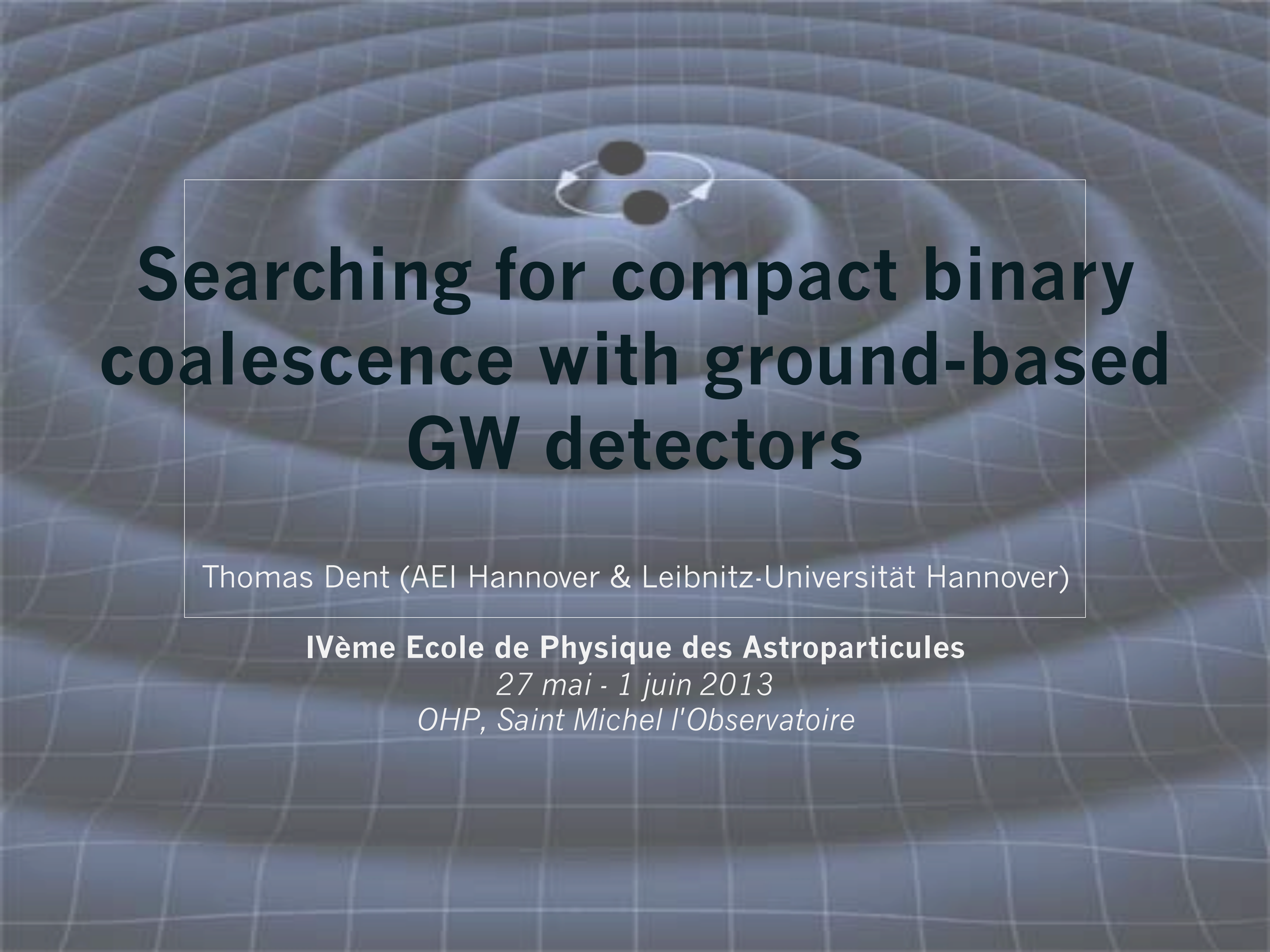

Searching for compact binary coalescence with ground-based GW detectors

Thomas Dent (AEI Hannover & Leibnitz-Universität Hannover)

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OHP, Saint Michel l'Observatoire

Plan of the lecture

① Motivation: Detection of gravitational waves from compact binary coalescence (CBC)

② Matched filter for a known signal in noise

③ Application of matched filter to CBC signals

④ Template banks and multi-detector analysis

⑤ Methods for non-Gaussian noise

⑥ Recent results of LIGO-Virgo searches

⑦ Challenges and expectations for future searches

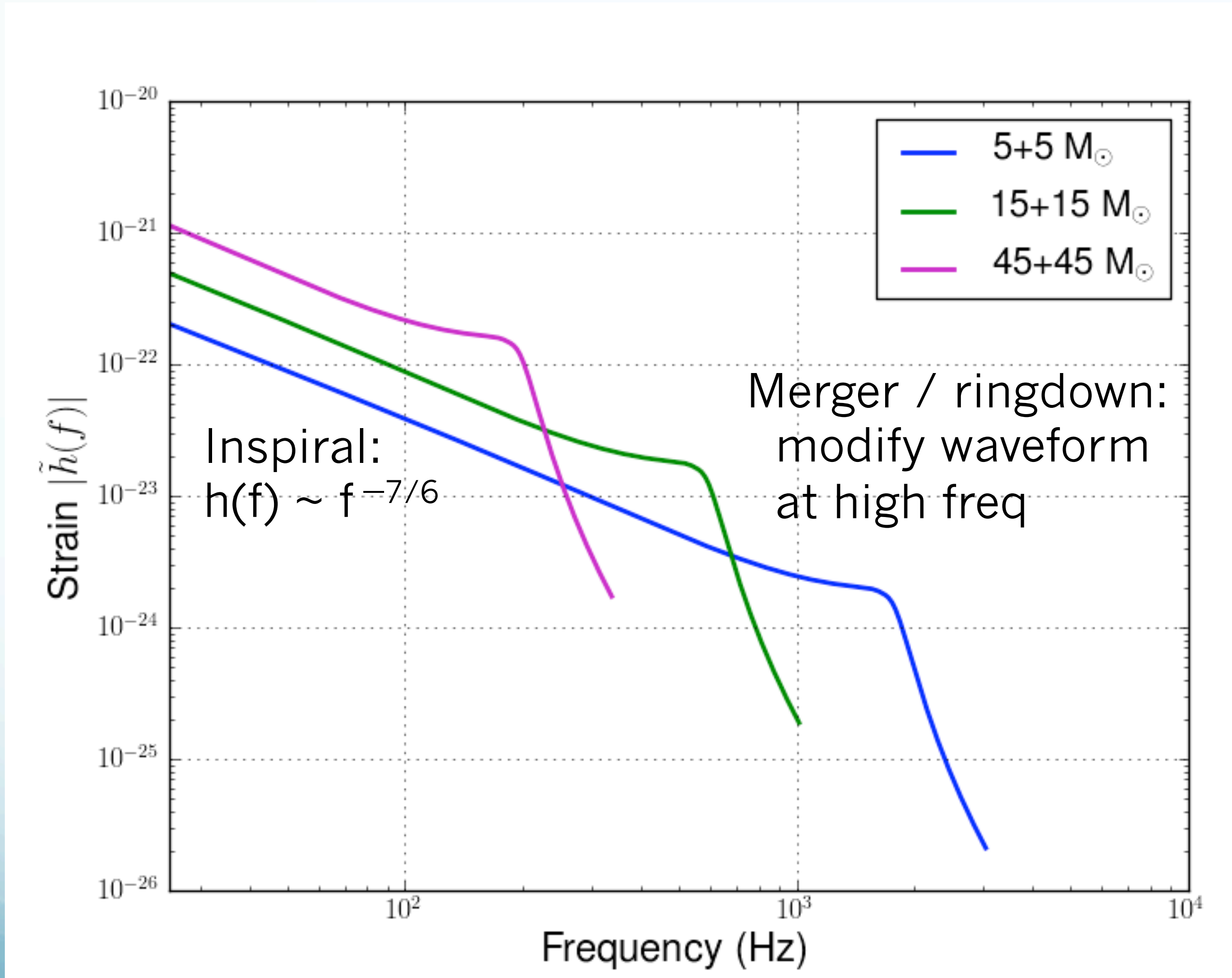
1. Detection of CBC gravitational wave signals

Motivation and statement of the problem

Statement of the problem

- Searching data from ground-based GW detectors
- Each detector provides a data stream
  - s(t) : **time series of measured GW strain**
  - Discrete series sampled at 16384 Hz
  - Data extends over several months
- Do the data show that a CBC signal is present?
- If yes, measure its physical properties
- If no, set limits on astrophysical rate of signals

Signal in frequency domain

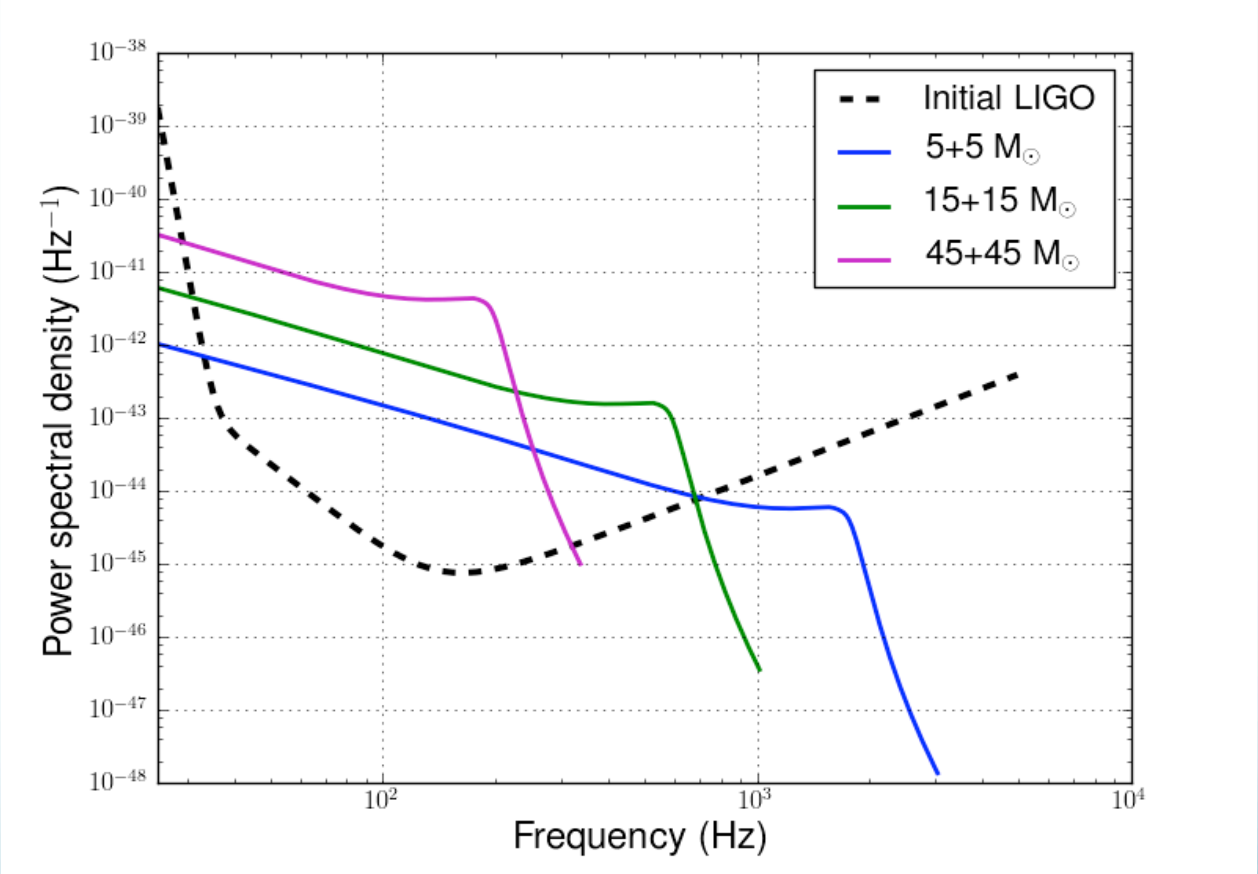


As M increases:

- |h(f)| grows
- Maximum frequency decreases

Signal vs. noise in freq domain

$|h(f)|^2 \times f$ plotted for optimally located signals at 30 Mpc



2. Theory of detection for a known signal

The statistical problem

- CBC signals arrive at the detector all the time!
- But the great majority are ‘too weak to detect’
  - Sources are not within sensitive volume of detector
  - Cannot extract any useful information
- Detector output is signal plus noise:

$$\mathbf{s(t) = h(t) + n(t)}$$

- Detection means:
  The data favour nonzero signal, relative to no signal

⇒ we can tell the difference between

$$s(t) = h(t) + n(t) \qquad \text{vs.} \qquad s(t) = 0 + n(t)$$

Signal and noise hypotheses

- Hypothesis $\mathbf{H_1}$: $s(t) = s_1(t) = h(t) + n(t)$ $h(t) \neq 0$
- Hypothesis $\mathbf{H_0}$: $s(t) = s_0(t) = n(t)$
- **Bayes' rule:**

$$\frac{P(\mathbf{H}_1|d)}{P(\mathbf{H}_0|d)} = \frac{P(d|\mathbf{H}_1)}{P(d|\mathbf{H}_0)} \times \frac{P_i(\mathbf{H}_1)}{P_i(\mathbf{H}_0)}$$

d, "data" → $s(t)$

Posterior Odds Ratio Likelihood Ratio ('Bayes Factor') Prior Odds Ratio

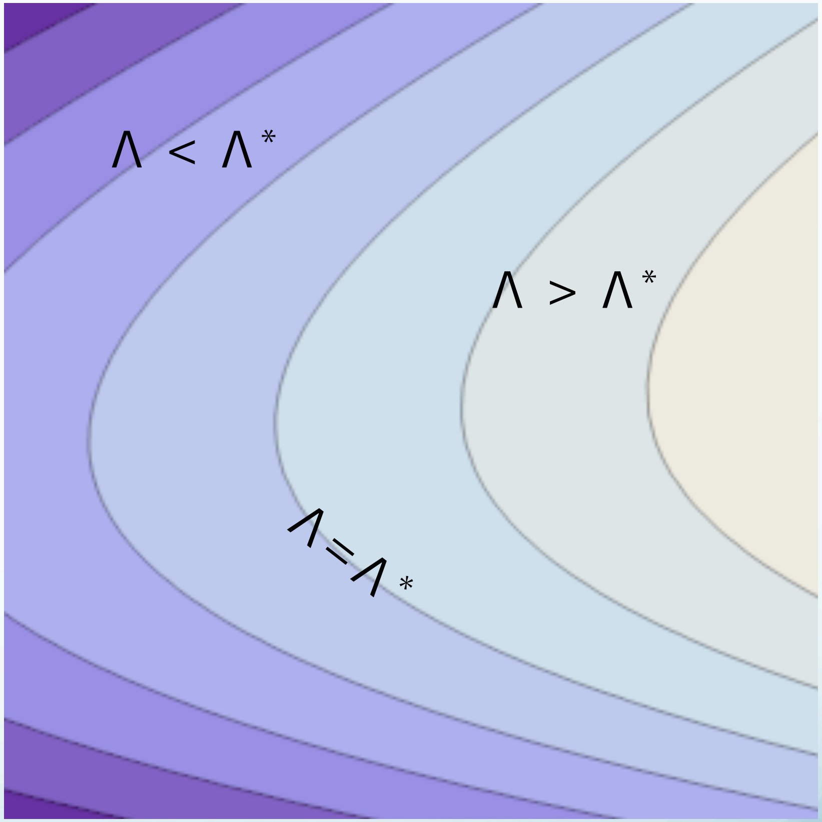
- Prior odds depends on astrophysical coalescence rate: highly uncertain!

Hypothesis testing

- Use a function of the data $\Lambda(d)$ to make decision
  - If $\Lambda(d) > \Lambda^*$ then we declare a detection
  - If $\Lambda(d) < \Lambda^*$ then we declare no detection
- Consider many different realizations of the data stream s(t) with or without signal : $s_{1,i}$, $s_{0,i}$
- Different possible outcomes:
  - $\Lambda > \Lambda^*$ for $s_{1,i}$ "true positive"
  - $\Lambda > \Lambda^*$ for $s_{0,i}$ "**false positive**" [= 'false alarm']
  - $\Lambda < \Lambda^*$ for $s_{1,i}$ "true negative"
  - $\Lambda < \Lambda^*$ for $s_{0,i}$ "**false negative**"

Ranking all possible outcomes

- N-dimensional space of possible data streams d=s(t)
  - N = number of time samples
- Different positions in space have different probabilities to occur under $\mathbf{H_0}$ and $\mathbf{H_1}$
- Constant levels of Λ(d) define contours
- Any monotonic function F(Λ(d)) has the same contours and produces the same decisions



Optimal decision function

- Define **false alarm probability** $Q_0 = P(\Lambda > \Lambda^* | \mathbf{H}_0)$
- Define **detection probability** $Q_d = P(\Lambda > \Lambda^* | \mathbf{H}_1)$
- "Neyman-Pearson optimal statistic" :
  $\Lambda(d)$ is optimal if it **maximizes detection probability Q_d at a fixed value of false alarm probability Q_0**
- Can be proved that **the likelihood ratio**

$$\Lambda(d) = \Lambda_{opt} = \frac{P(d|\mathbf{H}_1)}{P(d|\mathbf{H}_0)}$$

is an optimal statistic for a known signal h(t).

Statistics of (Gaussian) noise

- To calculate likelihood, need to describe statistics of noise P(n(t))
- Gaussian colored noise is easy to describe in the frequency (Fourier) domain n(f)
- **Power spectral density** $S_n(f)$

$$\langle n^*(f) n(f') \rangle = \delta(f - f') \frac{1}{2} S_n(f)$$

- Noise at different frequencies is not correlated
- Noise variance at frequency f: $\langle |n(f)|^2 \rangle = \frac{1}{2} S_n(f) \times T$

for observation time T [frequency resolution $\Delta f = 1/T$]

Likelihood for noise vs. signal

- Noise likelihood
  - under $\mathbf{H_0}$, n(f) = s(f)

$$P(s(f)|\mathbf{H}_0) = \mathcal{N} \exp\left\{ -\frac{1}{2}\int_{-\infty}^{\infty} df\, \frac{|s(f)|^2}{\frac{1}{2}S_n(f)} \right\}$$

- Signal likelihood
  - under $\mathbf{H_1}$, n(f) = s(f) – h(f)

$$P(s(f)|\mathbf{H}_1) = \mathcal{N} \exp\left\{ -\frac{1}{2}\int_{-\infty}^{\infty} df\, \frac{|s(f) - h(f)|^2}{\frac{1}{2}S_n(f)} \right\}$$

Scalar products and likelihood ratio

- Define **scalar product** of data streams a(t), b(t)

$$\langle a|b\rangle = \mathrm{Re}\int_{-\infty}^{\infty} df\, \frac{a^*(f)b(f)}{\frac{1}{2}S_n(f)}$$

- Usual properties: $\langle a|b\rangle = \langle b|a\rangle \qquad \langle a|a\rangle \geq 0$ etc.
- Rewrite likelihoods :

$$P(d|\mathbf{H}_0) = \mathcal{N} e^{-\frac{1}{2}\langle s|s\rangle}$$

$$P(d|\mathbf{H}_1) = \mathcal{N} e^{-\frac{1}{2}\langle s-h|s-h\rangle} = \mathcal{N} e^{-\frac{1}{2}\langle s|s\rangle + \langle s|h\rangle - \frac{1}{2}\langle h|h\rangle}$$

- Likelihood ratio $\Lambda_{\mathrm{opt}} = \dfrac{P(d|\mathbf{H}_1)}{P(d|\mathbf{H}_0)} = e^{\langle s|h\rangle - \frac{1}{2}\langle h|h\rangle}$

Optimal matched filter

- $\langle h|h \rangle$ is constant for a fixed signal, e^x is monotonic
- Therefore we can also use $\langle s|h \rangle$ as our statistic
  - Known as 'matched filter' or 'Wiener filter'
  - Linear in the detector output s

$$\langle s|h\rangle = \mathrm{Re}\int_{-\infty}^{\infty} df\, K^*(f)s(f), \quad K(f) = \frac{h(f)}{\frac{1}{2}S_n(f)}$$

- Expected value of $\langle s|h \rangle$ under $\mathbf{H_0}$ is = 0
- Expected value of $\langle s|h \rangle$ under $\mathbf{H_1}$ is = $\langle h|h \rangle$
- Variance of $\langle s|h \rangle$ is $\sigma^2 = \langle h|h \rangle$

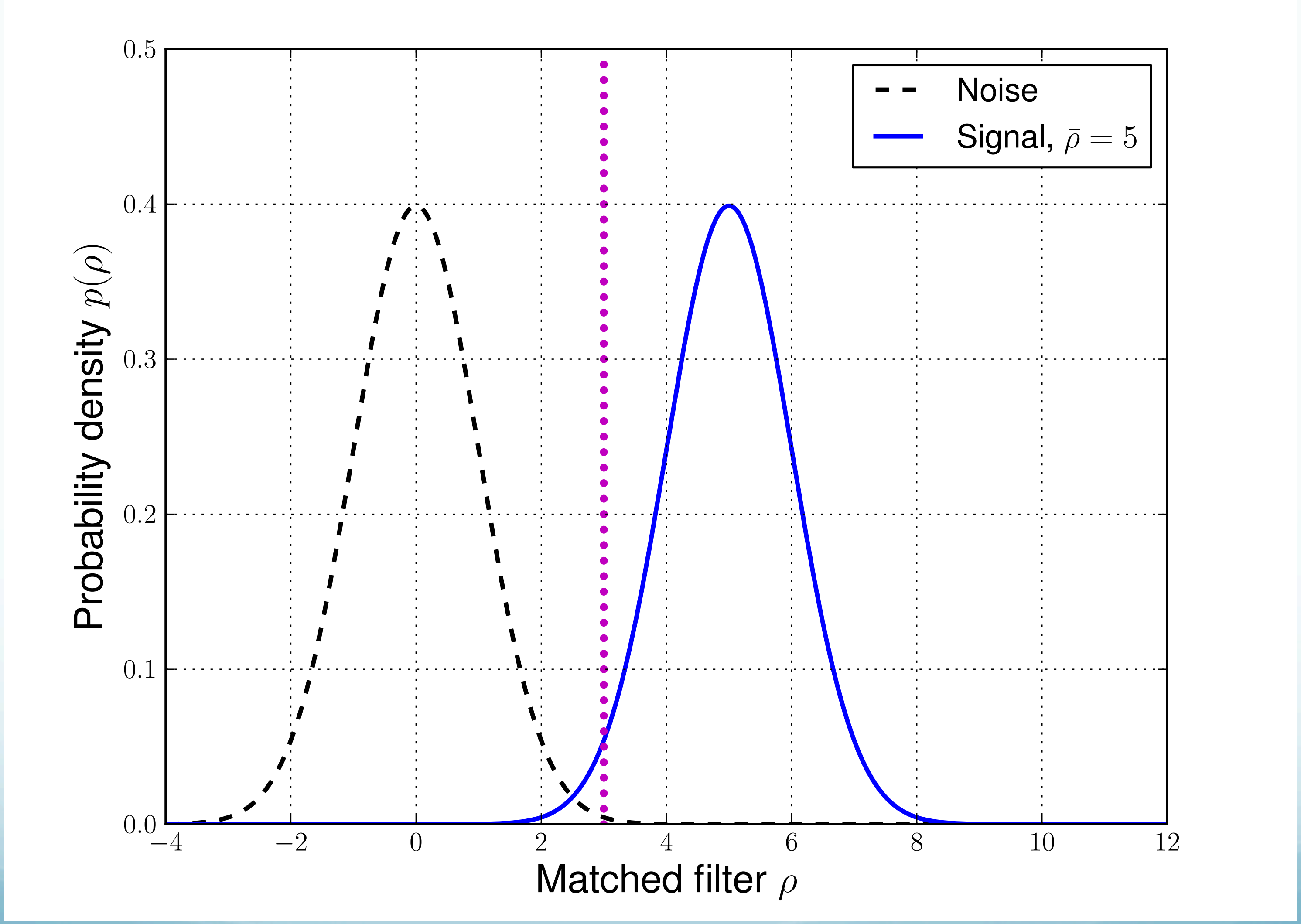
Signal-to-noise ratio (SNR)

- Rescale the matched filter : $$\rho = \frac{\langle s|h\rangle}{\sqrt{\langle h|h\rangle}}$$
- Variance $\sigma^2(\rho) = 1$
- Mean $\bar{\rho} = 0$ (noise)

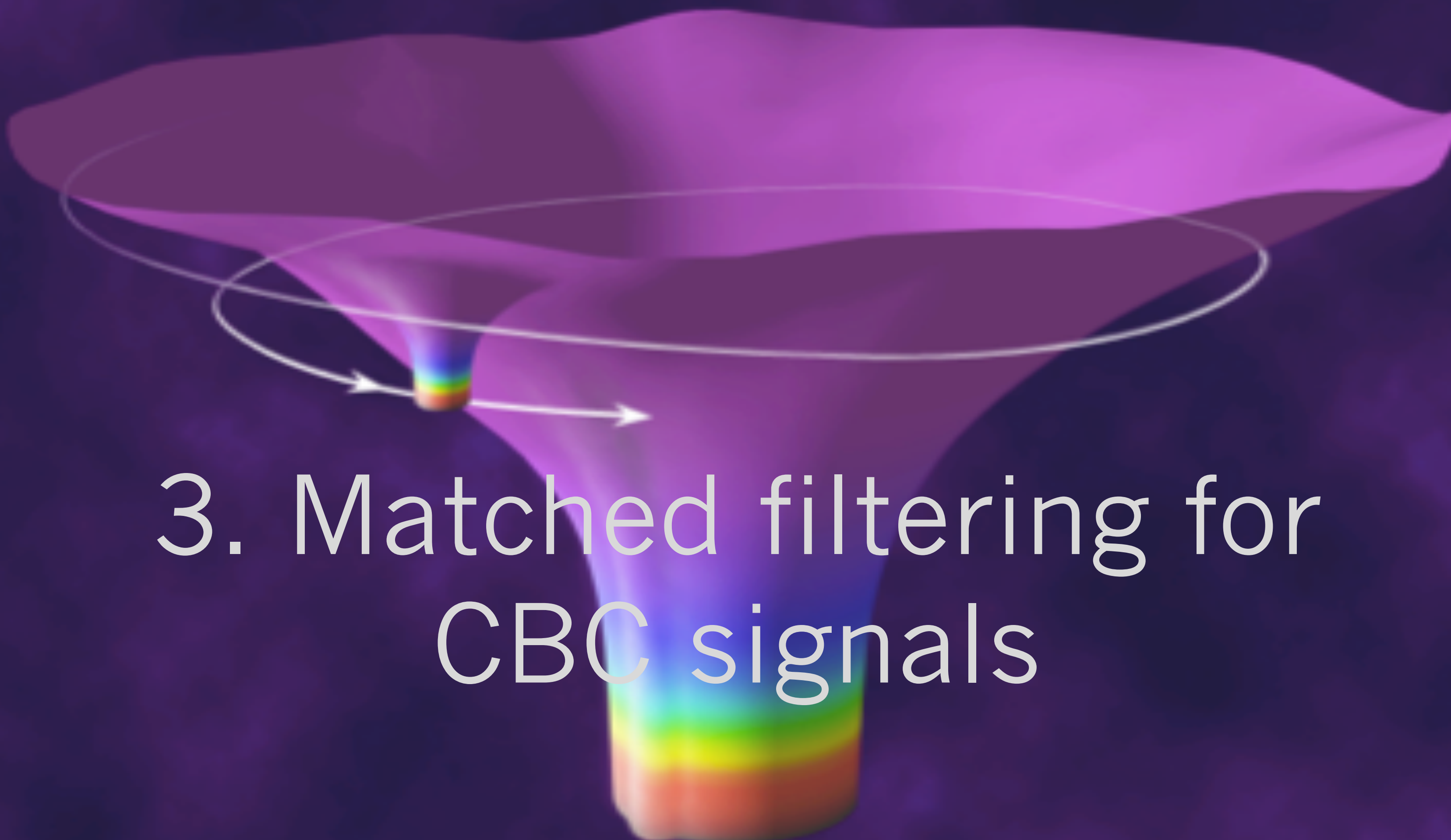
  $= \sqrt{\langle h|h\rangle}$ (signal)
- $\bar{\rho}$ is called "optimal SNR" of the signal h(t)
- Distribution of ρ :

$$p(\rho|\bar{\rho})\, d\rho = \frac{1}{\sqrt{2\pi}} e^{-(\rho-\bar{\rho})^2/2}\, d\rho$$

Filter output distribution



3. Matched filtering for CBC signals



Describing (simple) inspiral signals

- Time domain

$$h(t) = \frac{G\mathcal{M}}{c^2 D_{\mathrm{eff}}} \left(\frac{t_0 - t}{5G\mathcal{M}/c^3} \right)^{-1/4} \cos(2\phi(t) - 2\phi_0)$$

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{M^{1/5}}, \qquad M = m_1 + m_2$$

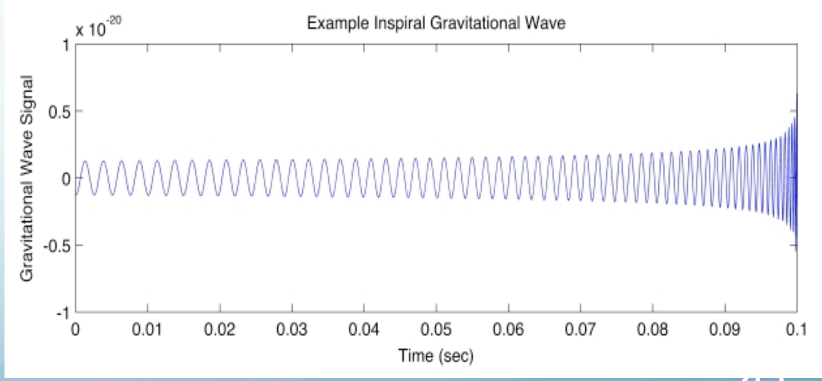
- Frequency domain

$$h(f) = \frac{1\,\mathrm{Mpc}}{D_{\mathrm{eff}}} \mathcal{A}_{1\mathrm{Mpc}}\, f^{-7/6} \exp(i\Psi(f; \mathcal{M}, M))$$

$$f \leq f_{\mathrm{max}}$$

- Ψ(f) is a series in f ("**P**ost-**N**ewtonian expansion") valid up to f_{max}

$$\Psi(f) = 2\pi f\, t_0 - 2\phi_0 + \cdots$$



Frequency limits

- Theoretical expression

$$\langle a|b\rangle = \mathrm{Re}\int_{-\infty}^{\infty} df\, \frac{a^*(f)b(f)}{\frac{1}{2}S_n(f)}$$

- In practice need to limit frequency range
  - $f > f_{min}$: detector data is not reliable and/or calibrated below a minimum frequency
  - also length of waveform $\propto f_{min}^{-8/3}$
  - $f < f_{max}$: inspiral waveform not reliable at large v/c
- Thus, use

$$\langle a|b\rangle = 2\,\mathrm{Re}\int_{f_{\min}}^{f_{\max}} df\, \frac{a^*(f)b(f)}{\frac{1}{2}S_n(f)}$$

Signal parameters seen in h(t)

- Signal h(t) is not unique (not a 'simple hypothesis')
- Described by parameters " θ "
  - Amplitude $\propto A_{1Mpc}/D_{eff}$
    Effective distance D_{eff} encodes physical distance D and binary geometry relative to the detector
  - Coalescence phase ϕ_0
  - Coalescence time t_0
  - Masses m_1, m_2
- Theoretically correct treatment:

  Evaluate likelihood $p(d|\mathbf{H_1}(\theta))$ for all θ, marginalize (integrate) over θ

Dealing with CBC parameters

1) Amplitude: Easy, the matched filter doesn't care about amplitude of h

$$\rho = \frac{\langle s|h\rangle}{\sqrt{\langle h|h\rangle}}$$

- The value of ρ is a *measurement* of expected SNR $\bar{\rho}$
- Proportional to $A_{1\mathrm{Mpc}}/D_{\mathrm{eff}}$ for a signal

2) Coalescence phase: Easy, use 'cos' and 'sin' filters

$$\langle s|f^{-7/6}e^{i\Psi(f)}\rangle = \cos 2\phi_0 \langle s|f^{-7/6}e^{i\Psi'(f)}\rangle + \sin 2\phi_0 \langle s|f^{-7/6}(-i)e^{i\Psi'(f)}\rangle$$

$$= \cos 2\phi_0 \cdot x + \sin 2\phi_0 \cdot y$$

$$\Psi(f) = -2\phi_0 + \Psi'(f)$$

- Can show that $|z| = |x+iy| = \sqrt{x^2+y^2}$

is an optimal statistic if the phase ϕ_0 is not known.

Dealing with CBC parameters II

- z is a *complex matched filter* :

$$z = \frac{2A}{D_{\text{eff}}} \int_{f_{\text{min}}}^{f_{\text{max}}} df \, \frac{s(f) f^{-7/6} e^{-i\Psi'(f)}}{\frac{1}{2} S_n(f)}$$

3) Coalescence time: Easy.

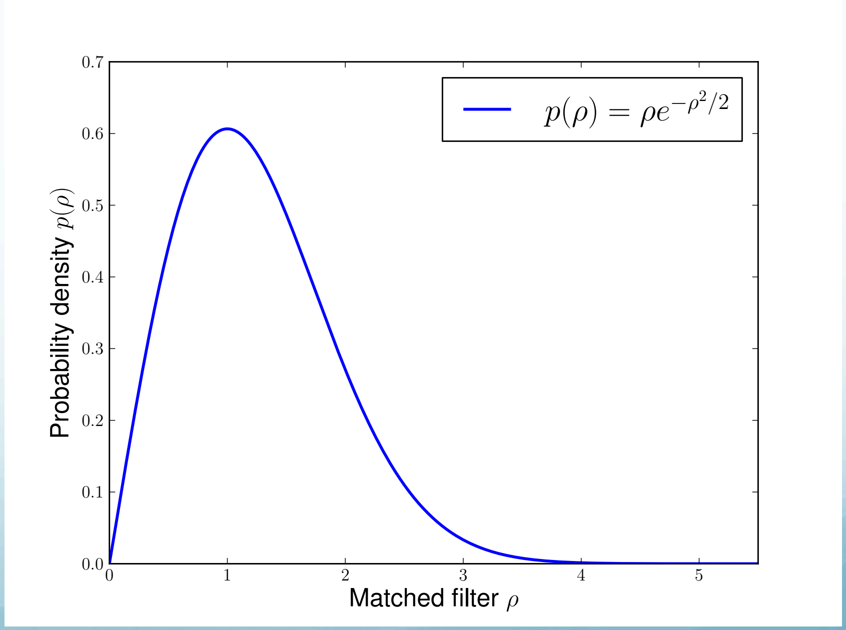
- Rewrite $\Psi'(t_0) = \Psi'(t_0 = 0) \cdot e^{2\pi i f t_0}$
- We get a matched filter *time series* :

$$z(t_0) = \frac{2A}{D_{\text{eff}}} \int_{f_{\text{min}}}^{f_{\text{max}}} df \, \frac{s(f) f^{-7/6} e^{-i\Psi'(f; t_0=0)}}{\frac{1}{2} S_n(f)} e^{-2\pi i f t_0}$$

- It's just a Fourier transform !

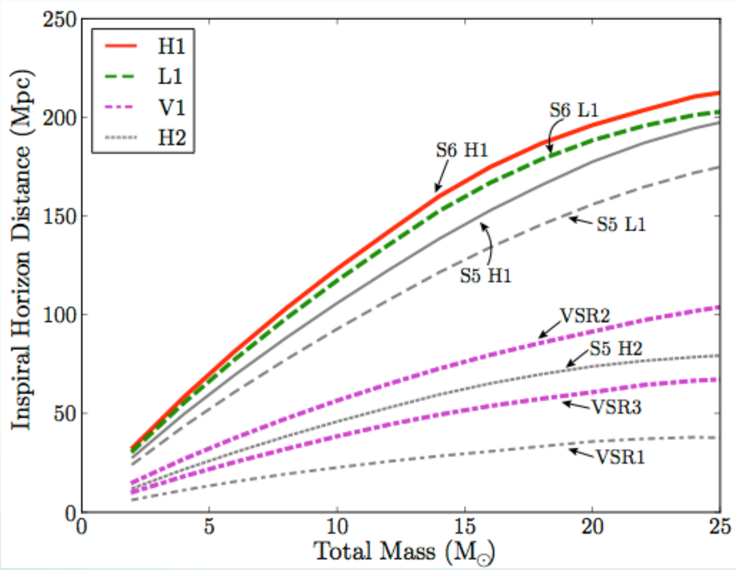
Statistics of the CBC filter

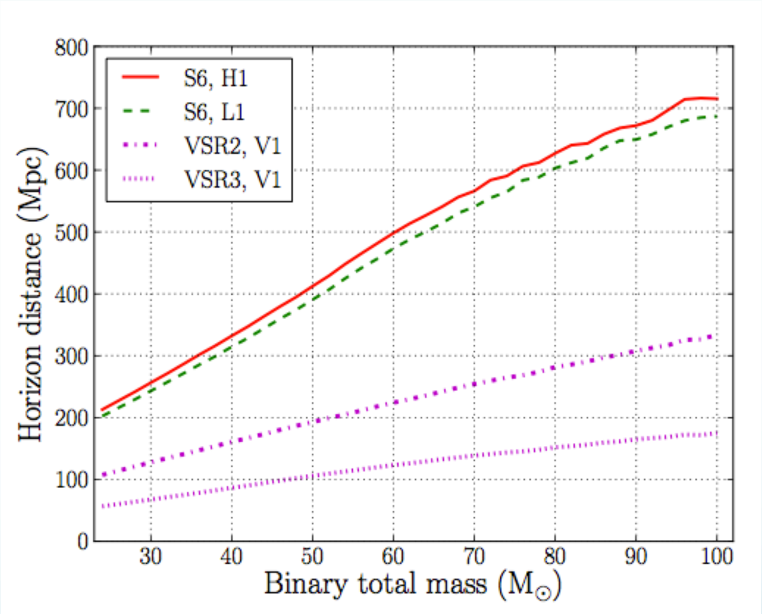
- x and y are independent filters with variance $\sigma^2 = \langle h|h\rangle$
- Define "SNR" as $\rho = |z|/\sigma = |x+i\cdot y|/\sigma$
- ρ has a 'Rayleigh' distribution in noise



Horizon distance

- Farthest distance D where a coalescing binary would produce a given expected SNR $\bar{\rho}$ (eg =8)
- We have $D \leq D_{eff}$ and $\bar{\rho} \propto A/D_{eff}$





- Depends on binary masses & detector noise spectrum $S_n(f)$

J. Abadie *et al.*, arXiv:1111.7314

J. Aasi *et al.*, arXiv:1209.6533

Time and masses in CBC search

- Cannot evaluate $\rho(t)$ for all time (computation)
- Instead filter short segments (order 1 min) and record time at **local maximum** values of ρ
  - Called a 'trigger' (t_0, ρ)
- Likelihood ratio Λ_{opt} varies exponentially with ρ
  - "Maximum likelihood" is a good approximation to to the optimum statistic
- But each set of masses (m_1, m_2) requires a different filter ...

4. Template banks and multi-detector analysis

How many filters do we need?

- Different masses $\theta_i = \{m_1, m_2\}$ require different filters
- If there is a signal with parameters θ and we use filter parameters $\theta' \neq \theta$ we do not have an optimal search
  - Given a fixed SNR ρ^* for detection, the probability that the signal exceeds ρ^* will be smaller for a mismatched template
  - How much 'lack of match' is acceptable?
- Define 'match' $M = \bar{\rho} / \rho_{opt}$ $(M \leq 1)$

= (SNR for template θ')/(SNR for optimal template θ)

Loss in search sensitive volume

- Assume binary mergers are uniform in space
- Volume of space where signals can be detected with $\bar{\rho} > \rho^*$ is $\propto D_{max}(\rho^*)^3$
- Optimal template: $$\bar{\rho}_{\mathrm{opt}} \propto \frac{\mathrm{A}}{D_{\mathrm{max}}}$$
- Non-optimal template: $$\bar{\rho} \propto M \frac{\mathrm{A}}{D_{\mathrm{max}}}, \quad M \leq 1$$
- M = 'match' of signal with non-optimal template
- Thus $D_{max}(\rho^*) \propto M/\rho^*$, sensitive volume $\propto (M/\rho^*)^3$

(Mis)match of templates

- Normalized templates $h(\theta, t_0, \phi_0)$: $\langle h|h \rangle = 1$
- Match M for small mass differences :

$$M(\theta, \Delta\theta) = \max_{t_0,\phi_0} \langle h(\theta) | h(\theta + \delta\theta) \rangle$$

  - max over t_0, ϕ_0 ensures differences due to $m_{1,2}$ only
- Expand near local maximum at $\Delta\theta = 0$:

$$M(\theta, \Delta\theta) = 1 + \frac{1}{2} \frac{\partial^2 M}{\partial\theta_i \partial\theta_j} \Delta\theta_i \Delta\theta_j + \mathcal{O}(\Delta\theta_i^3)$$

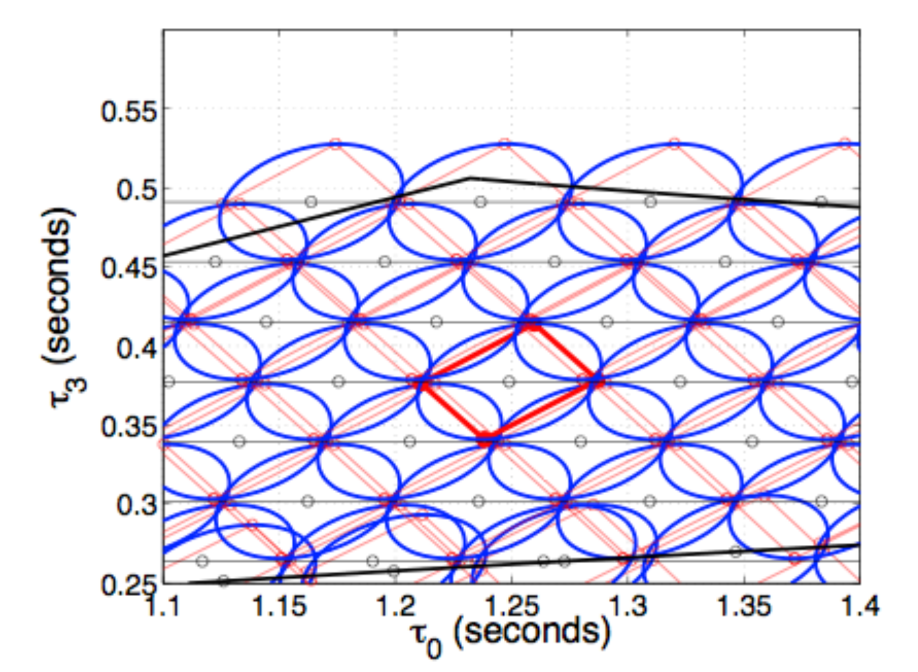
Mismatch metric

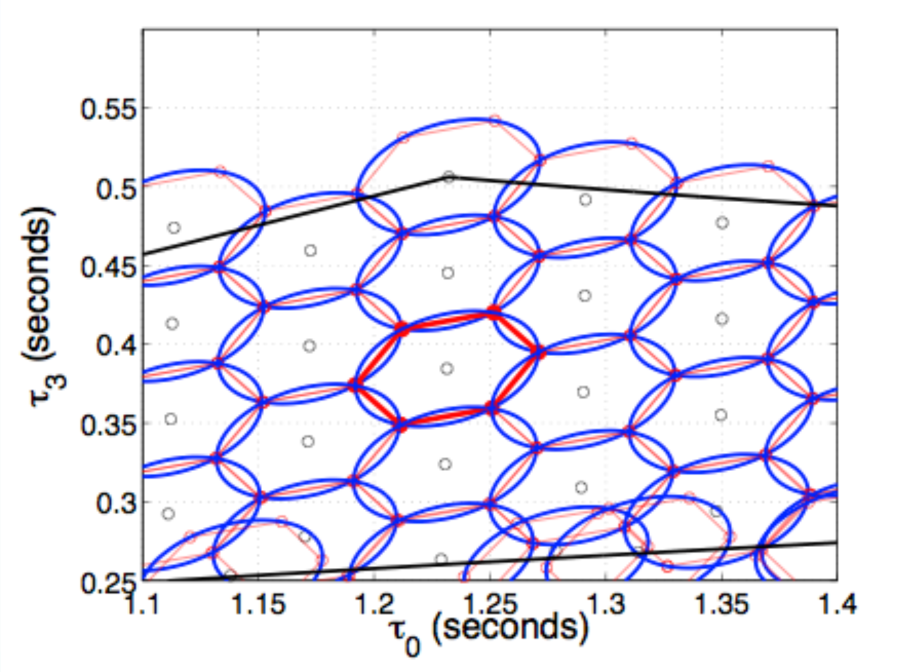
- Local deviation from M=1 defines a **metric** over θ_i

$$1 - M = \text{“}ds^2\text{”} = g_{ij}\Delta\theta_i\Delta\theta_j, \quad g_{ij}(\theta) = -\frac{1}{2}\frac{\partial^2 M}{\partial\theta_i\partial\theta_j}$$

- Calculating $M(\theta, \Delta\theta)$ explicitly → find g_{ij}
- Can find coordinates where g_{ij} is (nearly) constant
- Use a **regular lattice** of templates
  - Ensures that no point in space is further than some maximum distance from a template
  - ds^2_{max} : “maximal mismatch”

Template bank placement



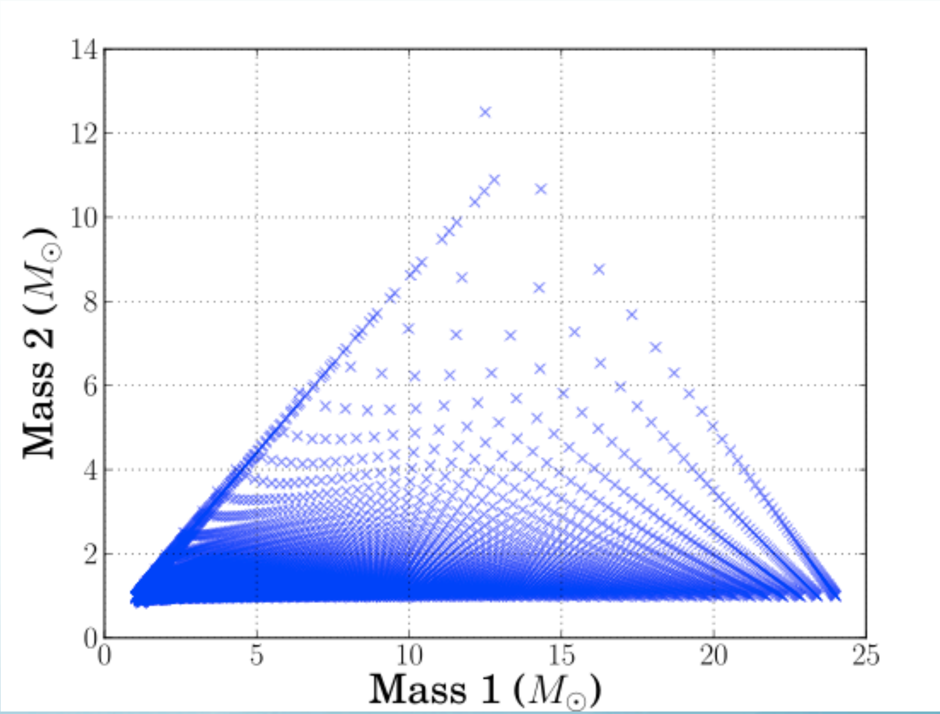


- Hexagonal bank is more efficient at covering space
- "Chirp time" coordinates τ_0 τ_3 : functions of $m_{1,2}$

T. Cokelaer, arXiv:0706.4437

A template bank

- Minimal match 0.97 (maximal mismatch 0.03)
  - ~10% maximum possible loss of sensitive volume
- Component masses $1 < m_{1,2}/M_\odot < 24$
- Max mtotal = 25 $M_\odot$
- Order 10,000 templates
- Computationally feasible to search ✓

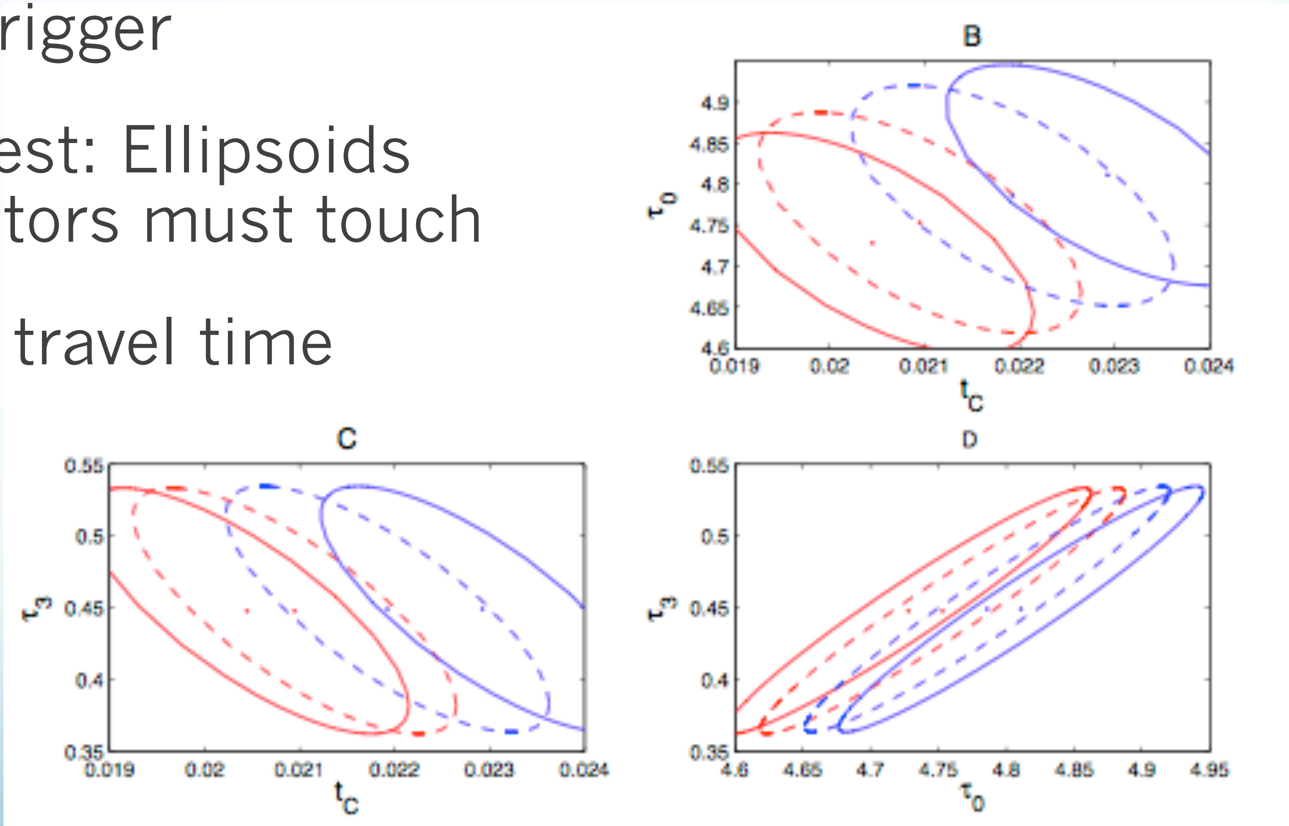


Multi-detector search

- Filter and record triggers (t_0,m_1,m_2, ρ) in each detector
  - Local maxima of likelihood using data in 1 detector
- Would like to combine information between several detectors
  - increase number of signals seen for a fixed number of false alarms
- Test triggers for **time and mass consistency** between detectors
- use a **combined detection statistic**

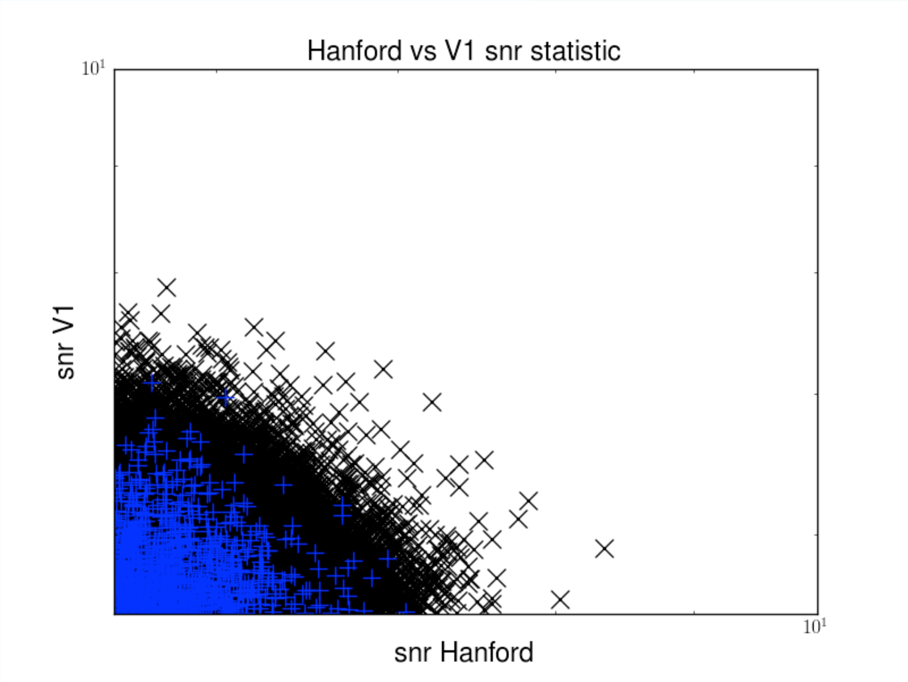
Coincidence testing

- Extend metric to $\theta_i = \{m_1, m_2, t_0\}$
- Construct ellipsoids of fixed size around each trigger
- Coincidence test: Ellipsoids from ≥2 detectors must touch
- Allow for light travel time



Combined detection statistic

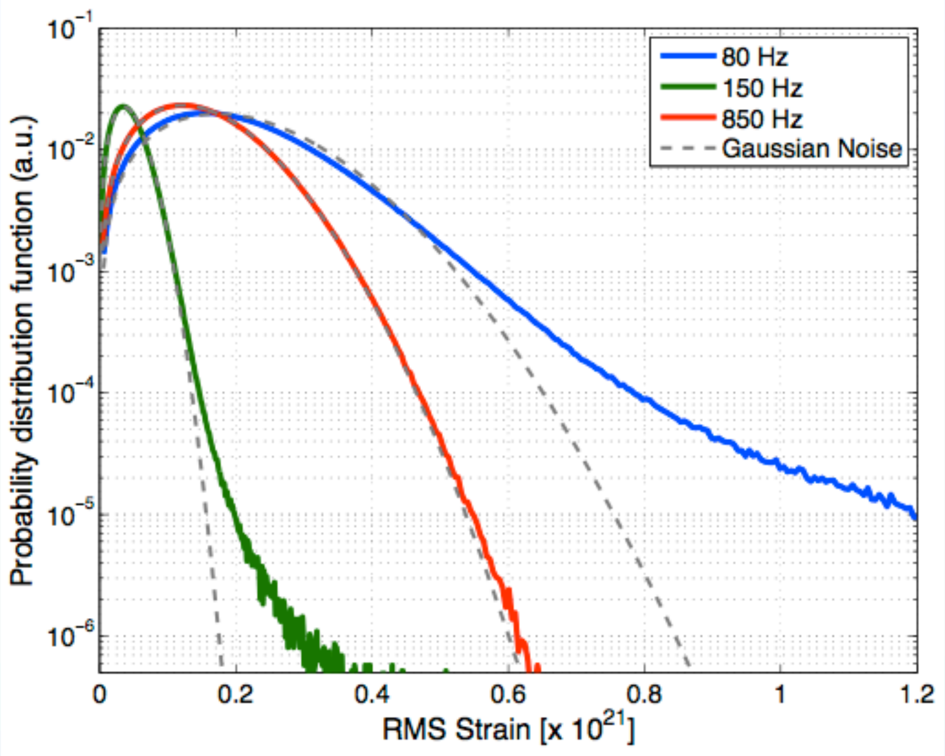
- Quadrature sum of ρ over detectors A, B, ...
- $\rho_c^2 = \rho_A^2 + \rho_B^2 + \cdots$
- Expect a Gaussian cumulative distribution



$$P(\rho_c \geq \rho_{\mathrm{th}}) = \text{const.} \times e^{-\rho_{\mathrm{th}}^2/2}$$

5. Non-Gaussian noise

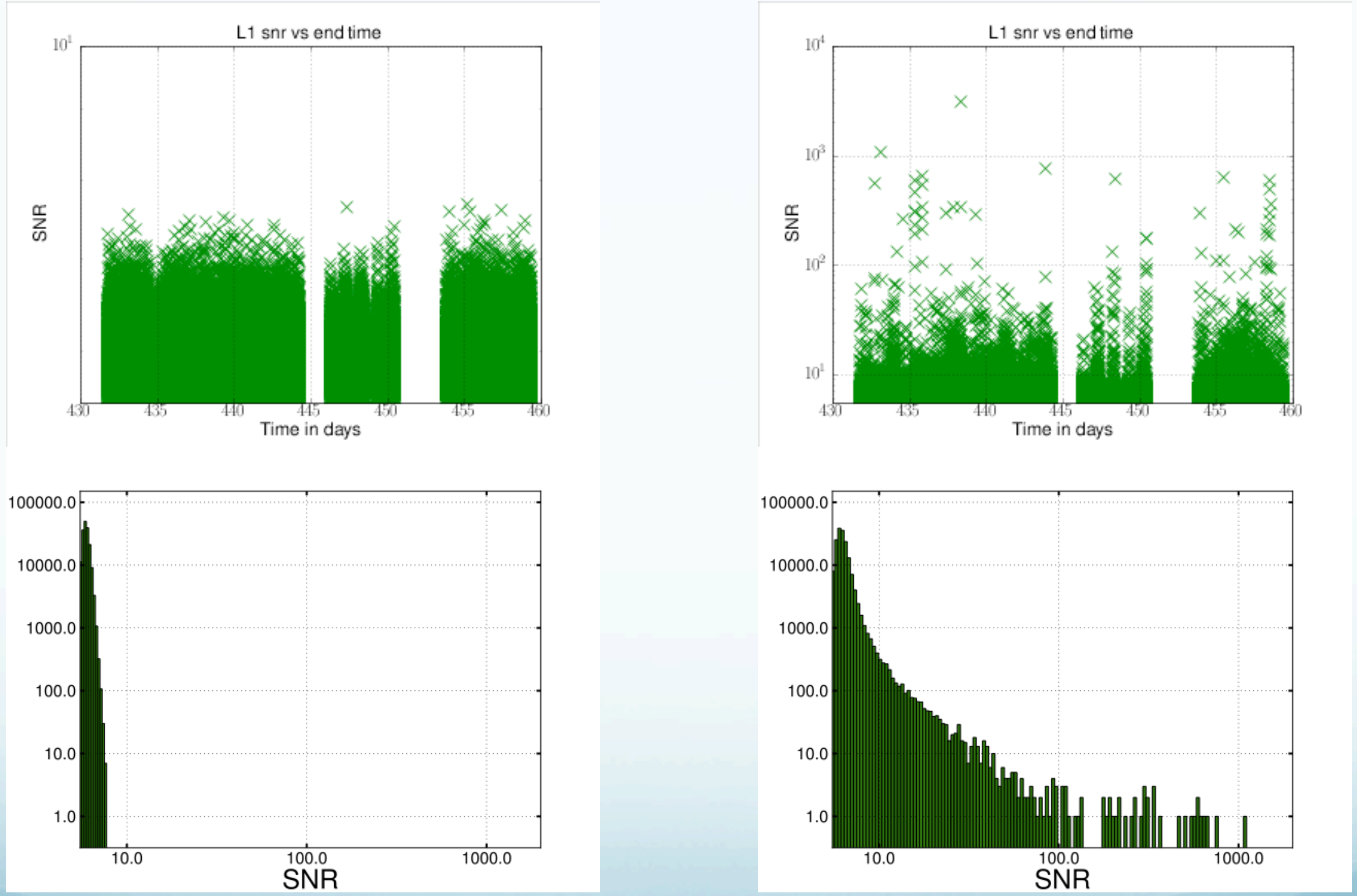
Real detector noise is not Gaussian



B. Abbott *et al.*, *Rep. Prog. Phys.* **72** 076901 (2009)

- Noise distribution is strongly non-ideal at mid/low frequencies

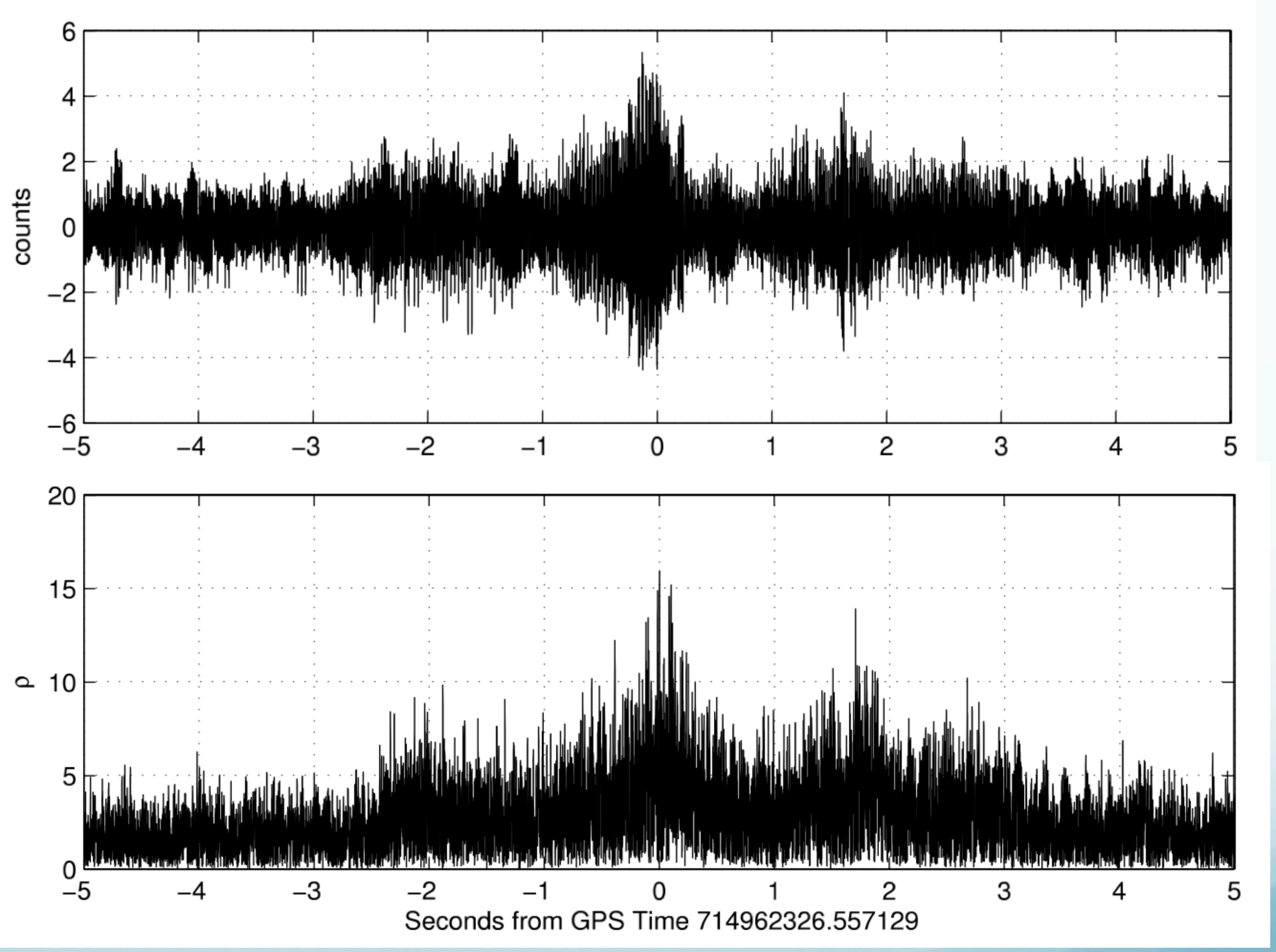
Matched filter in non-Gaussian noise



Non-Gaussian noise transients

Detector output

Matched filter ρ

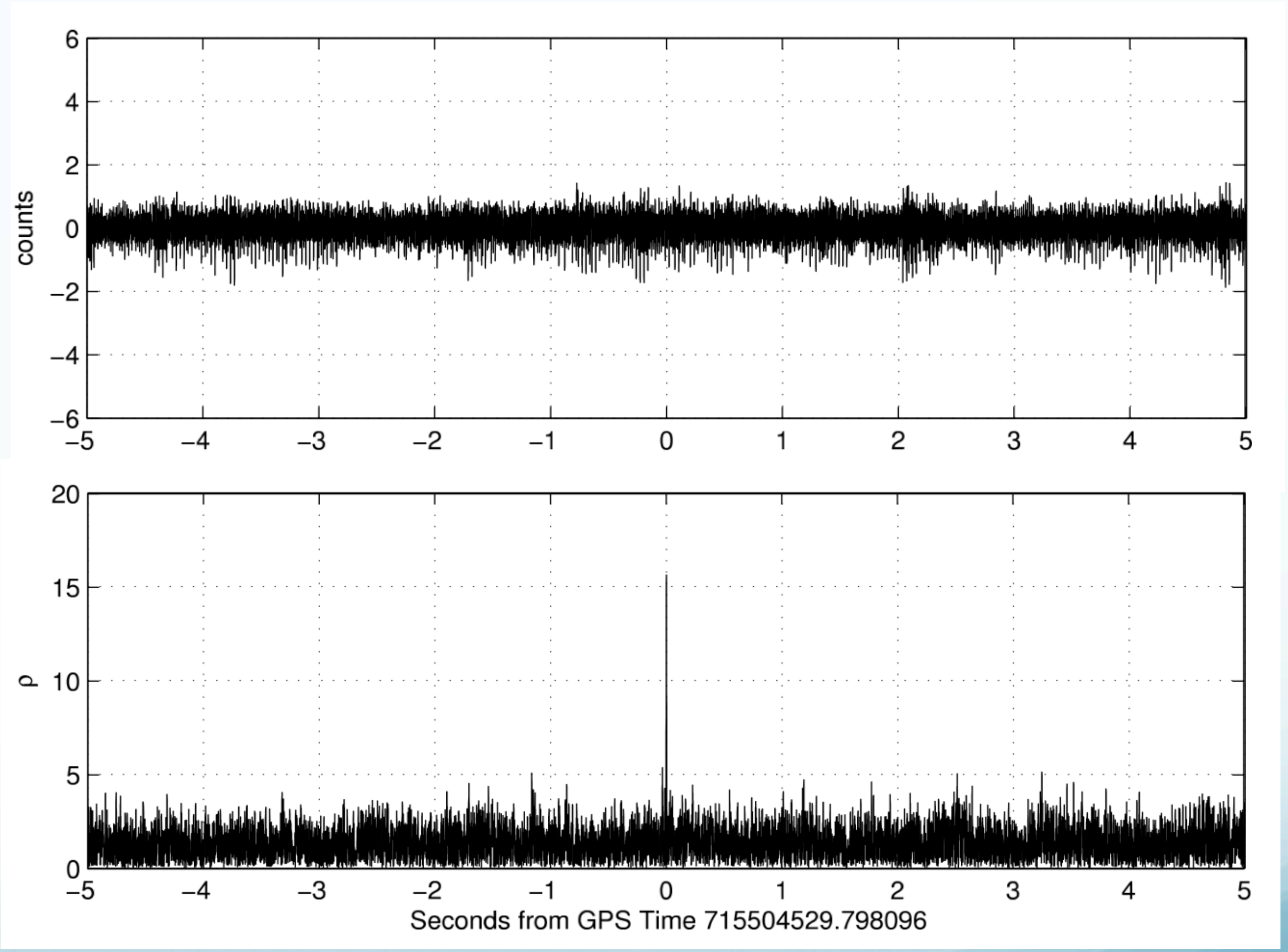


Abbott et al., arXiv:gr-qc/0308069

Simulated signal in real noise

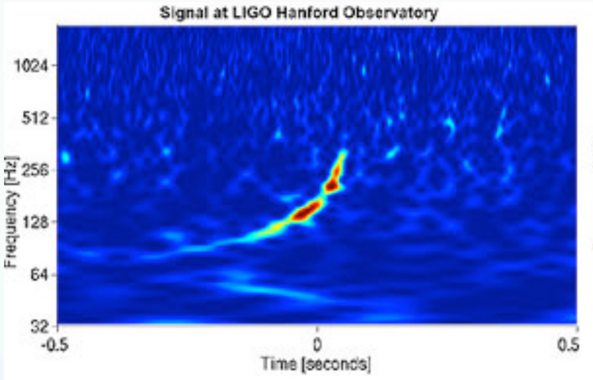
Detector output

Matched filter



Signal consistency vetos

- For large ρ values, check:
  - Does $\rho(t)$ behave as expected for a signal?
  - Does $\rho(m_1,m_2)$ behave as expected?
  - Is signal power distributed as expected over time / frequency?



- 'Chi-squared' (χ^2) tests
- Most widely used:
  - Divide up frequency range (f_{min},f_{max}) into p "sub-filters"
  - Find matched filter output ρ_i in each ...

Classic chisq test

$$\chi^2 \equiv p \sum_{i=1}^{p} \left(\rho_i - \frac{\rho}{p} \right)^2$$

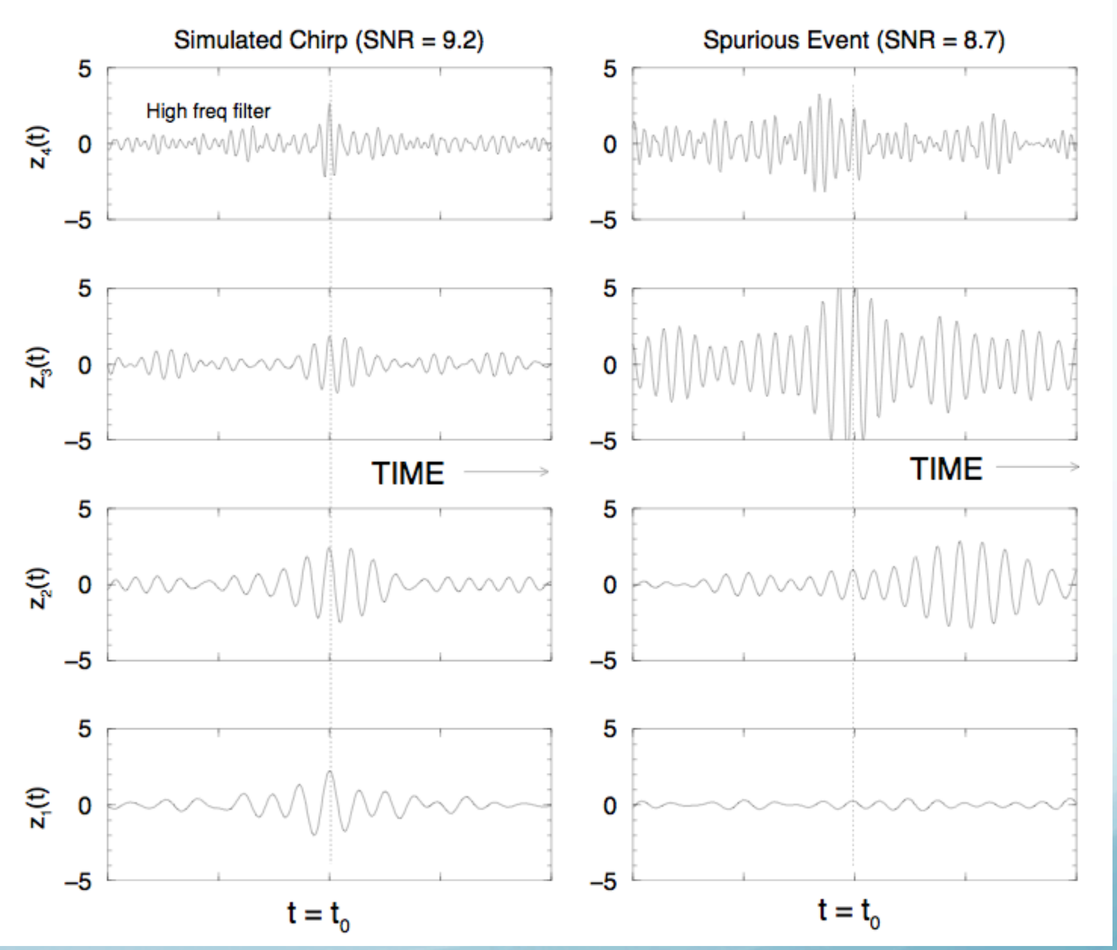
$\chi^2 = 1.30$ $\chi^2 = 68.4$

i=4

i=3

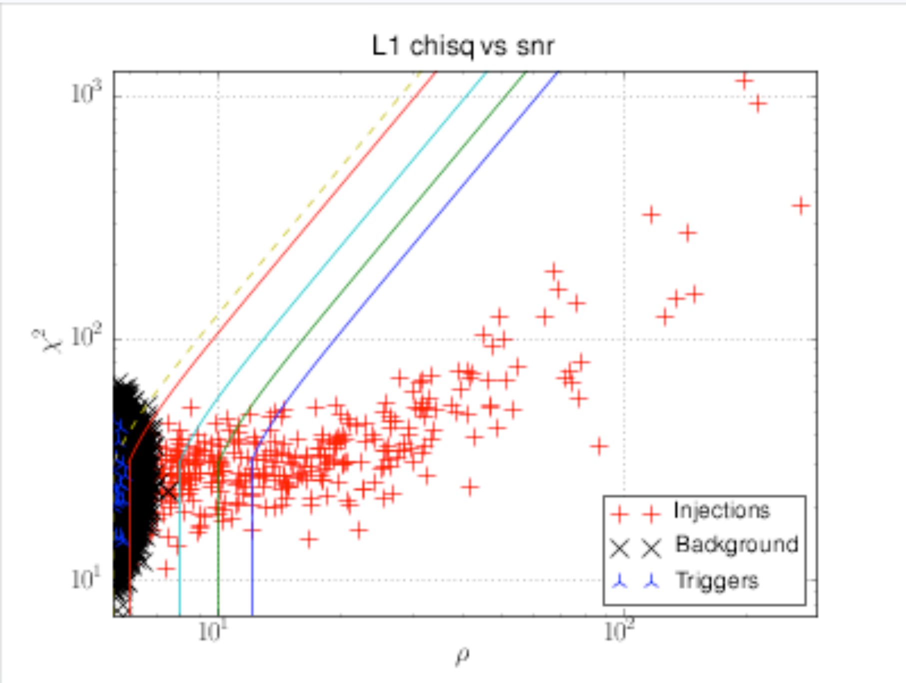
i=2

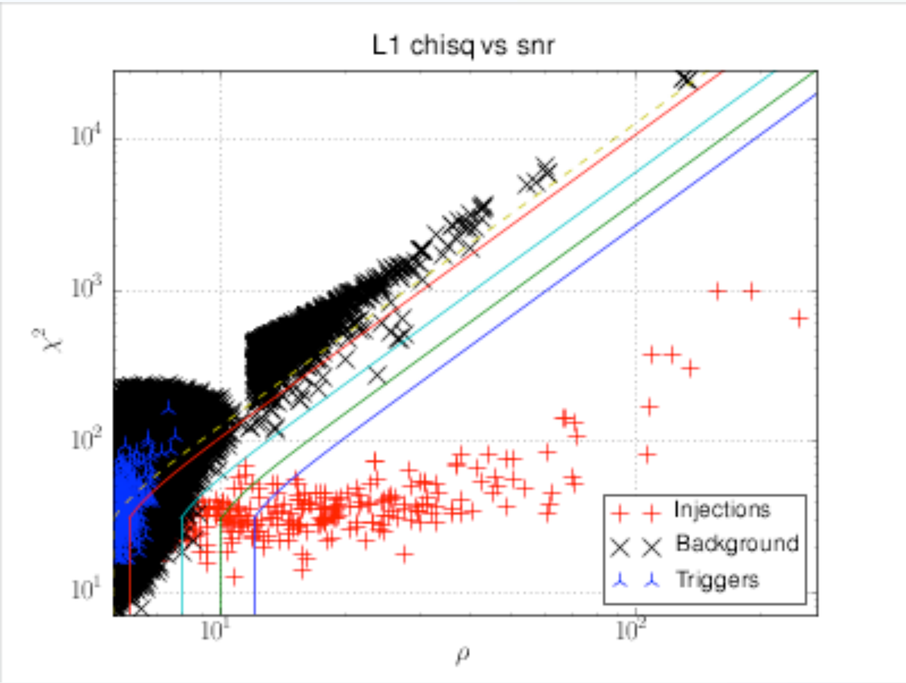
i=1



B. Allen, arXiv:gr-qc/0405045

SNR-chisq for noise vs. signal





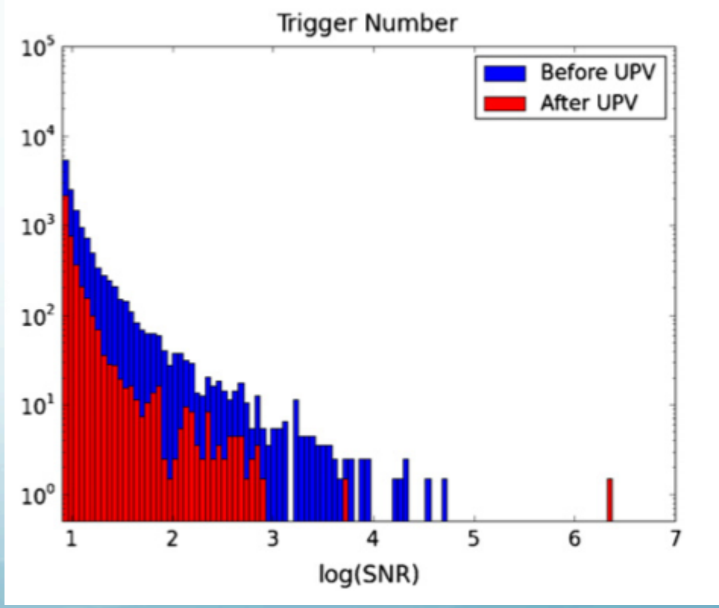
Gaussian noise

Real detector noise

- Contours : “reweighted” matched filter $\hat{\rho}$
- Detection statistic : $\rho_c^2 = \hat{\rho}_A^2 + \hat{\rho}_B^2 + \cdots$

Data quality vetoes

- Detector operation varies significantly over seconds/hours/days
- Many sources of environmental/ instrumental transient noise : “glitches”
- **Auxiliary channels** monitor potential sources of transient noise
- Identify times when noise coupled / correlated with GW strain s(t)
  & remove from analysis

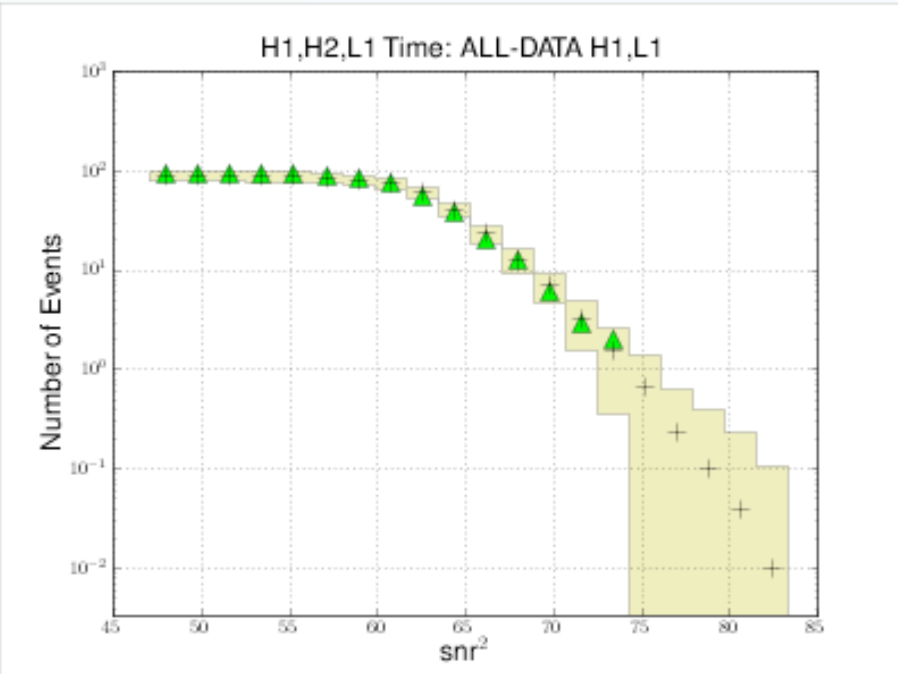


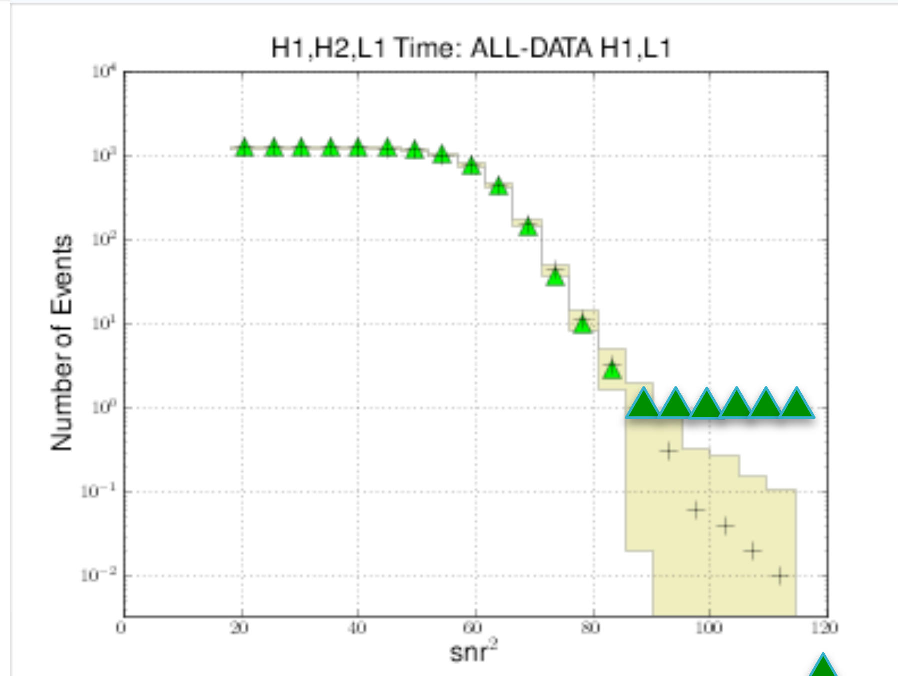
N. Christensen for LSC and Virgo, CQG 27 (2010) 194010

Background estimation

- Cannot predict distribution of noise triggers over ρ_c
- How to assign a false alarm probability (FAP) to an event with high ρ_c??
- Answer: Time-shifted analysis
  - Real signals occur with t_0 differences <0.1s between detectors
  - Artificially time shift $\rho_B(t) \rightarrow \rho_B(t + n \times 5s)$ in one detector, redo the coincidence tests
  - Result contains no real signals: use as background estimate

Identifying a candidate detection





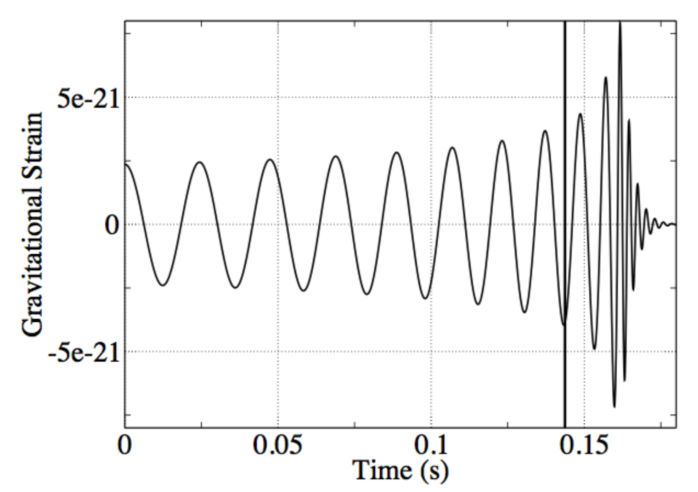
FAP = 0.01 candidate would look like this …

- “3 sigma evidence”: FAP ≤ 0.003
- “5 sigma discovery”: FAP ≤ 6e-7

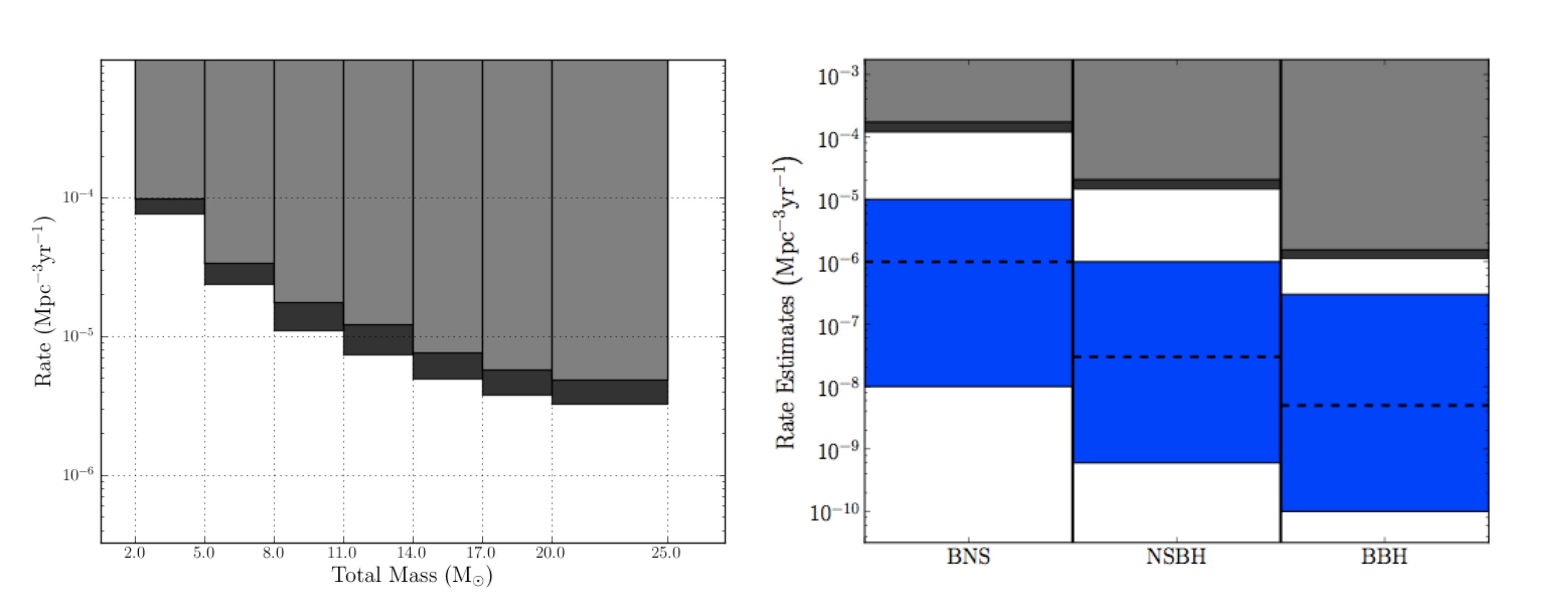
6. Recent results of CBC searches in LIGO-Virgo data

Summary of recent searches

- LIGO science run S6: July 2009–October 2010
- Virgo science runs VSR2 (7/2009–1/2010), VSR3 (8/2010–10/2010)
- Two main CBC searches performed
  - 'Lowmass': inspiral templates, $2 < (m_1+m_2)/M_\odot < 25$
  - 'Highmass': IMR templates, $25 < (m_1+m_2)/M_\odot < 100$
- No significant candidates found



Upper limits on merger rates

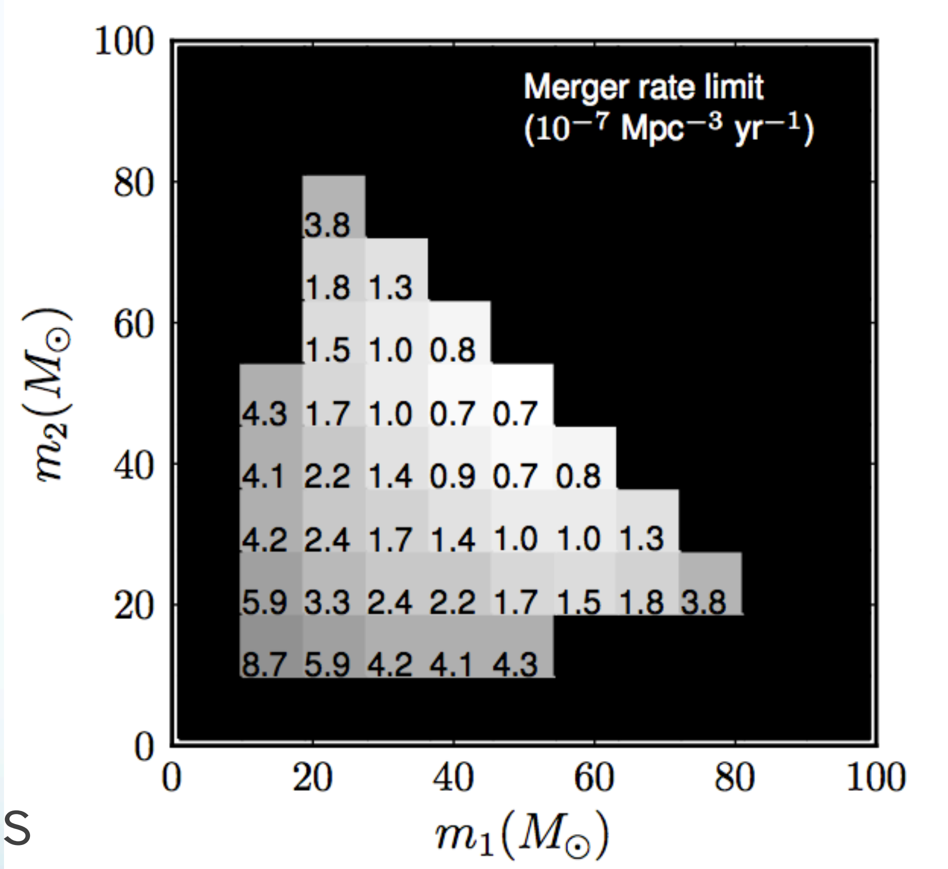


- Current limits from lowmass search some way above "realistic" astrophysical rates

J. Abadie et al. arXiv:1111.7314

‘Highmass’ BBH merger limits

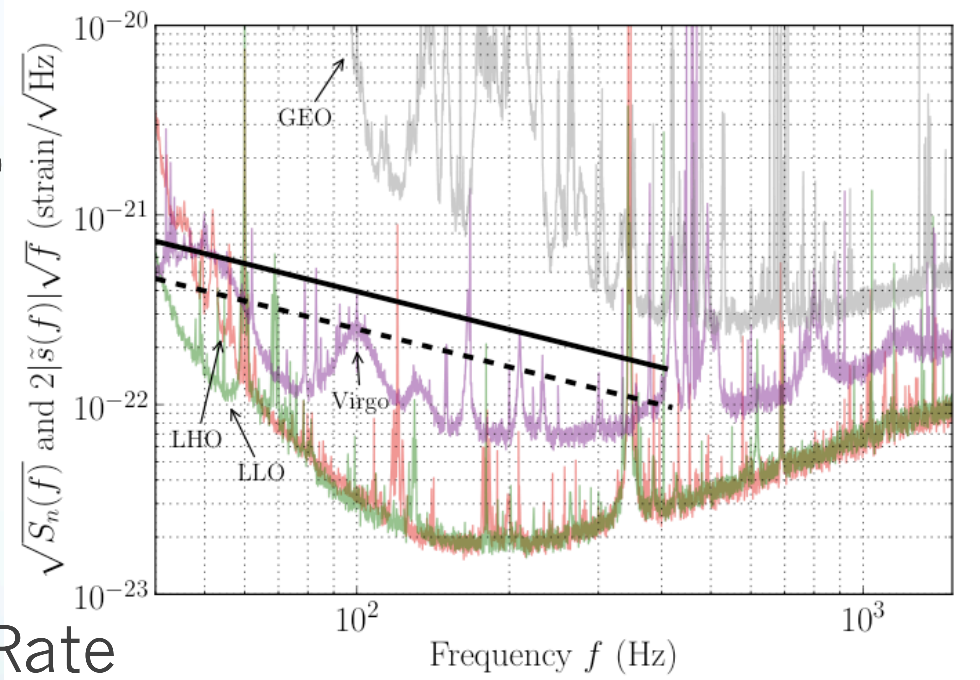
- Heavier systems are visible at larger distance
- Rate limit smallest at high m_1,m_2
- Astrophysical rates highly uncertain
- Rate limits from search are approaching “optimistic” astro models



J. Aasi et al., arXiv:1209.6533

Blind Injection Challenge

- Test of LSC–Virgo collaborations ability to claim detection in real data
- CBC signal injected into S6/VSR3 data: search groups were *unaware*
- Signal found by 'lowmass' search
- Estimated False Alarm Rate 1/7000 yr (2×10^{6} time shifts!)
- Draft paper prepared for submission to PRL ✓

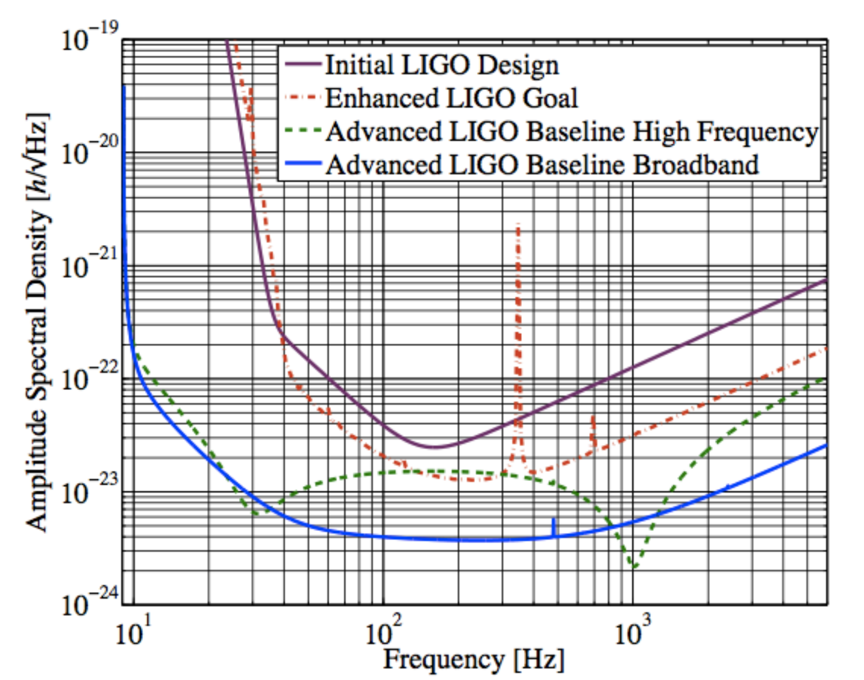


http://www.ligo.org/science/GW100916/

7. Future searches and analysis challenges

Advanced detectors

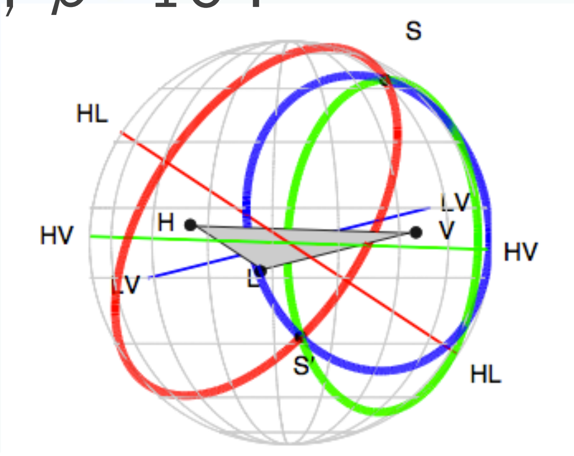
- Eventual ~factor 10 improvement in sensitivity over Initial LIGO
- Low-frequency sensitivity down to ~10 Hz
- Many advances in technology required
- Currently under construction
- Aim for first science run in 2015

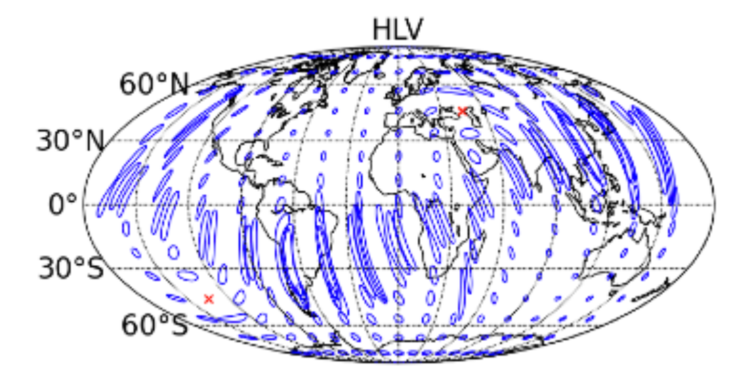


J.R. Smith for the LSC, CQG 26 114013

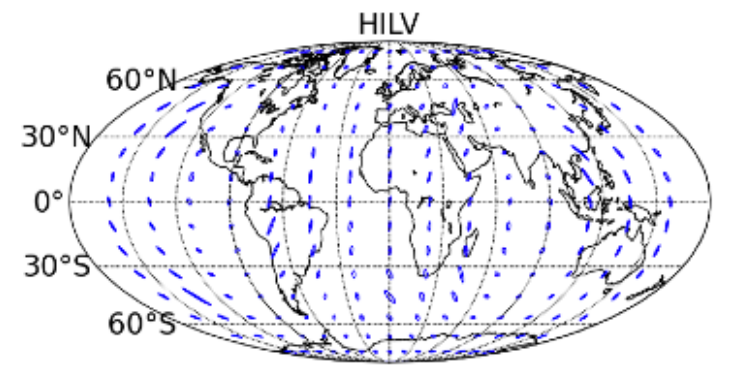
CBC sky localization

- Timing accuracy for single detector, $\rho \sim 10$: ~0.1ms
- Triangulation of time differences (for 3+ operating detectors)
- Localization depends on location!





HLV, projected 2017–18 : ~10% within 20 deg^2



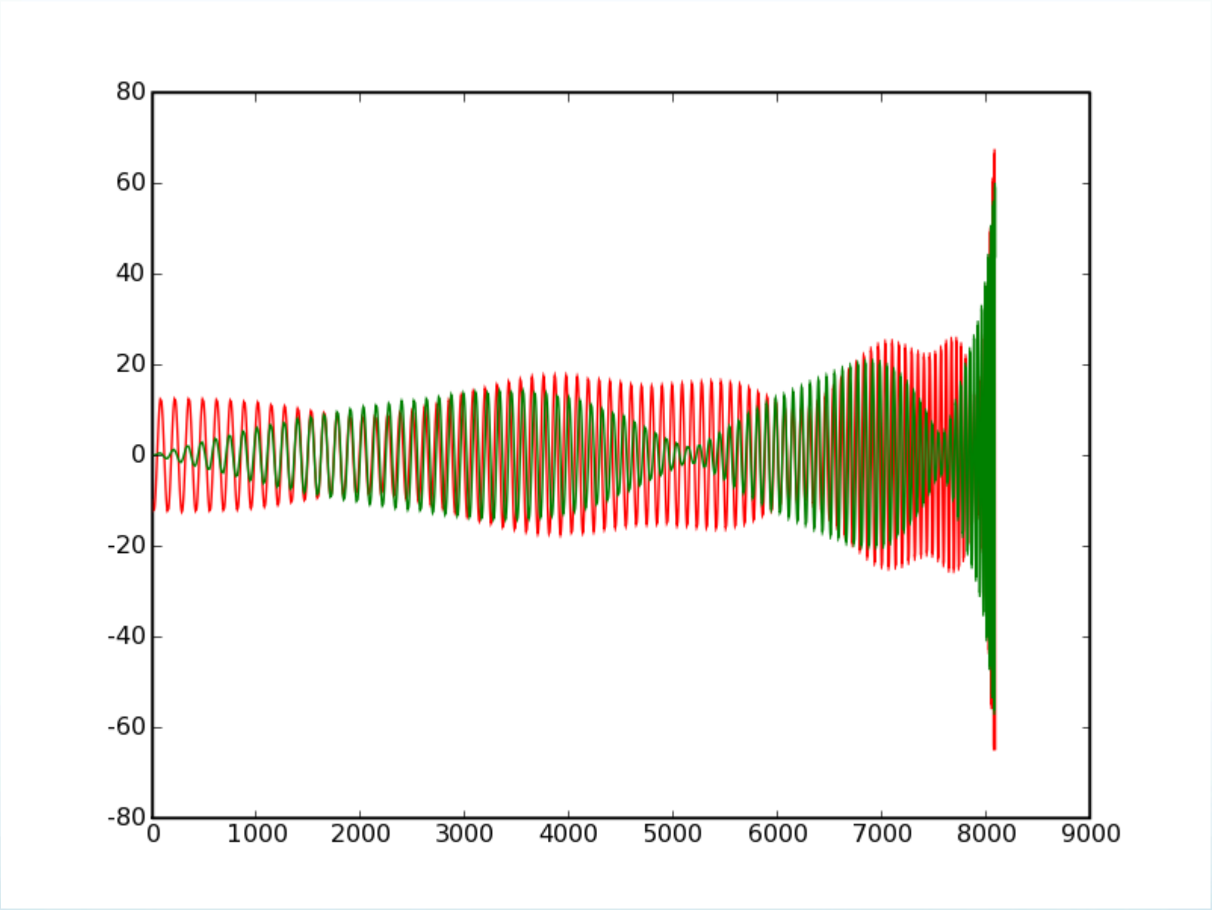
HILV, projected 2022+ : ~50% within 20 deg^2, 17% within 5 deg^2

J. Aasi et al., arXiv:1304:0670

Challenges from spinning CBCs

- Current searches use *non-spinning* template waveforms
  - Signals from systems with spinning BH seen with lower efficiency
  - Reduced match with templates ⇒ lower search SNR
  - Blind Injection signal was spinning – but also loud!
- Spinning systems have much larger parameter space
  - Challenge to increase search efficiency: find more signals without also finding more noise

Spinning precessing signals



Technical search challenges

- Number of templates
  - Increases as $f_{min}^{-8/3}$
  - Increased computing costs
- Length of templates
- Low latency
  - Cannot use simple Fourier transform – different filtering methods (multi-band, time domain, ...)
- More sophisticated treatment of detector variability
- Faster, more efficient background estimation

Summary

- Ground-based GW detectors are well adapted to detect signals from stellar mass CBC
- Current analysis techniques can search real (non-ideal) data efficiently
  - and make a strong case for first detection – given extra effort
- Advanced detectors expected to make several detections per year
- Next few years: improve analysis to make best use of coming data

Extra slides

Evidence for the Direct Detection of Gravitational Waves from a Black Hole Binary Coalescence
NOT A REAL DETECTION! Please see side note, and
`http://www.ligo.org/news/blind-injection.php`.

Background estimate for Blind Injection candidate