ν's in the CMB

Reviews:
- "Massive neutrinos in cosmology"
  J.L. & S. Pastor, Phys. Rep. 429 (2006) 307
  [astro-ph/0603494]
- "Neutrino physics from precision cosmology"
  S. Hannestad
  [1007.0658]

More specific aspects:
- planck forecasts with/without lensing:
  L. Perotto et al. JCAP 0610, 013 (2006)
  [astro-ph/0606227]

I Status of neutrino physics from laboratory experiments

* LEP ⇒ 3 left-handed flavor neutrinos

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} + \text{anti-particles}$$

* oscillations : ∃ mass eigenstates

$$\nu_F = U \nu_M \quad \text{with} \begin{cases} F = e, \mu, \tau \\ M = 1, 2, 3 \\ U = \text{unitary } 3\times 3 \text{ matrix} \end{cases}$$

[Dirac mass terms: ∃ right-handed neutrinos,
$$\mathcal{L}_\nu = -m \left(\overline{\nu_L} \nu_R + \overline{\nu_R} \nu_L \right)$$
Majorana mass state: $\mathcal{L}_\nu = -\frac{m}{2} \overline{\nu_L} \nu_L^c \left(-\frac{m}{2} \overline{\nu_R} \nu_R^c \right)$

If ν_M have Dirac masses,
$U \to$ 3 angles θ_{MN} + 1 phase

If ν_M have Majorana masses,
$U \to$ 3 angles θ_{MN} + 3 phases

Favored mechanism for mass generation (see-saw) requires the two mass terms (if this is true ν's would be first discovered Majorana particles)

Oscillation experiments (solar, atmospheric, accelerators) measure:

→ θ_{12}, θ_{23} (well constrained)

→ θ_{13} (upper bound)

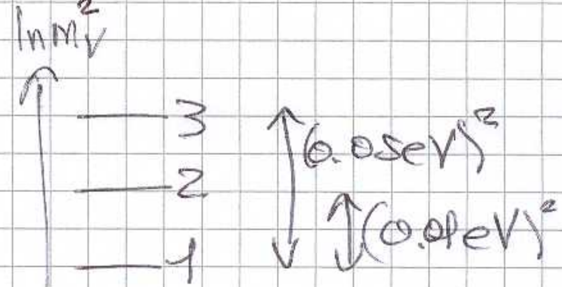
→ $\Delta m^2_{12} = (7.6 \pm 0.6)\, 10^{-5} \text{eV}^2 \sim (0.01 \text{eV})^2$

→ $\Delta m^2_{13} = \pm \underset{3\sigma}{(2.4 \pm 0.4)}\, 10^{-3} \text{eV}^2 \sim (0.05 \text{eV})^2$

* two possibilities:

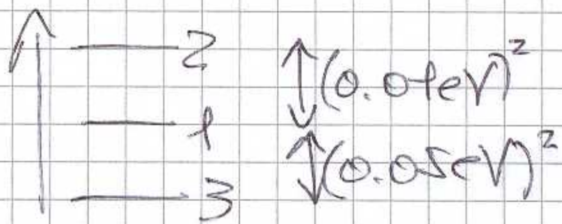
Normal hierarchy

$M_\nu \equiv \sum m_\nu \gtrsim 0.05\,\mathrm{eV}$



Inverted hierarchy

$M_\nu \gtrsim 0.1\,\mathrm{eV}$



In both cases, absolute mass scale is unknown. M_ν can be arbitrary large (then ν's nearly degenerate: $m_1 \sim m_2 \sim m_3$)

* neutrinoless double-beta decay:
  if majorana mass terms, probe

$$m_{\beta\beta} = \Big| \sum_j \underset{\substack{\uparrow \\ \text{flavor index}}}{U_e}{}^2_{\underset{\substack{\uparrow \\ \text{mass index}}}{j}} m_j \Big| \qquad < 0.27\,\mathrm{eV} \quad (2\sigma)$$

(Heidelberg-Moscow)

possible cancellations $\rightarrow$ no upper bound on $m_{\beta\beta}$

* ordinary β decay: probes

$$m_\beta = \Big[\sum_j U^*_{ej} U_{ej} m_j^2 \Big]^{1/2} \qquad < 2.3\,\mathrm{eV} \quad (2\sigma)$$

(Mainz)

$$\Rightarrow M_\nu < 3 m_\beta \sim 7\,\mathrm{eV}$$

Future: Katrin will probe $m_\beta \sim 0.2\,\mathrm{eV}$ (2σ)

Cosmology is and will be more efficient as a probe of $M_\nu = m_1 + m_2 + m_3$
(whatever the phases, Dirac/Majorana, θ_{MN}...)

II Neutrino decoupling and relic density

* As long as $T \gg \mathrm{MeV}$: neutrinos ultra-relativistic and maintained in thermal equilibrium through weak processes : $\nu_e + e^\pm \leftrightarrow \nu_e + e^\pm$

$$\nu_e + \bar{\nu}_e \leftrightarrow \nu_i + \bar{\nu}_i$$
$$\nu_e + \overset{(-)}{\nu}_i \leftrightarrow \nu_e + \overset{(-)}{\nu}_i \quad \text{etc...}$$

So, relativistic Fermi-Dirac phase space distribution

$$f_\nu(p) = \frac{1}{e^{p/T_\nu} + 1} \qquad \text{with } T_\nu = T_\gamma$$

$$\Rightarrow \begin{cases} n_\nu = \frac{\zeta(3)}{\pi^2} \frac{3}{4} \, 6 \, T_\nu^3 \\ \rho_\nu = \frac{\pi^2}{30} \frac{7}{8} \, 6 \, T_\nu^4 \end{cases}$$

This result is independent of existence of right-handed neutrinos: these are not weakly coupled, negligible density....

* Around $T \sim 1\,\mathrm{MeV}$: νs decouple (a bit later for ν_e). Exact value does not matter too much because f_ν just freezes-out (apart from cosmological redshift: $T_\nu \propto \frac{1}{a}$

* Around $T \sim 0.5\,\mathrm{MeV}$: electron-positron annihilate.

entropy conservation
instantaneous ν decoupling $\Big\}$ all e^+e^- pairs $\to \gamma$ and T_γ increases till

$$T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_\nu$$

In reality: $\frac{T_\gamma}{T_\nu}$ a bit smaller since a few e^+e^- annihilate into νs. Also f_ν gets small correction w.r.t. BB.

* At any time with $T \ll 0.5\,\mathrm{MeV}$:

$$T_\nu \propto \frac{1}{a}\ , \quad T_\gamma \propto \frac{1}{a}\ , \quad T_\nu \simeq \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

$$\rho_r = \frac{\pi^2}{30}\left[2T_\gamma^4 + \frac{7}{8}\times 6 \times T_\nu^4\right]$$

$$\equiv \underbrace{\frac{\pi^2}{30}2T_\gamma^4}_{\rho_\gamma}\left[1 + N_{eff}\times\frac{7}{8}\times\left(\frac{4}{11}\right)^{4/3}\right] \qquad \begin{pmatrix}\text{defines}\\ N_{eff}\end{pmatrix}$$

instantaneous dec. : $N_{eff} = 3$
in reality : $N_{eff} = 3.046$
... or larger if extrarelativistic degrees of freedom....

* For each mass eigenstate, when $T_\nu < m_i$, non-relativistic transition

$$n_\nu \propto \int dp\, p^2 f_\nu(p)$$

$$\rho_\nu \propto \int dp\, p^2 \sqrt{m^2+p^2}\, f_\nu(p) \longrightarrow m\, n_\nu$$

$$p_\nu \propto \int dp\, p^2 \frac{p^2}{\sqrt{m^2+p^2}} \sim \frac{\langle p^2\rangle}{m^2}\rho_\nu \ll \rho_\nu$$

* Today, $\begin{cases} n_\nu \sim 339\,\mathrm{cm}^{-3} \\ \text{at least 2 } m_{\nu_i} \ll T_\nu^0 \end{cases}$ since

$$T_\nu^0 \simeq \left(\frac{4}{11}\right)^{1/3} T_\gamma^0 \simeq 1.95\,\mathrm{K} \sim 10^{-4}\,\mathrm{eV}$$

If the 3 masses are $\gg T_\nu^0$, $\rho_\nu = \left(\sum_{i=1,2,3} m_{\nu i}\right) n_{\nu_1}$
$\Rightarrow \omega_\nu = \Omega_\nu h^2 = M_\nu / 93.14\,\mathrm{eV}$

If lighest mass $\lesssim T_\nu^0$, contr. so small that above formula remains true

(Note: 93.14 eV takes into account precise computation of ν decoupling)

III Impact of massless neutrinos on CMB

* For CMB physics, $T \ll$ MeV
  → decoupled species, impacts CMB only through gravitational coupling: $G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{\nu} + T_{\mu\nu}^{\text{others}}\right)$

  with $T_{\mu\nu} = \int dP_1\, dP_2\, dP_3 \sqrt{-g}\, \frac{P_\mu P_\nu}{P^0} f_\nu(x^i, P_j, \tau)$

  (P_μ = 4-momentum, $P_\mu P^\mu = m^2$)

* f_ν evolves through colisionless Boltzmann equation = Vlasov equation:

$$\frac{Df_\nu}{d\tau} = \left(\frac{\partial f_\nu}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f_\nu}{\partial x^i} + \frac{dP_i}{d\tau}\frac{\partial f_\nu}{\partial \tau}\right) = 0$$

* expansion of Vlasov at linear order:
  → perturbed FLRW metric
  → $\frac{dP_i}{d\tau}$ given by 1st order geodesic equation
  → f_ν decomposed in background + perturbation pieces: $f_\nu(x^i, P_j, \tau) = f_\nu^0(P, \tau)\left[1 + \Psi(x^i, P_j, \tau)\right]$

  ⇒ Equation of evolution for $\Psi(x^i, P_j, \tau)$

* change of variables: $x^i \longrightarrow k^i$ (Fourier)
  $P_j \longrightarrow (\hat{n}_j, P)$ direction momentum

  ⇒ Equation for $\Psi(k^i, \hat{n}_j, P, \tau)$

* does $f_\nu(x^i, P_j, \tau)$ remains a blackbody?
  YES for ultra-relativistic νs: when they go through potential wells/maxima, relative redshift is the same for $\forall p$
  So $f_\nu(x^i, P_j, \tau) = f_\nu^{\text{Fermi-Dirac}}$ with $T = T(\tau) + \delta T(\tau, x^i)$

So, the dependence of Ψ on P is very constrained and can be integrated out:

$$F_\nu(k^i, \hat{n}_j, \tau) \equiv \left[\frac{\int q^2 dq \; q f_\nu[1+\Psi]}{\int q^2 dq \; q f_{\nu_0}} - 1 \right] \quad \left(= 4 \frac{\delta T(k^i, \hat{n}_j, \tau)}{\bar{T}(\tau)} \right)$$

This F_ν is similar to the quantity $\Theta(x^i, n_j, \tau)$ of photons up to factor 4: gives temperature perturbation pattern in given point and direction

* FLRW is isotropic: $F_\nu(k^i, \hat{n}_j, \tau)$ depends on $\hat{n}_j$ only through angle $\hat{k}_i \hat{n}_j = \cos\theta$

→ Legendre expansion → $F_{\nu\ell}(k^i, \tau)$

* After all these steps we realize that the evolution is captured by a hierarchy of equations for $\dot{F}_{\nu\ell}$

* $T_{\mu\nu} = \int \cdots f_\nu \longrightarrow$

$$\delta_\nu = F_{\nu 0}$$
$$\theta_\nu = \frac{3}{4} k F_{\nu 1}$$
$$\sigma_\nu = -\frac{1}{2} F_{\nu 2}$$

metric $(+---)$

1st eq $\dot{F}_{\nu 0} = \cdots \Rightarrow$ continuity eq. for fluid with $w = c_s^2 = \frac{1}{3}$

$$\dot{\delta}_\nu = \frac{4}{3}\theta_\nu + 4\dot{\psi} \qquad \text{(Newtonian g.)}$$

2nd eq $\dot{F}_{\nu 1} = \cdots \Rightarrow$ Euler

$$\dot{\theta}_\nu = -\frac{k^2}{\delta_\nu} - k^2 \sigma_\nu - k^2 \phi$$

3rd eq. $\dot{F}_{\nu 2} = \cdots \Rightarrow$ evolution of shear/anisotropic stress

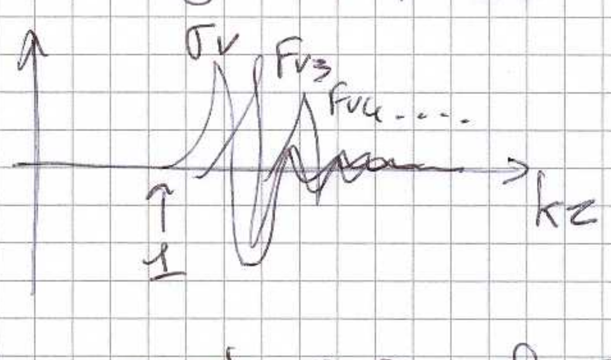
Higher moments do not appear in $T_{\mu\nu}$, but needed in order to close system up to some ℓ_{max} at which we truncate the system (see Ma & Bertschinger)

* Why do we need so many equations?

⇝ FOR PERFECT FLUID (many interactions)
Everything described locally by density, bulk velocity and isotropic pressure $\begin{cases} p = w\rho \\ \delta p = c_s^2 \delta\rho \end{cases}$
T_{ij} is diagonal

⇝ FOR COLLISIONLESS FLUID:
in each given point, superposition of flows of particles with $\forall$ momenta. Pressure anisotropic, σ = shear / anisotropic stress
= "quadrupole moment" of pressure
For ν's and decoupled photons, need ∞ of ℓ's ...

⇝ Still, ν's where behaving like perfect fluid at $T \gg 1$ MeV, and could be described with δ_ν, θ_ν only
At later times, remains true above the horizon: $F_{\nu \ell \geq 2} \simeq 0$ for $k\tau \ll 1$
Moments get populated later:



⇝ for accurate CMB, $\ell_{max} \sim 20$. But $\ell_{max} = 2$ gives a fair approximation. Then, system equivalent to "Generalized Dark Matter" (Hu98) with $(w, c_s^2, c_{vis}^2) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$$\dot{\sigma}_\nu + 3\frac{\dot{a}}{a}\sigma_\nu = 4\frac{c_{vis}^2}{w_\nu}\theta_\nu \quad \text{(factors to be checked)}$$

* effect of massless neutrinos in CMB:

① Background effect: shift in time of M/R equality:

$$\frac{a_{eq}}{a_0} = \frac{\rho_r^\circ}{\rho_m^\circ} = \frac{\omega_\gamma\left[1+(\dots)N_{eff}\right]}{\omega_m}$$

$\Rightarrow$ impacts CMB through early ISW and sound horizon

$N_{eff} \nearrow \Rightarrow$ later equality $\Rightarrow$ more early ISW $\Rightarrow$ higher peaks (mainly 1st peak)

$\Rightarrow a(t)$ different $\Rightarrow d_S = \int \frac{c_s dt}{a}$ smaller

$\Rightarrow \Theta = \frac{d_S}{d_A}$ smaller

$\Rightarrow \ell_{peaks}$ larger

② Perturbation effect: For $(k\tau) \gg 1$, ν_s free-stream over metric fluctuation. Add smooth component in Einstein equations. All sub-Hubble pert. are damped. (in fact sub-sonic: $\lambda \ll a\int \frac{c_s^2 dt}{a} \sim \frac{1}{\sqrt{3}} R_H$)

* impact of cosmic neutrino background in CMB is huge. For fixed $(\omega_m, \omega_b, \Omega_\Lambda, A, n, \tau)$:

$\to$ between $N_{eff}=0$ and $N_{eff}=3$: a_{eq}/z_{eq} shifted by 68% $\left(1+3\frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \simeq 1.68\right)$

$\to$ amplitude of 1st peak by $\sim 17\%$

$\to$ position " " " " $\sim 7\%$

* in practise, N_{eff} difficult to measure from CMB alone. Take model described by 7 parameters $(\omega_M, \omega_b, \Omega_\Lambda, N_{eff}, A, n, z)$.

⇝ Increase N_{eff} and ω_m in such way that z_{eq} is unchanged; then, no background effect. But more free-streaming: lower peaks. Now increase n a bit. Perturbation effect almost cancelled → degeneracy.

⇝ hardly resolved by WMAP.
WMAP7: $N_{eff} > 2.7$ (2σ) (0 excluded at few σ's)

⇝ removed when HST/LSS data → 4.3 ± 0.8 (1σ)

⇝ resolved by Planck alone:
Planck primary: $\sigma(N_{eff}) \sim 0.5$
" (with lensing extraction) $\sigma(N_{eff}) \sim 0.3$

[for 11 free params ΛCM + Y_{He}, M_ν, w, N_{eff}, α]

* if Planck confirms $N_{eff} > 3$, lots of new prospectives on cosmological models:
  - extra light ν's nearly thermalized (sterile ν's)?
  - light axions? Majorons? gravitinos?
  - dark radiation from braneworld?
  - GWs peaking around 10^{-15} Hz – 10^{-10} Hz $(\Omega_{gw} h^2 \sim 6.10^{-6} N_{eff})$

  (BBN bounds)

* can we probe the perturbation effect alone?
→ fit CMB with generalized DM with free c_{vis}^2
(0: perfect fluid; $\frac{1}{3}$: free-streaming ν's) ⇒ $c_{vis}^2 > 0.06$ (2σ) WMAP5 alone

IV Impact of massive neutrinos on CMB :

* Are neutrinos relativistic at time of γ decoupling?

$$T_\gamma^{dec} \sim 1089\, T_\gamma^0 \sim 3.0\ 10^3\,K$$

At that time $T_\nu \sim \left(\frac{4}{11}\right)^{1/3} T_\gamma \sim 2.1\ 10^3 K \sim 0.18\ eV$

So for each mass eigenstate, $m_i < 0.18 eV \Rightarrow$ relativistic at time of γ decoupling.

If at least one mass was $\geq 0.18 eV$ we would be in degenerate case with $M_\nu \geq 3 . 0.18 = 0.55 eV$

This is already excluded (although marginally) by CMB + LSS

(WMAP 7 + photometric SDSS + SNIA + BAO + HST : $M_\nu < 0.28 eV$ (2σ))

So we will not consider here the case of NR neutrinos at decoupling.

* overview of the section: effect of M_ν on CMB can be decomposed in:

$\rightsquigarrow$ background effect : despite parameter degeneracies, effect is important :

we will see that in 8-parameters model,

$$\left.\begin{array}{ll} \text{Planck :} & 0.2 eV\ (1\sigma) \\ \text{Inflation probe :} & [0.15 eV\ (1\sigma)] \end{array}\right\} \underbrace{\ll 0.55 eV}_{\text{Fisher matrix}}$$

$\rightsquigarrow$ perturbation effect : cannot affect primary CMB since νs still relativistic at decoupling.

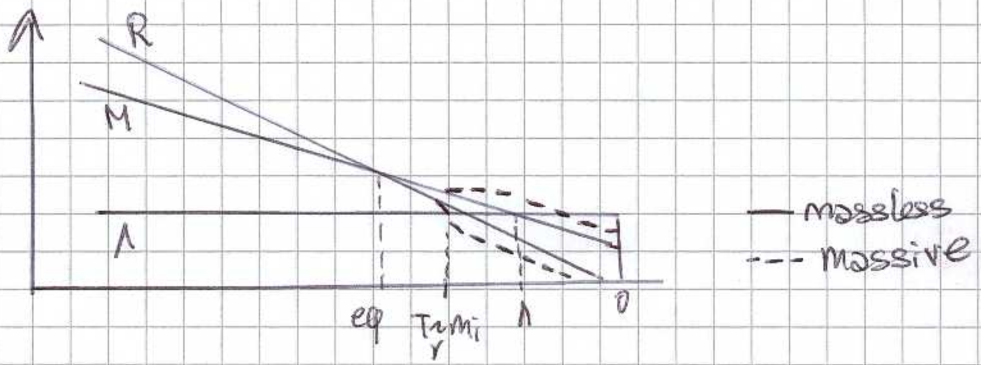
Important for $\left\{\begin{array}{l} \text{lensing} \\ \text{late ISW} \\ \text{SZ} \end{array}\right.$

Background effect

General idea: massive neutrinos included in

$\rightarrow \rho_R$ for $T_\nu > m_i$ } LEAKAGE FROM
$\rightarrow \rho_M$ for $T_\nu < m_i$ } R TO M

Surprisingly, this effect is not degenerate with other parameters.



→ if Increase M_ν with ω_m fixed: $\omega_b + \omega_{cdm} \searrow$ so M/R equality takes place later ⇒ { early ISW, scale of peaks

→ if increase M_ν with ω_b and ω_{cdm} fixed, M/Λ equality takes place later ⇒ { late ISW, scale of peaks

... in any case, in 7-parameters "ΛCDM + M_ν" = ΛMDM, effect of M_ν non-degenerate with other parameters.

Proof: 7 independent parameters

$\{\omega_b, \omega_m, \Omega_\Lambda, M_\nu, A, n, \tau\}$

7 independent effects

- b/cdm balance → odd/even peaks
- M/R equality → early ISW
- Λ/R equality → late ISW
- all these quantities → scale $\frac{d_s(\tau_{dec})}{d_H(\tau_{dec})}$ of peaks
- A → overall amplitude
- n_s → " slope
- τ → small-ℓ T and E modes

So, in principle, can all be measered independently. In practise, data have error-bars: need global fit to see if degeneracies resolved.

Same conclusion if N_{eff} is free: in principle, M_ν and N_{eff} can be distinguished since N_{eff} affects primary CMB while M_ν does not for $M_\nu < 0.55\,eV$.

For 7-param: WMAP7: $M_\nu < 1.3\,eV$ (2σ)
WMAP7+BAO+HST: " $< 0.58\,eV$ (")
→ but this could be attributed to perturbation effect

For 8-param (+Y_{He}): Planck primary: $\sigma(M_\nu) \sim 0.2\,eV$
" 11- " (+N_{eff}, W, α): " " : $\sigma(M_\nu) \sim 0.4\,eV$
... would degrade even more with Ω_k.

Perturbation effect (on secondary CMB / foregrounds)

* summary: during MD & ΛD, $\dot{\delta}_b$ and $\dot{\delta}_{cdm}$ ↘ if M_ν ↗. So,
  - less structures ⇒ less lensing
  - more $\dot{\phi}$ ⇒ non-trivial ISW
  - $P(k)$ lower ⇒ small signature on SZ

* <u>formalism</u>: νs cannot be described by $f_{\nu\ell}(\vec{k}, z)$ in N.R. regime. Geodesic equation involves m: neutrinos experience different redshift if they are deep/hardly in NR regime. So BLACKBODY GETS DISTORTED. Need to stick to variable $\Psi_{\nu\ell}(\vec{k}, P, z)$: many more equations.

* $\delta\rho_\nu, \delta p_\nu, \theta_\nu, \sigma_\nu$ etc... related to integrals over $\int p^2 dp \, (\ldots) f_0(p,\tau) \, \Psi(\vec{k},p,\tau)$

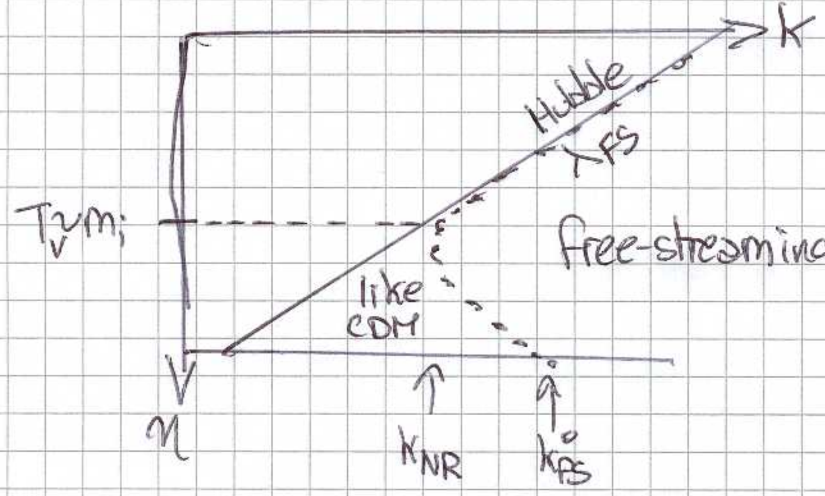
When $T_\nu < m_i$: * $p_\nu \ll \rho_\nu$
* $\delta p_\nu \ll \delta\rho_\nu$: $c_s^2 \longrightarrow 0$
* $\sigma_\nu \ll \delta_\nu$: asymptotically like pressureless perfect fluid

So ν cluster like CDM! However this is only possible on scales

$$\lambda \gg \lambda_{FS} = a(t) \int \frac{\langle v \rangle dt}{a} \quad \text{with} \ \langle v \rangle = \frac{\langle p \rangle}{m} \sim \frac{3T_\nu}{m} \propto \frac{1}{a}$$

so $\lambda_{FS}^{T_\nu < m_i} \sim a \int \frac{dt}{a^2} \sim a \int \frac{dt}{t^{4/3}} \sim a \, t^{-1/3}$

comoving λ_{FS} decreases



$\rightsquigarrow$ above $\frac{a}{k_{NR}}$, same power spectrum as in ΛCDM with same ω_m

$\rightsquigarrow$ below $\frac{a}{k_{FS}^0}$, $\begin{cases} \to \delta_\nu \text{ do not contribute to } \delta_m \\ \to \delta_b \ \& \ \delta_{cdm} \text{ grow at slower rate} \end{cases}$

Einstein (Mezaros) : $\ddot{\delta}_m + 3H\dot{\delta}_m = 4\pi G \rho_m \delta_m$
$= 4\pi G (\rho_{cdm}\delta_{cdm} + \rho_b \delta_b + \cancel{\rho_\nu \delta_\nu})$

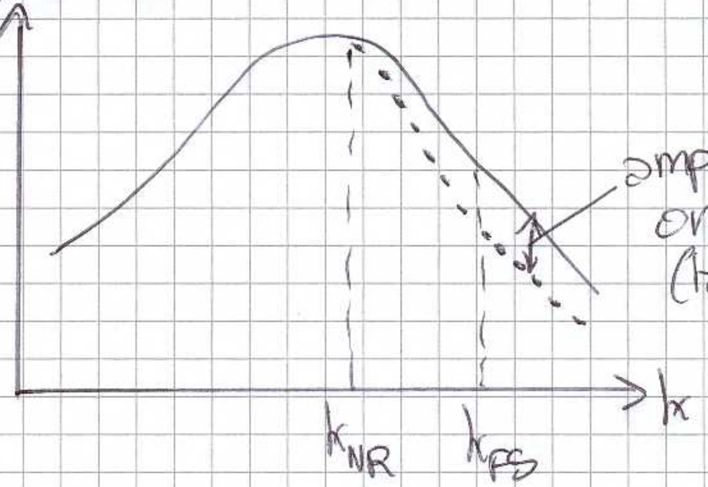
Friedman: $\rho_{cdm} + \rho_b + \rho_\nu$

$\delta_m \propto a^{1-\frac{3}{5}f_\nu}$

$f_\nu = \frac{\omega_\nu}{\omega_m}$

$$P(k,z) = \langle |\delta_m(k,z)|^2 \rangle$$

δ_0



amplitude depends on τ / z
(today, $\frac{\Delta P}{P} \sim -8 f_\nu \ldots$)
less in the past

$\Rightarrow$ impact on lensing:

$$\underset{\text{Poisson}}{\Delta\phi = 4\pi G\,\delta\rho} \Rightarrow k^2\phi = 4\pi G a^2 \rho_m \delta_m$$

$$\Rightarrow \langle|\phi|^2\rangle = \left(\frac{4\pi G a^2 \rho_m}{k^2}\right)^2 \underbrace{\langle|\delta_m|^2\rangle}_{P(k)}$$

Step-like reduction in $P(k)$ $\Rightarrow$ same in $\langle|\phi|^2\rangle$

$\Rightarrow$ same in $C_\ell^{\phi\phi}$ ← lensing potential

and C_ℓ^{dd} ← deflection angle

Negligible correction to $\tilde{C}_\ell^{TT}$, $\tilde{C}_\ell^{TE}$, etc....

... but detectable if $C_\ell^{\phi\phi}$ measured by lensing extraction

Forecasts (the error on lensing extraction is based on Hu & Okamoto's "quadratic estimator" formulas)

Planck + lensing extraction: $\sigma(M_\nu) \sim 0.1$ eV (8 params)

0.1 eV (11 params)

↳ degeneracy removed with w, N_{eff}, α

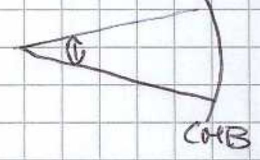
Inflation probe / cmbpol : $\sigma(M_\nu) \sim 0.03 - 0.04$ eV

Ideally this should be combined with tomographic cosmic shear survey to provide a "last redshift bin" $\rightarrow$ then for $\left.\begin{matrix}\text{Planck}\\ \text{cmbpol}\end{matrix}\right\}$ + lensing + LSST, $\sigma(M_\nu) \sim \begin{matrix}0.005\text{ eV}\\ \updownarrow\\ 0.02\text{ eV}\end{matrix}$

⇒ impact on ISW

* in C_ℓ^{TT} ISW component is difficult to see because primary anisotropies and late ISW mixed.
Best probe: correlate T map with galaxy map in redshift bin $z \simeq z_m$ ⇒ provides lISW at this redshift

* effect of ν free-streaming detectable on small scales.
lISW detectable for $\ell \ll 100$.
But small scales at small redshift projected on large angles! $\theta = \frac{\pi}{\ell} = \frac{(a/k)}{d_A(z)}$



So M_ν can impact $\ell \ll 100$

* what is the sign of the effect?

$$\Delta_T^{isw}(\hat{n}) = -2 \int_0^{z_{dec}} dz \, \frac{d\Phi}{dz}(\hat{n}, z)$$

(late ISW: contribution from $z \ll z_{dec}$)

⤳ neutrino reduce amplitude of fluctuations of Φ on small scales ⇒ reduction

⤳ neutrinos increase $|\dot{\Phi}|$ or $\frac{d\Phi}{dz}$ as soon as ν's are NR: $\delta_{m\,(b,cdm)} \sim a^{1-\frac{3}{5}f_\nu} \Rightarrow \Phi \sim a^{-\frac{3}{5}f_\nu}$

⇒ amplification

Second effect wins only when $z \geq 1$ and $\ell \geq 50$: hardly detectable.
First effect remains, but not such a good probe as galaxy auto-correlation spectrum.

[ends with remarks on SZ, on issue of non-linear perturbations]