Radiative Processes: Basics

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Overview

- Some definitions
- Bremsstrahlung
- Blackbody
- Synchrotron radiation
- Compton scattering
- Electron-positron pairs
- Hadronic processes
- Line emission and transition in atoms and molecules
- ....

Basic definitions

Luminosity [dE/dt dν] $L_{[\nu_1-\nu_2]} = \int_{\nu_1}^{\nu_2} L(\nu)\, d\nu$

Flux [dE/dA dt dν] $F(\nu) = \dfrac{L(\nu)}{4\pi R^2}$

Intensity [dE/dA dt dΩ dν] $I(\nu)/\nu^3 = I'(\nu')/\nu'^3$

Emissivity $j(\nu)$= dE/dV dt dΩ dν

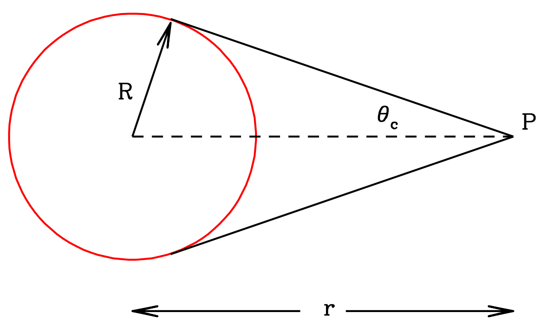
Energy density $u(\nu)$ = dE/c dt dA dν

Radiative transport: basics

Absorption coefficient $\alpha(\nu)$

$$dI_\nu = -\alpha_\nu I_\nu ds$$

$$\alpha_\nu = n\sigma_\nu$$



Intensity emission

$$dI_\nu = j_\nu ds$$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$I_\nu(\tau_\nu) = j_\nu R, \qquad (\tau_\nu \ll 1)$$

$$I_\nu(\tau_\nu) = \frac{j_\nu R}{\tau_\nu}, \qquad (\tau_\nu \gg 1)$$

Thermal and non-thermal plasma (1)

Thermal distribution (=Maxwellian)

$$F(v)dv \; = \; 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} dv$$

$$F(p)dp \; = \; \frac{p^2 e^{-\gamma/\Theta}}{\Theta m^3 c^3 K_2(1/\Theta)} \, dp$$

$p \equiv \gamma\beta mc$

$\Theta \equiv mc^2$

$K_2(1/\Theta)$

Modified Bessel function of the second type

- Velocities have to be isotropic
- Valid for non relat and relativistic
- Requires particle energy exchange
- $e^{-E/kT}$

Thermal and non-thermal plasma (2)

Energy exchange

* Collisions
* Exchange of photons

$$\frac{\#\ \text{of collisions of a single particle}}{\text{time}} \propto \text{density } n$$

$$\frac{\text{total}\ \#\ \text{of collisions}}{\text{time}} \propto n^2$$

Cross section decreases with particle energy
➔ difficult to have a Maxwellian in low density, hot plasma

Thermal and non-thermal plasma (3)

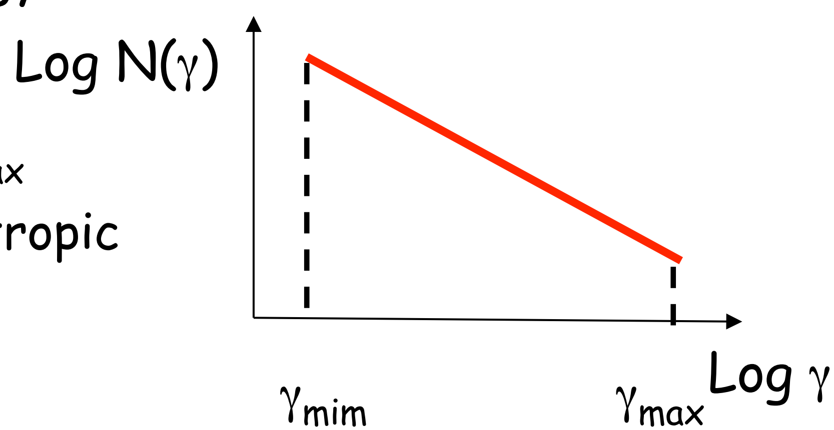
Rarefied, hot plasma ➔ $t_{rel} \gg t_{acc}, t_{cool}, t_{esc}$

Usually non-thermal particle distribution described by power-law ➔ no preferred energy

$$N(\gamma) = K\gamma^{-p}$$

$\gamma_{min} \ll \gamma \ll \gamma_{max}$

Can be anisotropic



Or broken power law p_1, p_2, γ_{break}

Particle accelerator: shock wave, reconnection?...

Total emitted power: the Larmor formula

Electric and magnetic field from a moving charge at retarded time

Non-relativistic case

Power per unit solid angle

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} a^2 \sin^2\theta$$

a= acceleration
q= electric charge
θ= n^a

$$P = \int \frac{dP}{d\Omega} d\Omega = \frac{2\pi}{4\pi c^3} \int_{-1}^{1} \sin^2\theta \, d(\cos\theta) = \frac{2q^2}{3c^3} a^2$$

Bremsstrahlung (1)

Charged particles: assume electron-proton plasma n_e, n_p
Impact parameter $b \sim n_p^{-1/3}$
Velocity of electrons v
Plasma temperature $kT \sim mv^2$

Interaction for $t \sim b/v$, i.e. $\omega \sim 1/t$, with acceleration ~ constant

The Larmor formula gives:

$$P = \frac{2e^2a^2}{3c^3} \approx \frac{e^2}{c^3}\frac{e^4}{m_e^2c^3b^4} = \frac{e^6}{m_e^2c^3b^4}$$

Bremsstrahlung (2)

Characteristic frequency $\omega \sim 1/\,t$ ➔

$$P(\omega) \approx \frac{P}{\omega} = \frac{e^6}{m_e^2 c^3 v b^3}$$

Emissivity $$j(\omega) \approx \frac{n_e n_p}{4\pi} \frac{e^6}{m_e^2 c^3} \left(\frac{m_e}{kT}\right)^{1/2}$$

Introducing the Gaunt factor g_{ff} (minimum impact factor), the contribution of the exponential part of a Maxwellian distribution the exact treatment gives:

$$j(\nu) = \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{n_e n_p e^6}{m_e^2 c^3} \left(\frac{m_e}{kT}\right)^{1/2} e^{-h\nu/kT}\, \bar{g}_{ff}$$

Absorption

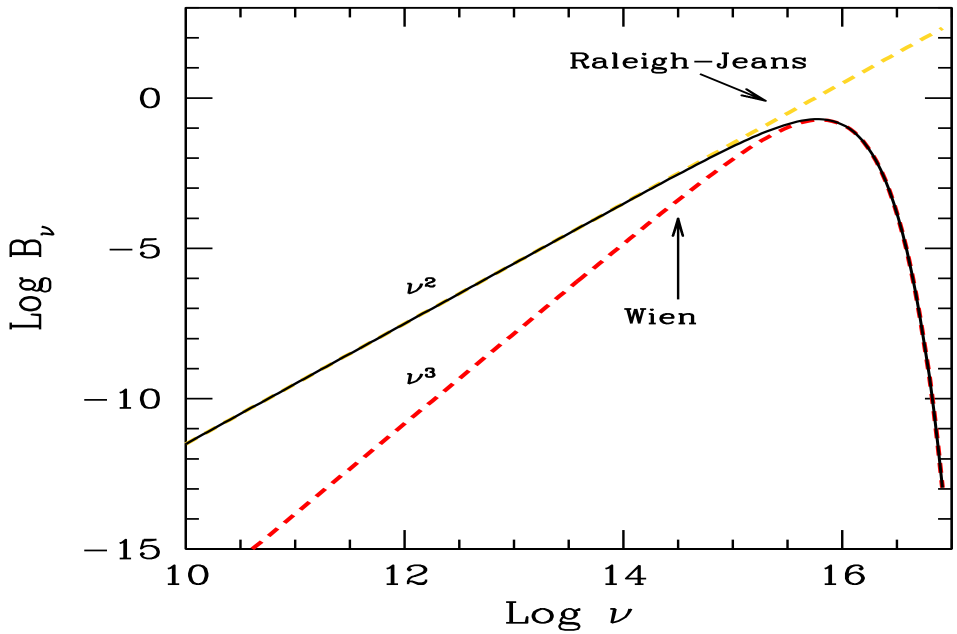
$$\alpha_\nu^{ff} = \frac{4}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{Z^2 n_e n_i e^6}{h m_e^2 c^2} \left(\frac{m_e c^2}{kT}\right)^{1/2} \frac{1 - e^{-h\nu/kT}}{\nu^3}\, \bar{g}_{ff}$$

Blackbody emission (1)

Increase in density until all spectrum is absorbed
And for Maxwellian particle distribution➔ blackbody

Blackbody emission (2)

Best emitter



$$B_\nu(T) = \frac{2}{c^2} \frac{h\nu^3}{e^{h\nu/kT} - 1}$$

$$h\nu_{\rm peak} = 2.82\, kT.$$

$$\int_0^\infty B_\nu d\nu = \frac{\sigma_{\rm MB}}{\pi} T^4, \qquad \sigma_{\rm MB} = \frac{2\pi^5 k^4}{15 c^2 h^3}$$

$$u = \frac{4\pi}{c} \int_0^\infty B_\nu d\nu = a\, T^4, \qquad a = \frac{4\sigma_{\rm MB}}{c}$$

If $T_2 > T_1$ ➔ $B_\nu(T_2) > B_\nu(T_1)$

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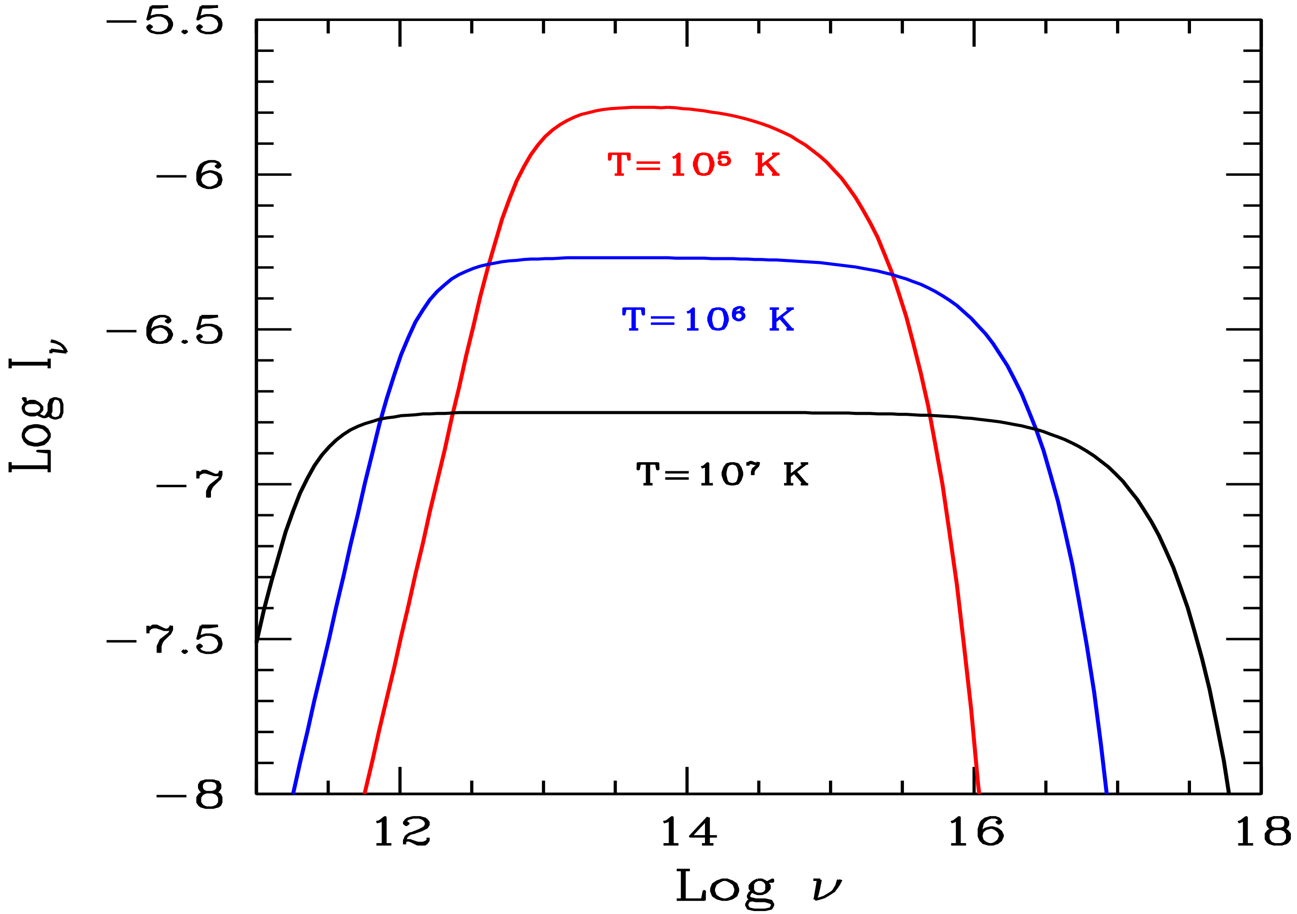
Bremsstrahlung (3)



Temperature behavior

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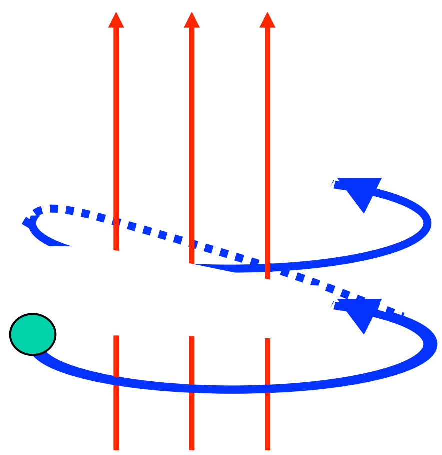
Synchrotron emission: single particle (1)



Lorentz force: no work

- $|v|$, v_{par}, $|v_{perp}|$ constant,
- helical motion
- $a_{perp} = \omega_L v_{perp}$ → radiation
- Pitch angle α

Larmor radius $r_L = (v \sin\alpha)\, mc\, \gamma / eB$

Larmor frequency $\omega_L = e B / \gamma m c$

- Non-relativistic particle → cyclotron
- Relativistic particle $(\gamma \sin \alpha)^3 \gg 1$ → synchrotron

Synchrotron emission: single particle (2)

aberration

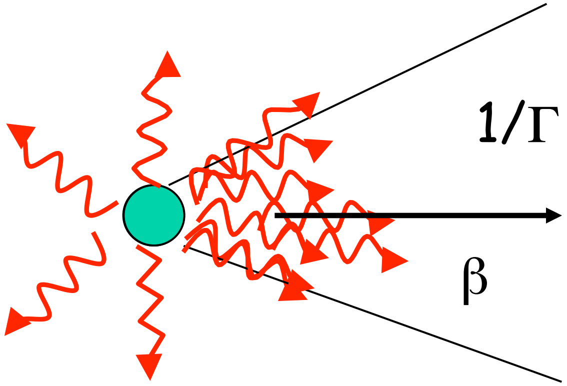
$\cos\theta = (\cos\theta' + \beta)/(1 + \beta\cos\theta')$

$\sin\theta = \sin\theta' / \Gamma(1 + \beta\cos\theta')$

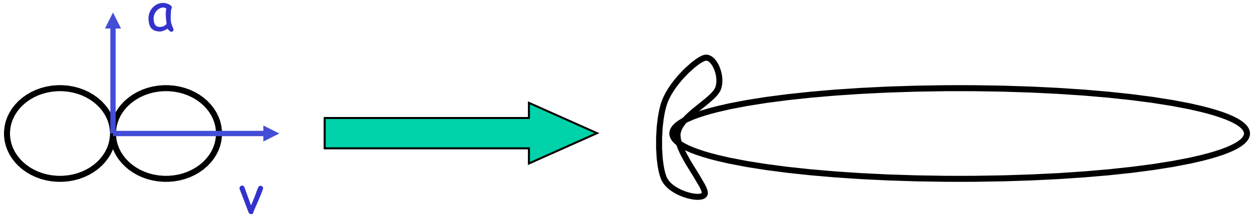
$\theta' = \pi/2$

$\cos\theta = \beta$

$\sin\theta = 1/\Gamma$



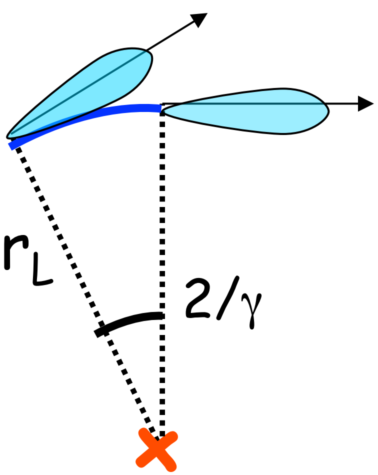
For $\underline{a}$ perp to $\underline{v}$ → emission pattern $dP/d\Omega = (e^2 a^2/4\pi c^3)\sin^2\Theta$

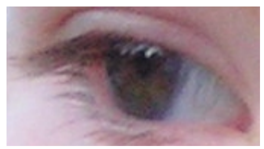


Synchrotron emission: single particle (3)

$\gamma >> 1$

Relativistic beaming





$r_L = (m c^2 / e) \ \gamma / B_{perp}$

$\Delta t \sim 2 r_L / c \gamma$

$\Delta t_{obs} \sim \Delta t / \gamma^2$

Characteristic frequency

$\nu_s = (e/2\pi mc) B \gamma^2 \sin \alpha$

average over pitch angles

$\langle \nu_s \rangle = (4/3) (e/mc) B \gamma^2$

$B_{perp} \sim$ const over r_L

Synchrotron emission: single particle (4)

Emitted power

Relativistic dipole emission – frame v=0

$P = P' = (2e^2/3c^3)\,(a'^2_{perp} + a'^2_{par}) = (2e^2/3c^3)\,\gamma^4\,(a_{perp}^2 + \gamma^2\, a_{par}^2)$

a_{par} small $\qquad a_{perp} \rightarrow \gamma$=const, but $\Delta\underline{v}$ can be large

$\dot{\gamma}_s = P/mc^2 = -(2\,e^2/3\,m\,c^3)\,\beta^2\,\gamma^2\,\omega_L^2\,\sin^2\alpha \quad \begin{cases} \sim \gamma^2 \sim E^2 & \gamma \gg 1 \\ \sim \beta^2 \sim E & \gamma \sim 1 \end{cases}$

$\gamma \gg 1$ Isotropic distribution pitch angles α

$\langle \dot{\gamma}_s \rangle = -(4/3)\,(\sigma_T\, c\,/\,m\,c^2)\,\gamma^2\, U_B$

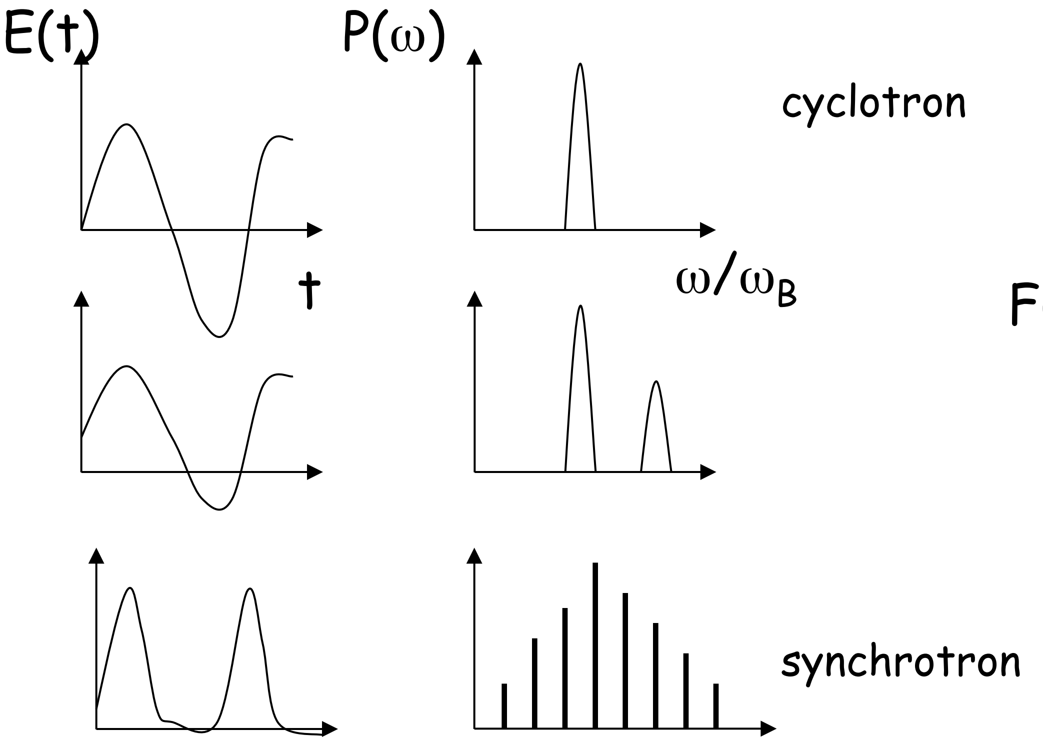
$U_B = B^2/8\pi$ magnetic energy density

Cooling time

$t_s \sim (\gamma-1)/\dot{\gamma} \sim 7.8\;10^8 / B^2\gamma \quad s$

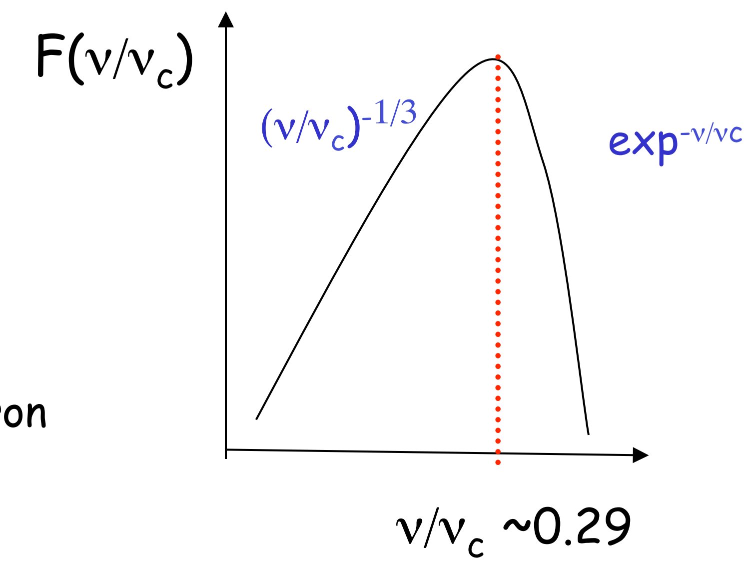
Synchrotron emission: single particle (5)

Spectrum: $P(\omega)$: Fourier transform of $\underline{E}(t)$



Emitted spectrum

('sum' of harmonics)



$P(\nu,\gamma,\alpha)=3^{1/2}\,(e^3/mc^2)\,\sin\alpha\;F(\nu/\nu_c)$

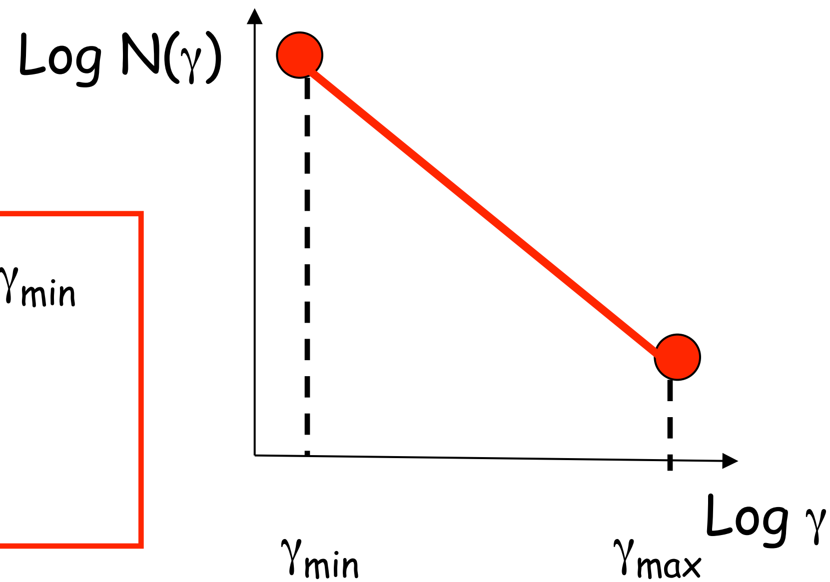
$\nu_c=(3/2)\,(eB/2\pi mc)\,\gamma^2\,\sin\alpha$

$F(\nu/\nu_c)=(\nu/\nu_c)\;K_{5/3}(x)dx$

Synchrotron: non-thermal particle distribution (1)

Non-thermal : out of thermodynamic equilibrium

$N(\gamma) = N_o \gamma^{-p}$; $\gamma_{min} < \gamma < \gamma_{max}$



for p ~2.5:

number density at γ_{min}

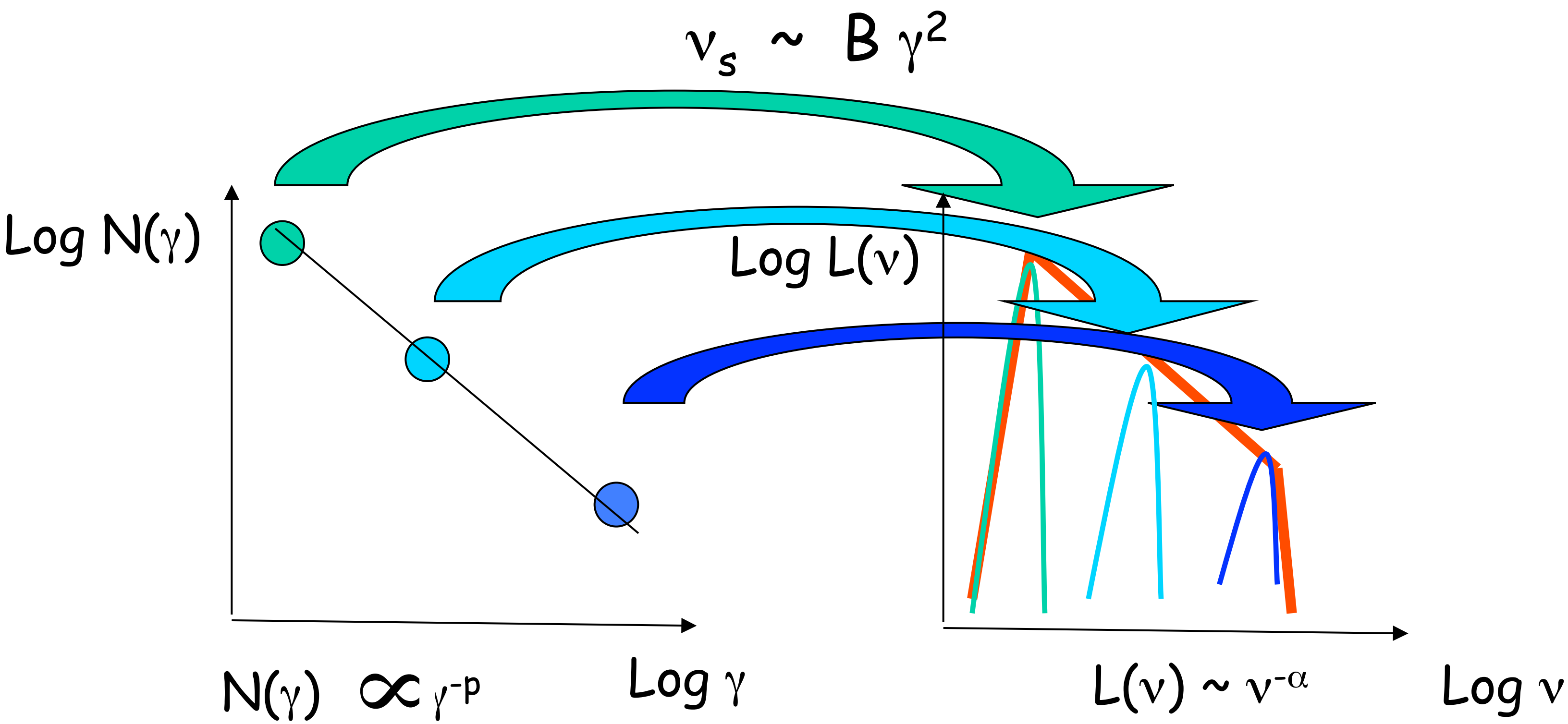
pressure at γ_{min}

emission at γ_{max}

Synchrotron: non-thermal particle distribution (6)

Particle spectrum

Photon spectrum



Synchrotron: non-thermal particle distribution (7)

$N(\gamma) = N_o \gamma^{-p}$ $\quad \gamma_{min} < \gamma < \gamma_{max}$ $\quad$ isotropic

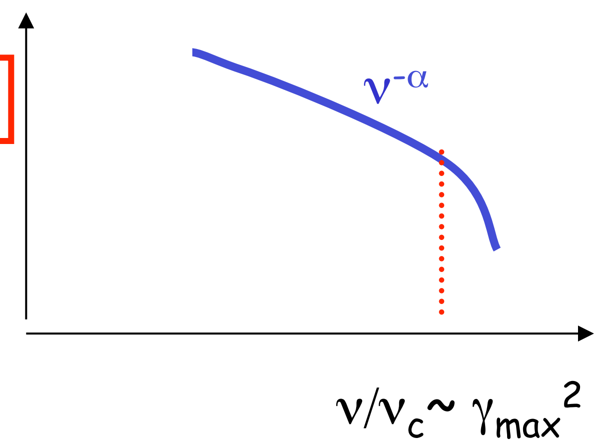
$$L(\nu) = \int N(\gamma)\, \dot{\gamma}\, d\gamma \quad \propto \quad \nu^{-\alpha}$$

$\gamma_{min}^2 \ll \nu/\nu_c \ll \gamma_{max}^2$

$\alpha = (p - 1)/2$

Emissivity $\quad [\mathrm{erg\ s^{-1}\ cm^{-3}\ sr^{-1}}]$

$$j_s(\nu) = (1/4\pi) \int N(\gamma)\, P(\nu,\gamma)\, d\gamma = C(\alpha)\, N_o\, B^{1+a}\, \nu^{-\alpha}$$



$C(a) = \pi^{1/2} e^2/4c\, (e/2\pi mc)^{1+\alpha}\, A(p)\, 3^{p/2}$

$A(p)$: product of Γ functions $\quad A(2) \sim 0.597$

Linear polarization $\quad \Pi = (p+1)/(p+7/3) \sim 70\ \%\ (p=2)$

Synchrotron: non-thermal particle distribution (4)

Self-absorption: absorption coefficient $\alpha_\nu \sim \nu^{-(p+4)/2}$ ➔

$I(\nu) = (2mf(p)/3^{1/2})\, \nu_B^{-1/2}\, \nu^{5/2}$

and

$\tau_s \sim 1$ @ $\nu_t = (4/3)\, \gamma_t^2\, \nu_B$

$\nu_t = [2^{-(p+8)} (3/\pi)^{p+1}\, g^2(p)\, e^{p+6}]^{1/(p+4)}\, (mc)^{-1}\, (N_o R)^{1/(p+4)}\, B^{(p+2)/(p+4)}$

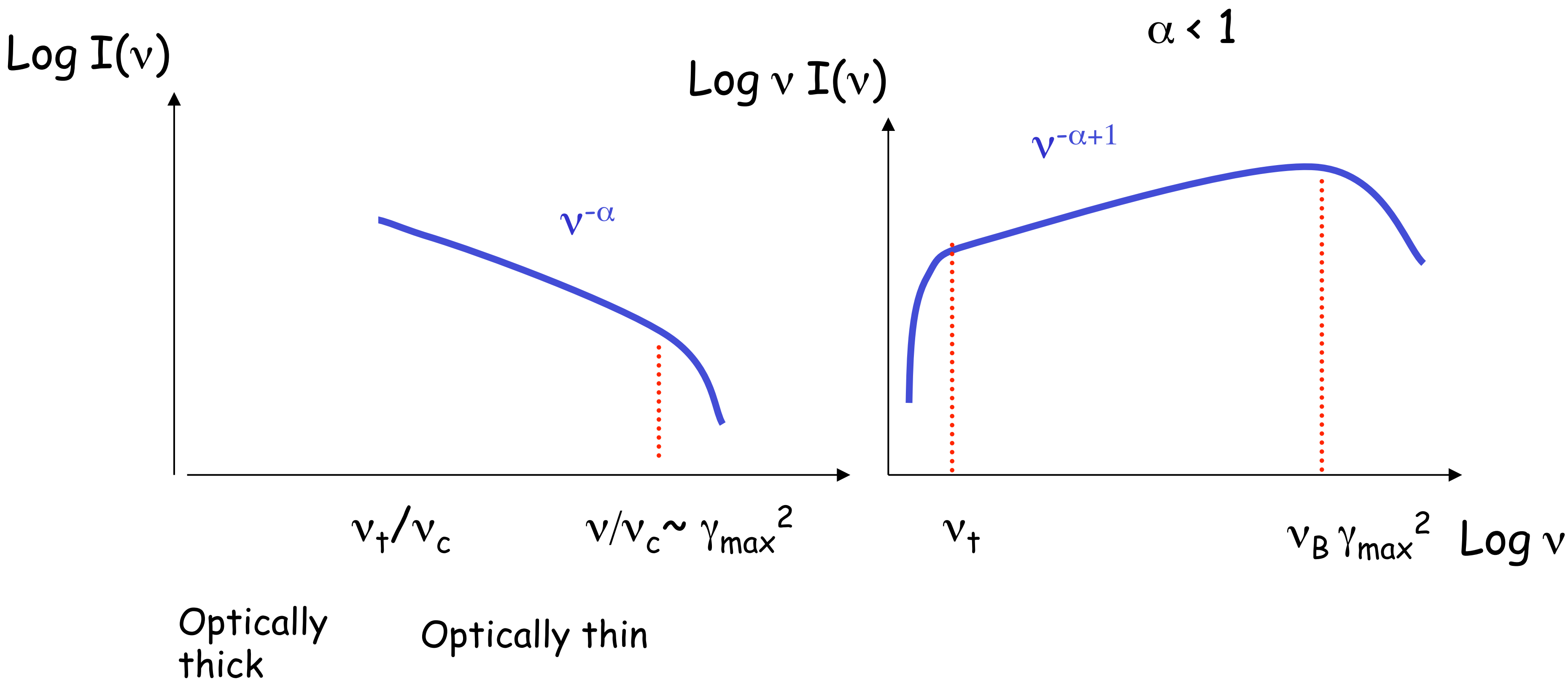
f(p),g(p): products of Γ functions f(2)~0.5, g(2)~ 1.213

Brightness temperature $k\, T_b(\nu) = (c^2/2)\ I(\nu)/\nu^2$

$k\, T_b \sim mc^2\ (\nu_t/\nu_B)^{1/2} \sim \gamma\, mc^2$

NOTE: If θ known, $I(\nu_t)$ & ν_t ➔ B, N_o

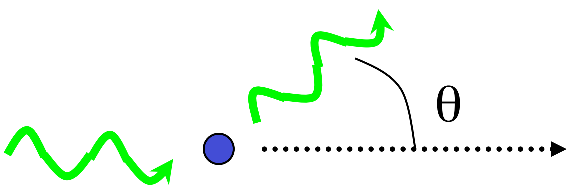
Synchrotron: non-thermal particle distribution (5)



Compton emission (1)

electron at rest

$m_e c^2 \gg h\nu$



elastic scattering $\nu' = \nu$

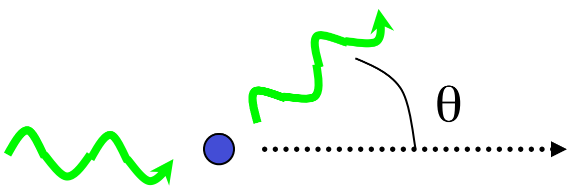
cross section $d\sigma/d\Omega = (3/16\pi)\, \sigma_T\, (1+\cos^2\theta)$

$\sigma_T = (8\pi/3)\, (e/mc^2)^2 \sim 6.7 \times 10^{-25}\ cm^2$

Compton emission (2)

electron at rest

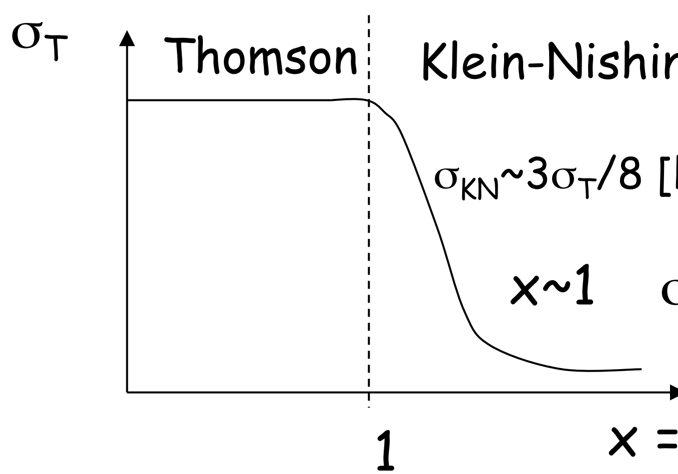
$m_e c^2 \ll h\nu$



recoil

$\nu' = \nu / [1 + (h\nu/mc^2)(1-\cos\theta)]$

cross section



$\sigma_{KN} \sim 3\sigma_T/8\ [\ln(2x)+0.5]/x \quad x \gg 1$

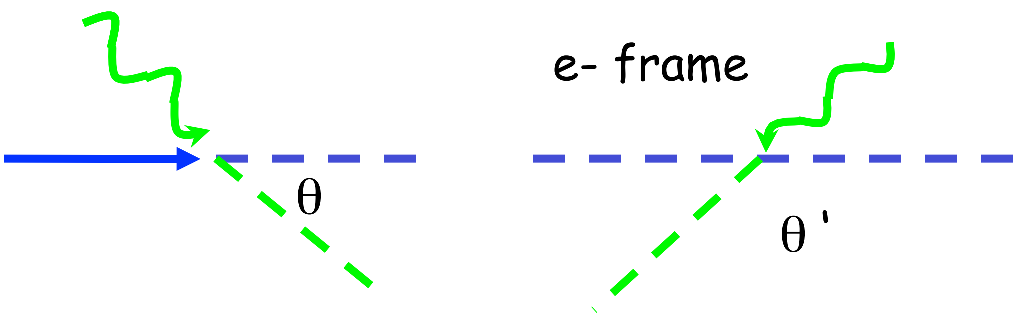
$x \sim 1 \quad \sigma \sim 0.43\ \sigma_T$

Inverse Compton emission (3)

direct Compton $h\nu > mc^2$; e^- recoil $\nu_1 < \nu_0$

"inverse" Compton $h\nu < mc^2$: transfer of energy to photon

$h\nu \ll mc^2$



$\nu' = \nu\,\gamma\,(1-\beta\cos\theta)$

$\nu'_c = \nu'$

$\nu_c = \nu'_c\,\gamma\,(1+\beta\cos\theta'_c) = \nu\,(1-\beta\cos\theta)/(1-\beta\cos\theta_c)$

$\theta'_c = 90°$ $\sin\theta_c \sim 1/\gamma$ $\nu_c = \gamma^2\,\nu\,(1-\beta\cos\theta)$

max $\theta=180°$ $\theta_c=0°$ $\nu_c \sim 4\gamma^2\,\nu$

min $\theta=0°$ $\theta_c=180°$ $\nu_c \sim \nu/4\gamma^2$

$\langle \nu_c \rangle = (4/3)\,\gamma^2\nu_o$

Inverse Compton emission (4)

Emitted power = initial power - scattered power

Isotropic distribution angles θ

$\langle \dot{\gamma}_c \rangle = -(4/3)\,(\sigma_T c / m c^2)\,\beta^2 \gamma^2 U_r$

[# collisions/s] x [average energy after scattering]

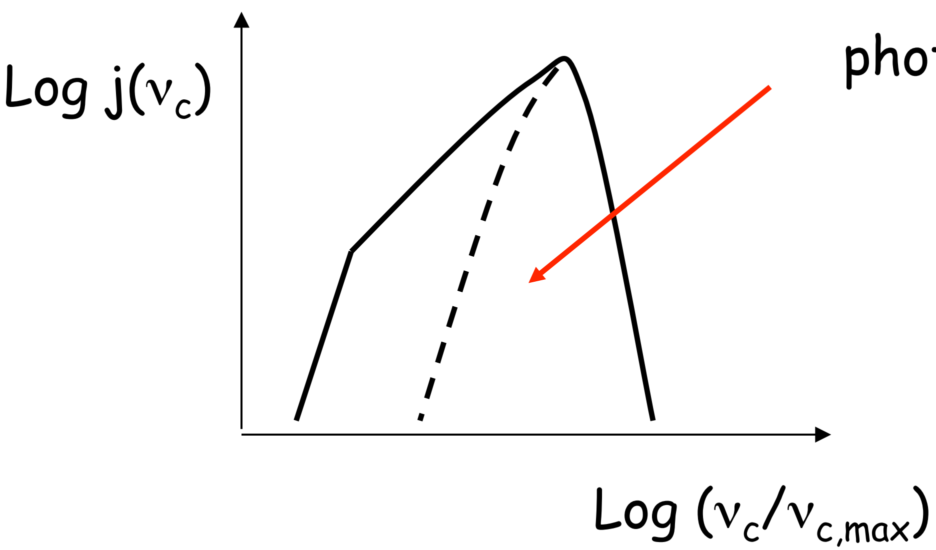
$[\sigma_T c U_r / \langle h\nu \rangle] \times [4/3 \langle h\nu \rangle \beta^2 \gamma^2]$

U_r seed photon energy density

Inverse Compton emission (5)

$\langle\nu_c\rangle = (4/3)\,\nu_0\,\gamma^2$ average over angles

Emitted spectrum: single e^- in isotropic, monochromatic ν_0 radiation field



photons within $1/\gamma$ ~ 75 % of power

$$P(\nu_c,\gamma)=\dot{\gamma}_c mc^2\,\delta(\nu - 4\gamma^2\nu_0/3)$$

Inverse Compton emission (6)

Emissivity

$$j(\nu_c) = (1/4\pi)\int N(\gamma)\, P(\nu_c,\gamma)\, d\gamma =$$

$$= (1/4\pi)\,[(4/3)^{\alpha}/2]\,[\sigma_T\, c\, N_o\, U_r / \nu_o]\,(\nu_c/\nu_o)^{-a}$$

$$j(\varepsilon_c) = (1/4\pi)\,[(4/3)^{\alpha}/2]\,(\tau_c/(R/c))\,(U_r/\varepsilon_o)\,(\varepsilon_c/\varepsilon_o)^{-a} \qquad [cm^{-3}\, s^{-1}\, sr^{-1}]$$

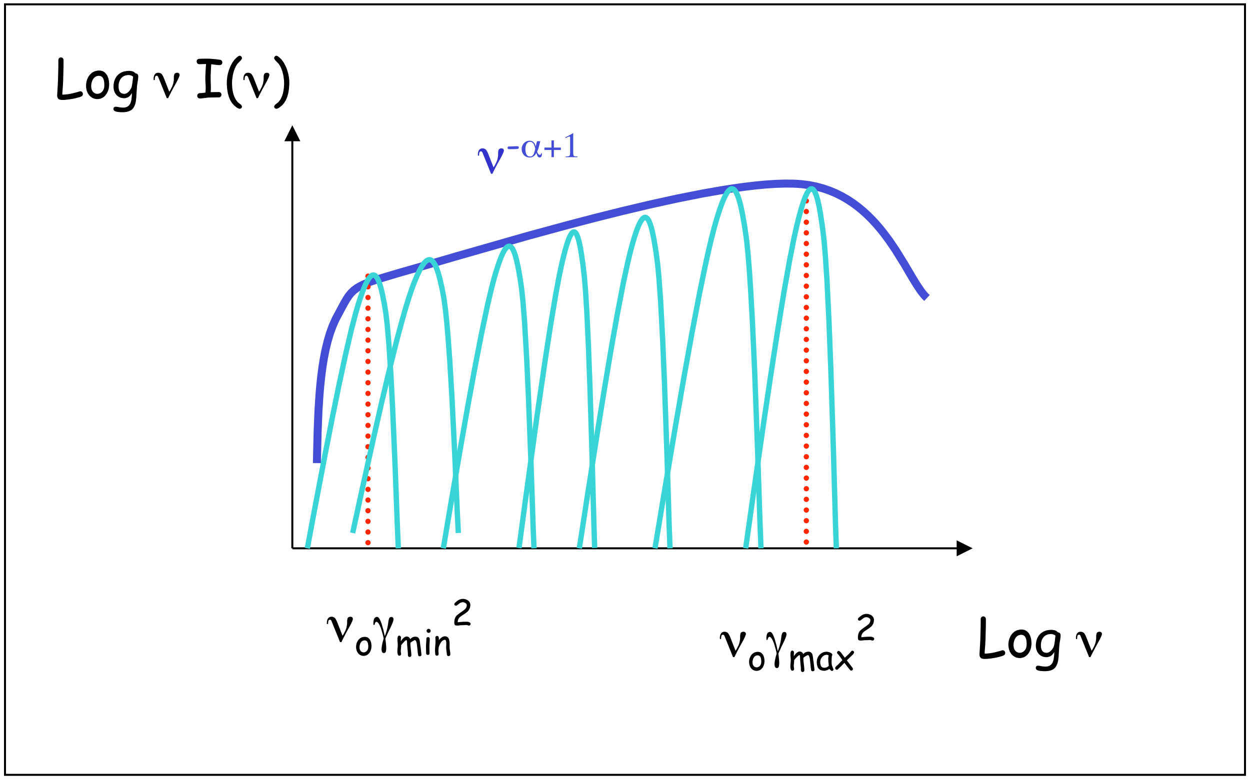
$\tau_c = \sigma_T N_o R$

t_{esc}

average gain/ scattering

photon density which contribute at ε_c and $h\nu\,\gamma < mc^2$

Inverse Compton emission (7)



Inverse Compton emission (8)

U_r seed photon energy density:

-"internal" photon field, synchrotron radiation (Synchrotron Self-Compton, SSC)

isotropy in source frame

- "external" photon field, e.g.
Broad Line Region photons,
Cosmic Microwave Background...

isotropy in observer frame

Inverse Compton vs synchrotron emission (1)

$< \nu_c > = (4/3)\, \nu_0\, \gamma^2$

$< \nu_s > = (4/3)\, (e/mc)\, B\, \gamma^2$

analogous processes

$< \dot{\gamma}_c > = -\, (4/3)\, (\sigma_T\, c\, /\, m\, c^2)\, \beta^2\, \gamma^2\, U_r$

$< \dot{\gamma}_s > = -\, (4/3)\, (\sigma_T c\, /\, m\, c^2)\, \beta^2\, \gamma^2\, U_B$

$\Rightarrow$ $L_c/L_s = U_r/U_B$

Unless: self-absorption

KN regime: $\gamma\, h\nu \;.\!> mc^2$

Inverse Compton vs synchrotron emission (2)

SSC $U_r = U_s$

$L_c/L_s = U_s/U_B$

if $U_r > U_B$ ➔ second inverse Compton order important etc…

As $U_s/U_B \sim T_b\, \nu_t^3/B^2 \sim T_b^5\, \nu_t$

$L_c < L_s$ ➔ $T_b < 10^{12}\,(1-\alpha)^{4/5}\, \nu_{max,9}^{(\alpha-1)/5}\, \nu_{t,9}^{-\alpha/5}$ K $\alpha<1$

$T_b > 10^{12}$ K ➔ "Compton catastrophe": higher Compton orders

➔ runaway cooling

BUT Klein-Nishina when $\gamma\, h\nu \sim mc^2$

Inverse Compton vs synchrotron emission (2)

<u>SSC</u>

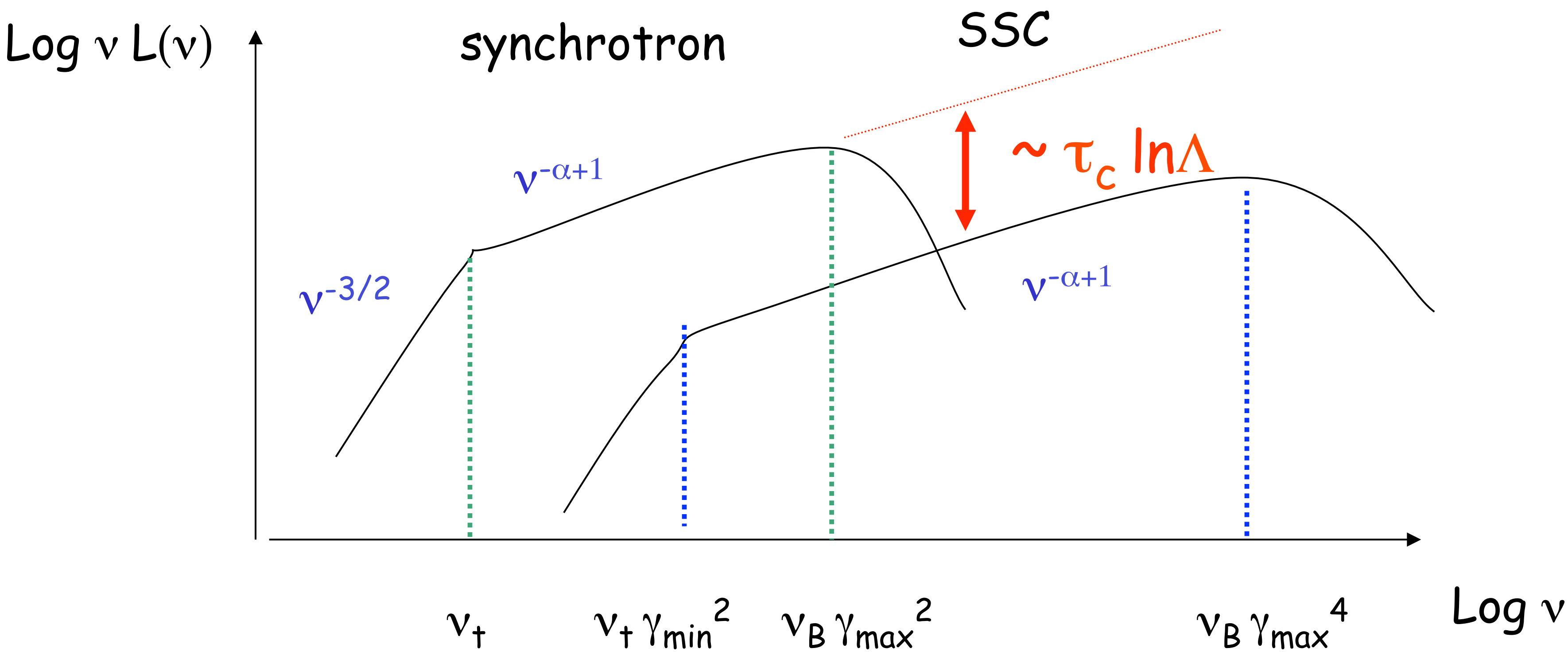
$U_r(\varepsilon)=U_s(\varepsilon) \sim 4\pi\ (3R/4c)\ j_s(\varepsilon)$

$$j_{ssc}(\varepsilon_c)= (4/3)^{\alpha-1}\ \tau_c\ j_s(\varepsilon_c) \int_{\varepsilon_1}^{\varepsilon_2} \varepsilon^{-1}\ d\varepsilon = (4/3)^{\alpha-1}\ \tau_c\ j_s(\varepsilon_c)\ \ln \Lambda$$

$$\Lambda = \varepsilon_2/\varepsilon_1$$

$$\sim R\ N_o^2\ B^{1+\alpha}\ \ln \Lambda\ \nu^{-\alpha}$$

Inverse Compton vs synchrotron emission (3)



Inverse Compton vs synchrotron emission (4)

Typical parameters

$R \sim 10\ R_s \sim 3\ 10^{15}\ M_9$ cm $\qquad L \sim L_{Edd} = 1.3\ 10^{47}\ M_9$ erg s^{-1}

$U_B \sim U_r \Rightarrow B \sim 10^3\ M_9^{-1/2}\ G$

$\gamma \sim 10^3$

$\nu_s \sim 2.8\ 10^6\ B\ \gamma^2$ Hz $\sim 5.5\ 10^{15}$ Hz

$\nu_c \sim \nu_s\ \gamma^2 \sim 7.3\ 10^{21}$ Hz ~ 30 MeV

$t_{cool} \sim 0.4$ s $\quad \Rightarrow$ injection or reacceleration

Thermal Comptonization (1)

Multiple scattering due to a thermal (or quasi thermal) electron distribution

$x = h\nu/m_e c^2$
$\Theta = kT/m_e c^2$

Comptonization parameter y =

[average # scatter]x[average fraction energy gain per scatt]

τ_T

$\langle x_1 \rangle = 4/3 \langle \gamma^2 \rangle x_0 = 16\Theta^2 x_0$ relativ

$\Delta x/x = 4\Theta x$ non relativ

y>1 Comptonization important

$\Delta x/x = 16\Theta^2 + 4\Theta x - x$

Thermal Comptonization (2)

$y = max(\tau_{T,} \tau_T{}^2) x [16\Theta^2 {}^+ 4\Theta x - x]$

Differential (neglecting downscattering) ➔

$x_f = x_0 \exp\{[16\Theta^2 {}^+ 4\Theta x] x [max(\tau_{T,} \tau^2)]\}$ ➔ $x_f = x_0 e^y$

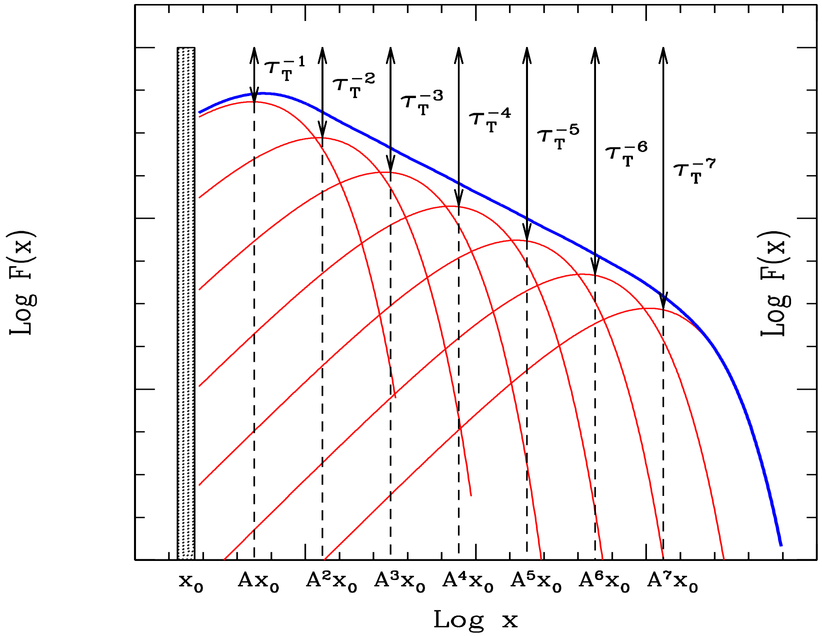
$L_f - L_o / L_o = e^y - 1$

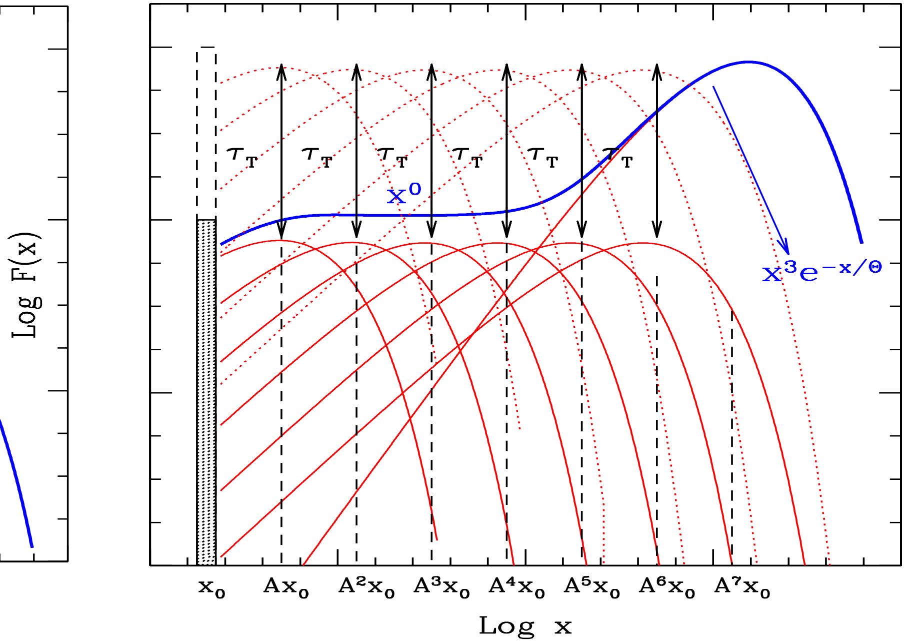
y>1 Comptonization important

Thermal Comptonization (3)

Spectrum = superposition of Compton scattering spectra

Different spectra depending on τ_T and T





$\tau_T < 1$ $\alpha(\tau_T, T)$

$\tau_T \gg 1$ saturation-Wien peak

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Hadronic processes

p+p	requires high n_p $L_{pic}/L_{ssc} \sim U_p/U_r$
p+B	proton synchrotron: requires high B
p+γ	proton induced cascade: pions ➔ e+, e-, γ synchrotron from e+ e- external photons

+ efficiency acceleration processes

- timescales: fast variability?
- overall efficiency ?
- pair cascade: flat X-ray spectra?

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The End