Probing dynamical spacetimes with gravitational waves

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Overview

- Was Einstein right?
  - General relativity has never been tested in the regime of strong, dynamical gravitational fields
- How does matter behave in strongly curved, dynamical spacetime?
  - Neutron star “equation of state” uncertain by factor ~10

Testing general relativity

with

gravitational waves

Testing general relativity

General Relativity has enjoyed important successes:

- Perihelium precession of Mercury
- Deflection of star light by the Sun
- Shapiro time delay
- Gravity Probe B
  - Geodetic effect
  - Frame dragging
- Expansion of the Universe
- Binary neutron stars

Testing general relativity

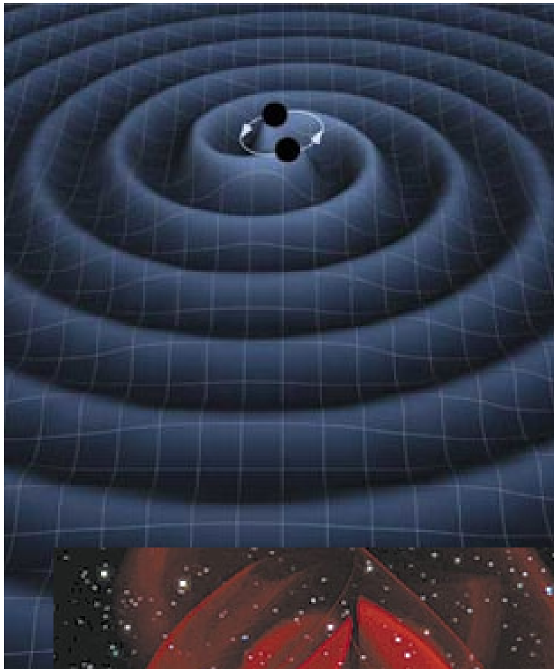
General Relativity has enjoyed important successes:

- Perihelium precession of Mercury [weak, static field]
- Deflection of star light by the Sun [weak, static field]
- Shapiro time delay [weak, static field]
- Gravity Probe B
  - Geodetic effect [weak, static field]
  - Frame dragging [weak, stationary field]
- Expansion of the Universe [dynamical but weak-field]
- Binary neutron stars [dynamical but weak-field]

No tests of genuinely strong-field dynamics of the gravitational field

Direct detection of gravitational waves will provide us with first empirical access

Coalescing binary neutron stars and black holes



- The Hulse-Taylor pulsar does not suffice to study the dynamics of spacetime at strong gravitational fields:

  $GM/(c^2 R) \sim 10^{-6}$, $v/c \sim 10^{-3}$

- Compare with binary neutron stars and/or black holes on the verge of merger:

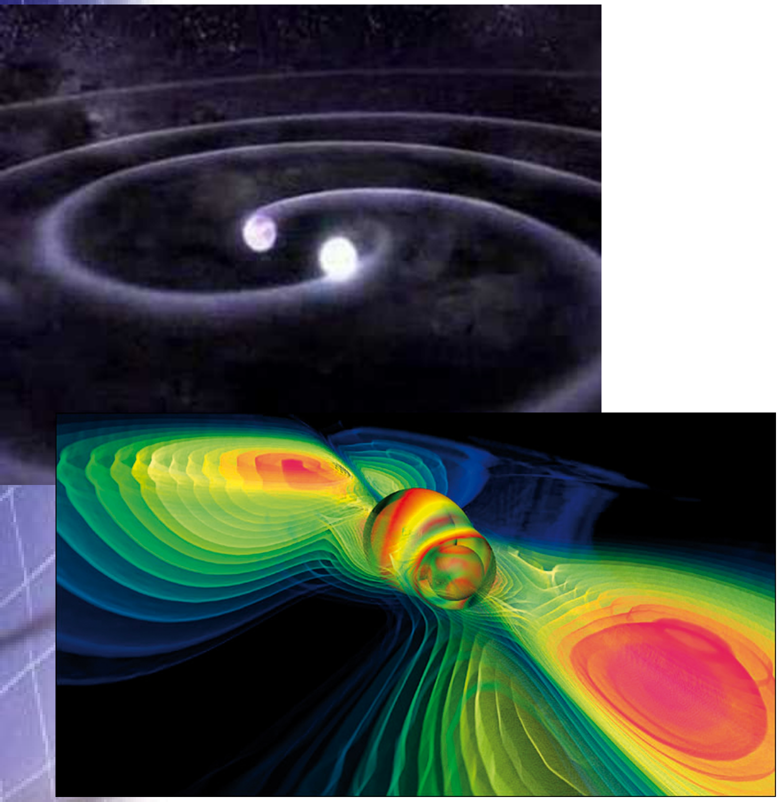
  $GM/(c^2 R) \sim 0.2$, $v/c \sim 0.4$

Genuinely strong-field dynamics of spacetime can only be studied with direct gravitational wave detection

Was Einstein right?

- Hulse-Taylor binary pulsar:
  can study dynamics up to leading order in *(v/c)*
- Use inspiraling and merging binary neutron stars or black holes to study extreme strong-field dynamics of spacetime

Was Einstein right?



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- Use inspiraling and merging binary neutron stars or black holes to study extreme strong-field dynamics of spacetime
- Most interesting effects occur starting at $(v/c)^3$:
    - Gravitational self-interaction: waves bouncing off the background spacetime

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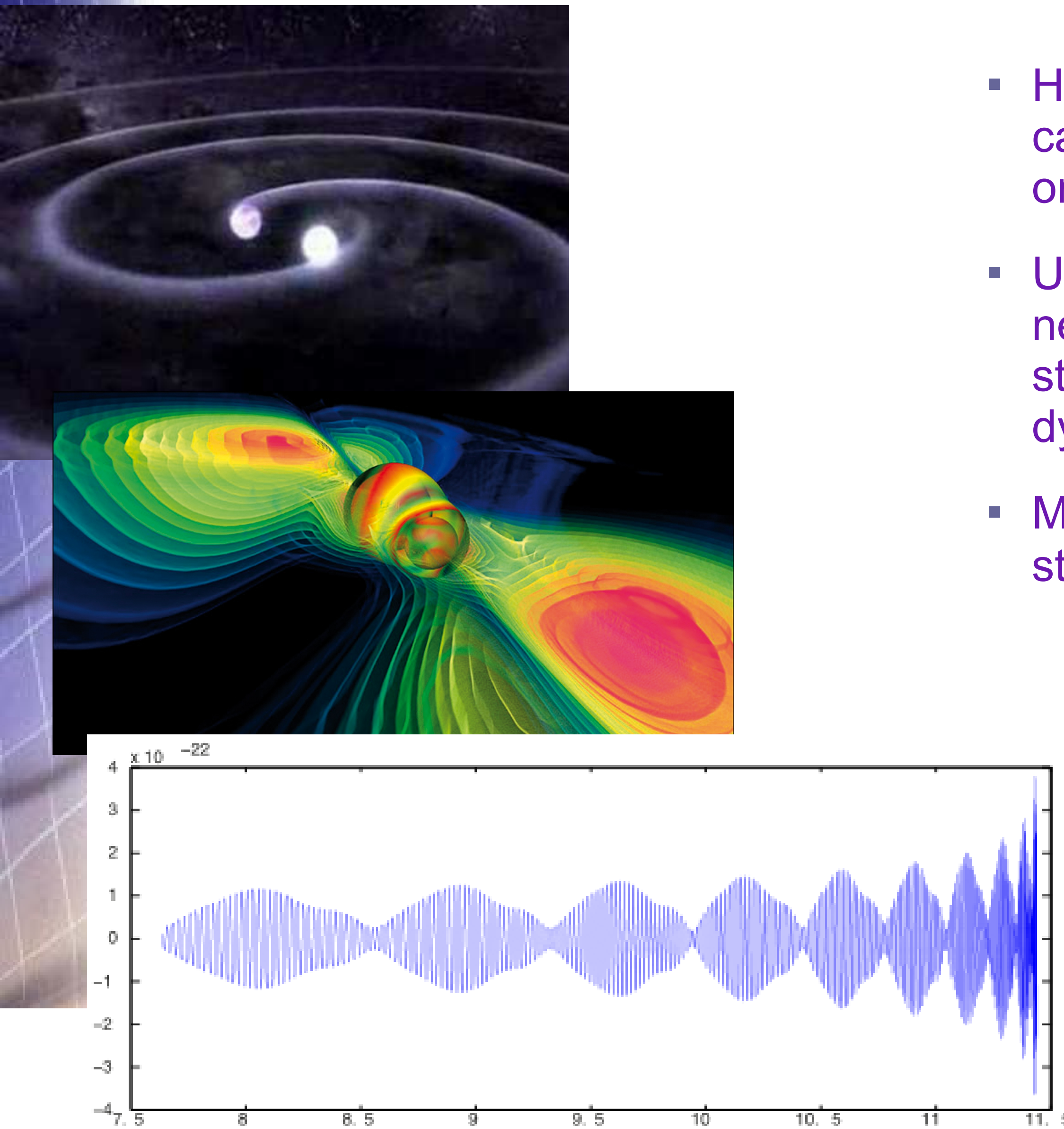

- Hulse-Taylor binary pulsar: can study dynamics up to leading order in *(v/c)*
- Use inspiraling and merging binary neutron stars or black holes to study extreme strong-field dynamics of spacetime
- Most interesting effects occur starting at $(v/c)^3$:
  - Gravitational self-interaction: waves bouncing off the background spacetime
  - Spin-orbit and spin-spin effects cause precession and even tumbling motion of orbital plane
  - ...

Testing GR with binary inspiral

- Schematically: inspiral phase can be expressed in terms of *characteristic speed v(t)*

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^n$$

where the ψ_n and $\psi_n^{(l)}$ depend on masses and spins of component objects

- Physical content:
  - ψ_3 incorporates lowest-order "tail effects" (non-linearity of GR), and spin-orbit interaction
  - ψ_4 has lowest-order spin-spin effects
  - $\psi_5^{(l)}$ is lowest-order non-zero "logarithmic" coefficient

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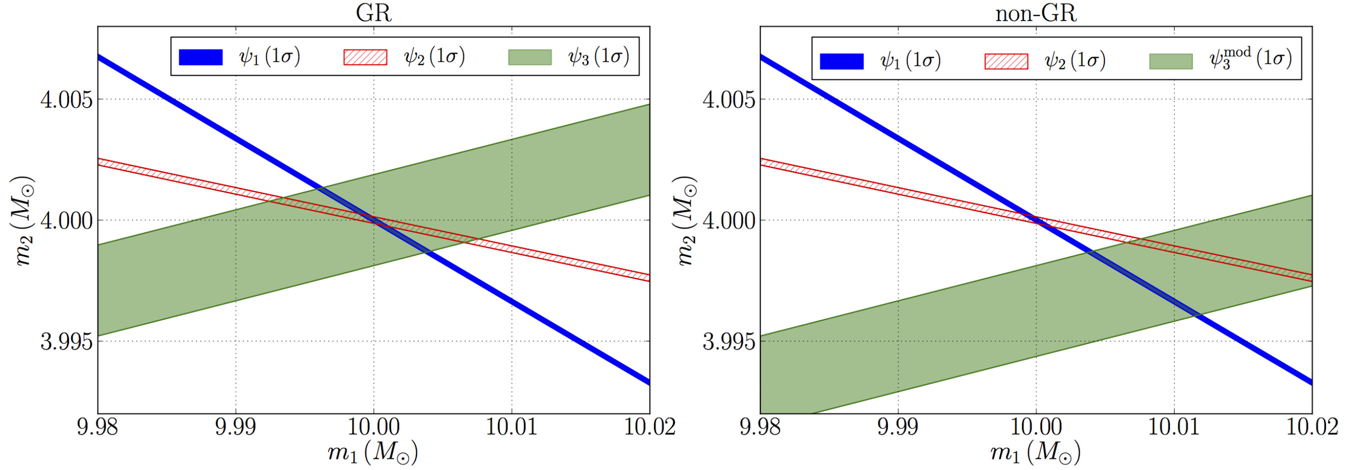
- Modifications to GR:
  - "Massive graviton" would modify ψ_2
  - Scalar-tensor theories add $\psi_{ST} v^{-2}$ inside the sum
  - Quadratic curvature terms in action add $\psi_{QC} v^4$
  - "Dynamical Chern-Simons theory" adds $\psi_{CS} v^9$
  - Variable G adds $\psi_{G(t)} v^{-8}$

Cornish et al., PRD **84**, 062003 (2011)

Testing GR with binary inspiral

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^n$$

- If no spins then coefficients only depend on the two component masses, hence only two independent coefficients
- Measure any two coefficients and see if third is consistent assuming GR correct:



Arun et al., PRD **74**, 024006 (2006)

In practice: parameter estimation not convenient, instead use Bayesian model selection

TIGER: Test Infrastructure for GEneral Relativity

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  where the ψ_n and $\psi_n^{(l)}$ depend on masses and spins of component objects

- After one or more detections, compare two hypotheses:
  - $\mathcal{H}_{\rm GR}$ the signal waveform is as predicted by GR
  - $\mathcal{H}_{\rm modGR}$ the signal waveform deviates from the GR prediction
- In practice not possible to let $\mathcal{H}_{\rm modGR}$ be the negation of $\mathcal{H}_{\rm GR}$
- Choice we make:

$\mathcal{H}_{\rm modGR}$ is the hypothesis that one or more of the ψ_n , $\psi_n^{(l)}$ are not as predicted by GR, without specifying which ones

TIGER: Test Infrastructure for GEneral Relativity

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^n$$

$\mathcal{H}_{\mathrm{modGR}}$ is the hypothesis that one or more of the ψ_n , $\psi_n^{(l)}$ are not as predicted by GR, without specifying which

- No waveform model associated to this!
- Split $\mathcal{H}_{\mathrm{modGR}}$ into mutually exclusive sub-hypotheses:

$$\mathcal{H}_{\mathrm{modGR}} = H_1 \vee H_2 \vee H_3 \vee H_{12} \vee H_{13} \vee H_{23} \vee H_{123}$$

  - H_i coefficient ψ_i is not as predicted by GR, but all others are
  - H_{ij} coefficients ψ_i and ψ_j are both not as in GR, but all others are
  - H_{ijk} coefficients ψ_i , ψ_j , ψ_k are not as in GR, but all others are
- Each of the sub-hypothesis does have associated waveform model
- Odds ratio:

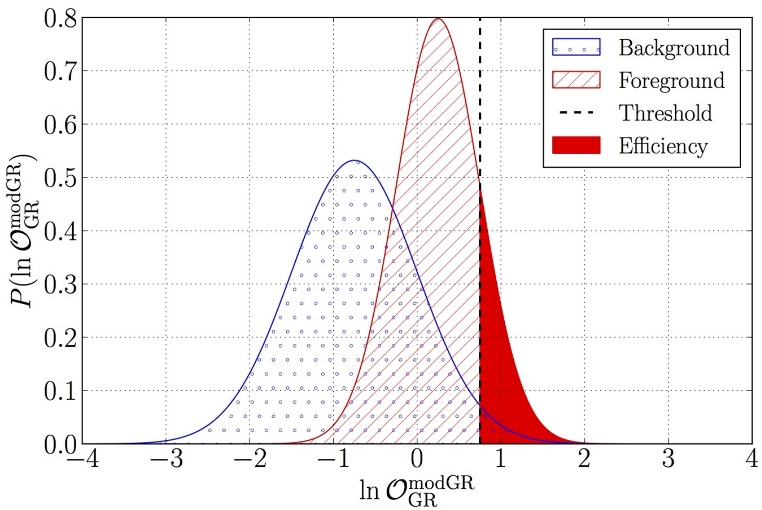
$$\boxed{O_{\mathrm{GR}}^{\mathrm{modGR}} = \frac{P(\mathcal{H}_{\mathrm{modGR}}|d, \mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|d, \mathrm{I})}}$$

d can be data from multiple detections

Dealing with noise

$$\boxed{O_{\mathrm{GR}}^{\mathrm{modGR}} = \frac{P(\mathcal{H}_{\mathrm{modGR}}|d,\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|d,\mathrm{I})}}$$

- Theoretically, should have *O* < 1 or *logO* < 0 if GR is correct
- *Noise can mimick GR violations!*
- Introduce a background distribution of log odds ratio for catalogs of GR signals
- Pick a maximum tolerable false alarm probability and set *logO* threshold
- For a given GR violation, efficiency is fraction of catalogs with *logO* above threshold

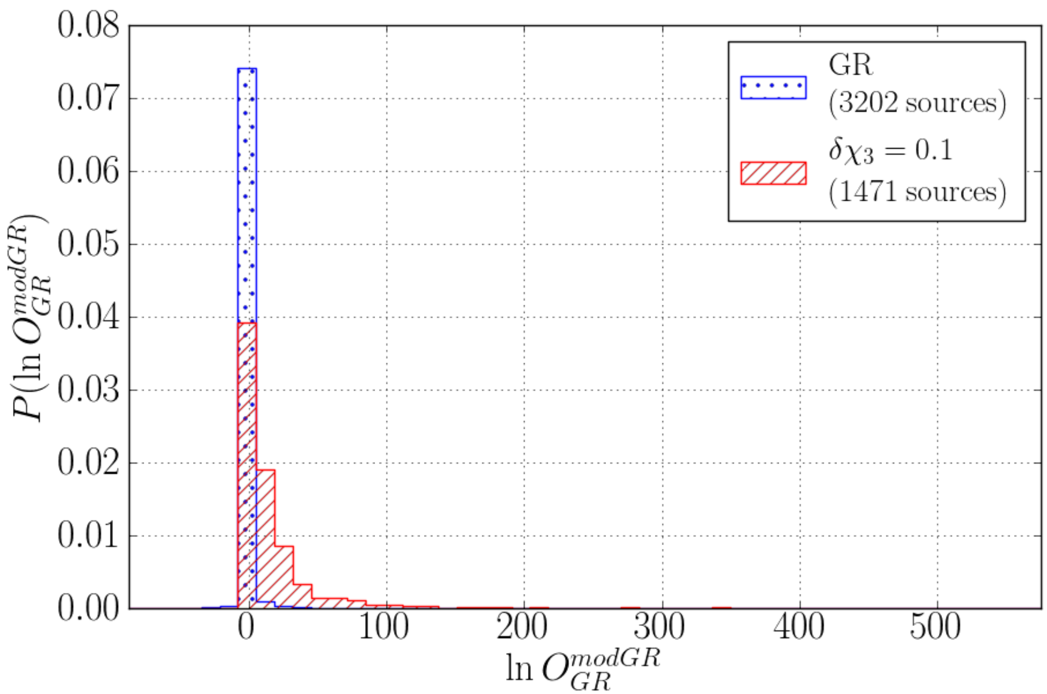


Li et al., PRD **85**, 082003 (2012)
Li et al., JPCS **363**, 012028 (2012)

Example 1: Constant 10% shift at $(v/c)^3$

- Binary neutron stars, masses uniform in [1, 2] M_{sun}
- Uniform in co-moving volume, uniform in orientation and sky position
- Distances [100, 400] Mpc
- Simulated stationary, Gaussian noise following advanced detector projections

Histogram of log odds ratios for individual sources



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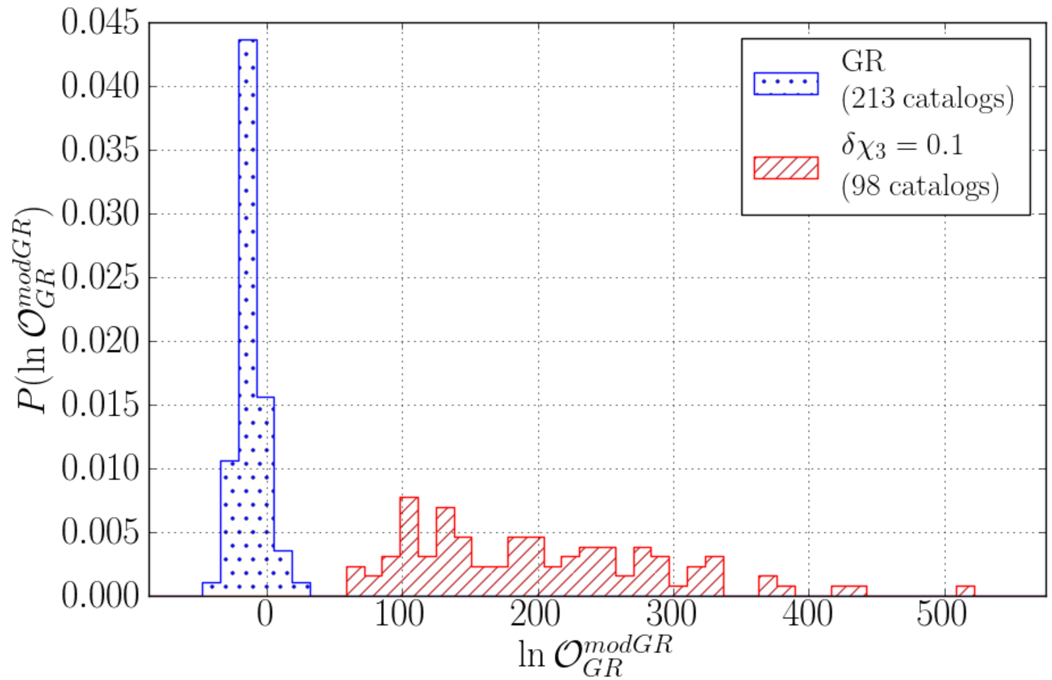
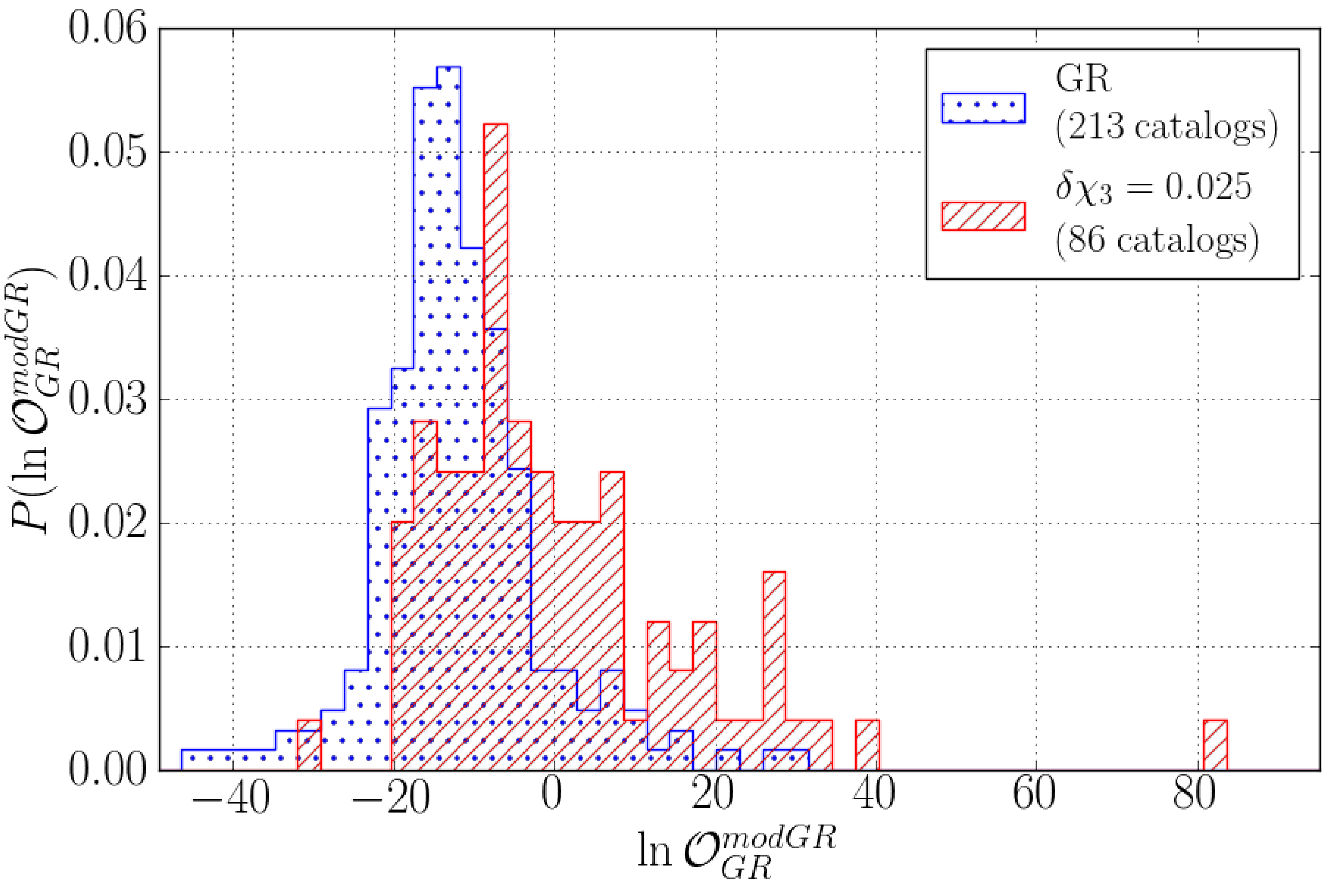
Histogram of log odds ratios for ***catalogs of 15 sources each***



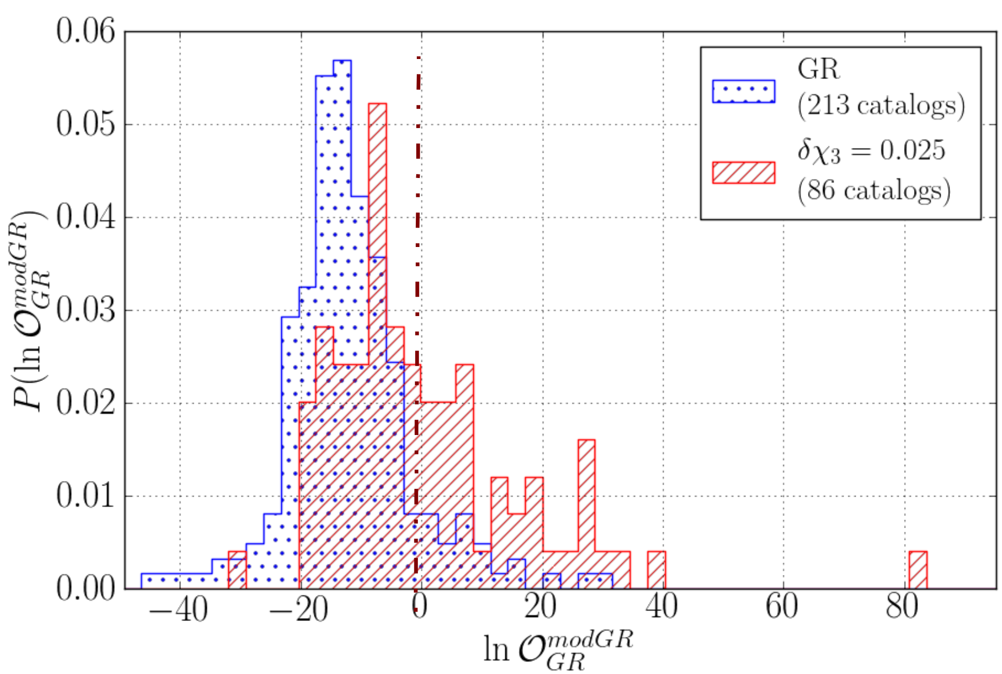
Example 2: Constant 2.5% shift at (v/c)^3

Histogram of log odds ratios for ***catalogs of 15 sources each***



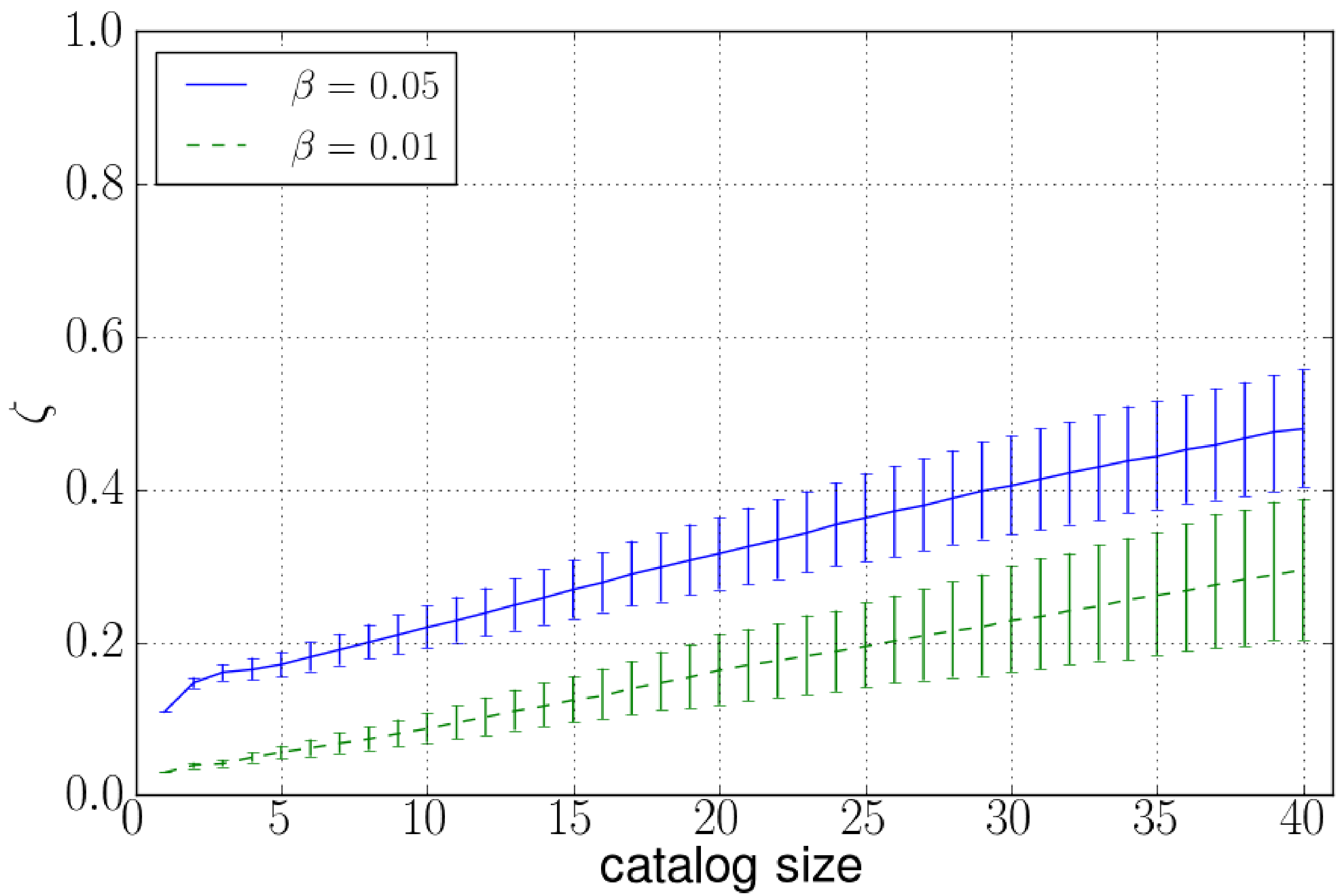
Example 2: Constant 2.5% shift at $(v/c)^3$

- Recall: pick a maximum tolerable false alarm probability β, and using the GR "background" distribution, set a threshold on *logO*
- Given the maximum false alarm probability, the efficiency ζ is the fraction of "foreground" above threshold



Example 2: Constant 2.5% shift at $(v/c)^3$

- Efficiency as a function of number of sources per catalog:



How general is the test?

- Inspiral phase:

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^n$$

- TIGER only looks for deviations in coefficients ψ_n , $\psi_n^{(l)}$:

$\mathcal{H}_{\mathrm{modGR}}$ is the hypothesis that one or more of the coefficients are not as predicted by GR

- In practice, we only look at lowest-order coefficients:

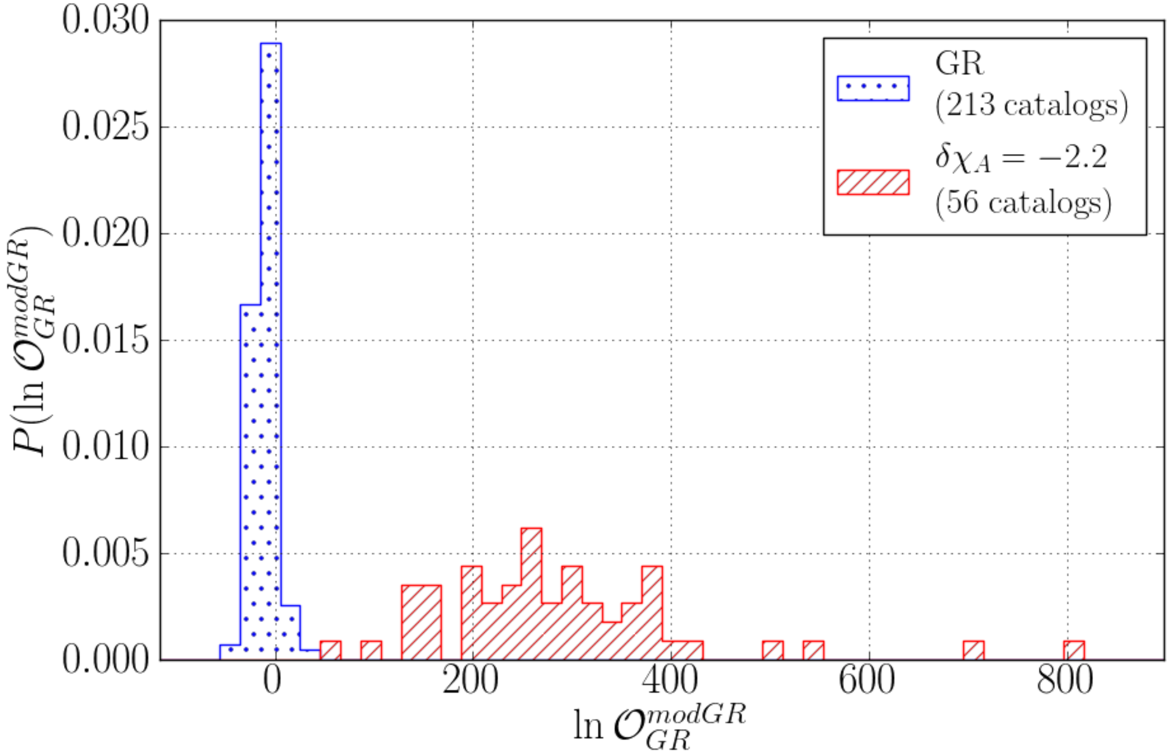
$$\mathcal{H}_{\mathrm{modGR}} = H_1 \vee H_2 \vee H_3 \vee H_{12} \vee H_{13} \vee H_{23} \vee H_{123}$$

What if GR is violated in a more complicated way?

Example 3: A deviation at $(v/c)^{2.5}$

- $\mathcal{H}_{\mathrm{modGR}} = H_1 \vee H_2 \vee H_3 \vee H_{12} \vee H_{13} \vee H_{23} \vee H_{123}$

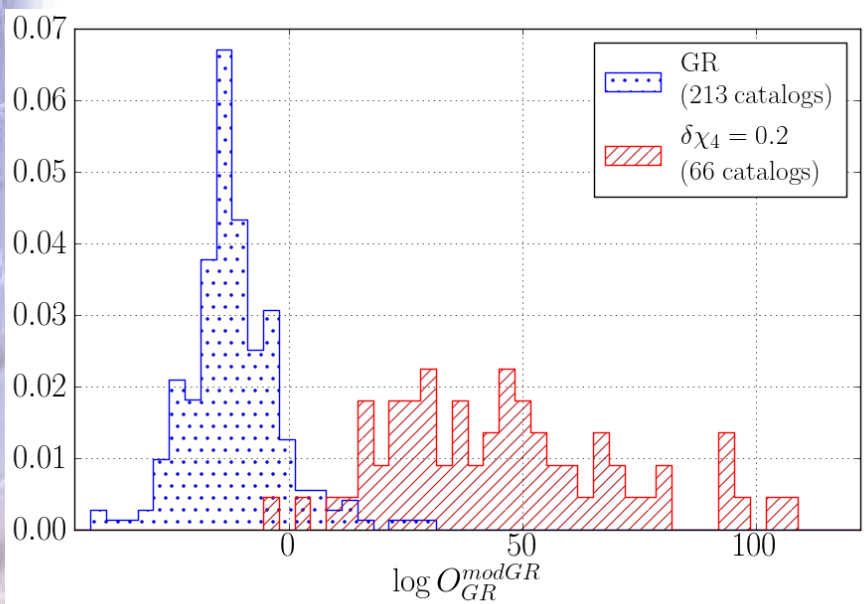
Histogram of log odds ratios for ***catalogs of 15 sources each***



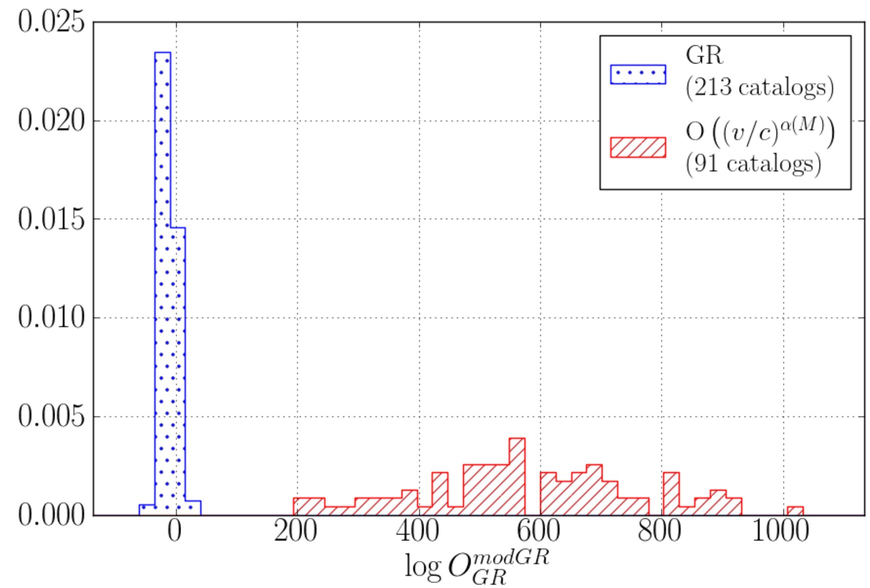
One or more of the sub-hypotheses H_1, ..., H_{123} will accommodate the signal better than the GR hypothesis!

Further examples

$(v/c)^4$



$(v/c)^{-1+M/Msun}$



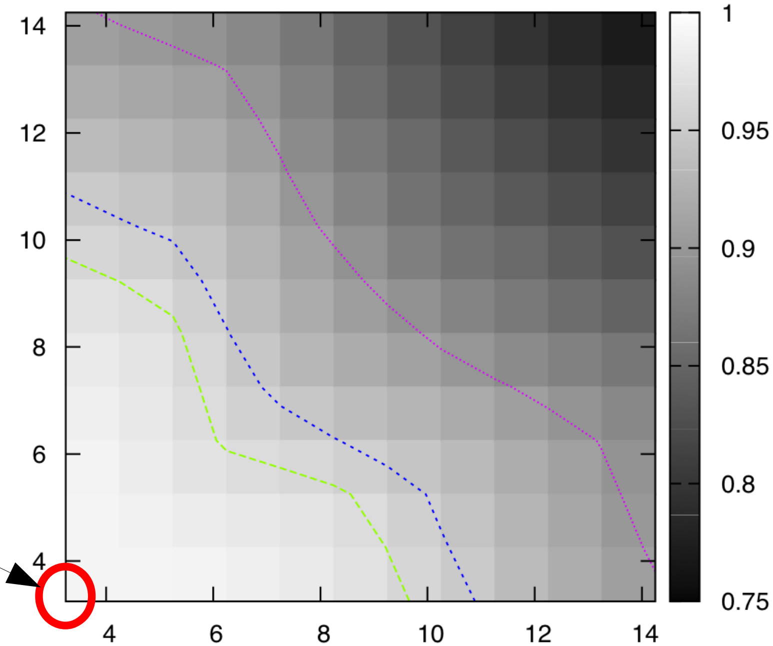
- Sensitive to very generic deviations

BNS versus NSBH, BBH

- Due to computational restrictions (background calculation!) we are currently forced to use TaylorF2-based waveforms
- Overlap of TaylorF2 with non-spinning EOB inspiral-merger-ringdown waveforms optimized using numerical simulations:

Buonanno et al.,
PRD **80**, 084043 (2009)

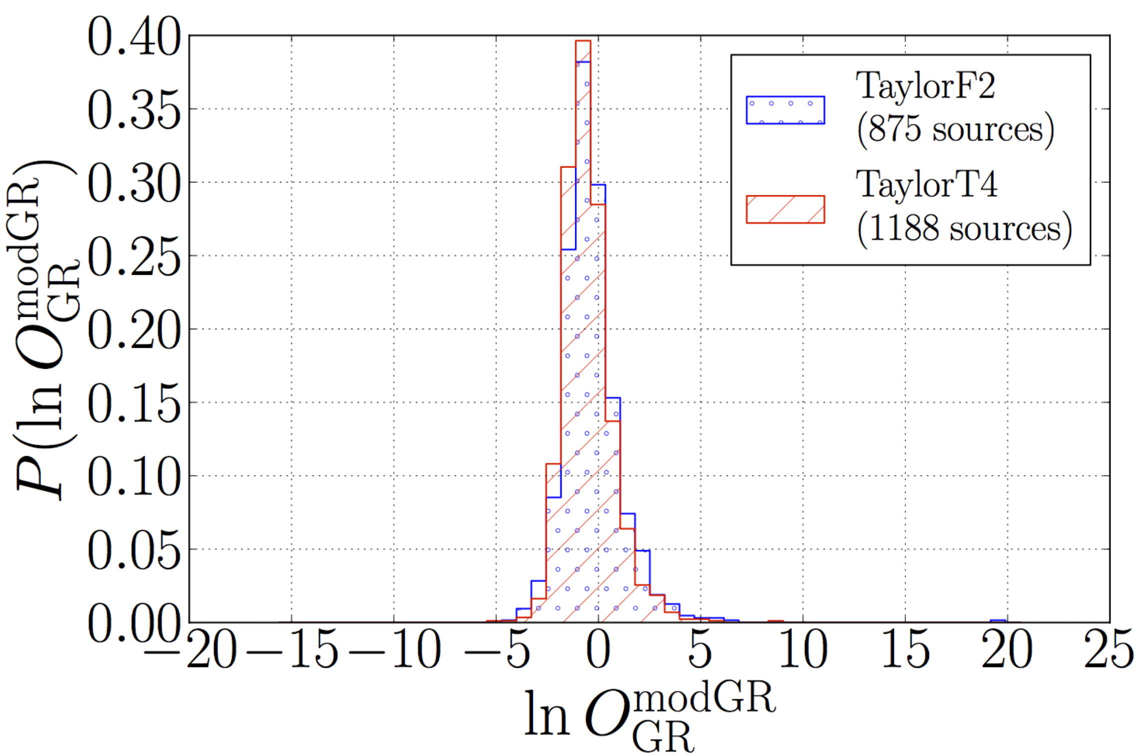
Most massive binary neutron stars



▶ For now, focus on BNS

Differences between waveform approximants

- How does background change if injected GR signals are TaylorT4 rather than TaylorF2?
- Odds ratio distribution for single sources:



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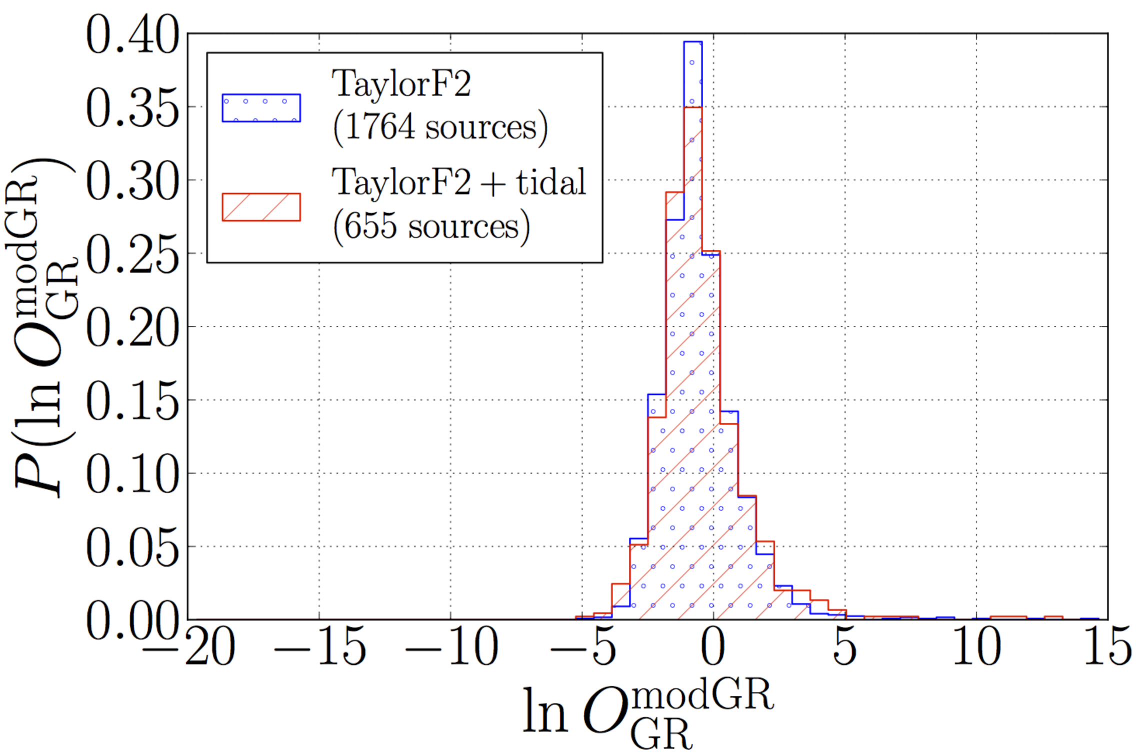
Finite size effects

- NS tidal deformability (depending on unknown equation of state) affects orbital motion and therefore gravitational waveform
- Only important at frequencies > 400 Hz
- Cut off recovery waveforms at that frequency
  - Loss in SNR < 1%
- Choose one of the "hardest" equations of state, check that this solves problem



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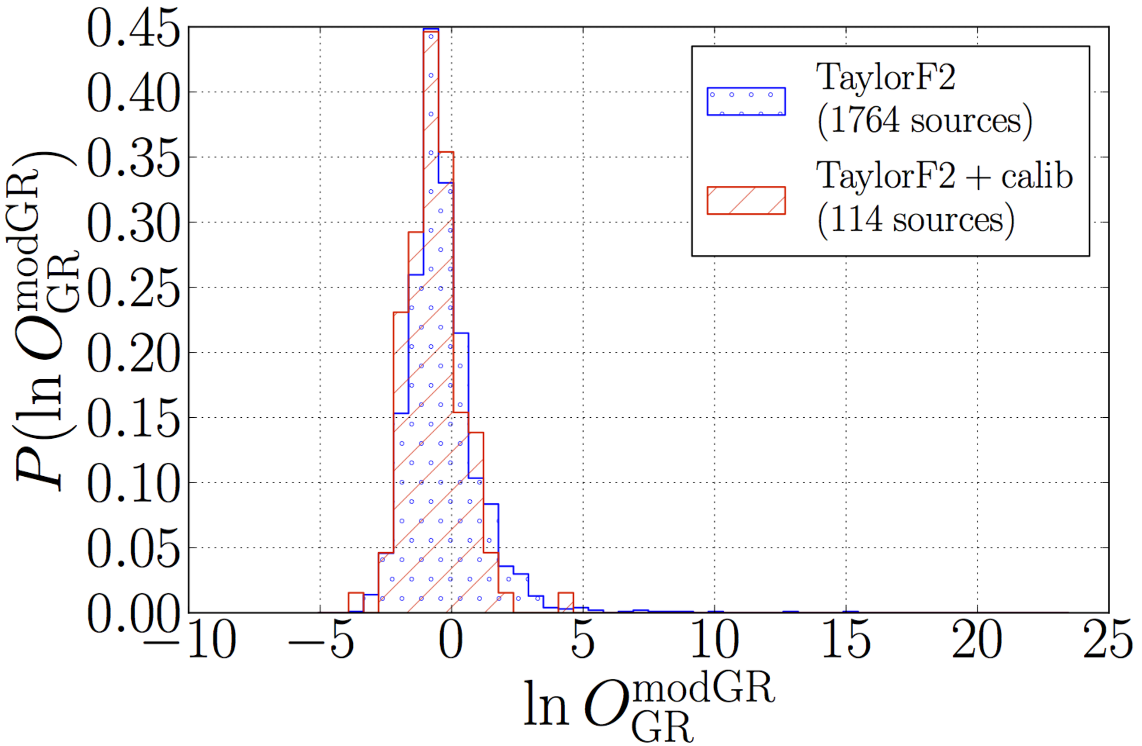
Instrumental calibration errors

- Frequency dependent amplitude and phase calibration errors

  Vitale et al., PRD **85**, 064034 (2012)

- Compare GR background with/without calibration errors



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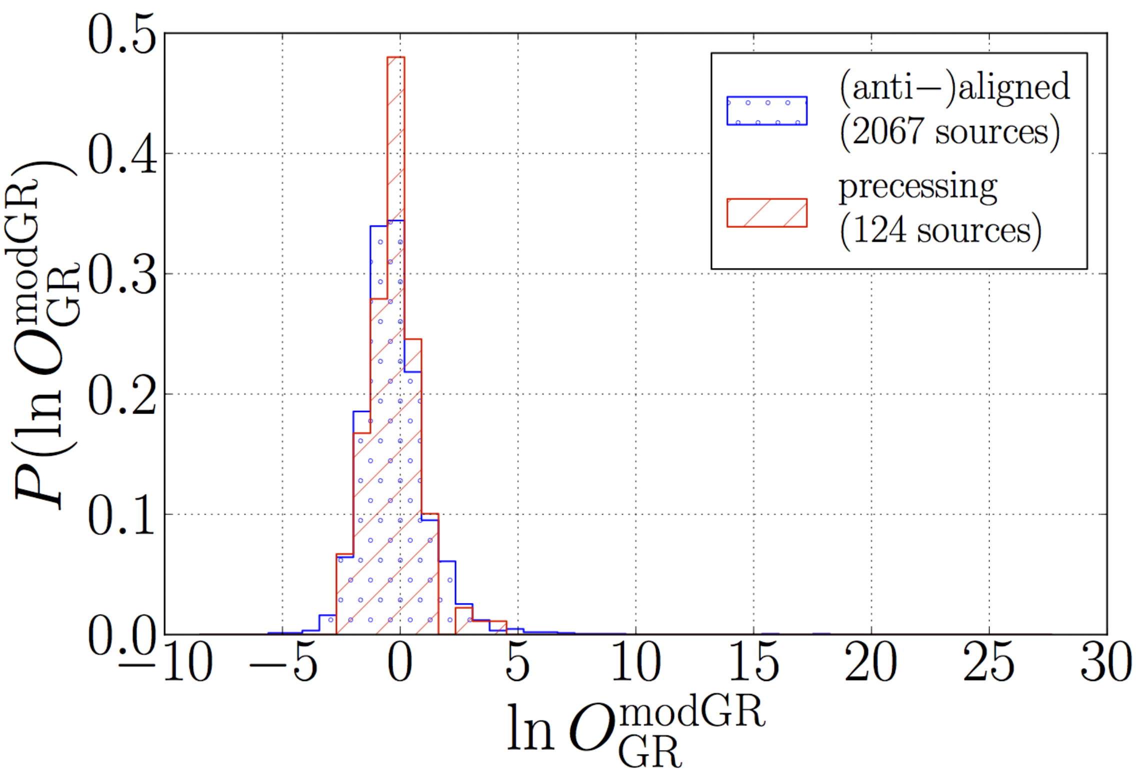
Precessing neutron star spins

- How does background change with precessing neutron star spins versus just (anti-)aligned?
- Compare background with SpinTaylorT4 versus TaylorF2 injections (magnitudes Gaussian with $\sigma = 0.05$), in both cases recovering with TaylorF2, (anti-)aligned spin



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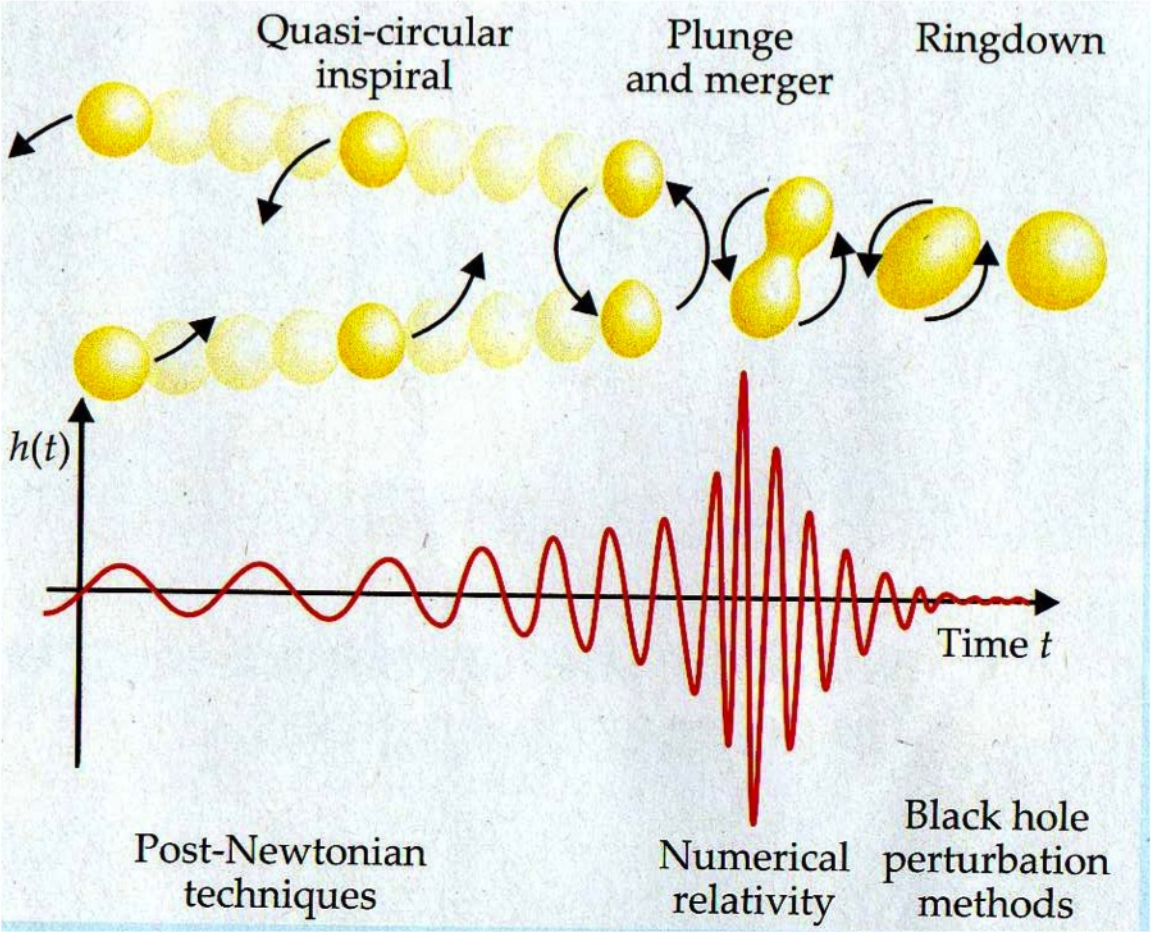
Testing GR with binary black holes?

- Reliable waveforms not yet available, or hard to use:
  - "Effective One-Body" approach and inspiral-merger-ringdown (IMR) extensions tuned to numerical relativity waveforms: Waveforms take a long time to generate
  - "IMRPhenom waveforms" are frequency domain, phenomenological IMR waveforms with high faithfulness to numerical waveforms: No precessing spins
  - "PhenSpin waveforms": IMR waveforms with precessing spins, but tuned against only a small number of numerical waveforms
- Could in principle plug (deformations of) any of these into TIGER for use as recovery waveforms
  - Unknown effects could be relegated to GR background
  - But: background likely to be very wide if recovery with waveforms that our codes can handle

Measuring the equation of state of neutron stars

Tidal effects during coalescence

- Consider BNS or NSBH



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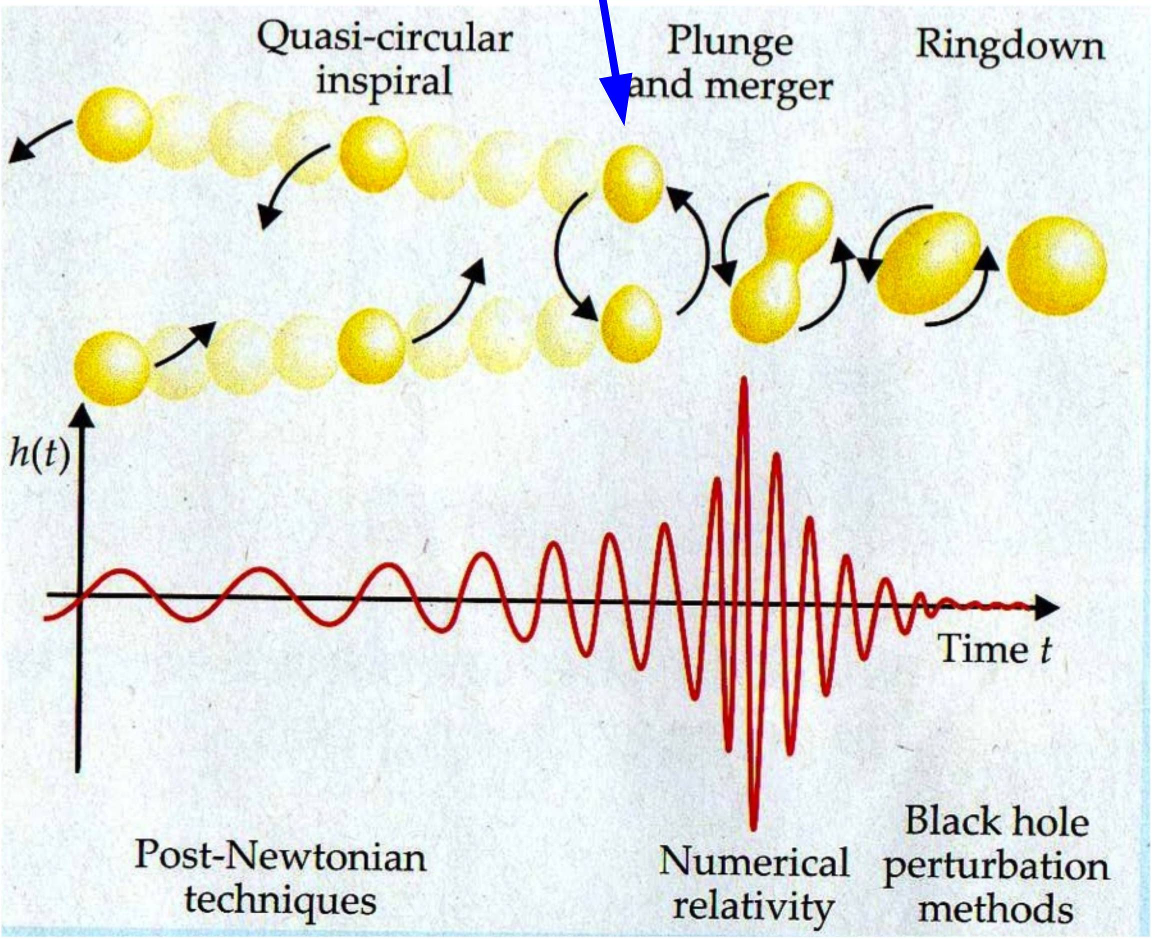
Tidal deformation



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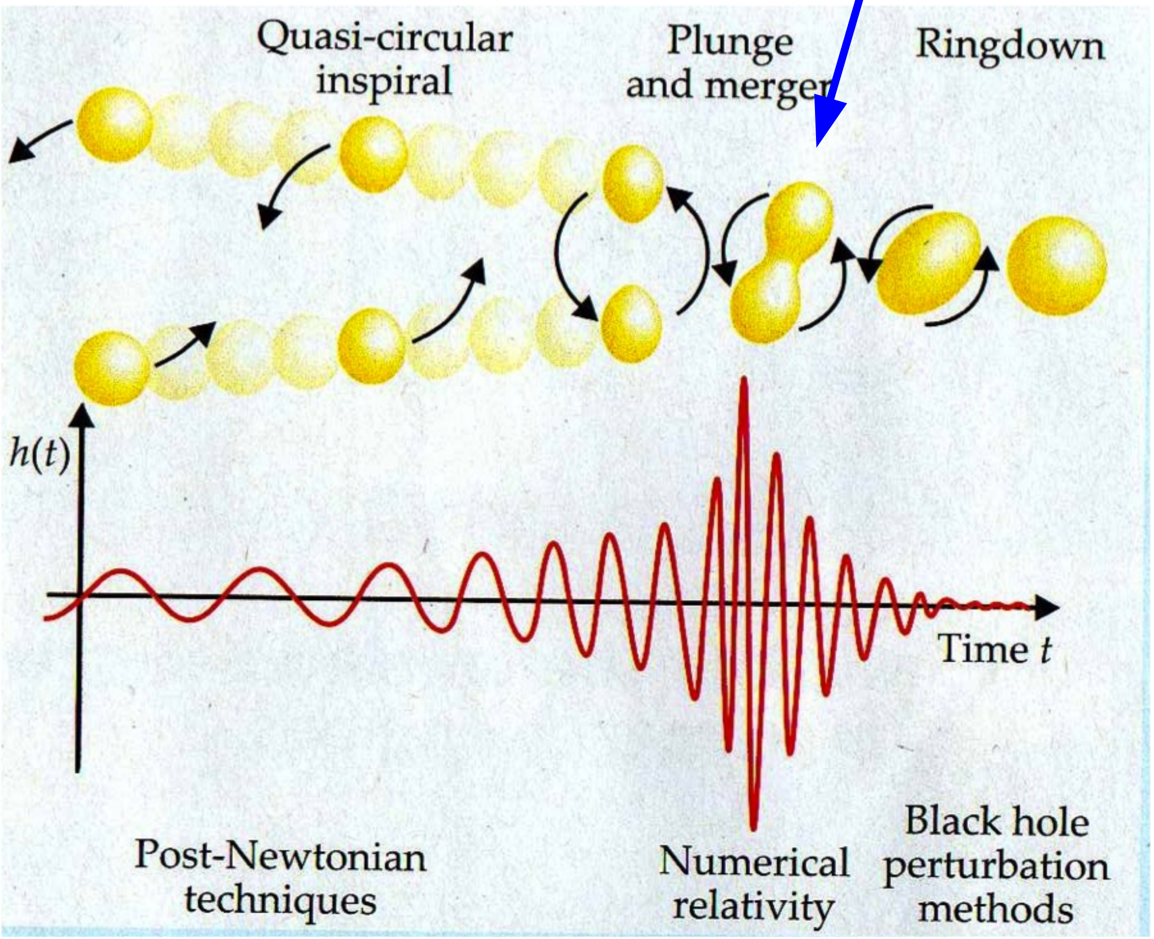
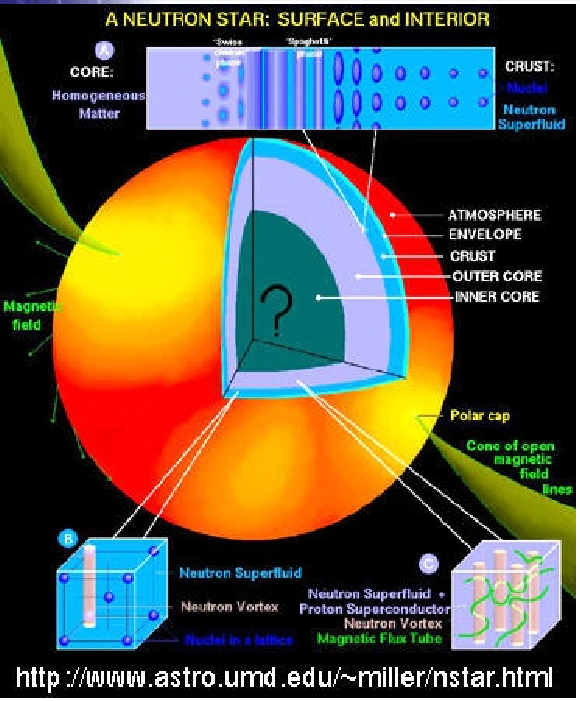
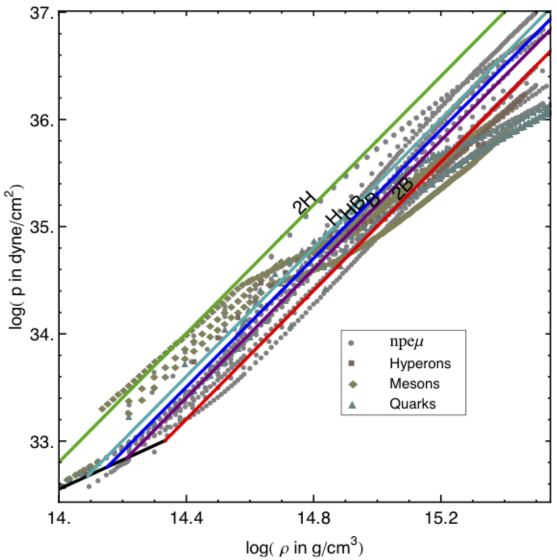
Tidal disruption



The equation of state of neutron stars





- Tidal deformability determined by equation of state (EOS)
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The equation of state of neutron stars

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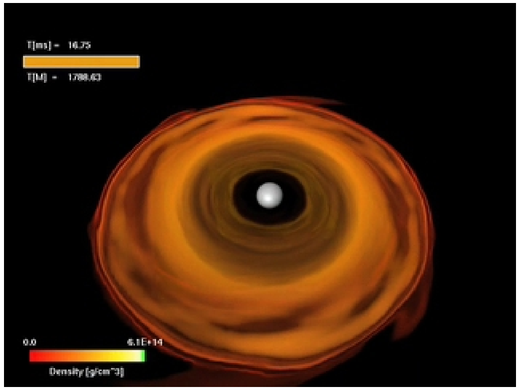
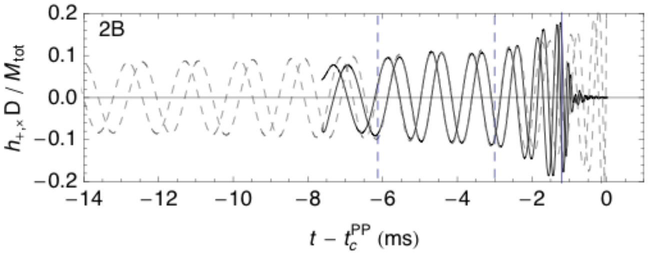
- Extremes:
    - "Soft" EOS: prompt collapse to a black hole





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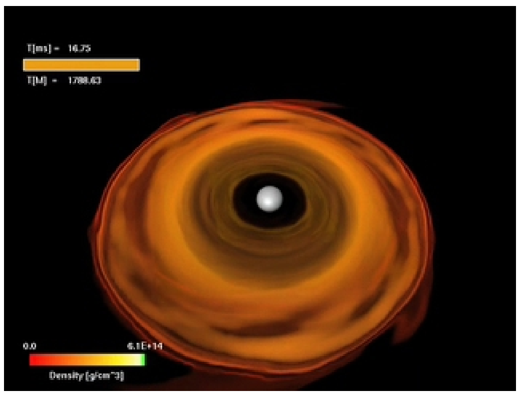
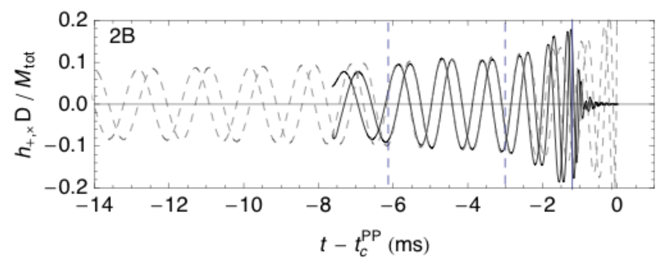
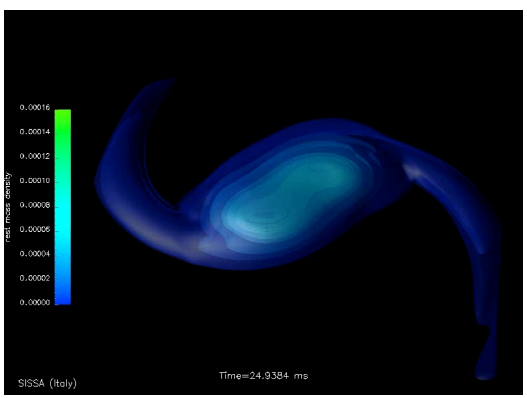
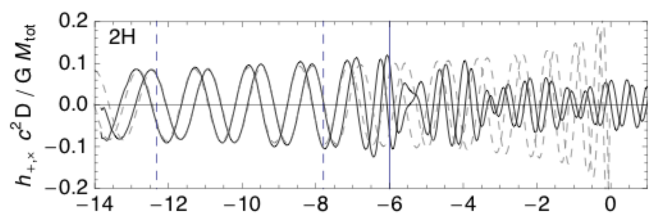


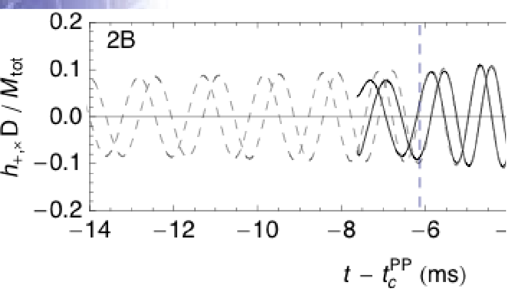


  - "Hard" EOS: unstable bar mode, eventually BH





Inspiral



- For now, focus on BNS
- Part of the signal in detector band is mostly inspiral
- Inspiral phase:

$$\Phi(t) = \underline{\Phi_{\mathrm{PP}}(t)} + \underline{\Phi_{\mathrm{tidal}}(t)}$$

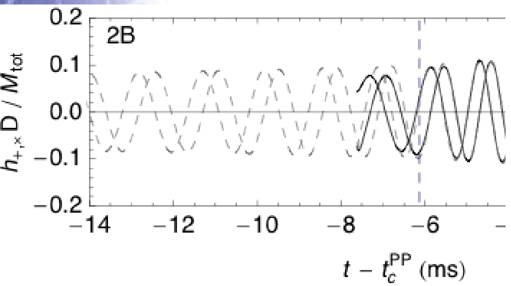
Point particle contribution   Tidal contribution

- Tidal contributions at $(v/c)^{10}$, $(v/c)^{12}$ beyond leading order
- But: proportional to $(R_{ns}/m_{ns})^5 \sim 10^5 \rightarrow$ huge prefactor!
- Tidal effects enter waveform through tidal deformability

$$Q_{ij} = -\lambda(\mathrm{EOS}; m)\, \underline{\mathcal{E}_{ij}}$$

Tidal field of companion star

Inspiral



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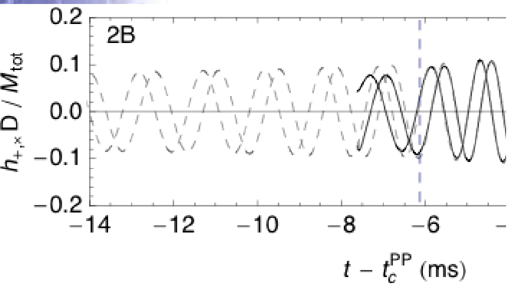
$$\underbrace{Q_{ij}}_{\text{Induced quadrupole moment}} = -\lambda(\mathrm{EOS}; m)\, \mathcal{E}_{ij}$$

Inspiral



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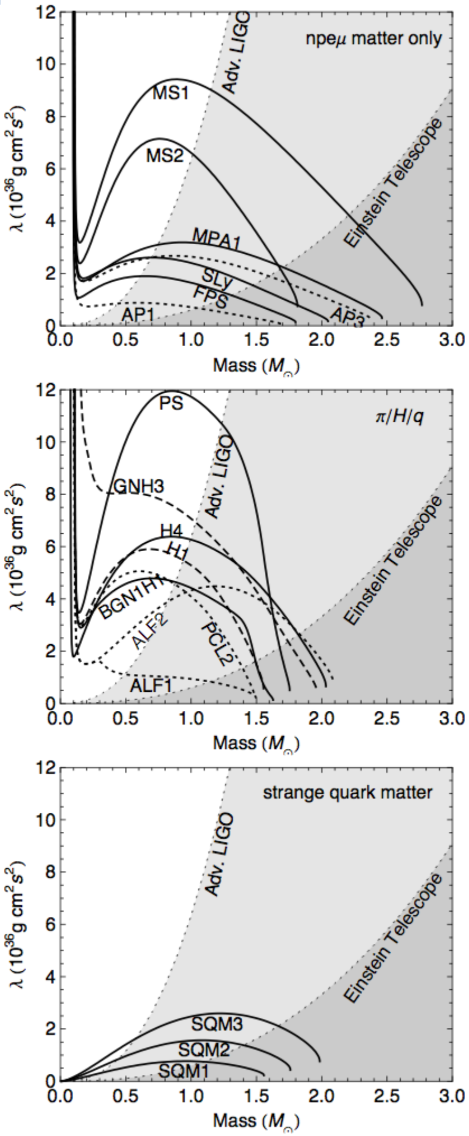
  - Depends on mass
  - Depends on EOS

Inspiral



- Tidal effects are high order in (v/c)
  - $\longrightarrow$ Need to combine information from multiple sources
- Two ways of inferring λ(m) and hence EOS:
  - Series expansion of λ(m):

$$\lambda(m) = \sum_j \frac{1}{j!} \underline{\lambda_j} \left(\frac{m - m_0}{M_\odot} \right)^j$$

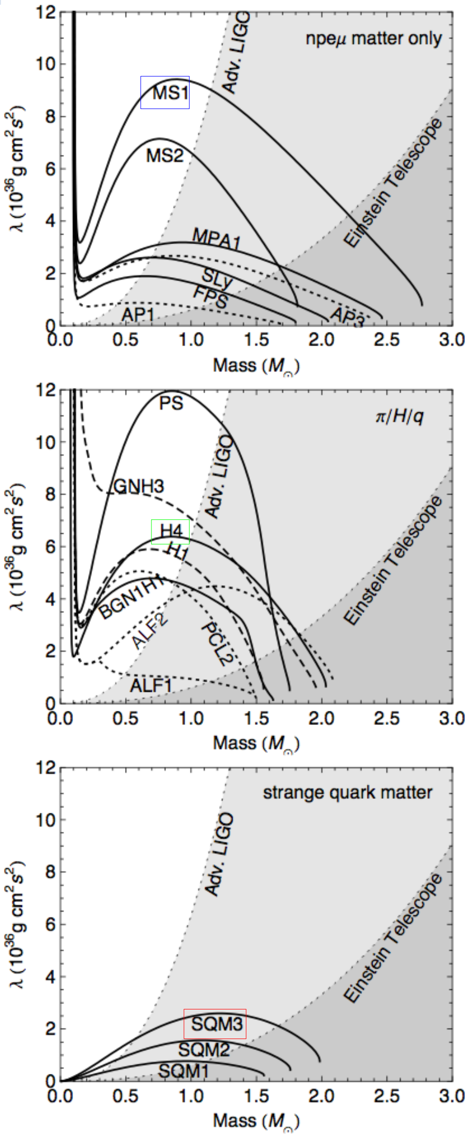
$\rightarrow$ Coefficients λ_j the same for all sources

  - Hypothesis ranking:

$$O_j^i = \frac{P(\mathcal{H}_i | d_1, d_2, \ldots, d_N, I)}{P(\mathcal{H}_j | d_1, d_2, \ldots, d_N, I)}$$

... for any pair of EOS $\mathcal{H}_i$, $\mathcal{H}_j$

Inspiral



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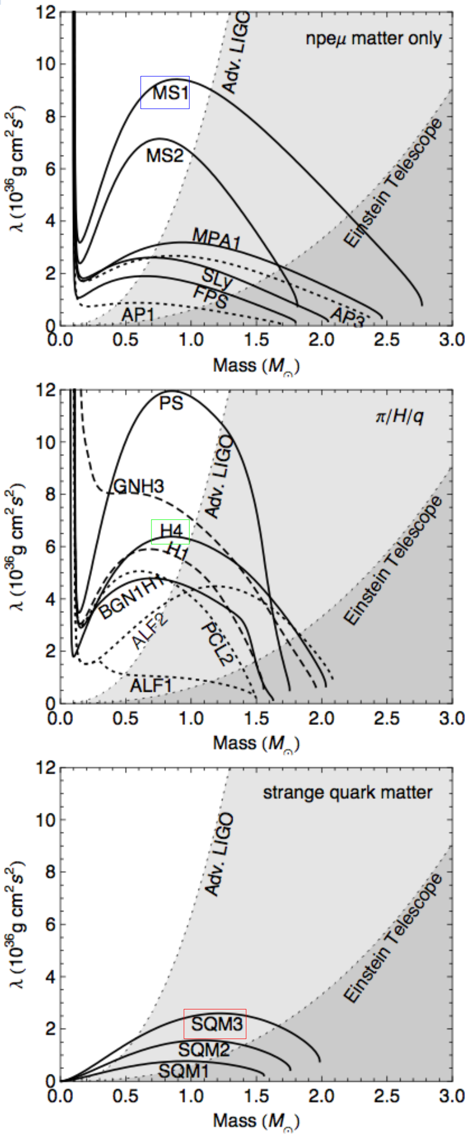
  - Hypothesis ranking:

$$O_j^i = \frac{P(\mathcal{H}_i | d_1, d_2, \ldots, d_N, I)}{P(\mathcal{H}_j | d_1, d_2, \ldots, d_N, I)}$$

... for any pair of EOS $\mathcal{H}_i$, $\mathcal{H}_j$

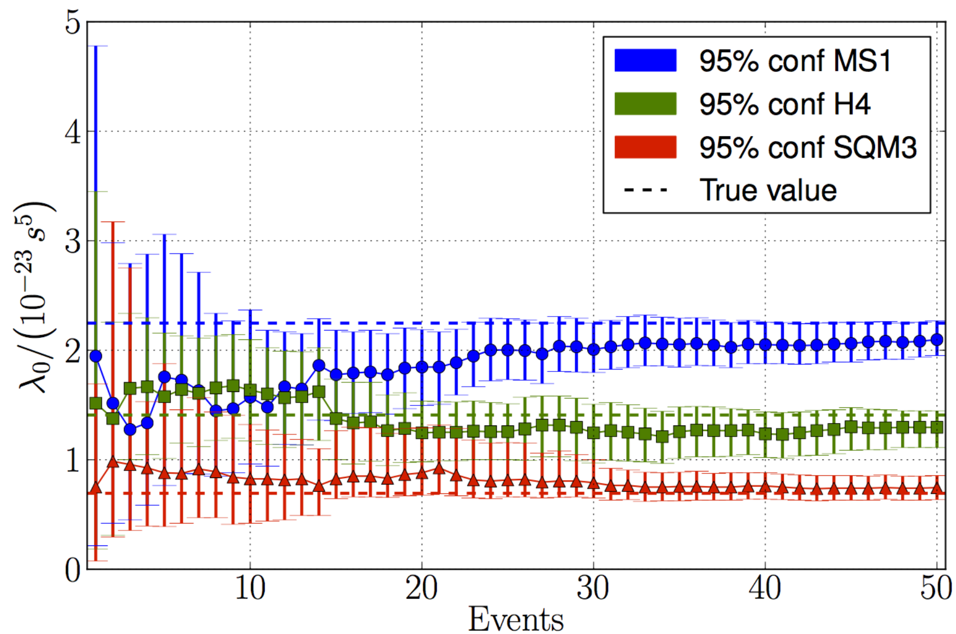
For proof of principle, pick a hard, moderate, and soft EOS

Series expansion of λ(m)

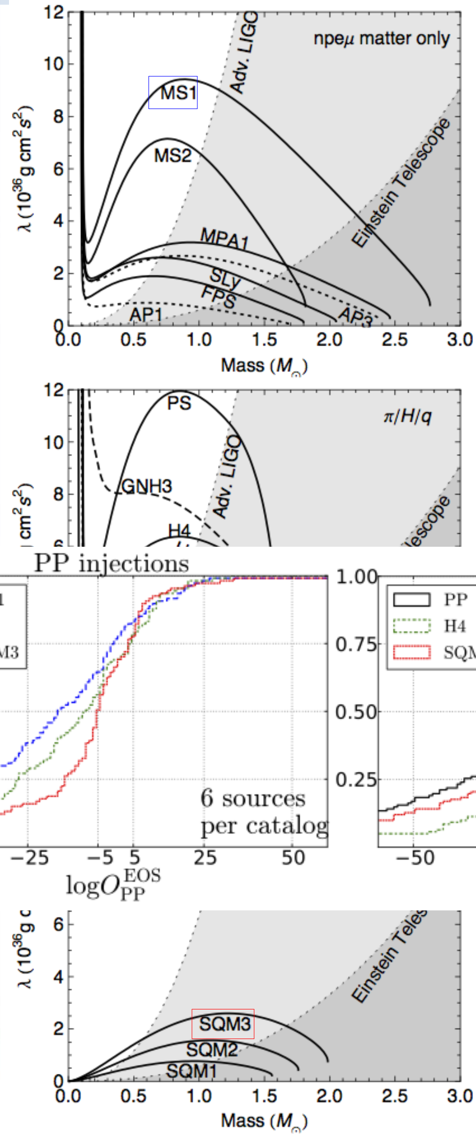


$$\lambda(m) = \sum_j \frac{1}{j!} \lambda_j \left(\frac{m - m_0}{M_\odot} \right)^j$$

In practice, only λ_0 can be measured well

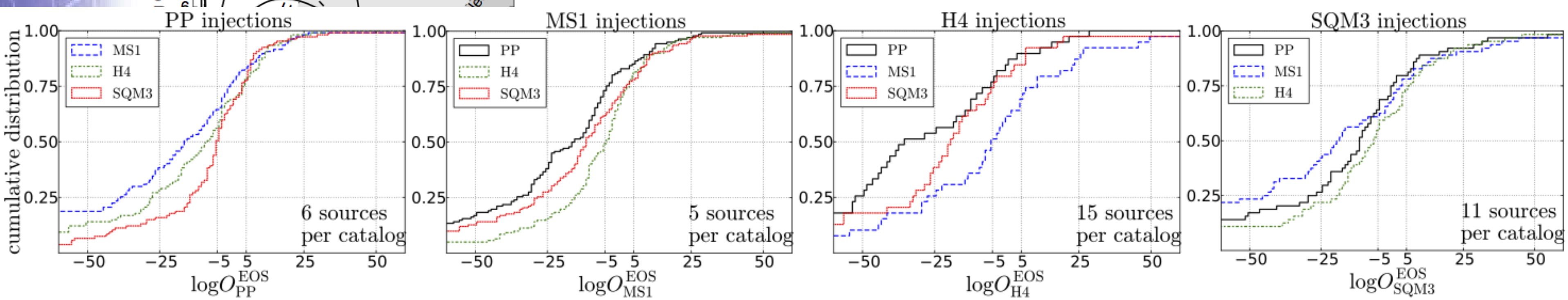


Hypothesis ranking



$$O^i_j = \frac{P(\mathcal{H}_i|d_1, d_2, \ldots, d_N, I)}{P(\mathcal{H}_j|d_1, d_2, \ldots, d_N, I)}$$

- Use *log* odds ratio (smaller numbers)
- $\ln O^k_j < 0$ means j preferred over k
- $\ln O^k_j < -5$ means decisive difference (1:150)



Summary

- Direct gravitational wave detection will allow us to test the genuinely strong-field dynamics of gravity
  - Binary neutron star coalescence: data analysis pipeline in advanced stage
    - Model-independent test
    - Well-suited for low-SNR sources
    - Robust against unknown instrumental and astrophysical effects
  - Binary black holes: sufficiently accurate and “fast” waveform models not yet available
- Behavior of matter in strong, dynamical gravitational fields
  - Neutron star equation of state currently uncertain by factor ~10
  - With O(10) BNS detections: can distinguish between soft, moderate, hard EOS (model selection, parameter estimation)
  - Effect of neutron star spins? Merger? Can we do better with NSBH?
    - Input from numerical relativity needed