Transport phenomena and acceleration processes for astroparticles

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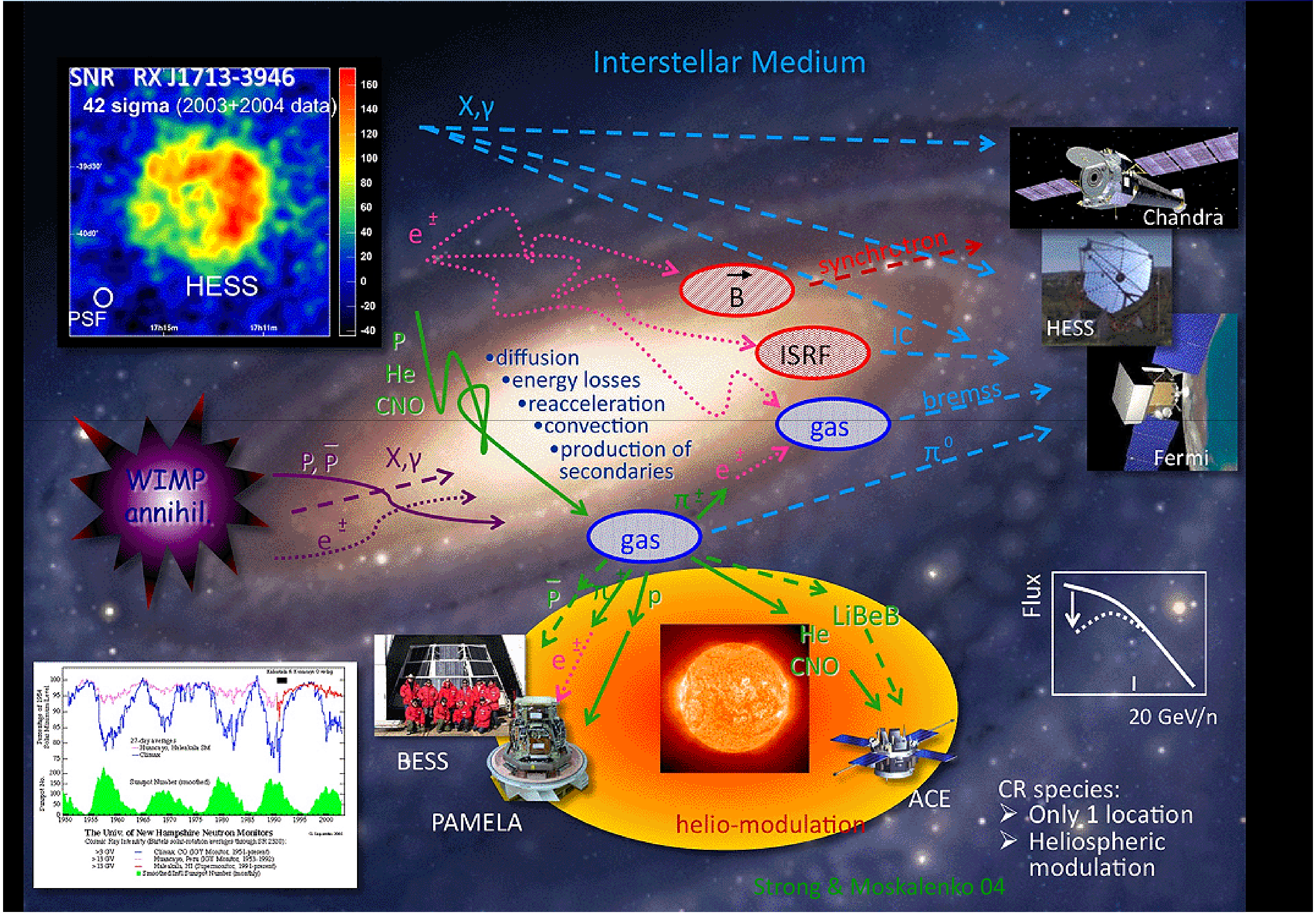
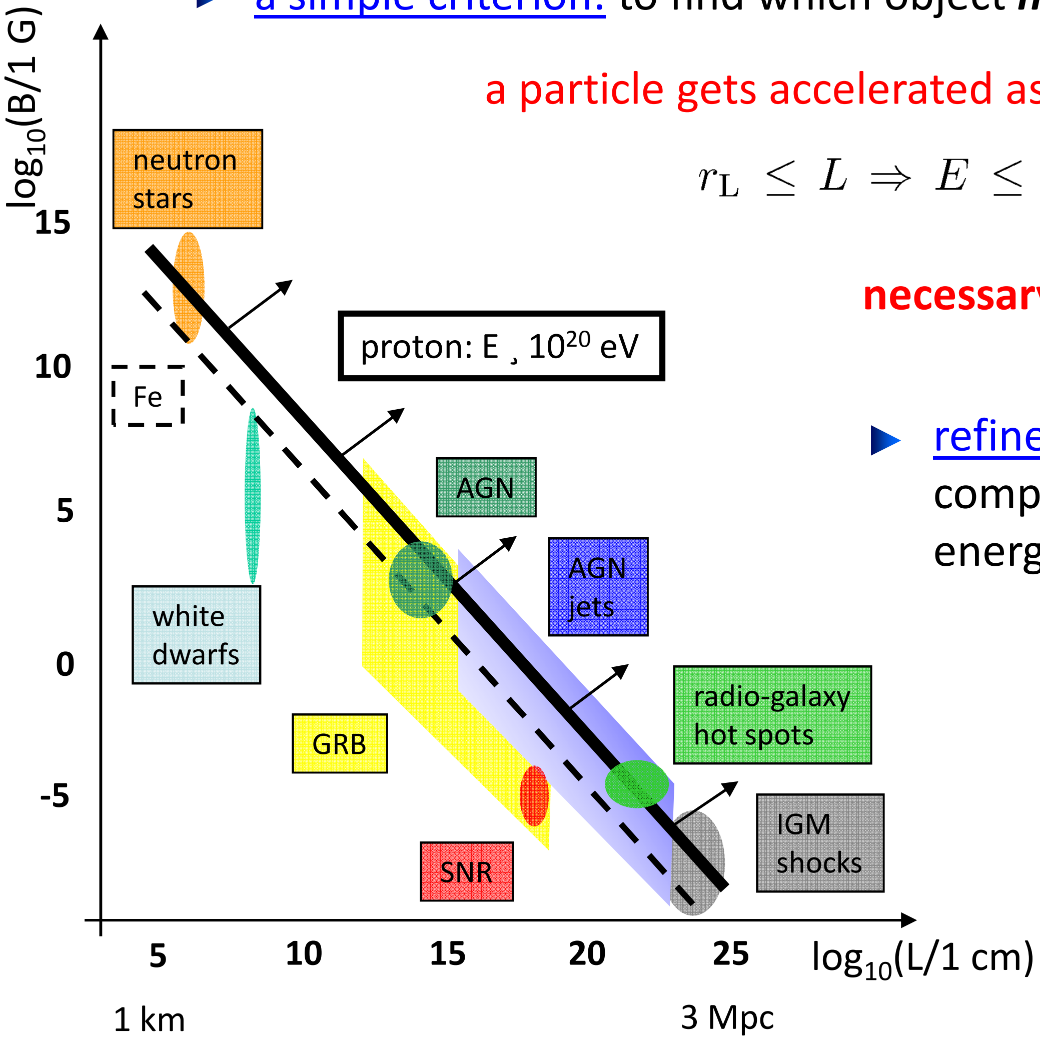
Introduction



Introduction - maximal energy at the source

Hillas 84

- <u>a simple criterion:</u> to find which object ***might*** be a source of UHE cosmic rays:



a particle gets accelerated as long as it is confined in the source:

$$r_{\rm L} \leq L \Rightarrow E \leq 10^{20}\,{\rm eV}\,Z B_{\mu{\rm G}} L_{100\,{\rm kpc}}$$

necessary, but by no means sufficient!

- <u>refined criterion:</u>
  compare acceleration timescale with
  energy loss timescale and escape timescale

$$t_{\rm acc} \leq t_{\rm loss},\quad t_{\rm esc}$$

t_{acc} depends on acceleration mechanism

t_{esc} depends on magnetic field

t_{loss} depends on environment

$\Rightarrow$ **requires an object by object study…**

Norman et al. 95

… magnetars, gamma-ray bursts and radiogalaxies are promising candidates…

Introduction - transport in phase space

Master equation: Fokker-Planck equation

$$\frac{\partial}{\partial t} f = -\frac{\partial}{\partial \alpha} (D_1 f) + \frac{\partial^2}{\partial \alpha^2} (D_2 f)$$

governs the evolution of the distribution function in stochastic Markovian process with drift D_1 and diffusion coefficient D_2

α = x $\Rightarrow$ spatial convection (D_1) and diffusion (D_2)
α = p $\Rightarrow$ energy loss or gain (D_1) and momentum diffusion (D_2)

Examples:

advection diffusion equation for particle acceleration (e.g. Drury 83):

$$\frac{\partial}{\partial t} f + \boldsymbol{U} \cdot \boldsymbol{\nabla} f - \frac{1}{3} \boldsymbol{\nabla} \cdot \boldsymbol{U}\, p \frac{\partial}{\partial p} f - \boldsymbol{\nabla} (\boldsymbol{\kappa} \boldsymbol{\nabla} f) - \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial f}{\partial p} \right) = 0$$

galactic cosmic ray propagation (e.g. Berezinsky et al. 90):

$$\frac{\partial}{\partial t} N_i - \nabla \cdot (D_i \nabla N_i) + \frac{\partial}{\partial E} \left(\frac{\mathrm{d}E}{\mathrm{d}t} N_i \right) = q_i - \frac{N_i}{\tau_{\ i}} + \sum_k \frac{N_k}{\tau_{k \to i}}$$

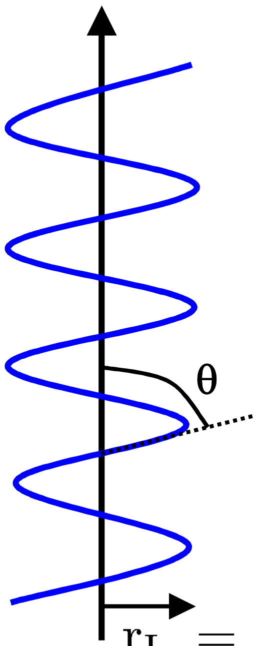
Outline

1. **Transport of charged particles in magnetized turbulence**
   a. quasi-linear theory
   b. applications and recent developments

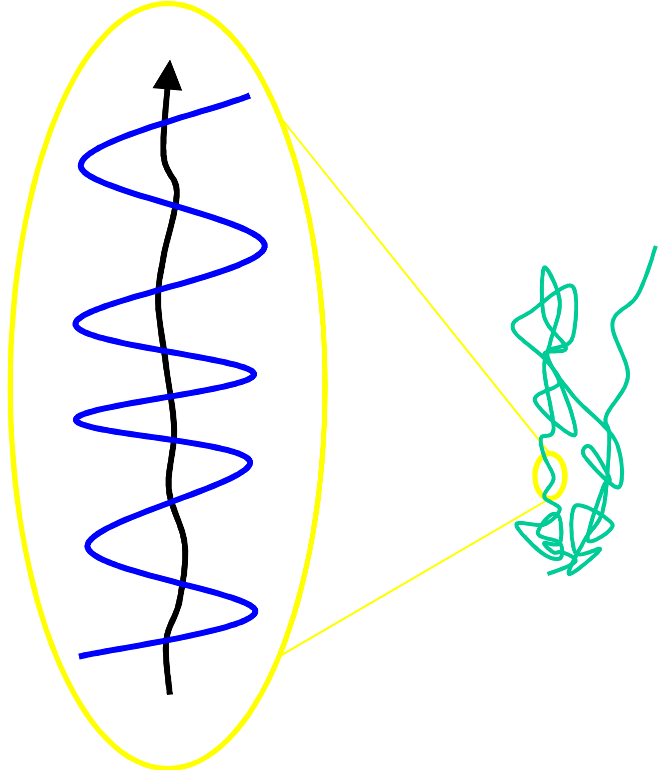
2. **Particle acceleration in non-relativistic flows**
   a. general principles
   b. particle acceleration in non-relativistic flows...
   c. applications and modern developments

3. **Particle acceleration at relativistic collisionless shocks**
   a. non-relativistic vs relativistic
   b. current picture

Particle transport in magnetic fields



$$r_L = \frac{p \sin(\theta)}{qB}$$



<u>regular field:</u>

pitch angle θ is conserved

transport across the field line is prohibited

<u>perturbed field:</u>

pitch angle θ is no longer conserved,
θ performs a random walk

transport across the field line is not prohibited but remains difficult

Turbulent magnetic fields

Generic magnetic fields comprise coherent $\mathbf{B_0}$ and turbulent components $\delta\mathbf{B}$: $\mathbf{B} = \mathbf{B_0} + \delta\mathbf{B}$

The properties of magnetic turbulence are defined by statistical averages:

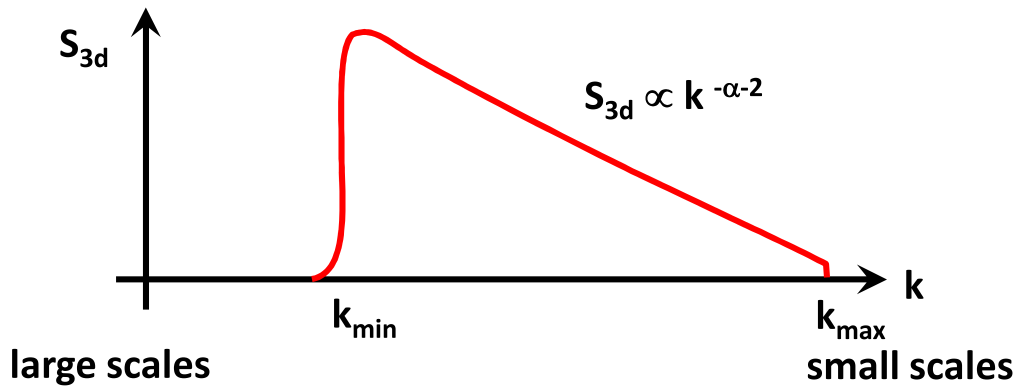
$$\langle \boldsymbol{B}_0 \rangle = \boldsymbol{B}_0, \quad \langle \boldsymbol{\delta B} \rangle = 0$$ (average taken over realizations or space)

but: $$\langle \boldsymbol{\delta B}(\mathbf{x}) \cdot \boldsymbol{\delta B}(\mathbf{x}+\mathbf{r}) \rangle = \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \mathrm{S}_{3\mathrm{d}}(\mathbf{k})$$ 3-d power spectrum of turbulence

$$\langle \boldsymbol{\delta B}^2 \rangle = \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\, S_{3d}(\mathbf{k})$$

Simple configuration: S_{3d} is isotropic (function of k only), δB is gaussian (fully characterized by S_{3d})

e.g., hydrodynamic turbulence is characterized by a Kolmogov spectrum (α=5/3):



Turbulent magnetic fields

In general (e.g. Kolmogorov), turbulence power lies on the largest scales:

$S_{3d} \propto k^{-\alpha-2}$ with $\alpha = 5/3$

Then the coherence length of the turbulent component is set by k_{min}:

$$\lambda_B = \frac{1}{\langle \boldsymbol{\delta B}^2 \rangle} \int_0^{+\infty} \mathrm{d}r \, \langle \boldsymbol{\delta B}(0) \cdot \boldsymbol{\delta B}(r) \rangle \sim k_{\mathrm{min}}^{-1}$$

The transport of charged particles in the turbulence is then governed by the following parameters:

$$\eta \equiv \frac{\langle \boldsymbol{\delta B}^2 \rangle}{\boldsymbol{B_0}^2 + \langle \boldsymbol{\delta B}^2 \rangle}$$ strength of the turbulence relative to B_0

$$\rho \equiv \mathrm{k_{min} r_L} \sim \frac{\mathrm{r_L}}{\mathrm{l_{coh}}}$$ ratio of Larmor radius to coherence length

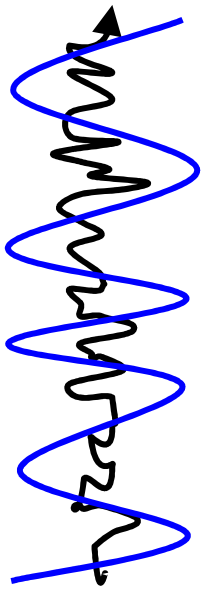
α spectral index of turbulence

Particle transport in magnetic fields

the Larmor radius of the particle sets a preferred spatial scale...

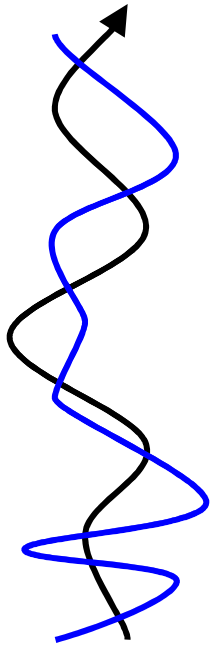
scale:

$\lambda \ll r_{\mathrm{L}}$



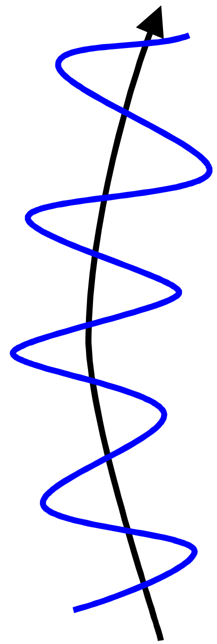
small scale modes provide ~ white noise with little effect on pitch angle

$\lambda \sim r_{\mathrm{L}}$



gyro-resonant modes provide efficient interactions that lead to pitch angle diffusion

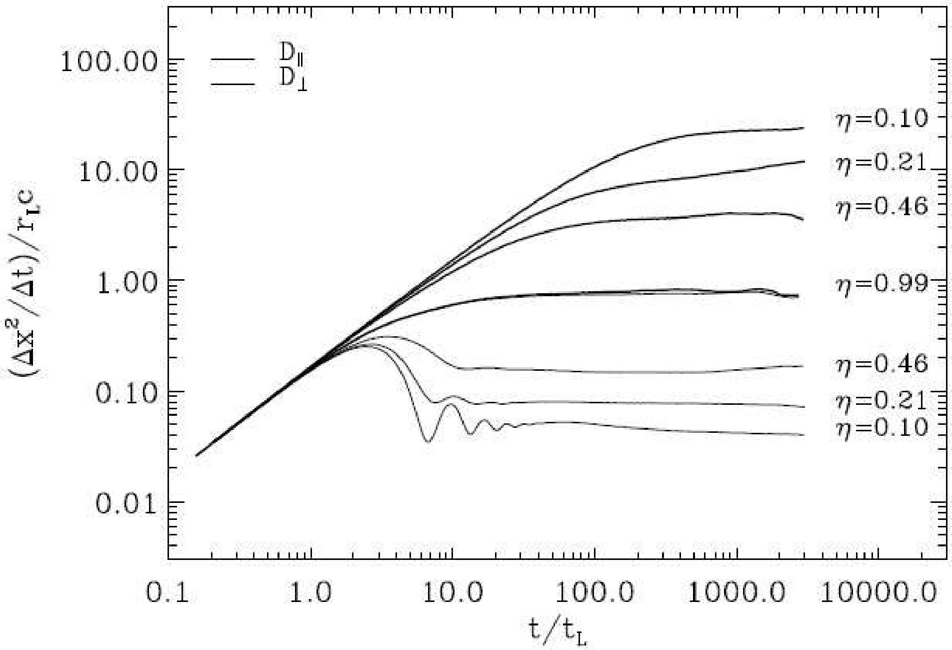
$\lambda \gg r_{\mathrm{L}}$

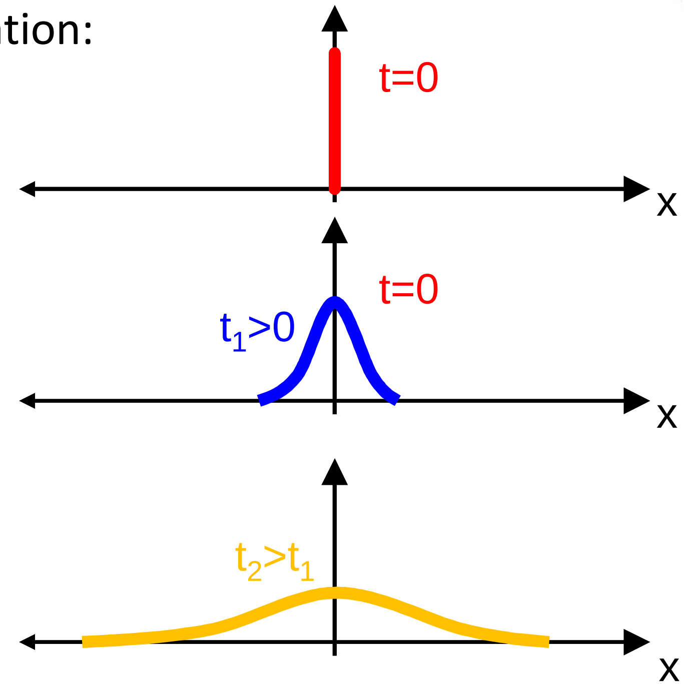


large scale modes make up the large scale magnetic field

Diffusion of pitch angle

Average square displacement over a particle population:





$$\langle \Delta x_{\parallel}^2 \rangle = \frac{2}{3} c^2 \, t_{\rm scatt} \, \Delta t$$

Particles are isotropized on timescale $t_{\rm scatt}$; condition for spatial diffusion:

$$t_{\rm travel} \gg t_{\rm scatt}$$

low energy particles always start to diffuse first

Diffusion of pitch angle

In the weak turbulence ($\delta B << B$) limit, it is possible to follow the random walk of the pitch angle and calculate the scattering time (quasi-linear theory):

$$\frac{1}{t_{\rm scatt}} = \frac{\langle \Delta\theta^2 \rangle}{\Delta t} \quad (\Delta t \to +\infty) \qquad \text{then} \qquad D_{\|} = \frac{1}{3} c^2 t_{\rm scatt}$$

$t_{\rm scatt}$:
time to achieve $\Delta\theta^2 \sim 1$

$\Delta\theta = \theta(\mathrm{t}+\Delta\mathrm{t}) - \theta(\mathrm{t})$
$\langle \Delta\theta^2 \rangle \propto \Delta\mathrm{t}$: random walk

parallel diffusion coefficient
(along B_0)

Quasilinear theory (e.g. Jokipii 1967, 1973):

$$\begin{aligned} \langle \Delta\theta^2 \rangle &= \int_0^{\Delta t}\int_0^{\Delta t} \mathrm{d}t_1 \mathrm{d}t_2 \left\langle \dot\theta(t_1)\dot\theta(t_2) \right\rangle \\ &= \frac{c^2}{r_{\rm L}^2}\frac{1}{\langle \delta B^2\rangle}\int_0^{\Delta t}\int_0^{\Delta t} \mathrm{d}t_1 \mathrm{d}t_2 \ \langle \boldsymbol{\delta B}_\perp(0)\cdot \boldsymbol{\delta B}_\perp(t_2-t_1)\rangle \cos\left(\frac{c(t_2-t_1)}{r_{\rm L}}\right) \\ &= 2\frac{c^2}{r_{\rm L}^2}\frac{1}{\langle \delta B^2\rangle}\int_0^{+\infty}\mathrm{d}t \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{\Delta x}} S_{\rm 3d}(\mathbf{k}) \cos\left(\frac{ct}{r_{\rm L}}\right) \end{aligned}$$

involves the correlation function of the turbulent component taken along the unperturbed trajectory of the particle in the background field

The integral brings the gyroresonant interactions: $\sim \delta\left[k_{\|} c\cos(\theta_0) \pm n\frac{c}{r_{\rm L}}\right], \quad n \in \mathbf{Z}$

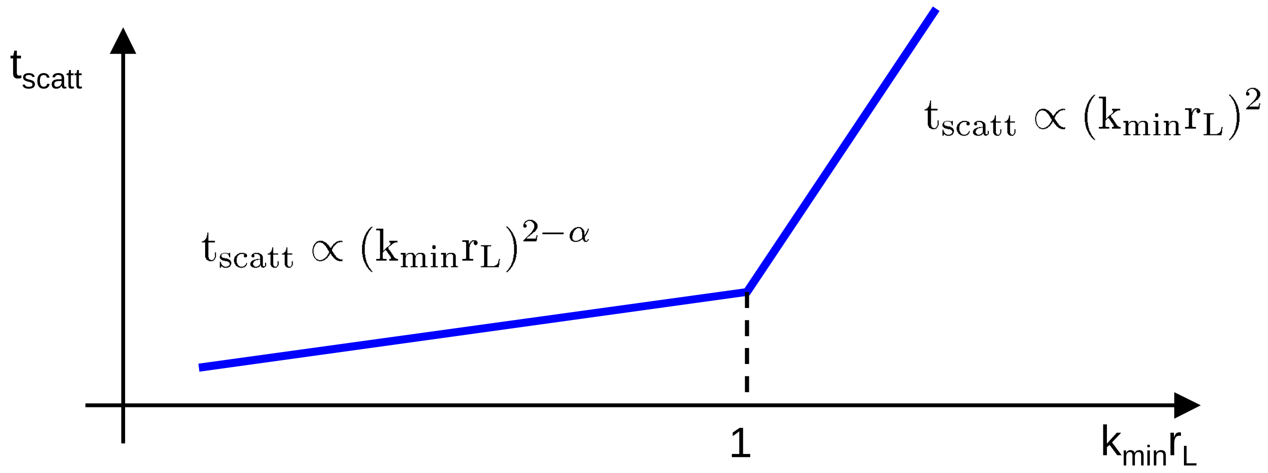
Diffusion of pitch angle

In the weak turbulence ($\eta \ll 1$) limit, the quasilinear calculation predicts:

$$\frac{1}{t_{\rm scatt}} = \frac{\langle \Delta\theta^2 \rangle}{\Delta t} \sim \eta \, (k_{\rm min} r_L)^{\delta} \, \frac{c}{r_L}$$

$t_{\rm scatt}$:
time to achieve $\Delta\theta^2 \sim 1$

$\delta = \alpha - 1$ for $k_{\rm min} r_L \ll 1$
$\delta = -1$ for $k_{\rm min} r_L \gg 1$

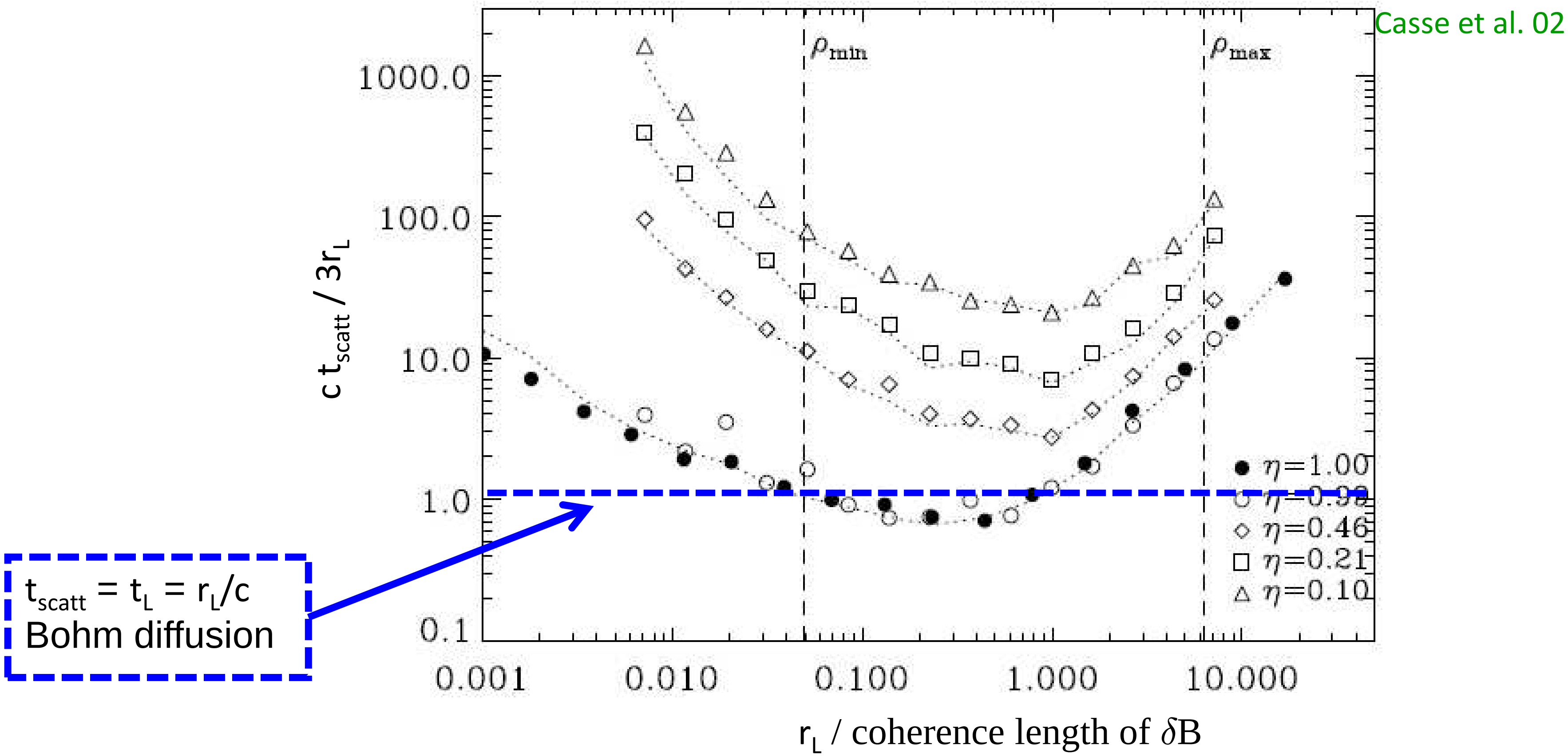


Diffusion timescale vs Larmor time

Perpendicular diffusion: more complex issue (e.g. Jokipii 1973), governed by the diffusion of the field lines themselves...

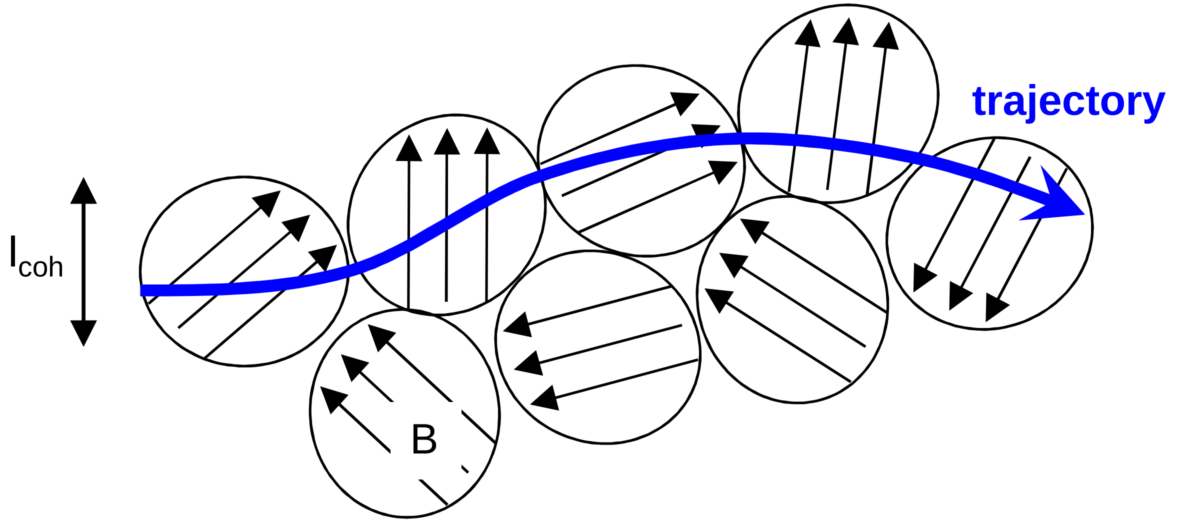
in general, $D_\perp \ll D_\parallel$ although $D_\perp \sim D_\parallel$ for full turbulence ($B_0 \ll \delta B$)

in a generic turbulence, the diffusion timescale is greater or much greater than r_L/c



High energy: ballistic motion, small deflection

Example for $r_L \gg l_{coh}$:



in each coherent cell, particle is deflected by: $\delta\theta \sim \frac{\lambda_B}{r_L}$

... deflections are independent, so after crossing N cells: $\delta\theta^2 \sim N \left(\frac{\lambda_B}{r_L}\right)^2$

... hence, for a distance L = N l_{coh} : $\delta\theta^2 \sim \frac{L\lambda_B}{r_L^2}$

... note that $\delta\theta^2 \sim 1$ after a scattering time: $t_{\rm scatt} \sim \frac{1}{c}\frac{r_L^2}{\lambda_B} \sim \frac{1}{c} k_{\rm min} r_L^2$

...as expected from quasilinear theory

... for transverse diffusion: see Plotnikov et al. 11

Applications

cosmic ray transport in the Galaxy:

$$r_L \sim 10^{-3}\,\mathrm{pc}\,(E/1\,\mathrm{TeV})Z^{-1} \qquad \Rightarrow t_{\rm scatt} \sim 1\,\mathrm{pc}\,(E/1\,\mathrm{TeV})^{1/3}Z^{-1/3}$$

$$\lambda_B \sim 10 - 100\,\mathrm{pc}$$

assuming Kolmogorov like turbulence

$\rightarrow$ does not fit well numerical fits to cosmic ray data...

$\rightarrow$ how to explain Ice Cube small angular scale anomalies with $t_{\rm scatt} \ll D_{\rm SNe}$??

ultrahigh cosmic ray transport in intergalactic magnetic fields:

$$r_L \sim 100\,\mathrm{Mpc}\,(E/10^{20}\,\mathrm{eV})Z^{-1}(B/10^{-9}\,\mathrm{G})^{-1}$$

$$\lambda_B \lesssim 1\,\mathrm{Mpc} \qquad \Rightarrow \delta\theta^2 \sim \frac{L\lambda_B}{r_L^2} \lesssim 10^{-2}Z^2(E/10^{20}\,\mathrm{eV})^{-2}(B/10^{-9}\,\mathrm{G})^2$$

small angular deflection expected for light nuclei

cosmic ray acceleration in supernovae remnants:

$$t_{\rm acc} \sim \mathcal{A}\frac{t_{\rm scatt}}{t_L}t_L \quad \left(\mathcal{A} \sim \frac{c^2}{v_{\rm sh}^2} \gg 1\right)$$

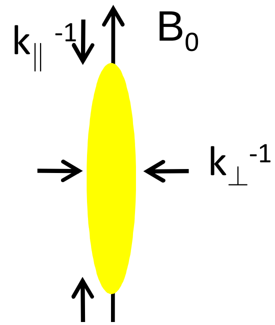
and maximal energy: $E_{\rm max}: \quad t_{\rm acc} \lesssim \frac{R}{v_{\rm sh}}$

to reach $E_{\rm max} \sim 1$ PeV, one needs Bohm scaling $t_{\rm scatt} \sim t_L$ + amplified field $B \sim 100\,\mu$G...
Bohm scaling corresponds to scale invariant turbulent spectra...

Modern developments: anisotropic MHD turbulence

Goldreich & Sridhar (95): preferred direction set by B_0...

$\rightarrow$ the cascade becomes anisotropic with typical relationships $k_{\|} \sim k_{\perp}^{2/3} k_{min}^{1/3}$



$\rightarrow$ energy spectra of the form $S_{3d} \sim k_{\perp}^{-\alpha} f\left(k_{\perp}, k_{\|}\right)$

e.g. Goldreich &Sridhar 95,97,
Ng & Battacharjee 96,
Galtier et al. 00, 02, ...

value of α depends on cascade conditions

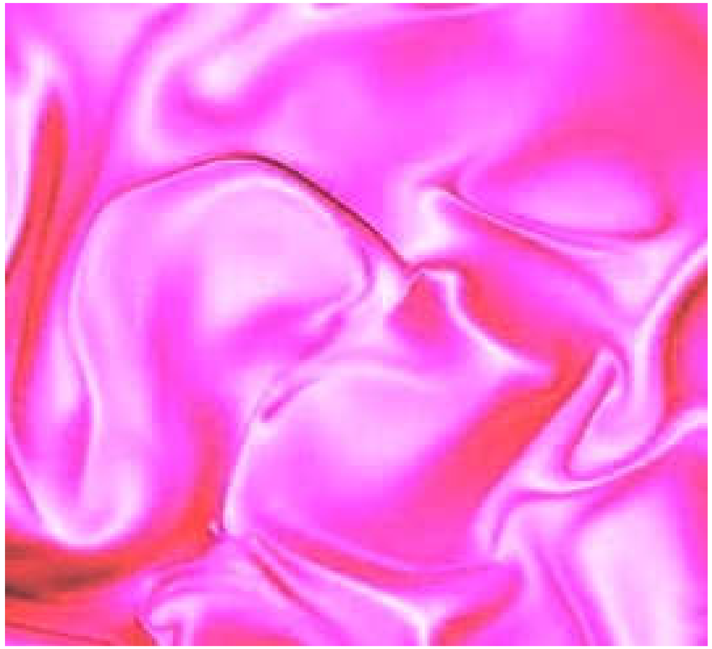
$f(k_{\perp},k_{\|})$ characterizes the anisotropy

$\rightarrow$ anisotropy is local wrt the local large scale magnetic field $\boldsymbol{B}_0 + \int_{k<l^{-1}} \mathrm{d}k \boldsymbol{\delta B_k}$

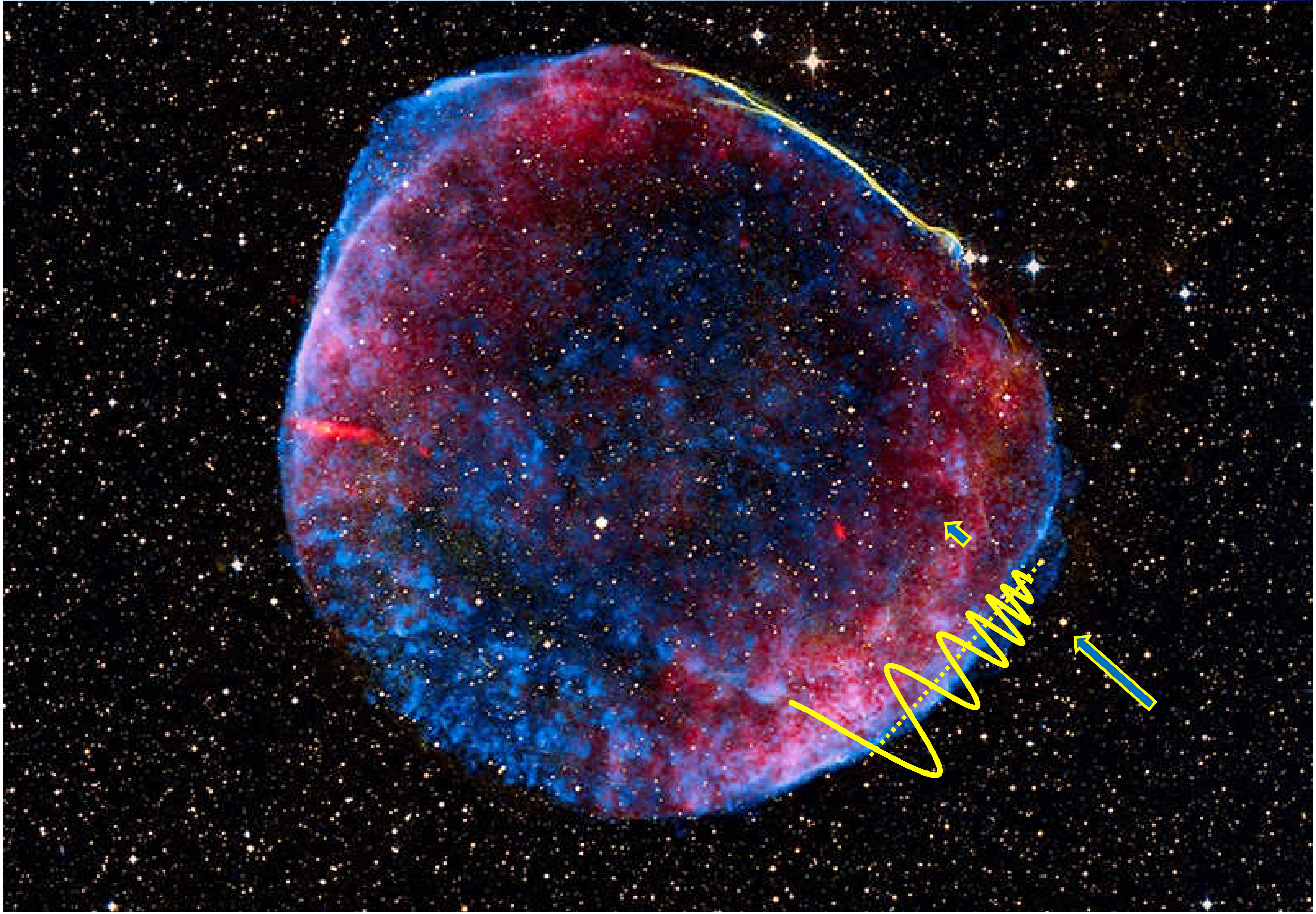
Cho & Vishniac 00, Cho & Lazarian 02

$\rightarrow$ consequences for cosmic ray transport still being debated...
e.g. Chandran 00, Marcowith et al. 06, Yan & Lazarian 04, 08, Beresnyak et al. 11

$\rightarrow$ note that theory + simulations usually assume test particle transport, whereas $\varepsilon_B \sim \varepsilon_{cr}$ in the ISM...



Part 2: Particle acceleration



Introduction

- many different acceleration mechanisms: Fermi 1, Fermi 2, shear, ...

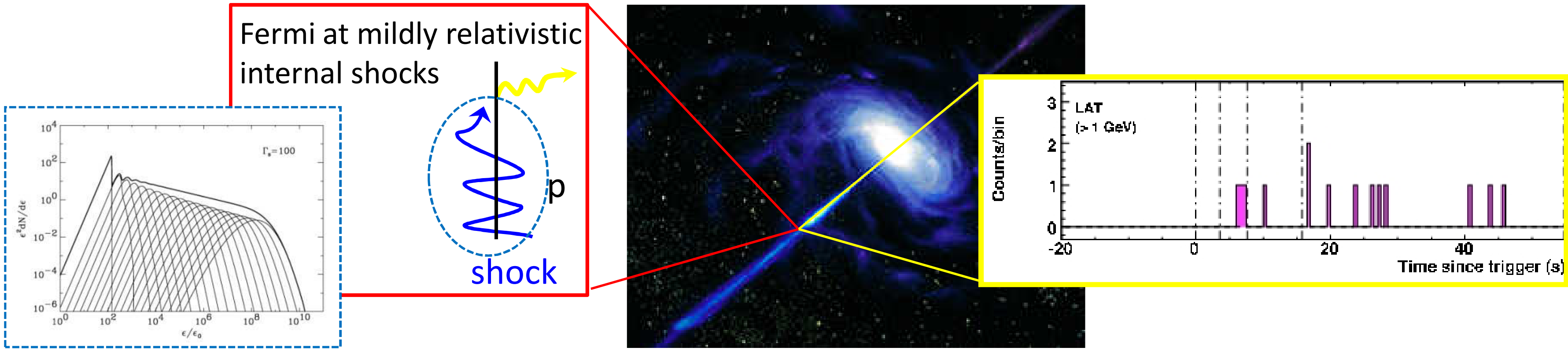
  however: Fermi acceleration at shock remains "the reference"...

  ... it is generic in outflows, it is energetically efficient, it generically produces (near) powerlaw spectra.... and its physics is determined by a minimum number of parameters...

- main signatures to be determined:

  E_{min} , E_{max} [timescale t_{acc}(E)], spectral slope α, running d α/ d ln E, radiative losses

- only secondary photon spectra are observed...



- different ways of addressing this problem:

  - acceleration physics: idealized source configurations $\rightarrow$ calculate t_{acc}(E), α(E)

  - data interpretation: most effort on source modelling ($t_{acc} \sim t_L$, $\alpha \leftarrow$ best fit)

Acceleration... general schemes

Particle dynamics: $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{p} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c\right)$ $\Rightarrow$ exploit electric fields...

MHD flows with electromotive force: $\mathbf{E} \simeq -\mathbf{v}_{\mathrm{pl}} \times \mathbf{B}/c$ (v_{pl} plasma velocity)

$\rightarrow$ guarantees **E**=0 in the plasma rest frame ($\leftarrow$ infinite conductivity + MHD)

$\rightarrow$ acceleration timescale $t_{acc} \geq r_L/v_{pl}$ **(+ subtle effects when $v_{pl} \sim c$)**

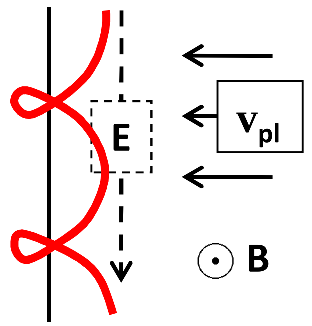
$E_{max} < Z\,e\,B\,L\,v_{pl}$ from equation of motion

$\rightarrow$ **need to 'push' particle along E, across B**

e.g., a force **F** implies a drifting motion of the guiding center:

$$\boldsymbol{v}_{\mathbf{gc}} = \frac{1}{q}\frac{\boldsymbol{F} \times \boldsymbol{B}}{B^2}$$

F can correspond to a force (e.g., qE),
to an average force ($\leftarrow$ pressure)
or to inhomogeneity in B...



$\rightarrow$ e.g.: shock drift acceleration e.g. Kirk 94

inhomogeneity in B transforms helix
into a cycloid with a net drift along E
$\Rightarrow$ energy gain until shock crossing

Acceleration... general schemes

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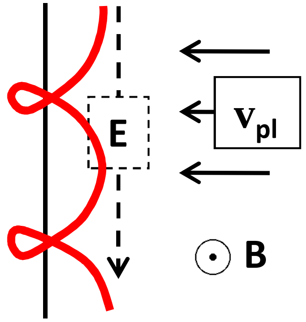


$\rightarrow$ e.g.: shock drift acceleration e.g. Kirk 94

shock surfing acceleration

magnetized rotators (inhomogeneous B, inertia effects)
e.g. Bell 92, Arons 03, Rieger & Aharonian 09, Istomin & Sol 09

shear acceleration (requires scattering or wave pressure)
e.g. Rieger et al. 07, Lyutikov & Ouyed 07

MHD flows, stochastic interactions:

$\rightarrow$ **shock diffusive acceleration**... Bell 78, Axford et al. 77, Krimskii 77, Blandford & Ostriker 78

... regular energy gain due to shock converging geometry... scattering $\Rightarrow$ large t_{acc}

$\rightarrow$ stochastic process: $\delta E_k \sim v_\phi\, \delta B_k /c$, with v_ϕ scattering center velocity in original Fermi mechanism (Fermi 49) or phase velocity for MHD waves...

... acceleration timescale $\propto (c/v_\phi)^2$ (+ time for scattering)

Beyond MHD:

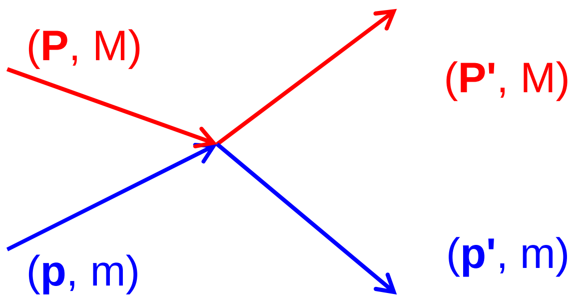
$\rightarrow$ e.g.: reconnection...

ponderomotive force of coherent waves (wakefield)... Katsouleas & Dawson 83

Basics of diffusive acceleration

e.g. Kirk 06

Two-body scattering:



e.g. light particle on magnetic cloud (Fermi 49)

conservation of energy and momentum implies:

$$\frac{\Delta E}{E} \simeq -\frac{\boldsymbol{\Delta p}\cdot\boldsymbol{P}}{EM} + \mathcal{O}\left(\frac{\Delta p^2}{EM}\right)$$

(Δp $\equiv$ p'-p, assuming Δp $\ll$ p, P $\ll$ M)

after averaging over outgoing directions:

energy gain if head-on collision **p . P** < 0
energy loss if tail-on collision **p . P** > 0

$\Delta E/E \simeq \pm P/M \equiv \pm \beta_{sc}$

$\Rightarrow$ systematic gain due to more frequent head-on than tail-on collisions: $\dfrac{\langle\Delta p\rangle}{p\Delta t} \simeq \dfrac{\beta_{\rm sc}^2}{t_{\rm scatt}}$

average energy gain $\Delta E/E \propto \beta_{sc}^{2}$ 2nd order process

$\Rightarrow$ includes diffusion in momentum space, with: $\dfrac{\langle\Delta p^2\rangle}{p^2\Delta t} \simeq \dfrac{\beta_{\rm sc}^2}{t_{\rm scatt}}$

Acceleration timescale: $t_{\rm acc} \simeq \dfrac{t_{\rm scatt}}{\beta_{\rm sc}^2}$

2^{nd} order Fermi acceleration

Acceleration timescale: $t_{\rm acc} \simeq \dfrac{t_{\rm scatt}}{\beta_{\rm sc}^2}$

for interactions with magnetized turbulence: t_{scatt} is the diffusion timescale, β_A Alfven velocity

however: $t_{scatt} > r_L/c$, $\beta_A \sim 10^{-5}$ (B/1μ G)(n/1 cm^{-3})$^{-1/2}$

$\Rightarrow$ requires relativistically moving scattering centers (high B, low density)

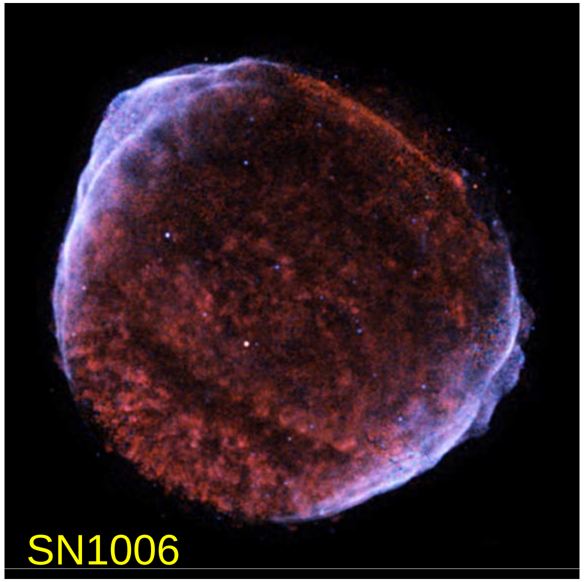
Application: acceleration of cosmic rays in the insterstellar medium? (Fermi 49)

typical parameters: n $\sim$ 1 cm^{-3}, B $\sim$ 1 μG

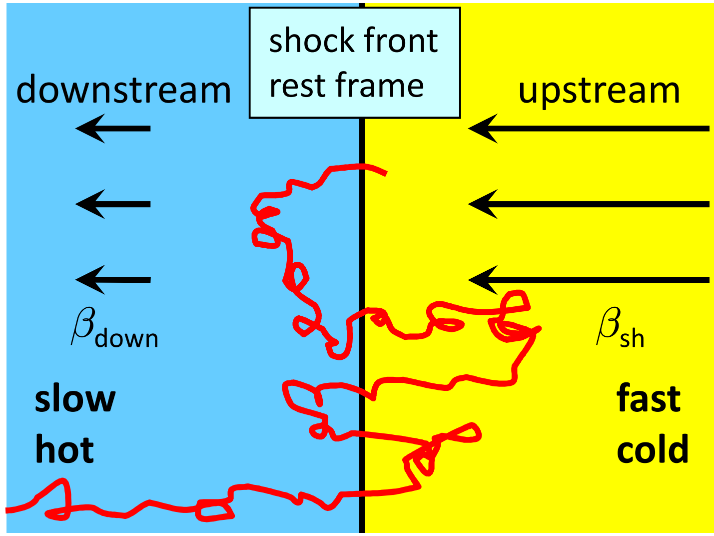
acceleration timescale: $t_{acc} \sim 3\ 10^4$ yrs (E / 1 GeV) (B / 1 μG)$^{-3}$ (n / 1 cm^{-3})

... too slow to accelerate particles to high energy on the cosmic ray escape timescale (10^7 yrs) and the energy loss timescale through ionization...

1st order Fermi acceleration



model for the forward shock



particles get accelerated as they bounce back and forth across the shock wave

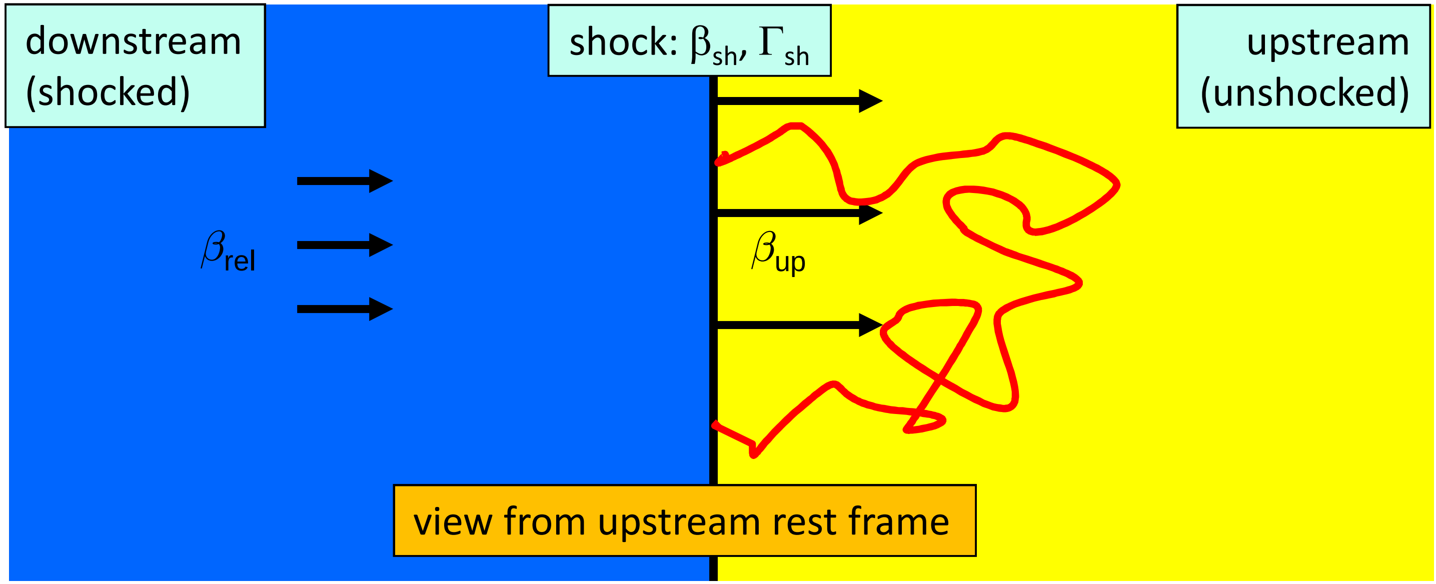
Shock jump conditions for strong shocks:

non-relativistic $\beta_{sh} \ll 1$: downstream velocity vs upstream $\beta_{down} = \beta_{sh}/4$,
relative velocity between up and down $\beta_{rel} = 3\beta_{sh}/4$,
compression ratio $n_{down} = 4\, n_{up}$

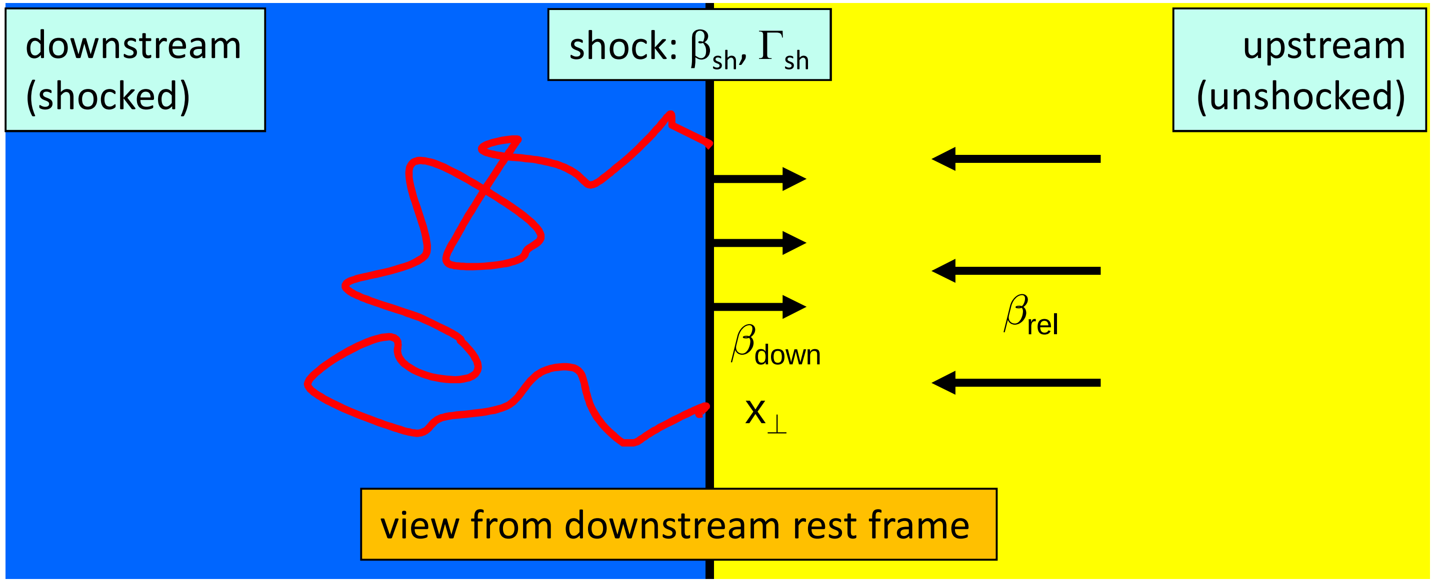
particles gain energy by bouncing back and forth:

... **regular gain through stochastic interactions**...
... regular thanks to the converging nature of the flow in each rest frame...
... source of energy: energy of the flow

1^{st} order Fermi acceleration

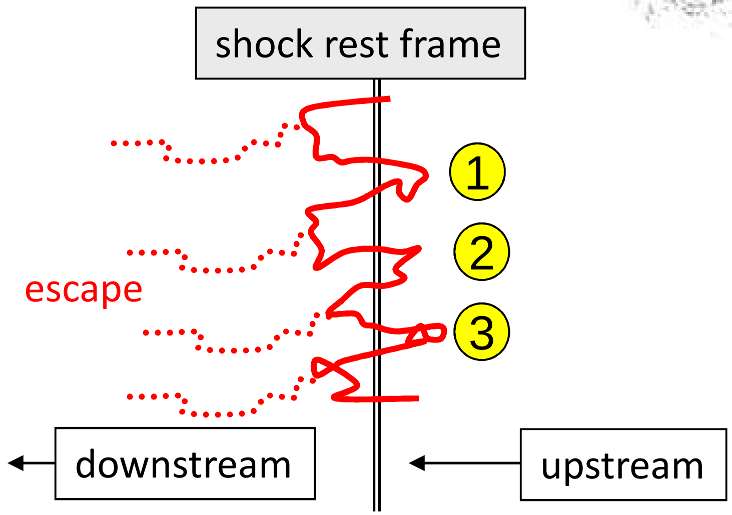


as the particle circulates upstream and downstream, it systematically experiences a converging flow when returning to the shock $\Rightarrow$ **head-on collisions**



Fermi acceleration in three steps

Step 1: particle interacts with scattering centers (=magnetic inhomogeneities); **in the rest frame of the scattering center, diffusion is elastic:**
$E^f = E^i$



Step 2: follow a cycle around the shock:
upstream $\rightarrow$ downstream $\rightarrow$ upstream
switching from one frame to the other and back

2.a: shock crossing from up- to downstream

$$E^{\rm in}_{\rm down} = \Gamma_{\rm rel} E^{\rm out}_{\rm up}\left(1 - \mu^{\rm out}_{\rm up}\beta_{\rm rel}\right)$$

$\beta_{\rm rel}$, $\Gamma_{\rm rel}$: velocity between up and down

cosine of angle to shock normal
(= velocity component along shock normal)

2.b: particle bounces back downstream

$$E^{\rm out}_{\rm down} = E^{\rm in}_{\rm down}, \qquad \mu^{\rm out}_{\rm down} \neq \mu^{\rm in}_{\rm down}$$

(elastic scattering)

2.c: particle crosses the shock toward upstream (if it does not escape)

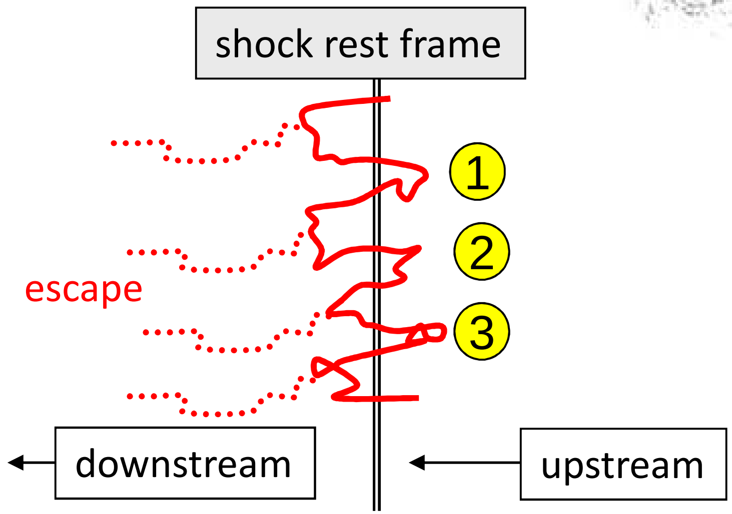
$$E^{\rm in}_{\rm up} = \Gamma_{\rm rel} E^{\rm out}_{\rm down}\left(1 + \mu^{\rm out}_{\rm down}\beta_{\rm rel}\right)$$

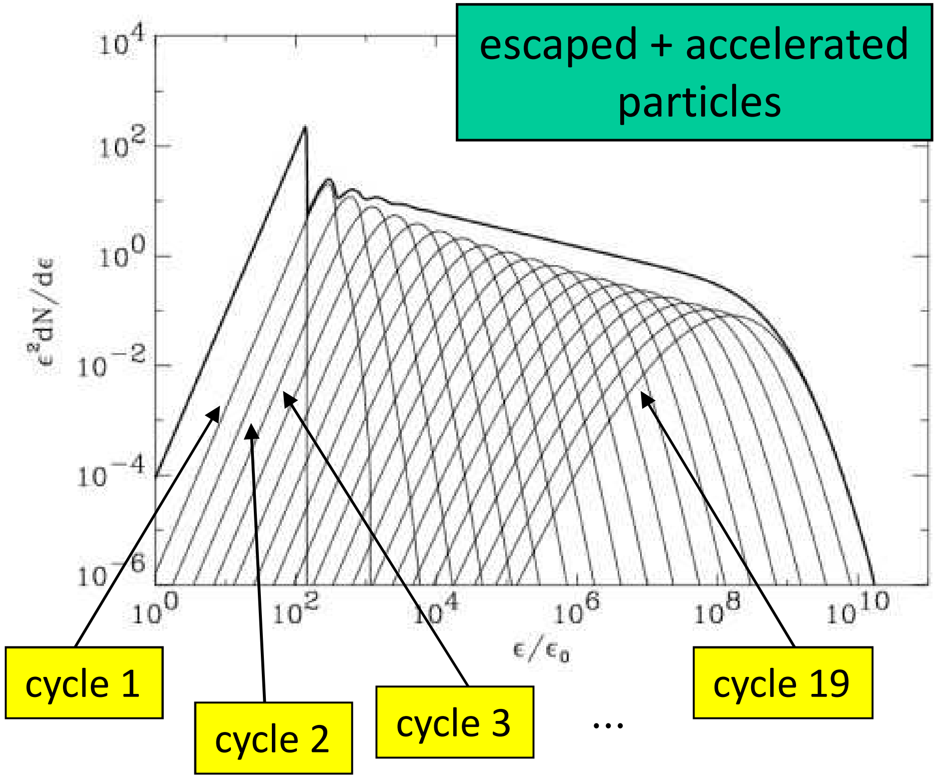
Fermi acceleration in three steps

Step 3: collect the results and average over angles (isotropic scattering)

$$\frac{E^{\rm in}_{\rm up}}{E^{\rm out}_{\rm up}} = \Gamma^2_{\rm rel}\left(1+\mu^{\rm out}_{\rm down}\beta_{\rm rel}\right)\left(1-\mu^{\rm out}_{\rm up}\beta_{\rm rel}\right)$$

$$\left\langle \frac{E^{\rm in}_{\rm up}}{E^{\rm out}_{\rm up}} \right\rangle \simeq 1+\beta_{\rm sh}$$





particle gains energy at each cycle, but has a non zero probability of escaping from the flow

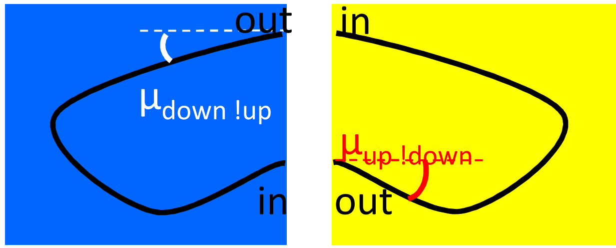
accelerated population: sum of populations escaped at each cycle ~ power law spectrum f(E) / E^{-s}

Diffusive shock acceleration

Energy gain:

interactions are elastic in the plasma rest frame, only the particle direction is altered

as viewed from the shock frame, the particle overturn is associated with a net change of energy



turn around toward the shock implies a net energy gain

after one cycle: $g \equiv \dfrac{\Delta E_{\rm up}}{E_{\rm up}} = \Gamma_{\rm rel}^2 \left(1+\beta_{\rm rel}\cos\theta_{\rm down\to up}\right)\left(1-\beta_{\rm rel}\cos\theta_{\rm up\to down}\right) - 1$

for non-relativistic shocks, particle populations are isotropic (due to diffusion), average of angles gives $g \simeq \beta_{\rm sh}$ (1st order)

Probability of escape: at each cycle, particles can escape downstream due to advection, with probability $P_{\rm esc} \simeq \beta_{\rm sh}$

Return timescale: $t_{\rm ret} \simeq \dfrac{t_{\rm scatt}}{\beta_{\rm sh}}$

Acceleration timescale: $t_{\rm acc} = \dfrac{t_{\rm ret,u} + t_{\rm ret,d}}{g}$

Spectrum of accelerated particles

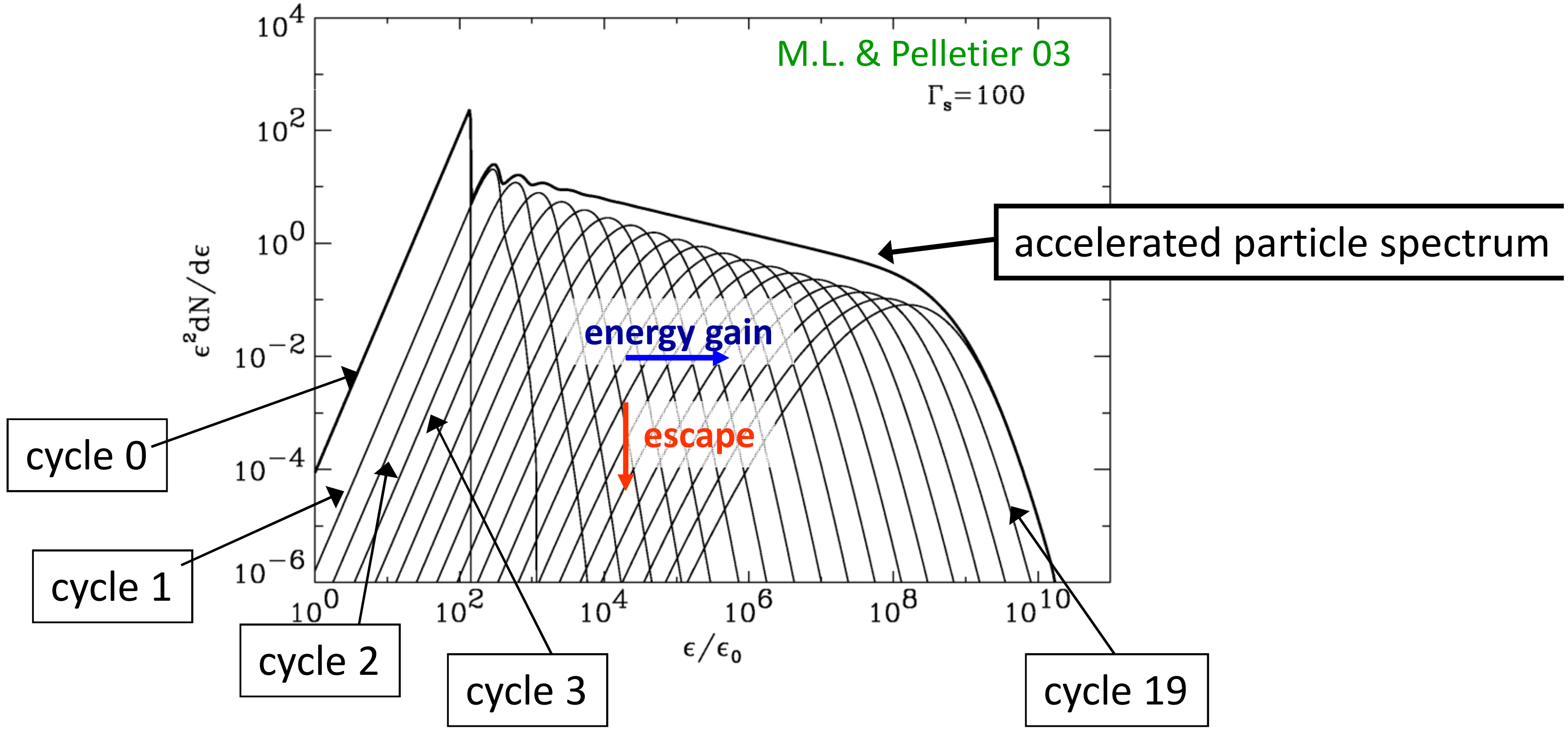
Energy spectrum: at each cycle, particles escape downstream with finite probability P_{esc} ; the final accelerated population is the sum of all escaped particles:

$\Rightarrow$ **spectral index results from a competition between energy gain and escape**

Spectrum formation: after N cycles, $E_N = (1+g)^N E_0$, $N(>E) = N(>E_0)\,(1-P_{esc})^N$

$\Rightarrow$ $N(>E) \propto E^{1-s}$, with $s = 1 + \ln(1-P_{esc})/\ln(1+g)$

$\Rightarrow$ $dN/dE \propto E^{-s}$, with $s = 1 + \ln(1-P_{esc})/\ln(1+g) \simeq 2.0$ in non-relativistic strong shocks

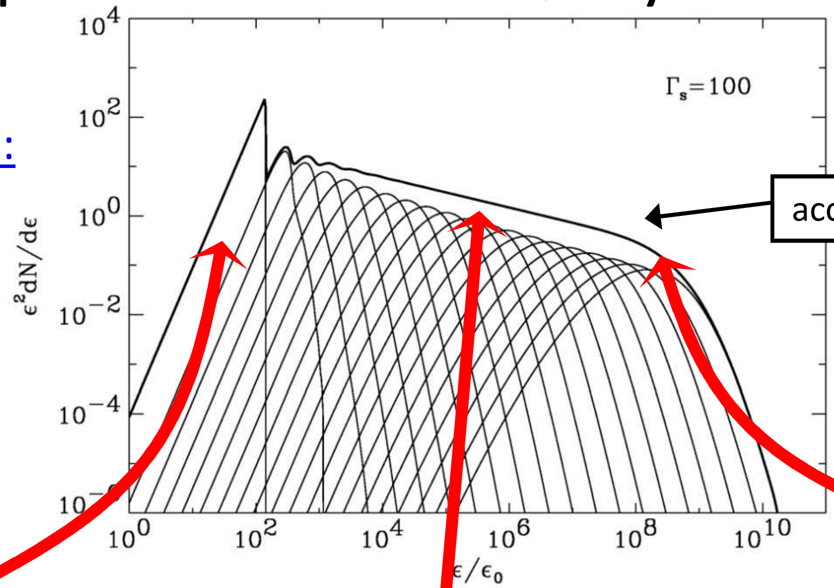


Modern developments... beyond test particles...

Test particle: as previously, one solves for the kinematics of the accelerated particle population in a predefined magnetized turbulence...

... **however cosmic rays back react on the flow, through their contribution to the pressure and through the amplification of MHD modes $\Rightarrow$ fully nonlinear problem...**

Main open questions:



accelerated particle spectrum

injection

crucial issue for e, w/o heating:

$r_{L,e} << r_{L,p} \sim \Delta_{shock}$

source of heating?

non-linear acceleration

$P_{cr} >> P_{up} \Rightarrow$ CR modify jump conditions in the precursor...

CR may also amplify B_{up} through streaming instabilities...

accelerated CR

shock structure

instabilities

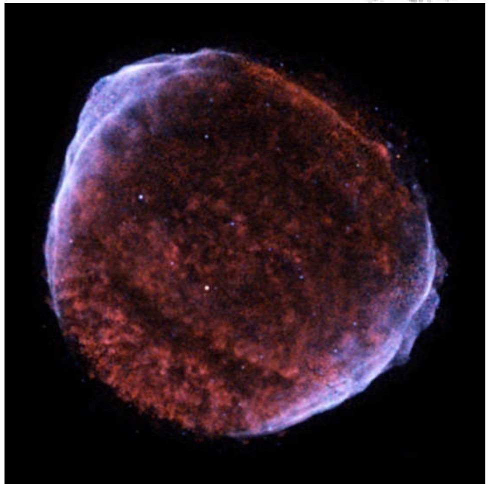
maximal energy and escape

how is escape realized?

maximal energy?

resulting spectrum integrated over SN evolution?

Supernovae remnants and Galactic cosmic rays



Supernovae remnants: efficient particle accelerators at young age, with $R \sim 1$ pc, $\beta_{sh} \sim 0.01 - 0.1$

Energetics: if each supernova (Galactic rate $r_{SN} \sim 2 / 100$yr) injects 10% of its shock kinetic energy ($\sim 10^{51}$ ergs) in cosmic rays, one recovers the observed low energy cosmic ray flux, accounting for the Galactic volume and escape timescale ($\sim 10^7$ yrs)

Spectrum:

- in the test particle approximation, the predicted spectral index is s=2.0
- accounting for the influence of accelerated particles on the shock structure, non-linear models predict concave spectra (i.e., soft to hard transition) due to the modification of the shock strength by the accelerated population

... however, this represents only a first order correction... the full wave - particle relationship is still not understood ...

Maximal energy:

- comparing the age of the remnant and the acceleration timescale, one finds **$E_{max} \sim 10$ TeV Z (B/3μ G)** (assuming Bohm scaling t_{scatt} / t_L)

- to reach the knee, one needs $B \sim 300\ \mu$G, as suggested by X-ray observations of supernovae remnants, and interpreted as a consequence of streaming instabilities of accelerated particles on the ambient magnetic field

Cosmological shock waves and UHECRs

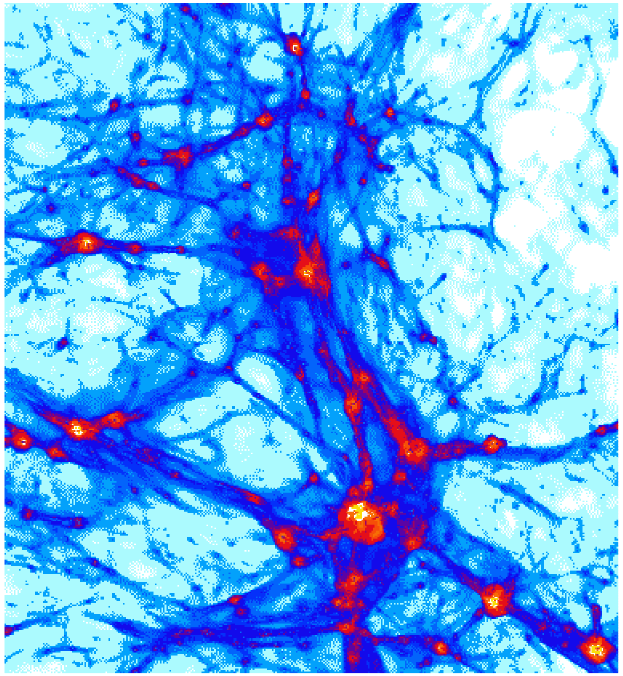
Generic acceleration timescale: $$t_{\rm acc} = \frac{t_{\rm ret,u} + t_{\rm ret,d}}{g}$$

with $t_{\rm ret} \simeq \dfrac{t_{\rm scatt}}{\beta_{\rm sh}}$, $g = \beta_{sh}$ and $t_{scatt} > t_L$

optimistic estimate (Bohm scaling always): $t_{scatt} = t_L$, and $B \sim 1\ \mu G$

Maximum energy:

$$t_{\rm acc} \approx 10^{18}\,{\rm sec}\,Z^{-1}\left(\frac{E}{10^{20}\,{\rm eV}}\right)\left(\frac{B}{1\,\mu{\rm G}}\right)^{-1}\left(\frac{v_{\rm sh}}{1000\,{\rm km/s}}\right)^{-2}$$



but, age of the Universe (and expansion losses):

$$t_{\rm loss} \approx H_0^{-1} \simeq 10^{17}\,{\rm sec}...$$

$\Rightarrow$ with optimistic assumptions, only heavy nuclei in the most powerful shocks around clusters of galaxies

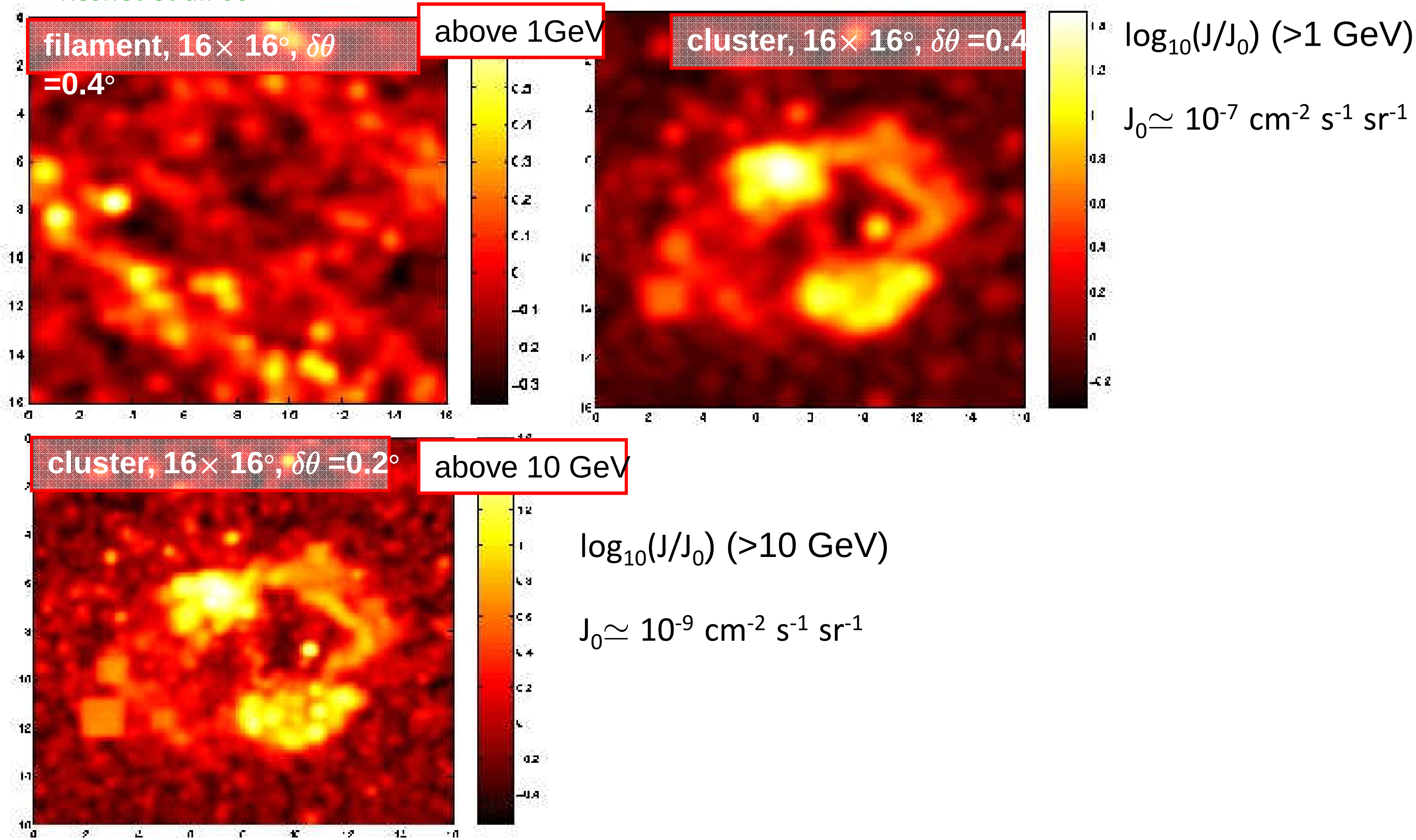
$\Rightarrow$ **relativistic sources are more promising (?)**

Acceleration at IGM shock waves

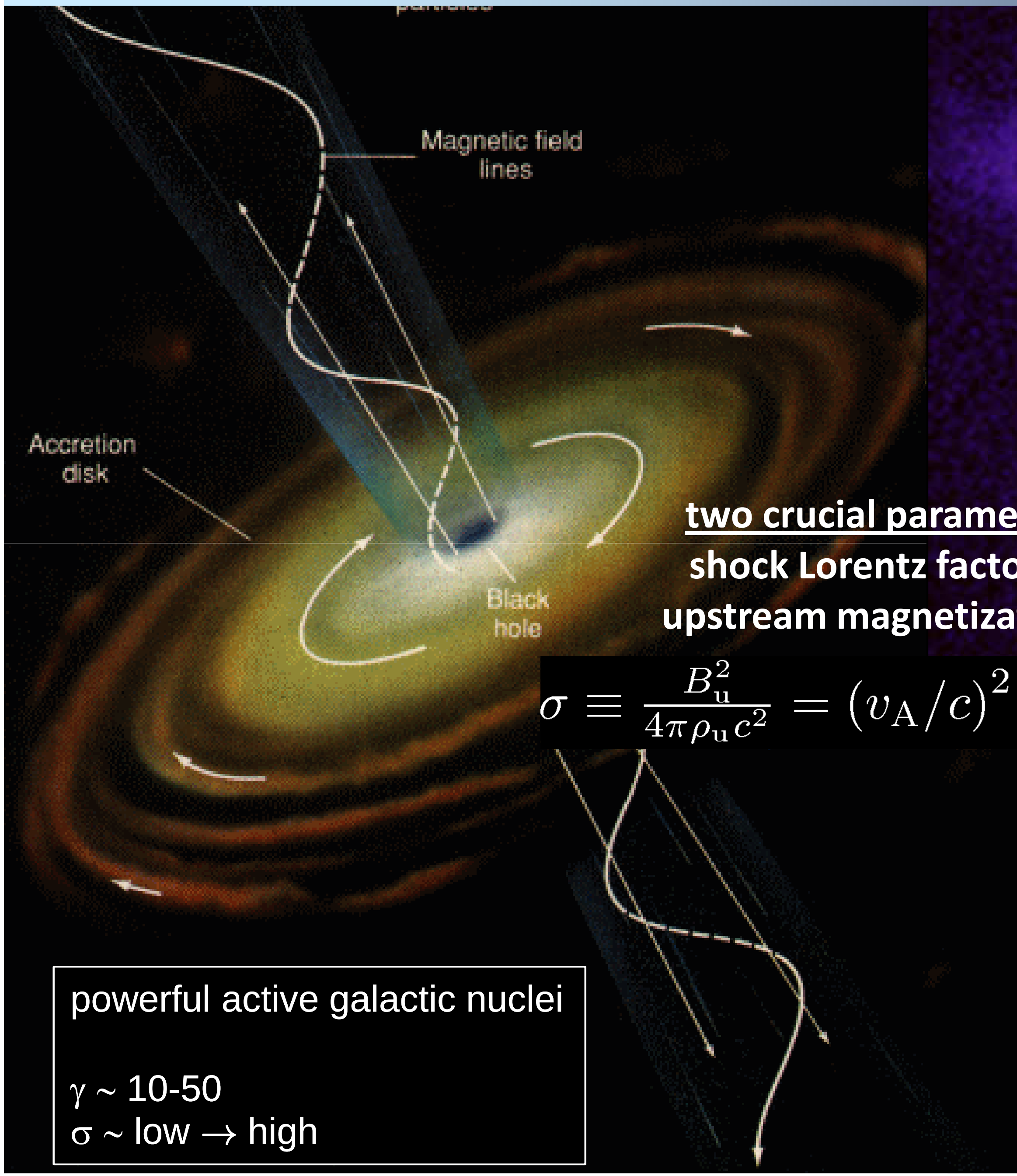
- IGM shock waves:

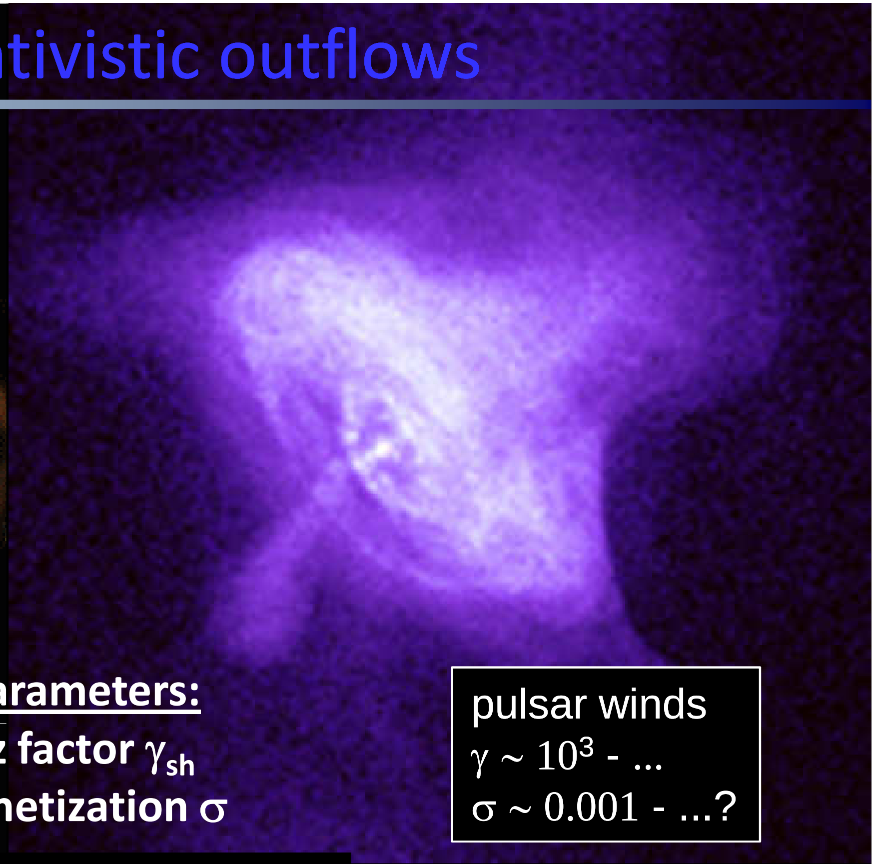
  acceleration can proceed if the unshocked medium is magnetized: gamma-ray observations would allow to measure this unshocked (primeval?) magnetic field and/or constrain the amplication mechanisms...

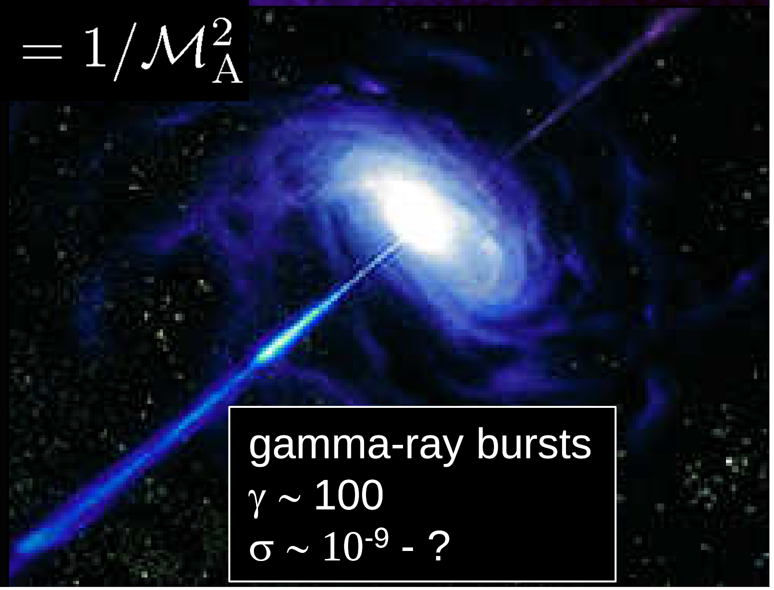
Keshet et al. 03



Part 3: particle acceleration in relativistic outflows







<u>two crucial parameters:</u>
shock Lorentz factor γ_{sh}
upstream magnetization σ

$$\sigma \equiv \frac{B_{\mathrm{u}}^2}{4\pi \rho_{\mathrm{u}} c^2} = (v_{\mathrm{A}}/c)^2 = 1/\mathcal{M}_{\mathrm{A}}^2$$

pulsar winds
$\gamma \sim 10^3$ - ...
$\sigma \sim 0.001$ - ...?

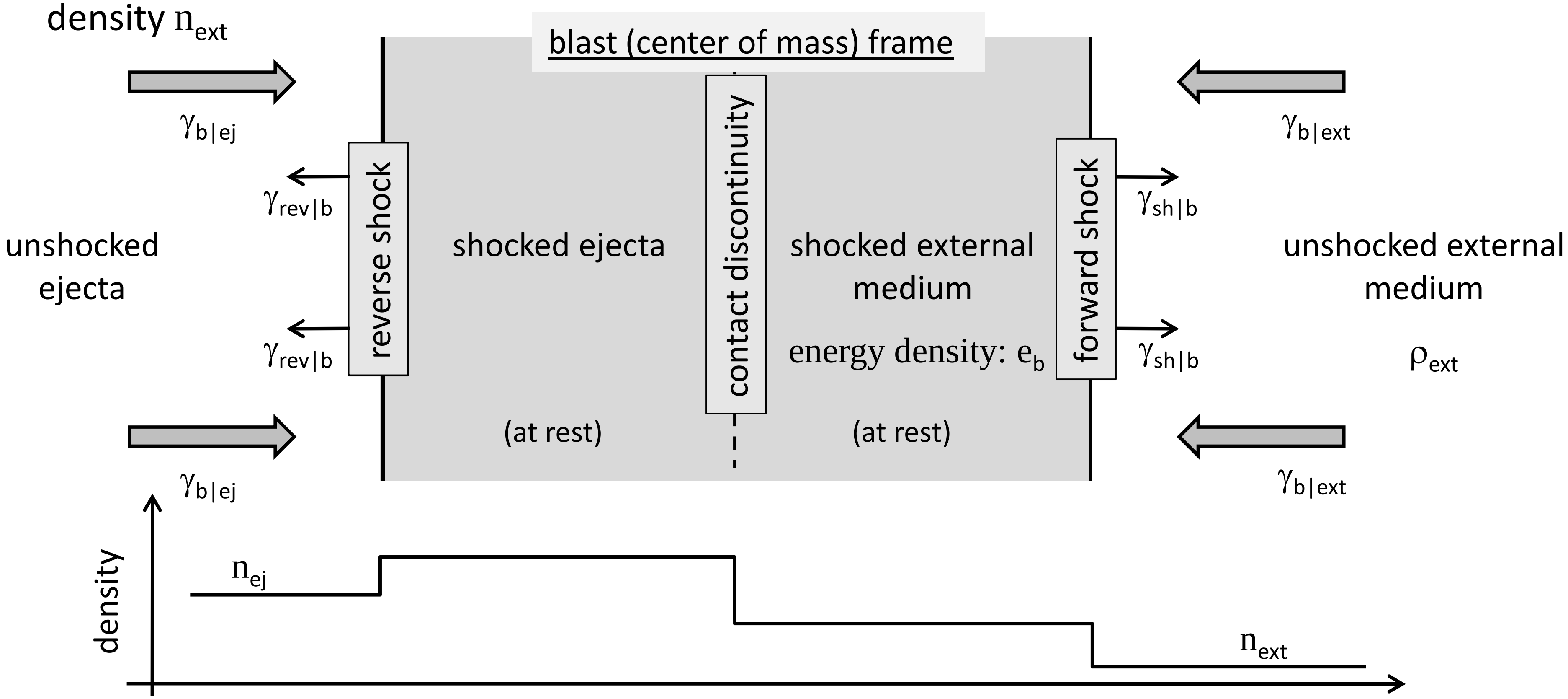
powerful active galactic nuclei

$\gamma \sim$ 10-50
$\sigma \sim$ low $\rightarrow$ high

gamma-ray bursts
$\gamma \sim 100$
$\sigma \sim 10^{-9}$ - ?

(Hydro-)Dynamics of relativistic shock waves

outflow of proper density n_{ej} moving at Lorentz factor γ_{ej} relative to medium at rest, density n_{ext}

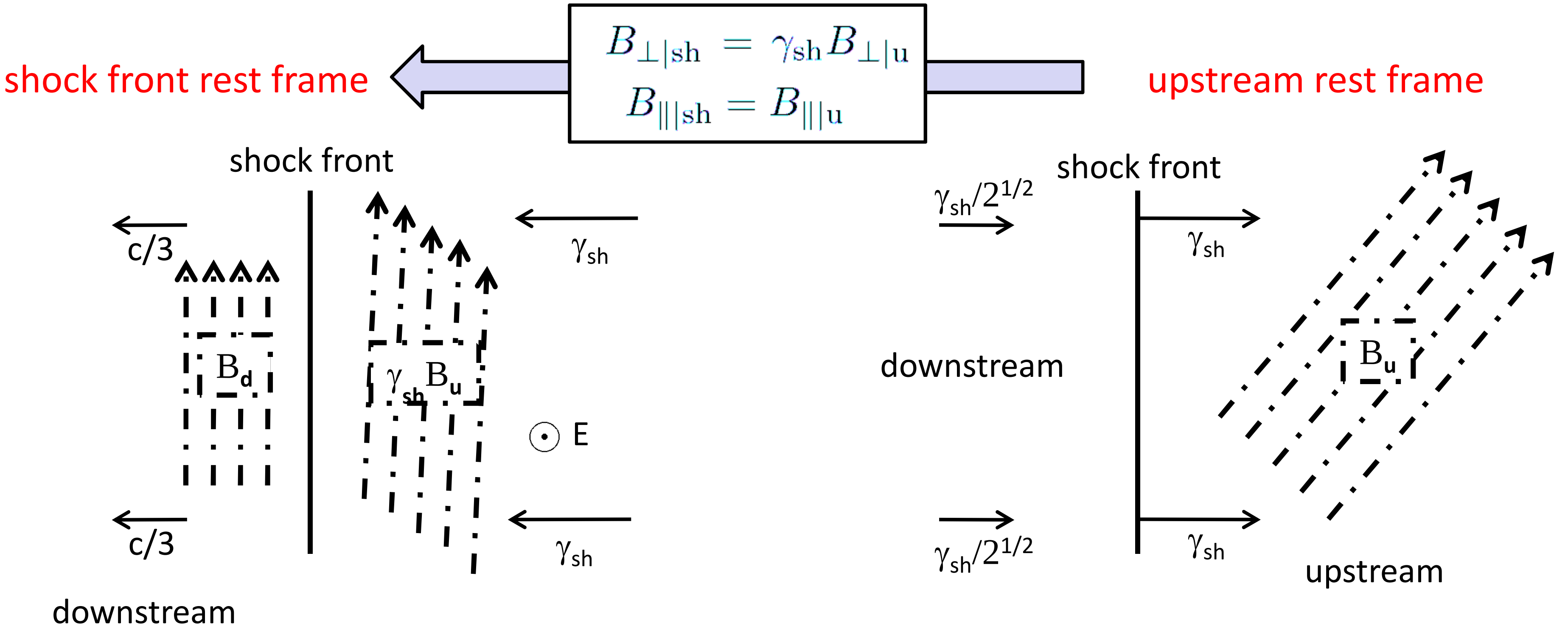


Continuity equations (strong, unmagnetized shock) with velocities β_+, β_- in shock frame:

$$\begin{aligned}
\beta_+\gamma_+ n_+ &= \beta_-\gamma_- n_- \\
\beta_+^2\gamma_+^2(e_+ + p_+) + p_+ &= \beta_-^2\gamma_-^2(e_- + p_-) + p_- \\
\beta_+\gamma_+^2(e_+ + p_+) &= \beta_-\gamma_-^2(e_- + p_-)
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
\beta_{\rm sh|b} &\simeq \frac{1}{3} \\
\gamma_{\rm sh} &\simeq \sqrt{2}\gamma_{\rm b|ext} \\
n_{\rm b} &\simeq 4\gamma_{\rm b|ext} n_{\rm ext} \\
e_{\rm b} &\simeq 4\gamma_{\rm b|ext}^2 \rho_{\rm ext} c^2
\end{aligned}$$

e.g. Blandford & McKee 76, Kirk &Duffy 99

Ultra-relativistic shock waves



⇒ ultra-relativistic shock waves are mostly perpendicular (superluminal)

incoming particle energy $\gamma_{sh} m_p\, c^2$ (shock frame) is converted into random motion, corresponding to an effective temperature $\sim \gamma_{sh} m_p\, c^2$, for protons and electrons!

Relativistic Fermi acceleration - energy gain, kinematics

Gallant & Achterberg 99, Achterberg et al. 01

- Crucial difference w.r.t. non-relativistic Fermi: **shock wave velocity ~ c ~ particle velocity**

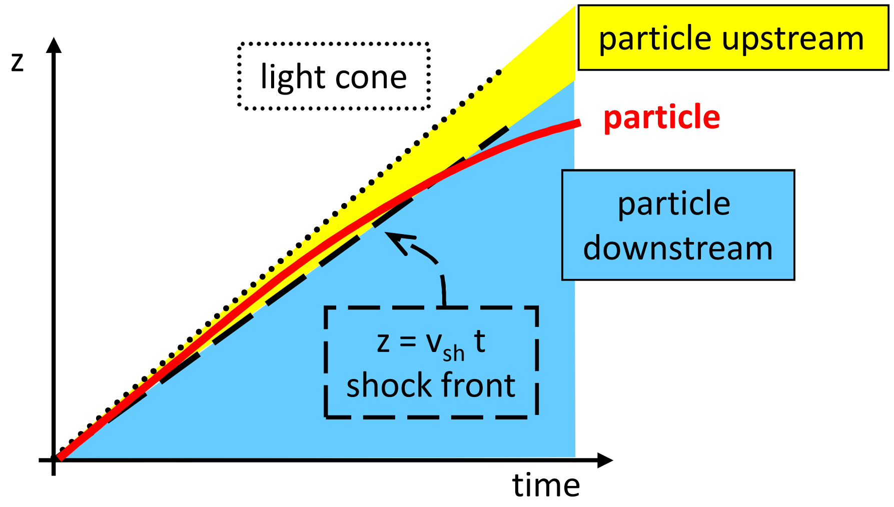
$\Rightarrow$ distance between particle and shock wave after time t: $(1-\beta_{sh})ct \sim$ **c t/(2γ_{sh}^2)**

$\Rightarrow$ particle caught back once its parallel velocity drops below β_{sh}

$\Leftrightarrow$ **deflection by 1/γ_{sh}**

$\Rightarrow$ energy gain $\sim \gamma_{sh}^2$ at first interaction, then ~ 2

$\Rightarrow$ precursor length scale $\sim r_L/(2\gamma_{sh}^3)$



$\rightarrow$ downstream is advected away from the shock at c/3, scattering needs to be extremely efficient to bring the particle back to the shock...
this requires very special conditions! (M.L. et al. 06, Niemiec et al. 06, Pelletier et al. 09)

- Energy gain: $$\frac{\Delta E_{\rm up}}{E_{\rm up}} = \gamma_{\rm b|ext}^2\left(1+\beta_{\rm b|ext}\cos\theta_{\rm down\to up}\right)\left(1-\beta_{\rm b|ext}\cos\theta_{\rm up\to down}\right)-1$$

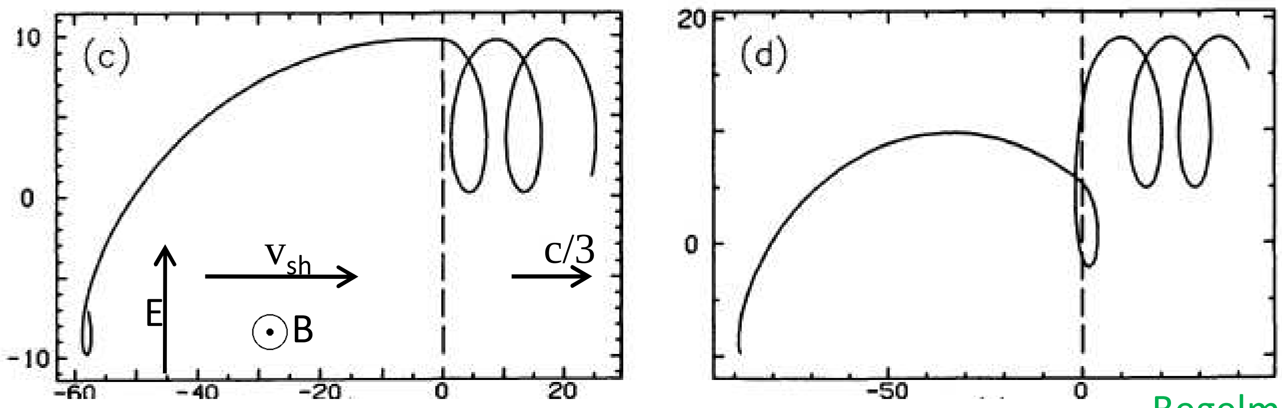
with $\gamma_{\rm b|ext} \simeq \gamma_{\rm sh}/\sqrt{2}$

(assumes efficient elastic scattering up- and down-stream)

Relativistic Fermi acceleration - oblique shock waves

- Superluminal shock waves:

$\rightarrow$ the intersection between a magnetic field line and the shock front moves faster than c
$\rightarrow$ if a particle is tied to a field line, this particle cannot return to the shock front...



Begelman & Kirk 90

- Scattering from turbulence:

$\rightarrow$ **large scale turbulence does not help:** the particle cannot execute more than 1 1/2 cycle...
up $\rightarrow$ down $\rightarrow$ up $\rightarrow$ down then advection to -∞ (ML et al. 06, Niemiec et al. 06)

$\rightarrow$ large scale : w.r.t. to typical Larmor radius (downstream frame) of accelerated particles $\gamma_p \sim \gamma_{sh}$

$r_L \sim 10^{12}\,\mathrm{cm}\,B_{u,-6}^{-1}\gamma_{sh,2.5}$ for shock heated particles, to be compared with the coherence length of interstellar turbulence $\lambda_B \sim 10^{20}$ cm!

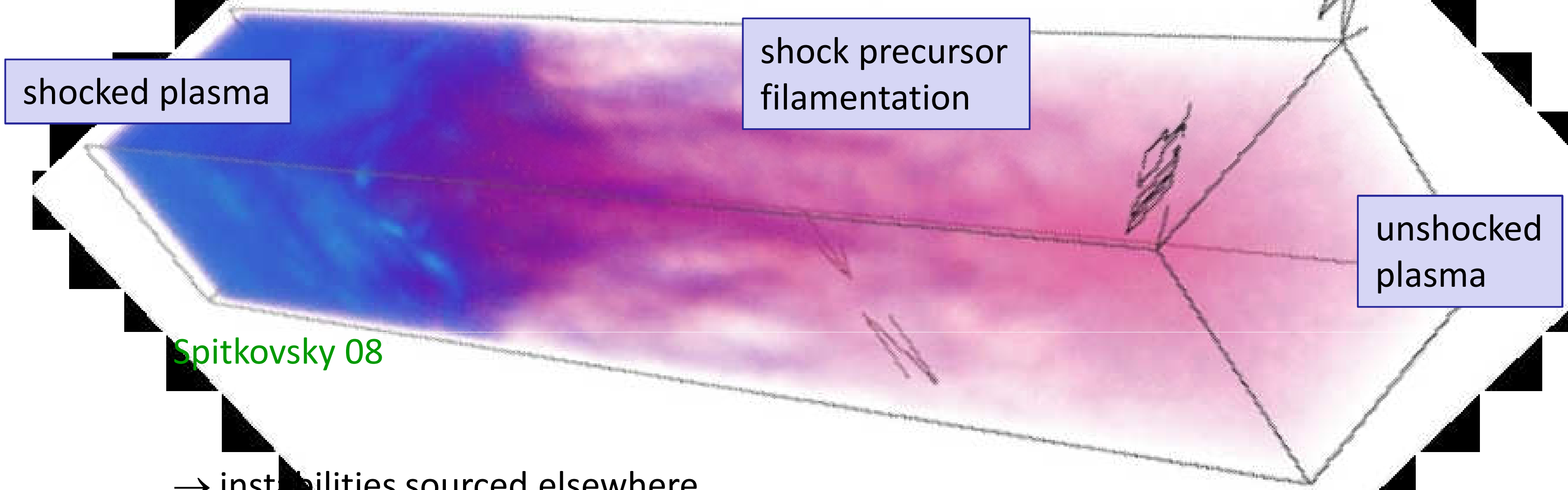
for reference, plasma scale: $c/\omega_p \sim 10^7\,\mathrm{cm}\,n_{u,0}^{1/2}$

$\Rightarrow$ **hierarchy of spatial scales: plasma << Larmor < blast << ISM coherence**

Relativistic Fermi acceleration - sources of turbulence

▶ Sources of small-scale turbulence:

$\rightarrow$ instabilities seeded by the accelerated particles upstream of the shock, e.g. streaming instabilities in the relativistic regime, micro-instabilities...



Spitkovsky 08

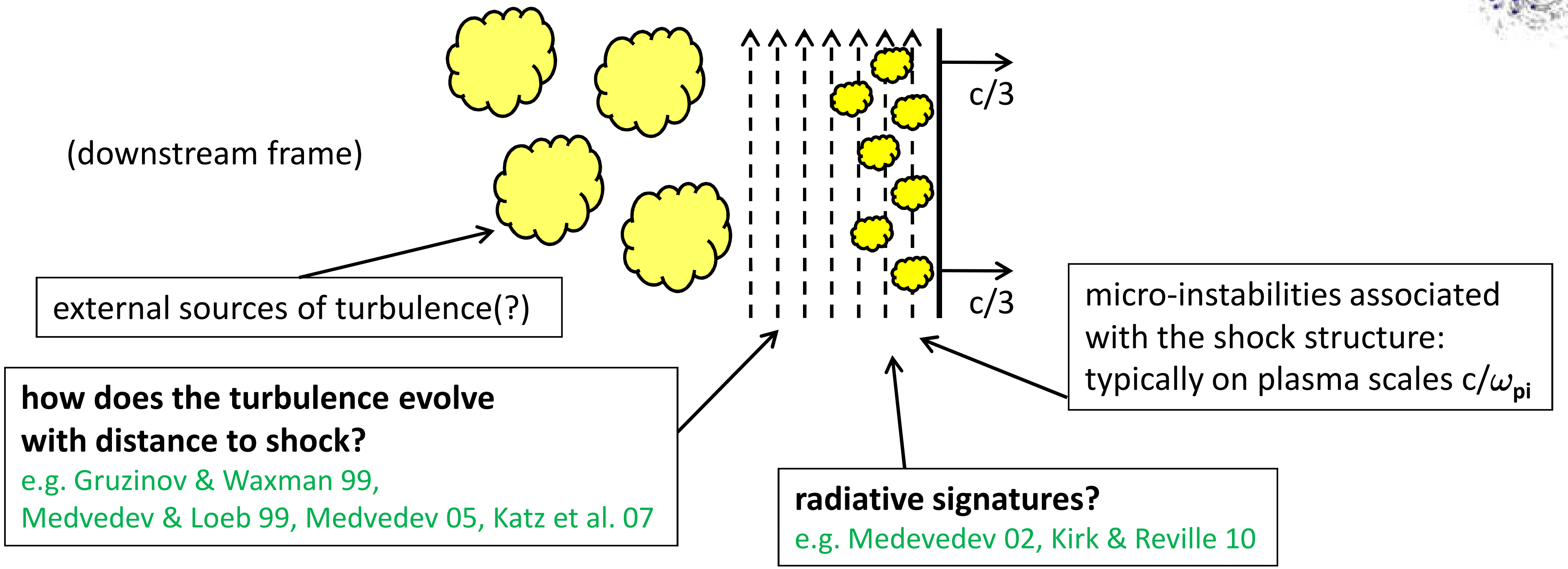
$\rightarrow$ instabilities sourced elsewhere, e.g. instabilities of the blast (Levinson 10), interactions of the shock with external inhomogeneities (Sironi & Goodman 07), ...

note: magnetic fluctuations must be sourced on scale $\leq$ Larmor radius (downstream) to allow Fermi acceleration

▶ Acceleration:

$\rightarrow$ once Fermi acceleration is operational, energy gain per cycle is ~ 2, escape probability is ~ 0.4 and spectral index $\sim 2 - 2.5$ (Achterberg et al. 01, M.L. & Pelletier 03, Keshet & Waxman 05)

Relativistic Fermi acceleration - sources of turbulence



- Micro-instabilities:

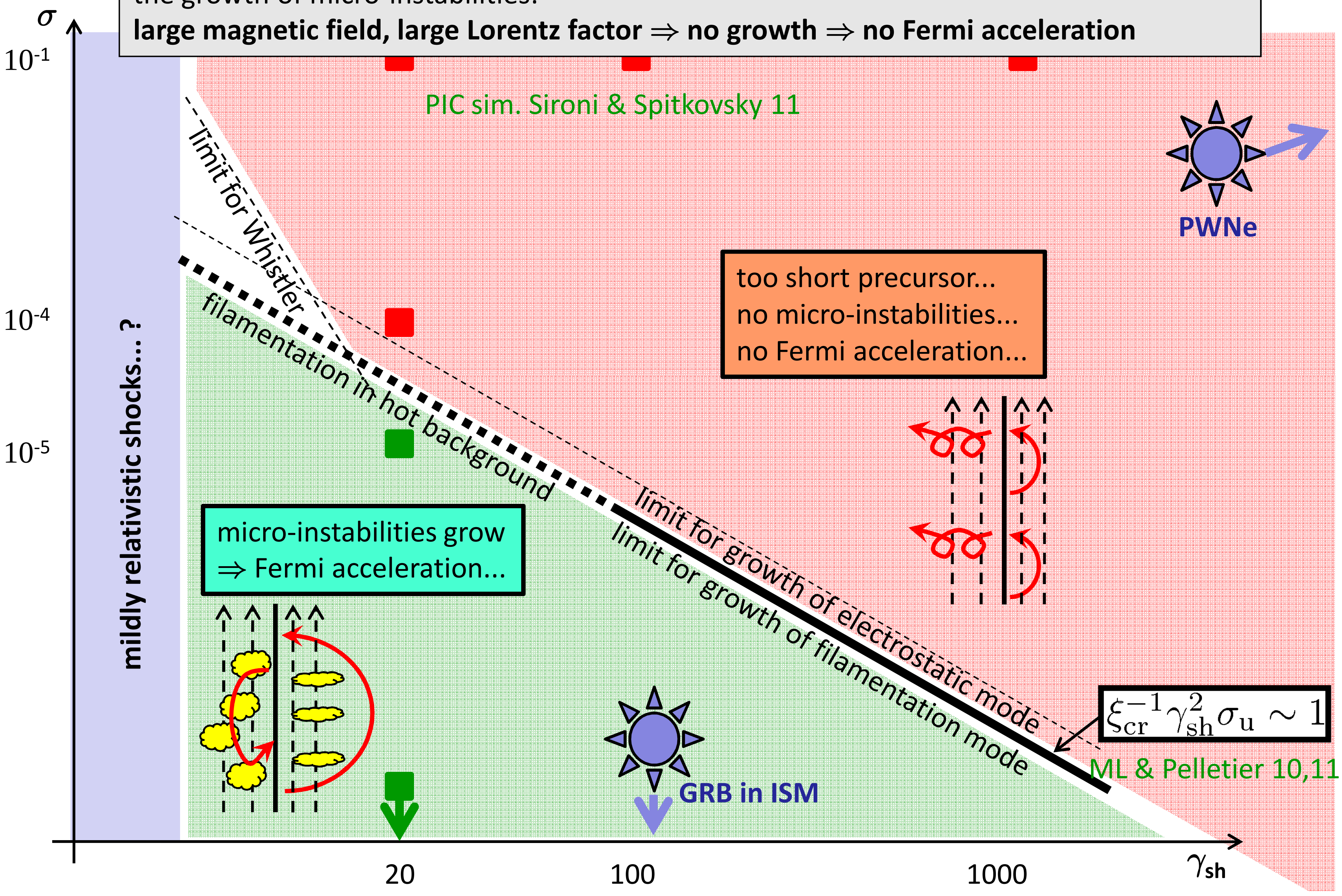
$\rightarrow$ for GRB+ISM, micro-instabilities can grow and allow Fermi acceleration...
control first cycles of Fermi generations $r_{\rm L}\omega_{\rm pi}/c \sim \epsilon_B^{-1/2}\gamma/\gamma_{\rm min}$

$\rightarrow$ if magnetization $\gg$ ISM, micro-instabilities are necessary to trigger Fermi cycles in the absence of fast growing instabilities downstream

$\rightarrow$ micro-instabilities may be a natural product of the dissipative processes in the shock formation, in particular triggered by the returning and accelerated particle populations... however, these microinstabilities can only develop i

Magnetization vs shock Lorentz factor...

the background magnetic field of the unshocked medium and the Lorentz factor control the growth of micro-instabilities:
large magnetic field, large Lorentz factor ⇒ no growth ⇒ no Fermi acceleration



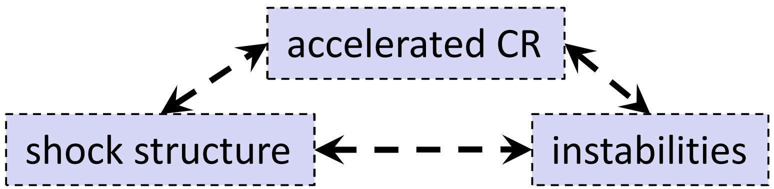
Summary

Transport:

→ much activity in this field at the theoretical level...
... yet, situation still very much idealized (e.g. isotropic + homogeneous turbulence vs ISM)...

Acceleration:

→ shock acceleration (non-relativistic to relativisitic) is well understood in the test particle limit, most studies focus now on the nonlinear scenario...



→ modern developments benefit from massive PIC simulations (+analytical calculations)

→ many open and important questions :

- → **injection mechanism**
- → **spectral index (if any)...**
- → **escape of particles from acceleration site + maximal energy ...**
- → **stationarity of shock fronts, radiative signatures, comparison to data...**