Perfect

- So now we just go to a telescope and measure
  - The magnification
  - The convergence
  - The shear
- And we perfectly understand cosmology
  - No?

Weak Lensing
Shape Measurement

Tom Kitching

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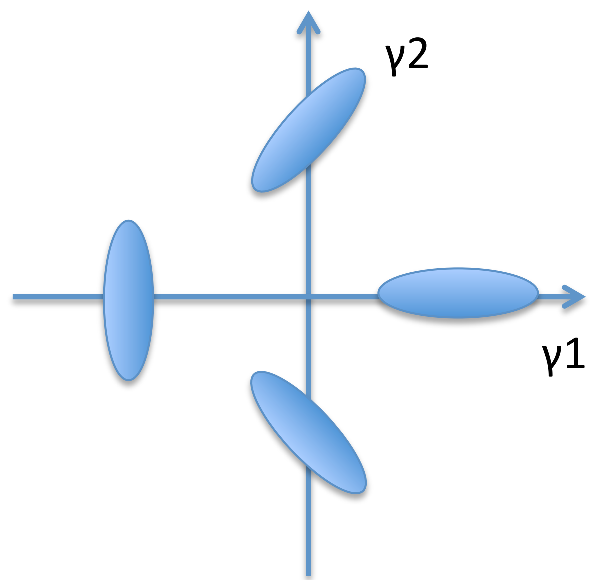
@tom_kitching

David	Tom	David	Tom
What do we want to measure	How do we measure	Statistics of measurement	Cosmological Parameter Extraction

photon → ... → Understanding

$$
\begin{aligned}
\beta &= \theta - \hat{\alpha}\frac{D_{ds}}{D_s} \\
\beta &= \theta - \alpha.
\end{aligned}
$$

- *Distortion matrix* implies that distortion is ***<u>elliptical</u>***



Chess Player/Mud Wrestler

ct mathematical

reasoning… in that

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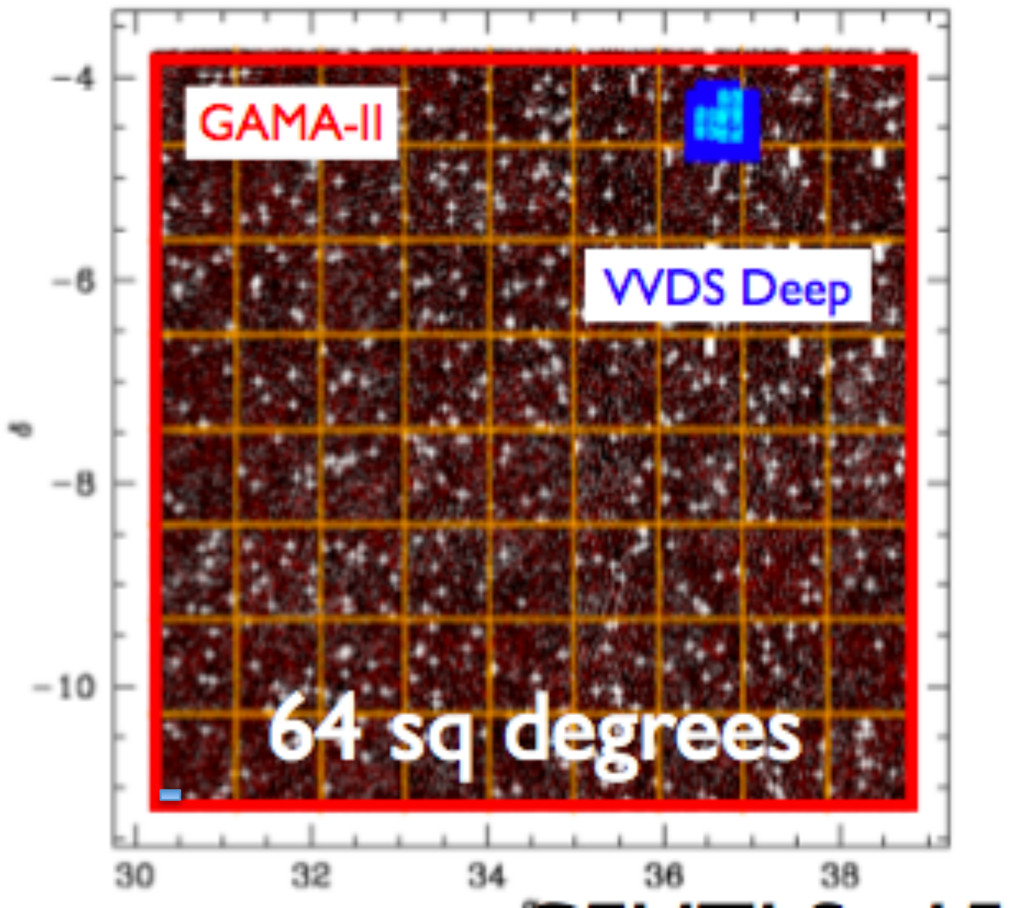
re, is more like **mud**

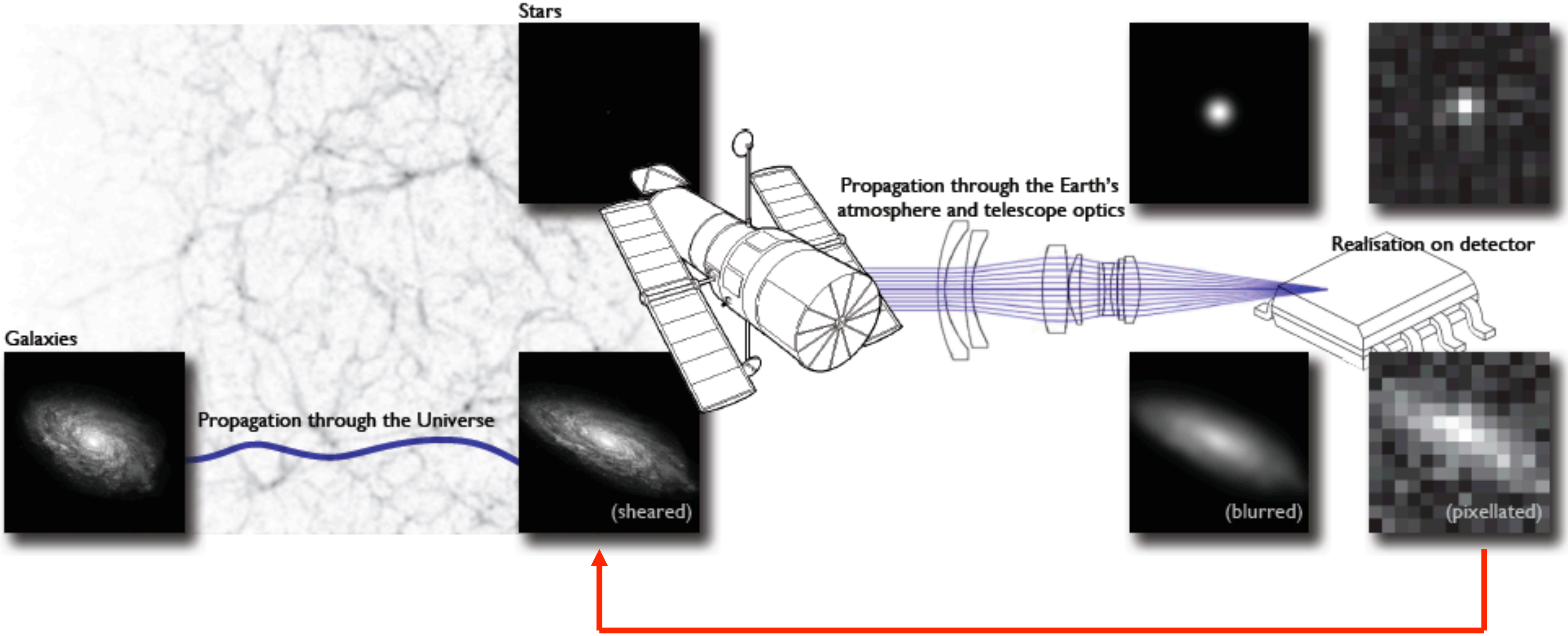
Martin Rees

Example of Real Data



- What will we learn?

  - Basics of shape measurement
  - Method Review
  - Simulations


Stars
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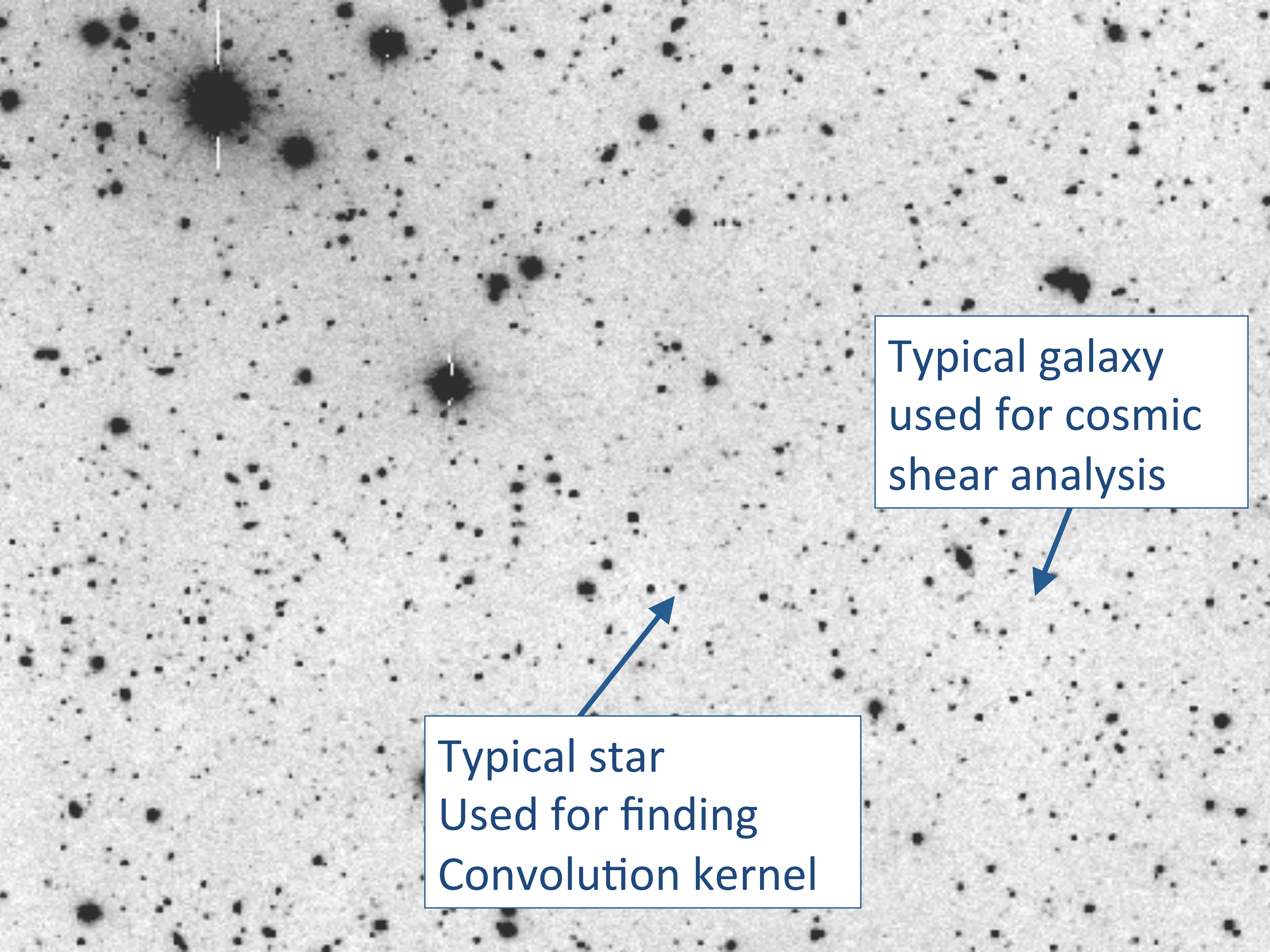
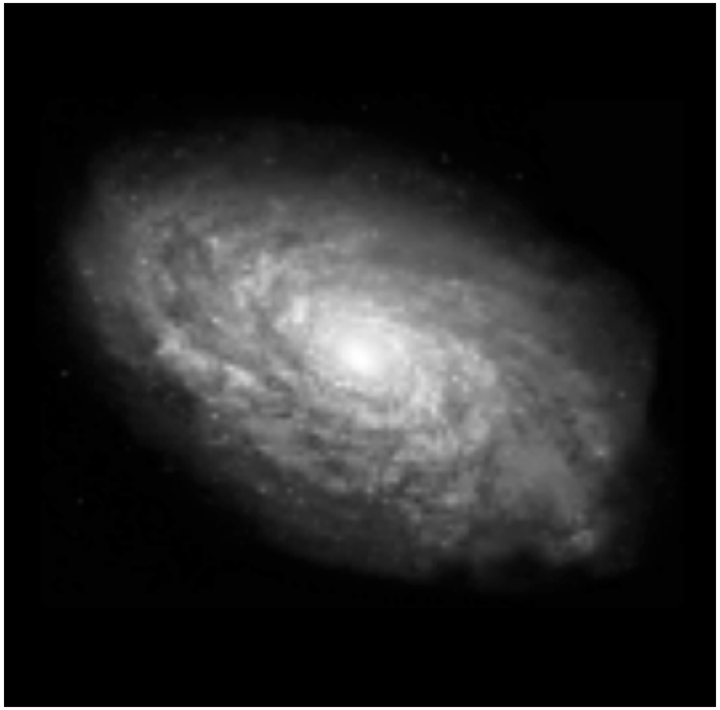

Typical galaxy used for cosmic shear analysis
Typical star Used for finding Convolution kernel

Cosmic Lensing



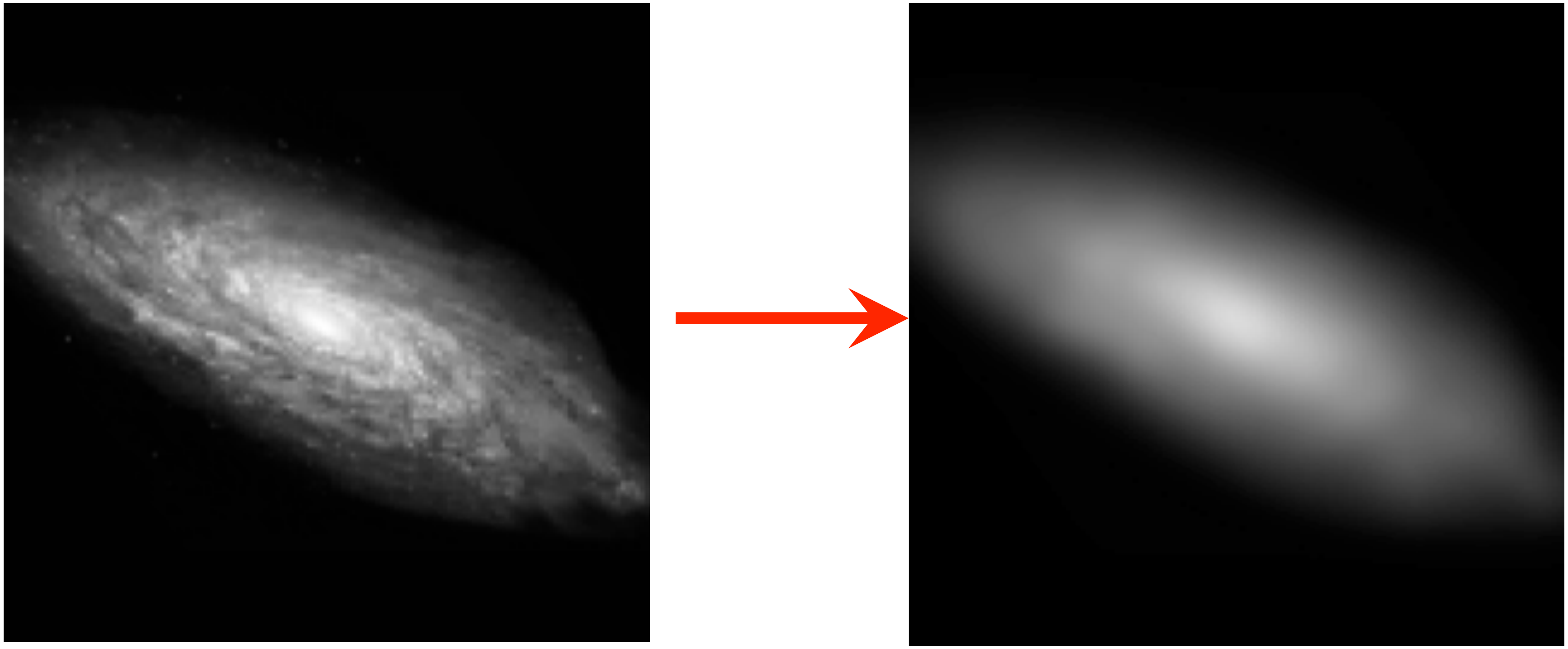
g_i~0.2

$$\begin{pmatrix} x_u \\ y_u \end{pmatrix} = \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix} \begin{pmatrix} x_l \\ y_l \end{pmatrix}$$

Real data: g_i~0.03

Atmosphere and Telescope

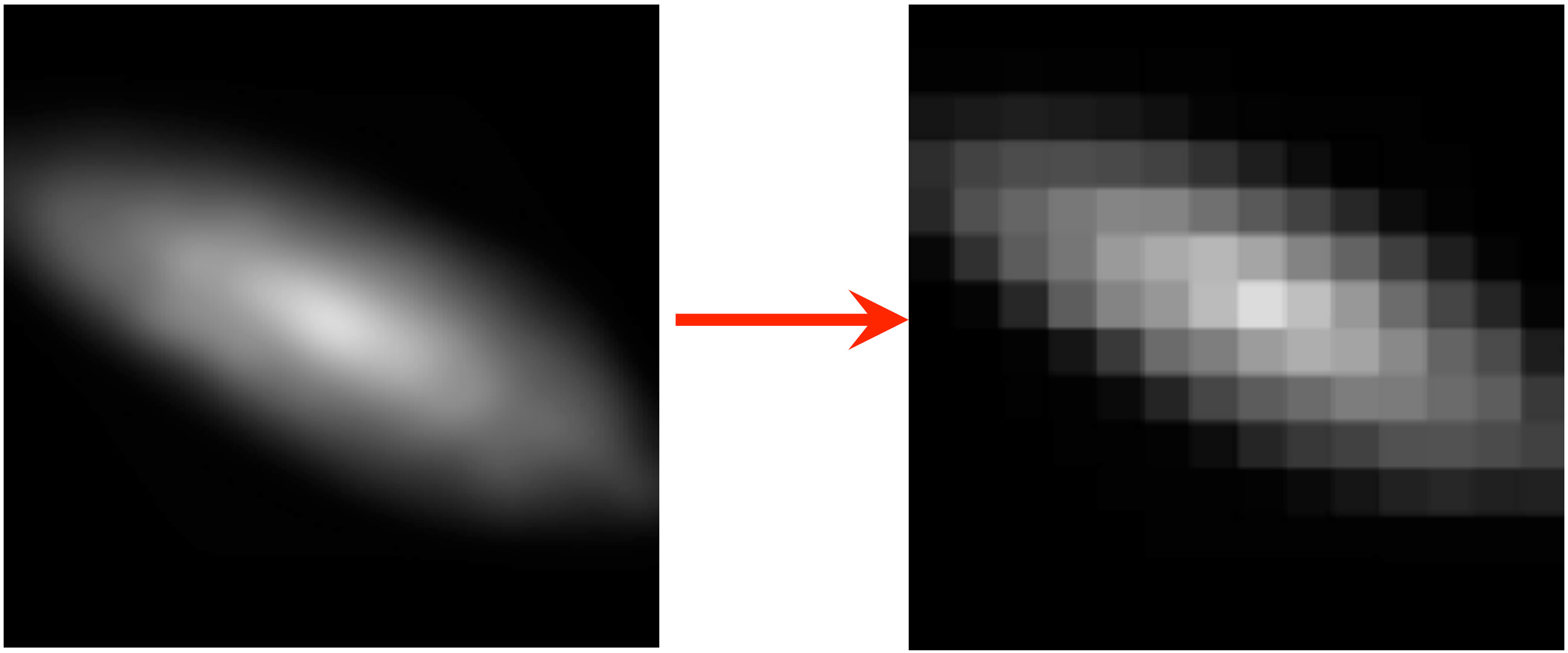
Real data: Kernel size ~ Galaxy size



Convolution with kernel

- PSF=Point Spread Function
- Fourier Transform call "Impulse Response"

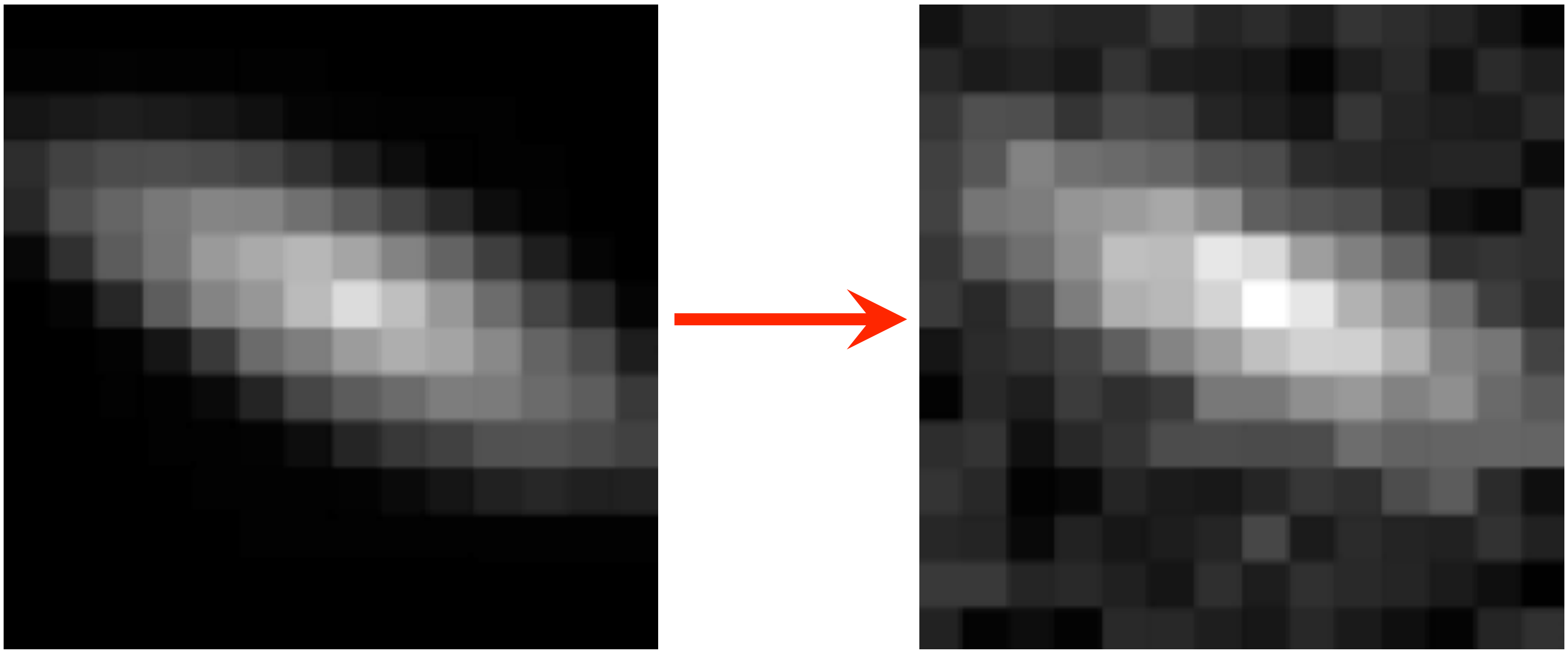
Pixelisation



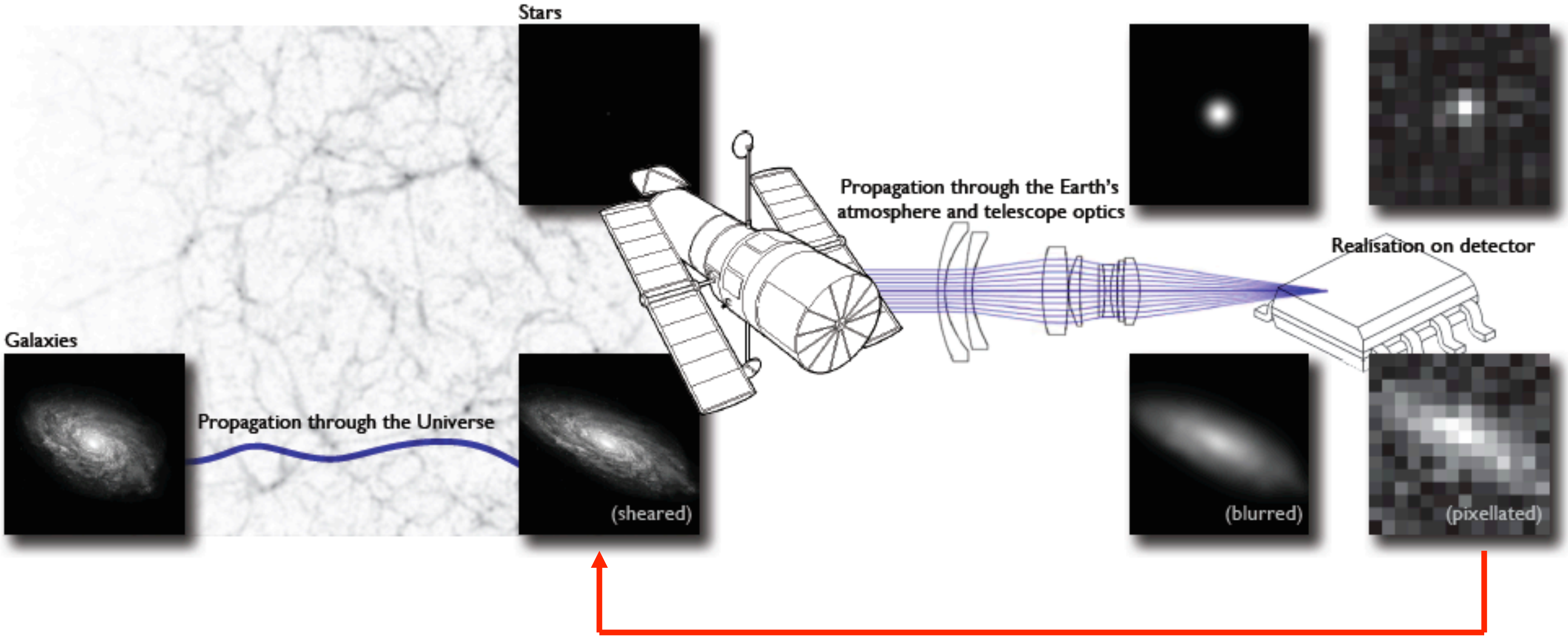
Sum light in each square

Real data: Pixel size ~ Kernel size /2

Noise



Mostly Poisson. Some Gaussian and bad pixels.
Uncertainty on total light ~ 5 per cent



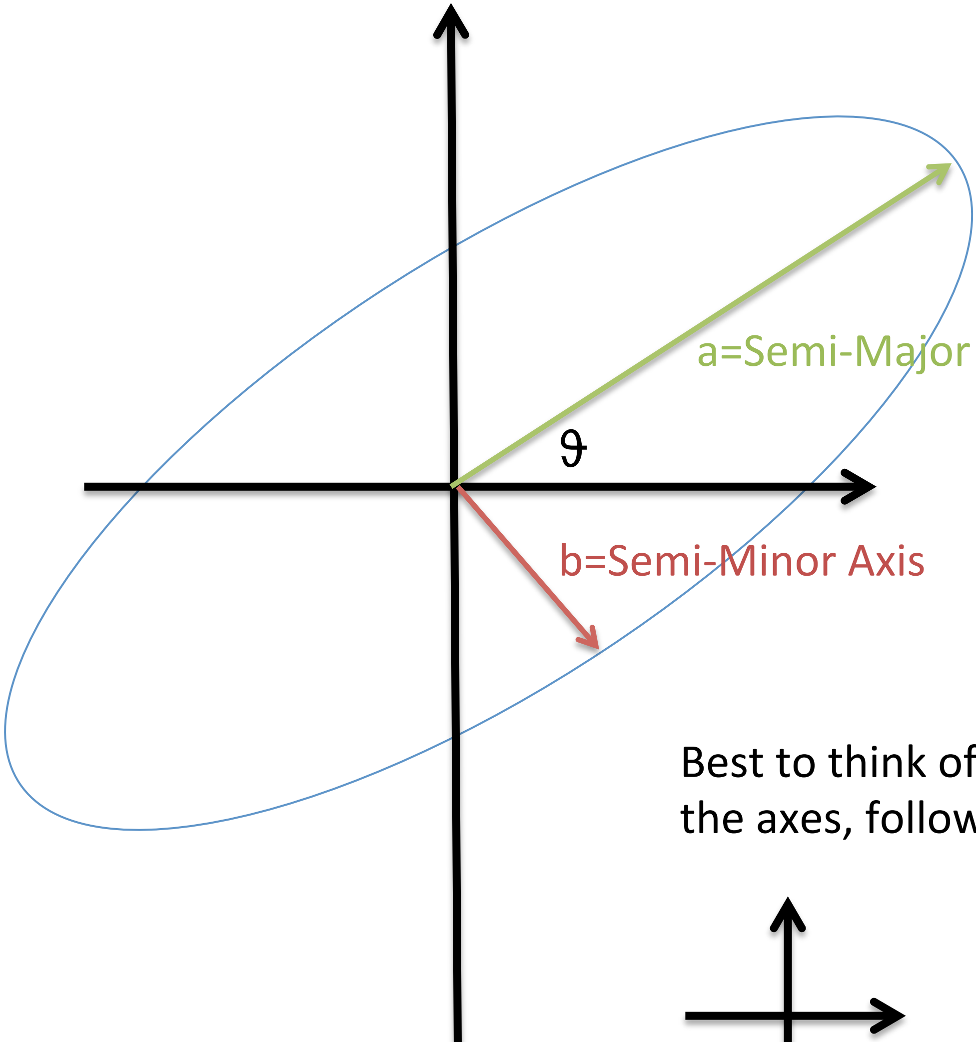
Need to :

- Raw data reductions
- Detect objects
- Classify Objects
- Astrometry (accurate positions)
- Photometry (accurate fluxes/magnitudes)

Need to :

- Measure the PSF
- Measure the galaxy ellipticities

What is ellipticity

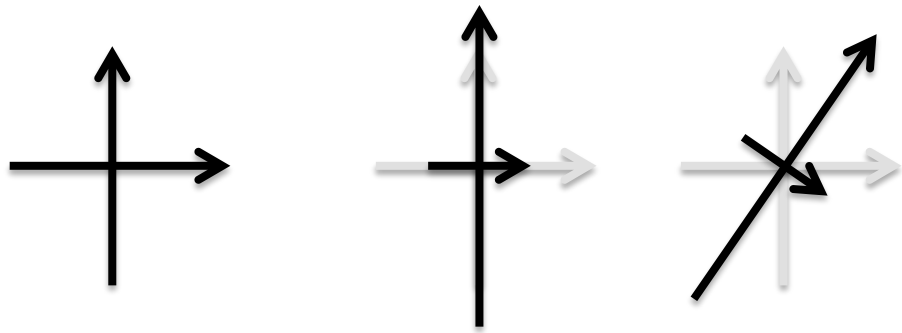


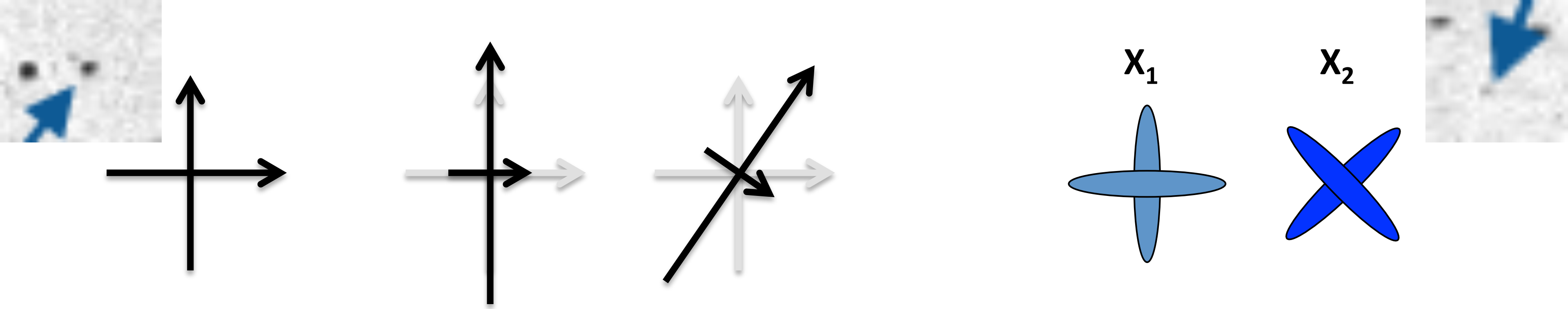
Specified by **3 numbers**

- Semi-Major axis
- Semi-Minor
- Angle

Many different equations for an ellipse use different parameterizations and coordinate systems

Best to think of an ellipse as a compression/stretching of the axes, followed by a rotation





- Create a single (complex) number “ellipticity”
  - Collapse the size information -> 2 numbers
  - Encodes orientation

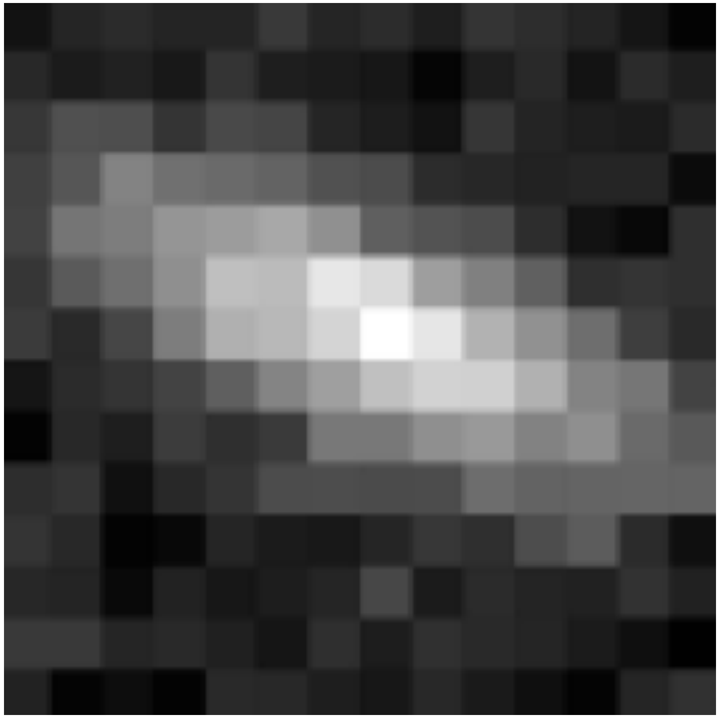
$$\chi = \frac{a^2 - b^2}{a^2 + b^2} = \frac{1 - r^2}{1 + r^2}$$

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1 - r^2}{1 + r^2} \begin{pmatrix} \cos(2\theta) \\ \sin(2\theta) \end{pmatrix}$$

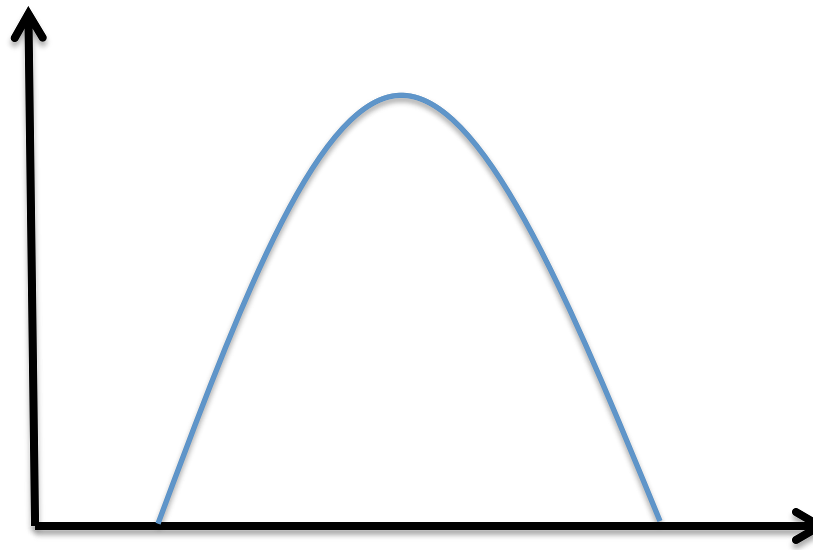
- r=1 means?
- why 2, what does this mean?

$$\boxed{\boldsymbol{\chi} = |\chi| \mathrm{e}^{\mathrm{i}2\theta} = \chi_1 + \mathrm{i}\chi_2}$$

How to measure ellipticity?



- Consider moments of a 1D distribution



$$m_i = \int x^i f(x) \mathrm{d}x$$

$$m_{i,c} = \int (x - m_1)^i f(x) \mathrm{d}x$$

0th Moment = ?
1st Moment = ?
2nd Moment = ?

Moments of a 2D Function

- Include a weight G

$$F = \int P(x,y)G(x,y,\sigma,\bar{x},\bar{y})\mathrm{d}x\mathrm{d}y$$

0^{th} Moment

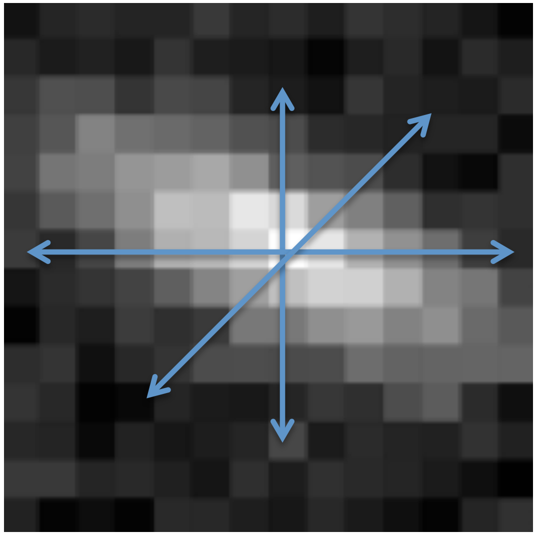
$$\begin{aligned}\bar{x} &= \int P(x,y)xG(x,y,\sigma,\bar{x},\bar{y})\mathrm{d}x\mathrm{d}y/F\\ \bar{y} &= \int P(x,y)yG(x,y,\sigma,\bar{x},\bar{y})\mathrm{d}x\mathrm{d}y/F\end{aligned}$$

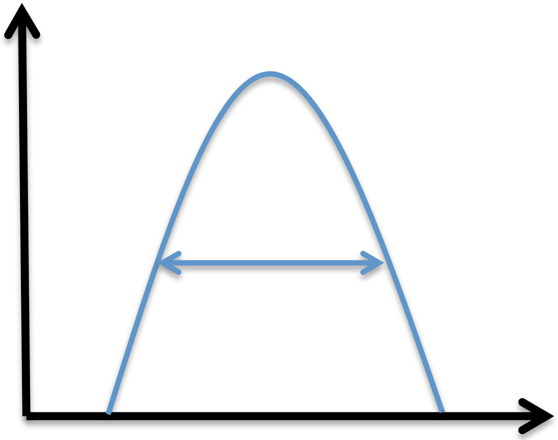
1^{st} Moments
Mean x
Mean y

2^{nd} Order Moments in 2D

$$
\begin{aligned}
Q_{xx} &= \int P(x,y)(x-\bar{x})^2 G(x,y,\sigma,\bar{x},\bar{y})\mathrm{d}x\mathrm{d}y/F \\
Q_{xy} &= \int P(x,y)(x-\bar{x})(y-\bar{y}) G(x,y,\sigma,\bar{x},\bar{y})\mathrm{d}x\mathrm{d}y/F \\
Q_{yy} &= \int P(x,y)(y-\bar{y})^2 G(x,y,\sigma,\bar{x},\bar{y})\mathrm{d}x\mathrm{d}y/F
\end{aligned}
$$

- <u>Three</u> 2^{nd} order moments (recall that ellipticity needs 3 numbers)

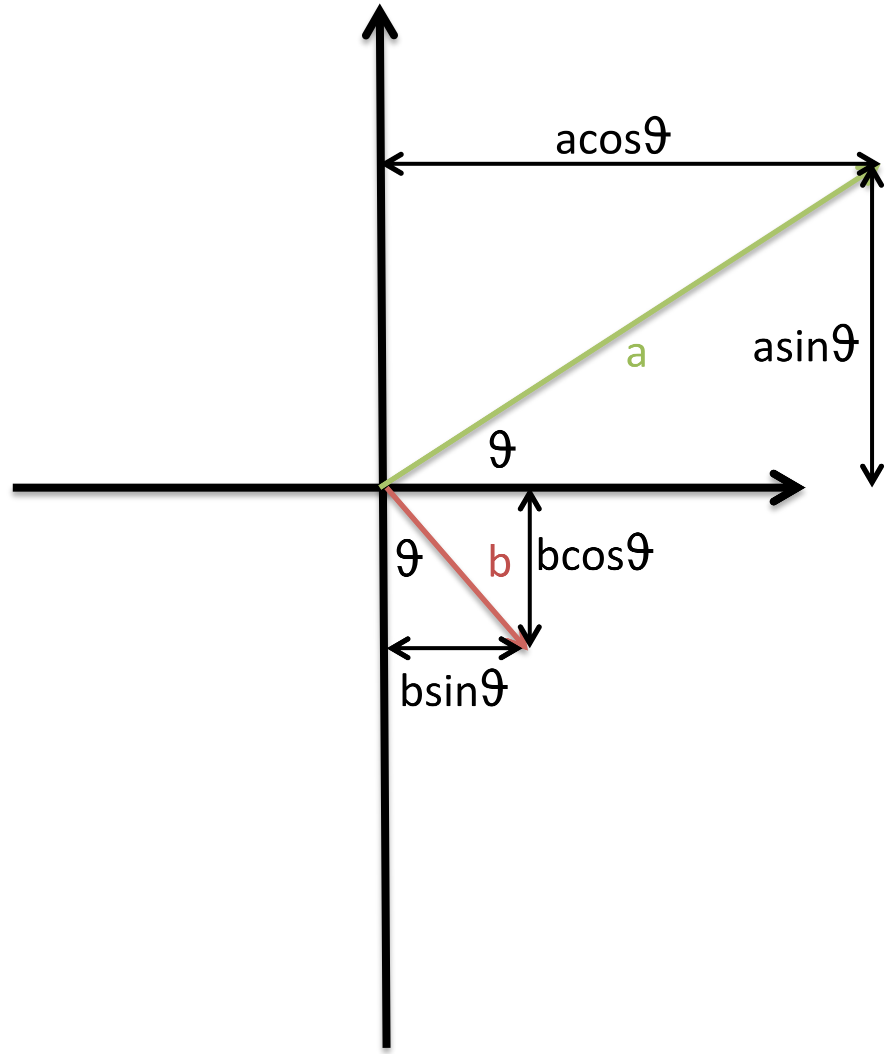




How are Q related to ellipticity?

Note:
Q are variances (so squared quantities)

Homework/Beachwork… prove the Q_{xy}



- $Q_{xx}=F(a^2\cos^2\vartheta+b^2\sin^2\vartheta)$
- $Q_{yy}=F(a^2\sin^2\vartheta+b^2\cos^2\vartheta)$
- $Q_{xy}=F(a^2-b^2)\sin\vartheta\cos\vartheta$

How are Q related to ellipticity?

- $Q_{xx}=F(a^2\cos^2\vartheta+b^2\sin^2\vartheta)$
- $Q_{yy}=F(a^2\sin^2\vartheta+b^2\cos^2\vartheta)$
- $Q_{xy}=F(a^2-b^2)\sin\vartheta\cos\vartheta$

$$\chi = \frac{a^2-b^2}{a^2+b^2} = \frac{1-r^2}{1+r^2}$$

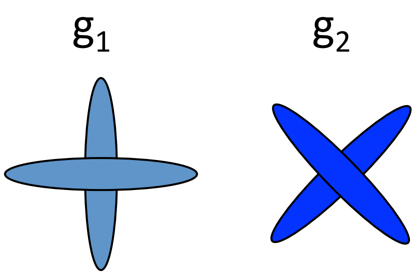
- Take combinations:
  - $Q_{xx}+Q_{yy}=F(a^2+b^2)$
  - $Q_{xx}-Q_{yy}=F(a^2-b^2)\cos2\vartheta$
  - $2Q_{xy}=F(a^2-b^2)\sin2\vartheta$

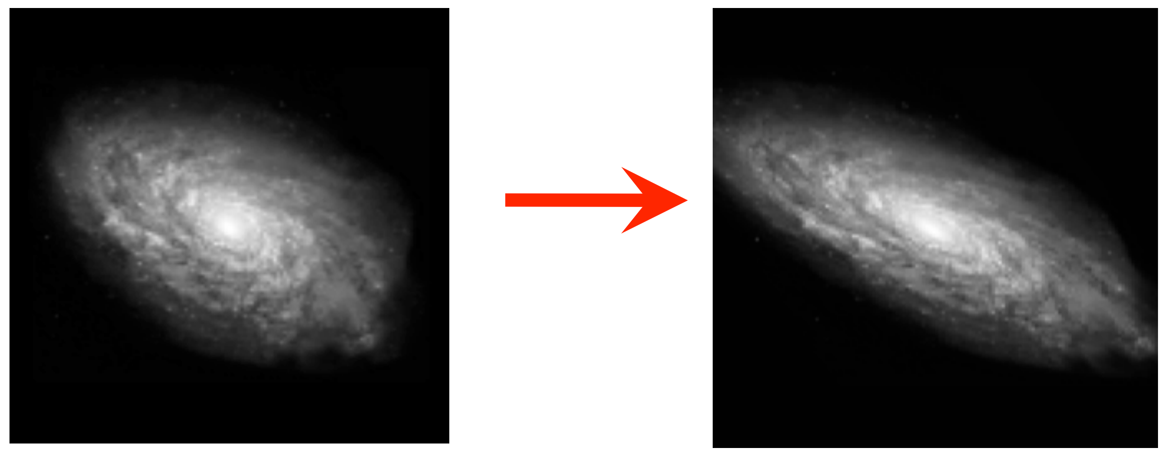
$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1-r^2}{1+r^2}\begin{pmatrix} \cos(2\theta) \\ \sin(2\theta) \end{pmatrix}$$

$$\boxed{\chi^S = \frac{(Q^S_{11}-Q^S_{22})+2\mathrm{i}Q^S_{12}}{Q^S_{11}+Q^S_{22}}}$$

Intrinsic Ellipticity

- Have introduced here the notion that the sources themselves are already elliptical





- *"Shear"* is an **additional ellipticity** imprinted by lensing
- How to related **observed** ellipticity to **shear**?

How to relate ellipticity to shear

- How to source moments transform/change under the influence of the lens equation?

$$\boxed{\begin{aligned} \beta &= \theta - \hat{\alpha}\frac{D_{ds}}{D_s} \\ \beta &= \theta - \alpha . \end{aligned}}$$

$$\boxed{\beta = \theta - \theta^* g}$$

$$\chi = \frac{\int \mathrm{d}\beta I(\beta)\beta^2}{\int \mathrm{d}\beta I(\beta)\beta\beta^*}$$

Image plane

$$\chi^S = \frac{\int \mathrm{d}\theta I(\theta)\theta^2}{\int \mathrm{d}\theta I(\theta)\theta\theta^*}$$

Source plane

1) Substitute lens equation into image plane moments
2) Rearrange (hint: multiply numerator and denominator by source plane denominator)
3) Recall 2Re(z)=z*+z for a complex number

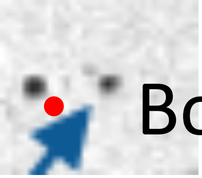
$$\boxed{\chi^S = \frac{(Q^S_{11} - Q^S_{22}) + 2\mathrm{i}Q^S_{12}}{Q^S_{11} + Q^S_{22}}}$$

$$\chi^{(\mathrm{s})} = \frac{2g + \chi + g^2\chi^*}{1 + |g|^2 + 2\mathcal{R}\mathrm{e}\,(g\chi^*)}$$

The inverse transformation is

$$\chi = \frac{\chi^{(\mathrm{s})} - 2g + g^2\chi^{(\mathrm{s})*}}{1 + |g|^2 - 2\mathcal{R}\mathrm{e}\left(g\chi^{(\mathrm{s})*}\right)}$$

- Schneider & Seitz (1995)
- Allows the *observed ellipticity* (moments) to be related to the intrinsic (unlensed) ellipticity and shear



- Bonnet & Mellier (1995)
- Different normalisation of the moments

$$\varepsilon \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}},$$

$$\varepsilon = \frac{\chi}{1 + (1 - |\chi|^2)^{1/2}}, \quad \chi = \frac{2\varepsilon}{1 + |\varepsilon|^2}$$

$$\varepsilon^{(s)} = \begin{cases} \dfrac{\varepsilon - g}{1 - g^* \varepsilon} & \text{for} \quad |g| \leq 1 \\[2ex] \dfrac{1 - g\varepsilon^*}{\varepsilon^* - g^*} & \text{for} \quad |g| > 1 \end{cases}$$

- More complicate denominator, better shear relation

The weak lensing limit

$$\boxed{\chi = \frac{\chi^S - 2g + g^2\chi^{S*}}{1 + |g|^2 - 2\mathrm{Re}(g\chi^{S*})}}$$

- Only linear in shear

$$\boxed{\begin{aligned} \chi &= \chi^S - 2g \\ \epsilon &= \epsilon^S - g \end{aligned}}$$

The Weak Lensing Assumption

- When we average over (enough) galaxies in the universe the intrinsic ellipticity is randomly orientated such that

$$\begin{aligned} \chi &= \chi^S - 2g \\ \epsilon &= \epsilon^S - g \end{aligned}$$

$$\boxed{\begin{aligned} \langle\epsilon\rangle &= \langle\epsilon^S\rangle - \langle g\rangle \\ \langle\epsilon\rangle &\approx \langle g\rangle. \end{aligned}}$$

KEY WEAK LENSING RESULT

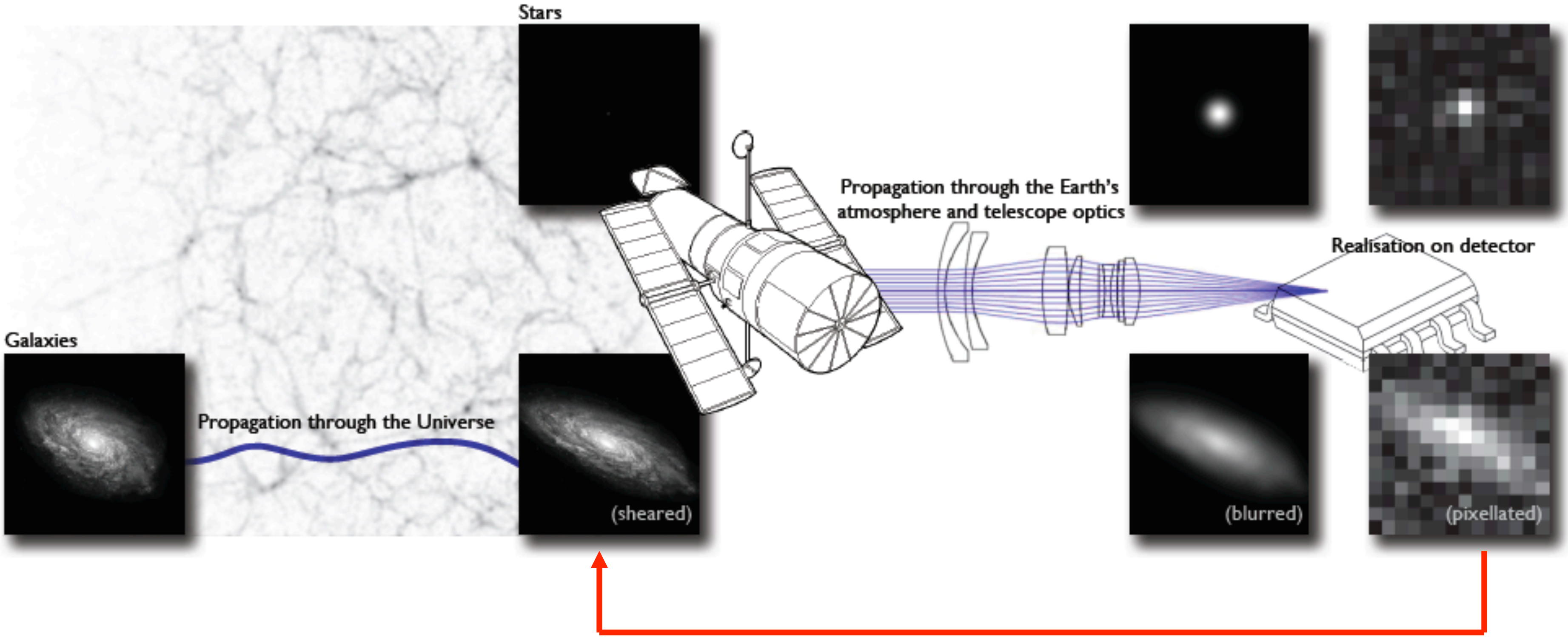
RELATES
OBSERVABLE (MOMENTS) to
PHYSICS (LENS EQUATION) and SHEAR

- Recap:

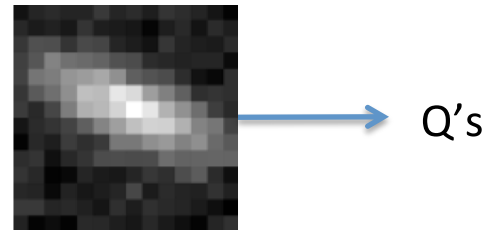
  - Basics of shape measurement
  - Method Review
  - Simulations

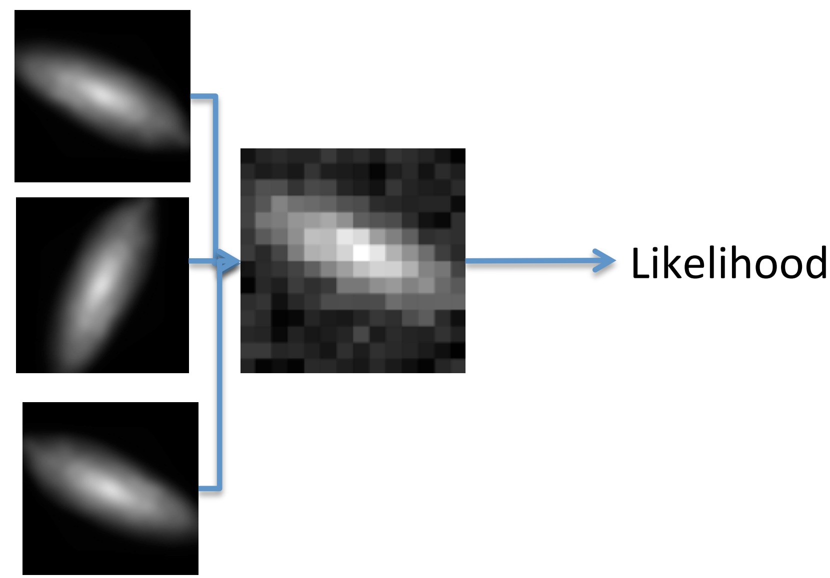
Practical Methods

"Shape Measurement Method"


Stars
Galaxies
Propagation through the Universe
(sheared)
Propagation through the Earth's
atmosphere and telescope optics
Realisation on detector
(blurred)
(pixellated)

- Two main approaches
  - “Moment Measurements”
    - Accounts for the PSF by mathematically changing moments
  - “Model Fitting”
    - Accounts for PSF by convolving models





Moments

- Taking into account the PSF
  - Additional Quadrupole

$$Q_{ij}^{(\mathrm{obs})} = P_{ij} + Q_{ij}$$

$$\chi^{(\mathrm{obs})} = \frac{\chi + T\chi^{(\mathrm{PSF})}}{1+T},$$

$$T = \frac{P_{11}+P_{22}}{Q_{11}+Q_{22}}; \quad \chi^{(\mathrm{PSF})} = \frac{P_{11}-P_{22}+2\mathrm{i}P_{12}}{P_{11}+P_{22}}$$

  - For practical implementation (KSB, 1995)
    - Also ask Henk Hoekstra over coffee
  - Also DEIMOS (Melchior, Viola et al.)

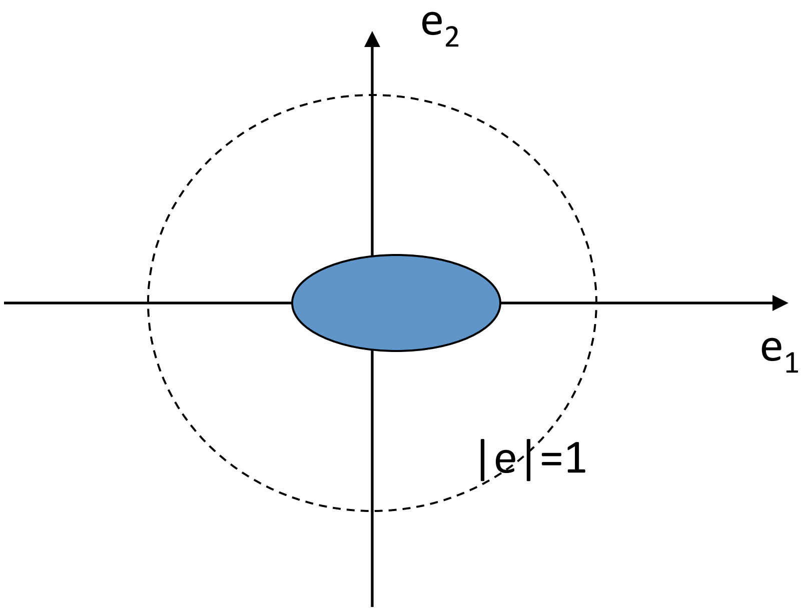
Model Fitting

- Idea of model fitting
  - Instead of measuring a quantity *from* the data we can fit a model *to* the data
- The model can contain elements that
  - Model the galaxy (intrinsic shape)
  - Model the PSF
  - The model can be convolved with the PSF

$$\ln P(e_1, e_2, \boldsymbol{\theta}) = \sum_{\mathrm{d,data}} (\mathrm{Galaxy} * \mathrm{PSF} - \mathrm{Data}) C^{-1} (\mathrm{Galaxy} * \mathrm{PSF} - \mathrm{Data})^T$$

Model Fitting

- Minimum set of parameters we need are
  - e_1, e_2, position (x,y), brightness, size



$$P(e_1, e_2) = \int \mathrm{d}\theta_\alpha P(e_1, e_2, \theta_\alpha)$$

Model Fitting

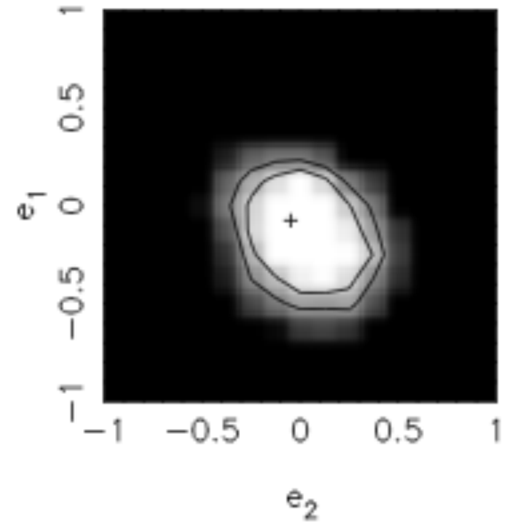
- Bayesian Model Fitting

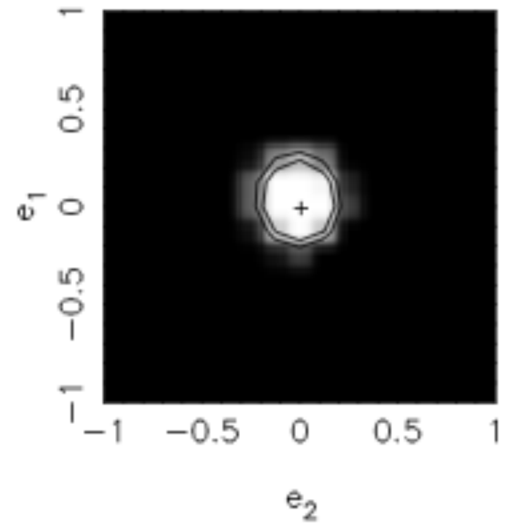
$$p_i(\mathbf{e}|\boldsymbol{y}_i) = \frac{\mathcal{P}(\mathbf{e})\,\mathcal{L}(\boldsymbol{y}_i|\mathbf{e})}{\int \mathcal{P}(\mathbf{e})\,\mathcal{L}(\boldsymbol{y}_i|\mathbf{e})\,d\boldsymbol{e}}$$

- Prior in this case is the probability distribution of the *intrinsic* ellipticity distribution

Model Fitting

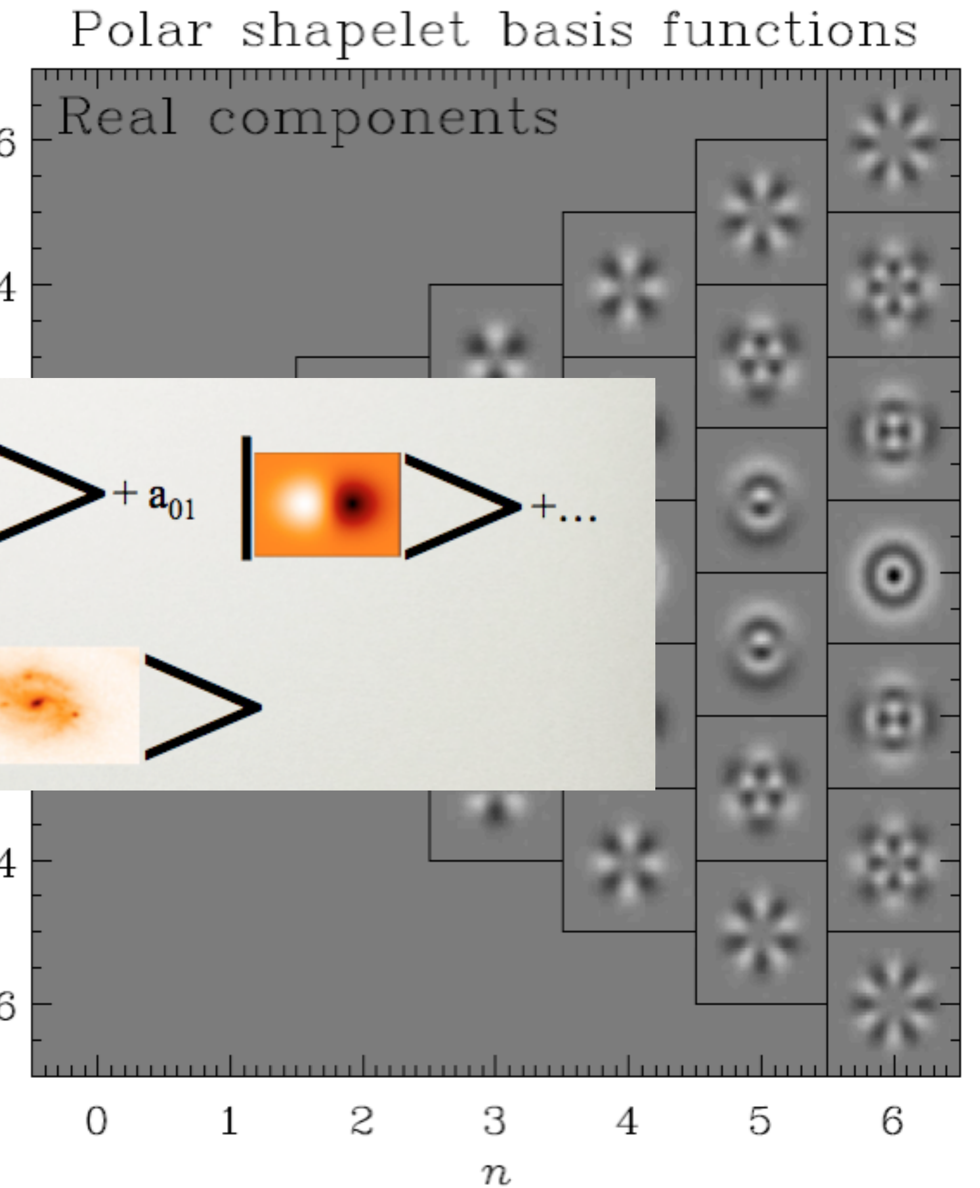
- Lensfit
  - Miller et al. (07) & Kitching et al. (08)
  - Bayesian Model fitting
  - Uses empirical models
    - bulge+disk
  - Analytically marginalises over brightness and galaxy position
- im3shape
  - Bridle et al.

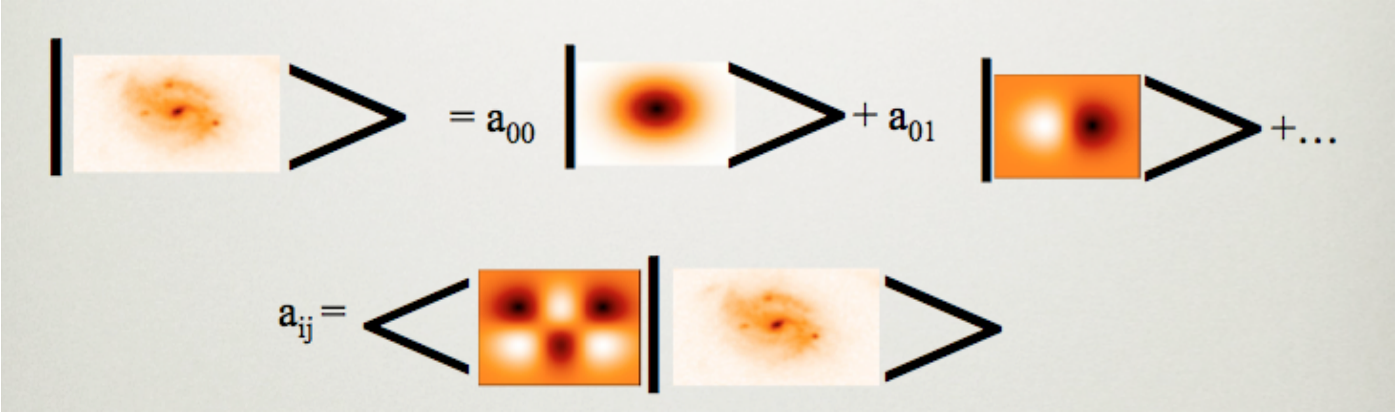




Model Fitting

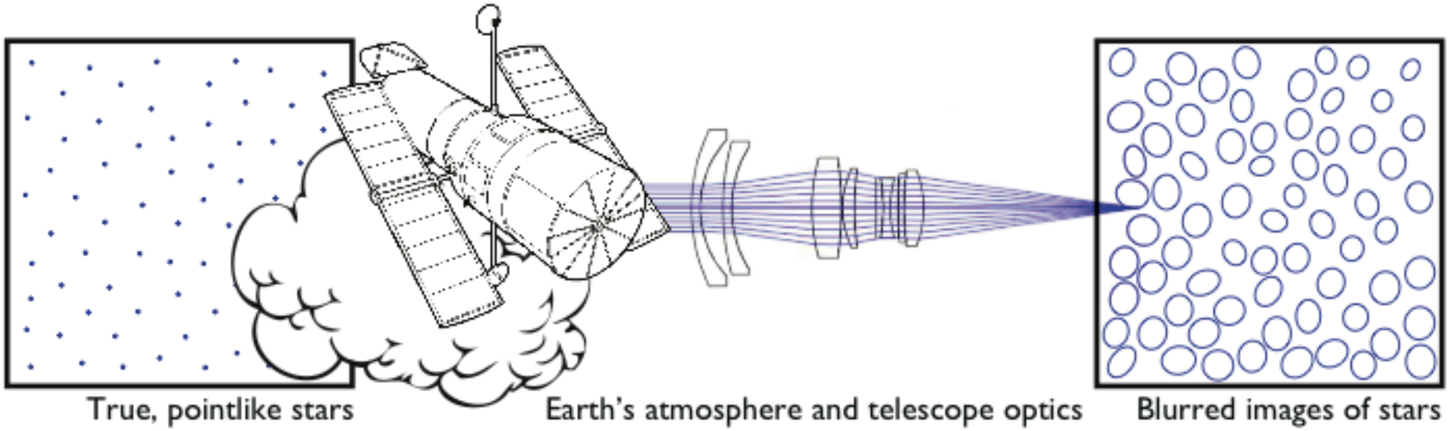
- Shapelets
  - Complex model based on a QM formalism
  - 2D basis set expansion
- Al
  -
- Ask David over coffee

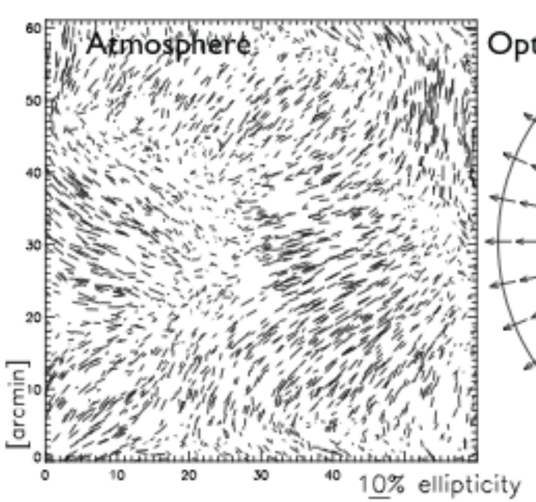


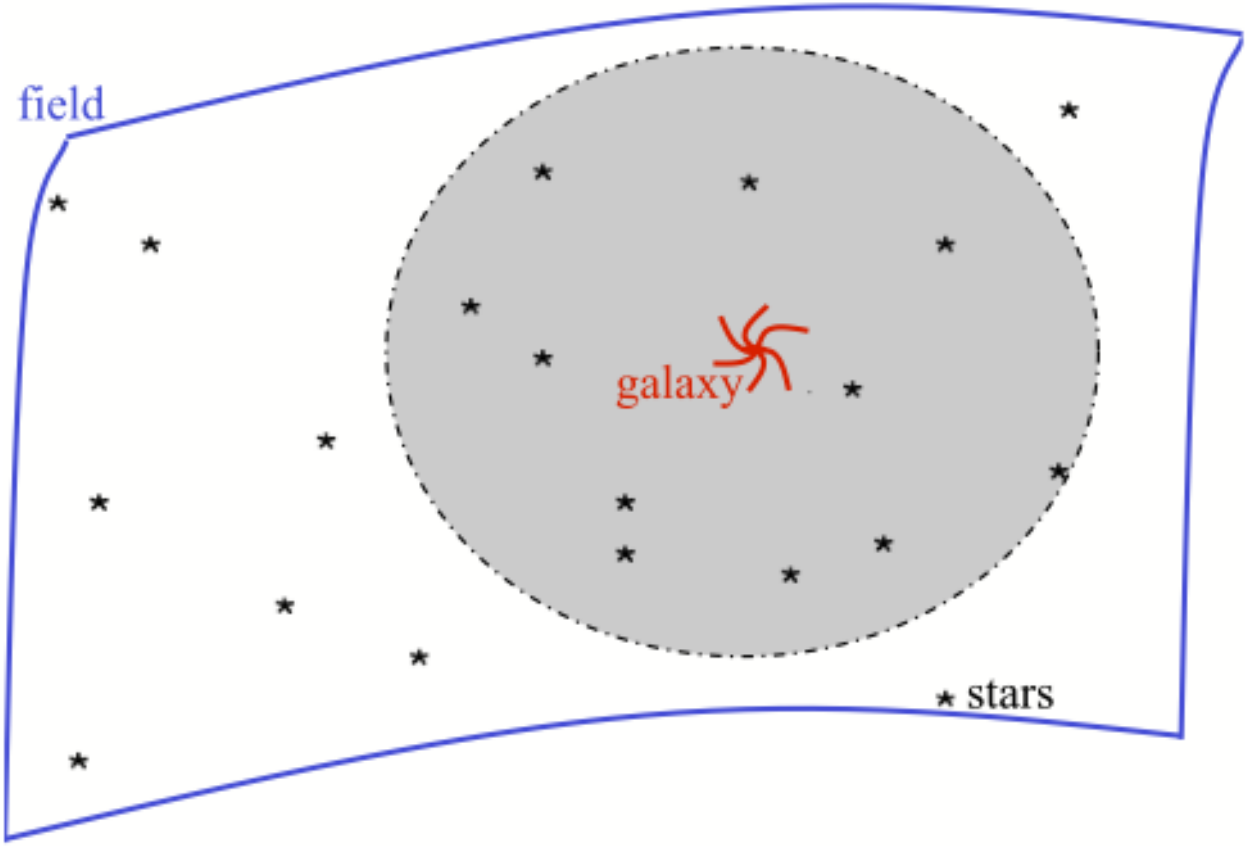


PSF Modelling

Need to model the PSF as well





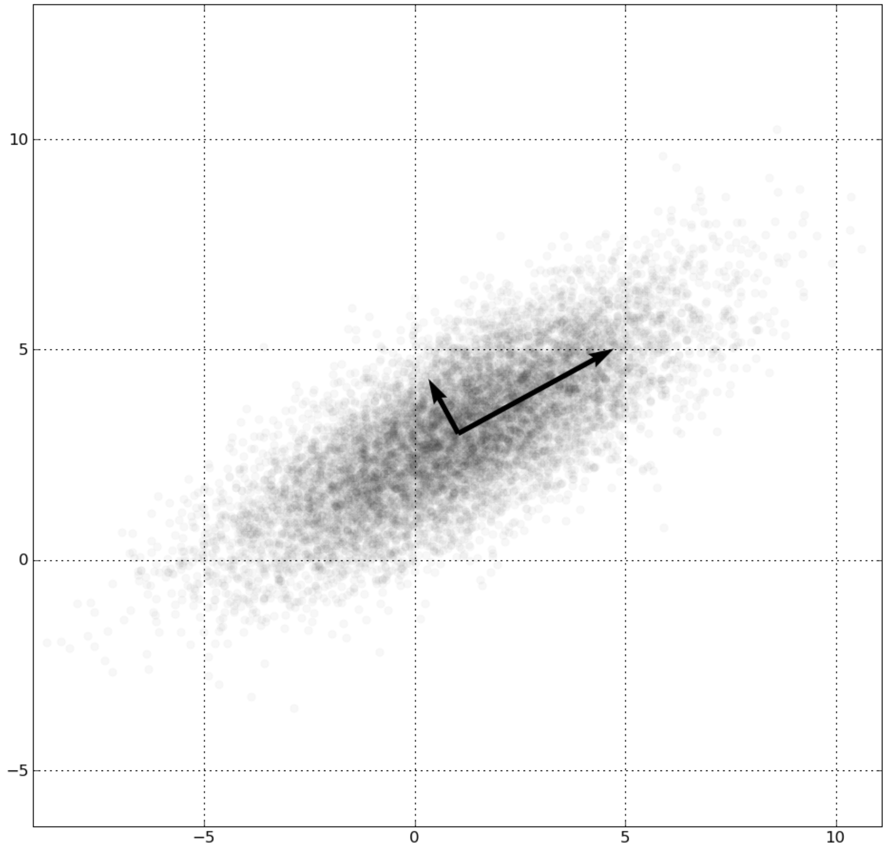


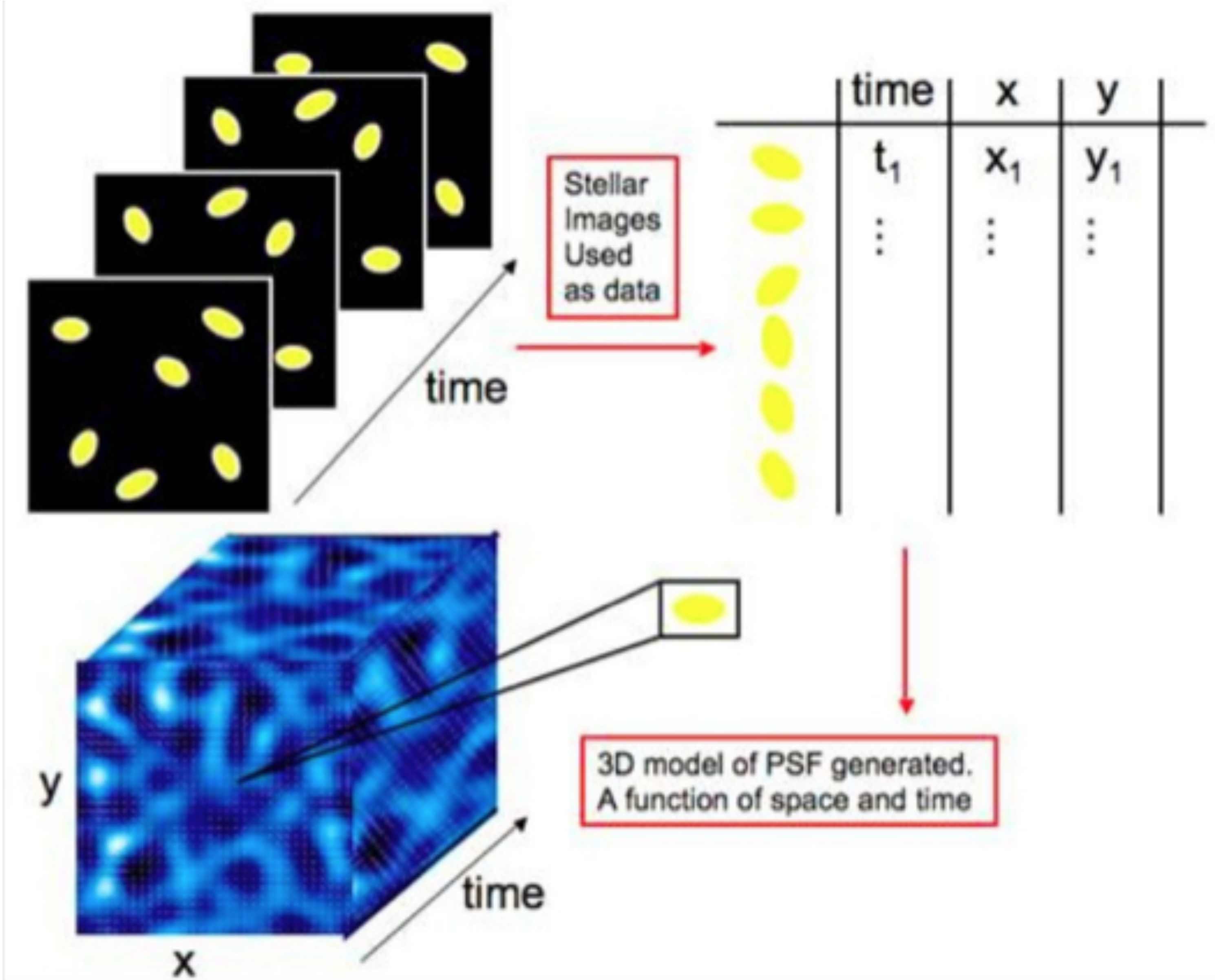
Want : PSF at the Galaxy Position
Have : PSF sampled at Star Position

PSF Modelling

- Two main ways of PSF modelling
  - Direct : Model the PSF in each exposure using a fitted model to either pixel intensity, ellipticity, size of stars
  - Indirect : Use multiple exposures to extract the model from the data -- a PCA-like approach
  - Also *deconvolution : remove the PSF from the data by deconvolving the data*

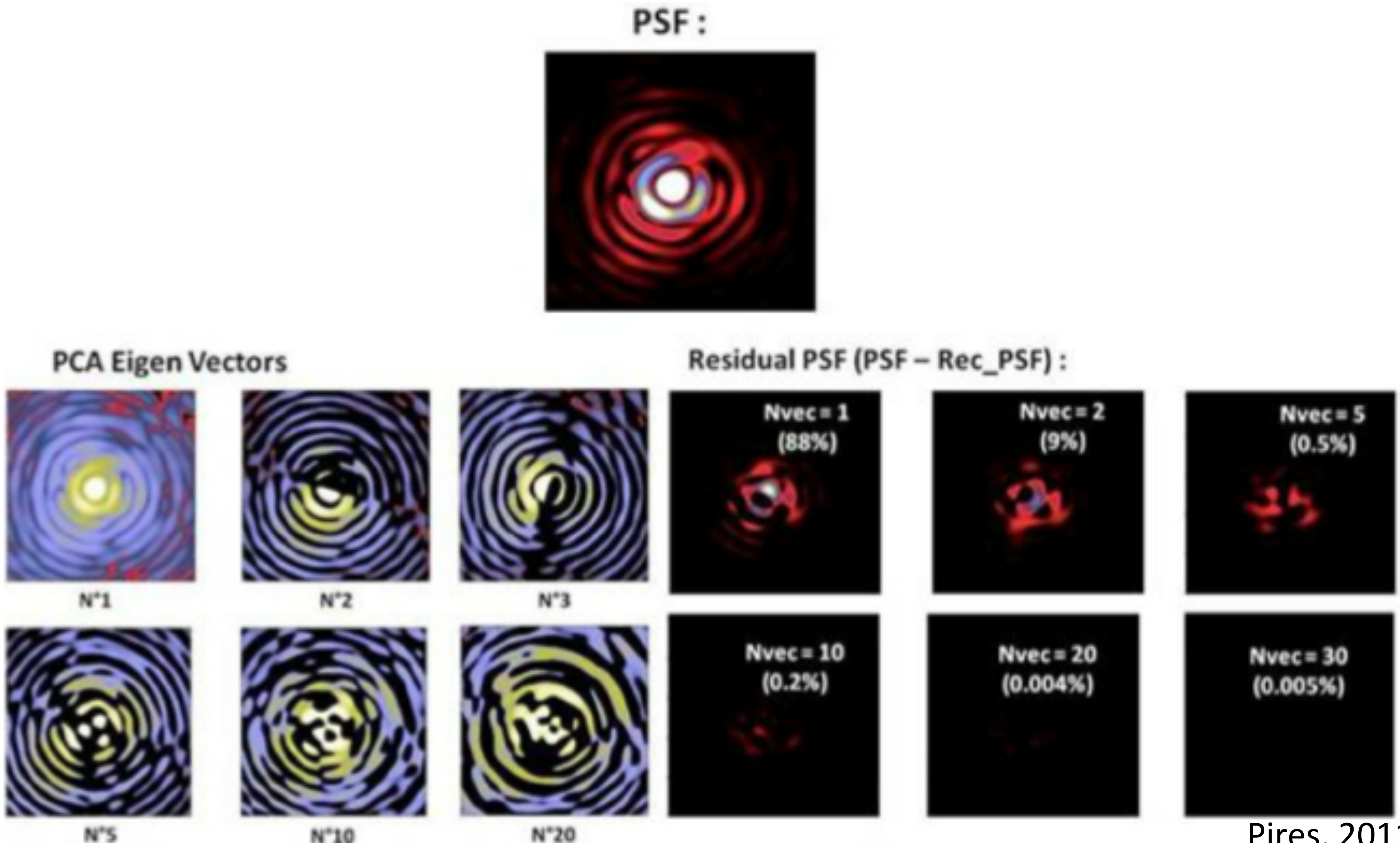
- Can use Principle Component Analysis





time
x
y
t_1
x_1
y_1
Stellar
Images
Used
as data
time
3D model of PSF generated.
A function of space and time
y
x
time

- Example from Euclid

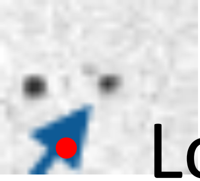


Pires, 2011

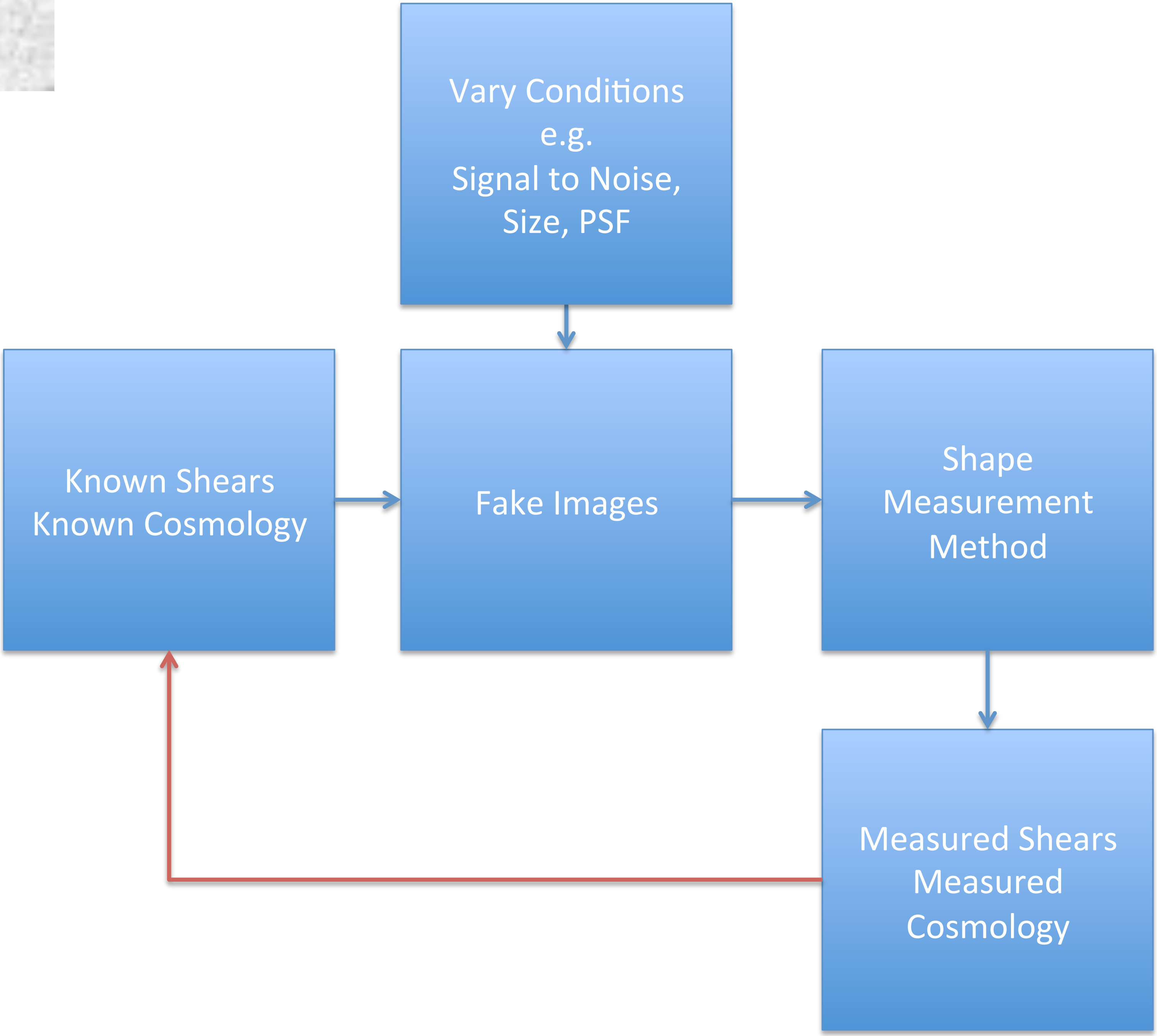
- Recap:
  - Basics of shape measurement
  - Method Review
  - Simulations

Simulations

- Shear Testing Programme (STEP)
- GRavitational lEnsing Accuracy Testing (GREAT)



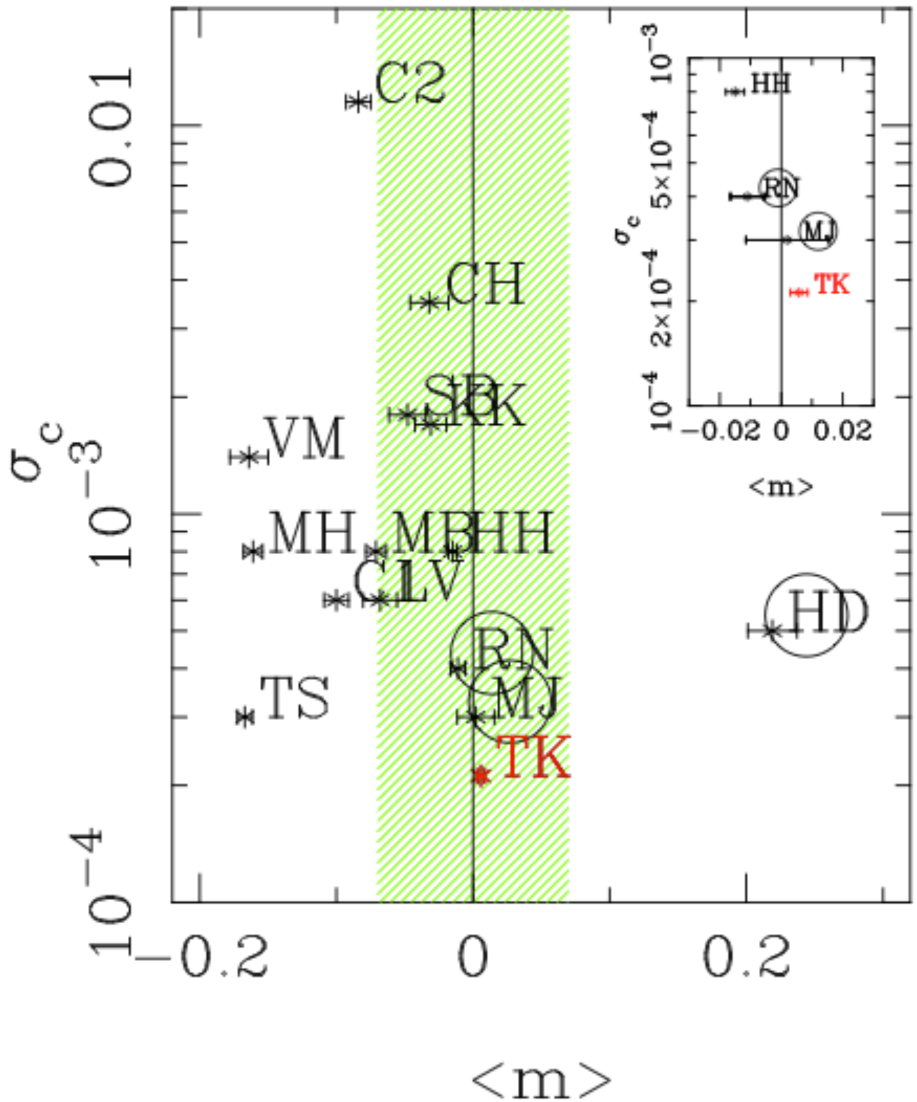
- Lots of shape measurement codes and approaches
    - KSB
    - Lensfit
    - Shapelets
    - DIEMOS
    - Seclets
    - Sersiclets
    - …
- Which one to use? And why?
- We don’t know the true shear (no “spectra”)
- So need simulations

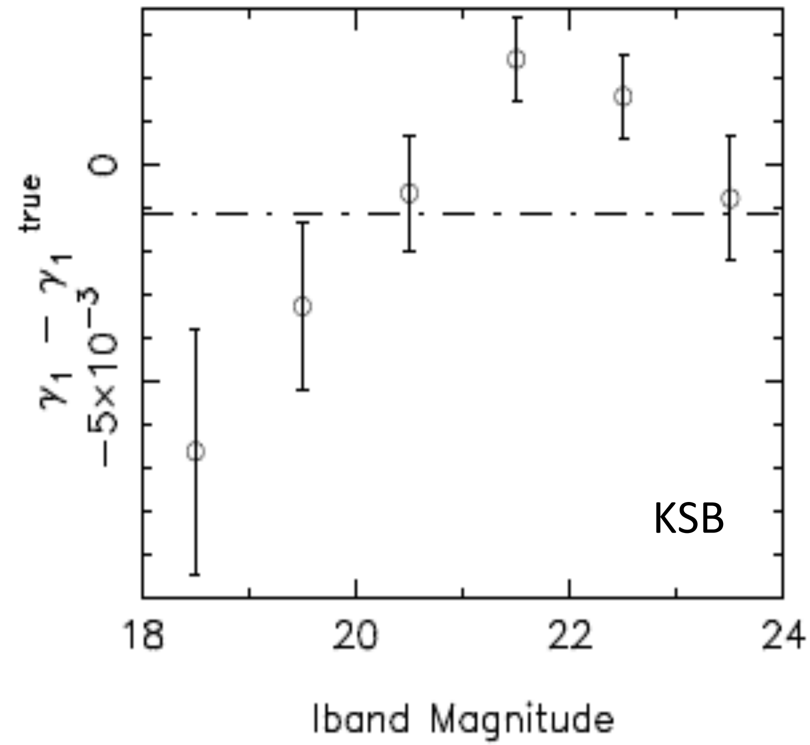

Vary Conditions
e.g.
Signal to Noise,
Size, PSF
Known Shears
Known Cosmology
Fake Images
Shape
Measurement
Method
Measured Shears
Measured
Cosmology

STEP

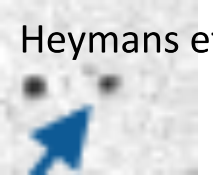
- Shear TEsting Programme
- Created in 2006 to compare existing methods
  - which disagreed on results of cosmological parameter measurements
- Blind simulations set to weak lensing community
- STEP 1:
  - Simple galaxy models
  - Complex unknown ground-based PSF
  - Constant unknown shear in each image
- STEP 2:
  - Complicated galaxy models (made using Shapelets)
  - Complex unknown ground-based PSF
  - Constant unknown shear in each image

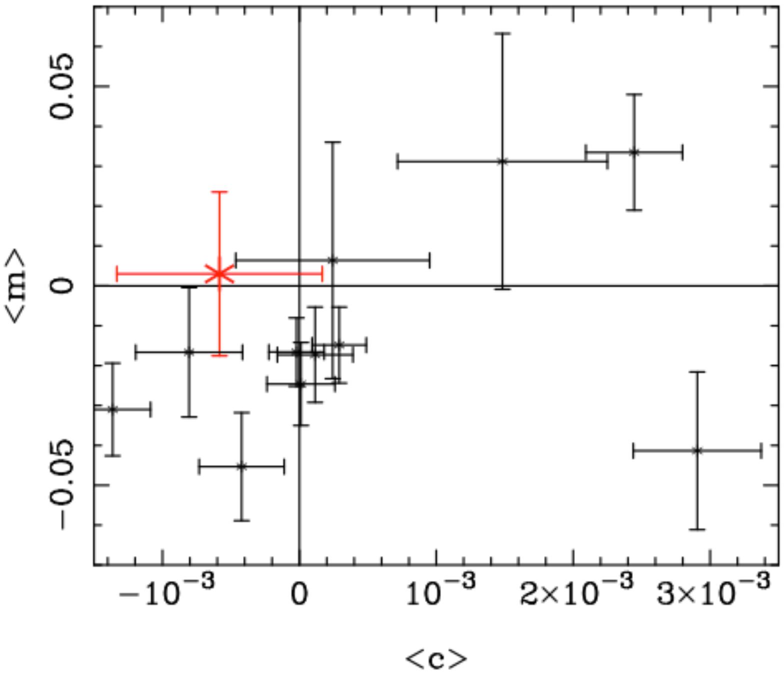
Heymans et al., 2006; Massey et al., 2007 & Kitching et al., 2008

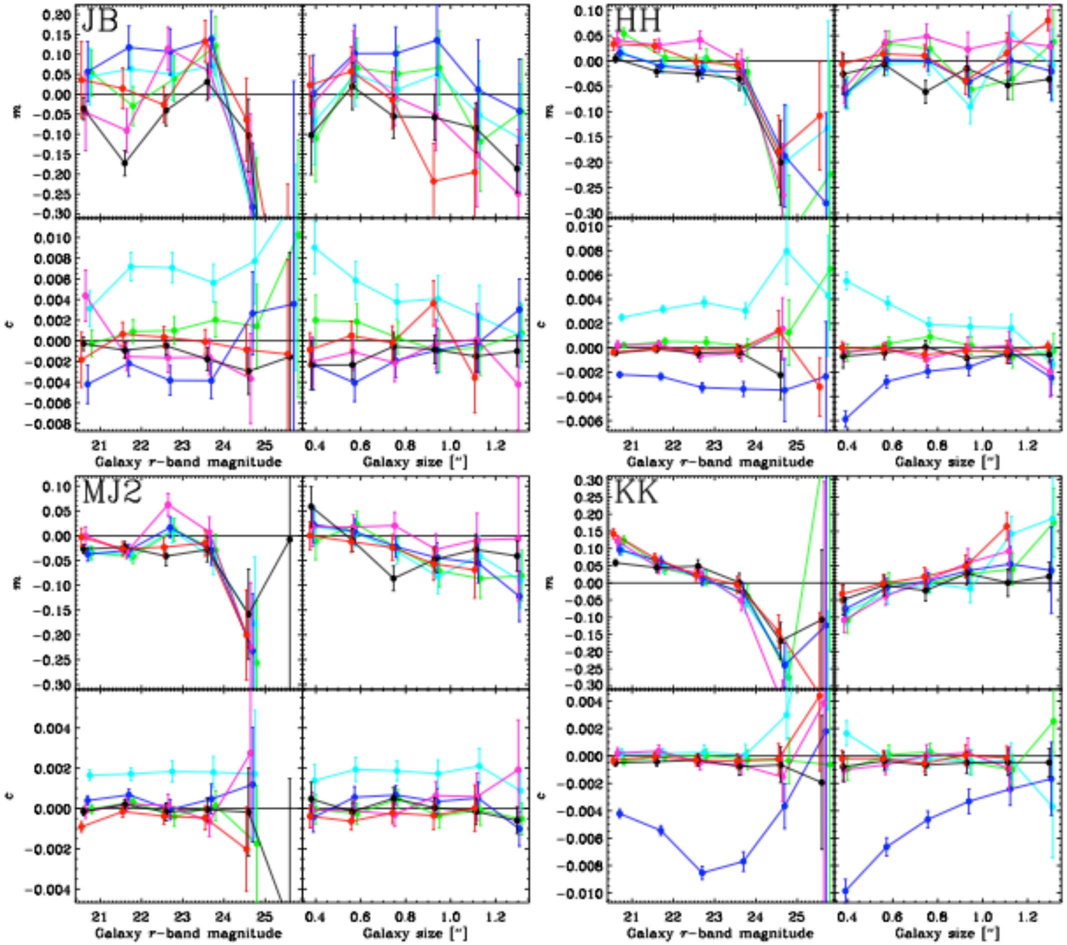




Heymans et al., 2006; Massey et al., 2007 & Kitching et al., 2008







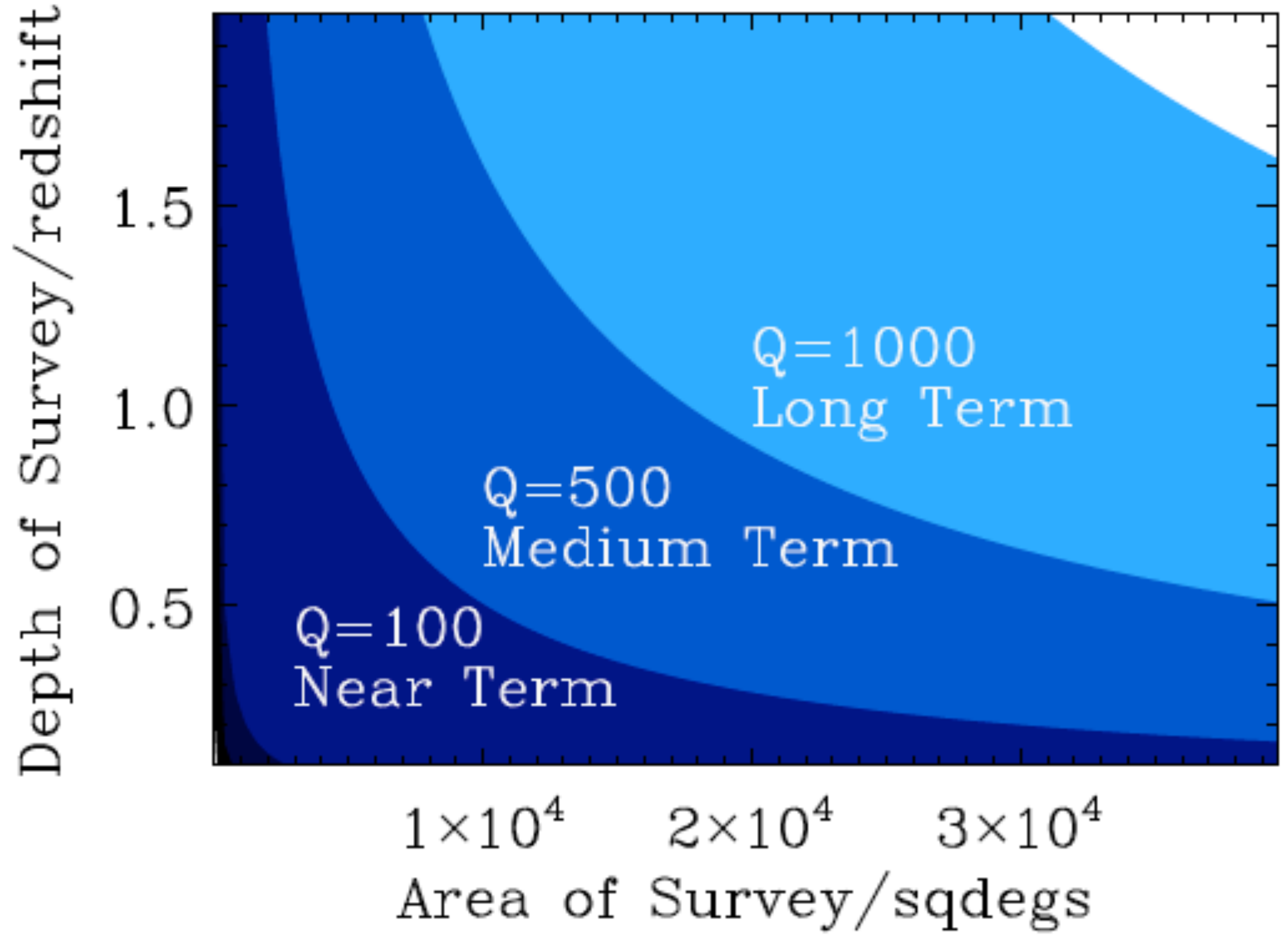
“dirty laundry”

- STEP results were complicated

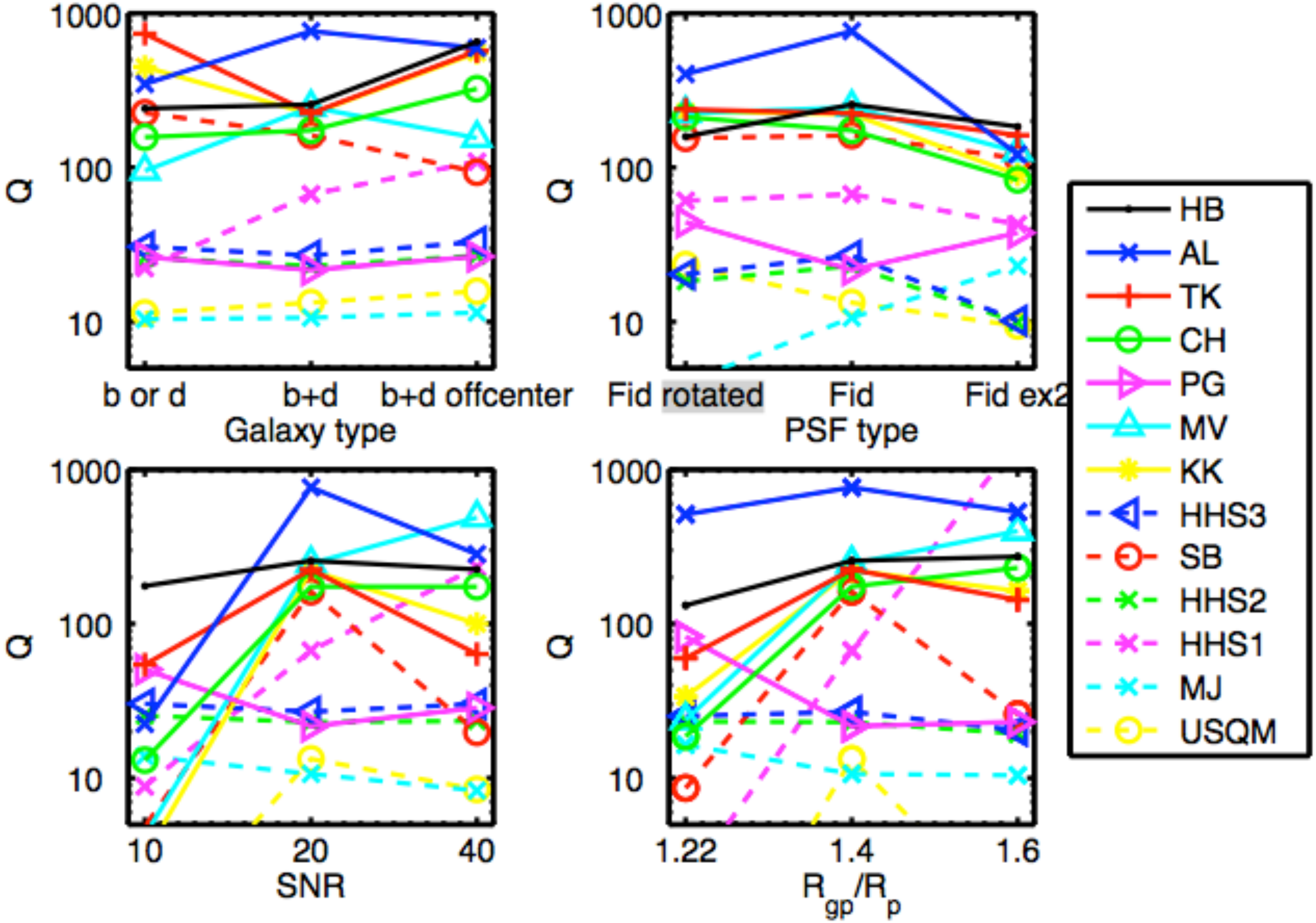
- Community decided to simplify the problem
  - Known PSFs
  - Simple galaxy models (exponentials)
  - Known galaxy positions on a grid
  - “Re-branded” as GREAT
    - GRavitational lEnsing Accuracy Testing
  - Made fully public
  - Constant unknown shear in each image

Quality Factor

$$Q = \frac{10^{-4}}{\langle (\langle g^m_{ij} - g^t_{ij} \rangle_{j \in k})^2 \rangle_{ik}}$$



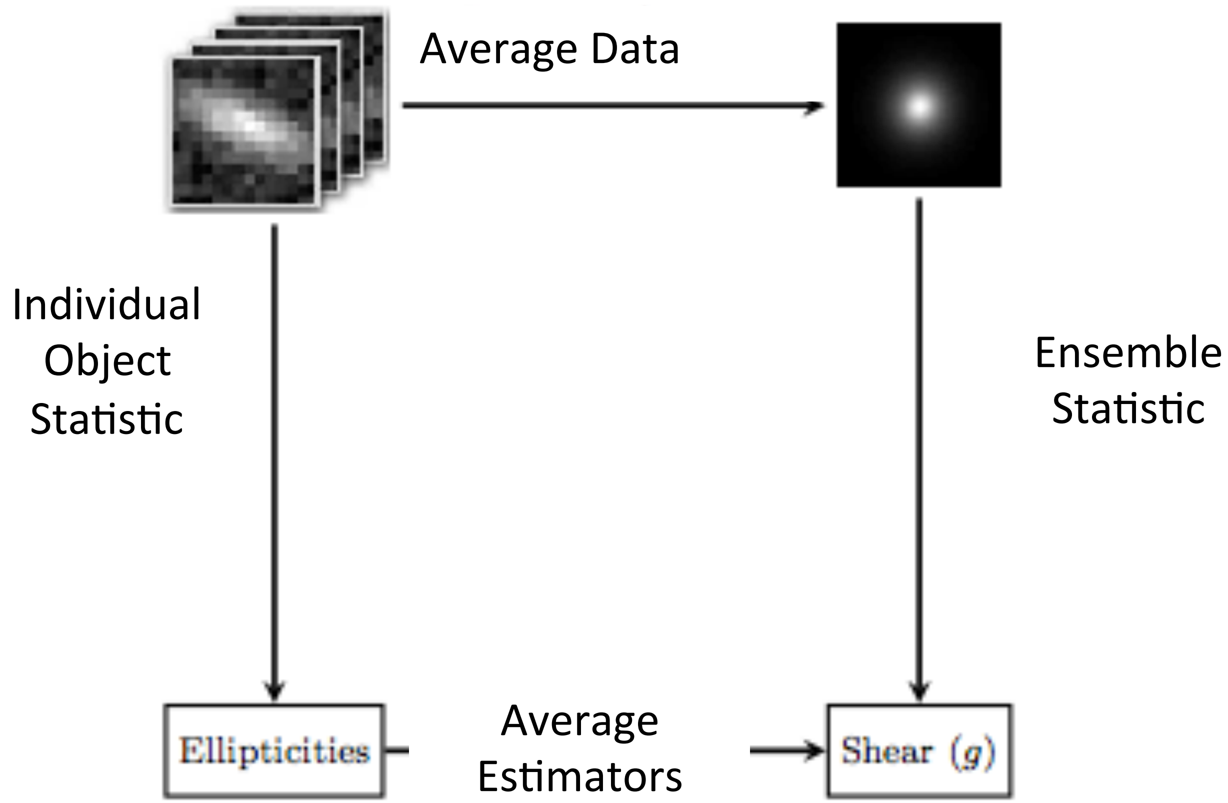
Kitching et al., 2008 (form filling functions); Amara & Refregier (2007)



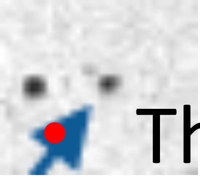
7 non-lensing participants
Q~1000 in some regimes

Bridle et al. 2011

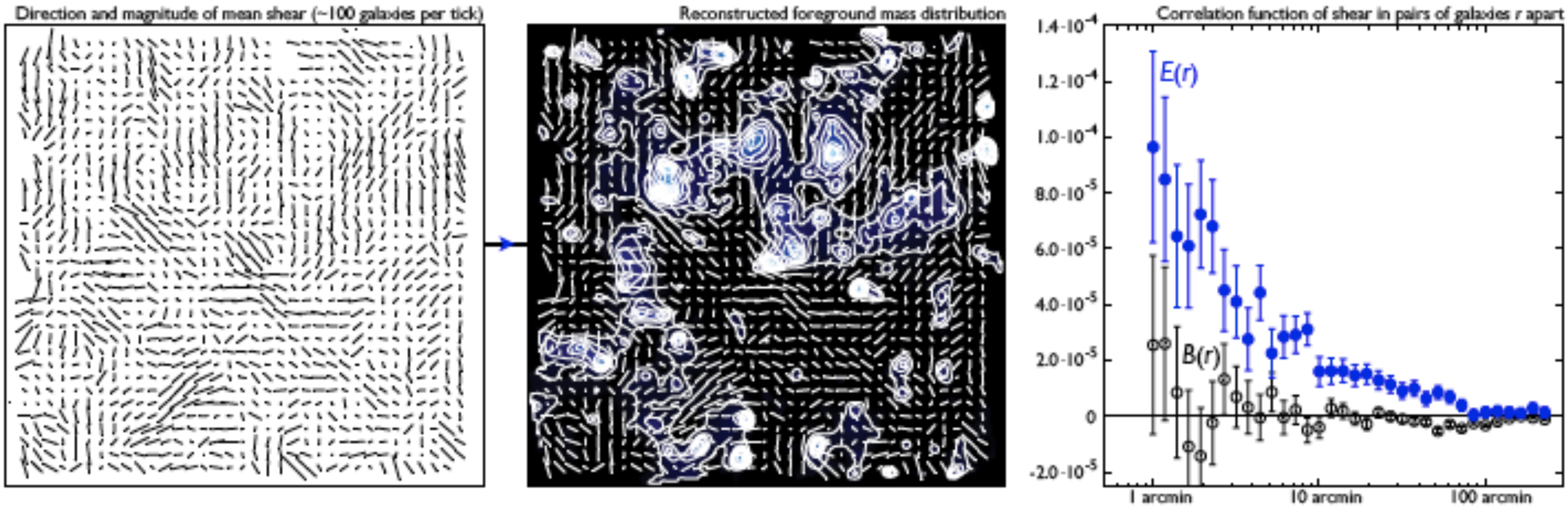
GREAT08 : Stacking Procedure is Important



Winning Methods (Q=1000) **Stacked the Data**



- The shear was not constant
- The PSF was not constant

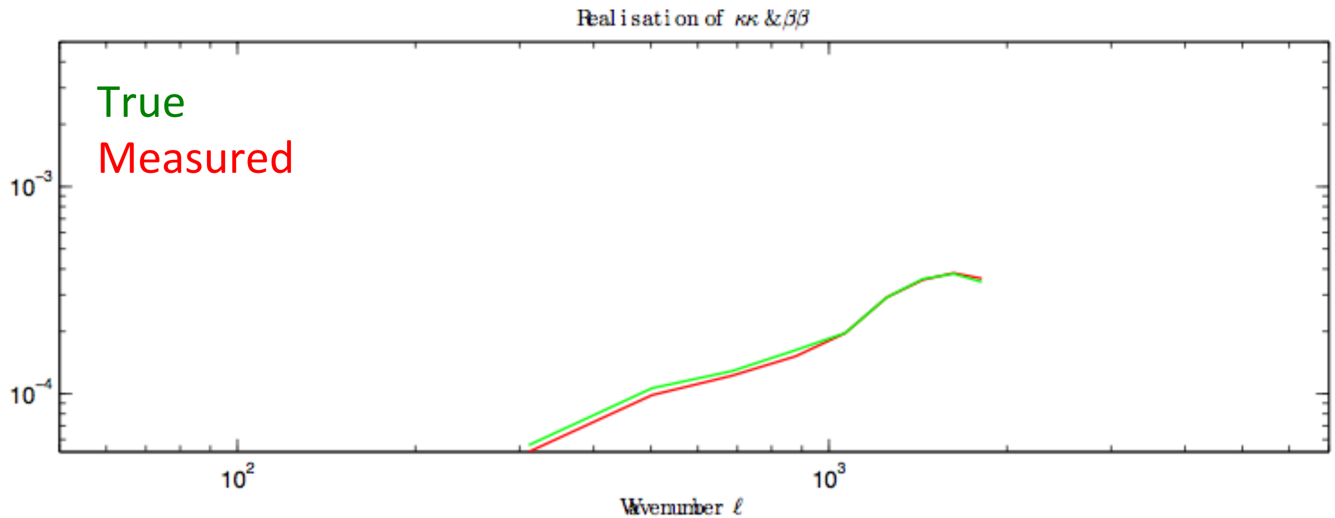


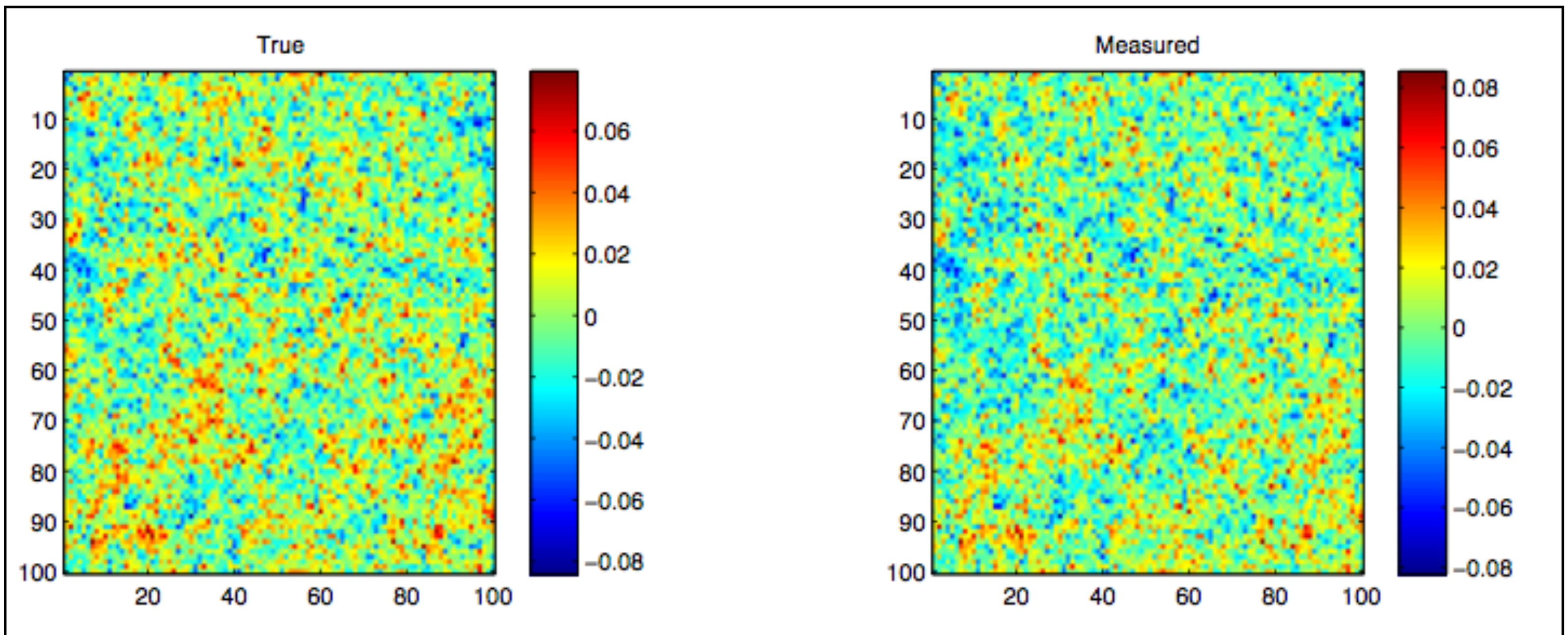
- GREAT10 introduced realistic variable fields into the simulations
    - Simple known spatially varying PSF
    - Simple galaxy models

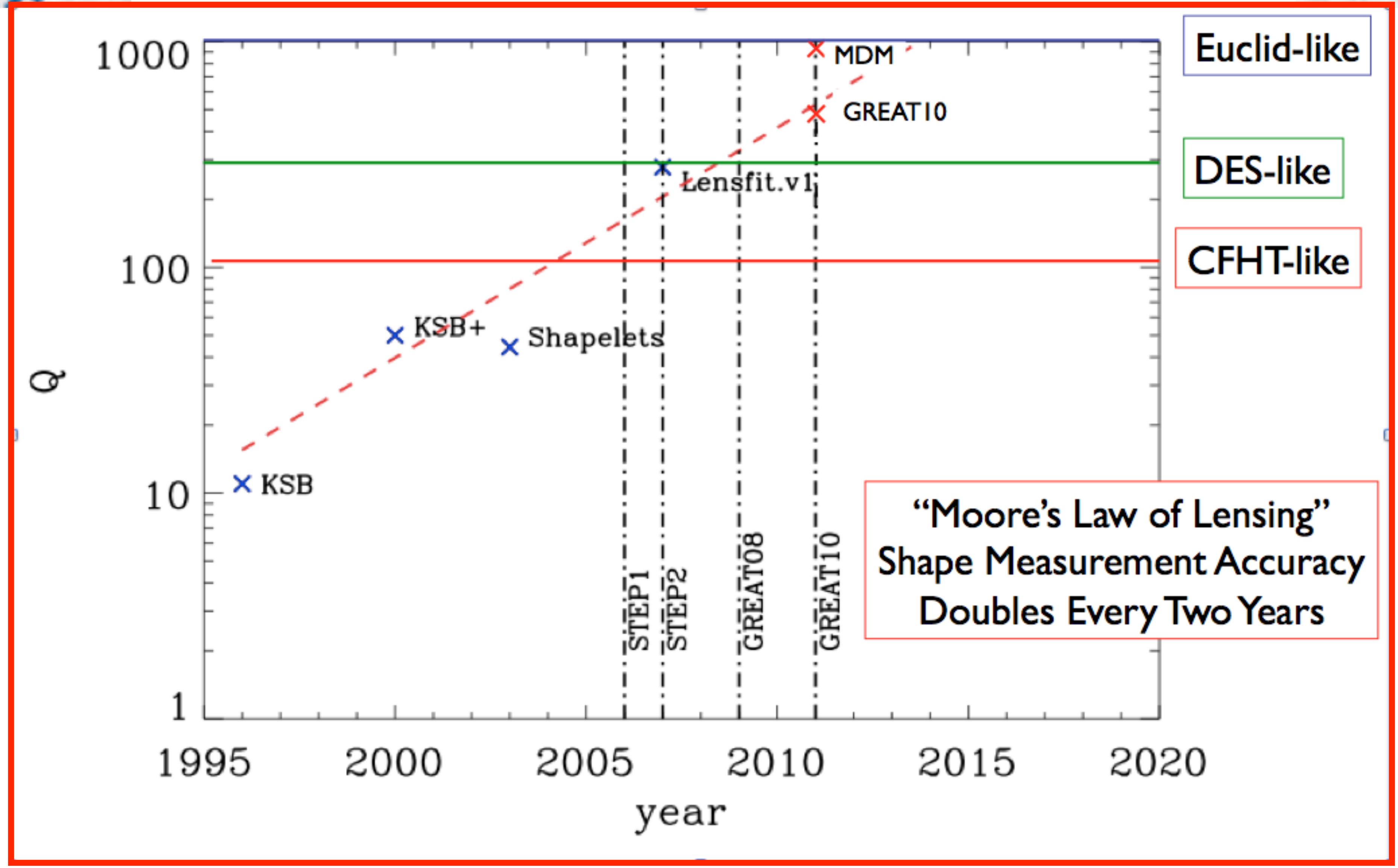
GREAT10 Galaxy

- KSB (as an example)

Power Spectrum

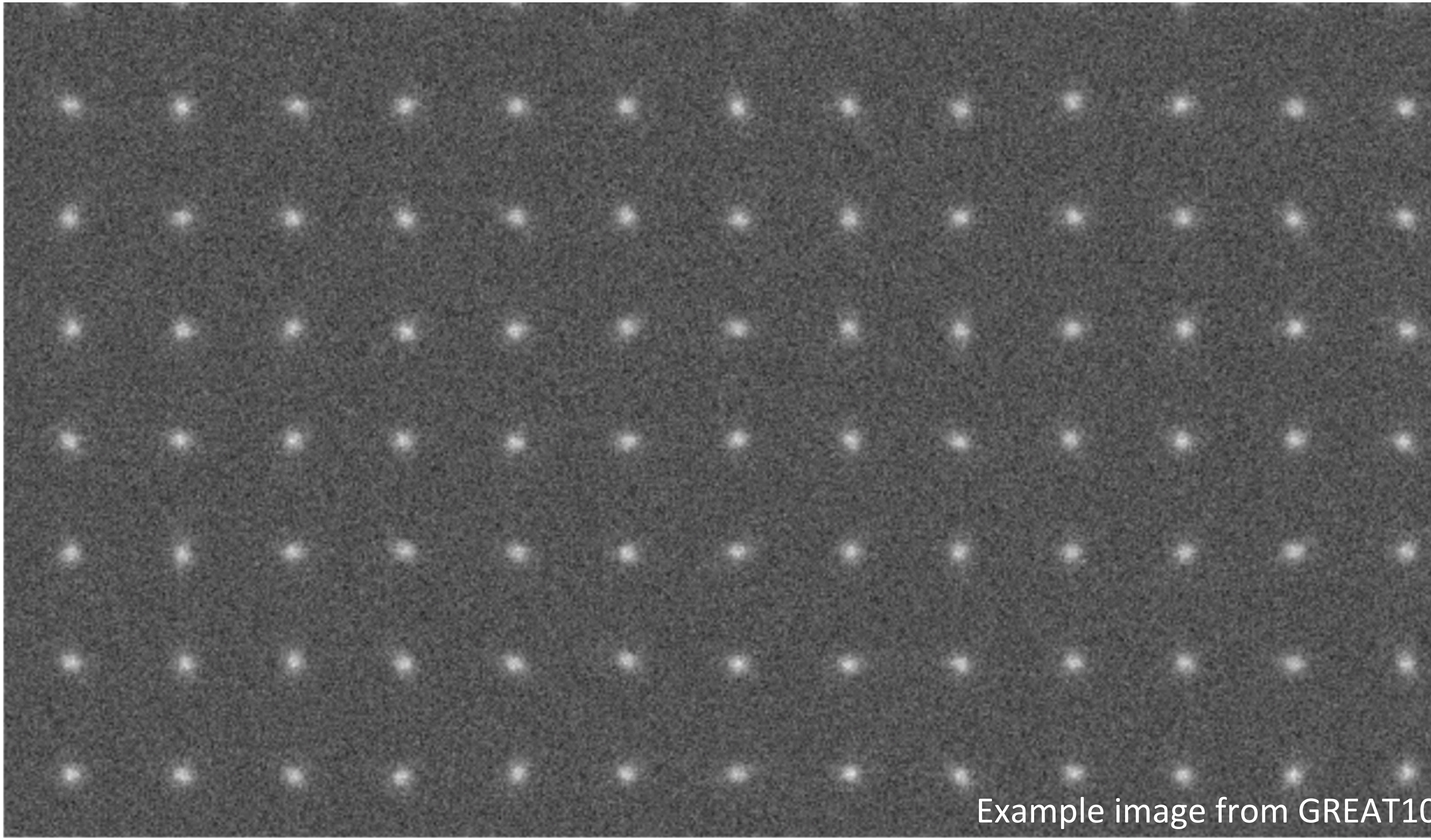






1000
100
10
1
Q
1995
2000
2005
2010
2015
2020
year
KSB
KSB+
Shapelets
Lensfit.v1
GREAT10
MDM
STEP1
STEP2
GREAT08
GREAT10
Euclid-like
DES-like
CFHT-like
“Moore’s Law of Lensing”
Shape Measurement Accuracy
Doubles Every Two Years

Next in Challenges: More Complexity



Example image from GREAT10

http://www.greatchallenges.info

- Recap:

    - Basics of shape measurement
    - Method Review
    - Simulations

- So now we have a catalogue :-)
    - Testing new cosmological models is easy!
    - Yes?
    - No?