Compact binaries as sources of gravitational waves

Philippe Grandclément

Laboratoire de l'Univers et de ses Théories (LUTH)
CNRS / Observatoire de Paris
F-92195 Meudon, France

philippe.grandclement@obspm.fr

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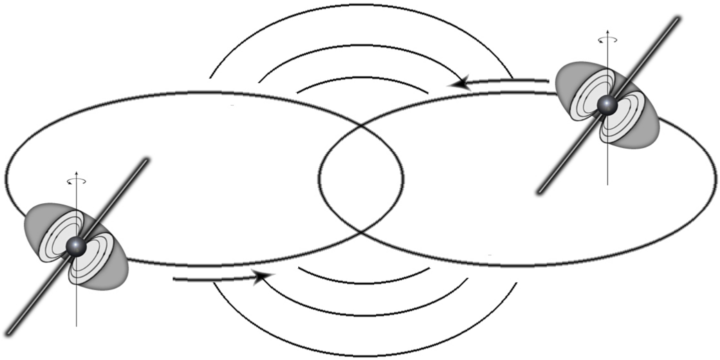
Generalities



The quadrupole

For sources that are not too compact and not too relativistic the emission of GW is governed by the time derivatives of the Newtonian quadrupole $\mathbf{Q}\left(t\right)$

$$Q_{kl}\left(t\right) = \int_{\text{source}} \rho\left(\mathbf{x}, t\right)\left(x_k x_l - \frac{1}{3} r'^2 \delta_{kl}\right) dV.$$

where ρ is the Newtonian matter density $\rho \approx T^{00}$.
In order to have an emission one must have a varying $\mathbf{Q}$ so that one must avoid spherical or axisymmetric sources.

Quadrupole formulae

The emitted quantities are then given by the so-called quadrupole formulae:

$$
\begin{aligned}
h_{ij}^{TT} &= \frac{2}{r} P_{ijkl} \frac{d^2 Q_{kl}}{dt^2}(t-r) \\
\frac{dE}{dt} &= \frac{1}{5} \frac{d^3 Q_{ij}}{dt^3} \frac{d^3 Q_{ij}}{dt^3} \\
\frac{dJ_j}{dt} &= \frac{2}{5} \varepsilon_{jkl} \frac{d^2 Q_{km}}{dt^2} \frac{d^3 Q_{ml}}{dt^3},
\end{aligned}
$$

where ε_{jkl} is the Levi-Civita symbol and $\mathbf{P}$ the TT projector operator

$$
P_{ijkl} = (\delta_{ik} - n_i n_k)(\delta_{jl} - n_j n_l) - \frac{1}{2}(\delta_{ij} - n_i n_j)(\delta_{kl} - n_k n_l)
$$

$\vec{n}$ is the unit vector from the observer to the source.

What is a good emitter ?

Consider an object of characteristic size R, mass M and velocity v. Its quadrupole is $Q \sim \varepsilon M R^2$, where $\varepsilon \sim 1$ is given by the geometry. The emitted power can then be approximated by :

$$\frac{dE}{dt} \sim \varepsilon^2 \left(\frac{GM}{Rc^2}\right)^2 \left(\frac{v}{c}\right)^6 10^{52}\,\mathrm{W}.$$

So a good source is compact, moves at relativistic speed and has an appropriate geometry (i.e. a varying quadrupole).

A compact binary system fulfills those requirements

Binary system in Newtonian mechanics (1)

Consider 2 point masses of mass m on a Newtonian circular orbit of radius d.

Newtonian dynamics implies

$$\omega^2 = \frac{2m}{d^3} \quad E = -\frac{m^2}{2d} \quad L^2 = \frac{m^3 d}{2}.$$

The varying quadrupole parts are

$$\begin{aligned} Q_{xx} &= \frac{md^2}{4}\cos(2\omega t) \\ Q_{xy} &= \frac{md^2}{4}\sin(2\omega t) \\ Q_{yy} &= -\frac{md^2}{4}\cos(2\omega t). \end{aligned}$$

Binary system in Newtonian mechanics (2)

The quadrupole formulae imply that

$$
\begin{aligned}
f_{\mathrm{GW}} &= 2 f_{\mathrm{orb}} \\
\frac{dE}{dt} &= -\frac{64}{5}\frac{m^5}{d^5} \\
\frac{dJ_z}{dt} &= -\frac{32}{5}\frac{m^4}{d^3}\sqrt{\frac{2m}{d}}.
\end{aligned}
$$

Binary system in Newtonian mechanics (3)

One can show that

$$\begin{aligned} \frac{\mathrm{d}E}{E} &= \frac{64}{5}\frac{m^3}{d^5} \\ \frac{\mathrm{d}L}{L} &= -\frac{32}{5}\frac{m^3}{d^5}. \end{aligned}$$

- It implies that $EL^2 = \mathrm{const.}$ so that the orbit stays circular.
- Moreover one could show that an initialy excentric orbit will be cricularized by GW emission.

Binary system in Newtonian mechanics (4)

- By demanding that the total energy variation is due to GW emission one can find a differential equation for the separation

$$d^3 d' = -\frac{128}{5} m^3$$

- It can be integrated as

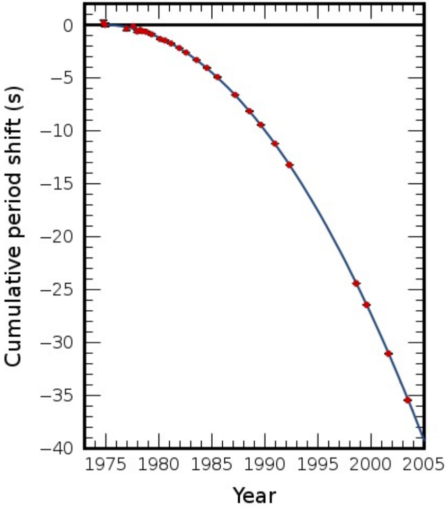
$$d\left(t\right) = \left(d_0^4 - \frac{512}{5} m^3 t\right)^{1/4}.$$

- The time at coalescence is defined as the time for which $d = 0$ and so is :

$$T = \frac{5 d_0^4}{512 m^3}.$$

PSR B1913+16 : Hulse and Taylor binary system

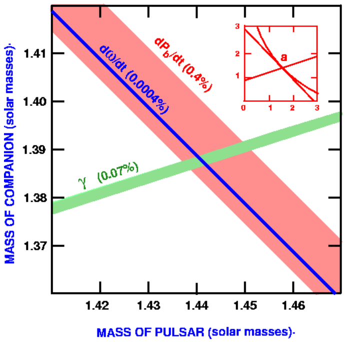
- In 1974, using Aricibo radio telescope, Hulse and Taylor observed a 59 ms. period pulsar, being a $1.4\,M_\odot$ neutron star.
- Observation of a smooth variation of the pulse on a period of 7.75 hours implies the existence of a companion star that is also a neutron star.
- Subsequent decay of the orbit is consistent with an emission of gravitational waves.



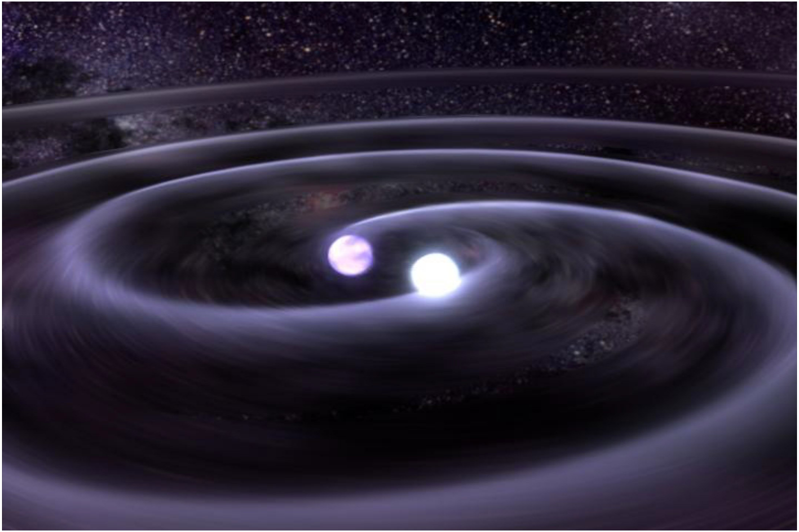
PSR B1913+16 : constraints on the masses

Various quantities lead to consistent masses

- Perihelion precession $\dot{\omega}$.
- Einstein effect γ.
- Variation of the period $\dot{P}$.



Stellar mass binaries



Emission frequency

Only compact objects reach the strong gravity regime without being tidally disrupted.
Typical frequency is given by the last stable orbit. Using Schwarzschild value of $R = 6M$ one finds that

$$f_{\mathrm{LSO}} = 220 \left(\frac{20\mathrm{M}_\odot}{M} \right) \mathrm{Hz}.$$

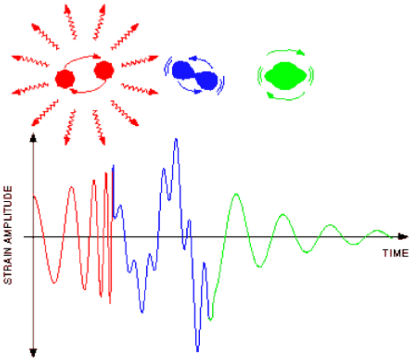
- Neutron stars emit in the $\mathrm{kHz.}$ regime.
- Stellar mass black holes emit in the $100\,\mathrm{Hz.}$ regime.

Amplitude orders of magnitude

Interferometric detectors measure a relative variation of length $h \sim \dfrac{\Delta L}{L}$.
Using the quadrupole formulae one can estimate the typical amplitude of a binary system :

$$h \sim 10^{-23} \left(\frac{2.8\mathrm{M}_\odot}{M}\right)^{5/3} \left(\frac{f}{100\mathrm{Hz}}\right)^{2/3} \left(\frac{200\mathrm{Mpc}}{D}\right).$$

Schematic evolution



- **Inspiral phase** : post-Newtonian expansion.
- **Merger phase** : numerical relativity.
- **Ringdown phase** : perturbation theory.

pN expansion

The inspiral phase is believed to be well described in the post-Newtonian framework.

Basic assumptions

- The source is compactly supported.
- The observer is far from the system.
- The source is not too relativistic i.e. $v/c < (<)1$.

Mathematical technique

- post-Minkowskian expansion : $h^{\alpha\beta} = \sum_n G^n h_n^{\alpha\beta}$.
- Each order is given by $\Box h_n^{\alpha\beta} = S_n\left(h_1, ..., h_{n-1}\right)$.
- Those equations are solved by making used of the appropriate Green's functions.

Comments on the pN expansion

- The expansion parameter is $(v/c)^2$ (so that 3.5 pN contains corrections up to $(v/c)^7$).
- Objects are point masses, described with Dirac functions. Diverging termes appear that must be regularized (typically using Hadamard regularization).
- The first order of the expansion gives the quadrupole formulae.
- Convergence of the expansion is difficult to assess.

Number of cycles in the Virgo band

	$2 \times 1.4\, M_\odot$	$10\, M_\odot + 1.4\, M_\odot$	$2 \times 10\, M_\odot$
Newtonian order	16031	3576	602
1PN	441	213	59
1.5PN (dominant tail)	-211	-181	-51
2PN	9.9	9.8	4.1
2.5PN	-11.7	-20.0	-7.1
3PN	2.6	2.3	2.2
3.5PN	-0.9	-1.8	-0.8

Additional approximations

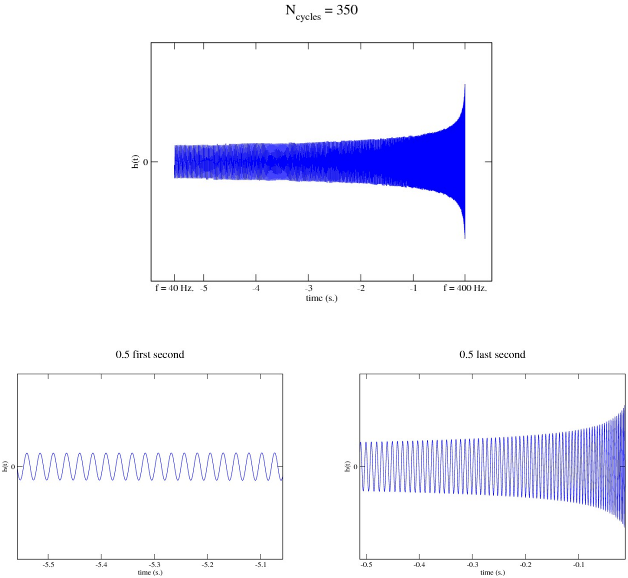
- Consider only circular orbits.
- Neglect the spin of the bodies.
- Amplitude of the wave at leading order.
- Phase known up to 3.5 order.

Chirp signal

$$\begin{aligned}
\tilde{h}(f) &= \mathcal{A} f^{-7/6} \exp\left[i\phi(f)\right] \\
\phi_{\mathrm{2PN}}(f) &= \phi_{\mathrm{const}} + 2\pi f t_c + \frac{3}{128}\left(\pi \mathcal{M} f\right)^{-5/3}\left[1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11\mu}{4M}\right)\left(\pi M f\right)^{2/3}\right. \\
&\quad \left. - 16\pi\left(\pi M f\right) + 10\left(\frac{3058673}{1016064} + \frac{5429\mu}{1008M} + \frac{617\mu^2}{144M^2}\right)\left(\pi M f\right)^{4/3}\right]
\end{aligned}$$

$\mathcal{A}$ is a constant amplitude, t_c the time at coalescence, M the total mass, μ the reduced mass and $\mathcal{M} = \mu^{3/5} M^{2/5}$ is known as the chirp mass.

Waveform for $M_1 = M_2 = 1.4 M_\odot$

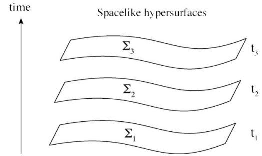


Merger phase : numerical relativity

In the merger phase, non-linearities are strong and one needs to solve the equations numerically.

3+1 formalism

- Defines a time by a foliation of spacetime.
- Transforms the 4-D equations into a set of evolutions equations for 3D quantities.



The 3+1 metric takes the form

$$\mathrm{d}s^2 = -\left(N^2 - B_i B^i\right)\mathrm{d}t^2 + 2B_i \mathrm{d}t\mathrm{d}x^i + \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j.$$

where the lapse N the shift $\vec{B}$ and the metric γ_{ij} are 3D quantities depending on the time.

3+1 equations

Using the 3+1 framework, Einstein equations can be put into the following form:

Evolution equations

- $\left(\frac{\partial}{\partial t} - \mathcal{L}_{\vec{B}}\right) \gamma_{ij} = -2NK_{ij}$, where K_{ij} is the extrinsic curvature tensor. It can be seen as the definition of the speed of the metric.
- $\left(\frac{\partial}{\partial t} - \mathcal{L}_{\vec{B}}\right) K_{ij} = S_{ij}$, is equivalent to $\frac{\mathrm{d}v}{\mathrm{d}t} = f/m$.

Constraint equations

- They only need to be solved at $t = 0$ and do not involve time derivatives.
- One scalar equation : the Hamiltonian constraint.
- One vectorial equation : the momentum constraint.

Decomposition of the problem

Initial data problem

- Generate initial data verifying the constraints.
- Can be tricky because it involves elliptic equations.
- How to control the physical content of such initial data before evolving them ?

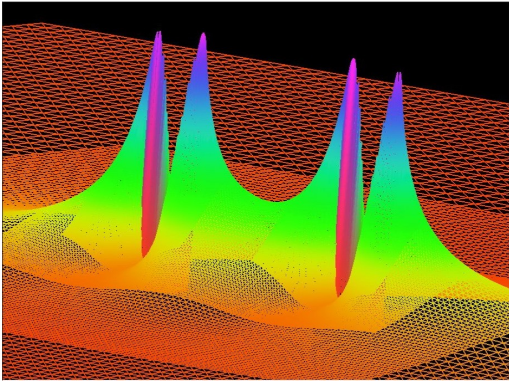
Evolve the data

- Find a stable formulation when chocks or horizon can appear.
- Choose appropriate gauge conditions.
- Check that the constraint equations are not violated.

post-processing

- Extract meaningful physical quantities.
- Generation of the emitted gravitational waves.

Example of initial data

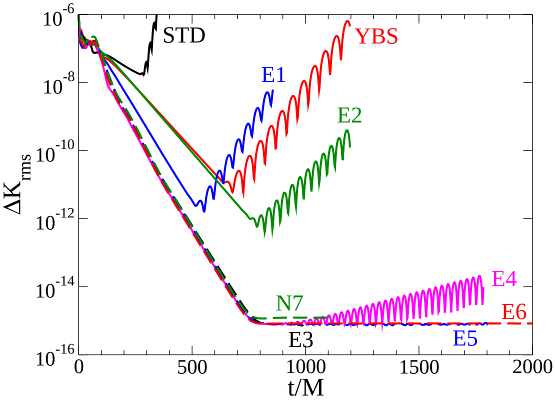


A stable formulation : BSSN formalism

Stands for Baumgarte, Shapiro, Shibata and Nakamura.

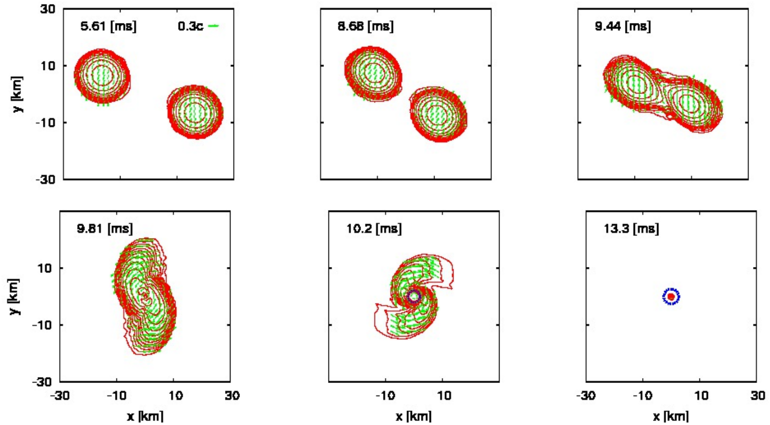
- The direct evolution of γ_{ij} and K_{ij} lead to the appearance of *constraint-violating modes*.
- Define the conformal metric as $\tilde{\gamma}_{ij} = \exp(-4\phi)\,\gamma_{ij}$ and such that $\det(\tilde{\gamma}) = 1$.
- Decomposition of the extrinsic curvature as $\tilde{A}_{ij} = \exp(-4\phi)\left[K_{ij} - 1/3K\gamma_{ij}\right]$.
- Define $\tilde{\Gamma}^i = \tilde{\gamma}^{jk}\tilde{\Gamma}^i_{jk}$.
- Write evolution equations for ϕ, $\tilde{\gamma}_{ij}$, K, $\tilde{A}_{ij}$ and $\tilde{\Gamma}^i$.

Many implementations of BSSN



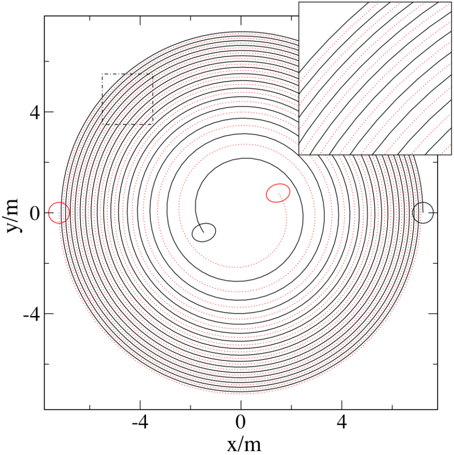
Single black hole evolutions from H-Y. Yo *et al.*, Phys. Rev. D **86**, 064027 (2012)

Binary NS simulations



K. Kiuchi *et al.*, Phys. Rev. D **80**, 064037 (2009)

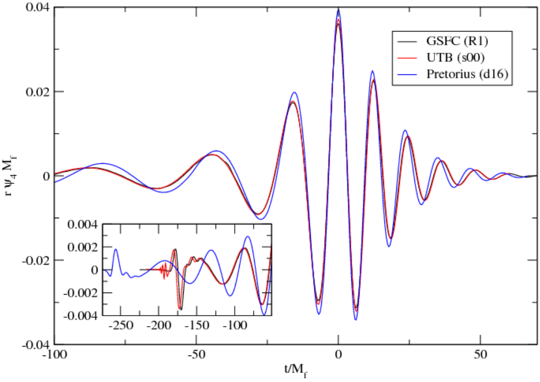
Binary BH simulations



Caltech-Cornell group

End of the evolution

- The newly formed BH relaxes to a Kerr one by emitting GW.
- Well described with a perturbative theory (dates from the 70's).
- Is also well captured by the numerical simulations.
- Rapid damping of the emitted waves.



Full waveform

post-Newtonian expansion

- describes well the inspiral phase.
- is expected to fail at late times.

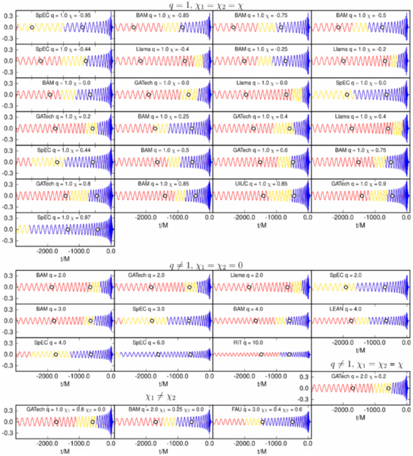
Numerical relativity

- provides a description of the merger and ringdown.
- is computationally expensive.
- can not evolve widely separated initial data.

Solution : matching the two methods but

- not the same coordinates.
- not a single way to proceed.
- matching is largely empirical.

Example of hybrid waveforms

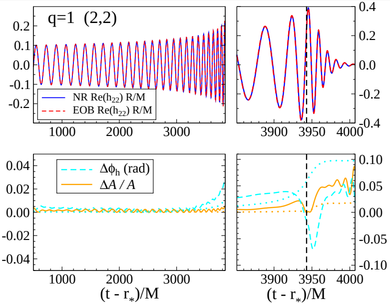


P. Ajit *et al.*, Class. Quant. Grav. **29**, 124001 (2012)

Effective One Body approach (EOB)

Principle

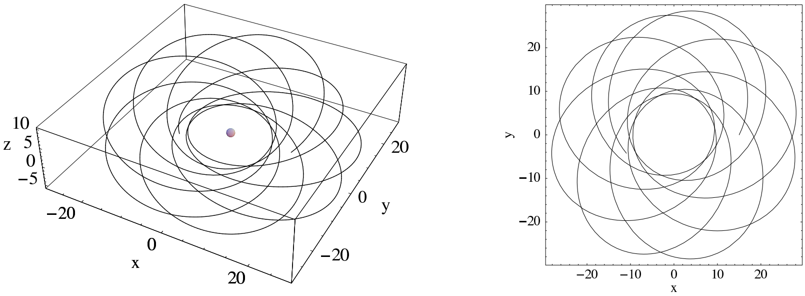
- Map the binary system m_1 and m_2 into a single particle of mass $\mu = m_1 m_2 / (m_1 + m_2)$ moving into an effective metric.
- The effective metric reads $\mathrm{d}s^2 = -A(r)\,\mathrm{d}t^2 + \frac{D(r)}{A(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega$.
- $A(r)$ and $D(r)$ are determined by an effective Hamiltonian and contain parameters that can be used to fit results from numerical simulations.



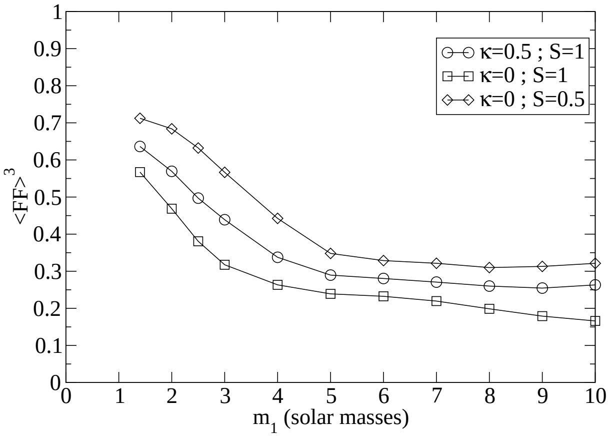
Y. Pan *et al.*, Phys. Rev. D **83**, 064003 (2011)

Effects of the spins

- Spins can probably be neglected for binary neutron stars.
- The same is no longer true for black holes.
- Spin effects induce many modulations in the waveforms (precession for instance).
- Requires many parameters to be modeled accurately.
- Need to rely on effective waveforms for detection.



Loss in detection rate due to the spins



P.G. *et al.*, Phys. Rev. D **67**, 042003 (2002)

Formation of binary compact objects

In short

- Compact objects of stellar mass are the endpoint of the evolution of stars.
- Many stars are in binary systems.
- Evolution of a binary system can lead to a formation of a compact binary.

More precisely, one needs to follow the different phases of the lifetime of the binary to ensure that it will end in a double compact star object.

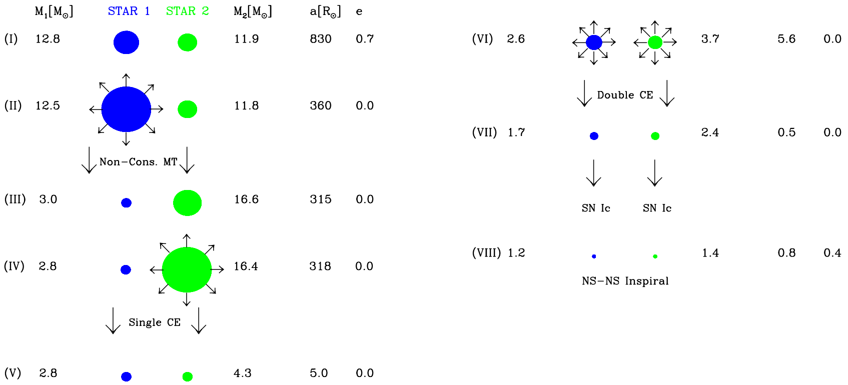
Population synthesis

- Generate a realistic initial population of binary systems.
- Follow their evolution as they undergo many different phases.
- Check how many compact binaries are left at the end.

Ingredients

- Many different star-type (around 20).
- Tidal disruption.
- Magnetic fields.
- Stellar winds.
- Roche overflow.
- Common envelop phases.
- and many more...

Example of NS-NS formation channel



K. Belczynski *et al.*, Astrophys. J. **550**, L183 (2001)

Channels for NS-NS binaries

Formation Channel	Relative Efficiency$^{\alpha}$	Evolutionary History$^{\beta}$
NSNS:01	20.3 %	NC:a$\rightarrow$b, SN:a, HCE:b$\rightarrow$a, HCE:b$\rightarrow$a, SN:b
NSNS:02	10.8 %	NC:a$\rightarrow$b, SCE:b$\rightarrow$a, NC:a$\rightarrow$b, SN:a, HCE:b$\rightarrow$a, SN
NSNS:03	5.5 %	SCE:a$\rightarrow$b, SN:a, HCE:b$\rightarrow$a, HCE:b$\rightarrow$a, SN:b
NSNS:04	4.0 %	NC:a$\rightarrow$b, SCE:b$\rightarrow$a, SCE:b$\rightarrow$a, SN:b, HCE:a$\rightarrow$b, S
NSNS:05	3.2 %	DCE:a$\rightarrow$b, SCE:a$\rightarrow$b, SN:a, HCE:b$\rightarrow$a, SN:b
NSNS:06	2.5 %	SCE:a$\rightarrow$b, SCE:b$\rightarrow$a, NC:a$\rightarrow$b, SN:a, HCE:b$\rightarrow$a, S
NSNS:07	2.2 %	NC:a$\rightarrow$b, NC:a$\rightarrow$b, SN:a, HCE:b$\rightarrow$a, HCE:b$\rightarrow$a, SN
NSNS:08	2.0 %	NC:a$\rightarrow$b, DCE:b$\rightarrow$a, SN:a, HCE:b$\rightarrow$a, SN:b
NSNS:09	2.0 %	DCE:a$\rightarrow$b, DCE:a$\rightarrow$b, SN:a, SN:b
NSNS:10	1.6 %	NC:a$\rightarrow$b, SCE:b$\rightarrow$a, SN:b, HCE:a$\rightarrow$b, SN:a
NSNS:11	1.5 %	NC:a$\rightarrow$b, SCE:b$\rightarrow$a, DCE:b$\rightarrow$a, SN:a, SN:b
NSNS:12	1.5 %	NC:a$\rightarrow$b, SCE:b$\rightarrow$a, DCE:a$\rightarrow$b, SN:a, SN:b
NSNS:13	1.0 %	DCE:a$\rightarrow$b, SN:a, HCE:b$\rightarrow$a, SN:b
NSNS:14	3.0 %	all other

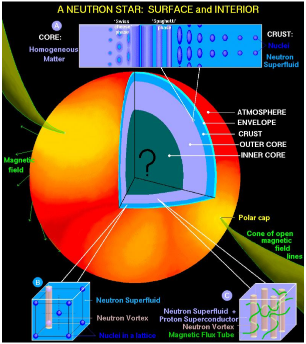
Channels for NS-BH and BH-BH

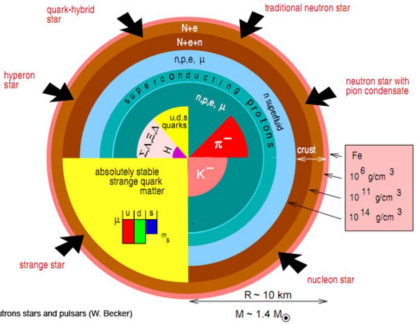
BHNS:01	4.5 %	NC:a→b, SN:a, HCE:b→a, SN:b
BHNS:02	1.6 %	NC:a→b, SCE:b→a, SN:a, SN:b
BHNS:03	1.3 %	SCE:a→b, SN:a, HCE:b→a, NC:b→a, SN:b
BHNS:04	2.0 %	all other
BHBH:01	17.7 %	NC:a→b, SN:a, HCE:b→a, SN:b
BHBH:02	10.5 %	NC:a→b, SCE:b→a, SN:a, SN:b
BHBH:03	1.4 %	all other

V. Kalogera *et al.*, Phys. Rept. **442**, 75 (2007)

Structure of neutron stars

The structure of the interior of NS is largely unknown. The physical conditions are impossible to study on earth $\Rightarrow$ one needs to rely on astrophysical observations.

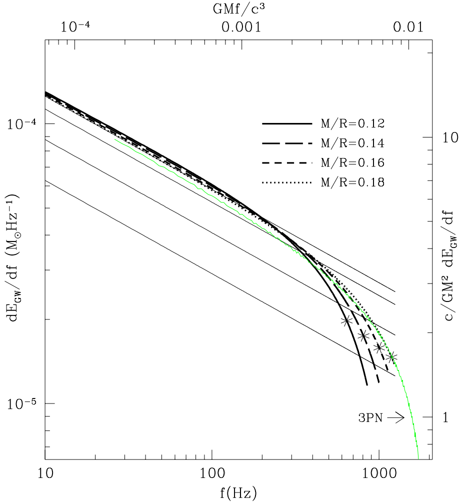




Source : Neutrons stars and pulsars (W. Becker)

Measuring the compactness

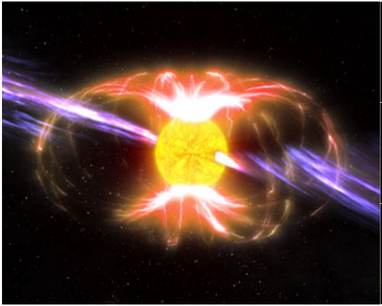
Gravitational waves can help constraining the NS structure.

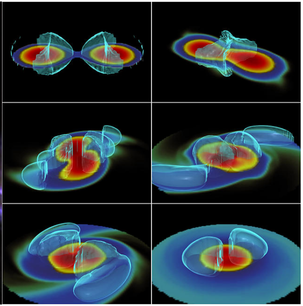


Multi-messenger astronomy

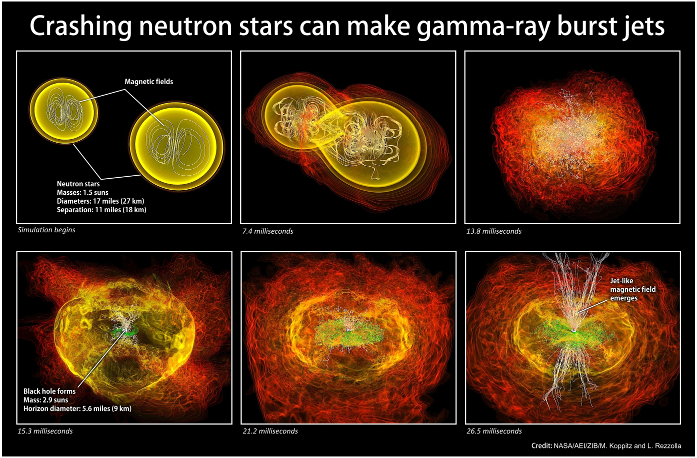
Example of gamma-ray bursts

- The jet is believed to be powered by an accretion disk around a black hole.
- The formation of the BH-disk system is still uncertain.
- Different mechanism could explain the various classes of GRB.
- Emission of GW could be very different in one case or the other.

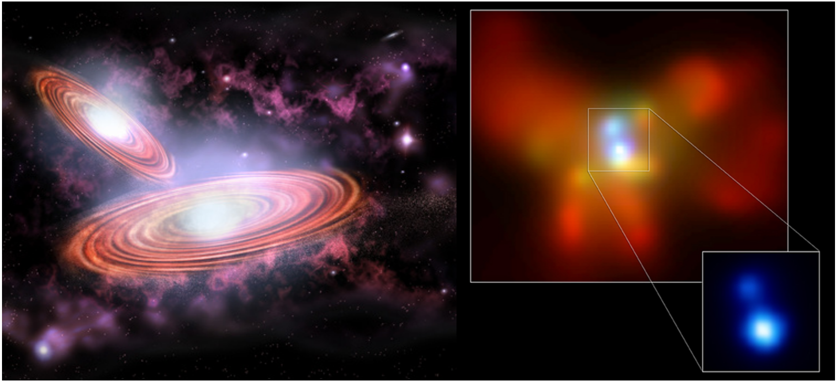




NS-NS mergers can launch jets



Supermassive black holes



Link with galaxy formation

Formation scenario

- Most, if not all, galaxies, are believed to host a supermassive black hole at the center.
- The central object mass is in the range $M \in \left[10^6 M_\odot, 10^9 M_\odot\right]$.
- Current galaxies are formed by successive mergers of smaller ones.
- When a merger occurs, dynamic friction can bring the two black holes on a bound orbit.
- The binary object emits gravitational waves and finally merges.

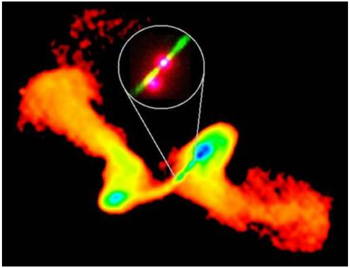
Emitted gravitational waves

- The same physics as for stellar mass black holes.
- Only a different scaling in terms of amplitude, timescales, frequencies...

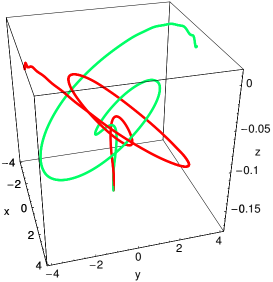
$$h_{\rm amp} \approx 10^{-19} \left(\frac{2 \times 10^6 M_\odot}{M} \right)^{5/3} \left(\frac{f}{10^{-3}{\rm Hz}} \right)^{2/3} \left(\frac{5{\rm Gpc}}{D} \right)$$

Influence of the spins

- Indications that SMBH can have large spins.

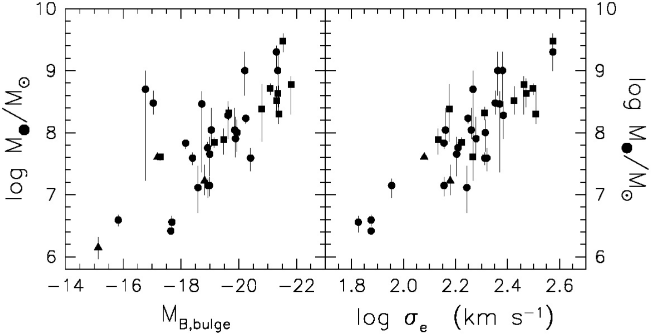


- It influences the trajectories.
- It affects the waveforms.
- It can have a strong impact on the final state (like kicks).



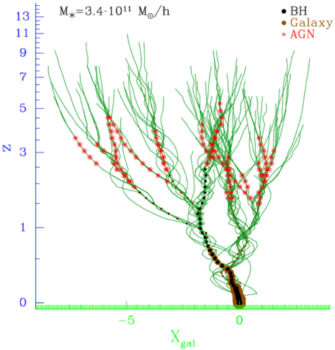
Relation between SMBH and galaxy

- Strong correlation between the SMBH mass and the mass of the bulge of the host galaxy.
- The formation of the BH is closely linked to the formation of the galaxy itself.



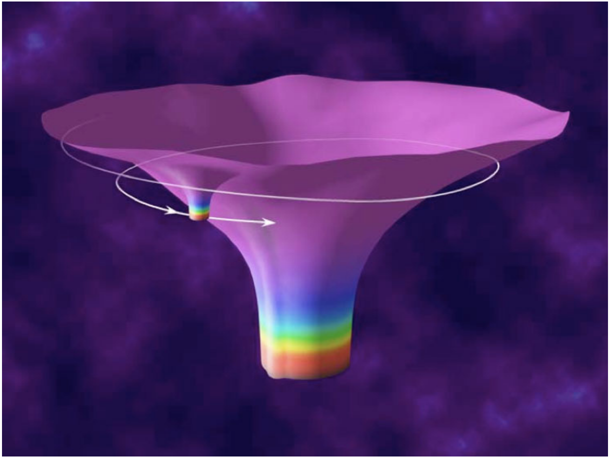
Forming the first black hole

- The first generation of stars provides the first BH.
- A seed BH is formed by collapse of gas at the center of the proto-galaxy.
- A cluster of stars can undergo collapse at the center of the galaxy.



Observing GW from SMBH will probe the history of galaxy formation and thus put strong constraints on cosmological models for structure formation.

Extreme mass ratio inspiral



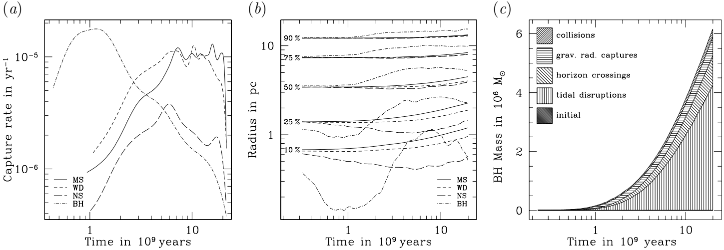
Formation scenario

- At the center of galaxies, a stellar mass object can be captured by the central massive black hole.
- This can be caused 3-body interaction.
- At periastron, emitted waves are emitted.
- The orbit shrinks and somewhat circularized.
- The small object must be compact not to be destroyed by tidal forces when passing close to the SMBH.

$$h_{\mathrm{amp}} \approx 4 \times 10^{-24} \left(\frac{m}{1 M_\odot}\right) \left(\frac{1\mathrm{Gpc}}{D}\right) \left(\frac{M}{10^6 M_\odot}\right)^{2/3} \left(\frac{f}{10^{-4}\mathrm{Hz}}\right)^{2/3}$$

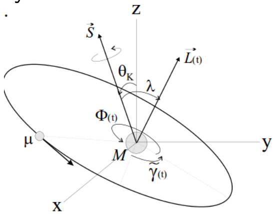
Computing the number of events

- Study the dynamics of stellar mass CO in the dense cluster surrounding the SMBH.
- An object is said to be captured if $t_{\mathrm{GW}} < t_{\mathrm{scat}}$.
- Depending on the physical parameters one can use different approaches :
  - Semi-analytical considerations.
  - Fokker-Planck
  - direct N-body simulations.
- Results are very uncertain and depend on many unknown parameters (initial distribution of stars, model of the SMBH)



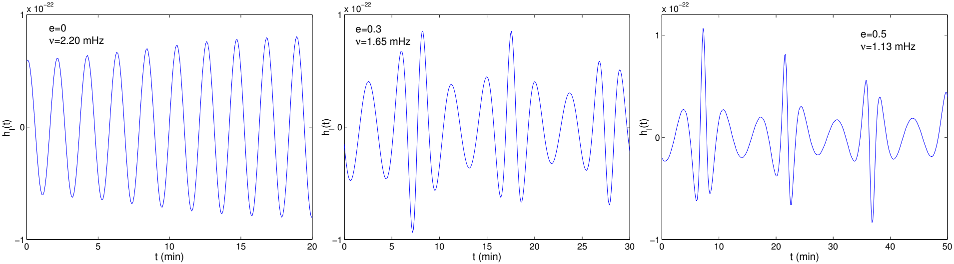
Complicated waveforms

Even when neglecting the spin of the small object, the orbits depend on **14 parameters** and many different timescales are involved.



- Distance D, masses m and M, spin of the SMBH S.
- Direction of the detector θ_D, ϕ_S and of the spin θ_S, ϕ_S.
- Angle between $\vec{S}$ and $\vec{L}$.
- Reference time t_0 and
  - Initial eccentricity e_0, direction of the pericenter $\tilde{\gamma}_0$ and ean anomaly Φ_0.
  - Position of $\vec{L}$ around $\vec{S}$: α_0.

Examples of waveforms : influence of the eccentricity



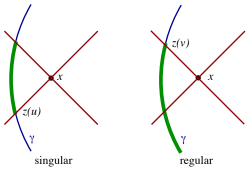
L. Barack and C. Cutler, Phys. Rev. D **69**, 082005 (2004)

Gravitational self-force

Principle

- Let m be the mass of the small object and M the mass of the SMBH.
- The objects move on geodesic curves of the whole spacetime (including emitted waves...)
- Instead consider that :
  - The geometry is given by the SMBH : Kerr spacetime of mass M and spin S.
  - m follows a trajectory that is not a geodesic of Kerr.
  - Gravitational radiation is described by a force acting on it.

- Computing the self-force involves many technical difficulties.
- For instance one must compute path integrals diverging at the location of m.
- One can rely on approximated techniques.



Testing the “Kerness” of the central object

- The small CO acts as a probe of the geometry generated by the SMBH.
- One can define
  - the mass multipoles $M_\ell = \int \rho r^\ell \mathrm{d}\tau$.
  - the mass-current multipoles $\vec{S}_\ell = \int \rho r^{\ell-1} \vec{r} \wedge \vec{v} \mathrm{d}\tau$.
- For a black hole, the uniqueness theorem implies that

$$M_\ell + iS_\ell = M\,(ia)^\ell$$

.

- This is not true for alternative models, like boson stars for instance.

Example of test

Principle

- Various multipoles induce corrections in the phase of the waveform.
- One can use those correction to extract the multipoles.

In practice

- Extracting all the moments is not possible.
- Used M, S, and M_2 as independent variables.
- Assume all the other moments verify the Kerr relation.
- Check whether M_2 is consistent with a Kerr BH.
- For SNR of 10 the precision on M_2 would be of order 10^{-2}.

F.D. Ryan, Phys. Rev. D **56**, 1845 (1997)

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What if GR is not the correct theory ?

One can use an extended waveforms like the *parametrized post-Einsteinian*.

Look at theories that

- are metric theories.
- are consistent with tests in the weak field regime.
- are *inconsistent* in the strong field regime.

Guidelines of PPE

Decompose the signal in three epochs : Inspiral phase, Merger and Ringdown

Modifications of GR

- Inspiral : modification the Hamiltonian and the Energy balance law.
- Merger phase : use a phenomenological fit from numerical simulations and assume interpolation between inspiral and ringdown phases.
- Ringdown : Keep main features of GR but modify the tails (somewhat ad-hoc)

Example of PPE waveform

$$
\begin{aligned}
f < f_{\mathrm{I}} \qquad & \tilde{h}(f) = \tilde{h}_I^{\mathrm{GR}}(1 + \alpha u^a)\exp\left(i\beta u^b\right) \\
f_{\mathrm{I}} < f < f_{\mathrm{RD}} \qquad & \tilde{h}(f) = \gamma u^c \exp\left(i\left(\delta + \epsilon u\right)\right) \\
f_{\mathrm{RD}} < f \qquad & \tilde{h}(f) = \zeta \frac{\tau}{1 + 4\pi^2\tau^2\kappa\left(f - f_{\mathrm{RD}}\right)^d}.
\end{aligned}
$$

where $u = \pi\mathcal{M}f$.

Parameters

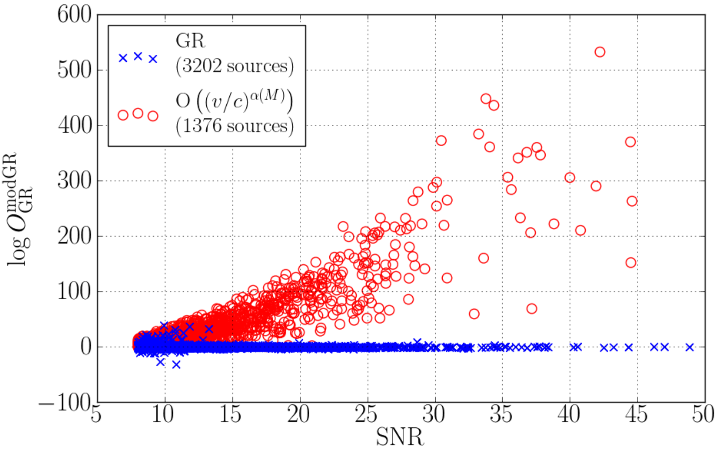
- γ, δ, τ and ζ are obtained by continuity.
- 4 parameters in the inspiral phase (α, a, β, b).
- 2 parameters in the merger phase (ϵ, c).
- 2 parameters in the ringdown phase (κ, d).
- The two frequencies f_{I} and f_{RD} can also be treated as parameters.

Alternative theories

GR is recovered for $(\alpha, a, \beta, b) = (0, 0, 0, 0)$, $(\epsilon, c) = (1, -2/3)$ and $(\kappa, d) = (1, 2)$

- Scalar-tensor theories ; Brans-Dicke for $(\alpha, a, \beta, b) = (0, 0, \beta_{\mathrm{BD}}, -7/3)$.
- Massive graviton for $(\alpha, a, \beta, b) = (0, 0, \beta_{\mathrm{MG}}, -1)$.
- Chern-Simons for $(\alpha, a, \beta, b) = (\alpha_{\mathrm{CS}}, 1, 0, 0)$.
- Einstein-Aether theory.
- MOND theory.
- Higher number of dimensions (DGP)

What are the odds that the right theory is not GR ?



T.G.F. Li *et al.*, Phys. Rev. D **85**, 082003 (2012)

The end...

- Coalescing binaries are expected to be the strongest gravitational wave sources.
- Inspiral phase is well described by pN theory.
- Breakthrough in numerical relativity allowed to have completed waveforms.
- Still some work to be done (hybrid waveforms, self-force).
- Detection will provide unique tests of the physics (testing GR in the strong field regime).
- Observation will open a new window on the universe with implications on NS structure, cosmology and many more...