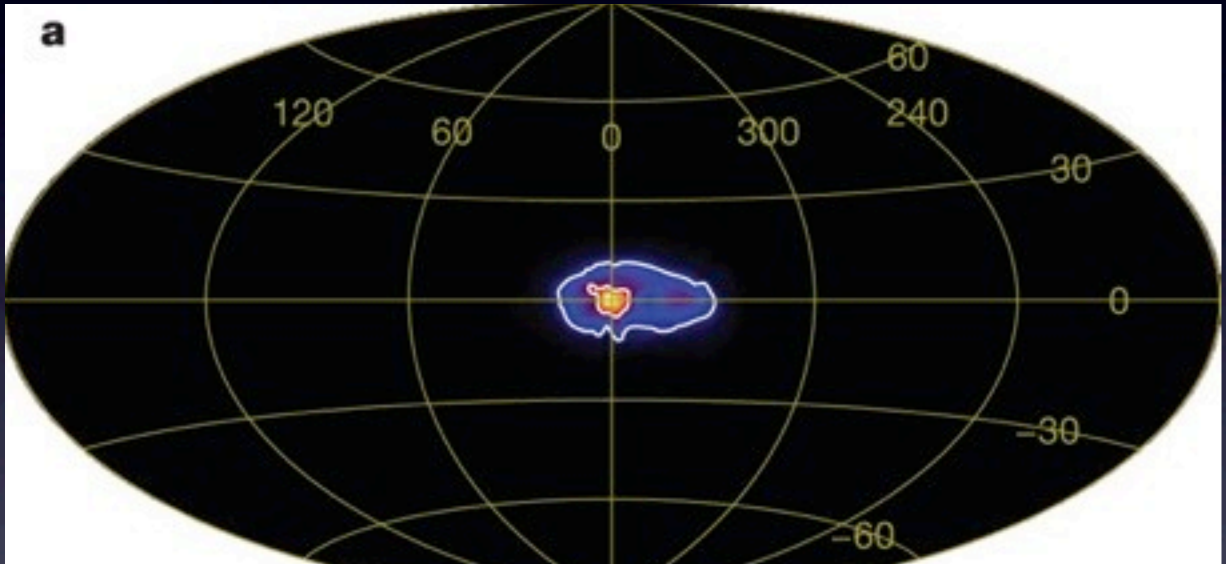


Annihilation of cosmic ray positrons

- ✓ Diffuse emission at 511 keV from the Galactic center



Churazov et al.

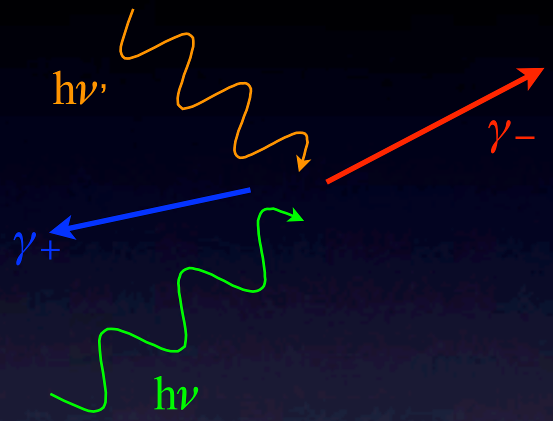


Weidenspointner et al. 2008

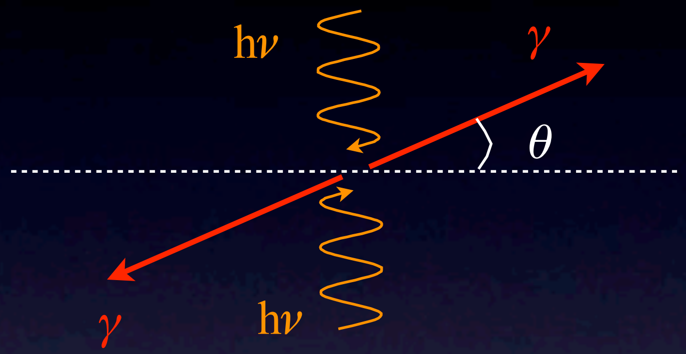
- ✓ No confirmed detection of point sources CR positron emitters
- ✓ However, might be important for pair equilibrium...

Pair production/Photon annihilation

In the lab frame:



In the center of momentum frame:



- In the center of momentum of the 2 incoming photons
- Conservation of momentum and energy: $\omega = \gamma$
  - 2 photons of energy $h\nu$
  - 2 leptons of energy: γ (for all production direction)
  - Production threshold $\omega>1$

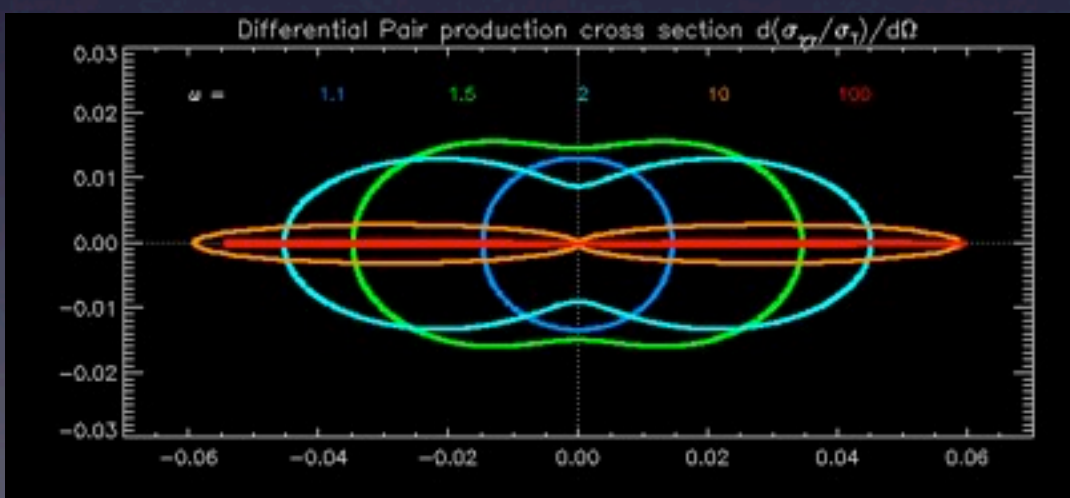
Differential and Total cross sections

In the center of momentum frame

Solid angle cross section:

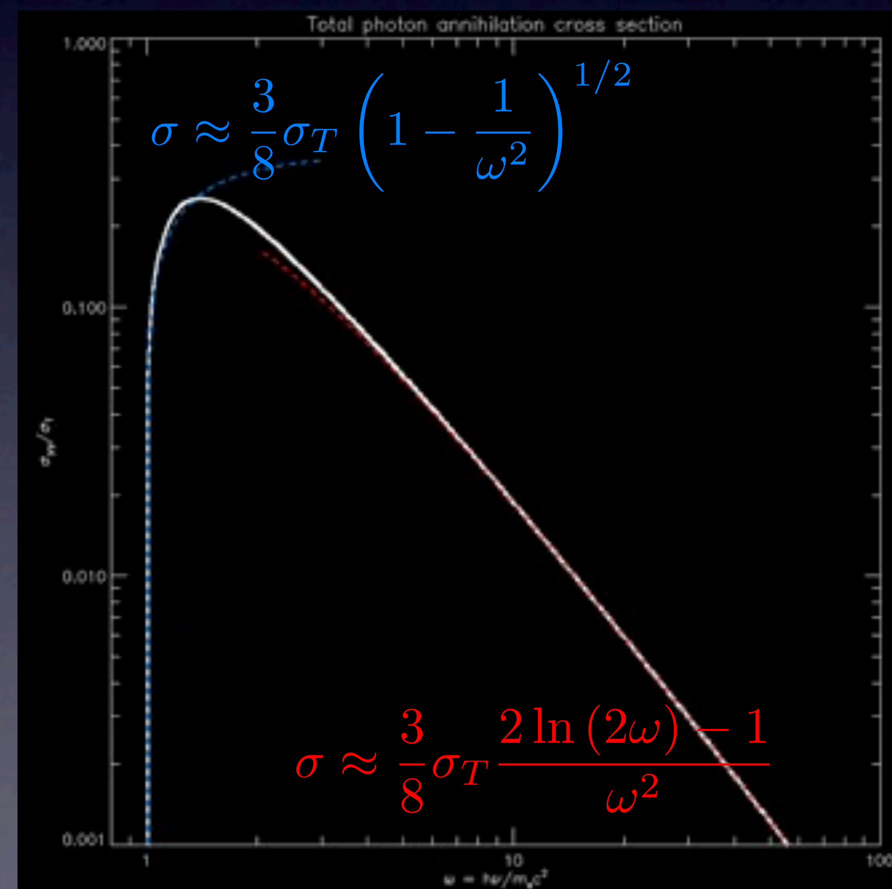
$$\frac{\partial \sigma_{\gamma\gamma}}{\partial \Omega} = \sigma_T \frac{3}{32\pi}\frac{\beta}{\omega^2}\frac{1-\beta^4\cos^4\theta + 2\beta^2\sin^2\theta/\omega^2}{(1-\beta\cos\theta)^2(1+\beta\cos\theta)^2}$$

$$\beta^2 = 1 - 1/\omega^2$$



Total cross section:

$$\sigma_{\gamma\gamma} = \sigma_T \frac{3}{8}\frac{1}{\omega^2}\left[\left(2+\frac{2}{\omega^2}-\frac{1}{\omega^4}\right)\cosh^{-1}\omega - \left(1+\frac{1}{\omega^2}\right)\left(1-\frac{1}{\omega^2}\right)^{1/2}\right]$$



$$\sigma \approx \frac{3}{8}\sigma_T\left(1-\frac{1}{\omega^2}\right)^{1/2}$$

$$\sigma \approx \frac{3}{8}\sigma_T\frac{2\ln(2\omega)-1}{\omega^2}$$

Total cross section

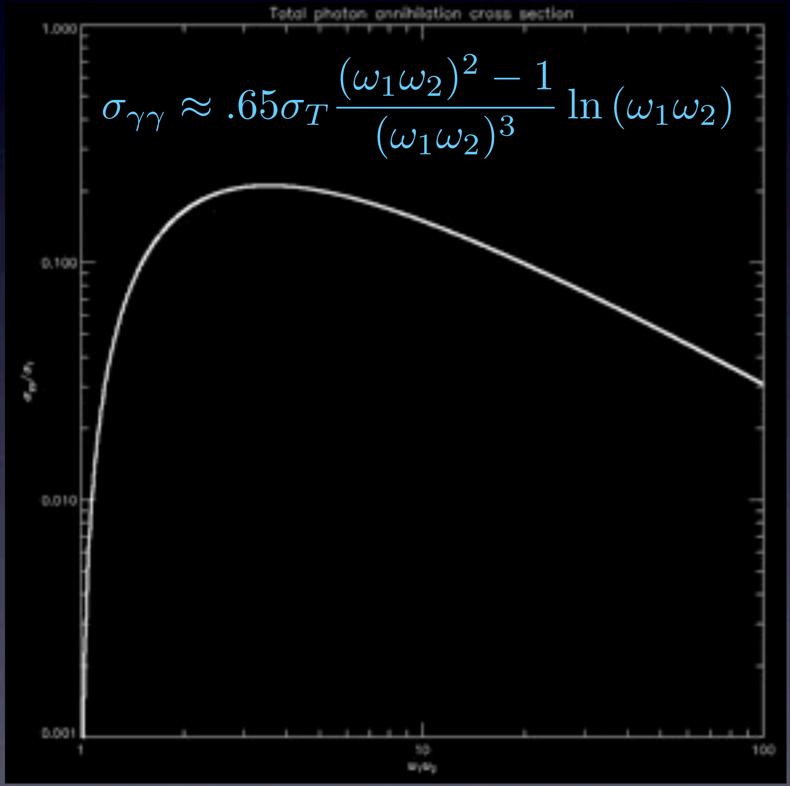
In the lab frame

- Threshold: $\gamma_{cm} = \omega_1 \omega_2 \dfrac{1-\cos\theta}{2} \geq 1$
  - Head-on collisions (θ=π): $\omega_1\ \omega_2 > 1$
  - Trailing collisions (θ=0): $\omega_1\ \omega_2 \to \infty$

Total cross section

In the lab frame

- Threshold: $\gamma_{cm} = \omega_1\omega_2 \frac{1-\cos\theta}{2} \geq 1$
  - Head-on collisions ($\theta=\pi$): $\omega_1\,\omega_2 > 1$
  - Trailing collisions ($\theta=0$): $\omega_1\,\omega_2 \rightarrow \infty$
- Total cross section for isotropic photon field
  - Analytical (Gould&Schreder67, approx: Coppi&Blandford90)
  - Maximal absorption for: $\omega_2 \approx 1/\omega_1$
  - TeV $\leftrightarrow$ 0.1 eV
  - GeV $\leftrightarrow$ 100 eV



Total cross section

In the lab frame

- ✓ Threshold: $\gamma_{cm} = \omega_1\omega_2 \frac{1-\cos\theta}{2} \geq 1$
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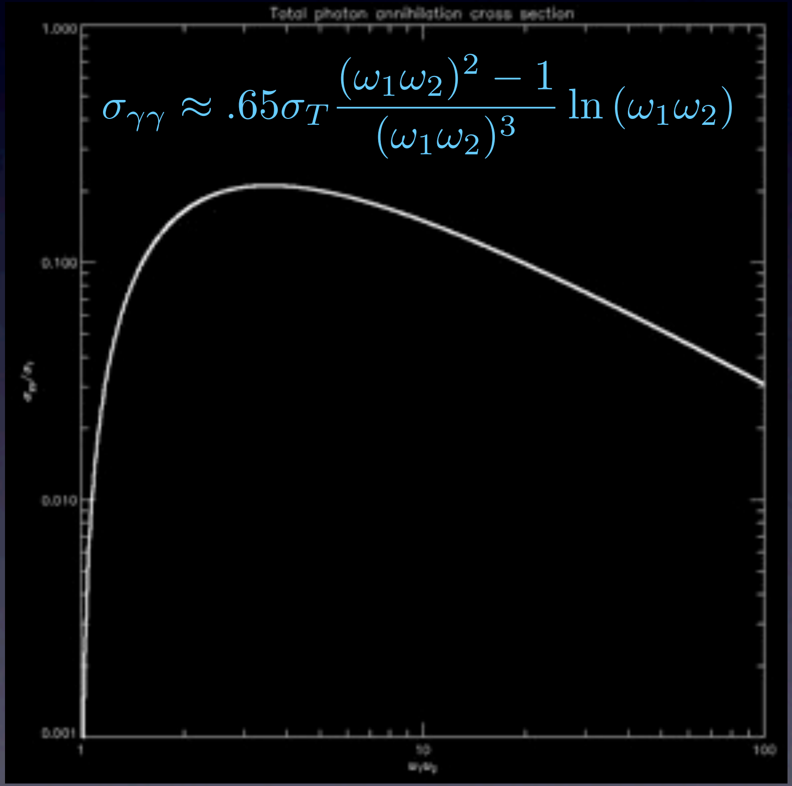
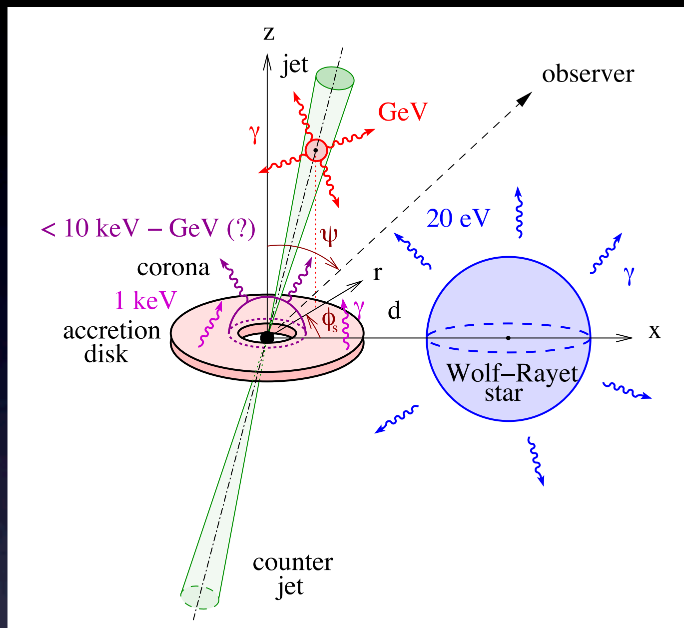
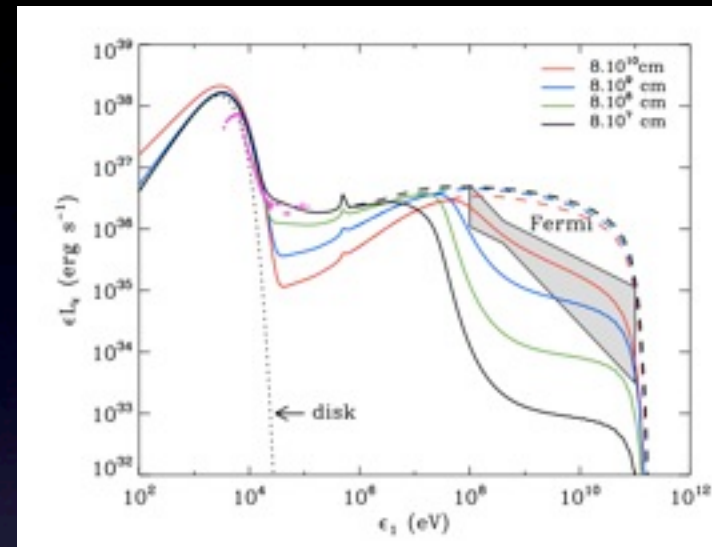


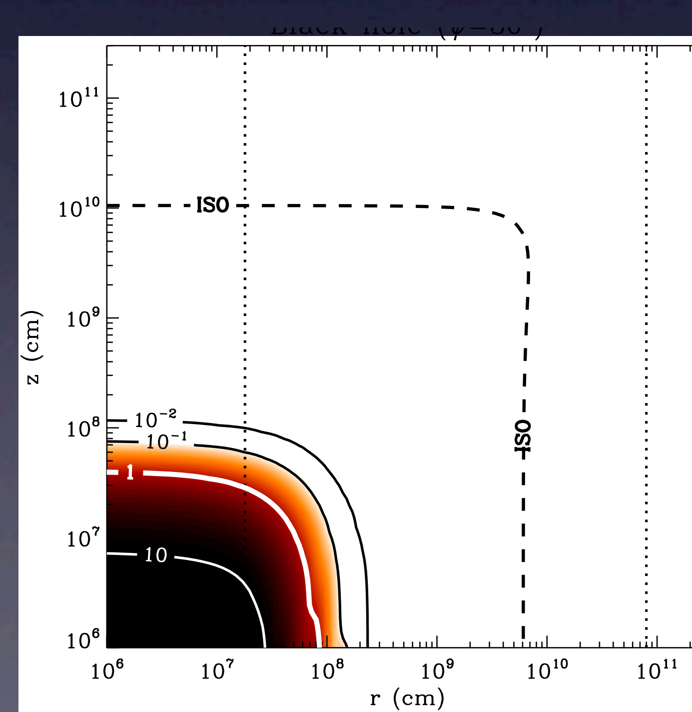
- ✓ Photon-photon absorption

$$\tau_{\gamma\gamma}(\omega) = L \int N_\omega(\omega_0)\sigma_{\gamma\gamma}(\omega_0,\omega)d\omega_0 \approx 0.2\sigma_T L \frac{N_\omega(1/\omega)}{\omega}$$

Photon absorption in gamma binaries





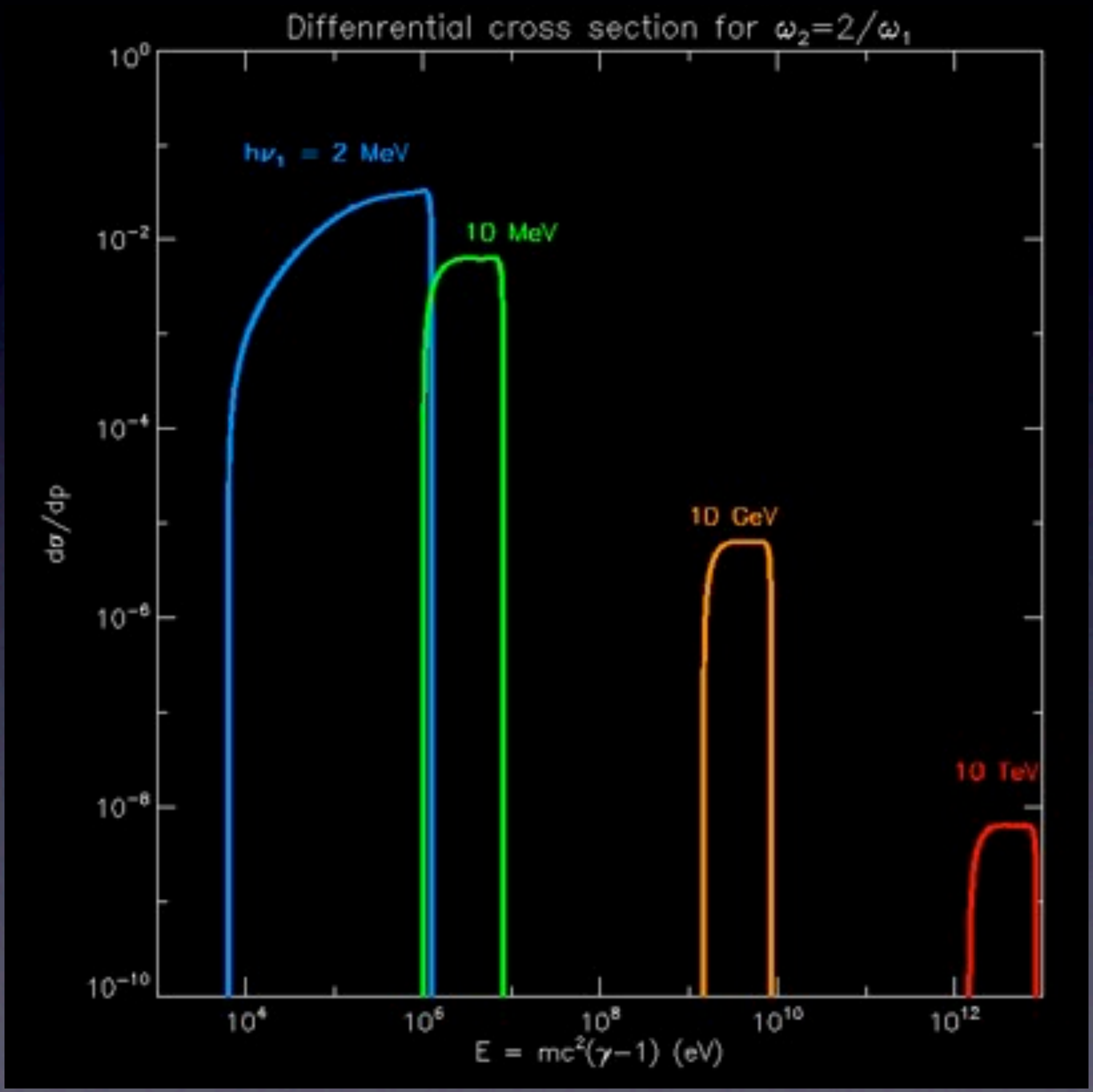


- ✓ Strong photon field
  - ✓ From the companion star
  - ✓ The accretion disk
- ✓ Efficient photon-photon absorption
- ✓ Ex: Cyg -X3:
  - ✓ GeV detection by Fermi (Abdo et al. 2009)
  - ✓ Anisotropic Absorption maps (Cerutti et al. 2011)
  - ✓ => GeV production far from the BH (not coronal)

Spectrum of produced pairs

In the lab frame

- Analytical for isotropic photon field = lengthy (Boettcher&Schilkeiser97):
- For high energy photons :
  - pairs produced with $h\nu/10 < E < h\nu$
- Pair cascade:
  - strong γ radiation field
  - production of secondaries
  - radiation of produced pairs
  - additional gamma emission
  - Cascade efficiency depends on radiative cooling



Numerical methods

- I. Simpler models:
  - No geometry, particle distribution set, simple photon production and escape
  - Simple computation of the emissivity of each process
  - e.g.: radio jets, blazars (single Compton scattering...)

- II. Models with multiple photon interactions
  - photon distribution = solution of an equation
  - Kinetic codes/Transfer codes
  - Linear Monte Carlo codes

Kinetic codes

- 1-zone problem: homogeneous, isotropic
- Time dependent kinetic equation with isotropic cross sections

$$\partial_t N_\nu = \sum_{processes} [\partial_t N_\nu(N_p, N_\nu)]_i$$

  - ex: Kompaneets equation = kinetic equation for photons comptonized by a thermal distribution of particles

$$\frac{\partial n(\omega)}{\partial(tc/L)} = \tau\theta \, \frac{1}{\omega^2}\frac{\partial}{\partial\omega}\left\{\omega^4\left(\lambda_\theta\frac{\partial n}{\partial\omega} + \mu_\theta n(1+n)\right)\right\}$$

- General case: non-linear integro-differential equation
- Use of an escape formalism
  - Simple for only absorption/emission problems
  - Very complicated for scattering + emission/absorption
  - Approximate escape rates

$$\mu\partial_z I_\nu = j_\nu - \alpha I_\nu$$

- For 1D geometry, steady problems: transfer codes (Poutanen&Svenssen96)
- Applied to the corona of X ray binaries and AGN, to galaxy clusters...

Linear Monte Carlo codes

- ✓ Photon trajectories are traced one by one
- ✓ Interactions are computed one by one
- ✓ Simple to write
- ✓ Slow for simple geometries (0D, 1D) compared to kinetic codes (stern 95)
- ✓ Most efficient for complex geometries
- ✓ used in X-ray binaries, AGN

Particle interactions

- II. Codes with computation of the particle properties
  - Evolution of particles is of increasing interest
    - injection of accelerated particles/continuous acceleration
    - Evolution, cooling
  - Non linear Monte Carlo: compute the effect of photon interaction on the particle properties
  - Kinetic codes:
    - coupled kinetic equations
    - particle equation often = Fokker-Planck equation

$$\partial_t N_p = S_p - \frac{N_p}{T_p} + \frac{\partial}{\partial p}\left(A_p N_p\right) + \frac{\partial^2}{\partial p^2}\left(D_p N_p\right)$$

  - X-ray binaries, microquasars, Blazars

Conclusions

- At high energy
  - Total cross sections drop off
  - Differential cross sections become highly anisotropic
- Particle cooling:
  - Synchrotron: $P \propto \sigma_T p^2 U_B$
  - Compton in the Thomson regime: $P \propto \sigma_T p^2 U_{ph}$
  - Bremsstrahlung: $P \propto \sigma_T \alpha_f \, p \, U_i$ (with $U_i = n_i m_e c^2$)
- Photons:
  - Synchrotron:
    - Thin spectrum of 1 particle peaks at $\nu_c \propto \gamma^2 B$
    - Thin spectrum of a power-law distribution is a power-law
    - Absorption => Thick spectrum at low frequency
  - Compton
    - Amplification factor in the Thomson regime: $A = \gamma^2$
    - Mildly relativistic particles: power-law spectrum
    - Comptonization by a relativistic power-law distribution is a PL spectrum
  - $\gamma-\gamma$ pair production:
    - Threshold at $\omega_1\omega_2 \approx 1$
    - Most efficient photon absorption for $\omega_1\omega_2 \approx 1$