POLARIZATION AND WEAK GRAVITATIONAL LENSING OF THE CMB

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OUTLINE

CMB polarization

- Polarization observables
- Physics of CMB polarization
- Observational status and applications
- (Near-)Future of CMB research

CMB lensing

- Basics of CMB lensing
- Lensing of CMB power spectra
- Lensing reconstruction
- Applications of lensing reconstruction
- Lensing and primordial non-Gaussianity

Useful references

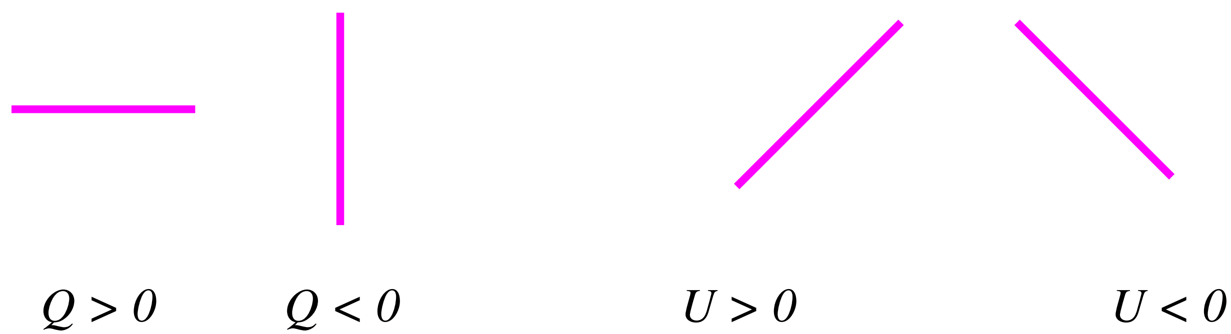
- CMB polarization
  - Wayne Hu's website (`http://background.uchicago.edu/~whu/`)
  - Hu & White's "Polarization primer" (arXiv:astro-ph/9706147)
  - AC's summer school lecture notes (arXiv:0903.5158 and arXiv:astro-ph/0403344)
  - Kosowsky's "Introduction to Microwave Background Polarization" (arXiv:astro-ph/9904102)
- CMB lensing
  - Lewis & AC's "Weak gravitational lensing of the CMB" (arXiv:astro-ph/0601594)
  - Hanson, AC & Lewis's "Weak lensing of the CMB" (arXiv:0911.0612)
- Applications of CMB lensing
  - Smith et al. CMBPol document (arXiv:0811.3916)
- Textbooks covering most of the above
  - *The Cosmic Microwave Background* by Ruth Durrer (CUP)

Introduction to CMB polarization

CMB POLARIZATION: STOKES PARAMETERS

- For plane wave along z, symmetric trace-free correlation tensor of electric field $\boldsymbol{E}$ defines (transverse) linear polarization tensor:

$$\mathcal{P}_{ab} \equiv \begin{pmatrix} \frac{1}{2}\langle E_x^2 - E_y^2 \rangle & \langle E_x E_y \rangle \\ \langle E_x E_y \rangle & -\frac{1}{2}\langle E_x^2 - E_y^2 \rangle \end{pmatrix} \equiv \frac{1}{2} \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$



- Under right-handed rotation of x and y through ψ about propagation direction (z)

$$Q \pm iU \rightarrow (Q \pm iU)e^{\mp 2i\psi} \quad \Rightarrow \quad Q + iU \text{ is spin -2}$$

E AND B MODES: VECTOR FIELDS

- As a warm-up, can always write vector field in 2D as

$$\begin{aligned} V_a &= \text{gradient} + \text{divergence-free vector} \\ &= \nabla_a V_E + \epsilon_a{}^b \nabla_b V_B \end{aligned}$$

- Consider spin-± 1 components of $\boldsymbol{V}$ on *null basis* $\boldsymbol{m}_\pm \equiv (\partial_\theta \pm i\text{cosec}\theta \partial_\phi)$
  - Since $\epsilon_a{}^b m_\pm^a = \mp i m_\pm^b$ have

$$\begin{aligned} \boldsymbol{m}_\pm \cdot \boldsymbol{V} &= (\partial_\theta \pm i\text{cosec}\theta\partial_\phi)V_E \mp i(\partial_\theta \pm i\text{cosec}\theta\partial_\phi)V_B \\ &= (\partial_\theta \pm i\text{cosec}\theta\partial_\phi)(V_E \mp iV_B) \end{aligned}$$

- Define spin-weight derivatives via

$$\begin{aligned} \eth {}_s\eta &= -\sin^s\theta(\partial_\theta + i\text{cosec}\partial_\phi)(\sin^{-s}\theta \, {}_s\eta) \\ \bar{\eth} {}_s\eta &= -\sin^{-s}\theta(\partial_\theta - i\text{cosec}\partial_\phi)(\sin^s\theta \, {}_s\eta) \end{aligned}$$

  - Then spin components of $\boldsymbol{V}$ are spin-weight derivatives of complex potential:

$$\boldsymbol{m}_+ \cdot \boldsymbol{V} = -\eth(V_E - iV_B), \qquad \boldsymbol{m}_- \cdot \boldsymbol{V} = -\bar{\eth}(V_E + iV_B)$$

E AND B MODES FOR POLARIZATION

- Generalisation of E-B decomposition to 2nd-rank STF tensors

$$\mathcal{P}_{ab}(\hat{\boldsymbol{n}}) = \nabla_{\langle a}\nabla_{b\rangle}P_E + \epsilon^c{}_{(a}\nabla_{b)}\nabla_c P_B$$

- Components of $\mathcal{P}_{ab}$ on null basis are complex combinations of Stokes parameters (defined in $(\theta, -\phi)$ basis following IAU)

$$m_\pm^a m_\pm^b \mathcal{P}_{ab} = Q \mp iU$$

- Evaluating null components of covariant derivatives gives

$$Q + iU = \bar{\eth}\bar{\eth}(P_E - iP_B), \qquad Q - iU = \eth\eth(P_E + iP_B)$$

- P_E and P_B are scalar fields $\Rightarrow$ can expand in usual spherical harmonics:

$$P_E(\hat{\boldsymbol{n}}) = \sum_{lm}\sqrt{\frac{(l-2)!}{(l+2)!}}E_{lm}Y_{lm}(\hat{\boldsymbol{n}}), \qquad P_B(\hat{\boldsymbol{n}}) = \sum_{lm}\sqrt{\frac{(l-2)!}{(l+2)!}}B_{lm}Y_{lm}(\hat{\boldsymbol{n}})$$

  – l-dependent factors “undo” $\sim l^2$ factors from double derivatives to give

$$Q \pm iU = \sum_{lm}(E_{lm} \mp iB_{lm})\sqrt{\frac{(l-2)!}{(l+2)!}}\left\{\begin{array}{c}\bar{\eth}\bar{\eth}\\ \eth\eth\end{array}\right\}Y_{lm} = \sum_{lm}(E_{lm} \mp iB_{lm})_{\mp 2}Y_{lm}$$

FLAT-SKY LIMIT

- Work near North pole and can use global $x, -y$ basis now
- Since $\boldsymbol{m}_\pm = e^{\mp i\phi}(\hat{\boldsymbol{x}} \pm i\hat{\boldsymbol{y}})$

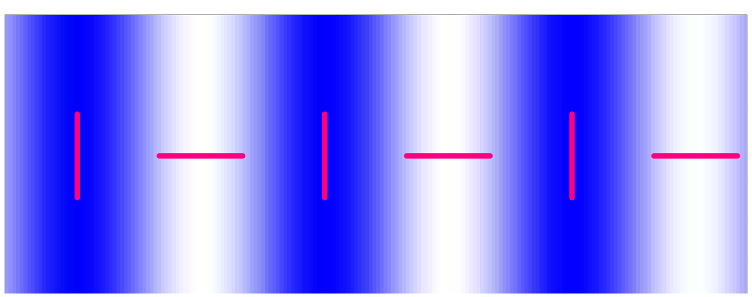
$$
\begin{aligned}
(Q \pm iU)_{\rm flat} &= e^{\mp 2i\phi}(Q \pm iU) \\
&= e^{\mp 2i\phi} \left\{ \begin{array}{c} \bar{\eth}\bar{\eth} \\ \eth\eth \end{array} \right\} (P_E \mp iP_B) \\
&\approx e^{\mp 2i\phi} e^{\pm 2i\phi} (\partial_x \mp i\partial_y)^2 (P_E \mp iP_B)
\end{aligned}
$$

- In Fourier space, with e.g. $P_E(\boldsymbol{l}) = E(\boldsymbol{l})/l^2$ have

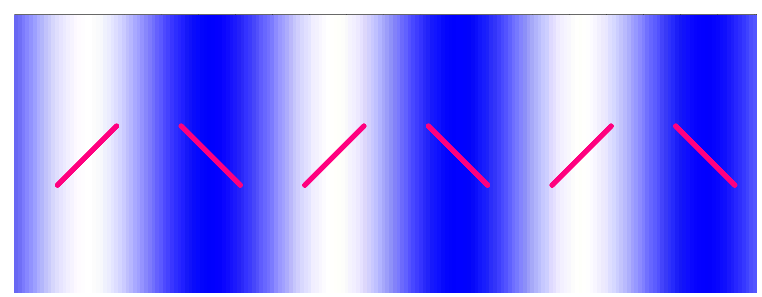
$$
(Q \pm iU)_{\rm flat}(\boldsymbol{l}) = -e^{\mp 2i\phi_l}(E \mp iB)(\boldsymbol{l})
$$

  - Fourier modes $E(\boldsymbol{l})$ produces Q polarization on basis adapted to $\boldsymbol{l}$:

Pure E mode



Pure B mode



Two-point statistics

- Statistical isotropy demands 2-point correlations of form

$$\langle E_{lm} E^*_{l'm'} \rangle = C_l^E \delta_{ll'} \delta_{mm'}$$

- For *Gaussian* fluctuations all information in *power spectrum* C_l
- Under parity transformations $Q(\hat{\boldsymbol{n}}) \to Q(-\hat{\boldsymbol{n}})$ and $U(\hat{\boldsymbol{n}}) \to -U(-\hat{\boldsymbol{n}})$ so

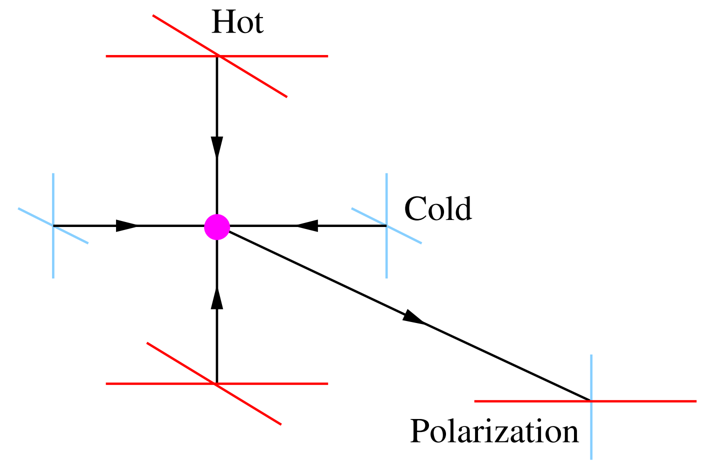
$$\begin{aligned} \hat{P}(Q \pm iU)(\hat{\boldsymbol{n}}) &= (Q \mp iU)(-\hat{\boldsymbol{n}}) \\ \Rightarrow \quad \sum_{lm} \hat{P}(E_{lm} \mp iB_{lm})_{\mp 2}Y_{lm}(\hat{\boldsymbol{n}}) &= \sum_{lm} (E_{lm} \pm iB_{lm})_{\pm 2}Y_{lm}(-\hat{\boldsymbol{n}}) \\ &= \sum_{lm} (-1)^l (E_{lm} \pm iB_{lm})_{\mp 2}Y_{lm}(\hat{\boldsymbol{n}}) \end{aligned}$$

  - Follows that $E_{lm} \to (-1)^l E_{lm}$ under parity but $B_{lm} \to -(-1)^l B_{lm}$
  - Cannot have E-B or T-B correlations if parity respected in mean

- Flat-sky limit of power spectra: e.g. $\langle E(\boldsymbol{l}) E^*(\boldsymbol{l}') \rangle = C_l^E \delta(\boldsymbol{l} - \boldsymbol{l}')$

Physics of CMB polarization

CMB POLARIZATION: THOMSON SCATTERING



- Photon diffusion around recombination $\to$ local temperature quadrupole
  - Subsequent Thomson scattering generates (partial) linear polarization with r.m.s. $\sim 5\,\mu$K from density perturbations

- Thomson scattering of radiation quadrupole produces linear polarization (dimensionless temperature units!)

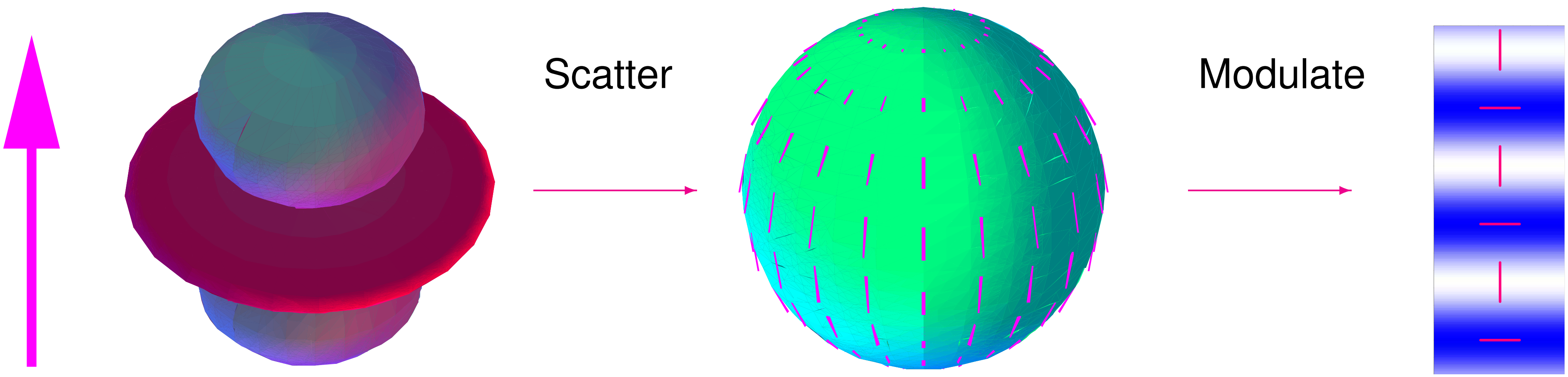
$$d(Q \pm iU)(\boldsymbol{e}) = \frac{3}{5} a n_e \sigma_T d\eta \sum_m {}_{\pm 2}Y_{2m}(\boldsymbol{e}) \left(E_{2m} - \sqrt{\frac{1}{6}} \Theta_{2m} \right)$$

  - Purely electric quadrupole ($l = 2$)

- In linear theory, generated $Q + iU$ then conserved for free-streaming radiation
  - Suppressed by $e^{-\tau}$ if further scattering at reionization

PHYSICS OF CMB POLARIZATION: SCALAR PERTURBATIONS

- Single plane wave of scalar perturbation has $\Theta_{2m} \propto Y_{2m}^*(\hat{\boldsymbol{k}}) \Rightarrow$ with $\hat{\boldsymbol{k}}$ along z, $dQ \propto \sin^2\theta$ and $dU = 0$



Plane-wave scalar quadrupole   Electric quadrupole ($m = 0$)   Pure E mode

- Linear scalar perturbations produce only E-mode polarization

Quadrupole Source Term

- Consider scales large compared to diffusion-damping scale (few$\times 10\,\mathrm{Mpc}$)
  - Temperature fluctuation seen by electron determined by conditions at previous scattering l_p away:

$$
\begin{aligned}
\Theta(\boldsymbol{e}) + \psi &\approx (\Theta_0 + \psi)(-l_p \boldsymbol{e}) + \boldsymbol{e} \cdot \boldsymbol{v}_{(b)}(-l_p \boldsymbol{e}) \\
&\approx (\Theta_0 + \psi) - l_p e^i \partial_i (\Theta_0 + \psi) + \frac{1}{2} l_p^2 e^i e^j \partial_i \partial_j (\Theta_0 + \psi) \\
&\quad + \boldsymbol{e} \cdot \boldsymbol{v}_{(b)} - l_p e^i e^j \partial_j v_{(b)i} + \cdots
\end{aligned}
$$

  - Dominant temperature quadrupole for each source from trace-free part of $e^i e^j$ component [$e^{\langle i} e^{j \rangle} = e^i e^j - \delta^{ij}/3$]:

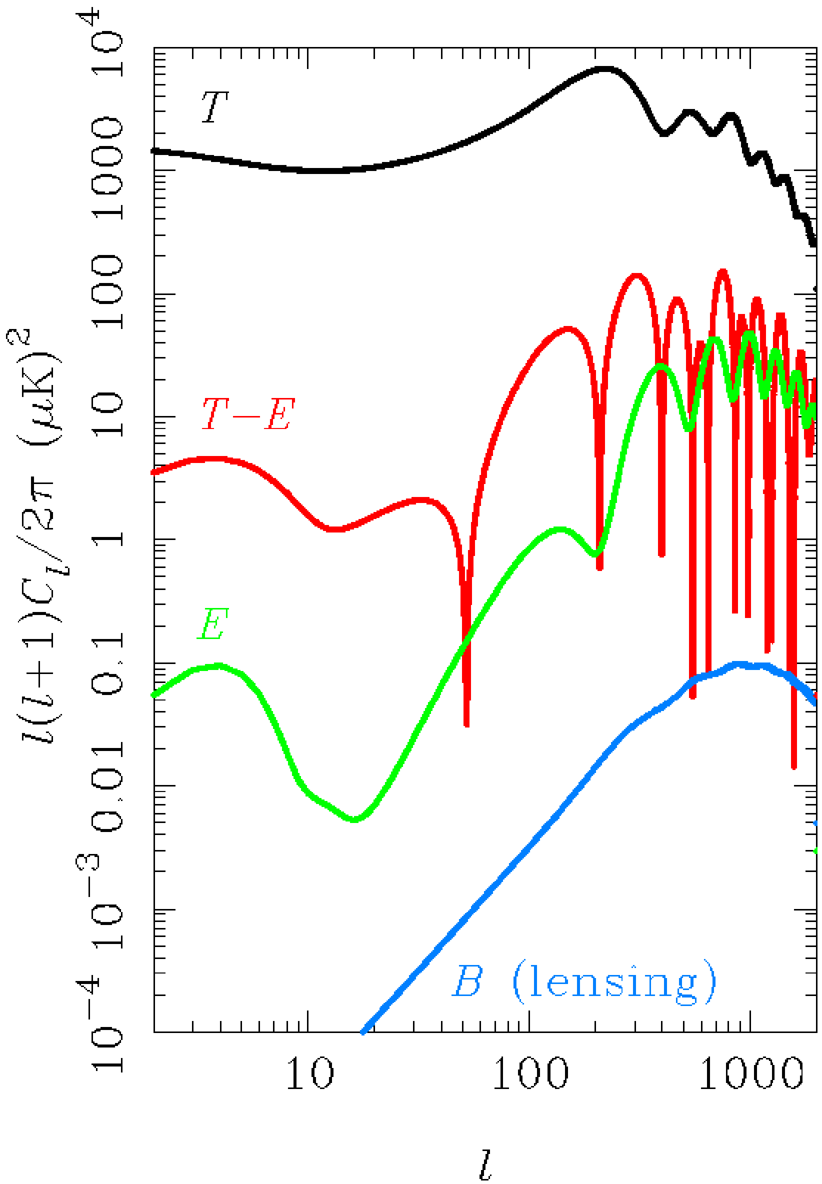
$$
\sum_m \Theta_{2m} Y_{2m}(\boldsymbol{e}) \sim \frac{1}{2} l_p^2 e^{\langle i} e^{j \rangle} \partial_i \partial_j (\Theta_0 + \psi) - l_p e^{\langle i} e^{j \rangle} \partial_j v_{(b)i}
$$

  - Intrinsic temperature contribution suppressed by factor $\sim k l_p$ cf Doppler

- Polarization traces baryon velocity at recombination $\Rightarrow$ peaks at troughs of ΔT

- Large-angle polarization from recombination small since quadrupole source generated causally

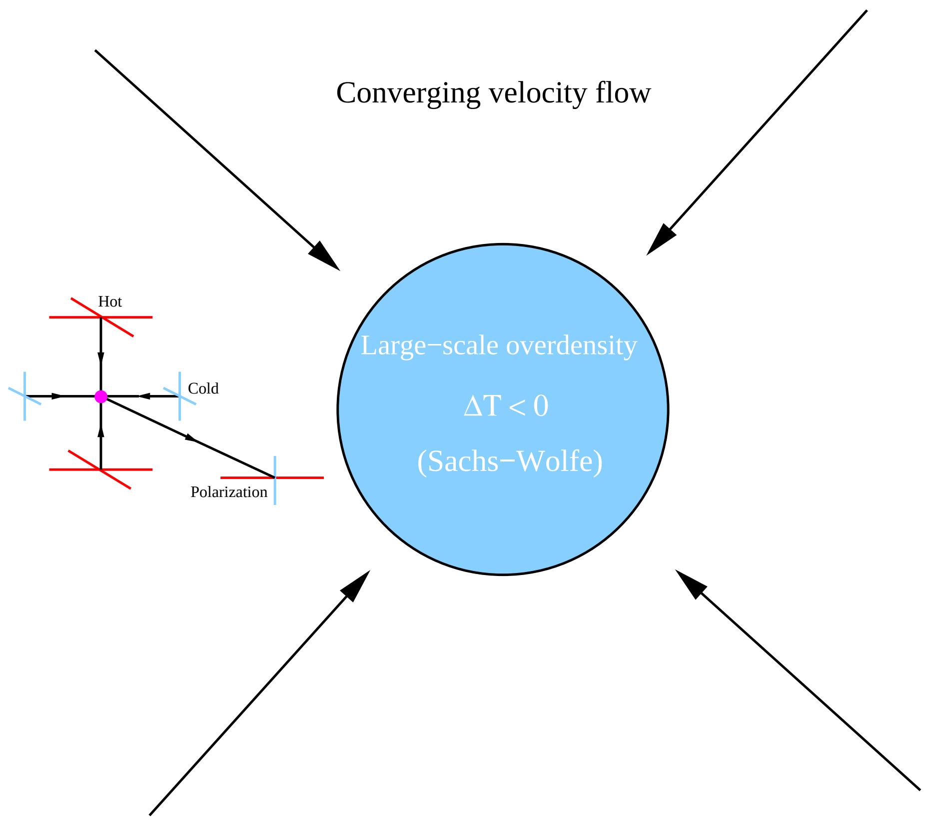
Scalar Polarization Power Spectra



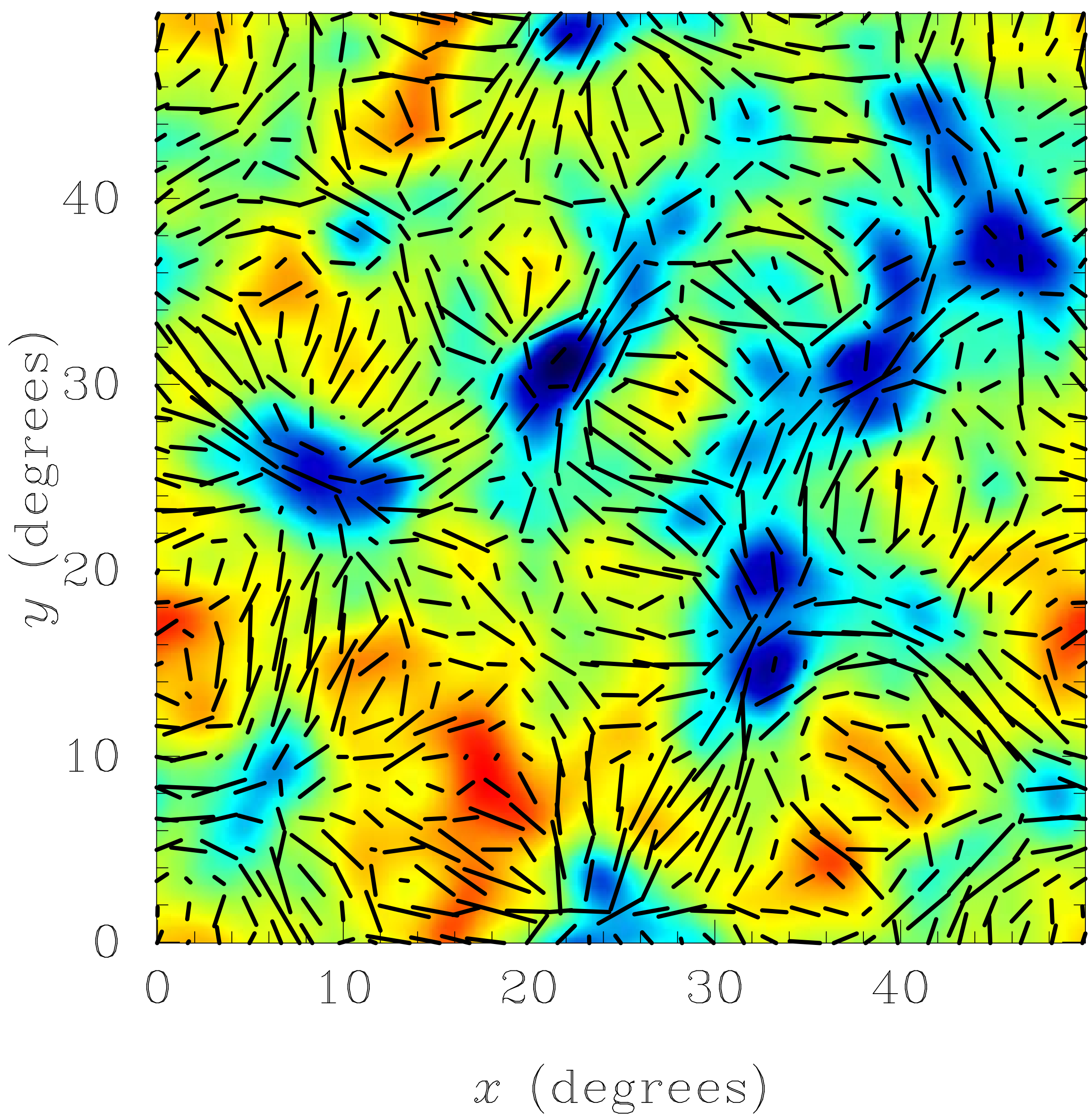
- Polarization mostly probing $\boldsymbol{v}_b$ at last scattering
  - C_l^E peaks at minima of C_l^T
- Correlations between T and E
- Additional large-angle polarization from scattering around reionization
- B-modes are generated at second order, e.g. by lensing (see later)

CORRELATED POLARIZATION IN REAL SPACE

- On largest scales, infall into potential wells at last scattering generates e.g. radial polarization around large-scale cold spots
- Sign of correlation scale-dependent inside horizon

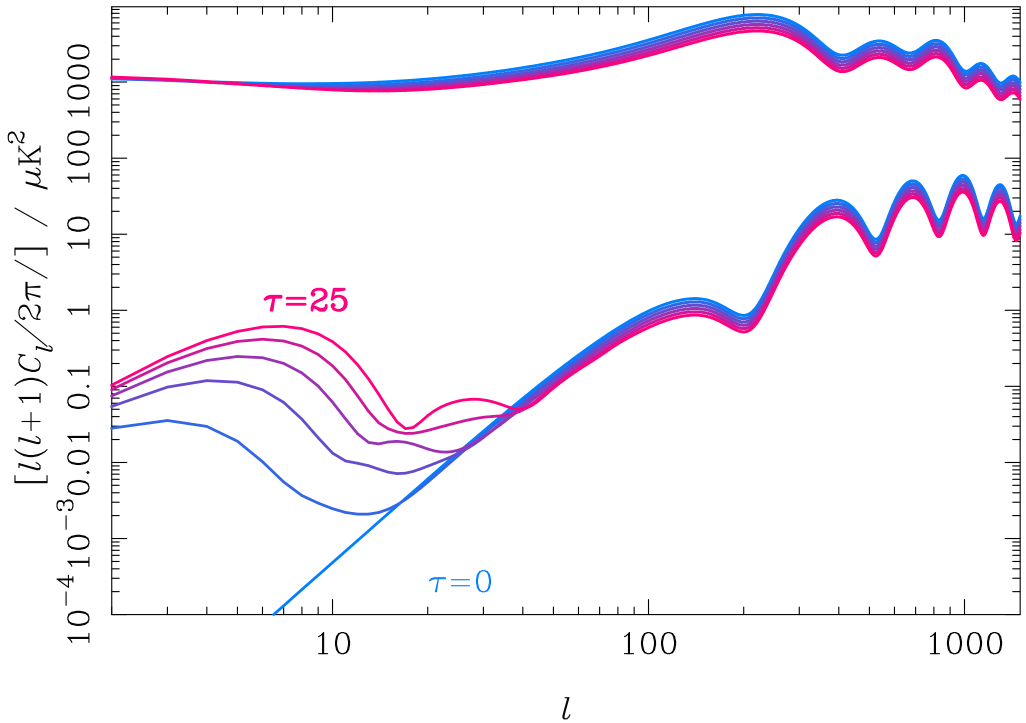


Correlated polarization in real space



LARGE-ANGLE POLARIZATION FROM REIONIZATION

- Temperature quadrupole at reionization peaks around $k(\eta_{\rm re} - \eta_*) \sim 2$
  - Re-scattering generates polarization on this linear scale $\rightarrow$ projects to $l \sim 2(\eta_0 - \eta_{\rm re})/(\eta_{\rm re} - \eta_*)$
  - Amplitude of polarization $\propto$ optical depth through reionization $\rightarrow$ best way to measure τ with CMB

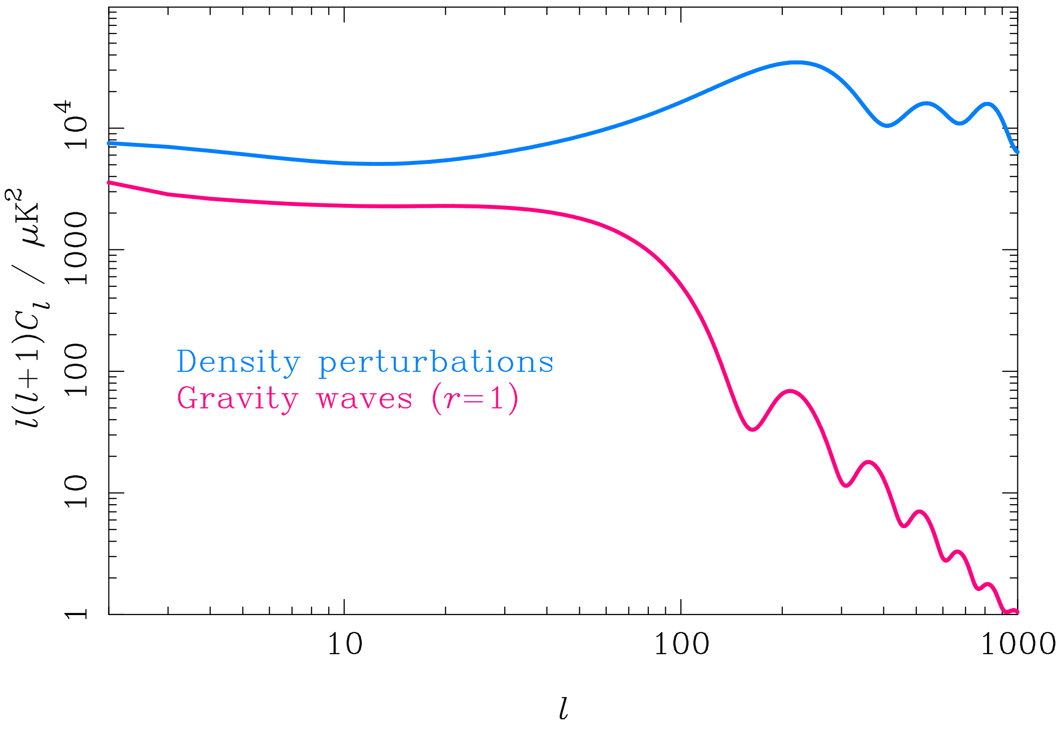


Gravity waves and the CMB

- Tensor metric perturbations $ds^2 = a^2[d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j]$ with $\delta^{ij}h_{ij} = 0$
  - Shear (anisotropic expansion) $\propto \dot{h}_{ij}$ gives anisotropic redshifting $\Rightarrow$

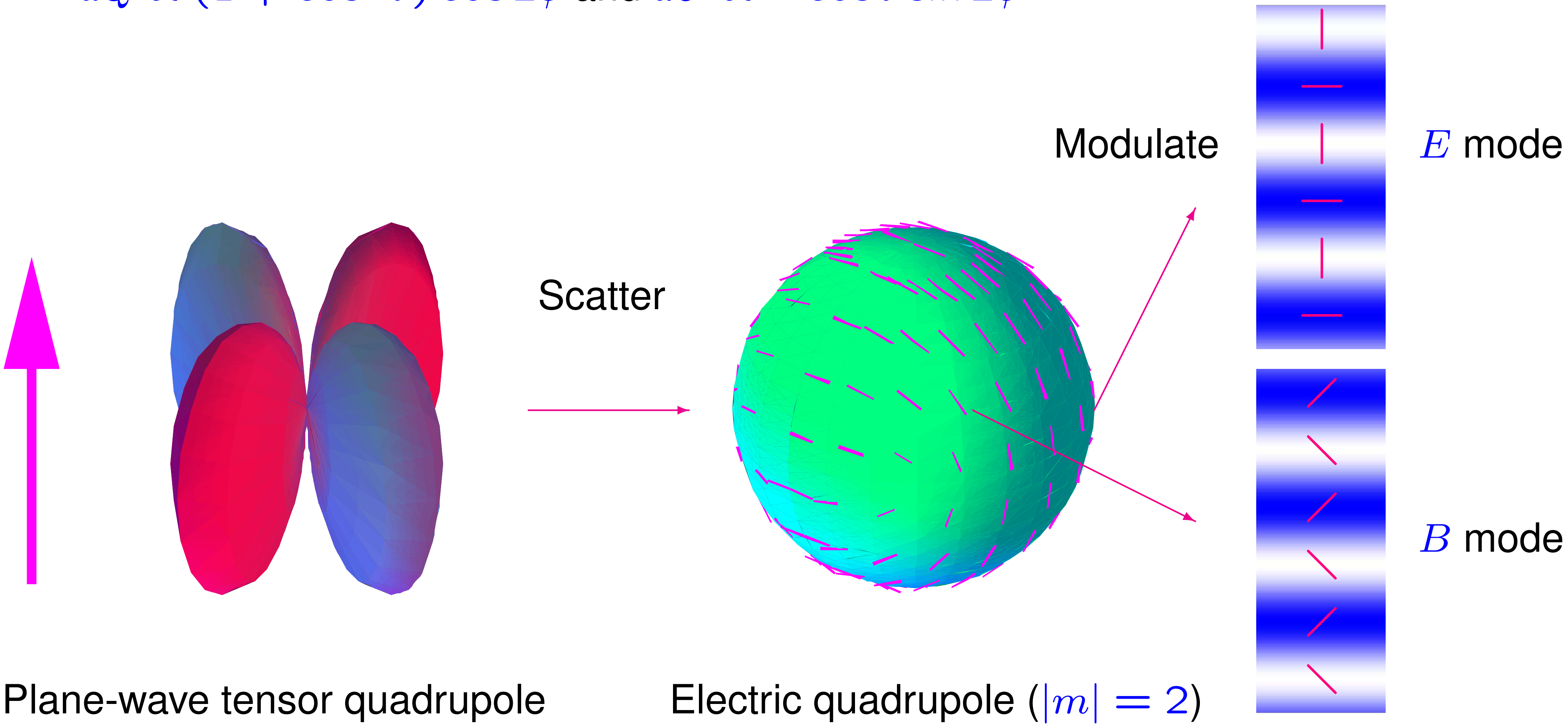
$$\Theta(\hat{\boldsymbol{n}}) \approx -\frac{1}{2}\int d\eta\, \dot{h}_{ij}\hat{n}^i\hat{n}^j$$

  - Only contributes on large scales since h_{ij} decays like a^{-1} after entering horizon



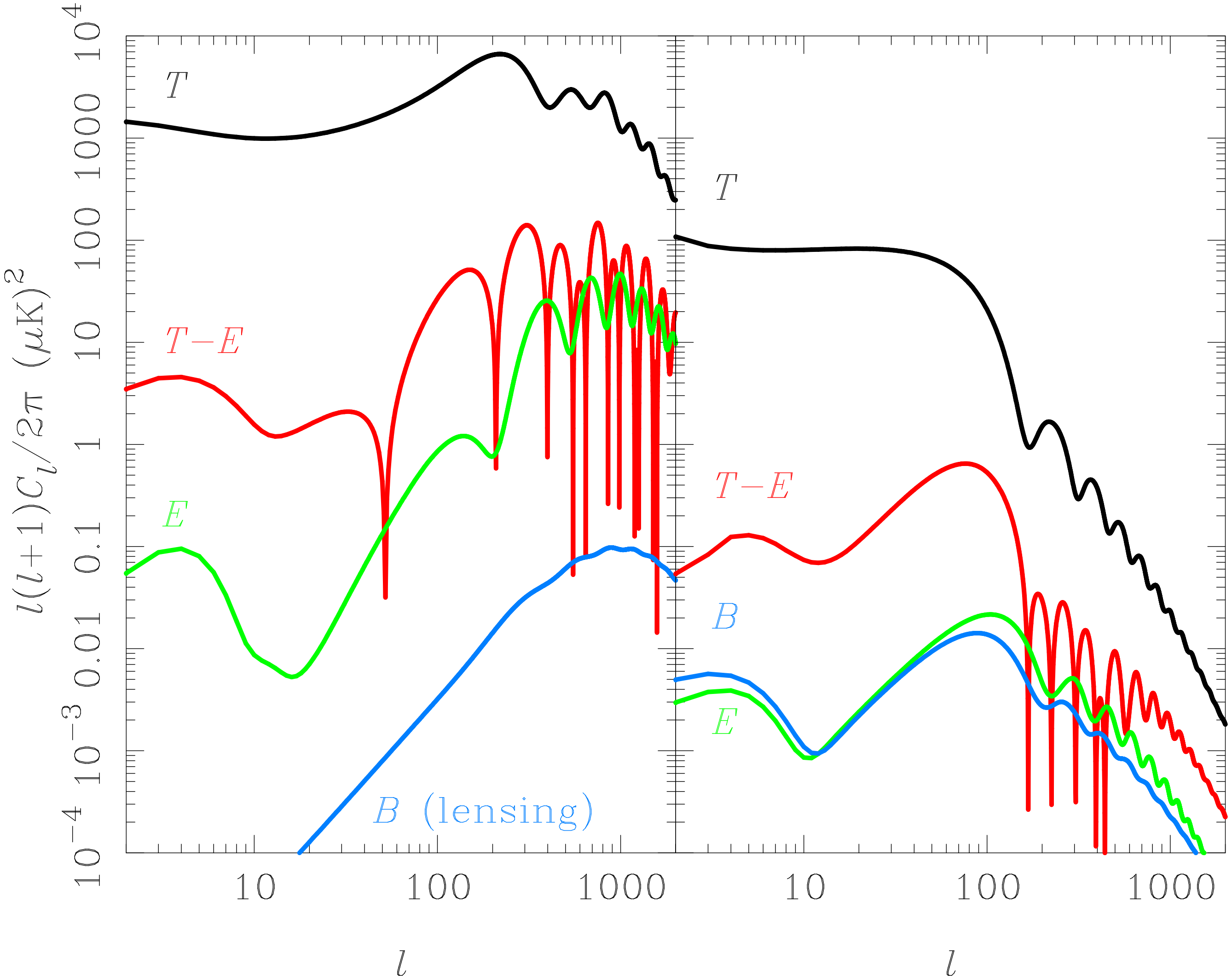
CMB POLARIZATION FROM GRAVITY WAVES

- For single $+$-polarized gravity wave with $\hat{\boldsymbol{k}}$ along z, $\Theta_{2m} \propto \delta_{m2} + \delta_{m\,-2}$ so $dQ \propto (1 + \cos^2\theta)\cos 2\phi$ and $dU \propto -\cos\theta \sin 2\phi$



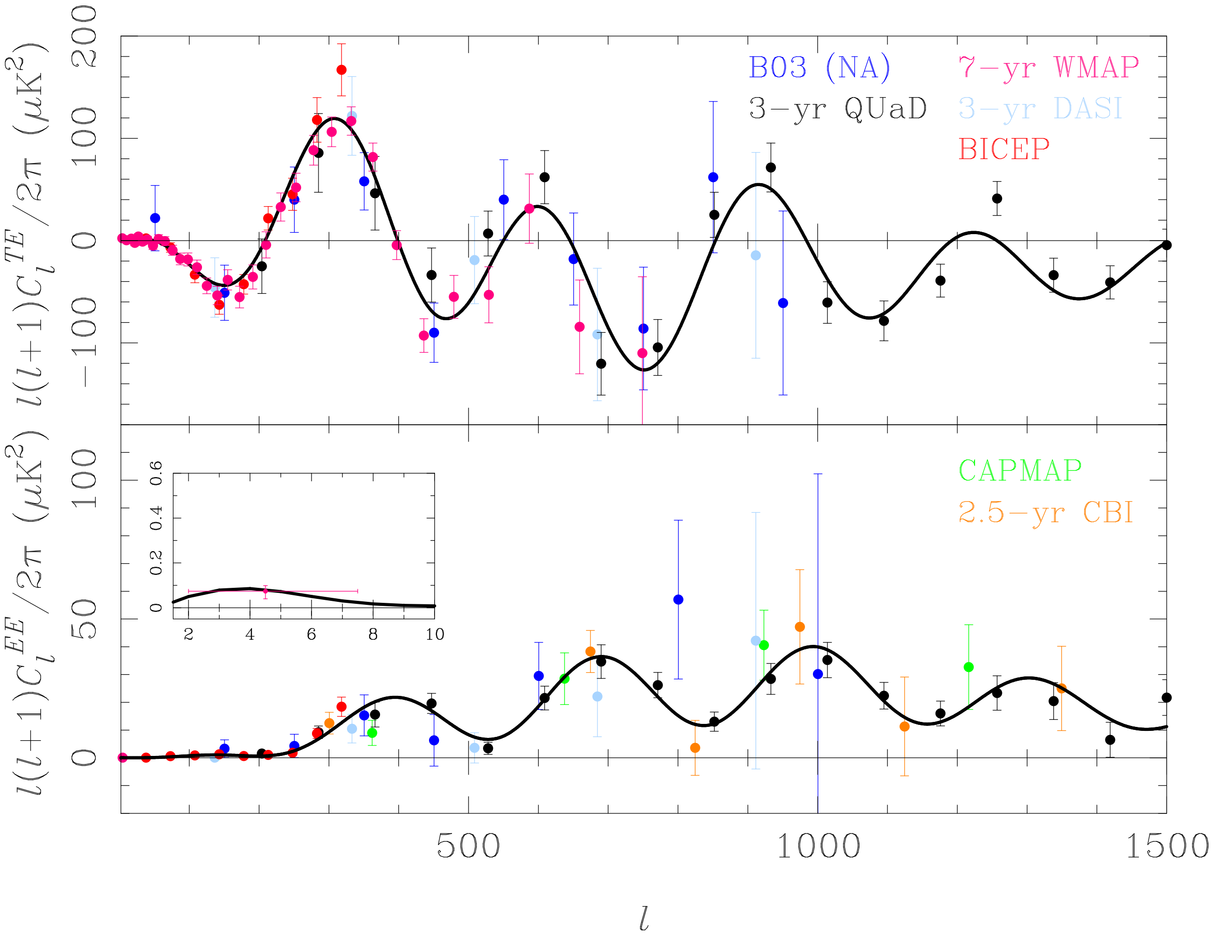
- Gravity waves produce both E- and B-mode polarization (with roughly equal power)

SCALAR AND TENSOR POWER SPECTRA ($r = 0.2$)



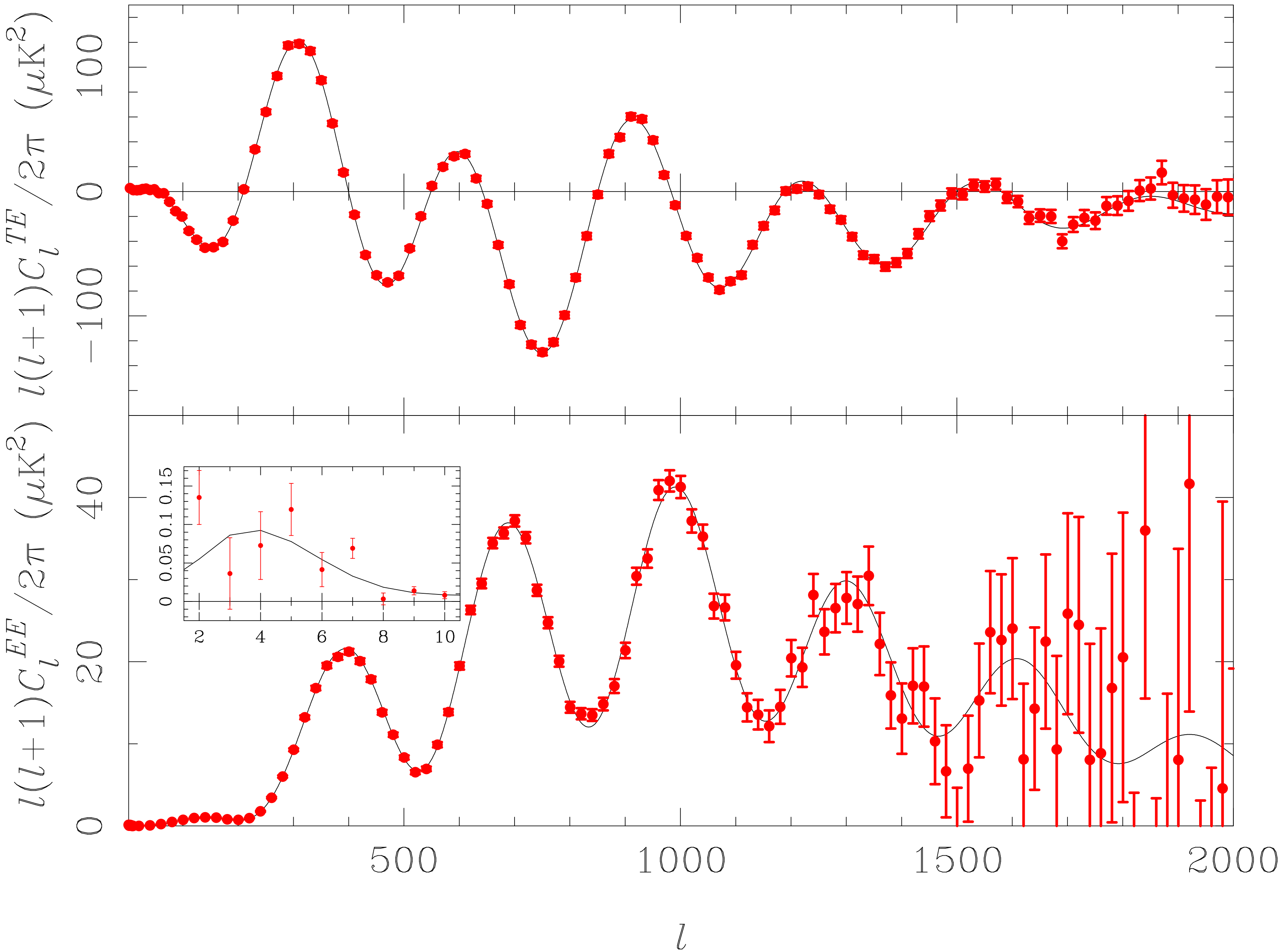
Observational status and applications

CURRENT MEASUREMENTS: TE AND EE



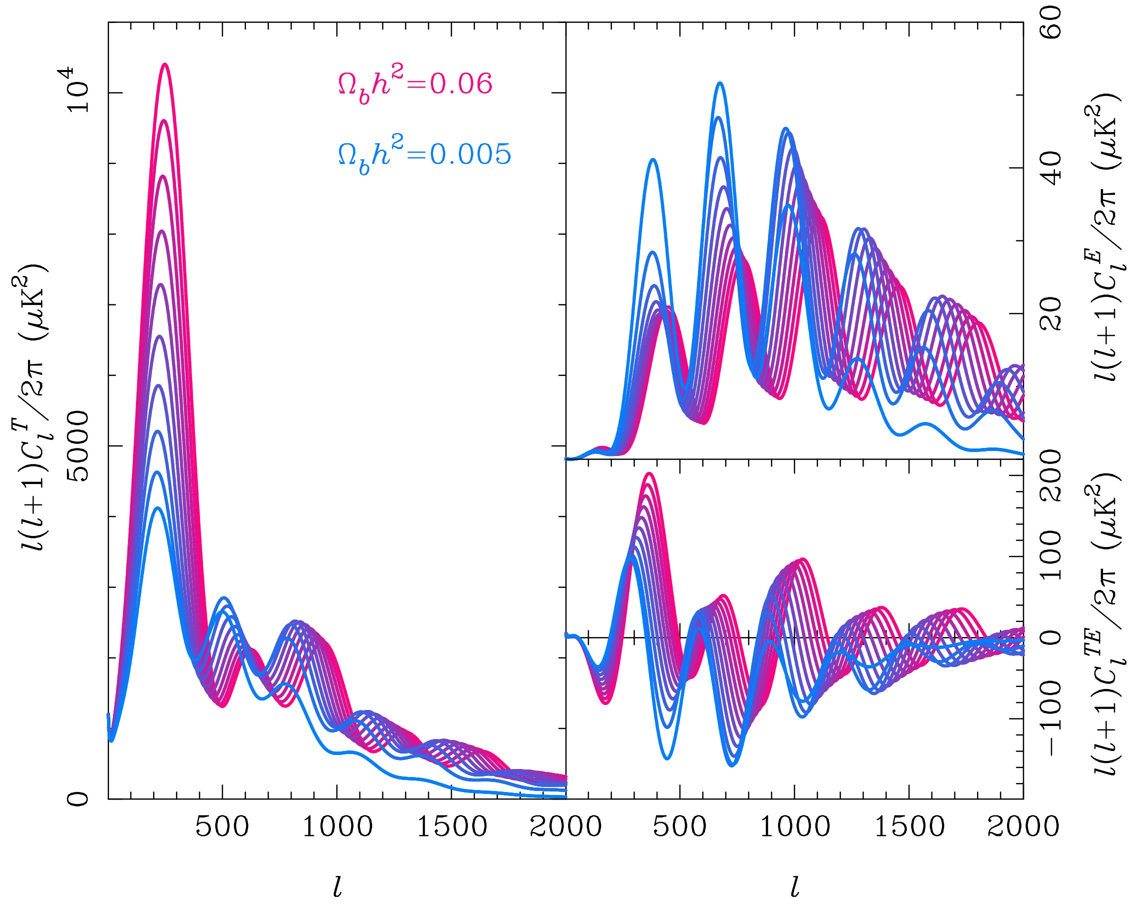
- Super-horizon, adiabatic fluctuations from TE anti-correlation for $5° > \theta > 1°$
- Optical depth through reionization: $\tau = 0.087 \pm 0.014$ from WMAP7

TE AND EE FORECASTS FOR PLANCK



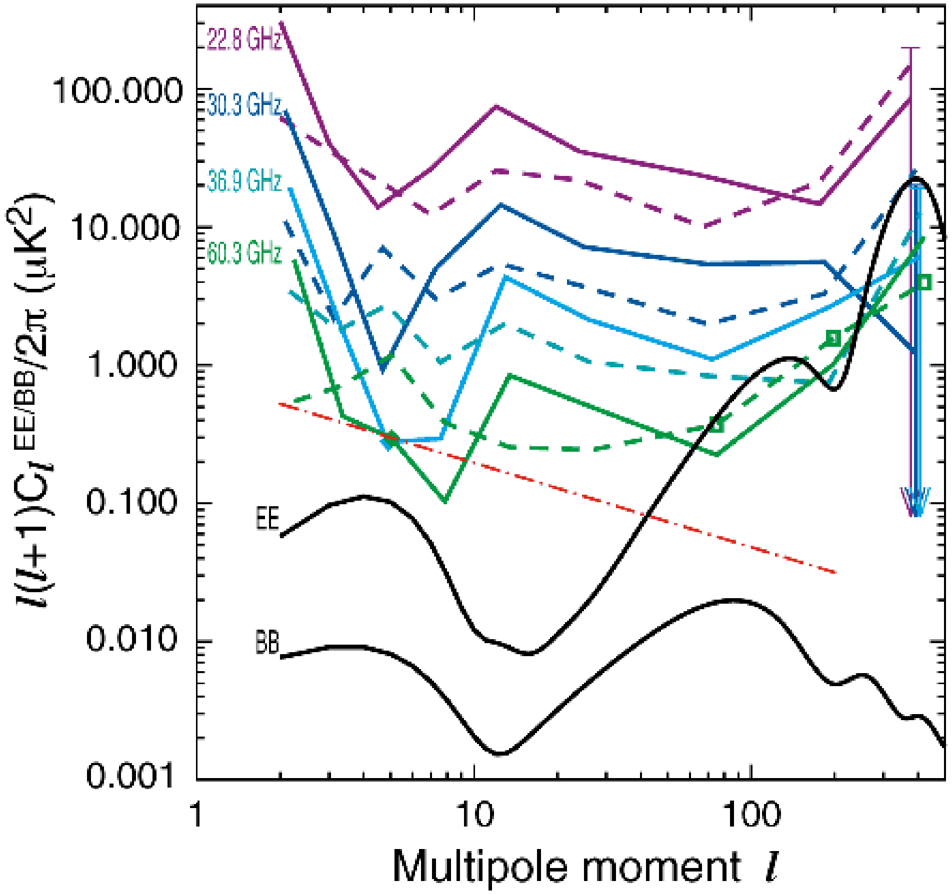
PARAMETERS FROM POLARIZATION: ACOUSTIC PEAKS AND DAMPING TAIL

- 30% improvement on $\Omega_b h^2$, $\Omega_m h^2$, h and n_s from Planck E modes at $l > 20$
- Other beneficiaries of polarization: isocurvature modes, Helium abundance etc.



Parameters from large-angle polarization

- Reionization: $\tau = 0.087 \pm 0.014$ from WMAP7
  - Expect $\Delta\tau = 0.004$ from nominal Planck mission
- Constraints on gravity waves (next)

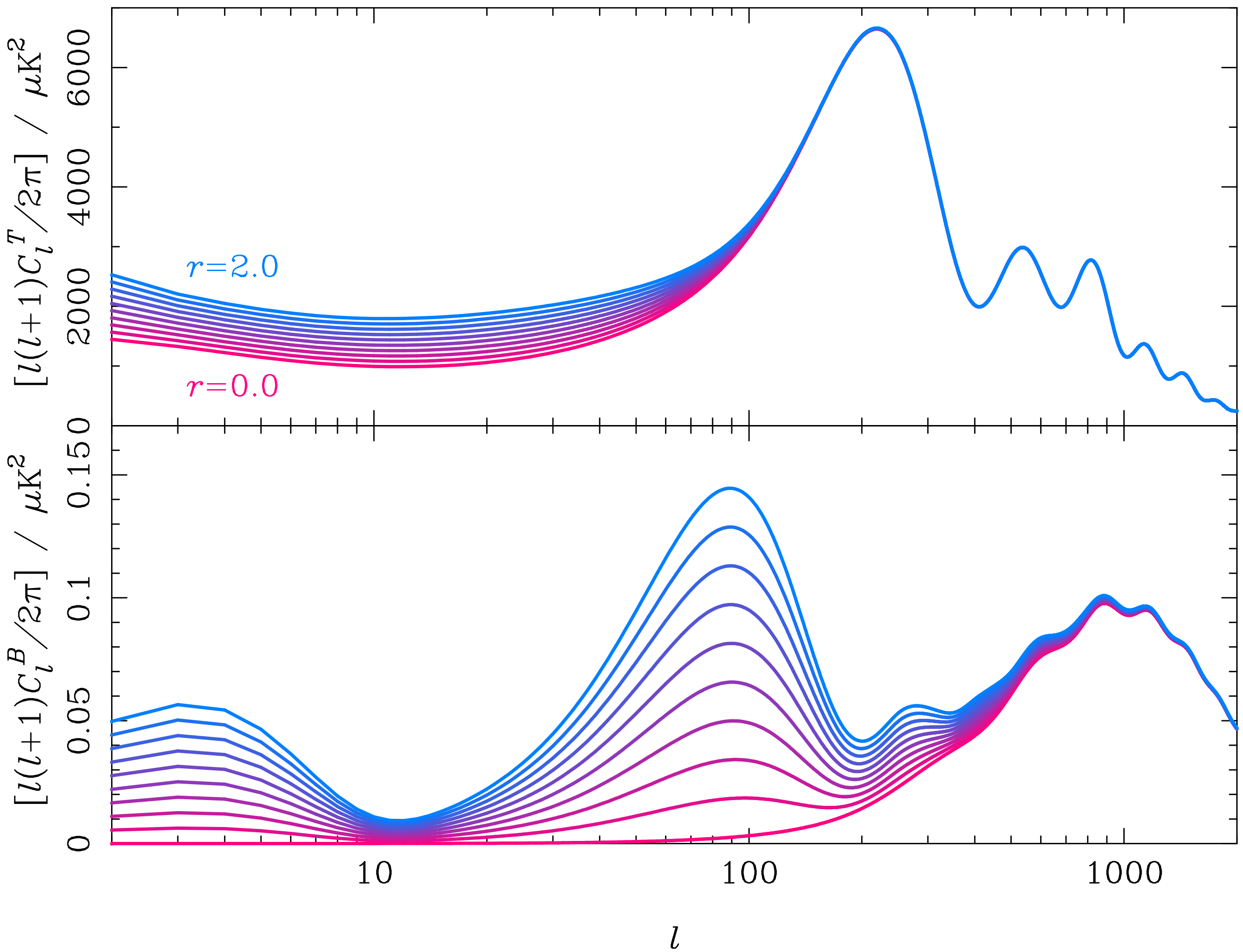


Page et al. (2007)

PRIMORDIAL GRAVITATIONAL WAVES AND B-MODES

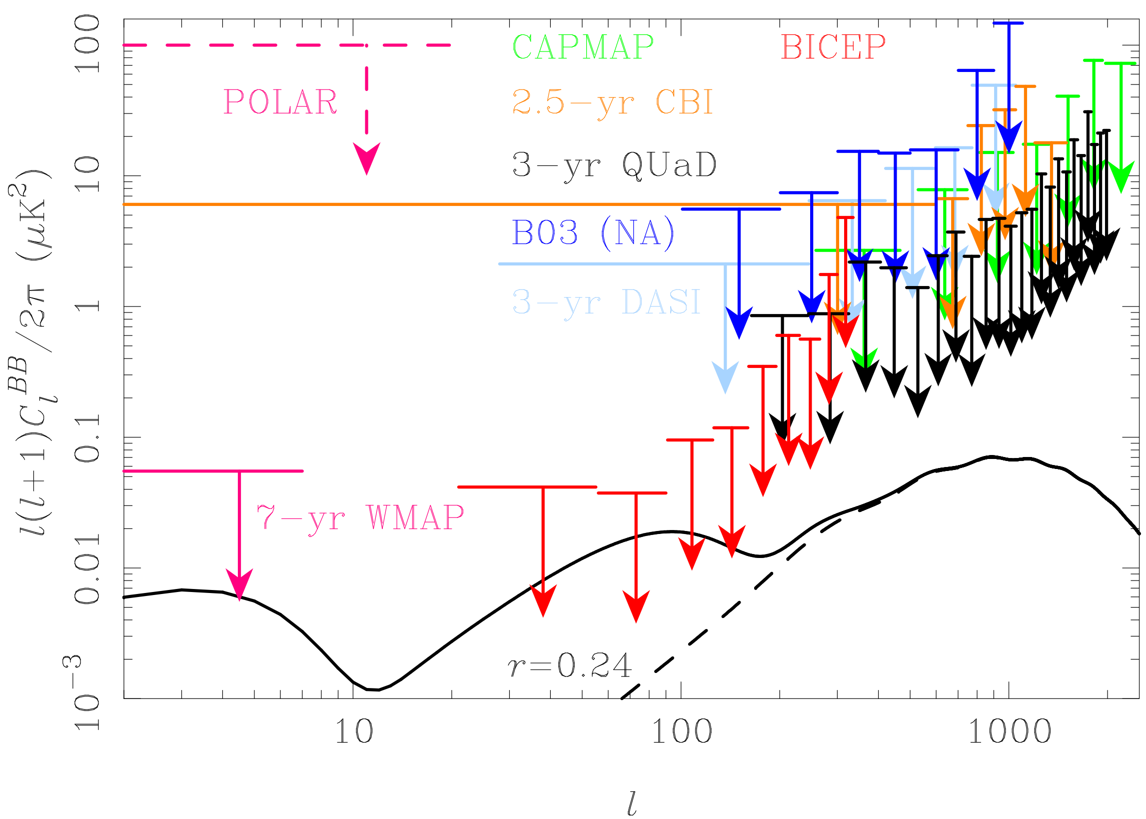
- Well motivated by inflation models
  - Amplitude depends *only* on Hubble parameter during inflation
  - Should be detectable in CMB in large-field models ($\Delta\phi > M_{\mathrm{Pl}}$ requiring GUT-scale inflation)
- Detection would rule out some models (e.g. cyclic)
  - Also problematic for many string-inspired models
- Current best limit from WMAP7 (ΔT and E) alone: $r < 0.36$ (95% CL)
- Improves to $r < 0.24$ with inclusion of BAO and H_0 (degeneracy breaking)

Gravitational waves in the CMB



- Cosmic variance of dominant scalar fluctuations limits $\Delta r = 0.07$ from T and $\Delta r = 0.02$ if include E
  - Degeneracies make actual limits worse; WMAP7 alone $r < 0.36$ (95% CL)

CURRENT CONSTRAINTS: BB



- BICEP limit (Chiang et al. 2009) on r from B-modes alone: $r < 0.73$ (95% CL)
- B-modes will improve on $r < 0.24$ in next generation of experiments (Planck, BICEP2, EBEX, SPIDER etc.)

THE (NEAR-)FUTURE OF CMB

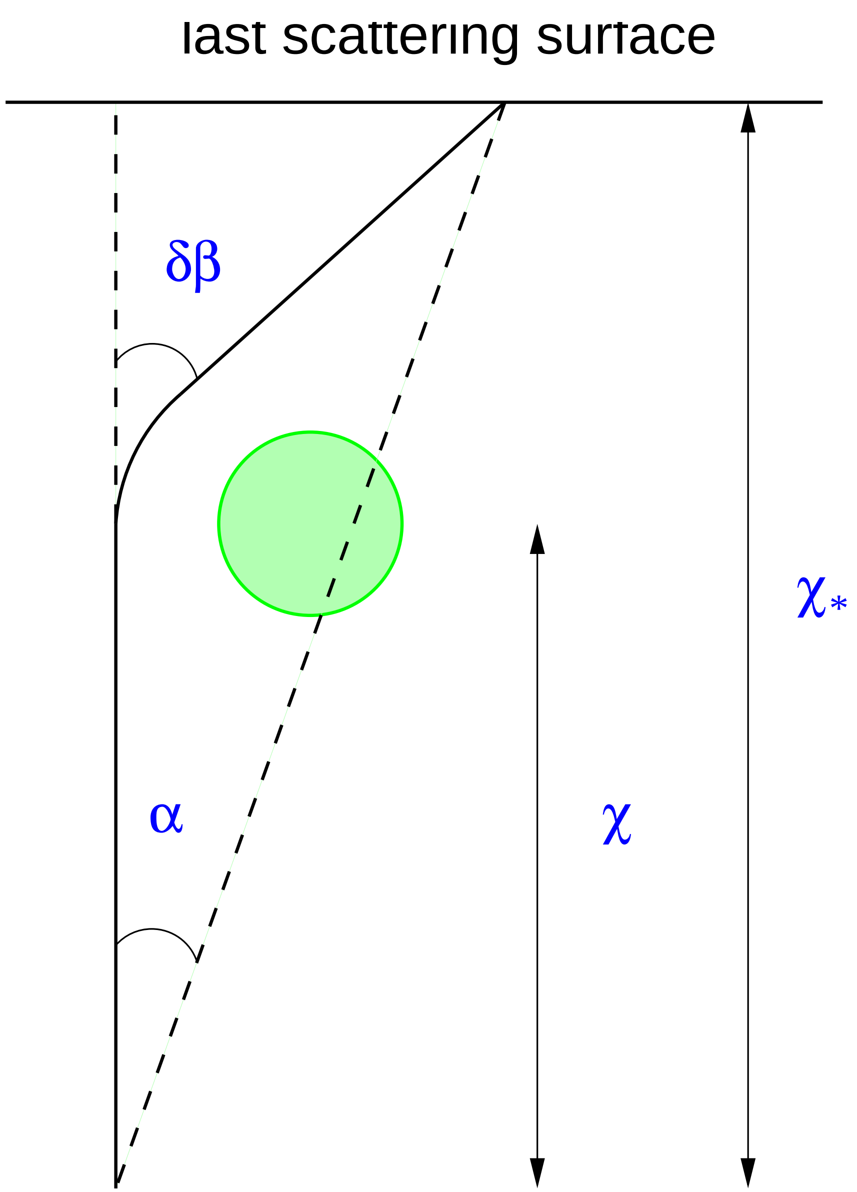
- Better measurements of higher peaks and damping tail
  - Sub-percent errors in acoustic parameters from Planck and shape of primordial power spectrum
- Better E-mode polarization
  - Some improvements in parameters and tests of large-angle anomalies
- Direct detection of weak lensing effect in CMB temperature and polarization
- Physics from scattering secondaries (reionization and clusters) and lensing reconstruction
- Tighter constraints on non-Gaussianity ($\Delta f_{NL} \sim 5$ from Planck)
  - Polarization improves Δf_{NL} by up to factor 2
- Gravity waves from B-mode polarization (E_{inf} and improved inflation phenomenology)?

Introduction to CMB lensing

CMB LENSING

- CMB photons gravitationally deflected by LSS in propagating from last-scattering surface
- Geometric deflections conserve surface brightness so no effect on uniform $2.275\,\mathrm{K}$ CMB
- CMB anisotropies and polarization mostly sourced around last scattering over distance $\sim 100\,\mathrm{Mpc}$
  - Narrow compared to $14000\,\mathrm{Mpc}$ distance to last scattering: approximate CMB as single source plane
- Only consider transverse displacements here
  - Radial displacements are suppressed geometrically

CMB Lensing Orders of Magnitude



- Bending angle $\delta\beta = -\delta\chi\nabla_\perp(\phi+\psi)$
  - Typical potential $\sim 2\times10^{-5}$ so expect $\delta\beta\sim10^{-4}$
- Coherence size of potentials $\sim$ 300 Mpc so random walk with $14000/300 \sim 50$ steps from last scattering
  - Net deflection typically $\sqrt{50}\times10^{-4}\sim 2\,\mathrm{arcmin}$
  - Coherent over $300/7000\sim2^\circ$ for lens midway to last scattering

QUALITATIVE EFFECTS ON CMB OBSERVABLES

- Acoustic peaks on degree scales:
  - Deflections much smaller (2 arcmin) but coherent over size of acoustic features
  - Typical CMB hotspot has its size increased/decreased by $2/60 \sim 3\%$ smoothing out primary acoustic peaks
- Primary CMB smooth on arcmin scales (diffusion damping):
  - Arcmin scale lenses imprint (non-Gaussian) small scale power exceeding primary CMB
- Only *relative deflections* are important!
- Transforms E-mode polarization to B (and vice versa)
- Introduces non-Gaussianity to CMB

So why should you care?

- Must include effect on power spectra to avoid biases in parameter determination
  - Can use this to break (some!) degeneracies
- Lens-induced B-modes act like white noise for primordial gravity wave searches
- Use non-Gaussianity to reconstruct deflections
  - Further constraints on dark parameters
- Must account for "local" bispectrum in f_{NL}^{local} searches
- Constrain cluster masses at high redshift

COMPARISONS WITH GALAXY LENSING

- Single source plane at known distance (fixed by background cosmology)
- Statistics of sources on source plane well understood
  - Given by linear-theory power spectrum
  - Magnification and shear equally useful so work with deflection angles directly
- Sources are large, i.e. CMB is smooth on small scales
  - CMB features do not "point-like" sample shear and magnification
- Source plane very distant – most efficient lenses at $z \sim 2$ – and large linear lenses
- Full-sky observations so account for spherical geometry for accurate correlation results

Lensing Deflections

- Lensing preserves brightness; simply re-maps temperature and polarization from recombination

$$\tilde{\Theta}(\hat{\boldsymbol{n}}) = \Theta(\hat{\boldsymbol{n}} + \boldsymbol{\alpha}) \qquad (\tilde{Q} \pm i\tilde{U})(\hat{\boldsymbol{n}}) = (Q \pm iU)(\hat{\boldsymbol{n}} + \boldsymbol{\alpha})$$

- Lensing (angular) deflection field from summing contributions of lenses along line of sight:

$$\begin{aligned} \boldsymbol{\alpha}(\hat{\boldsymbol{n}}) &= -2\int_0^{\chi_*} d\chi \frac{d_A(\chi_* - \chi)}{d_A(\chi_*)} \boldsymbol{\nabla}_\perp \Psi(\chi\hat{\boldsymbol{n}}; \eta_0 - \chi) \qquad 2\Psi \equiv \psi_N + \phi_N \\ &= -2\int_0^{\chi_*} d\chi \frac{d_A(\chi_* - \chi)}{d_A(\chi_*) d_A(\chi)} \boldsymbol{\nabla}_{\hat{\boldsymbol{n}}} \Psi(\chi\hat{\boldsymbol{n}}; \eta_0 - \chi) \end{aligned}$$

  - To $O(\Psi)$ can take integral along background line of sight (Born approximation)

- Deflection is then angular gradient of *deflection potential*, $\boldsymbol{\alpha} = \boldsymbol{\nabla}_{\hat{\boldsymbol{n}}}\psi$:

$$\psi(\hat{\boldsymbol{n}}) \equiv -2\int_0^{\chi_*} d\chi \frac{d_A(\chi_* - \chi)}{d_A(\chi_*) d_A(\chi)} \Psi(\chi\hat{\boldsymbol{n}}; \eta_0 - \chi)$$

Power spectrum of lensing potential

$$\psi(\hat{\boldsymbol{n}}) = -2\int_0^{\chi_*} d\chi \frac{d_A(\chi_* - \chi)}{d_A(\chi_*)d_A(\chi)} \Psi(\hat{\boldsymbol{n}}\chi; \eta_0 - \chi)$$

- Fourier expand (Weyl) potential as $\Psi(\boldsymbol{x};\eta) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}} \Psi(\boldsymbol{k};\eta) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$ and use plane-wave expansion to get

$$\psi_{lm} = -8\pi i^l \int d\chi \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}} \frac{d_A(\chi_* - \chi)}{d_A(\chi_*)d_A(\chi)} \Psi(\boldsymbol{k};\eta_0 - \chi) j_l(k\chi) Y_{lm}^*(\hat{\boldsymbol{k}})$$

- In terms of unequal-time power spectrum of $\Psi(\boldsymbol{k};\eta)$,

$$\langle \Psi(\boldsymbol{k};\eta)\Psi^*(\boldsymbol{k}';\eta') = \frac{2\pi^2}{k^3} \mathcal{P}_\Psi(k;\eta,\eta')\delta(\boldsymbol{k}-\boldsymbol{k}')$$

get power spectrum of deflection potential

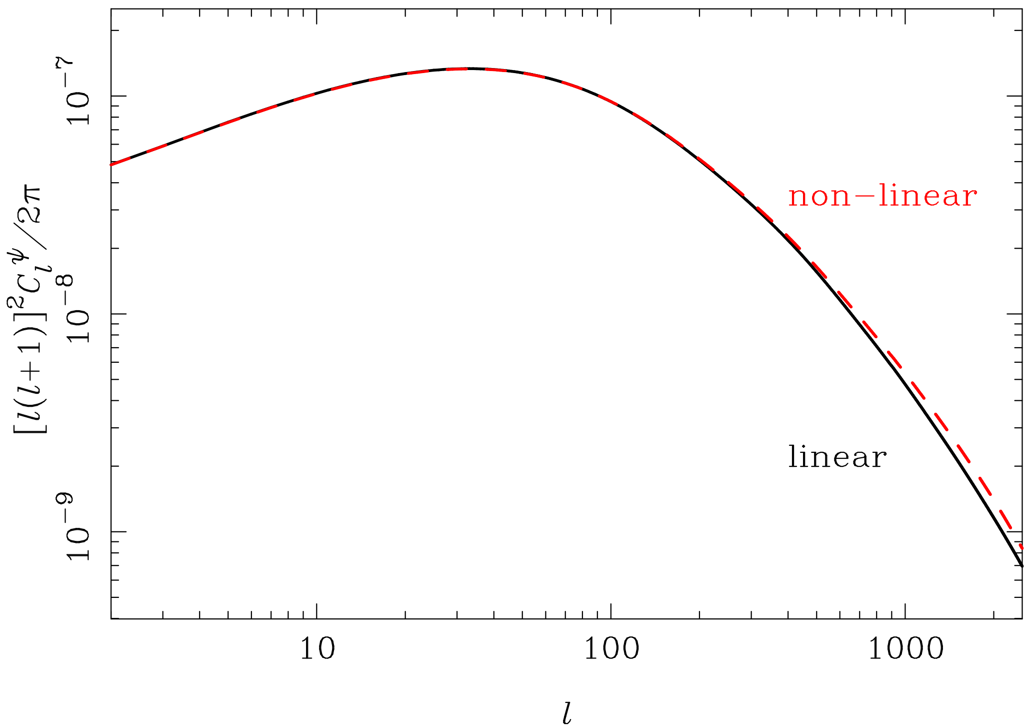
$$C_l^\psi = 16\pi \int d\ln k \int_0^{\chi_*} d\chi \int_0^{\chi_*} d\chi' \, \mathcal{P}_\Psi(k;\eta_0 - \chi, \eta_0 - \chi') j_l(k\chi) j_l(k\chi') \frac{d_A(\chi_* - \chi)}{d_A(\chi_*)d_A(\chi)} \frac{d_A(\chi_* - \chi')}{d_A(\chi_*)d_A(\chi')}$$

DEFLECTION POWER SPECTRUM

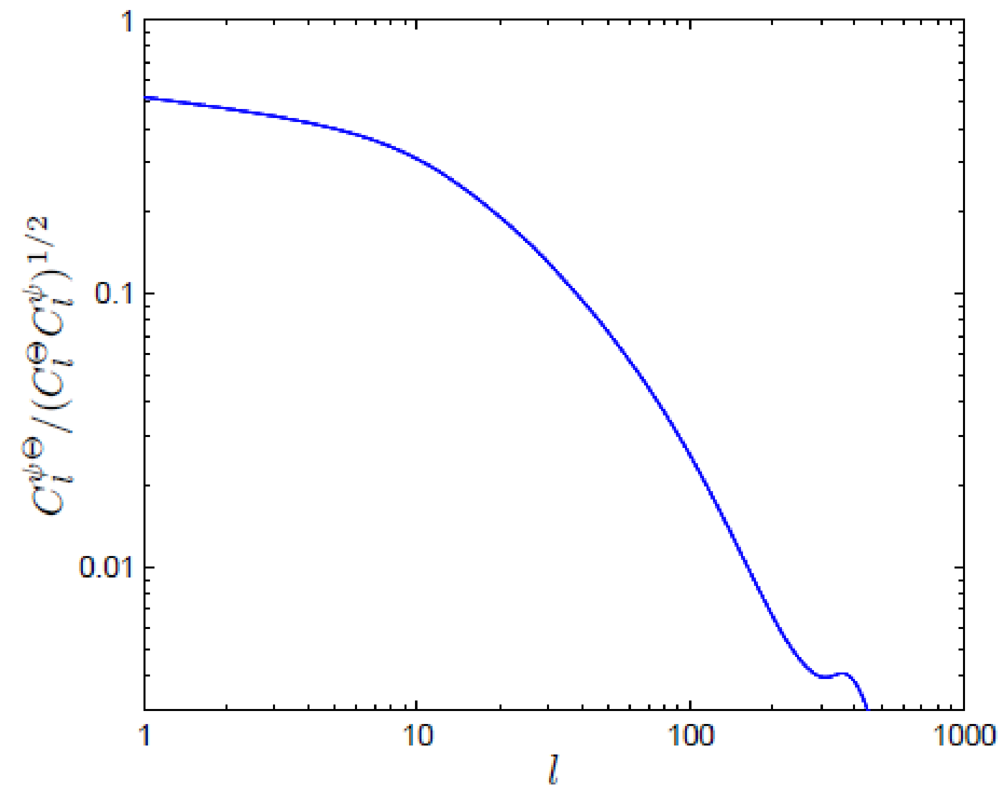
- Limber approximation ok except on large scales:

$$C_l^{\psi} \approx \frac{8\pi^2}{l^3} \int_0^{\chi_*} \chi d\chi \, \mathcal{P}_{\Psi}(l/\chi; \eta_0 - \chi) \left(\frac{d_A(\chi_* - \chi)}{d_A(\chi_*) d_A(\chi)} \right)^2$$

- Deflection angle power spectrum is $l(l+1)C_l^{\psi}$
  - $d\langle \boldsymbol{\alpha}^2 \rangle / d \ln l \approx [l(l+1)]^2 C_l^{\psi} / 2\pi$ peaks at $l \sim 40$ (few degrees coherence)
  - Receives contributions out to high redshift



Θ-ψ CORRELATION

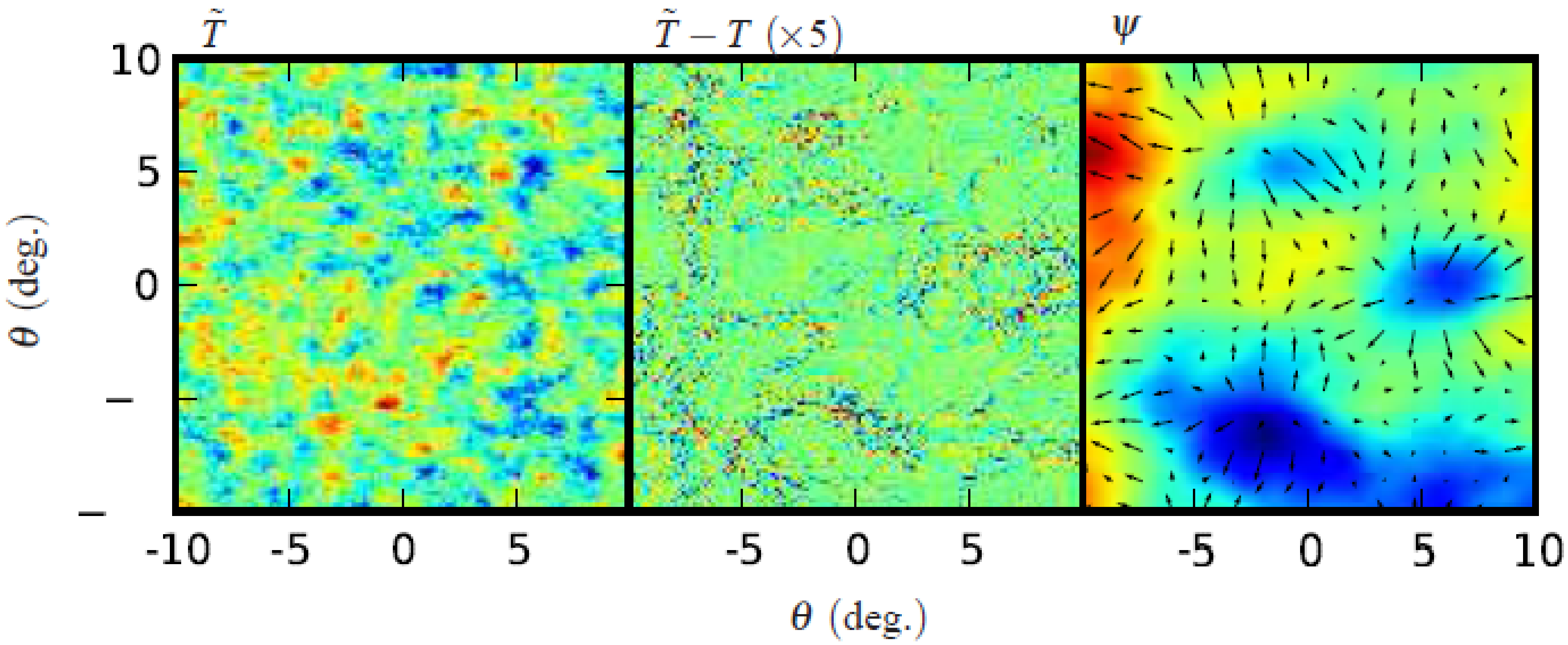


- ISW ($z < 1$) sourced by LSS that also lenses CMB
  - Produces *positive* large-angle Θ-ψ correlation, important for non-Gaussianity

LENSING OF CMB POWER SPECTRA

LENSING EFFECT ON TEMPERATURE

$$\tilde{\Theta}(\boldsymbol{x}) = \Theta(\boldsymbol{x}+\boldsymbol{\alpha}) = \Theta(\boldsymbol{x}) + \boldsymbol{\alpha}(\boldsymbol{x})\cdot\nabla\Theta(\boldsymbol{x}) + \cdots$$



Hanson, AC & Lewis (2009)

- Antony Lewis's temperature and polarization re-mapping tool LensPix:
  `http://cosmologist.info/lenspix/`

CALCULATING LENSED SPECTRA: APPROXIMATIONS

- Lensing potential uncorrelated to temperature
  - Good, except on large scales (ISW) but ignoring is harmless
- Gaussian lensing potential
  - Breaks down on non-linear scales but even then ok for calculating lensed power spectra
- Optional simplifying assumptions:
  - Work in flat-sky limit (induces percent level errors in lensed BB)
  - Series expansion to leading order (will have to relax later)

Flat-sky temperature anisotropies

- Expand unlensed temperature in Fourier modes:

$$\Theta(\boldsymbol{x}) = \int \frac{d^2\boldsymbol{l}}{2\pi}\Theta(\boldsymbol{l})e^{i\boldsymbol{l}\cdot\boldsymbol{x}} \quad , \quad \langle\Theta(\boldsymbol{l})\Theta^*(\boldsymbol{l}')\rangle = C_l\delta(\boldsymbol{l}-\boldsymbol{l}')$$

- Since only relative lensing displacements important, convenient to work with correlation functions

$$\xi(r) = \langle\Theta(\boldsymbol{x})\Theta(\boldsymbol{x}+\boldsymbol{r})\rangle = \int \frac{d^2\boldsymbol{l}}{(2\pi)^2}C_l e^{i\boldsymbol{l}\cdot\boldsymbol{r}}$$

$$\Rightarrow \quad C_l = \int d^2\boldsymbol{r}\,\xi(r)e^{-i\boldsymbol{l}\cdot\boldsymbol{r}} = 2\pi\int r dr\,\xi(r)J_0(lr)$$

- Lensed correlation function by $\boldsymbol{r} \to \boldsymbol{r} + \boldsymbol{\alpha}(\boldsymbol{x}+\boldsymbol{r}) - \boldsymbol{\alpha}(\boldsymbol{x}) \equiv \boldsymbol{r} + \boldsymbol{\alpha}' - \boldsymbol{\alpha}$ and averaging over lenses:

$$\tilde{\xi}(r) = \int \frac{d^2\boldsymbol{l}}{(2\pi)^2}C_l\langle e^{i\boldsymbol{l}\cdot\boldsymbol{r}}e^{i\boldsymbol{l}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})}\rangle$$

  – Only depends on relative displacement $\boldsymbol{\alpha}' - \boldsymbol{\alpha}$!

LEADING-ORDER CALCULATION

- Expand expectation of exponential as

$$\langle e^{i\boldsymbol{l}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})}\rangle = 1 + i\langle \boldsymbol{l}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})\rangle - \frac{1}{2}\langle[\boldsymbol{l}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})]^2\rangle + \cdots$$
$$= 1 - \langle(\boldsymbol{l}\cdot\boldsymbol{\alpha})^2\rangle + l_i l_j \langle \alpha_i \alpha'_j\rangle + \qquad \text{(statistical isotropy)}$$

- Correlation tensor of displacements is

$$\langle \alpha_i \alpha'_j \rangle = \int \frac{d^2\boldsymbol{L}}{(2\pi)^2} L_i L_j C_L^{\psi} e^{i\boldsymbol{L}\cdot\boldsymbol{r}}$$

  – Follows that

$$\langle(\boldsymbol{l}\cdot\boldsymbol{\alpha})^2\rangle = \frac{1}{2} l^2 \int \frac{L\,dL}{2\pi} L^2 C_L^{\psi} = \frac{1}{2} l^2 \langle \boldsymbol{\alpha}^2\rangle \qquad \text{(independent of } \boldsymbol{r}\text{)}$$
$$\langle(\boldsymbol{l}\cdot\boldsymbol{\alpha})(\boldsymbol{l}\cdot\boldsymbol{\alpha}')\rangle = \int \frac{d^2\boldsymbol{L}}{(2\pi)^2} C_L^{\psi} (\boldsymbol{L}\cdot\boldsymbol{l})^2 e^{i\boldsymbol{L}\cdot\boldsymbol{r}}$$

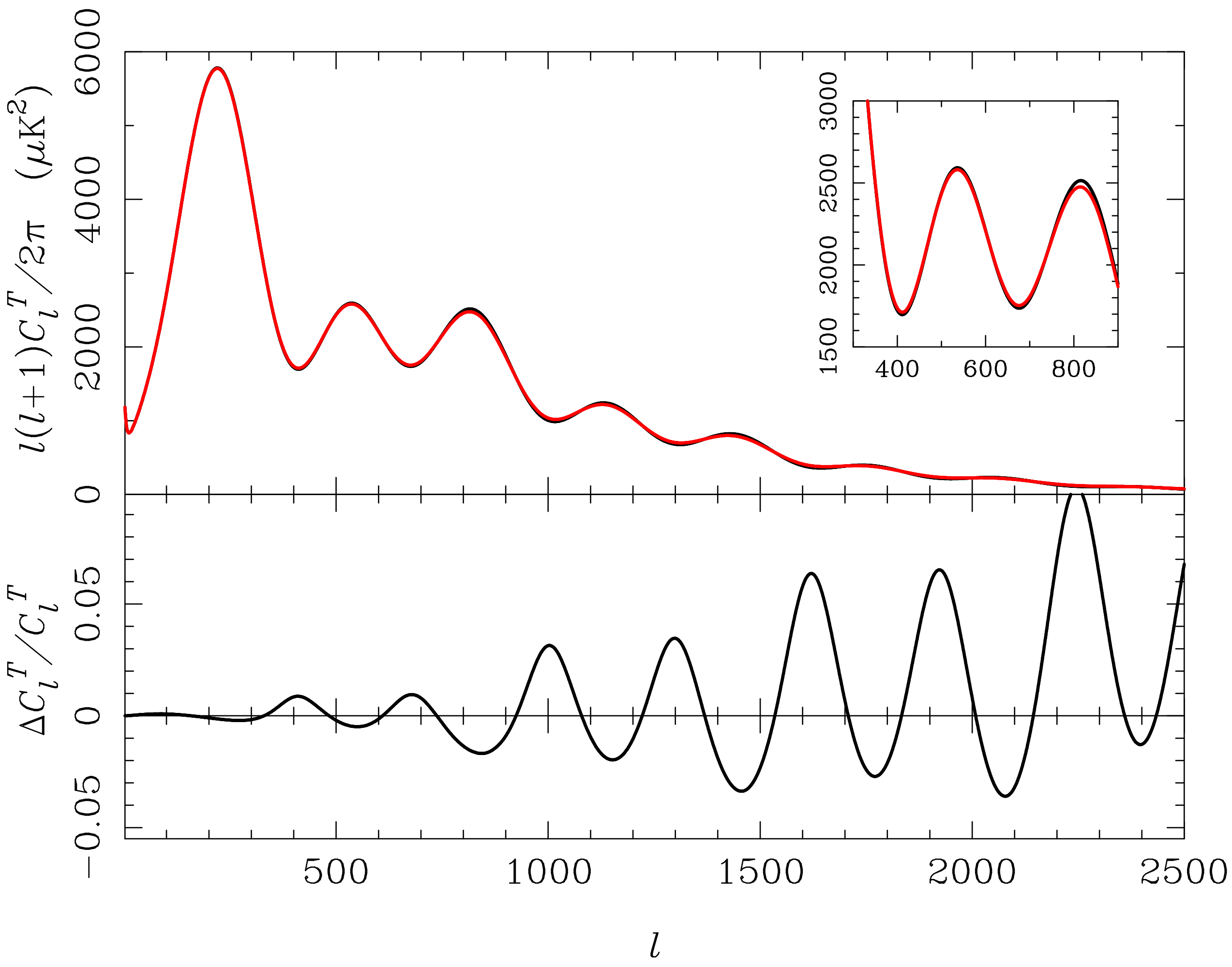
- Extracting lensed C_l from correlation function gives

$$\tilde{C}_l = \left[1 - \frac{1}{2} l^2 \langle \boldsymbol{\alpha}^2 \rangle\right] C_l + \underbrace{\int d^2\boldsymbol{r} \int \frac{d^2\boldsymbol{l}'}{(2\pi)^2} \int \frac{d^2\boldsymbol{L}}{(2\pi)^2} C_{l'} e^{-i\boldsymbol{l}\cdot\boldsymbol{r}} e^{i\boldsymbol{l}'\cdot\boldsymbol{r}} C_L^{\psi} (\boldsymbol{L}\cdot\boldsymbol{l}')^2 e^{i\boldsymbol{L}\cdot\boldsymbol{r}}}_{=\int \frac{d^2\boldsymbol{l}'}{(2\pi)^2} C_{l'} C^{\psi}_{|\boldsymbol{l}-\boldsymbol{l}'|} [\boldsymbol{l}'\cdot(\boldsymbol{l}-\boldsymbol{l}')]^2}$$

LENSED TEMPERATURE POWER SPECTRUM

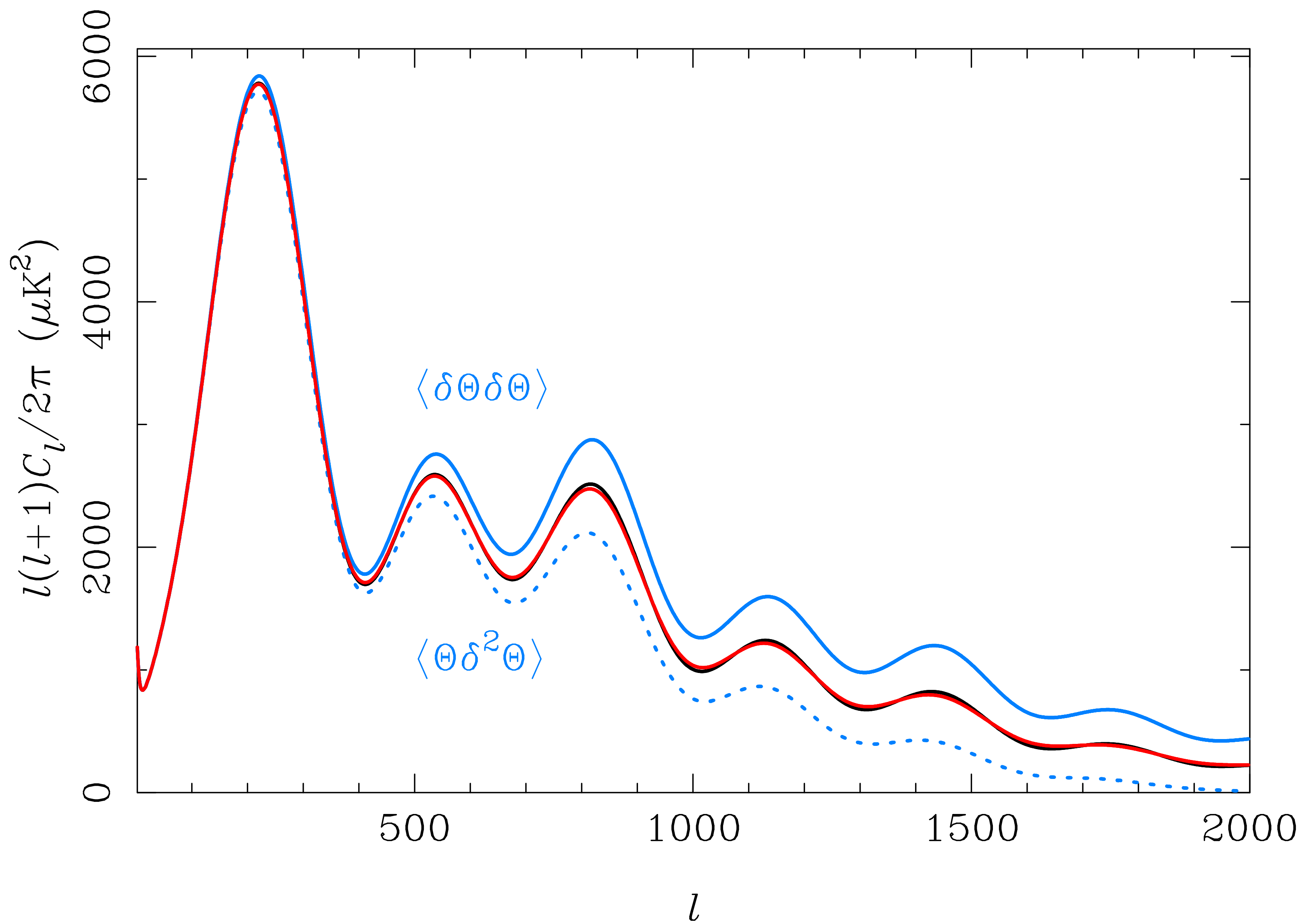
$$\tilde{C}_l = \left[1 - \frac{1}{2} l^2 \langle \boldsymbol{\alpha}^2 \rangle \right] C_l + \int \frac{d^2 \boldsymbol{l}'}{(2\pi)^2} C_{l'} C^{\psi}_{|\boldsymbol{l}-\boldsymbol{l}'|} [\boldsymbol{l}' \cdot (\boldsymbol{l} - \boldsymbol{l}')]^2$$

- Second term is a convolution: smooths acoustic peaks and generates small-scale power in damping tail



Cancellations between terms

- Taylor expansion out of control in map at $l \sim O(1000)$ for 3-arcmin deflections but only *relative* deflections matter for statistics



SMALL-SCALE LIMIT

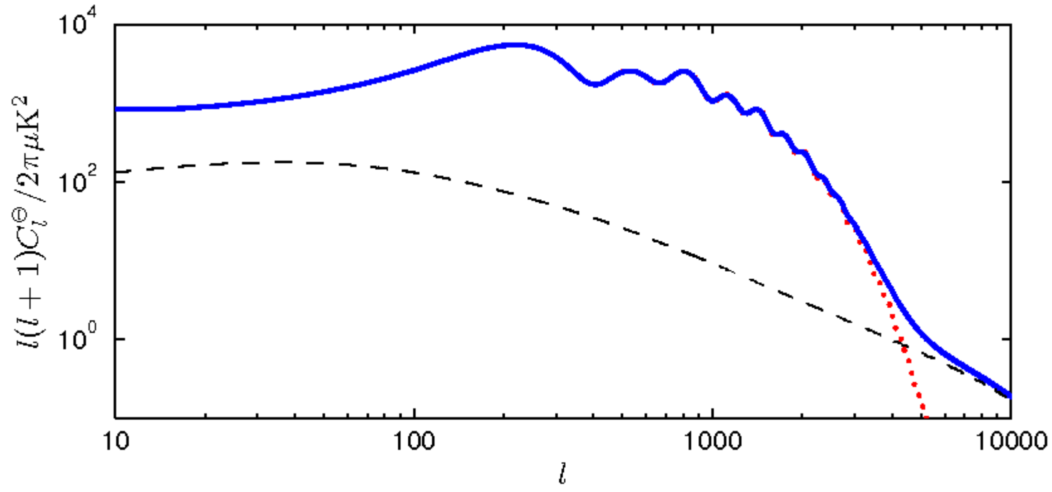
- For $l \gg 1000$ primary CMB has little power (diffusion damping) so drop first term:

$$\tilde{C}_l \approx \int \frac{d^2\boldsymbol{l}'}{(2\pi)^2} C_{l'} C^{\psi}_{|\boldsymbol{l}-\boldsymbol{l}'|} [\boldsymbol{l}' \cdot (\boldsymbol{l} - \boldsymbol{l}')]^2$$

- Integral restricted to $l' \ll l$ so $\boldsymbol{l} - \boldsymbol{l}' \approx \boldsymbol{l}$:

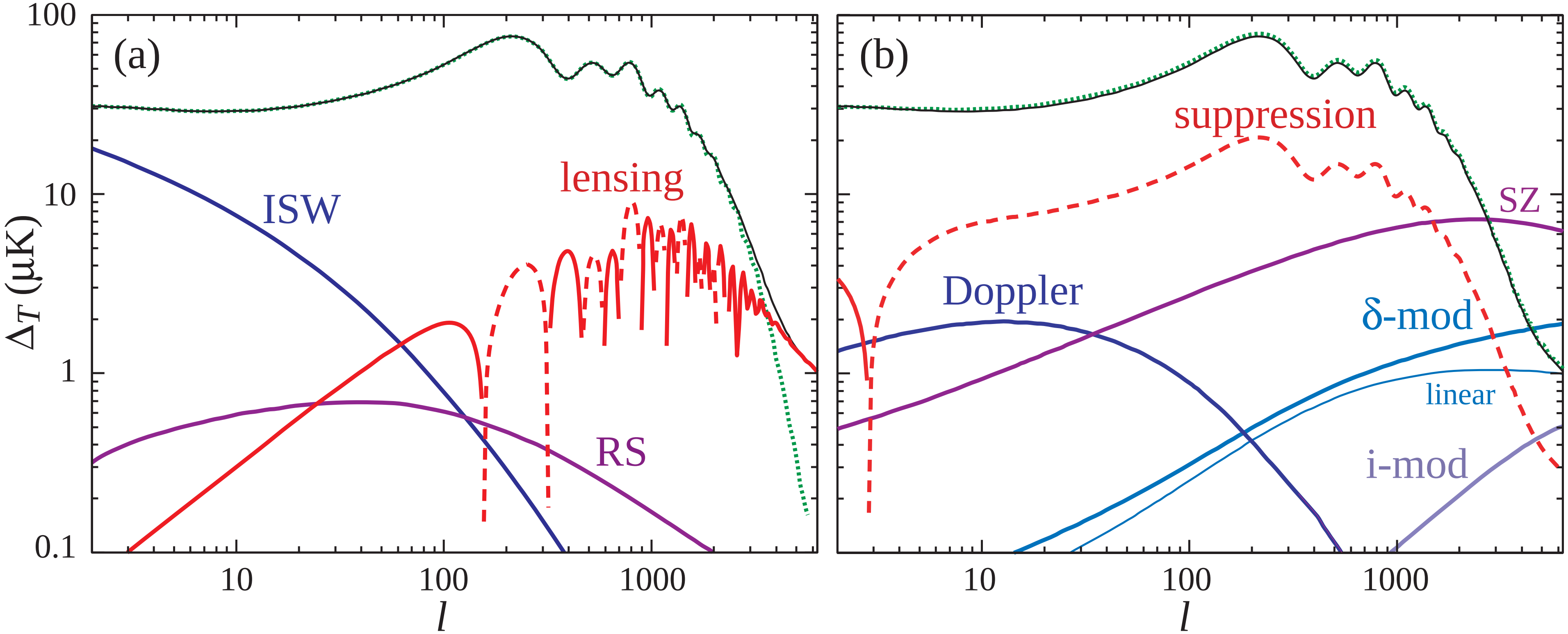
$$\tilde{C}_l \approx C^{\psi}_l \int \frac{d^2\boldsymbol{l}'}{(2\pi)^2} C_{l'} (\boldsymbol{l}' \cdot \boldsymbol{l})^2$$
$$= \frac{1}{2} l^2 C^{\psi}_l \int \frac{l' dl'}{2\pi} l'^2 C_{l'} = \frac{1}{2} \langle (\nabla\Theta)^2 \rangle l^2 C^{\psi}_l$$

- Small-scale lenses imprinting structure at same scale by displacing local gradient ($\langle (\nabla\Theta)^2 \rangle \sim 2 \times 10^9 \, \mu\text{K}^2$)



Comparison with other secondary anisotropies

- Lensing correction significant for $500 < l < 3000$
  - Sub-dominant to thermal SZ on small scales



Hu & Dodelson (2002)

A MORE ACCURATE CALCULATION

$$\tilde{\xi}(r) = \int \frac{d^2\boldsymbol{l}}{(2\pi)^2} C_l e^{i\boldsymbol{l}\cdot\boldsymbol{r}} \langle e^{i\boldsymbol{l}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})}\rangle$$

- Expansion of $e^{i\boldsymbol{l}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})}\rangle$ slow to converge at high l
- Avoid by making Gaussian assumption for deflections so

$$\langle e^{i\boldsymbol{l}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})}\rangle = e^{-\frac{1}{2}\langle[\boldsymbol{l}\cdot(\boldsymbol{\alpha}-\boldsymbol{\alpha}')]^2\rangle}$$

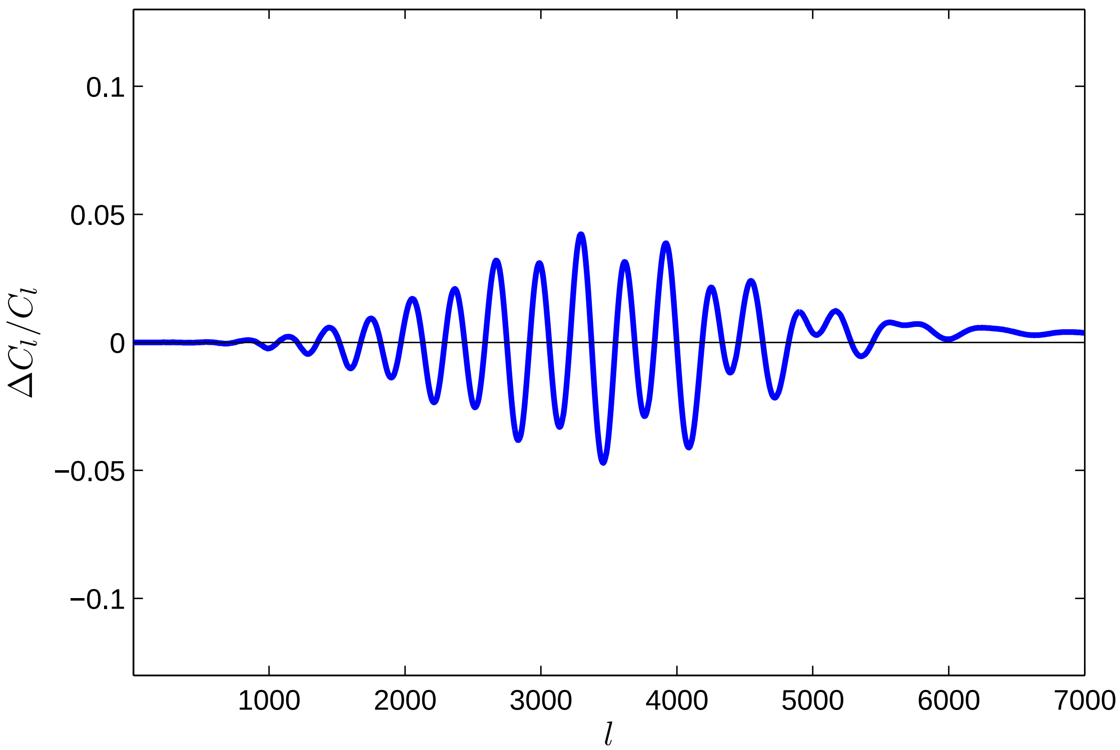
- Expectation value here is $\langle[\boldsymbol{l}\cdot(\boldsymbol{\alpha}-\boldsymbol{\alpha}')]^2\rangle = l^2\sigma^2(r) + l^2\cos 2\phi\, C_{\mathrm{gl},2}(r)$
  - $\sigma^2(r) = \int \frac{ldl}{2\pi} l^2 C_l^{\psi}[1 - J_0(lr)] = \langle(\boldsymbol{\alpha}-\boldsymbol{\alpha}')^2\rangle/2$ is variance of $\boldsymbol{\alpha}-\boldsymbol{\alpha}'$
  - $C_{\mathrm{gl},2}(r) = \int \frac{ldl}{2\pi} l^2 C_l^{\psi} J_2(lr)$ is *small* correction due to anisotropy of $\langle\alpha_i\alpha_j'\rangle$
- Lensed correlation function becomes

$$\begin{aligned}\tilde{\xi}(r) &= \int \frac{ldl}{2\pi} C_l e^{-l^2\sigma^2(r)/2} \int \frac{d\phi}{2\pi} e^{ilr\cos\phi} e^{-l^2 C_{\mathrm{gl},2}(r)\cos(2\phi)/2} \\ &\approx \int \frac{ldl}{2\pi} C_l e^{-l^2\sigma^2(r)/2}\left[J_0(lr) + \frac{1}{2} l^2 C_{\mathrm{gl},2}(r) J_2(lr) + \cdots\right]\end{aligned}$$

- Note exponential is non-perturbative in C_l^{ψ}

ERRORS IN LEADING-ORDER RESULT

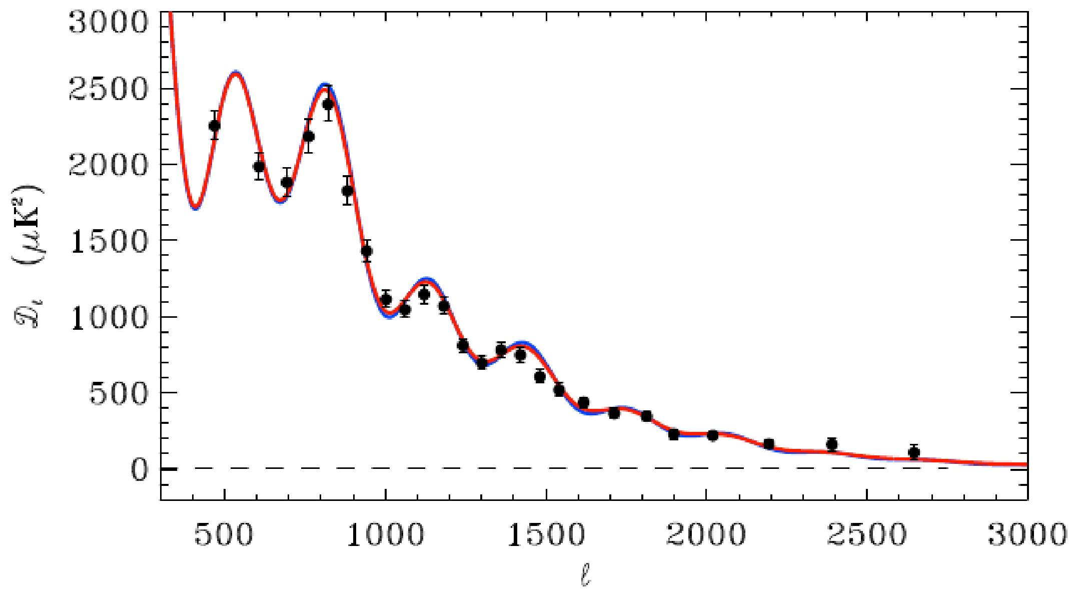
- Few percent errors in lensed C_l around $l \sim 3000$
  - Significant fraction of lensing effect on these scales
- Leading-order calculation accurate on large scales ($l^{-1} \gg$ typical deflection) and on small scales (CMB accurately a gradient)

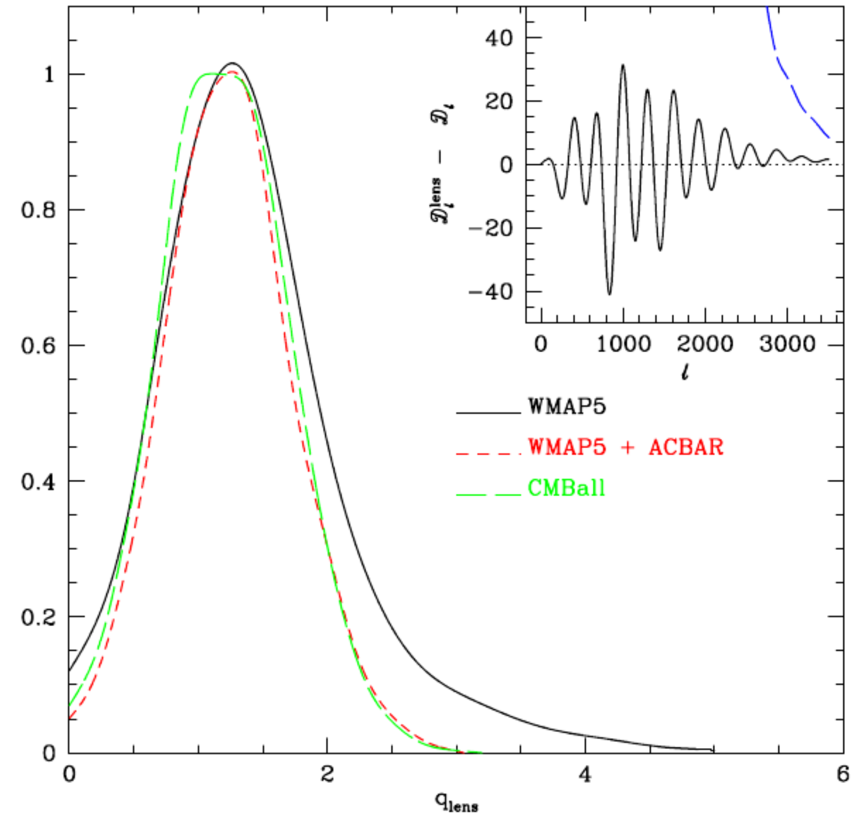


DETECTING LENSING IN POWER SPECTRA

$$C_l = C_l^{\mathrm{no-lens}} + q_{\mathrm{lens}} \Delta C_l^{\mathrm{lens}}$$

- $q_{\mathrm{lens}} = 1.23^{+0.83}_{-0.76}$ (95% C.L. from WMAP5 + ACBAR)





Reichardt et al. (2008)

LENSED POLARIZATION POWER SPECTRA

- Calculation for polarization spectra similar to temperature
- Assuming no primordial B modes, leading-order results are

$$\tilde{C}_l^E = \left(1 - \frac{1}{2} l^2 \langle \boldsymbol{\alpha}^2 \rangle\right) C_l^E + \int \frac{d^2 \boldsymbol{l}'}{(2\pi)^2} C_{l'}^E C_{|\boldsymbol{l}-\boldsymbol{l}'|}^{\psi} [\boldsymbol{l}' \cdot (\boldsymbol{l} - \boldsymbol{l}')]^2 \cos^2 2(\phi_{\boldsymbol{l}} - \phi_{\boldsymbol{l}'})$$

$$\tilde{C}_l^{TE} = \left(1 - \frac{1}{2} l^2 \langle \boldsymbol{\alpha}^2 \rangle\right) C_l^{TE} + \int \frac{d^2 \boldsymbol{l}'}{(2\pi)^2} C_{l'}^E C_{|\boldsymbol{l}-\boldsymbol{l}'|}^{\psi} [\boldsymbol{l}' \cdot (\boldsymbol{l} - \boldsymbol{l}')]^2 \cos 2(\phi_{\boldsymbol{l}} - \phi_{\boldsymbol{l}'})$$

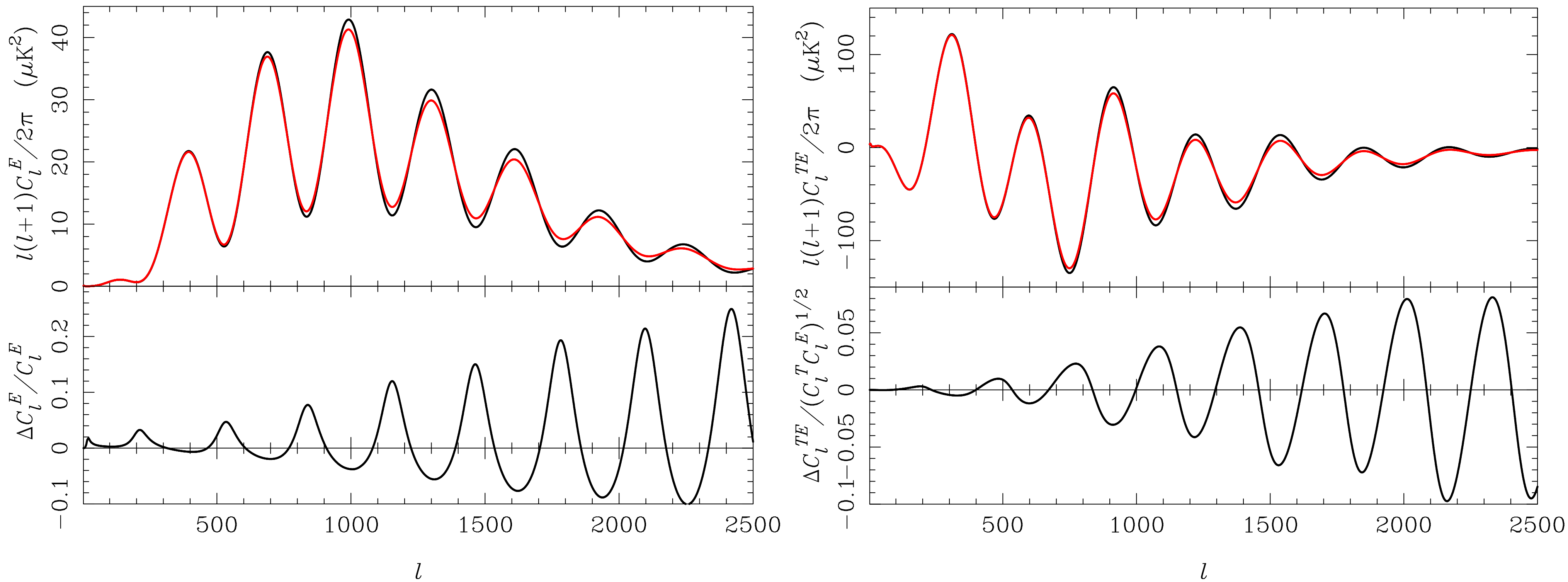
- Qualitatively new feature is generation of B-modes from lensing of E:

$$\tilde{C}_l^B = \int \frac{d^2 \boldsymbol{l}'}{(2\pi)^2} C_{l'}^E C_{|\boldsymbol{l}-\boldsymbol{l}'|}^{\psi} [\boldsymbol{l}' \cdot (\boldsymbol{l} - \boldsymbol{l}')]^2 \sin^2 2(\phi_{\boldsymbol{l}} - \phi_{\boldsymbol{l}'})$$

  - Geometric term ensures broad mode-coupling of E-mode power to B
  - Non-linearities in ψ change low-l B modes by 10%!

LENSED E-MODE SPECTRA

- Similar effect to temperature
  - Smoothing of acoustic peaks but more pronounced since sharper
  - Transfer of power to small scales

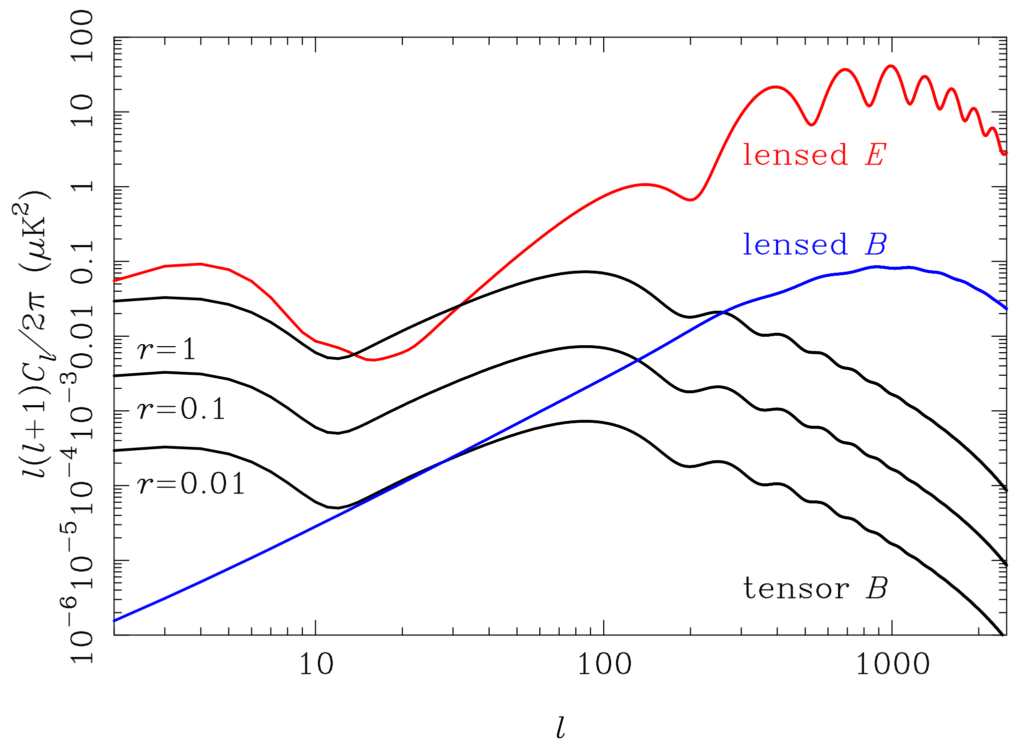


B-MODES FROM LENSING

- E-mode power peaks on small scales so, for $l \ll 1000$ have $\boldsymbol{l}' - \boldsymbol{l} \approx \boldsymbol{l}'$ and

$$\tilde{C}_l^B \approx \int \frac{d^2\boldsymbol{l}'}{(2\pi)^2} (\boldsymbol{l}' \cdot \boldsymbol{l}')^2 C_{l'}^{\psi} C_{l'}^{E} \sin^2 2\phi_{\boldsymbol{l}'} = \frac{1}{2} \int \frac{l' dl'}{2\pi} l'^4 C_{l'}^{\psi} C_{l'}^{E}$$

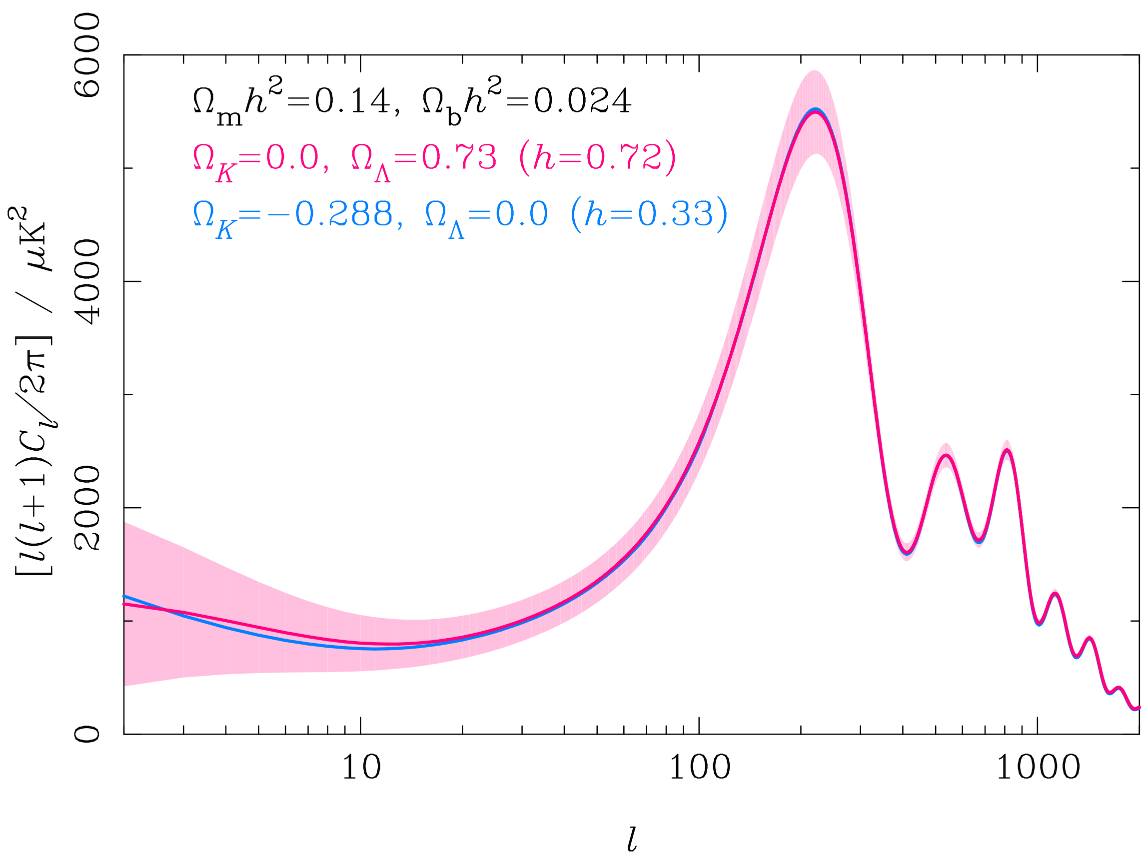
- White noise spectrum with $C_l^B \sim 2 \times 10^{-6} \mu\,\mathrm{K}^2$
  - Additional source of confusion for primordial B-mode searches comparable to $\Delta_P \sim 5\,\mu$K-arcmin



COSMOLOGICAL PARAMETERS FROM LENSED SPECTRA

- Important to include lensing to avoid parameter biases at high l (Lewis 2005)
  - C_l covariance due to lensing non-Gaussianity ignorable expect for BB
- Lensed spectra break geometric degeneracy
  - Models with same $d_A(z_*)$ generally have different C_l^{ψ}
- Lensed *spectra* contain essentially two new pieces of information (Smith et al. 2006)
  - One from T and E about C_l^{ψ} for $l < 300$
  - One from B about C_l^{ψ} over a broad range of l
  - More information can be mined with lens reconstruction (next!)

ASIDE: GEOMETRIC DEGENERACY



- Primary CMB fluctuations only constrain $d_A(z_*) = 14116 \pm 160\,\mathrm{Mpc}$ (WMAP7)
  - Need external data or secondaries (e.g. lensing) to break geometric degeneracies beyond flat, ΛCDM

LENSING RECONSTRUCTION

Basics of Reconstruction

- For fixed lenses, lensing correlates lensed CMB $\tilde{\Theta}$ with gradient of *unlensed* CMB:

$$\tilde{\Theta}(\boldsymbol{x}) = \Theta(\boldsymbol{x}) + \boldsymbol{\alpha}(\boldsymbol{x}) \cdot \boldsymbol{\nabla}\Theta(\boldsymbol{x}) + \cdots$$

$$\Rightarrow \qquad \langle \tilde{\Theta} \partial_i \Theta \rangle_\Theta = \alpha_j \langle \partial_j \Theta \partial_i \Theta \rangle_\Theta = \frac{1}{2} \alpha_i \langle \Theta \boldsymbol{\nabla}^2 \Theta \rangle_\Theta$$

  - Can estimate unlensed CMB by Wiener filtering observed CMB (see later)

- Chance correlations between unlensed CMB and its gradient introduce statistical noise to any reconstruction (similar to shape noise in galaxy lensing)

- Reconstructs the projection of dark mater on 100 Mpc scales back to high redshift

- Can estimate C_l^ψ from reconstruction by looking for excess power over and above that due to Gaussian CMB

RECONSTRUCTION IN FOURIER SPACE

- Fourier transform of $\tilde{\Theta}(\boldsymbol{x}) = \Theta(\boldsymbol{x}) + \boldsymbol{\alpha}(\boldsymbol{x}) \cdot \boldsymbol{\nabla}\Theta(\boldsymbol{x}) + \cdots$ is

$$\tilde{\Theta}(\boldsymbol{l}) = \Theta(\boldsymbol{l}) - \int \frac{d^2\boldsymbol{l}'}{2\pi} \boldsymbol{l}' \cdot (\boldsymbol{l} - \boldsymbol{l}')\Theta(\boldsymbol{l}')\psi(\boldsymbol{l} - \boldsymbol{l}') + \cdots$$

- Fixed lenses produce anisotropic (off-diagonal) correlations in lensed CMB:

$$\langle \tilde{\Theta}(\boldsymbol{l})\tilde{\Theta}^*(\boldsymbol{l} - \boldsymbol{L})\rangle_\Theta = C_l \delta(\boldsymbol{L}) + \frac{1}{2\pi}\left[\boldsymbol{l} \cdot \boldsymbol{L}' C_l + (\boldsymbol{L} - \boldsymbol{l}) \cdot \boldsymbol{L} C_{|\boldsymbol{l}-\boldsymbol{L}|}\right] \psi(\boldsymbol{L})$$

  - Estimate lensing potential for $\boldsymbol{L} \neq 0$ with a weighted-average of off-diagonal terms:

$$\hat{\psi}(\boldsymbol{L}) = N(\boldsymbol{L}) \int \frac{d^2\boldsymbol{l}}{2\pi} \tilde{\Theta}(\boldsymbol{l})\tilde{\Theta}^*(\boldsymbol{l} - \boldsymbol{L}) g(\boldsymbol{l}, \boldsymbol{L})$$

  - Want an unbiased estimate averaged over realisations of CMB – fixes normalisation:

$$\langle \hat{\psi}(\boldsymbol{L})\rangle_\Theta = \psi(\boldsymbol{L}) \quad \Rightarrow \quad N(\boldsymbol{L})^{-1} = \int \frac{d^2\boldsymbol{l}}{(2\pi)^2}\left[(\boldsymbol{L} - \boldsymbol{l}) \cdot \boldsymbol{L} C_{|\boldsymbol{l}-\boldsymbol{L}|} + \boldsymbol{l} \cdot \boldsymbol{L} C_l\right] g(\boldsymbol{l}, \boldsymbol{L})$$

"OPTIMAL" QUADRATIC ESTIMATOR

$$\hat{\psi}(\boldsymbol{L}) = N(\boldsymbol{L}) \int \frac{d^2\boldsymbol{l}}{2\pi} \tilde{\Theta}(\boldsymbol{l}) \tilde{\Theta}^*(\boldsymbol{l} - \boldsymbol{L}) g(\boldsymbol{l}, \boldsymbol{L})$$

- Free to choose weights $g(\boldsymbol{l}, \boldsymbol{L})$ to minimise statistical noise in reconstruction; leading order result gives

$$g(\boldsymbol{l}, \boldsymbol{L}) = \frac{(\boldsymbol{L} - \boldsymbol{l}) \cdot \boldsymbol{L} C_{|\boldsymbol{l} - \boldsymbol{L}|} + \boldsymbol{l} \cdot \boldsymbol{L} C_l}{2 \tilde{C}_l^{\text{tot}} \tilde{C}_{|\boldsymbol{l} - \boldsymbol{L}|}^{\text{tot}}}$$

  where $\tilde{C}_l^{\text{tot}}$ is total *observed* CMB power including instrument noise

- Statistical noise on reconstruction is

$$\langle |\hat{\psi}(\boldsymbol{L}) - \psi(\boldsymbol{L})|^2 \rangle \approx \delta(\boldsymbol{0}) N(\boldsymbol{L}) = \left(\int \frac{d^2\boldsymbol{l}}{(2\pi)^2} \frac{\left[(\boldsymbol{L} - \boldsymbol{l}) \cdot \boldsymbol{L} C_{|\boldsymbol{l} - \boldsymbol{L}|} + \boldsymbol{l} \cdot \boldsymbol{L} C_l \right]^2}{2 \tilde{C}_l^{\text{tot}} \tilde{C}_{|\boldsymbol{l} - \boldsymbol{L}}^{\text{tot}}} \right)^{-1}$$

  – Noise from both instrument noise and CMB sample variance
  – Requires high resolution – small CMB blobs can be used to reconstruct lenses on all scales

ESTIMATOR IN REAL SPACE

$$\hat{\psi}(\mathbf{L}) = N(\mathbf{L})\mathbf{L} \cdot \int \frac{d^2\mathbf{l}}{2\pi} \frac{\mathbf{l} C_l \tilde{\Theta}(\mathbf{l})}{\tilde{C}_l^{\text{tot}}} \frac{\tilde{\Theta}(\mathbf{L}-\mathbf{l})}{\tilde{C}_{|\mathbf{l}-\mathbf{L}|}^{\text{tot}}}$$

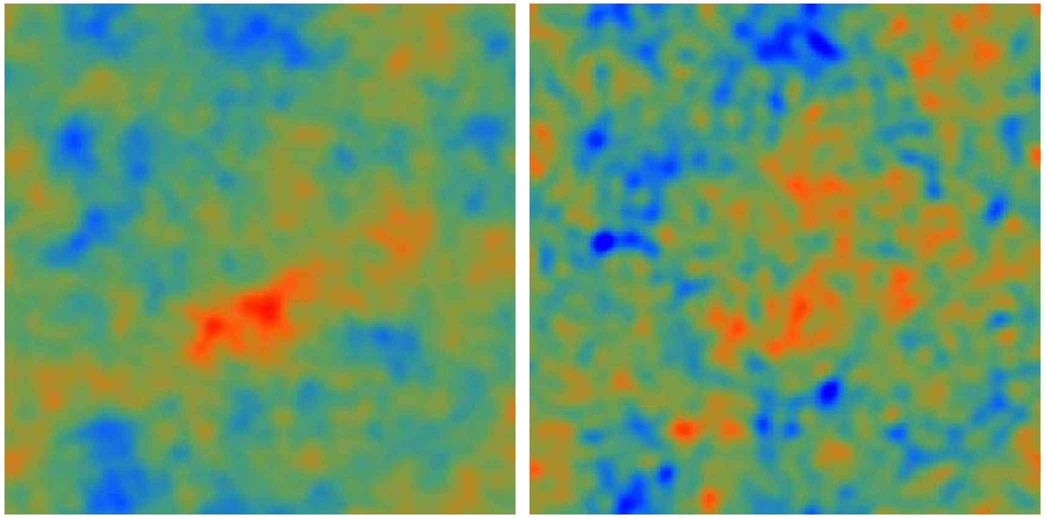
- Integral is a convolution so has local real-space form:

$$\hat{\psi}(\mathbf{L}) = -N(\mathbf{L}) \int \frac{d^2\mathbf{x}}{2\pi} e^{-i\mathbf{L}\cdot\mathbf{x}} \boldsymbol{\nabla} \cdot [F_1(\mathbf{x}) \boldsymbol{\nabla} F_2(\mathbf{x})]$$

where filtered fields in Fourier space are

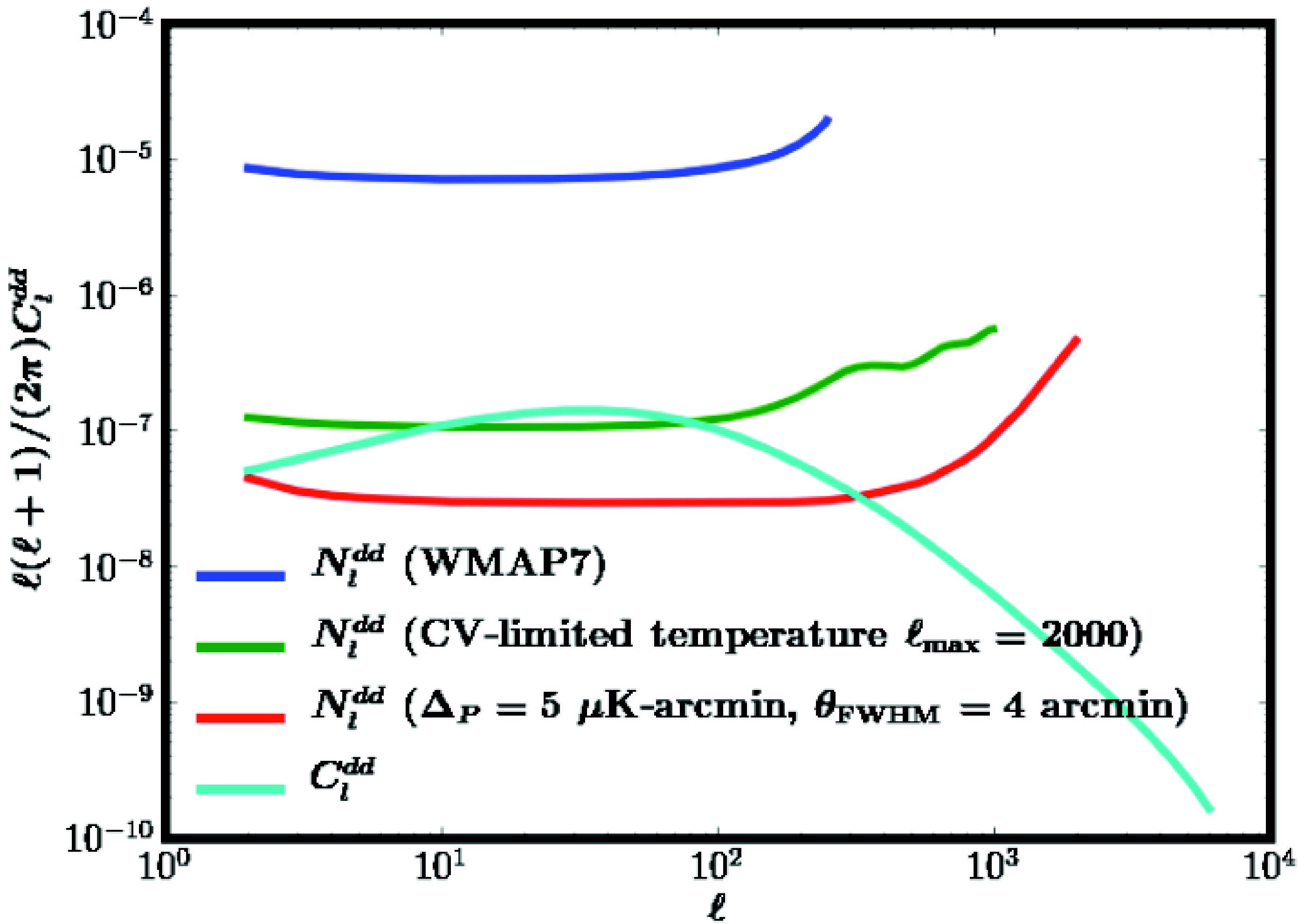
$$F_1(\mathbf{l}) \equiv \frac{\tilde{\Theta}(\mathbf{l})}{\tilde{C}_l^{\text{tot}}} \quad \text{and} \quad F_2(\mathbf{l}) \equiv \frac{C_l \tilde{\Theta}(\mathbf{l})}{\tilde{C}_l^{\text{tot}}}$$

  - $F_2(\mathbf{x})$ is Wiener reconstruction of unlensed CMB so $\boldsymbol{\nabla}\hat{\psi} \sim \tilde{\Theta} \boldsymbol{\nabla} \Theta$



Hu & Okamoto (2002)

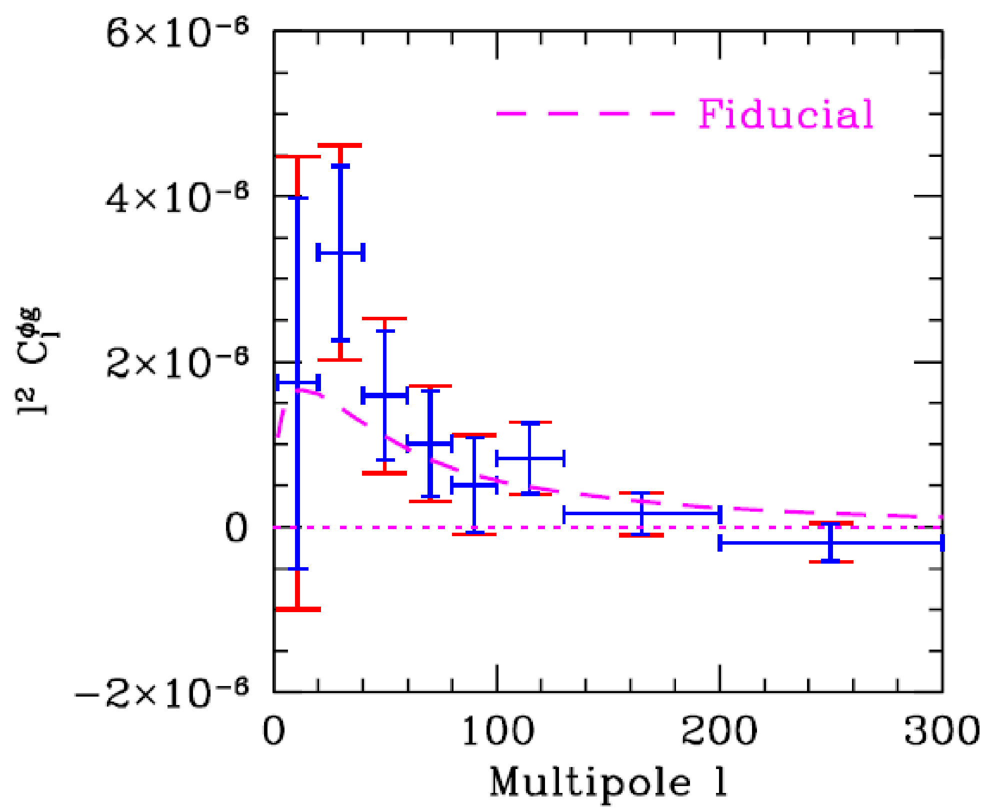
Statistical noise levels in reconstruction



Kendrick Smith

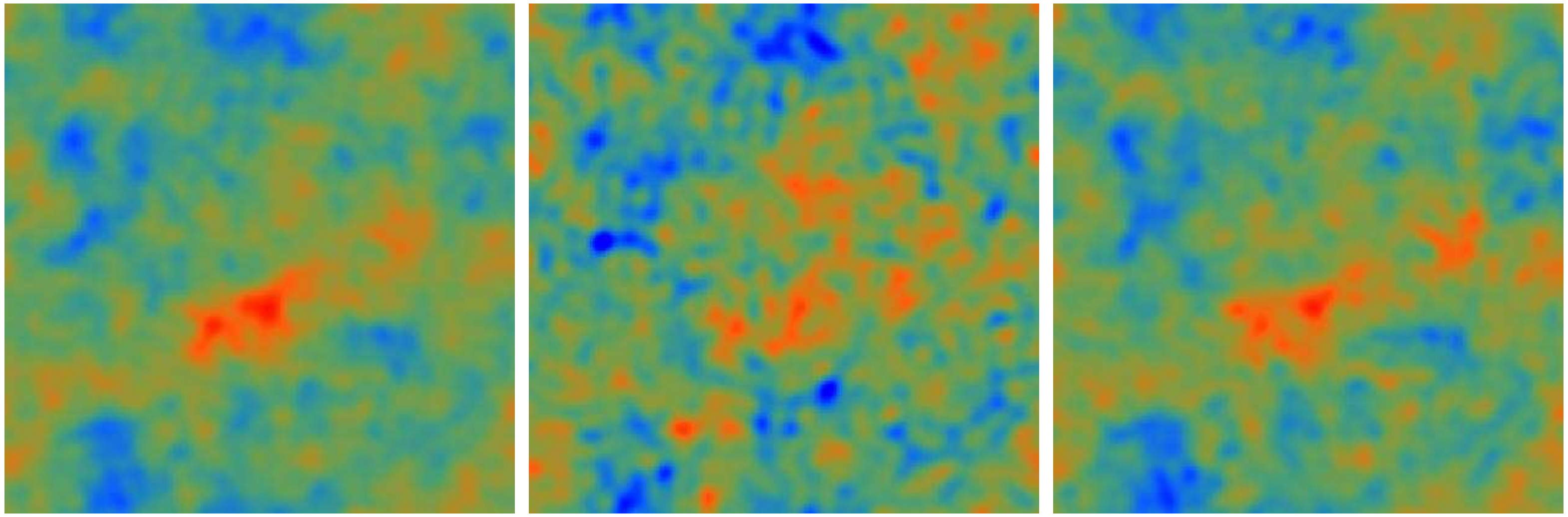
DETECTION OF LENSING IN CROSS-CORRELATION

- Smith et al. (2007) reconstruct (very noisy!) deflection map from WMAP3
  - Statistical noise too high (WMAP resolution 15 arcmin) for direct detection of lensing but . . .
  - Detect signal power at 3.4σ by cross-correlating reconstruction with (less noisy!) LSS tracer (NVSS radio galaxies)



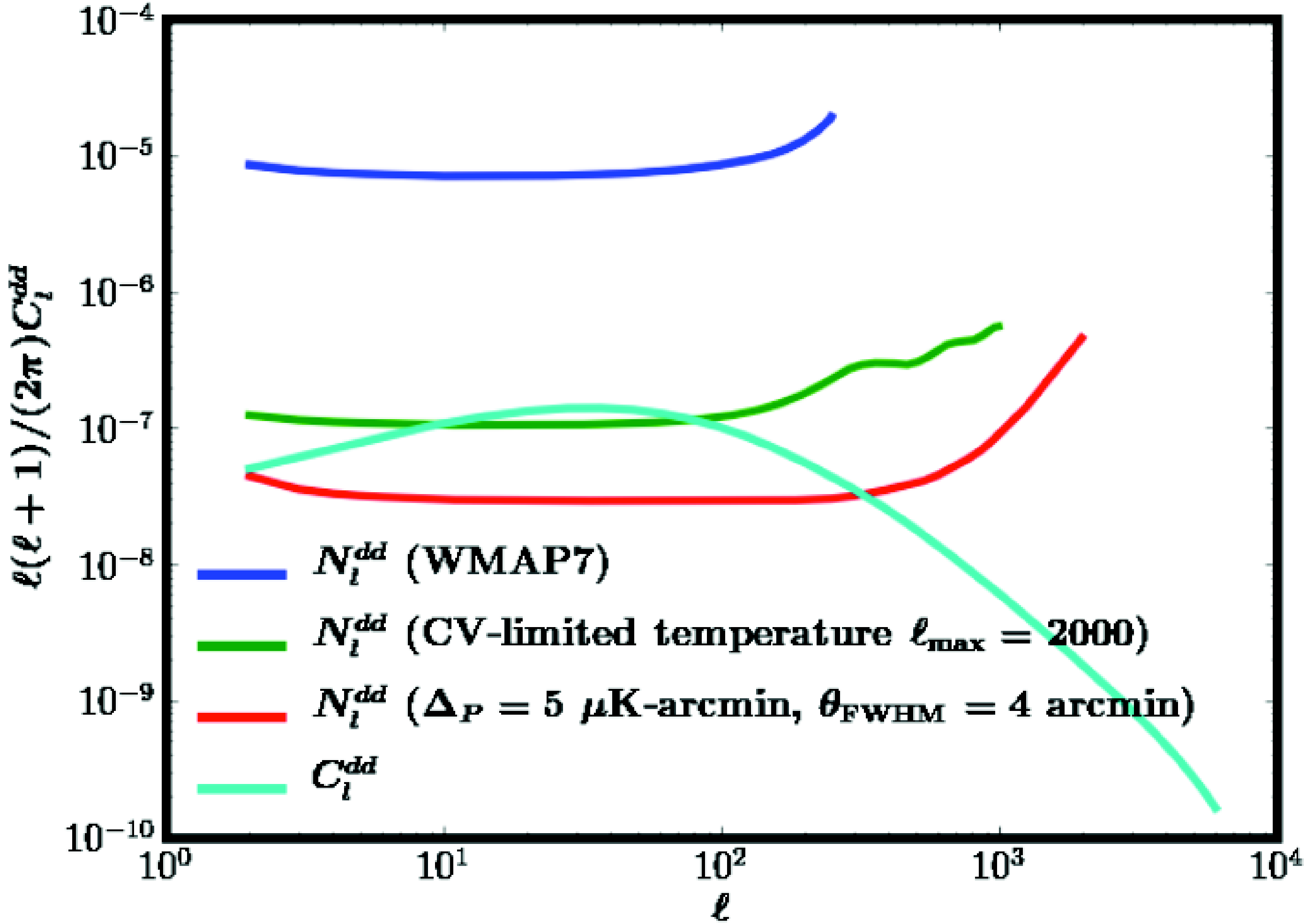
Reconstruction with CMB polarization

- Quadratic estimators generalise to polarization
  - Helpful since more small-scale power and, for TB and EB estimators, less confusion from chance off-diagonal correlations
  - Needs high sensitivity – imaging lens-induced B-modes requires $\Delta_p < 5\,\mu\mathrm{K\,arcmin}$
- Can improve significantly on quadratic estimator for polarization (Hirata & Seljak 2003)



Hu & Okamoto (2002)

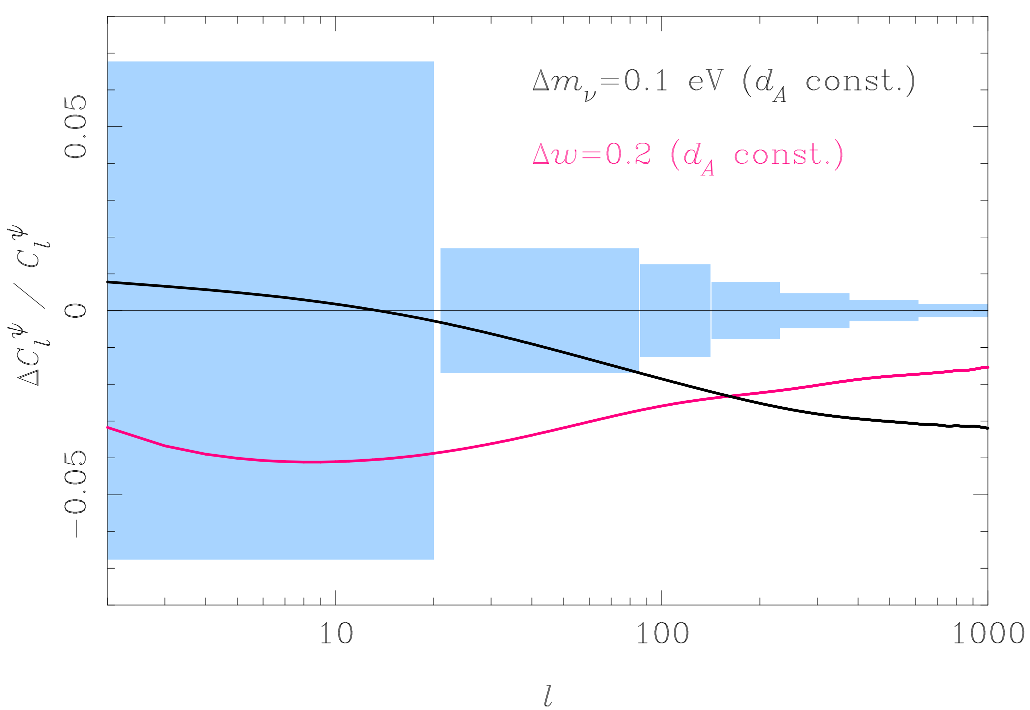
Statistical noise levels in reconstruction



Kendrick Smith

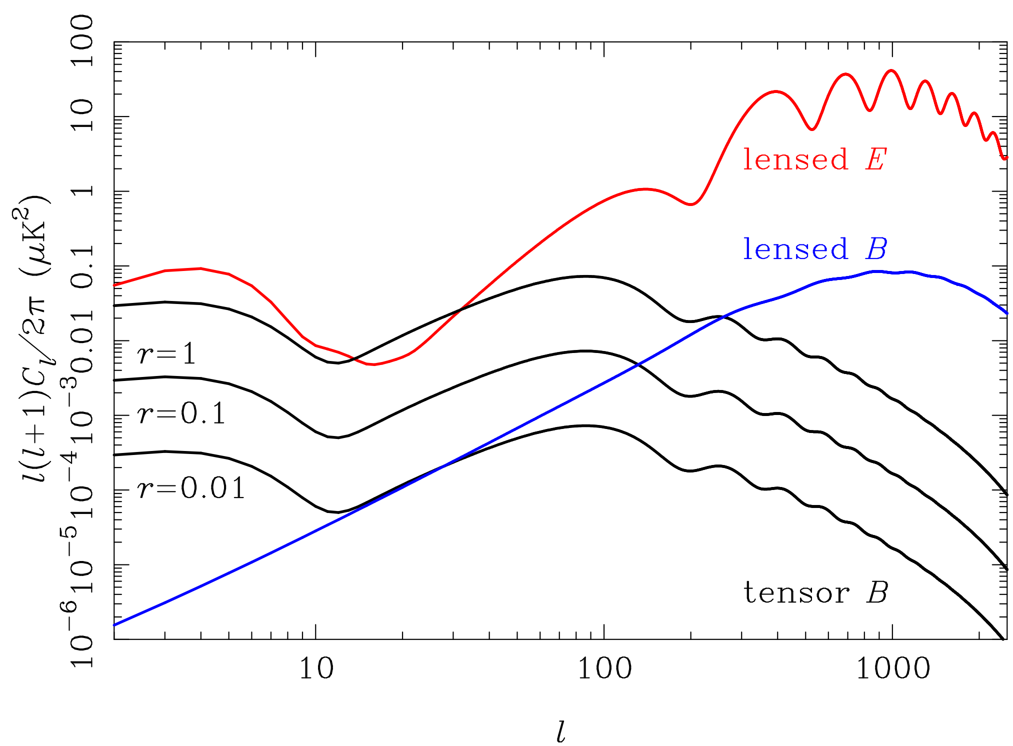
APPLICATIONS OF LENS RECONSTRUCTION: C_l^{ψ}

- Primary CMB provides limited information on sub-eV neutrino masses and dark energy – only through d_A (and ISW)
- Reconstruction gives full C_l^{ψ}: much more information than the lensing effect on CMB power spectra
  - Error on $\sum_\nu m_\nu \sim 0.04\,\mathrm{eV}$ (c.f. $\sum_\nu m_\nu > 0.05\,\mathrm{eV}$ from oscillation data)
  - Not very constraining for dark energy – $\sigma(w) > 0.08$ – since mostly sensitive to $z \sim 2$ but good probe of *early dark energy* models



DELENSING B-MODES

- Lensing acts like $5\,\mu$K-armin noise for detecting primordial B-modes
  - Limits $r > 3 \times 10^{-4}$ for $l > 40$ and $r > 3 \times 10^{-5}$ for all l
- Delens by remapping *observed* polarization with (noisy) reconstructed $\hat{\psi}$
  - Up to factor ~ 10 improvement on r but requires $\sim 1\,\mu$K-arcmin polarization imaging and < 5 arcmin resolution
  - Small-scale T observations alone insufficient; ideal LSS to $z = 3$ gives factor ~ 2 improvement



CMB lensing and primordial non-Gaussianity

Bispectrum from lensing-ISW correlation

- Reduced bispectrum of lensed CMB:

$$\langle \tilde{\Theta}(\boldsymbol{l}_1)\tilde{\Theta}(\boldsymbol{l}_2)\tilde{\Theta}(\boldsymbol{l}_3)\rangle = \frac{1}{2\pi} b_{l_1 l_2 l_3} \delta(\boldsymbol{l}_1 + \boldsymbol{l}_2 + \boldsymbol{l}_3)$$

- To leading order

$$\begin{aligned}
\langle \tilde{\Theta}(\boldsymbol{l}_1)\tilde{\Theta}(\boldsymbol{l}_2)\tilde{\Theta}(\boldsymbol{l}_3)\rangle &= \frac{1}{2}\langle \Theta(\boldsymbol{l}_1)\Theta(\boldsymbol{l}_2)(\nabla\Theta\cdot\nabla\psi)(\boldsymbol{l}_3)\rangle + 5\text{ perms} \\
&= -\frac{1}{2}\int \frac{d^2\boldsymbol{l}_3'}{2\pi}\langle \Theta(\boldsymbol{l}_1)\Theta(\boldsymbol{l}_2)\Theta(\boldsymbol{l}_3')\psi(\boldsymbol{l}_3 - \boldsymbol{l}_3')\rangle \boldsymbol{l}_3'\cdot(\boldsymbol{l}_3 - \boldsymbol{l}_3') \\
&= -\frac{1}{2\pi} C_{l_1}^{\Theta\psi} C_{l_2}^{\Theta} \boldsymbol{l}_1\cdot\boldsymbol{l}_2 \delta(\boldsymbol{l}_1 + \boldsymbol{l}_2 + \boldsymbol{l}_3) + 5\text{ perms}
\end{aligned}$$

- Gives reduced bispectrum

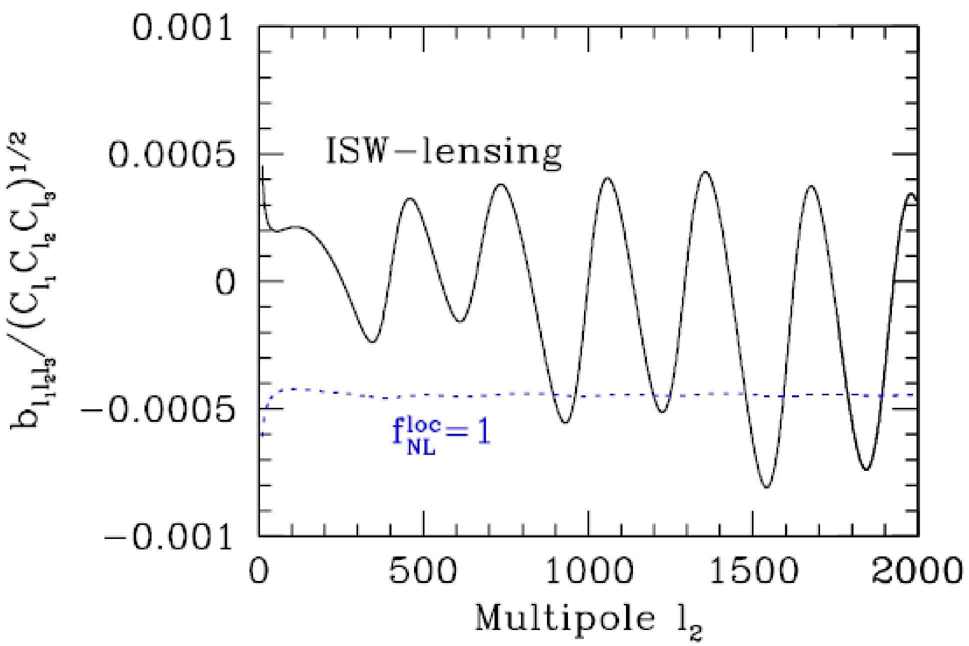
$$b_{l_1 l_2 l_3} = -\frac{1}{2}\left[l_3(l_3+1) - l_1(l_1+1) - l_2(l_2+1)\right] C_{l_1}^{\Theta\psi} C_{l_2}^{\Theta} + 5\text{ perms}$$

  – Peaks in squeezed limit ($\langle$ISW $\times$ small-scale Θ $\times$ small-scale $\tilde{\Theta}\rangle$)

IMPACT ON f_{NL}^{local} SEARCHES

$$b_{l_1 l_2 l_3} = -\frac{1}{2}\left[l_3(l_3+1) - l_1(l_1+1) - l_2(l_2+1)\right] C_{l_1}^{\Theta\psi} C_{l_2}^{\Theta} + 5\,\text{perms}$$

- +25% correlation with bispectrum of local model
  - Both peak in squeezed limit
- For Planck ($l_{\text{max}} = 2000$), gives spurious (local-model) $f_{NL} = 9.3$ if uncorrected; negligible for WMAP ($l_{\text{max}} = 750$)



Smith & Zaldarriaga (2006)

Lensing effect on primordial non-Gaussianity

- Lensing can also modify observed shape of primordial non-Gaussianity
- Analytic calculation to $O(C_l^\phi)$:

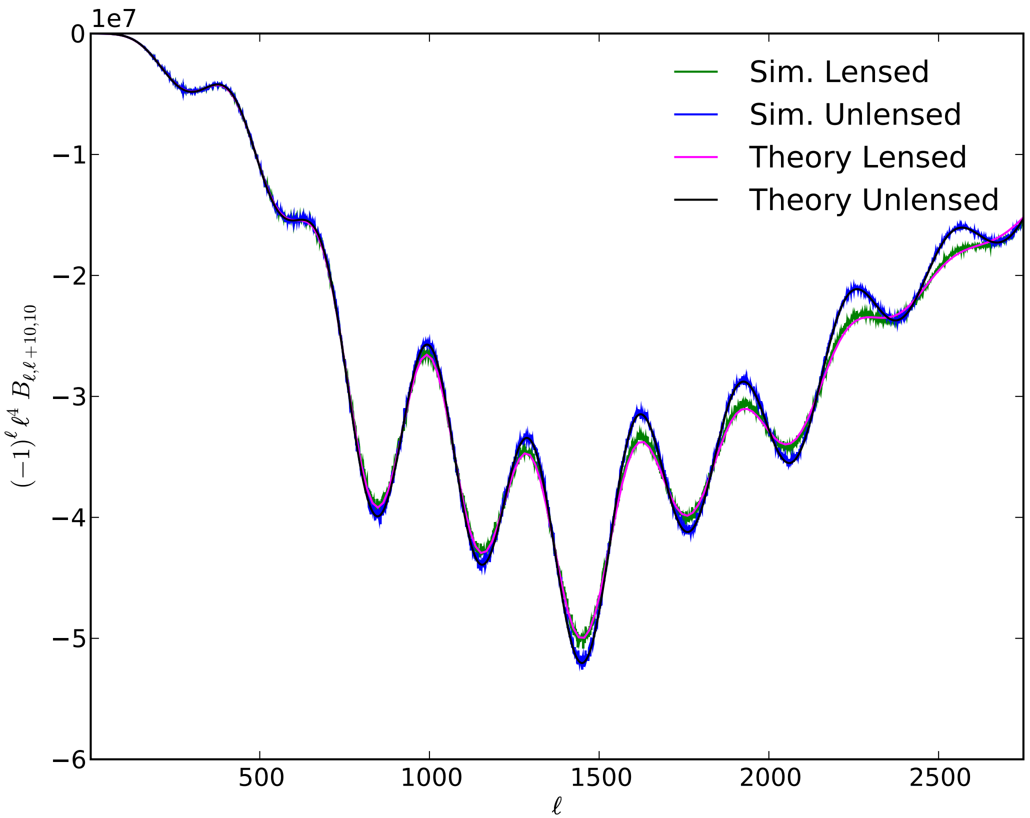
$$\delta b_{l_1 l_2 l_3} \sim \underbrace{-\frac{1}{4}\left[l_1(l_1+1) + l_2(l_2+1) + l_3(l_3+1)\right]\langle \boldsymbol{\alpha}^2 \rangle b_{l_1 l_2 l_3}}_{\langle \delta^2 \Theta\Theta\Theta \rangle} + \underbrace{\mathcal{B}_{l_1 l_2 l_3}[C^\phi, b]}_{\langle \delta\Theta\delta\Theta\Theta \rangle}$$

- Also check against simulations:
  - Generate non-Gaussian maps (Smith & Zaldarriaga 2006; Ligouri et al. 2007) with correct power spectrum and (at least) bispectrum

$$\Theta_{lm} = \Theta^{\mathrm{G}}_{lm} + f_{NL}\Theta^{\mathrm{NG}}_{lm}$$

  - Perform lensing displacement with LensPix (Lewis 2005)
  - Estimate bispectrum on full sky from lensed maps (reduce MC error by subtracting 3-pt from Θ^{G}_{lm})

Lensed local-model bispectrum



Hanson, Smith, AC, Ligouri (2009)

- Only 0.05% effect hence negligible:
  - If Planck found $f_{NL} = 60 \pm 5$, bias if ignore lensing only $\Delta f_{NL} = 0.03$

SUMMARY

- Weak lensing of CMB is important
  - Several percent corrections through acoustic peaks
  - Generates small-scale power
  - Lens-induced B-modes confuse primordial for $r < 0.01$
  - Non-Gaussian signal
  - All generally well-understood and can be modelled accurately in linear theory with small non-linear corrections
- Potential uses
  - Mapping distribution of dark matter to high redshift
  - Improve parameter constraints and break degeneracies
  - De-lens primordial B-modes
  - Others not covered: cluster masses at high redshift etc.
- Watch out for direct detections in next two years (Planck, ACT, SPT etc)