Lensing by galaxies

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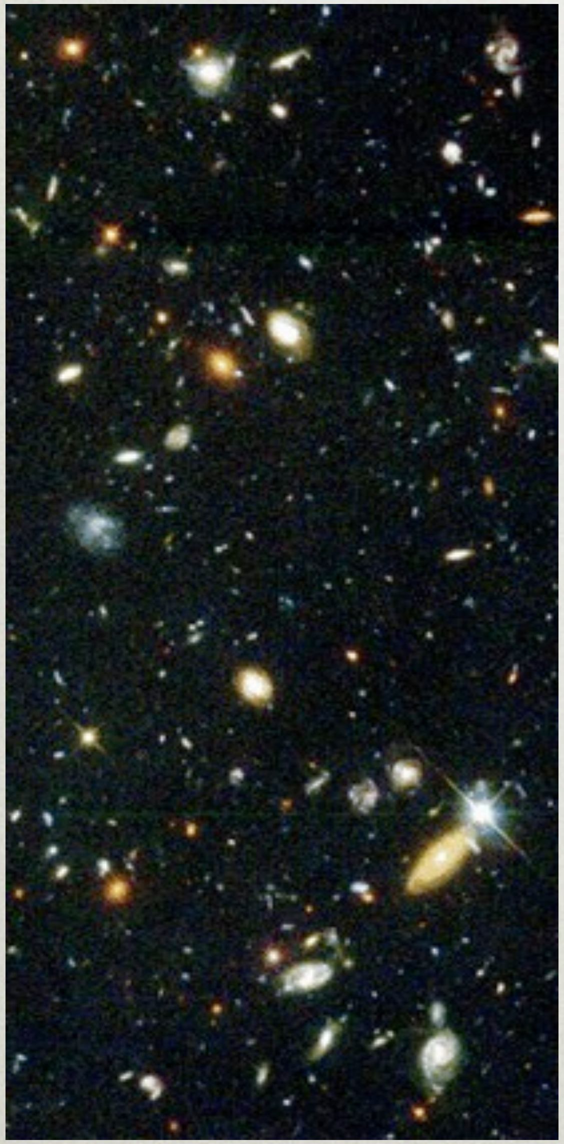
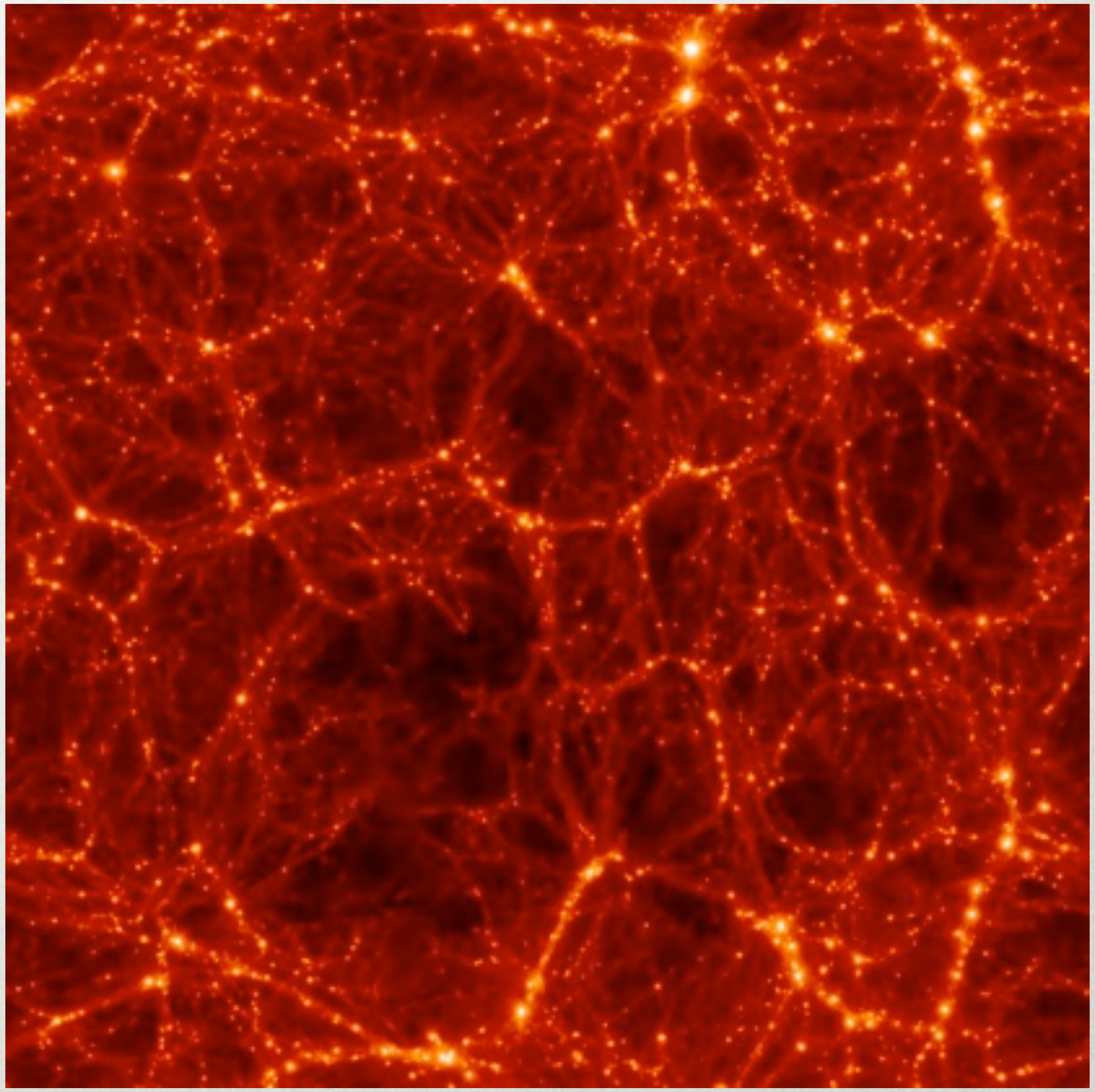
TWO VIEWS OF THE UNIVERSE

real

simulated



?

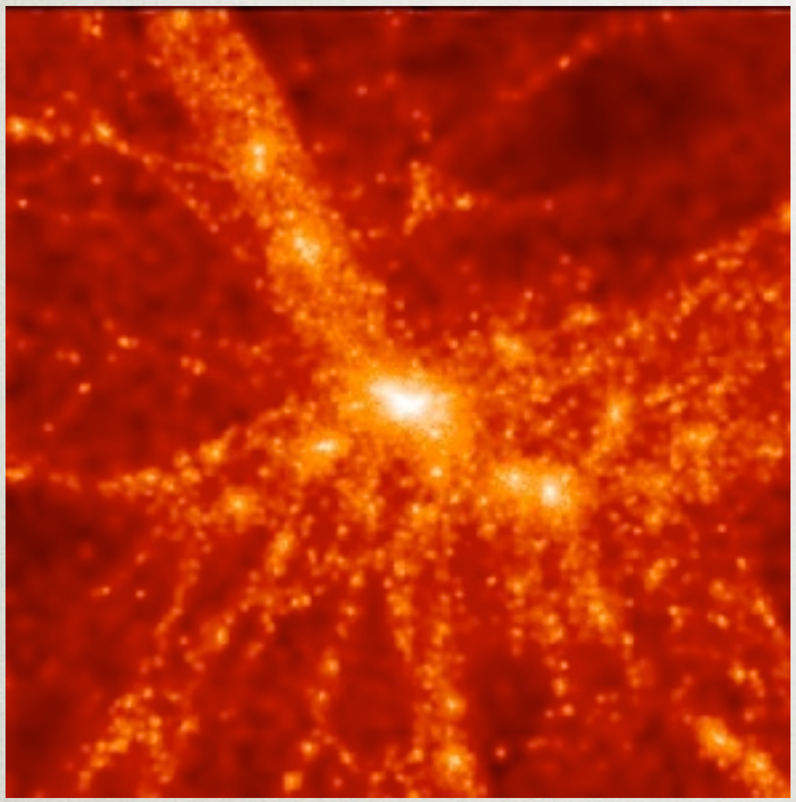


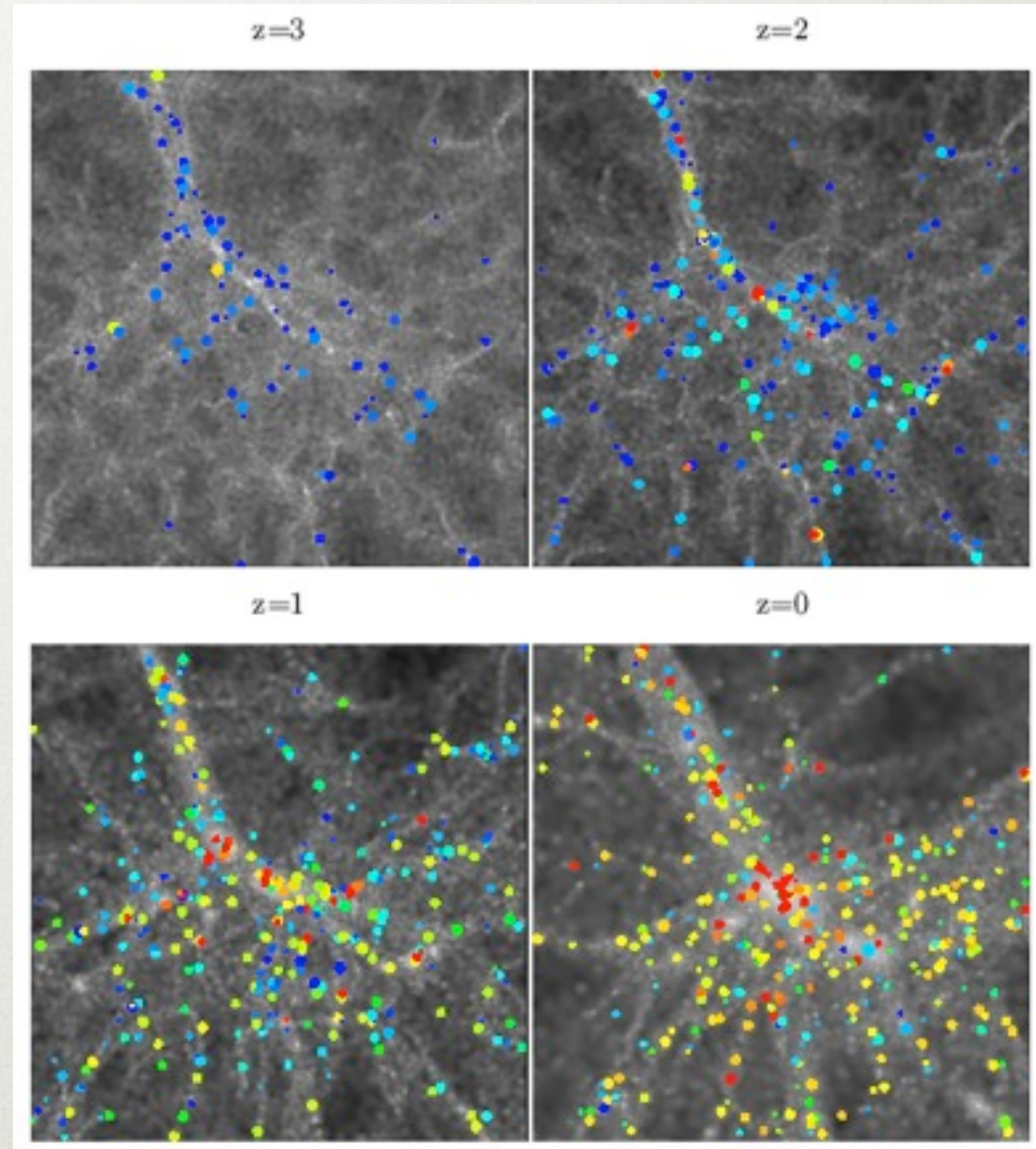
observable

invisible

TWO VIEWS OF THE UNIVERSE

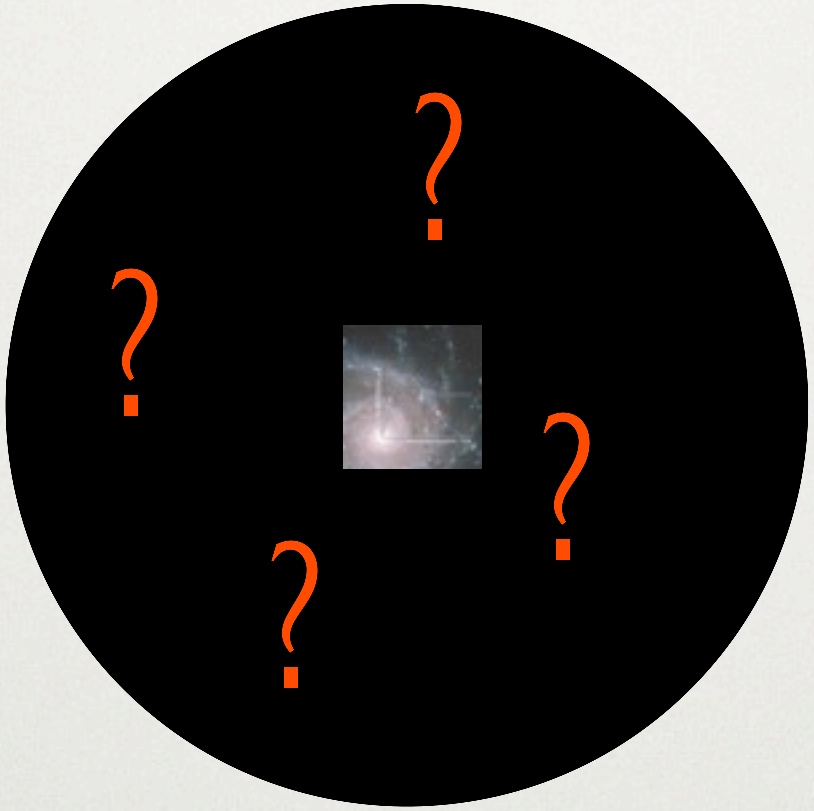
GIF collaboration





Make the simulations “look” like the observations

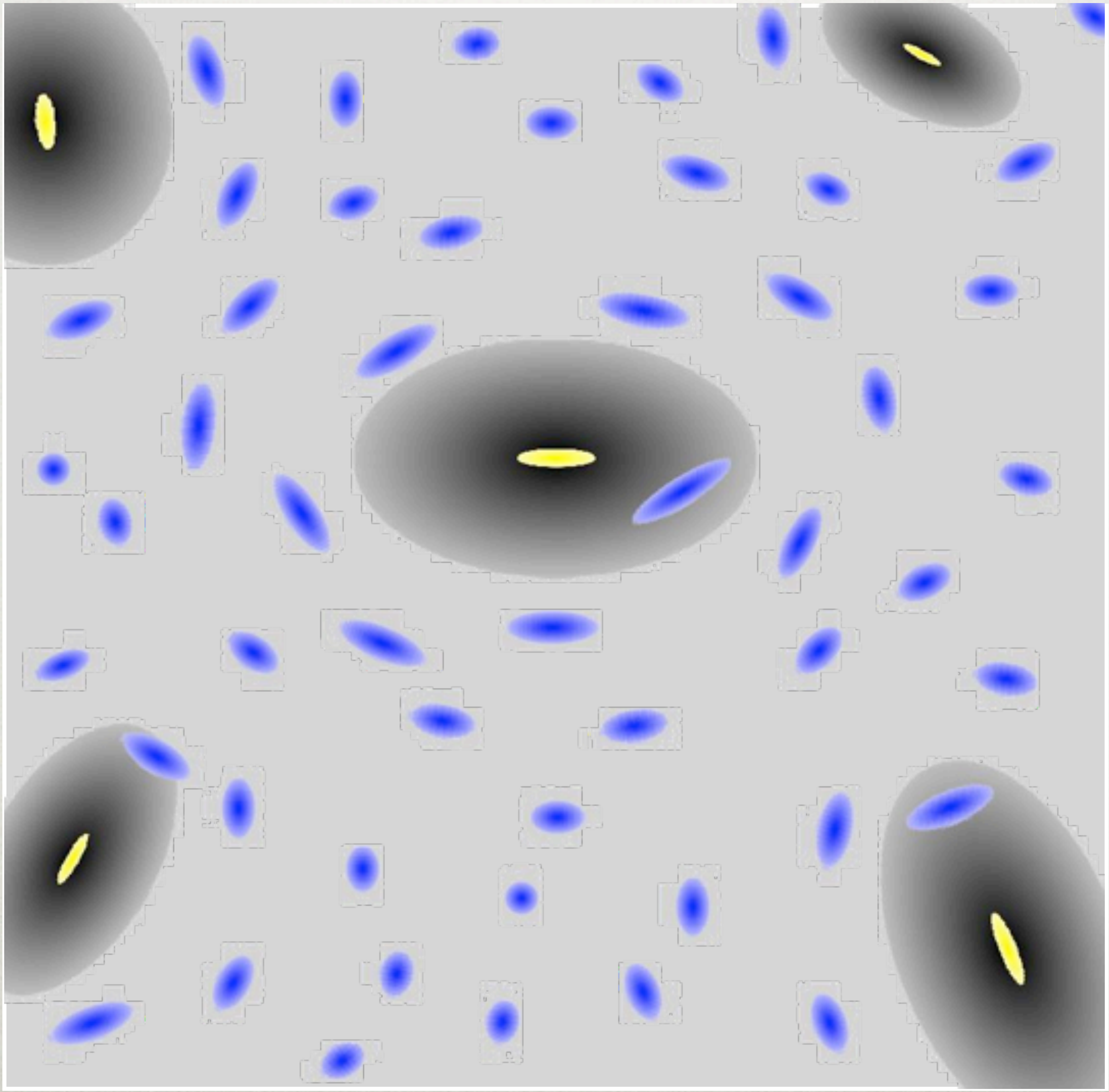
DARK MATTER AROUND GALAXIES



Dynamical and strong lensing studies provide important constraints on the mass distribution on scales of a few tens of kpc.

But what do we know about the mass distribution on scales larger than 100kpc? How can we study this (as a function of redshift)?

Weak gravitational lensing



Weak gravitational lensing

The signal induced by a typical galaxy is dwarfed by the intrinsic shapes of the sources and can only detected by averaging over many lenses: *only ensemble averages*

$$\frac{\mathrm{S}}{\mathrm{N}} \approx 4 \left(\frac{n}{20\,\mathrm{arc\ min}^{-2}}\right)^{1/2} \left(\frac{\sigma_\epsilon}{0.3}\right)^{-1} \left(\frac{\sigma_v}{600\,\mathrm{km\,s}^{-1}}\right)^2 \frac{\langle D_{\mathrm{ds}}/D_{\mathrm{s}}\rangle}{0.6} \,.$$

- Clusters of galaxies ($\sigma_v \gtrsim 600\,\mathrm{km/s}$) can be detected with $\mathrm{S/N} \gtrsim 4$ from weak lensing;
- individual galaxies ($\sigma_v \sim 200\,\mathrm{km/s}$) are too weak as lenses to be detected individually

Some history

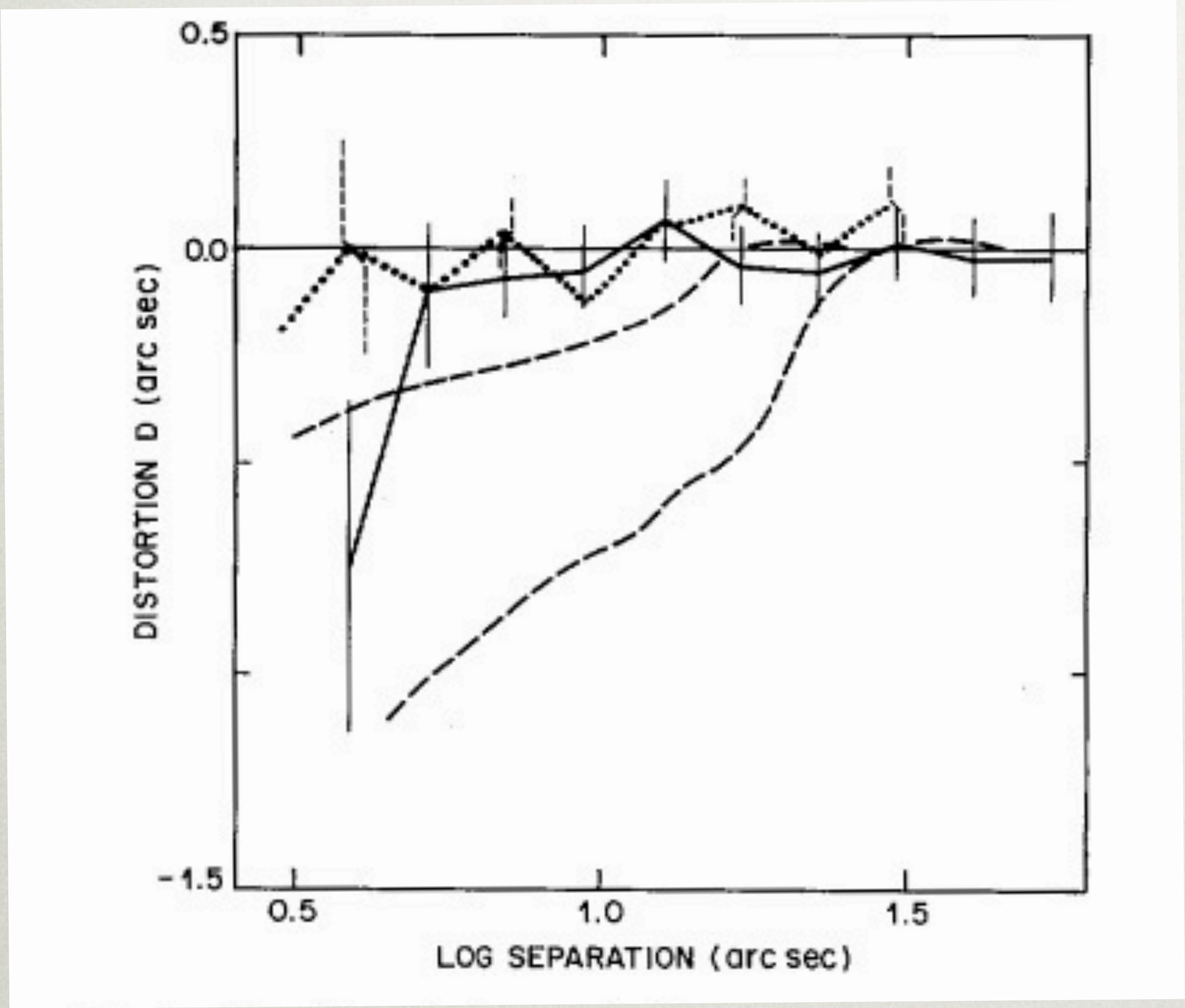
Just like cosmic shear, galaxy-galaxy lensing is a fairly new area of research, albeit the oldest application of weak lensing.

larger surveys

- ❑ 1984: first attempt to measure the signal (Tyson et al.)
- ❑ 1996: first detection (Brainerd et al.)
- ❑ 2000: first accurate measurement from SDSS (Fischer et al.)

Since then several results, for instance from SDSS (e.g., McKay et al.; Guzik & Seljak, Mandelbaum et al.), RCS (Hoekstra et al. 2004; 2005) and CFHTLS (Parker et al. 2007) have been published.

In the beginning...



Tyson et al. (1984)

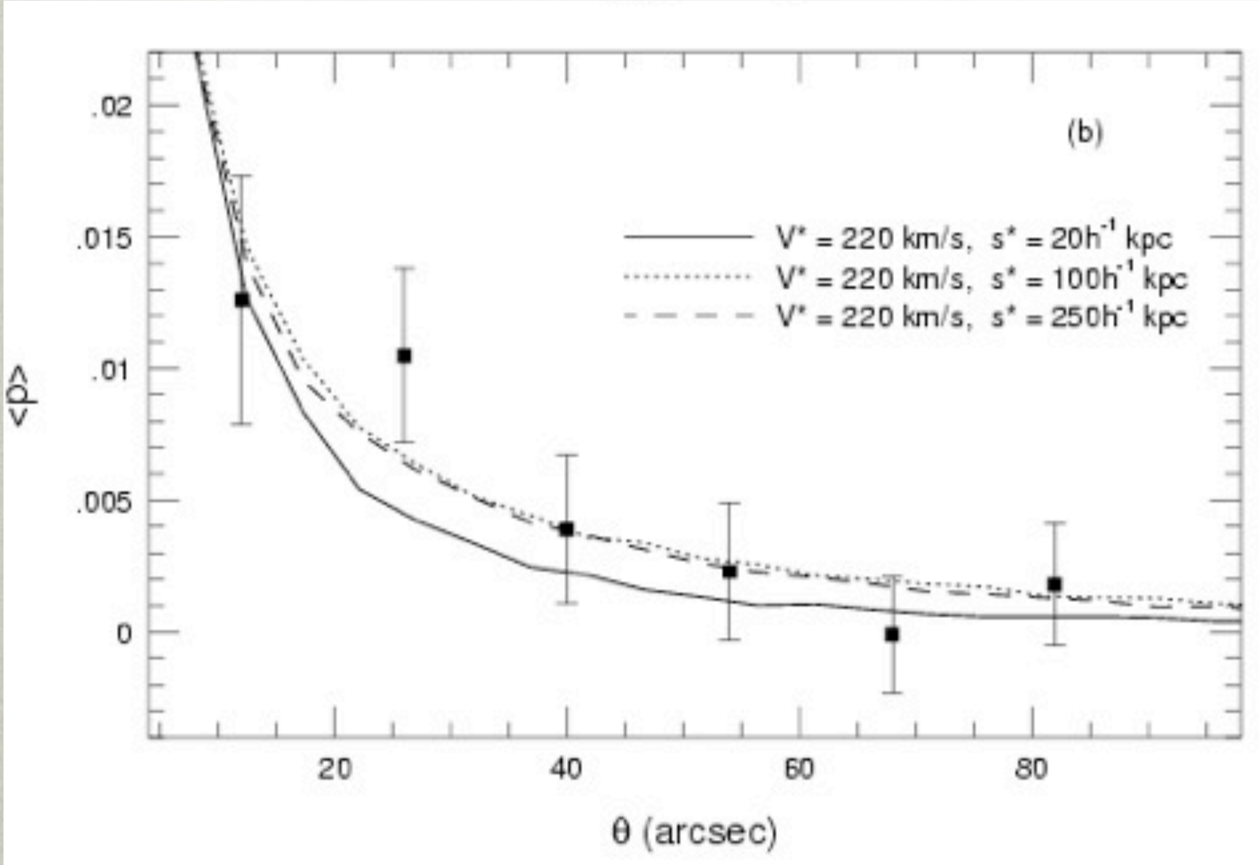
Photographic plates
~12000 lenses
~47000 sources

Circular velocity <170km/s

will it ever work?

A lot of progress...

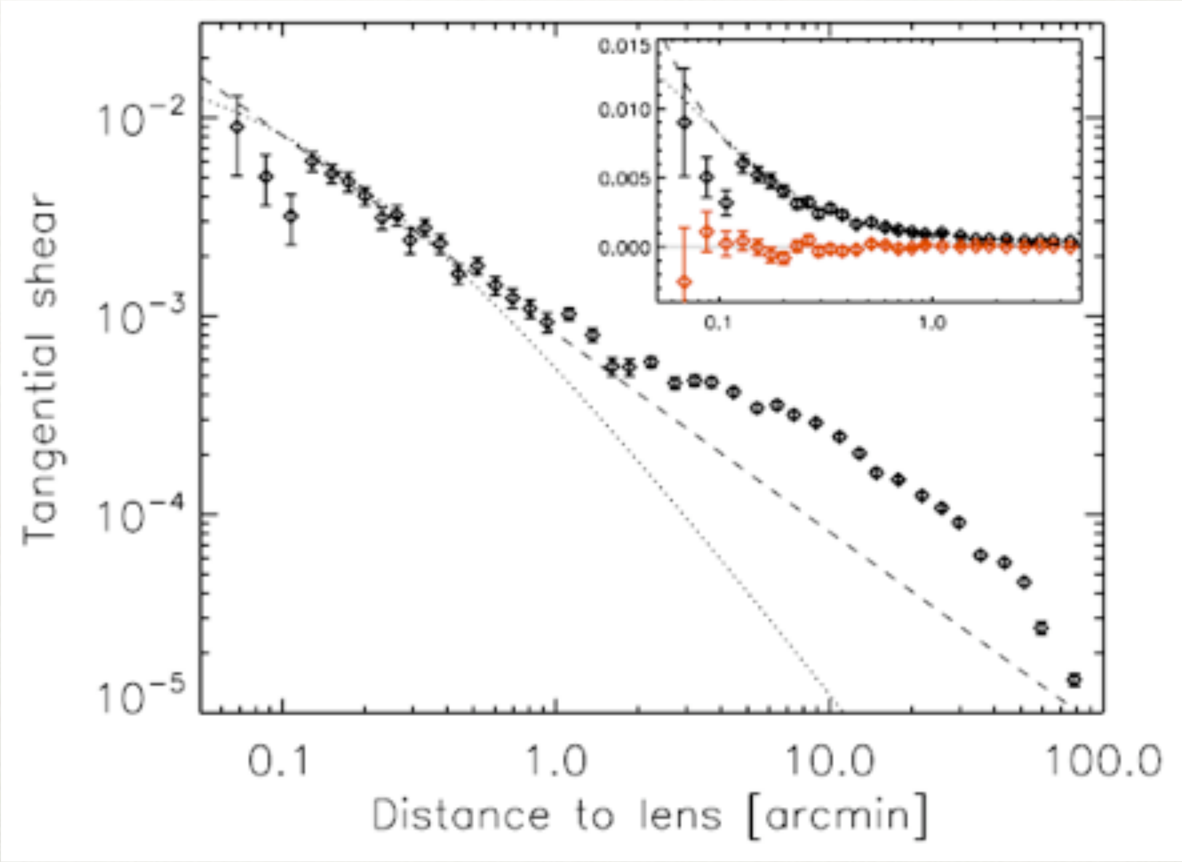
Brainerd et al. (1996)



~439 lenses

~511 sources

van Uitert et al. (2011)



~2x10⁶ lenses

~15x10⁶ sources

... AND MUCH MORE TO COME

Early studies observed relatively small areas of sky and lacked redshift information for the lenses.

large errors and limited interpretation

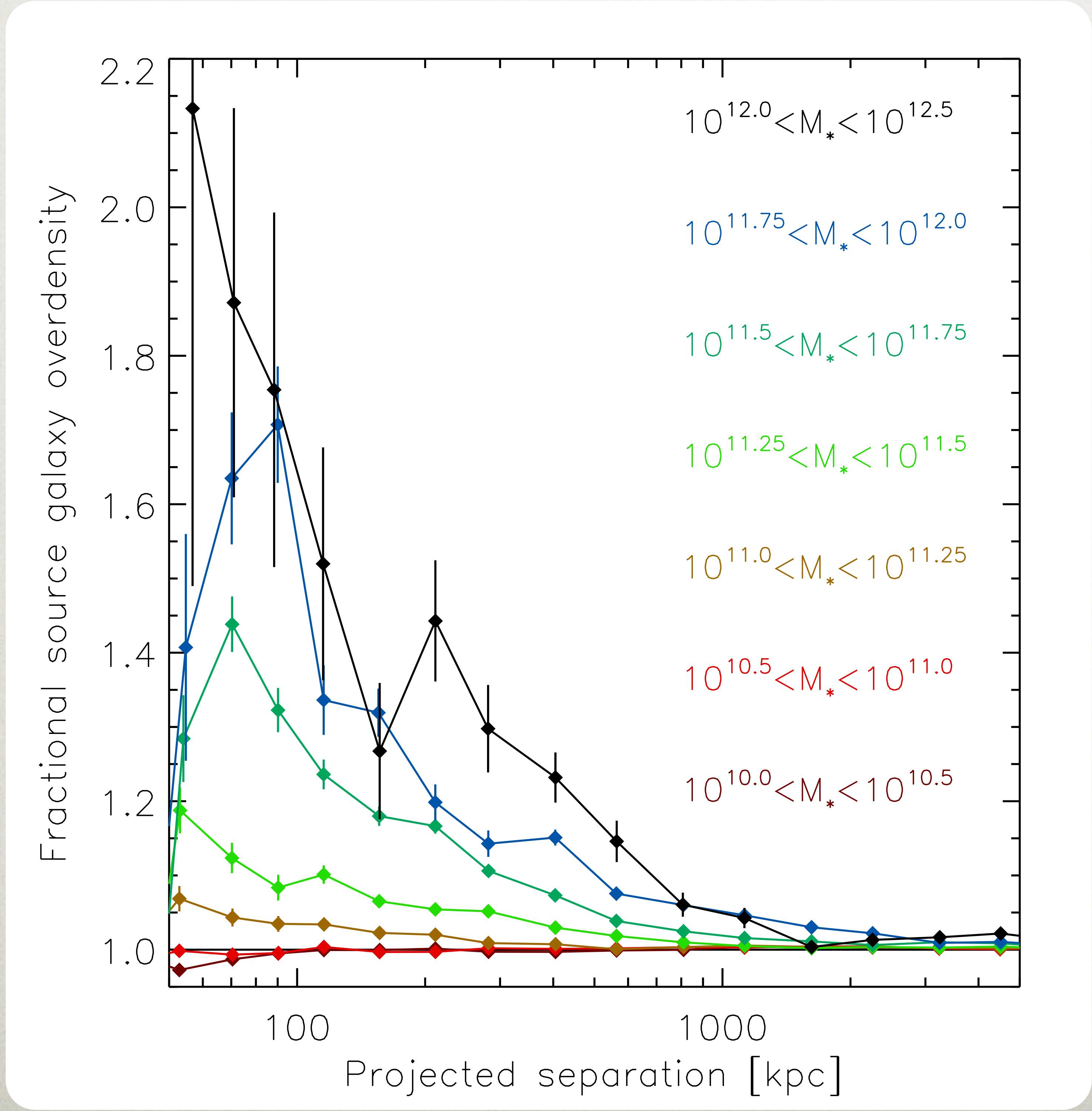
NEED FOR REDSHIFTS

Without redshift information we can only select on apparent magnitude: brighter galaxies are “lenses” and fainter galaxies are “sources”.

A fraction of the faint galaxies are in fact satellites that are physically associated with the lenses. When they are included in the source catalog they dilute the signal.

We can estimate the level of contamination by looking at *excess counts* of sources around the lens.

NEED FOR REDSHIFTS



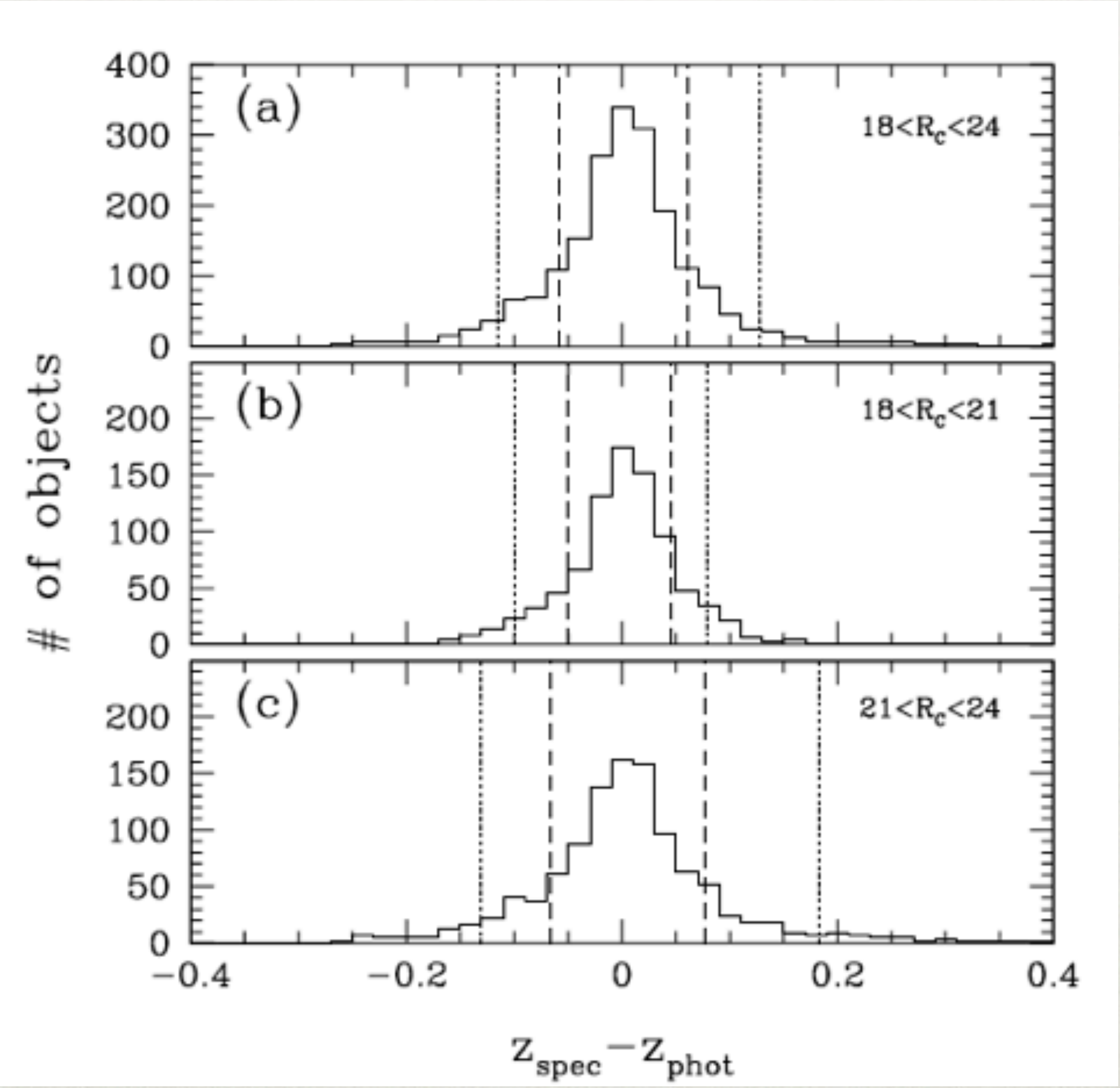
van Uitert et al. (2011)

NEED FOR REDSHIFTS

The SDSS provides a wealth of information about the baryonic content of galaxies. In particular the availability of spectroscopic redshifts is useful for a clear division of lenses.

Current surveys observe >1000 deg^2 in multiple bands, which yield photometric redshift information for the lenses. In this case we need to account for the redshift errors in the analysis.

NEED FOR REDSHIFTS



Photometric redshift errors have a bigger impact on low redshift lenses.

The future is here

The Kilo Degree Survey (KiDS) is will be an important step forward for galaxy-galaxy lensing. Observations have started and the survey will be completed in ~3-4 years.

survey area: 1500 deg^2
filter coverage: ugri ZYJHK

excellent photometric redshifts

Key science drivers

- study halos as a function of baryonic content

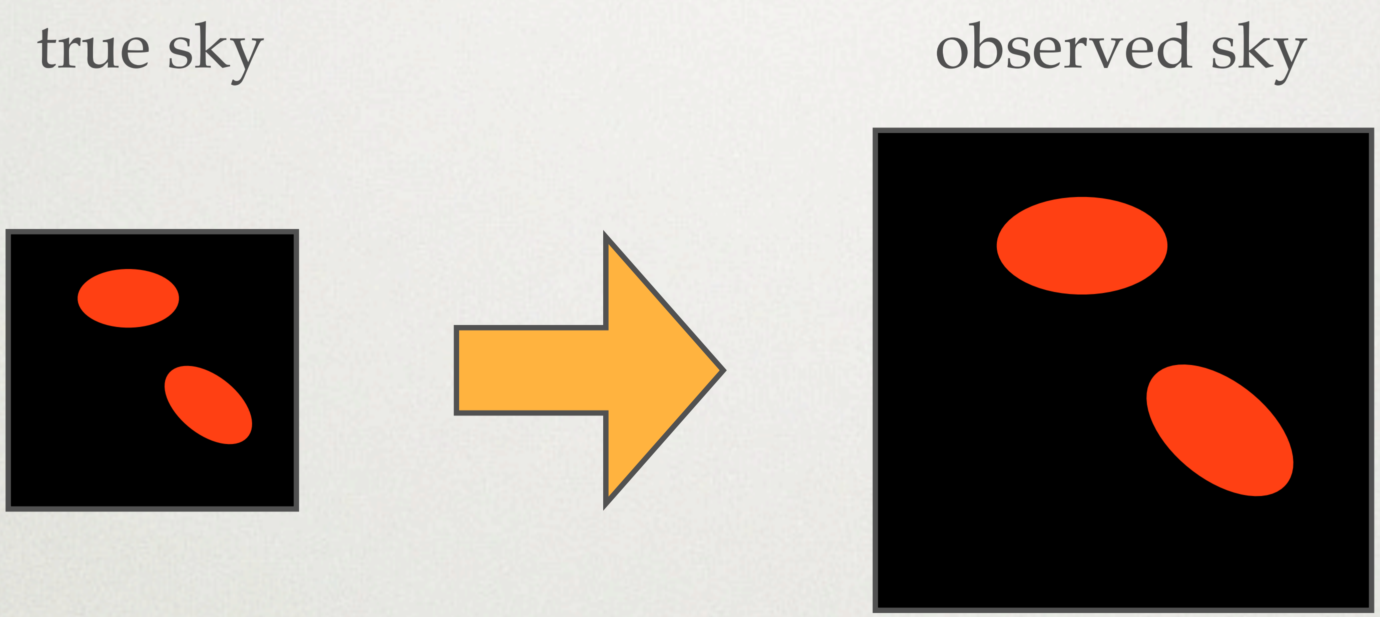
- study properties as a function of environment

- study these relations as a function of redshift

Can we study galaxies at $z>1$?

Magnification

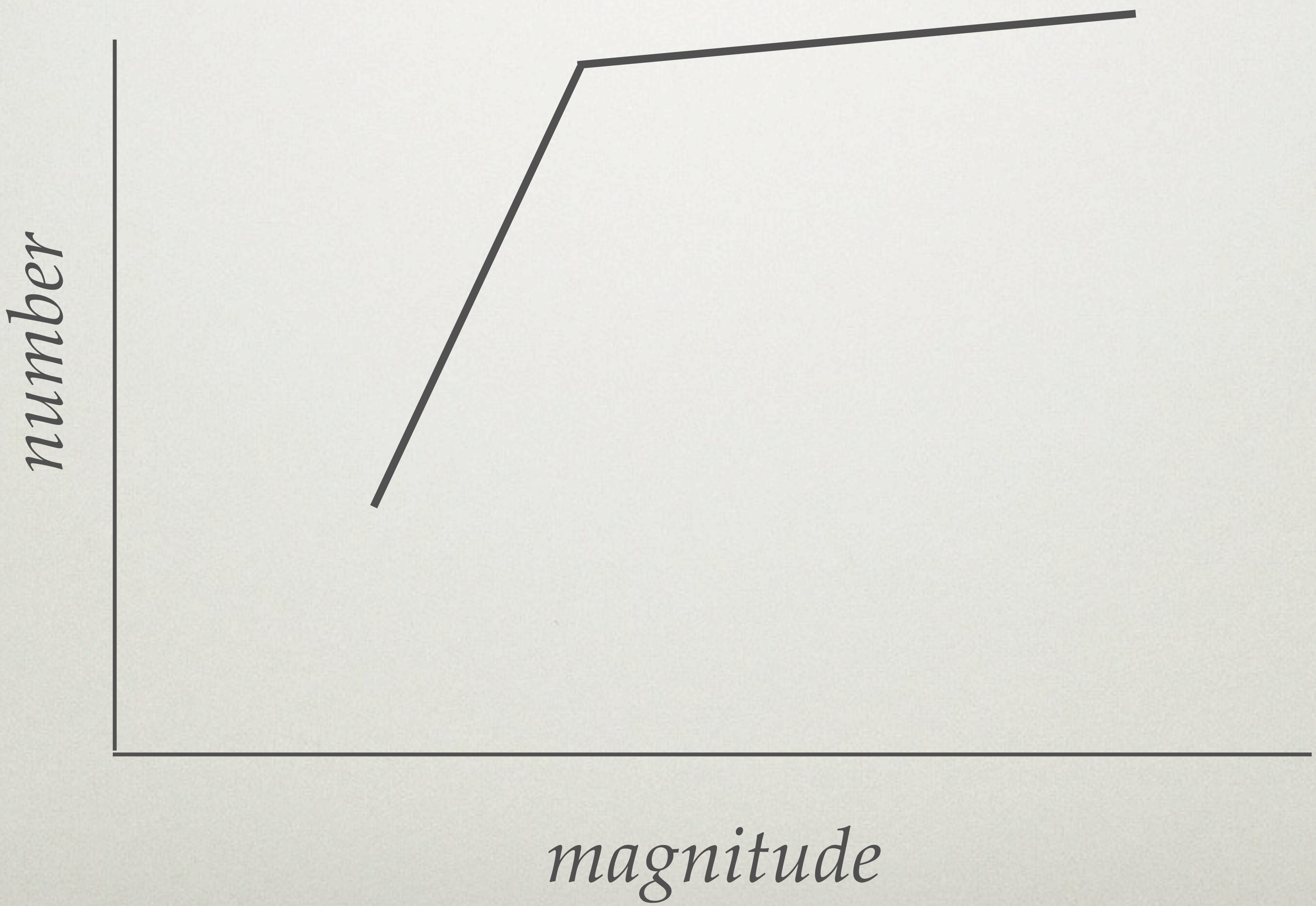
Magnification has two effects:



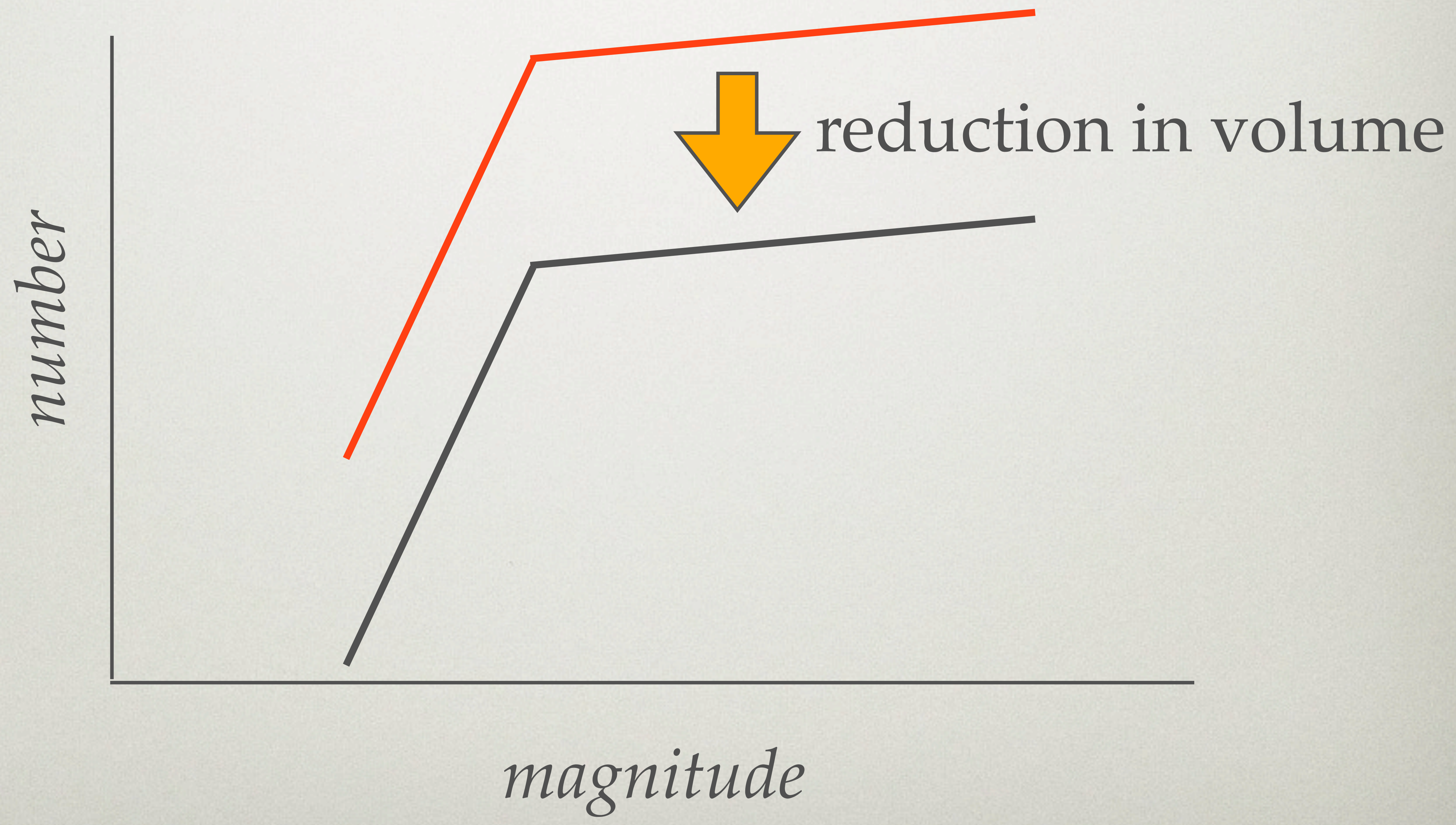
- true survey area is 1/μ times larger
- objects are μ times larger/brighter

$$n(> S, z) = \frac{1}{\mu(\boldsymbol{\theta}, z)} n_0 \left(> \frac{S}{\mu(\boldsymbol{\theta}, z)}, z \right)$$

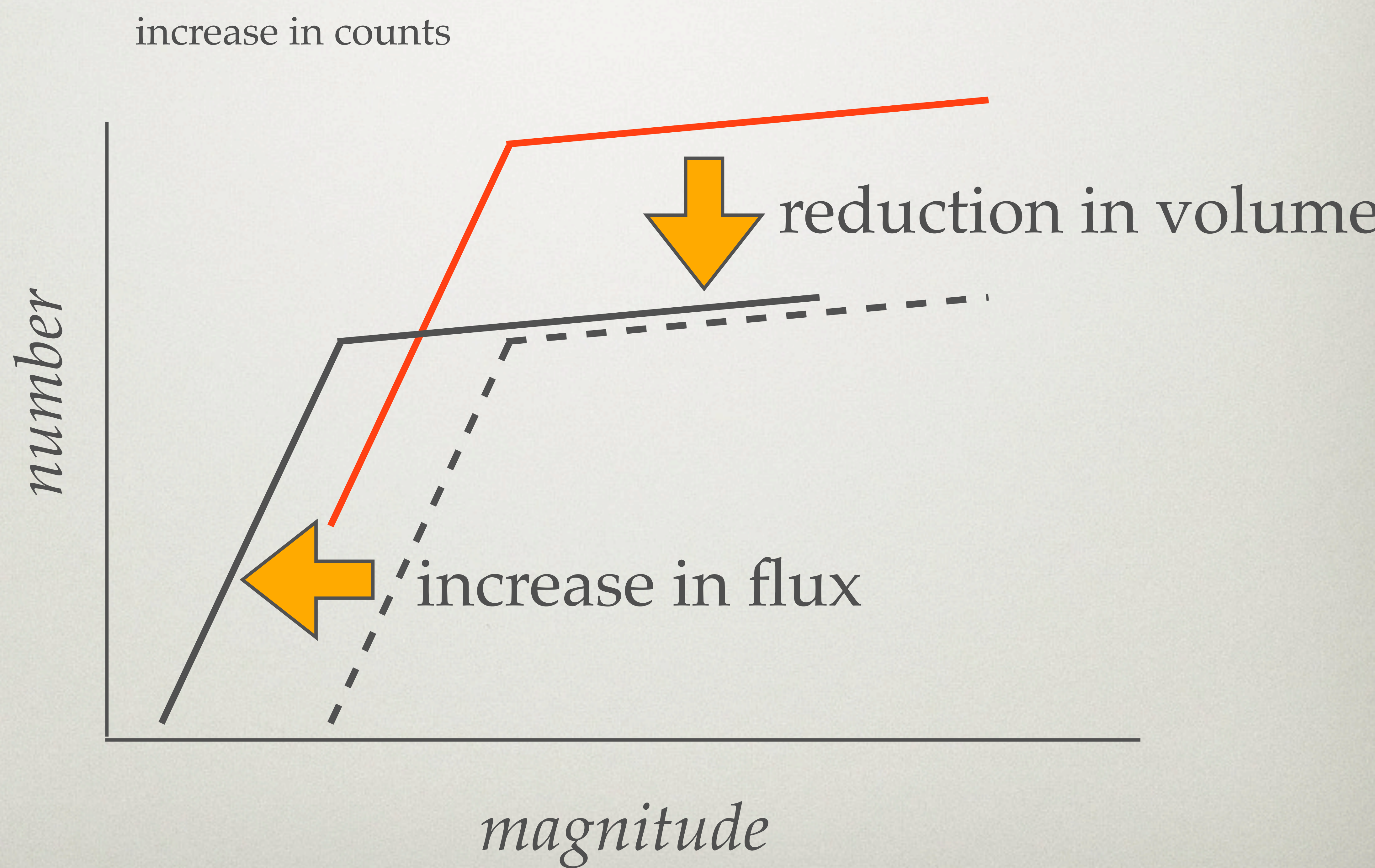
Magnification



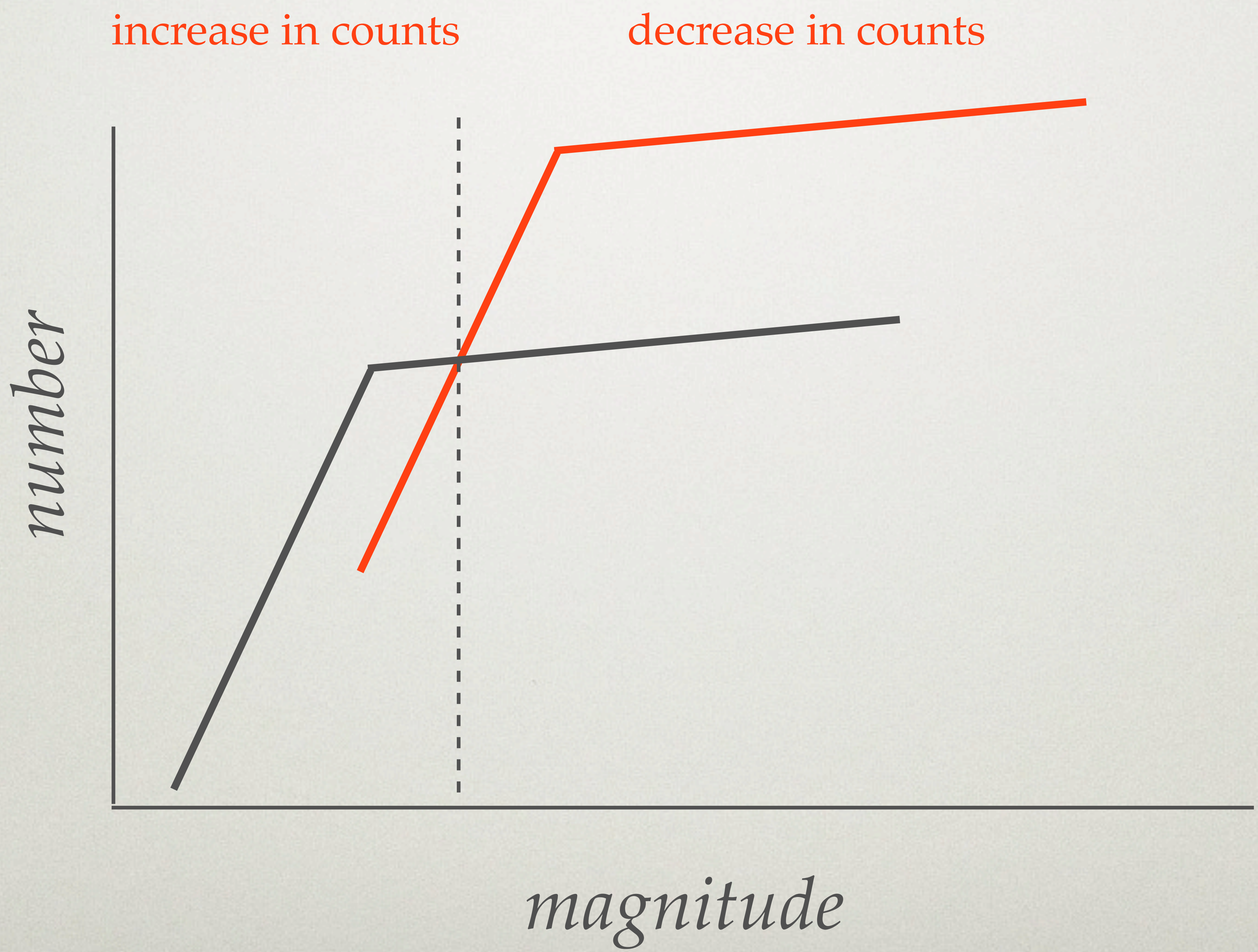
Magnification



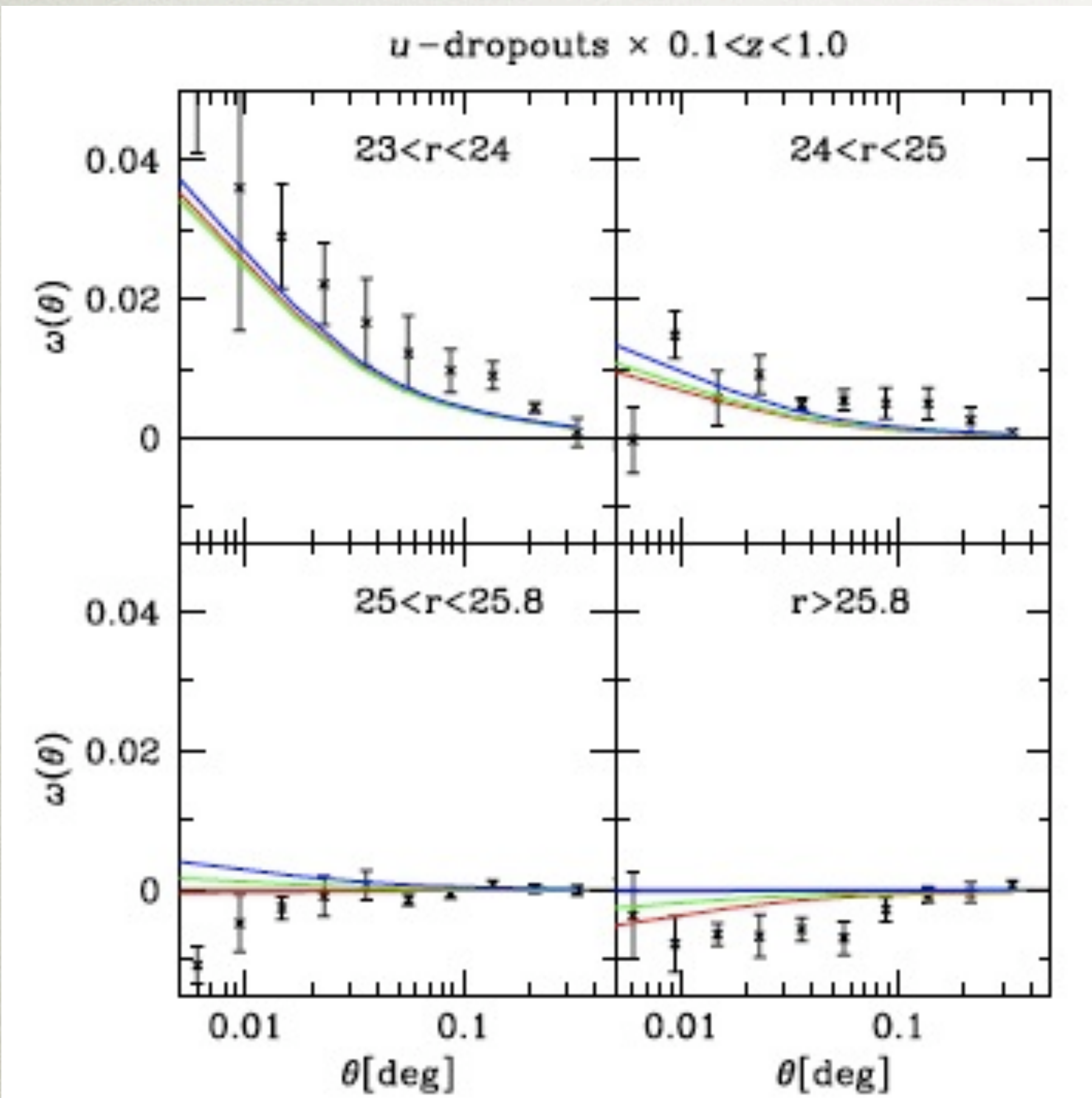
Magnification



Magnification



magnification of LBGs

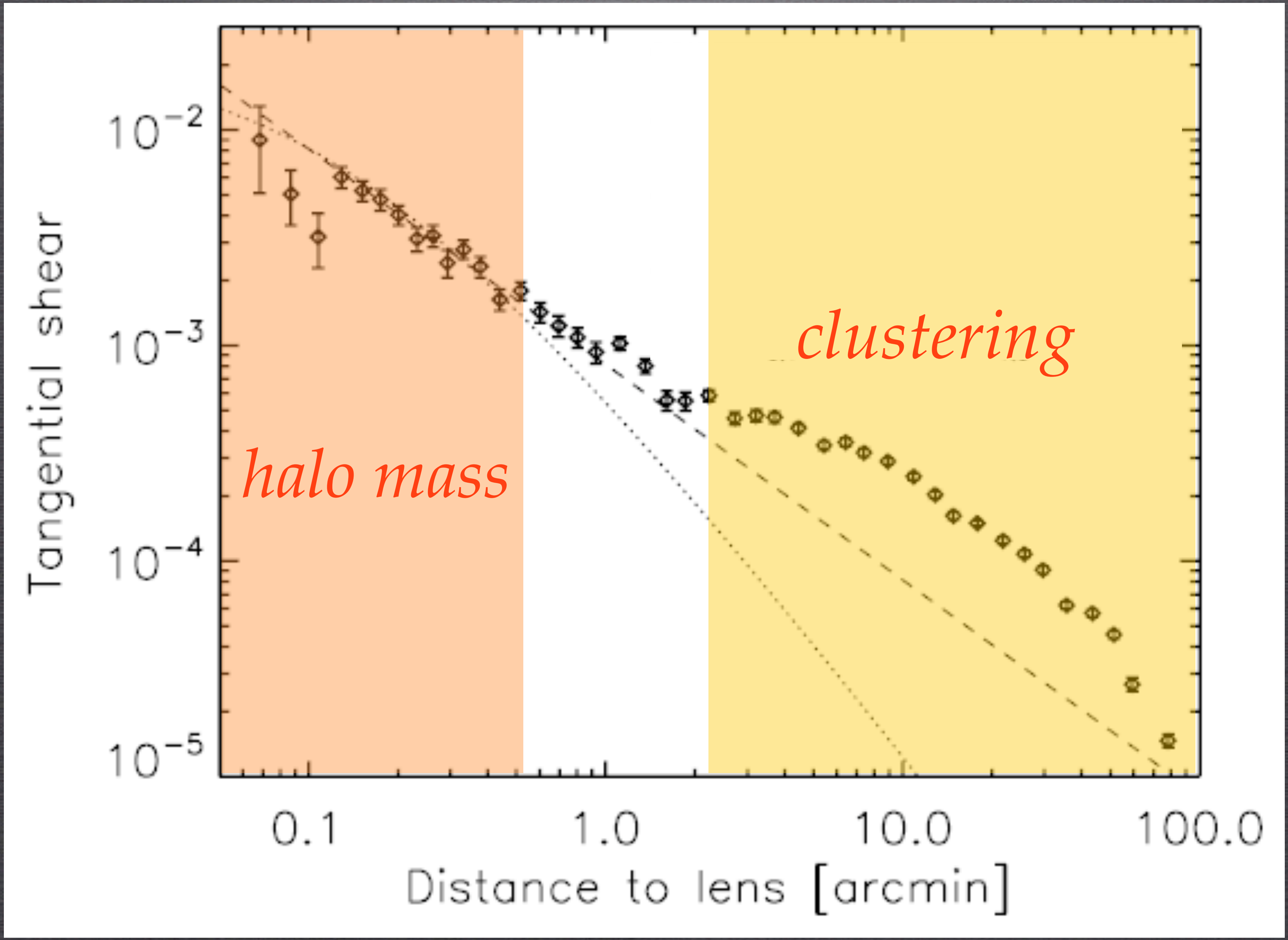


Hildebrandt et al. (2009):
CFHTLS DEEP: 4 sq. deg.

CFHTLS Wide: ~150 sq. deg
shear + magnification!

We can now start to study z>1 halos!

How to interpret the signal?



HOW TO INTERPRET?

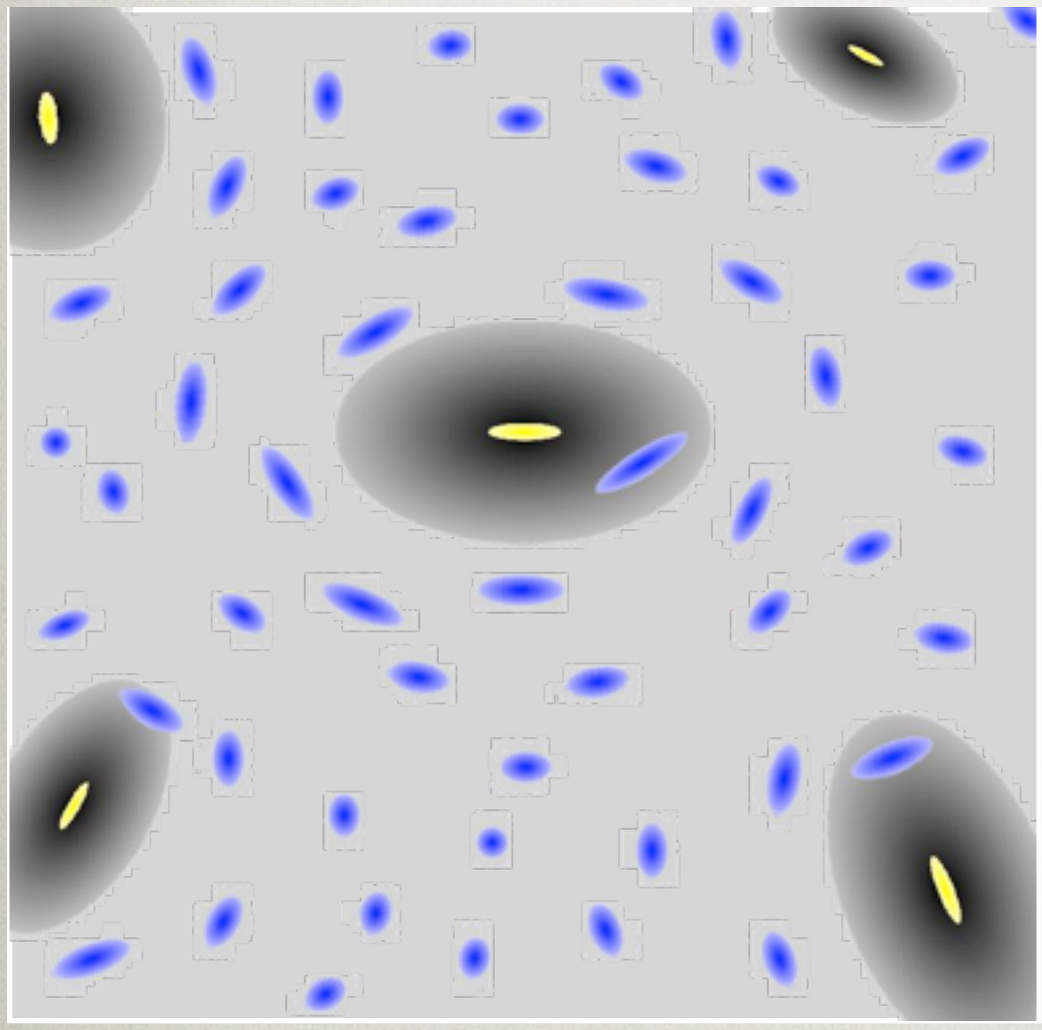
The signal *(the galaxy-mass cross-correlation function)* is the convolution of the dark matter distribution around galaxies and the clustering properties of the lenses.

We have some options to infer information about the properties of the dark matter halos around galaxies:

- interpret the data in the context of a model (simulations/analytical)
- deconvolve the correlation function
- look at isolated halos/small scales

- use the GMCC to learn about cosmology

‘Deconvolution’

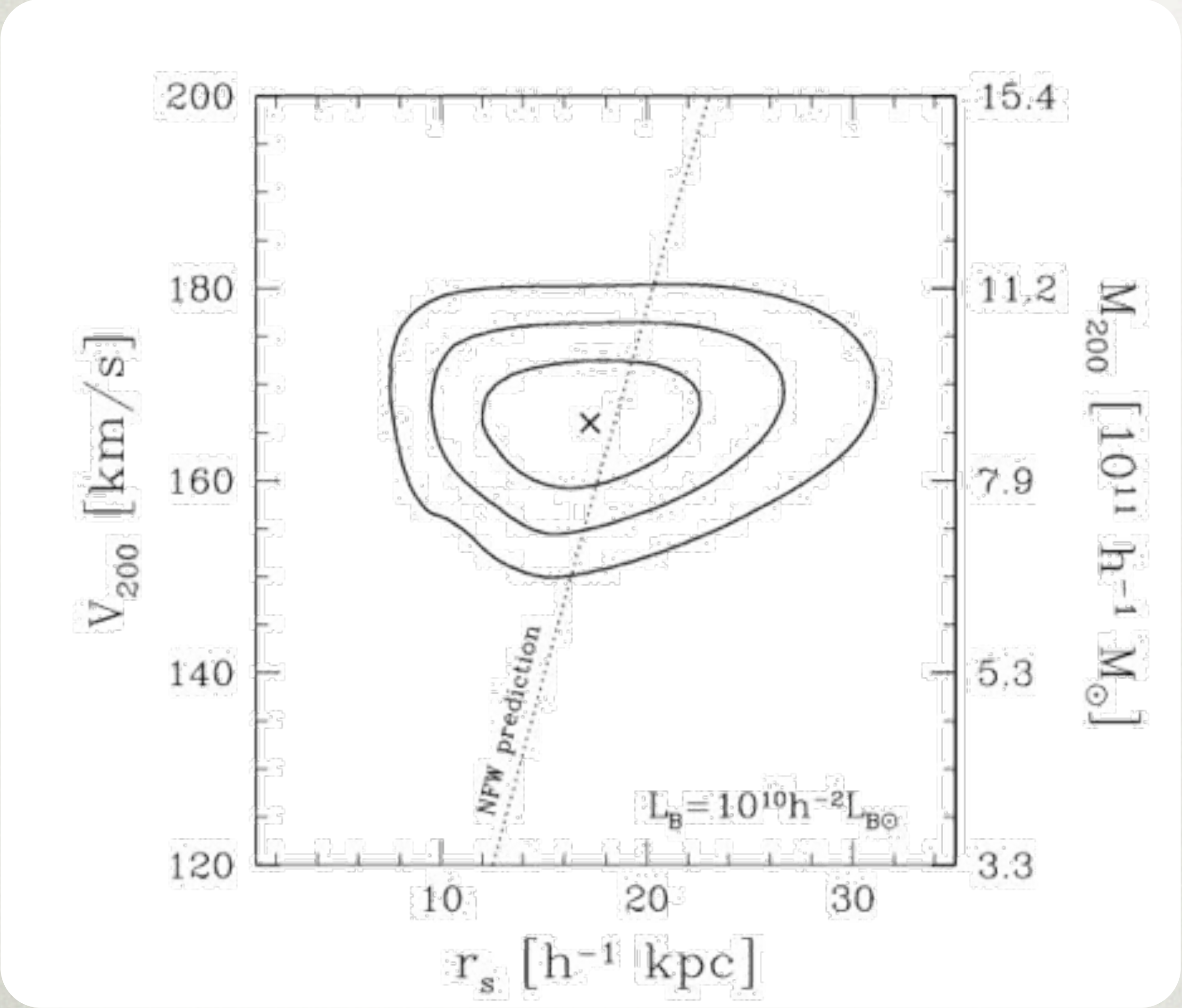


- Assign a halo to each galaxy
- Compute the lensing signal
- Compare to the data

best fit halo parameters

(also see Schneider & Rix 1997)

DECONVOLUTION: HALO SIZES



Hoekstra et al. (2004)

42 sq. deg. RCS

120,000 lenses

1.5 million sources

No redshifts for the lenses

M_{200}=(1.3±0.1) x 10^{12} M☼ for "Milky Way" halo

Deconvolution: issues

The maximum-likelihood has a number advantages:

- it uses that actual clustering of lenses
- it uses the 2-d shear signal (optimal use of data)

The maximum-likelihood has a number disadvantages:

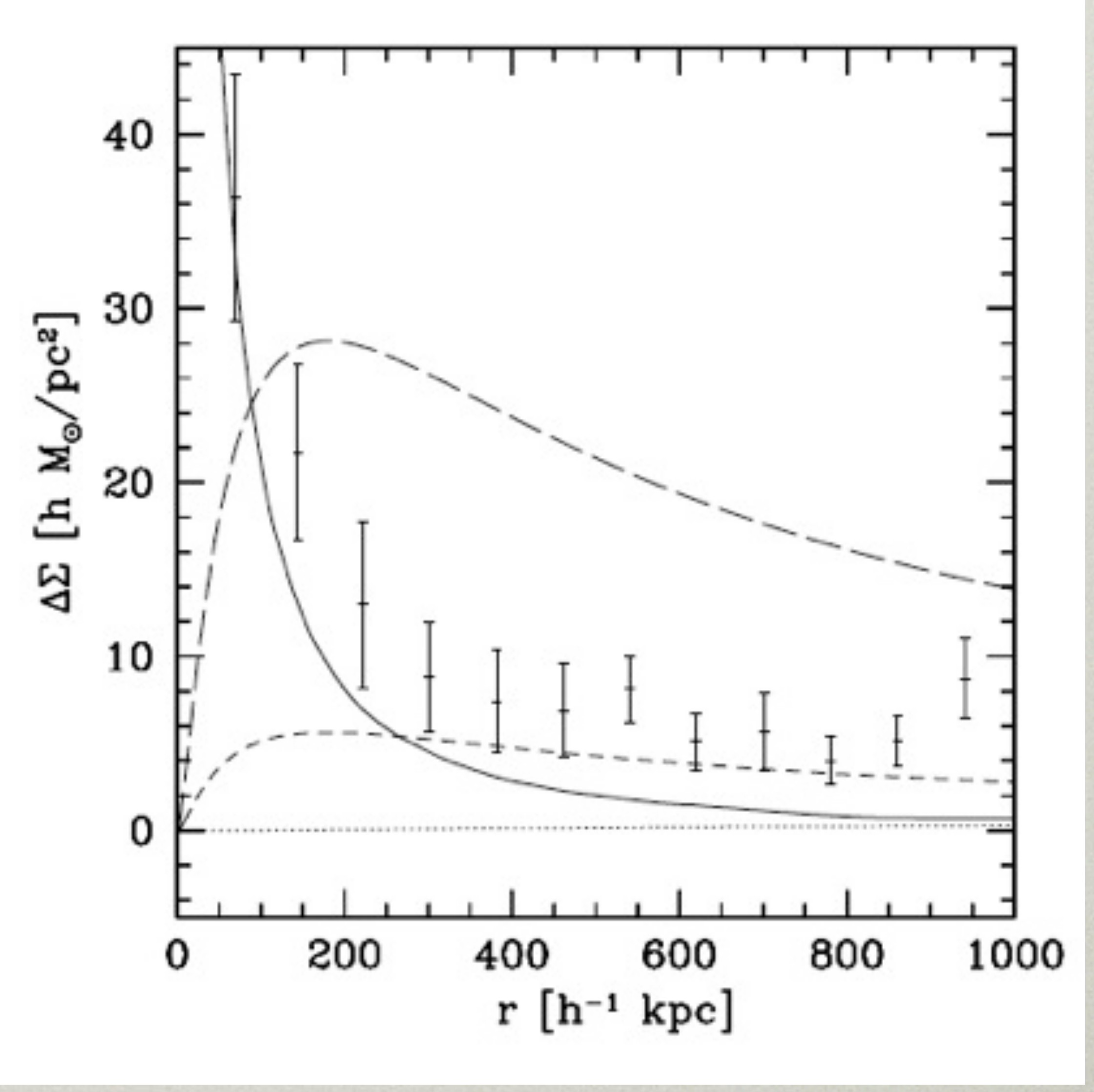
- it assumes that all mass is associated with galaxies
- it always gives an answer

Halos of cluster galaxies

It is possible to study the properties of galaxies in dense environments.

We need to separate the contribution from the cluster halo from the galaxy signal.

The latter dominates on very small scales only.

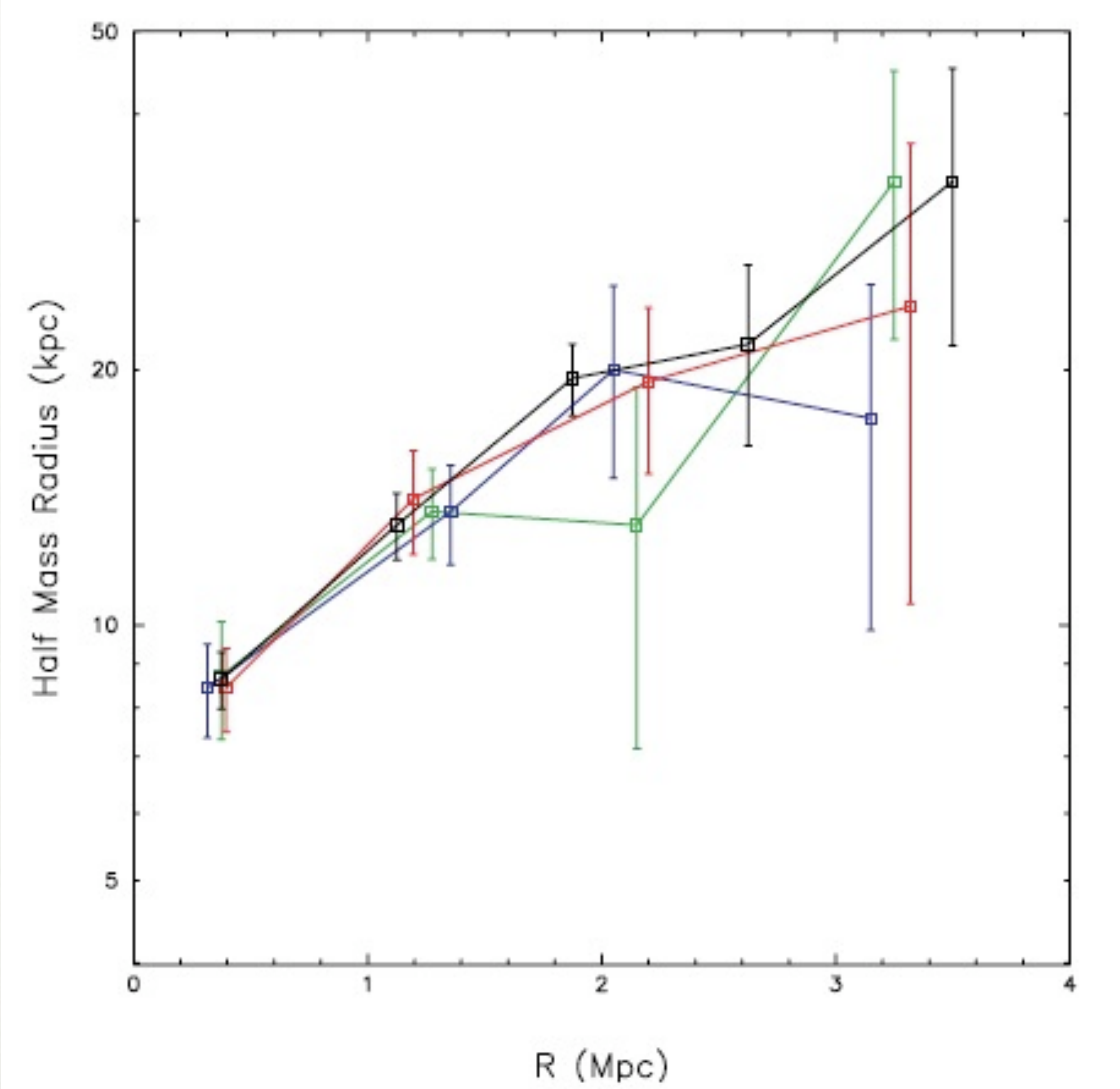


Guzik & Seljak (2002)

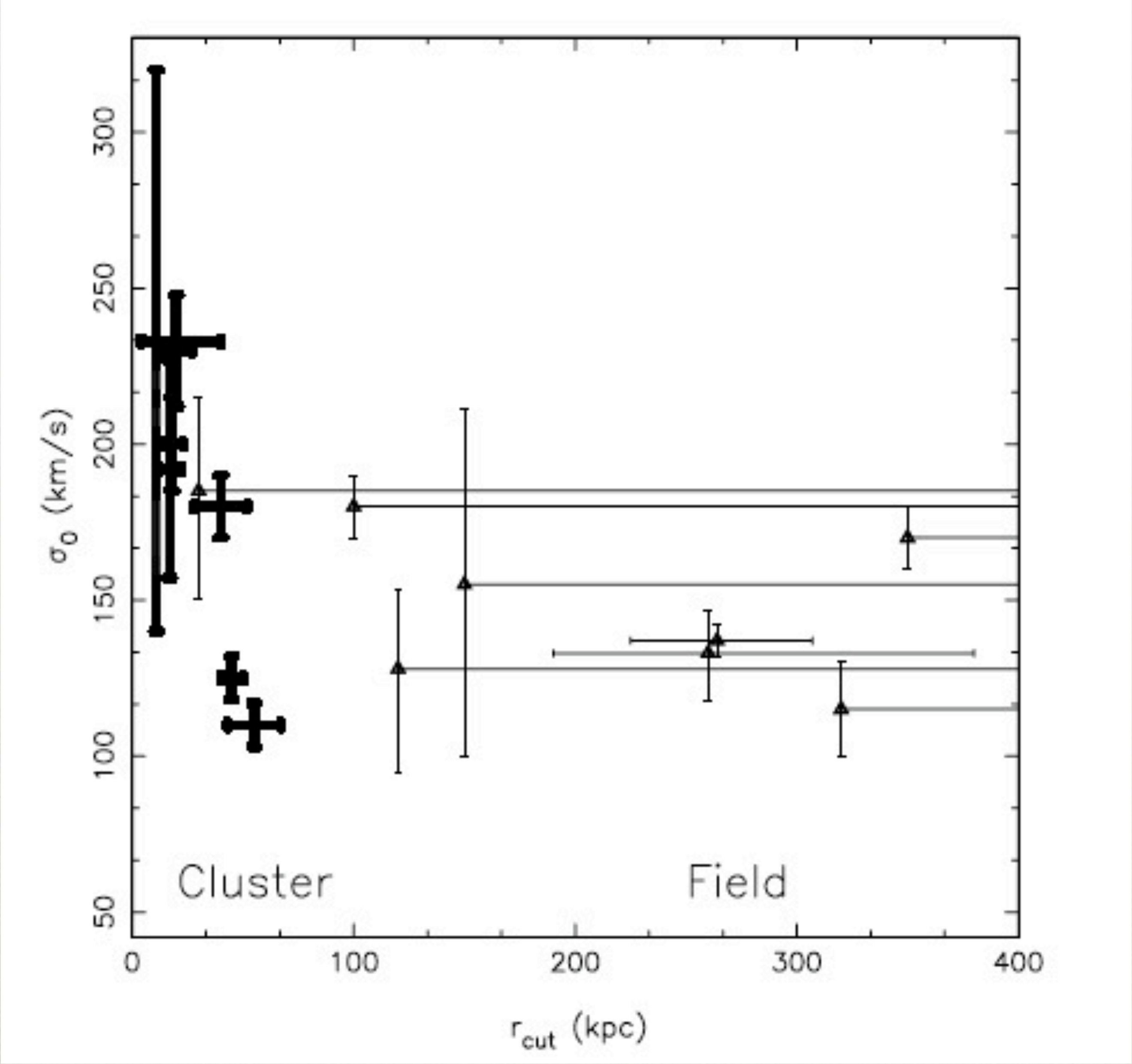
Halos of cluster galaxies

Expected signal for a Virgo mass cluster

Limousin et al. (2009)



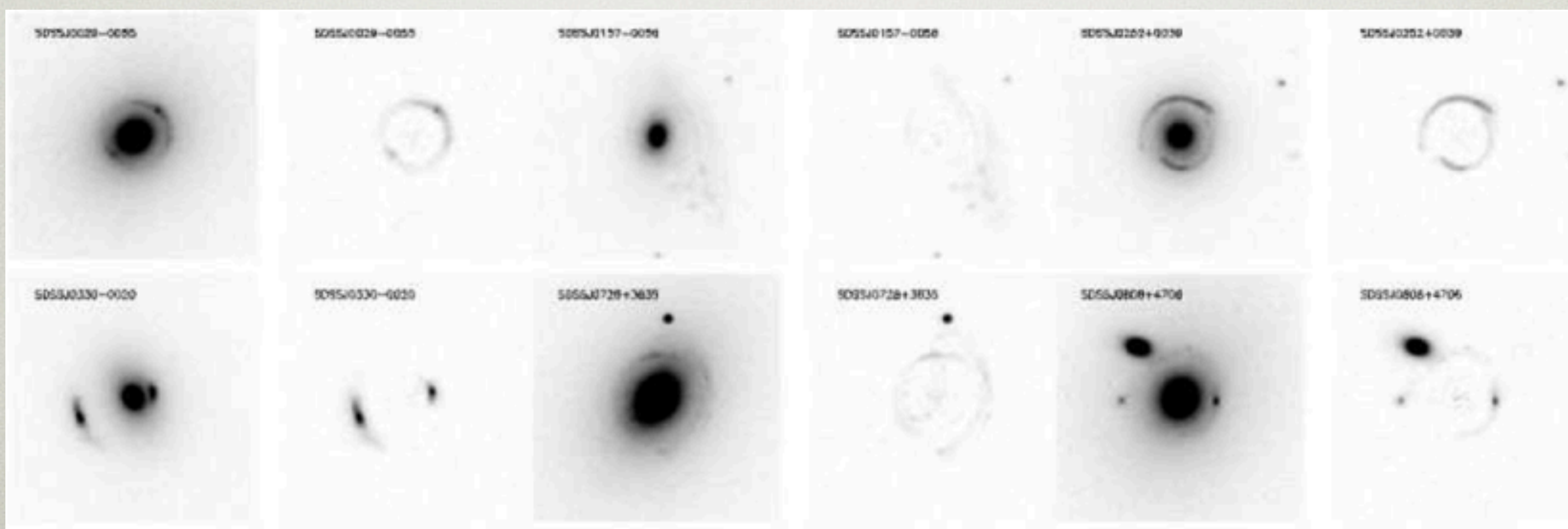
Halos of cluster galaxies



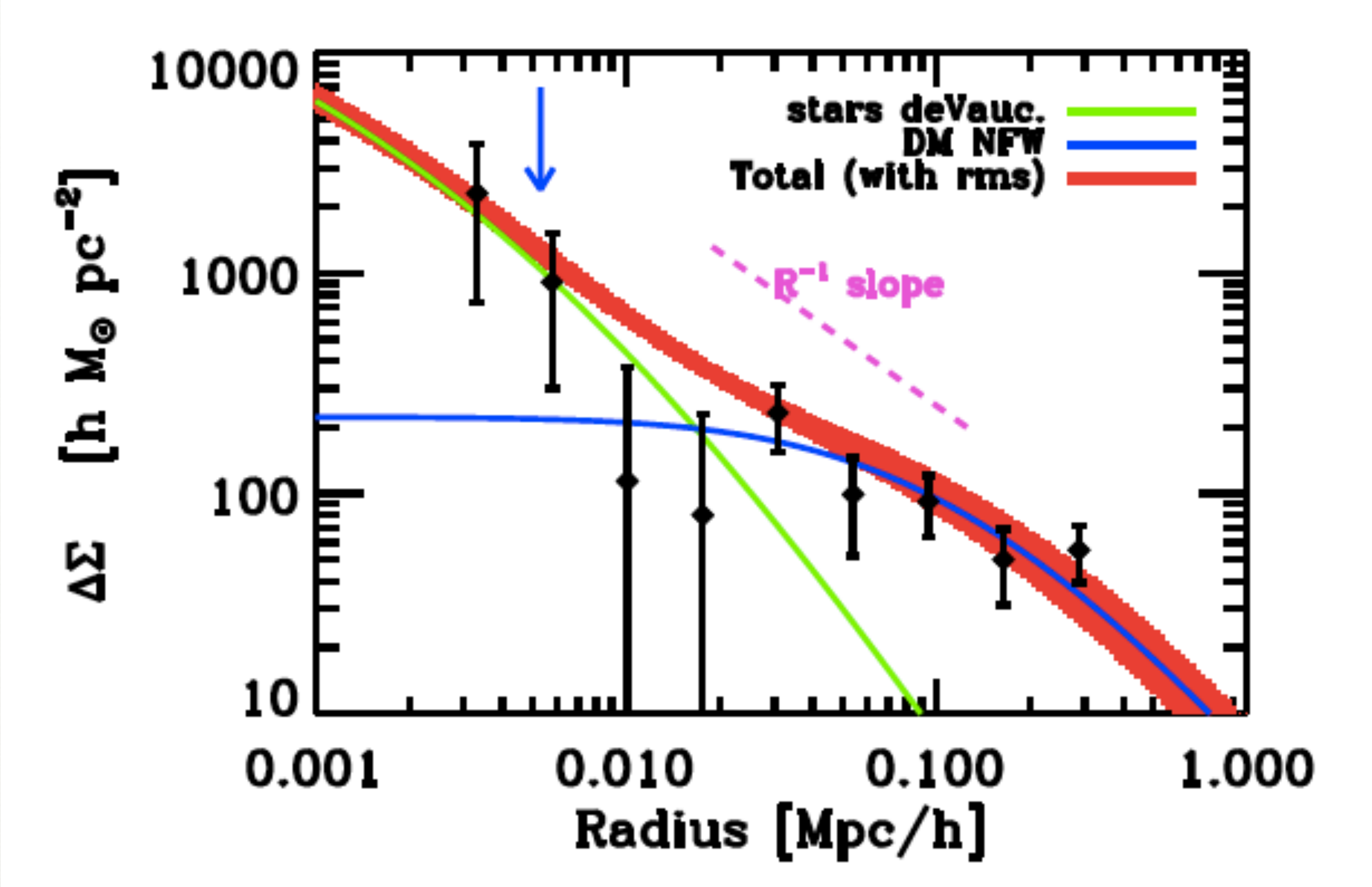
Limousin et al. (2007): limited to small radii

Focus on small radii

One small scales the weak galaxy lensing signal is sensitive to the density profile of the primary lens. On those scales it is also interesting to combine with strong lensing.



Focus on small radii



Gavazzi et al. (2007)

Focus on small radii

Close to the lens the shear is no longer contant across the source: *we need to consider the higher derivative of the deflection potential*.

We need to extend the Taylor expansion of the lensing mapping:

$$\beta_i \simeq A_{ij}\theta_j + \frac{1}{2}D_{ijk}\theta_j\theta_k$$

$$D_{ijk} = \partial_k A_{ij}$$

Source image is mapped to:

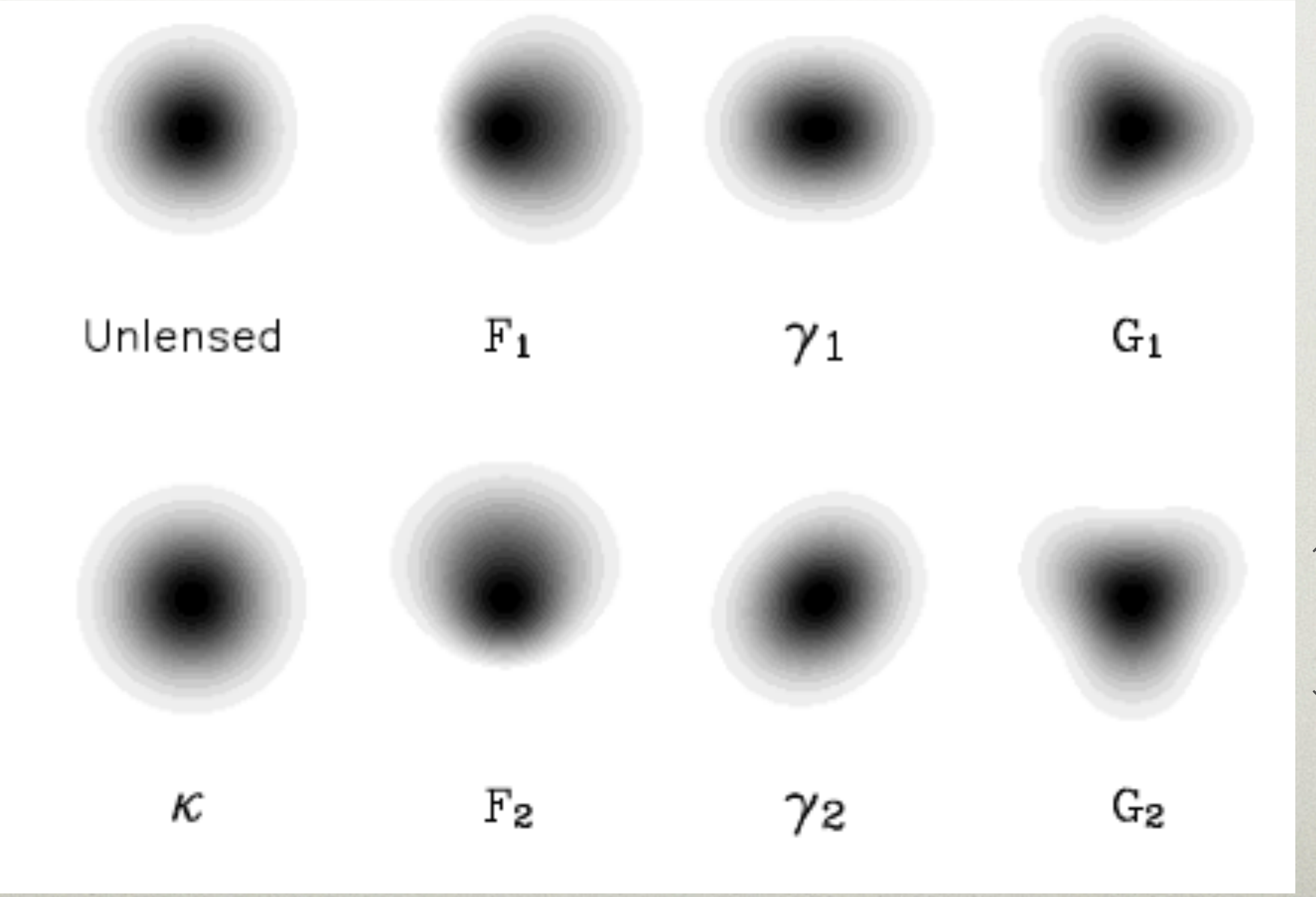
$$f(\theta) \simeq \left\{1 + \left[(A-I)_{ij}\theta_j + \frac{1}{2}D_{ijk}\theta_j\theta_k\right]\partial_i\right\} f^s(\theta)$$

Flexion

$$D_{ijk} = \mathcal{F}_{ijk} + \mathcal{G}_{ijk}$$

$$\mathcal{F} = \frac{1}{2}\partial\partial^*\partial\psi = \partial^*\gamma = \partial\kappa$$ spin-1

$$\mathcal{G} = \frac{1}{2}\partial\partial\partial\psi = \partial\gamma$$ spin-3



Bacon et al. (2006)

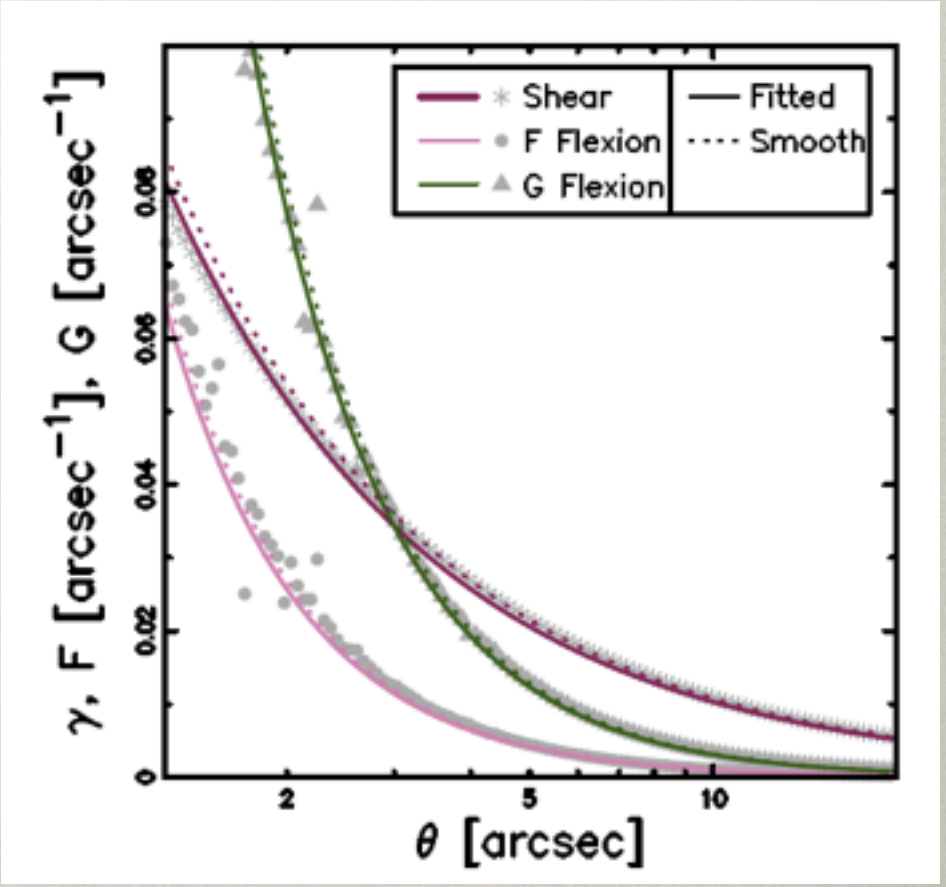
Flexion

- Flexion has units 1/length
- It probes small scales

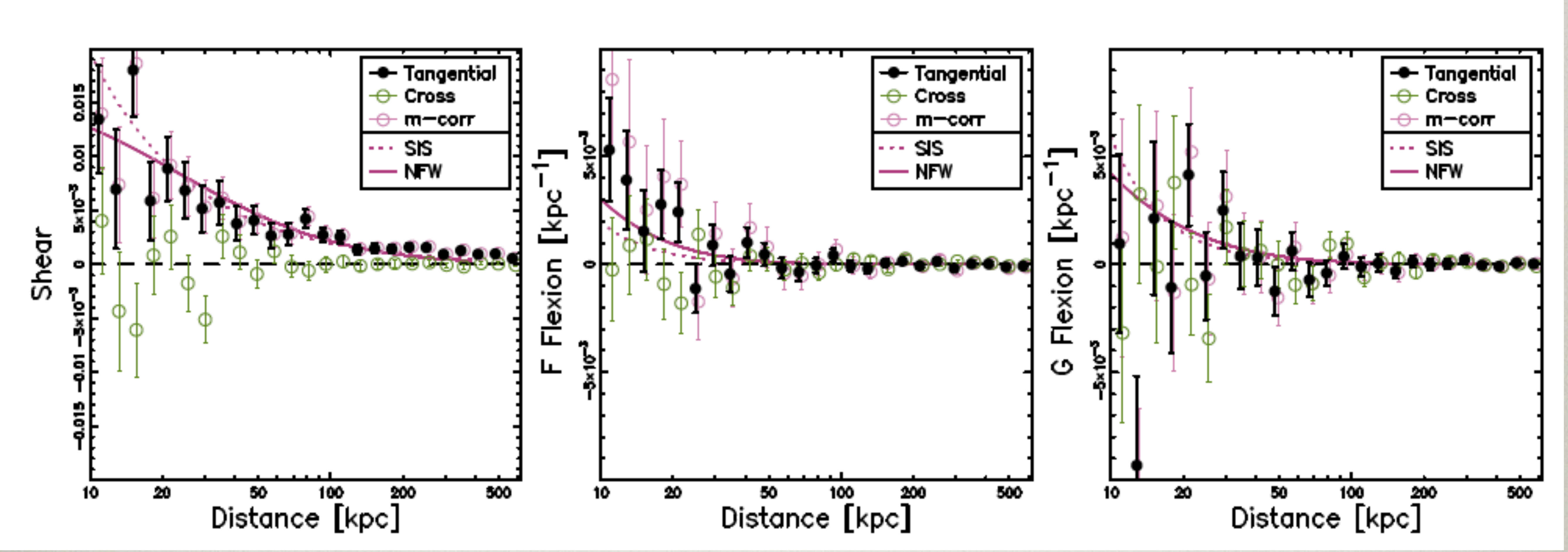
consider SIS: $$\kappa(\theta) = \frac{\theta_E}{2\theta}$$

$$\mathcal{F} = -\left[\frac{\theta_E}{2\theta^2}\right] e^{i\phi}$$

$$\mathcal{G} = \frac{3\theta_E}{2\theta^2} e^{3i\phi}$$



Flexion



Velander et al. (2011)

Avoid the clustering signal

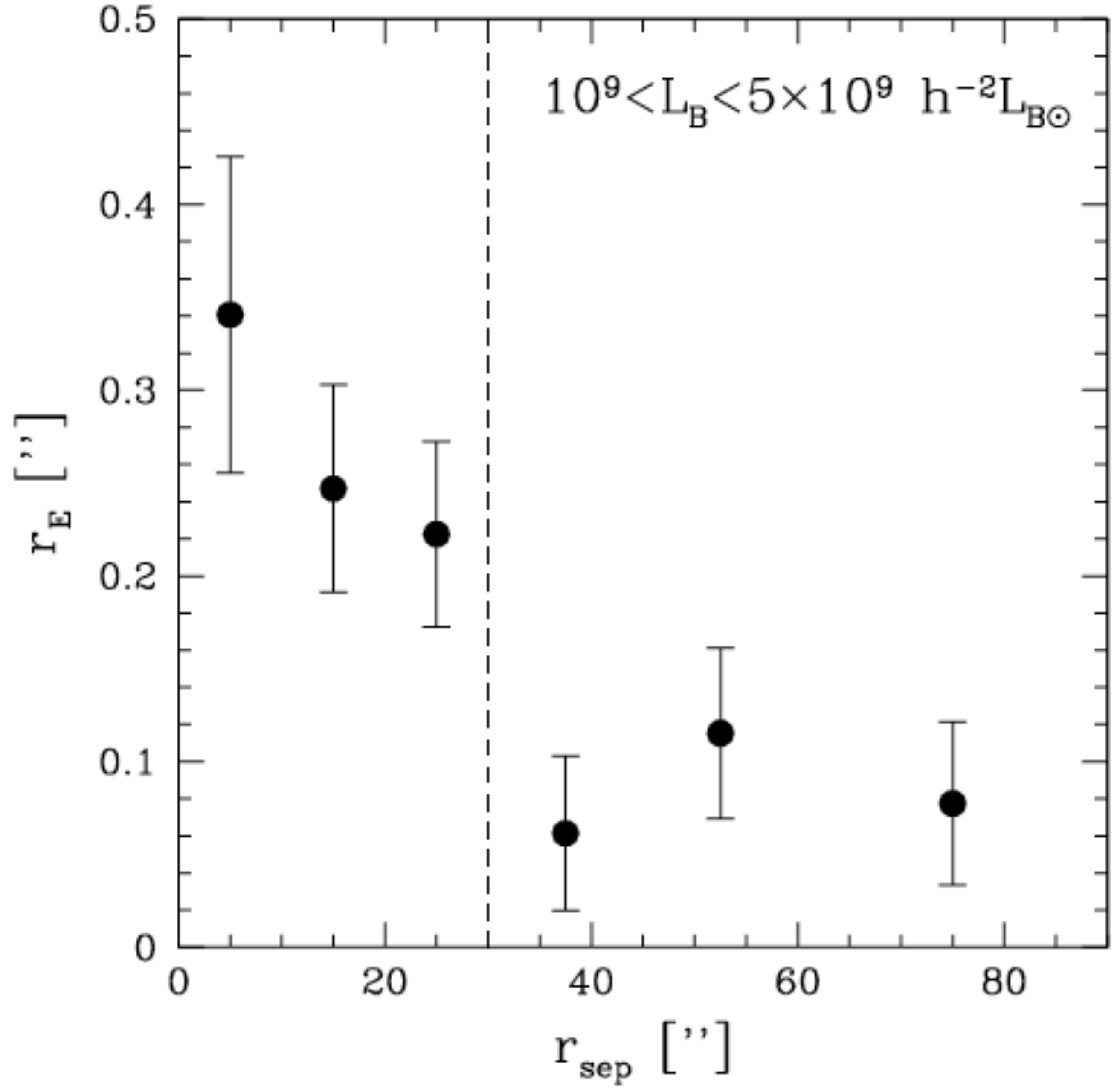
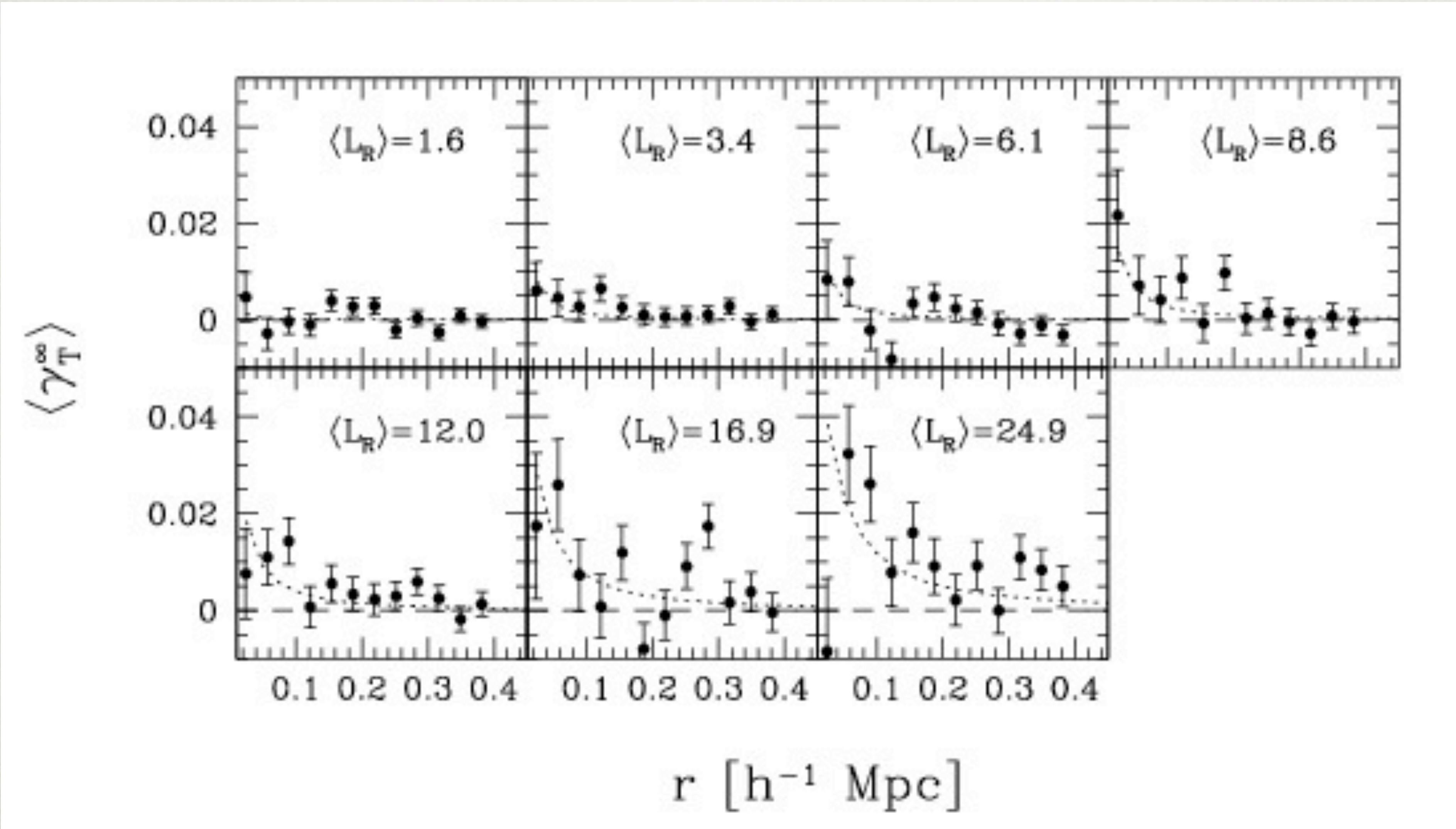


FIG. 5.— Einstein radius for 'faint' lenses as a function of projected distance to the nearest 'bright' lens r_{sep}. The faint galaxies have luminosities $10^9 < L_B < 5 \times 10^9 h^{-2} L_{B\odot}$, whereas the bright galaxies have $L_B > 5 \times 10^9 h^{-2} L_{B\odot}$.

Hoekstra et al. (2005)

Avoid the clustering signal

"isolated" galaxies with 0.2<z<0.4 from RCS using photometric redshift for the lenses



Hoekstra et al. (2005)

WHAT DO WE MEASURE?

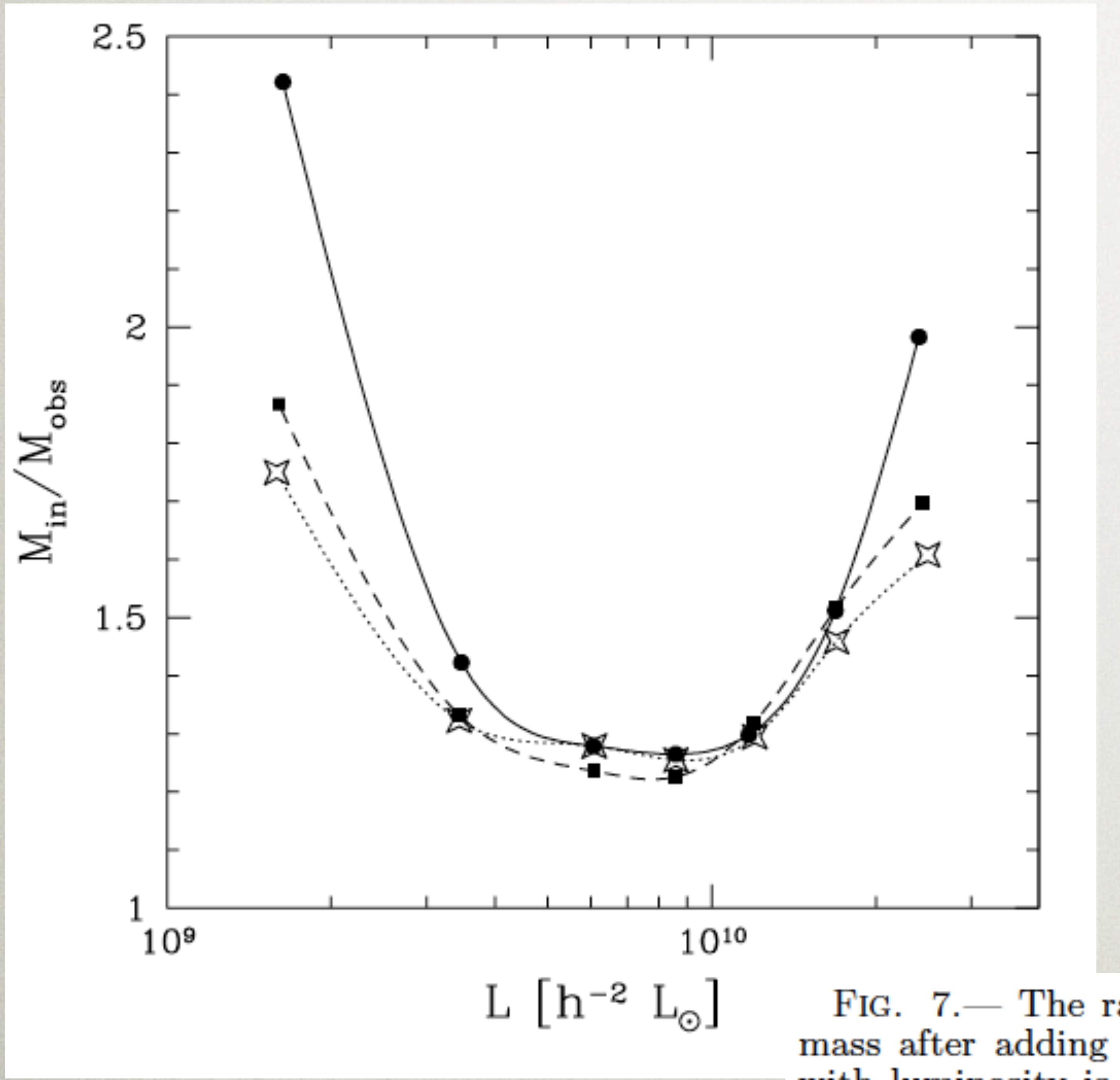
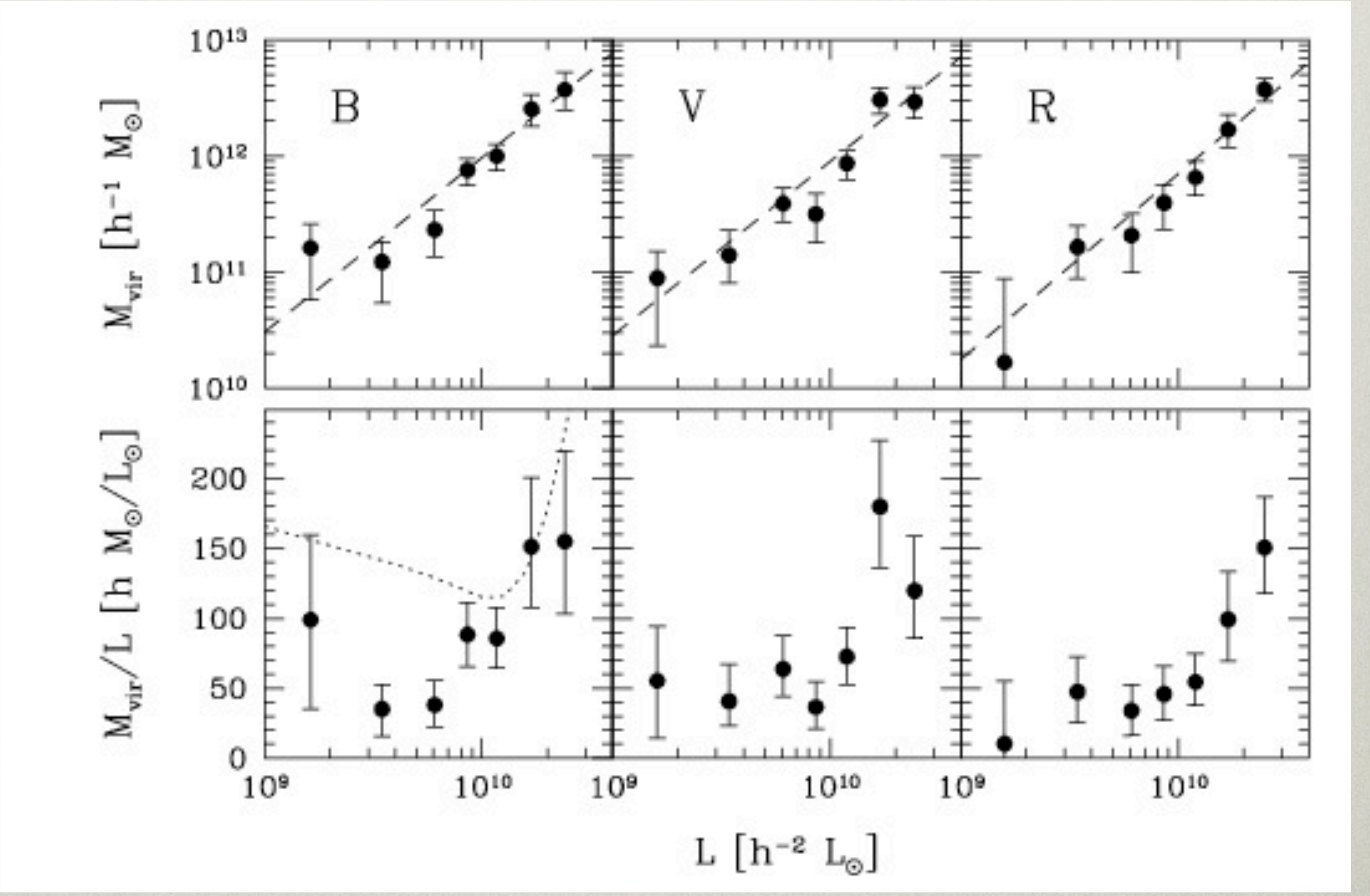


FIG. 7.— The ratio of the input virial mass and the observed mass after adding photometric redshift errors. The dependence with luminosity is dominated by how the redshift errors depend on brightness. The resulting curves depend only very weakly on

Scaling relation

$M_{vir} \propto L^{1.5}$



Hoekstra et al. (2005)

What do we measure?

We bin in terms of an observable, but if there is intrinsic scatter between this property and the mass, what do we end up measuring?

The distribution of halo masses for a certain luminosity (or stellar mass) is given by the conditional probability function, which is usually described by a log-normal function of the form

$$P(m_h|l) \propto \exp\Big(-\frac{(m_h - m_{h,cent})^2}{2\sigma_{m_h}^2}\Big) \qquad \text{(B.1)}$$

where $l = \log(L)$, $m_h = \log(M_h)$ and σ_{m_h} is the scatter in m_h. In this Appendix we study how the best fit lensing

We need to account for this in order to connect to predictions from theory.

WHAT DO WE MEASURE?

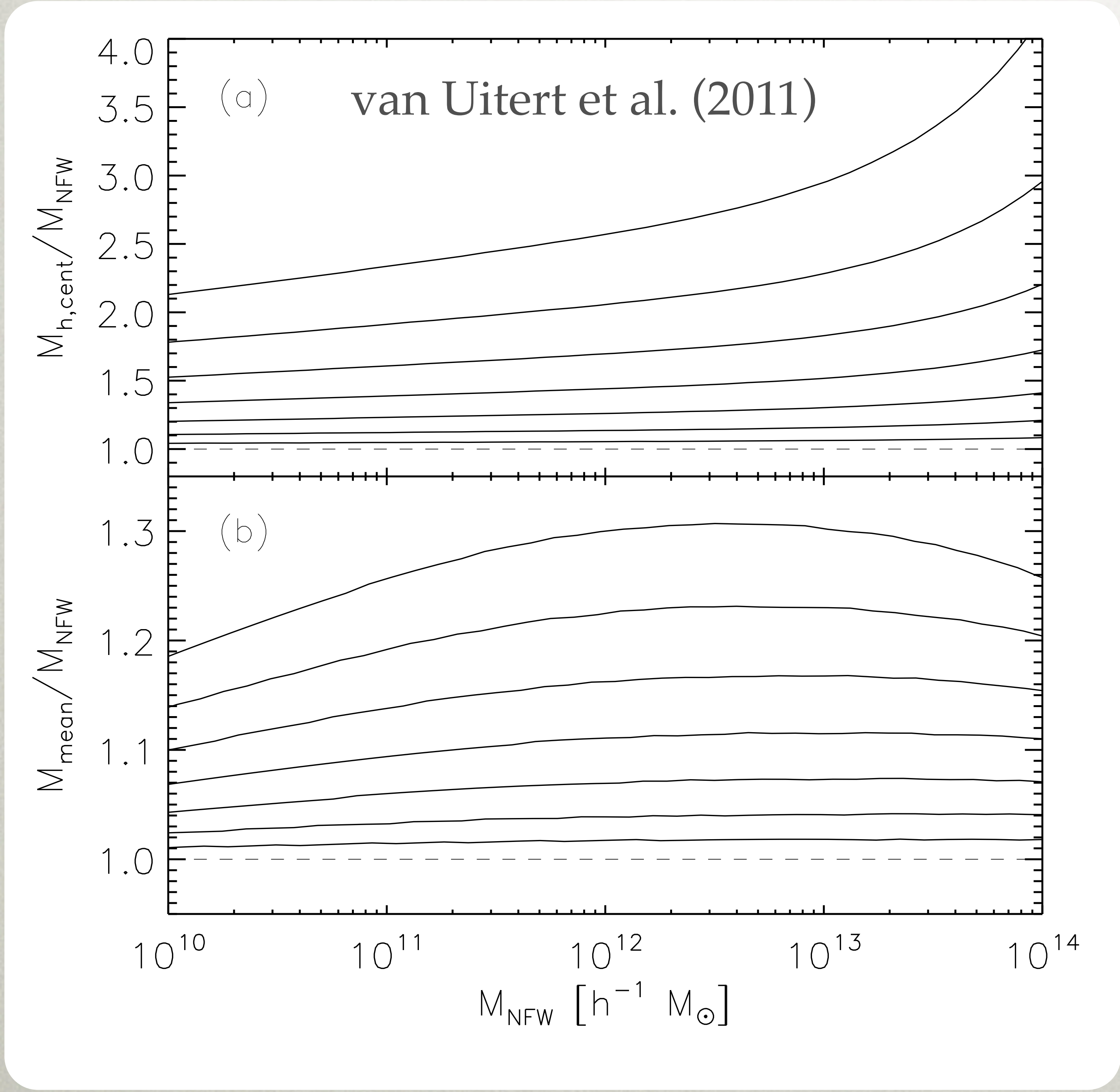
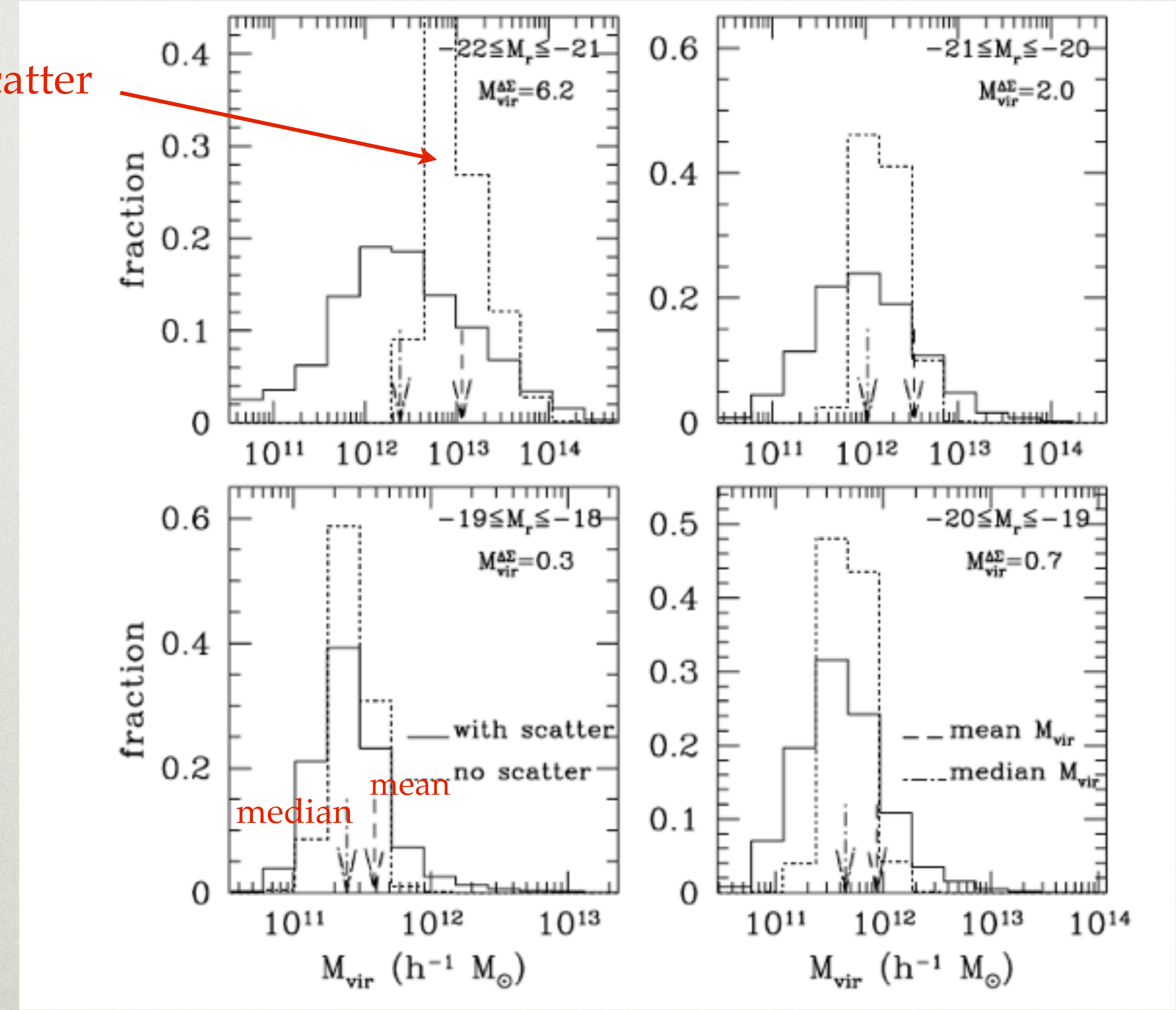


Fig. B.1. The ratio of the central mass of the halo mass distribution, $m_{h,cent}$, and the best fit NFW mass (*top*) and the ratio of the mean halo mass and the best fit NFW mass (*bottom*) as a function of best fit NFW mass. Different lines correspond to values of σ_{m_h} 0.10 (bottom line), 0.15, 0.20, 0.25, 0.30, 0.35 and 0.40 (top line). The lensing mass is converted to the mean halo mass using the corrections from the bottom panel.

What do we measure?

without scatter



Tasitsiomi et al. (2004)

Best fit mass somewhere between mean and median

Comparison with other probes

Although lensing provides the most direct measurement of the halo mass, it is interesting to consider other mass indicators:

- (central) velocity dispersion
- stellar mass
- combination of stellar mass and structural parameters

$$\boxed{\sigma_{\rm mod} = \sqrt{\frac{GM_*}{0.557 K_V(n) R_e}}}$$

$$K_V(n) \cong \frac{73.32}{10.465 + (n - 0.94)^2} + 0.954.$$

Which one is better?

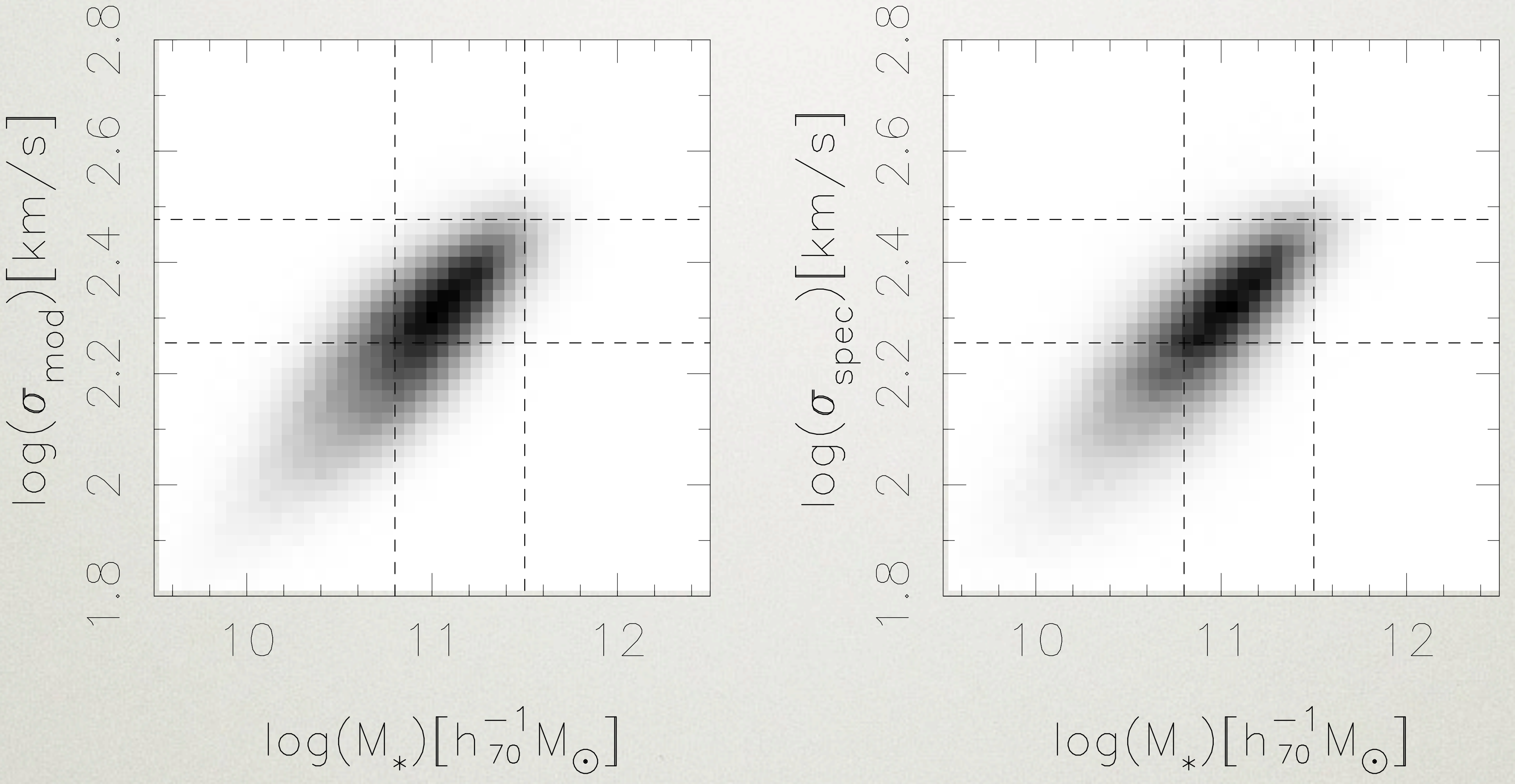
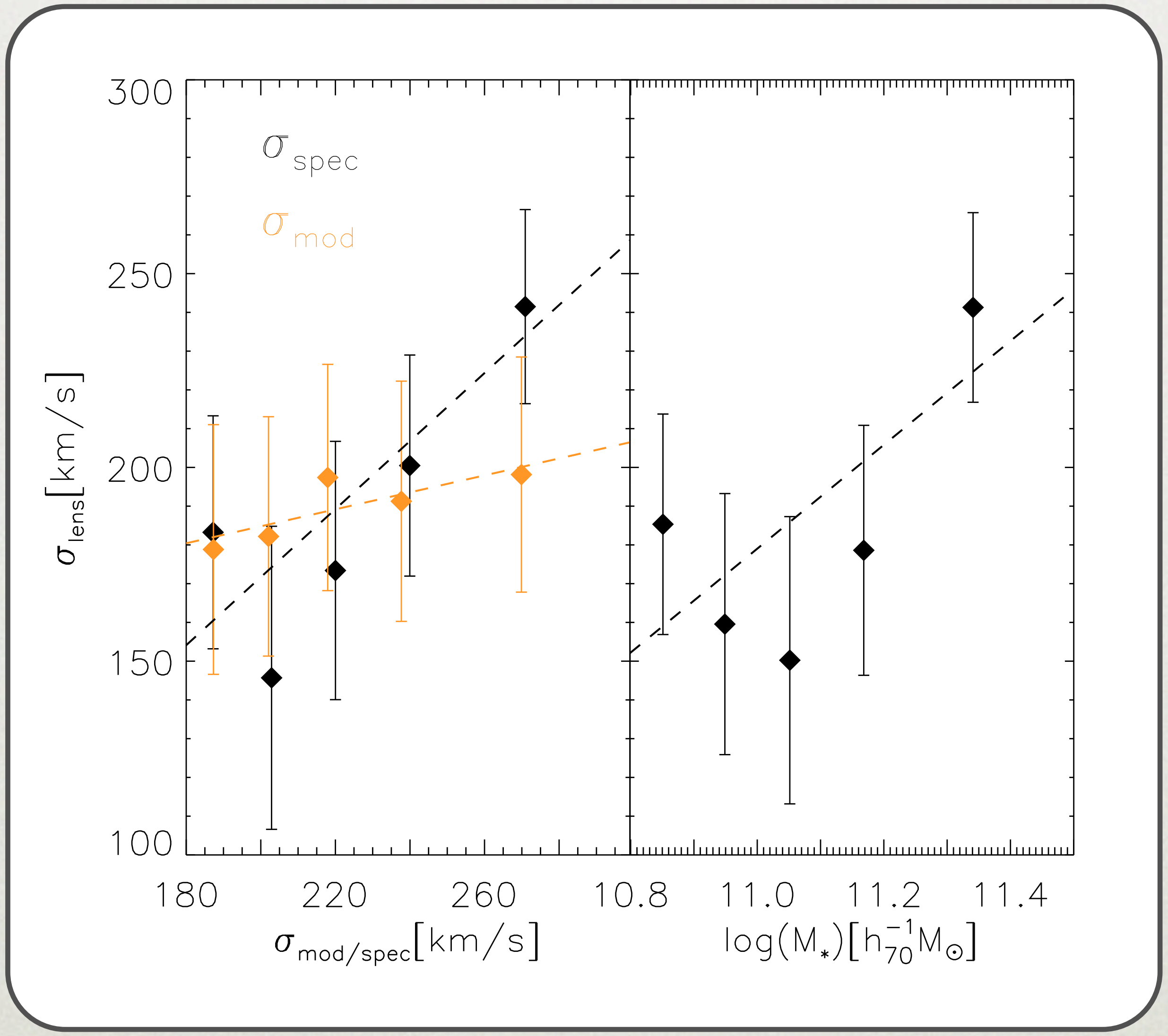


Fig. 3. Model velocity dispersion (*left*) and spectroscopic velocity dispersion (*right*) as a function of stellar mass. The dashed lines indicate the selection cuts for the lenses.

Which one is better?



van Uitert et al. (in prep): spectroscopic velocity dispersion and stellar mass trace halo mass equally well, but model dispersion is not a good indicator.

Shapes of DM halos

Numerical simulations show that halos are not spherical. This is an important prediction that can be tested using gravitational lensing. It can also be used to rule out alternative models of gravity that do not require dark matter.

Dynamical and strong lensing studies can only probe the inner regions where baryons are important.

Shapes of DM halos

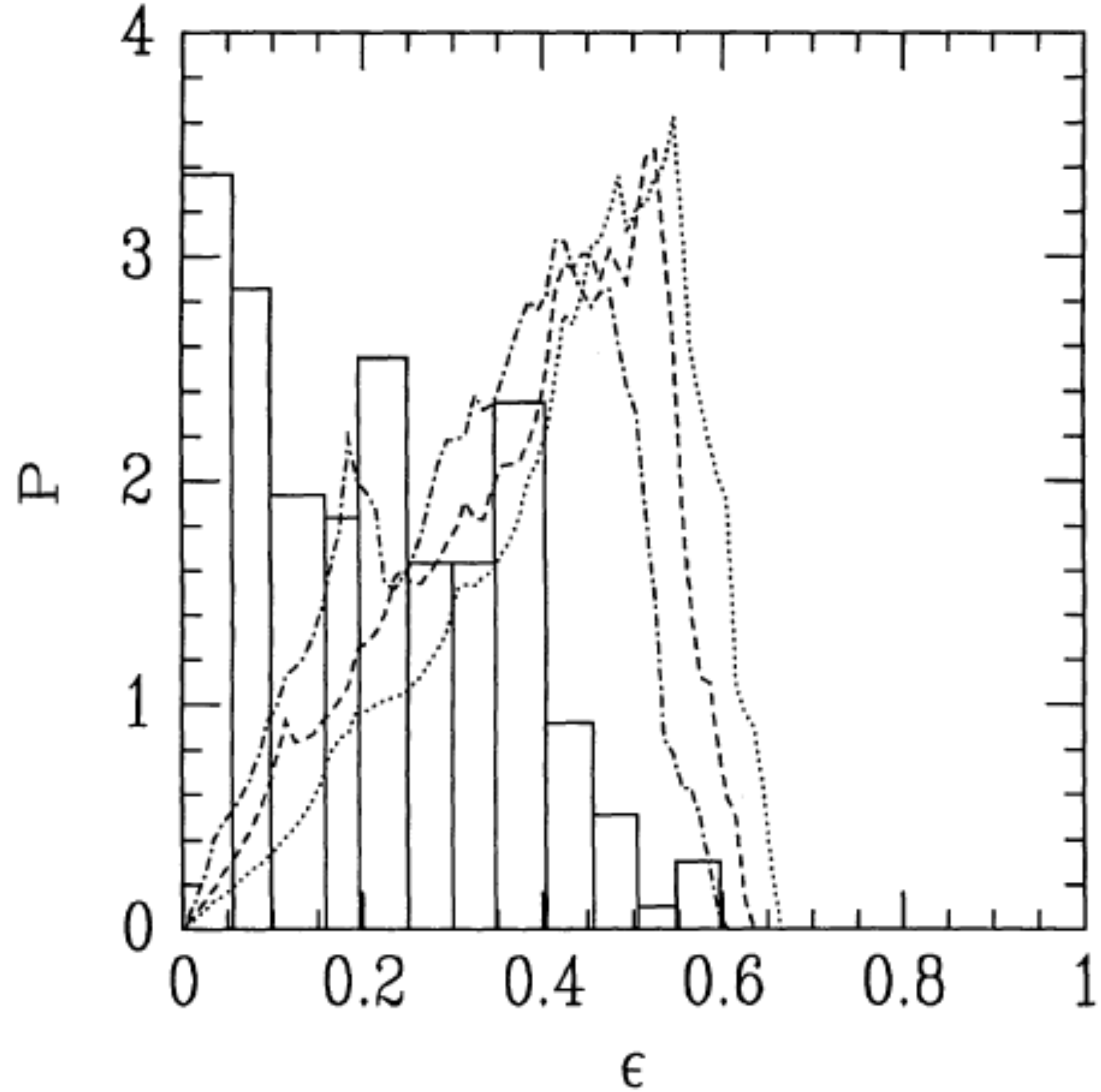


FIG. 8.—Probability distribution of ellipticities for elliptical galaxies and dark halos. The histogram is the renormalized data by Binney & de Vaucouleurs (1981). The three curves are the probability distributions derived from axial ratios measured out to 25 kpc (*dotted line*), 50 kpc (*dashed line*) and 100 kpc (*dashed-dot line*) from the center of the dark halos. Note that the elliptical galaxies are considerably rounder than the dark halos.

Dubinski & Carlberg (1991)

Shapes of DM halos

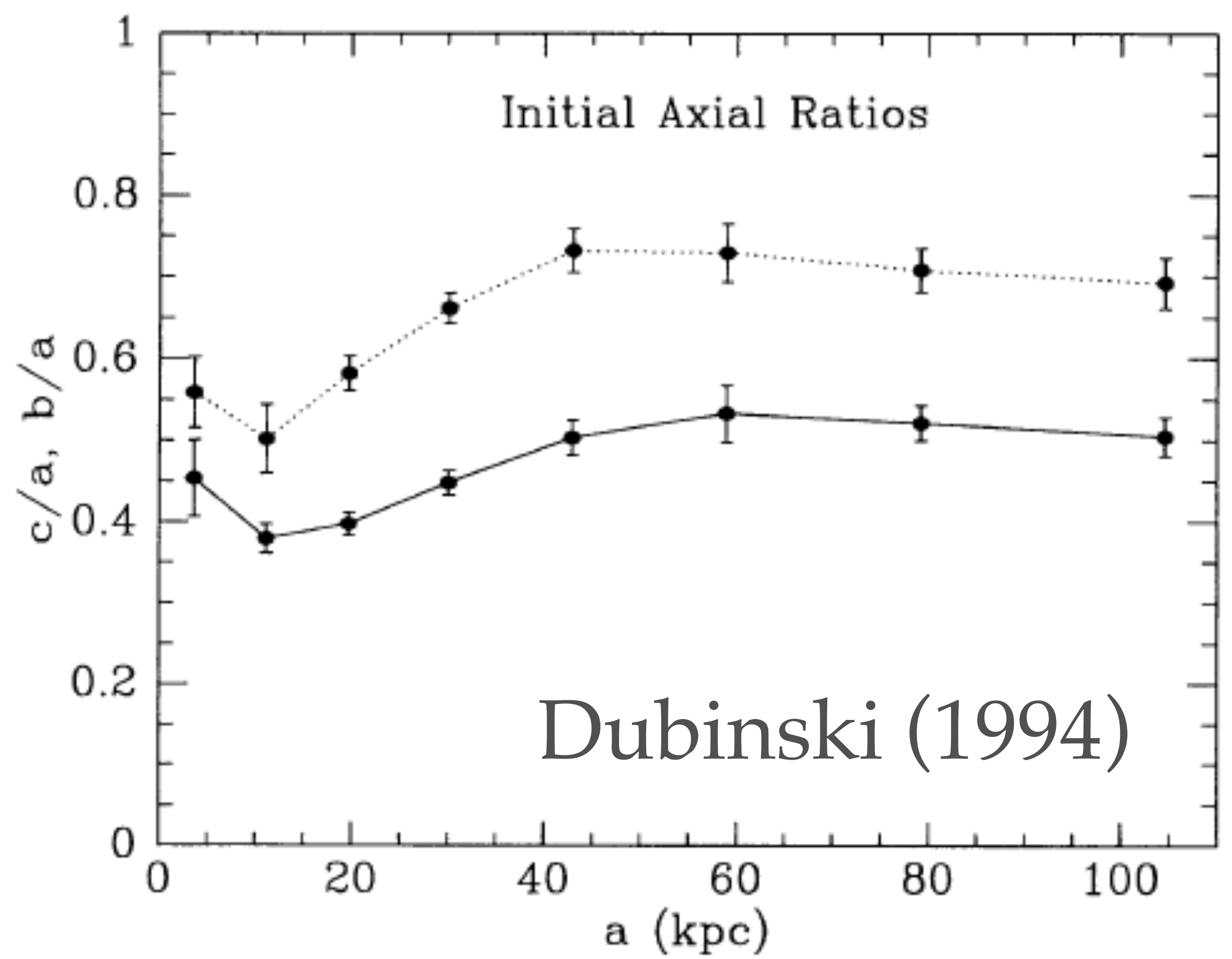
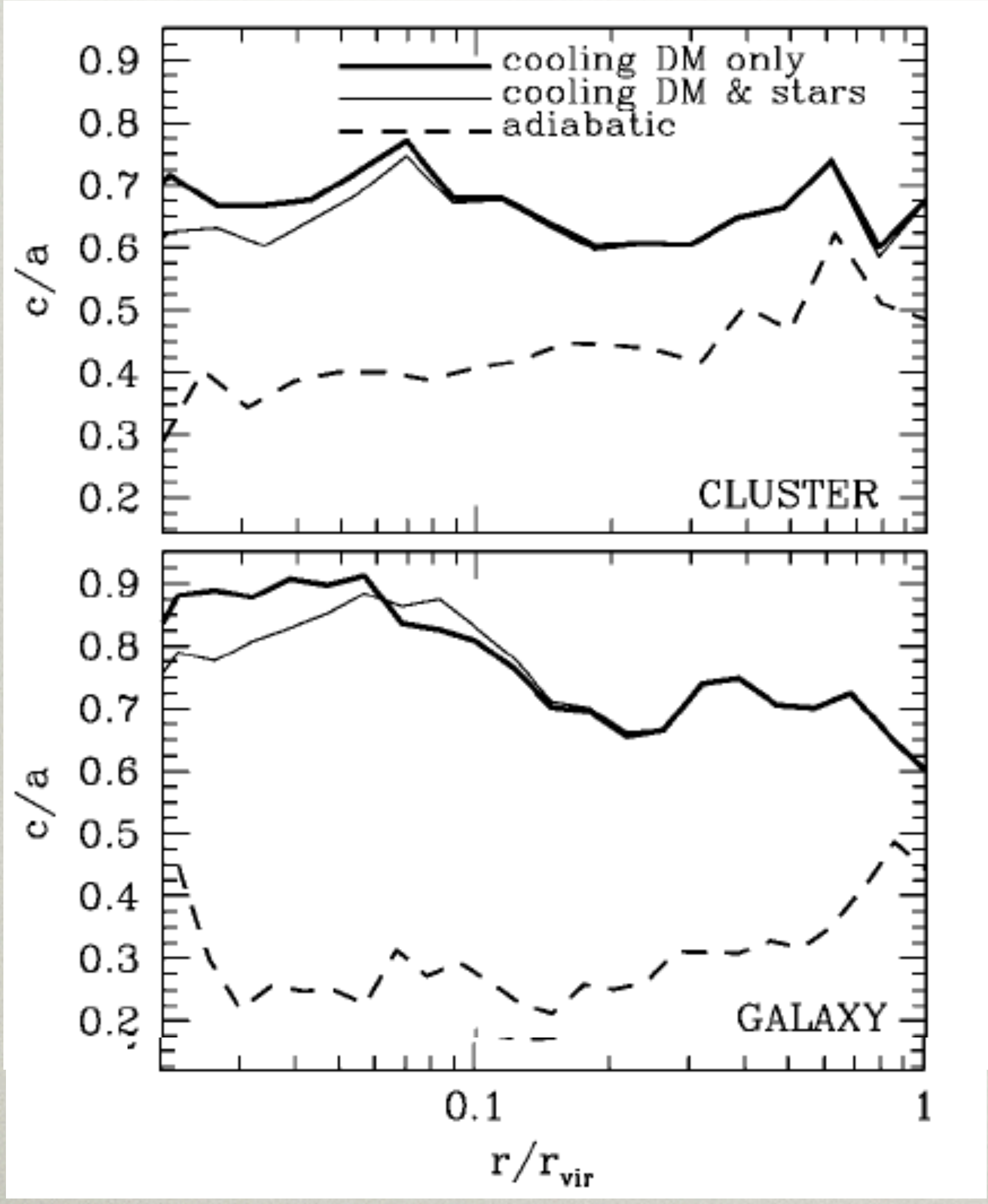


FIG. 1.—Axial ratio profiles of the dark halo. The solid and dotted lines, respectively, trace the ratios c/a and b/a. The error bars represent the rms variation in the axial ratios for measurements from 20 time intervals spanning 2.0 Gyr at the end of the simulation. The errors in the axial ratios are small showing that the dark halo is in equilibrium.

Shapes of DM halos



Kazantzidis et al. (2004):

Baryon physics can lead to more spherical halos.

This needs to be tested obsrvationally!

Shapes of DM halos

$$\gamma_T^{\text{lens}}(r,\ \theta) = [1 + \gamma_f \cos(2\theta)]\langle\gamma_T\rangle(r),$$

$$f_{\text{mm}}(r) = \frac{\gamma_{t,\text{B}}(r)}{\gamma_{t,\text{A}}(r)}$$

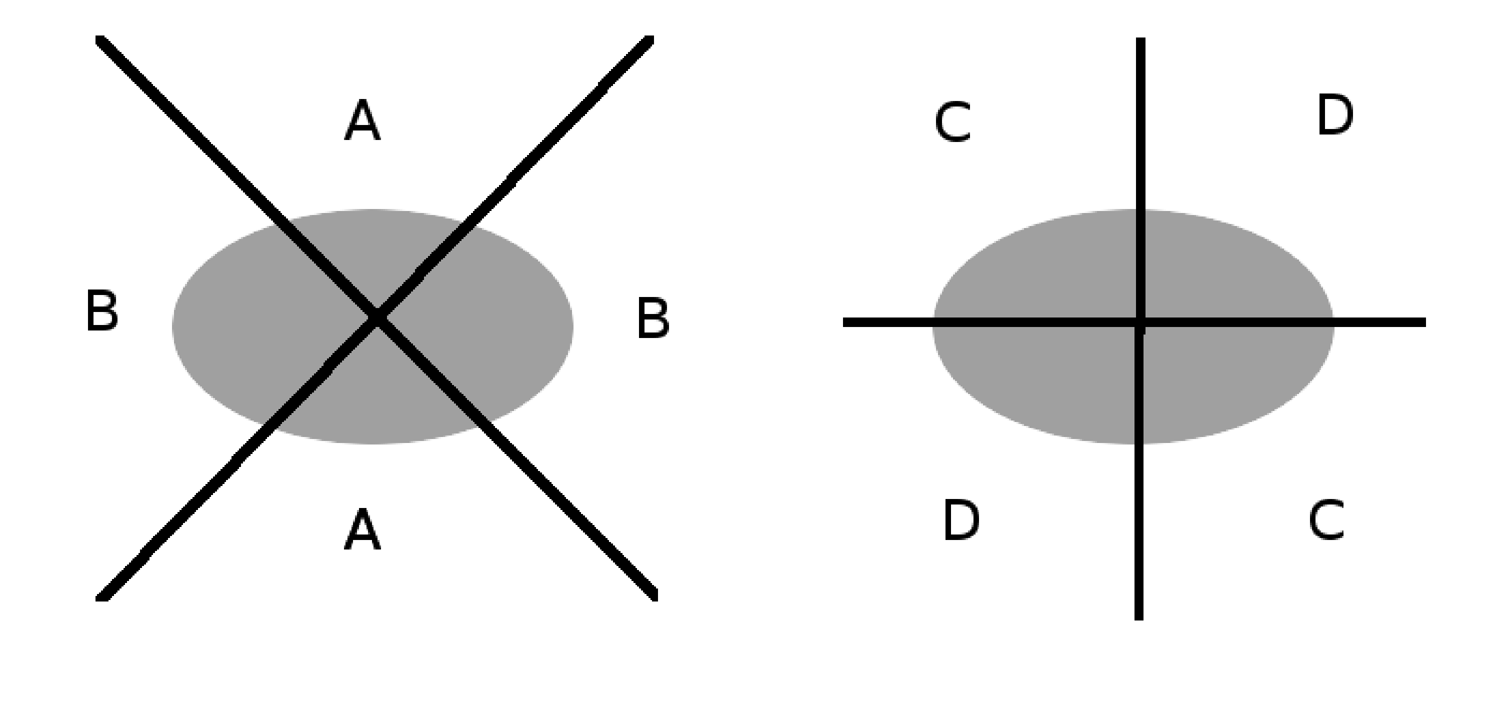
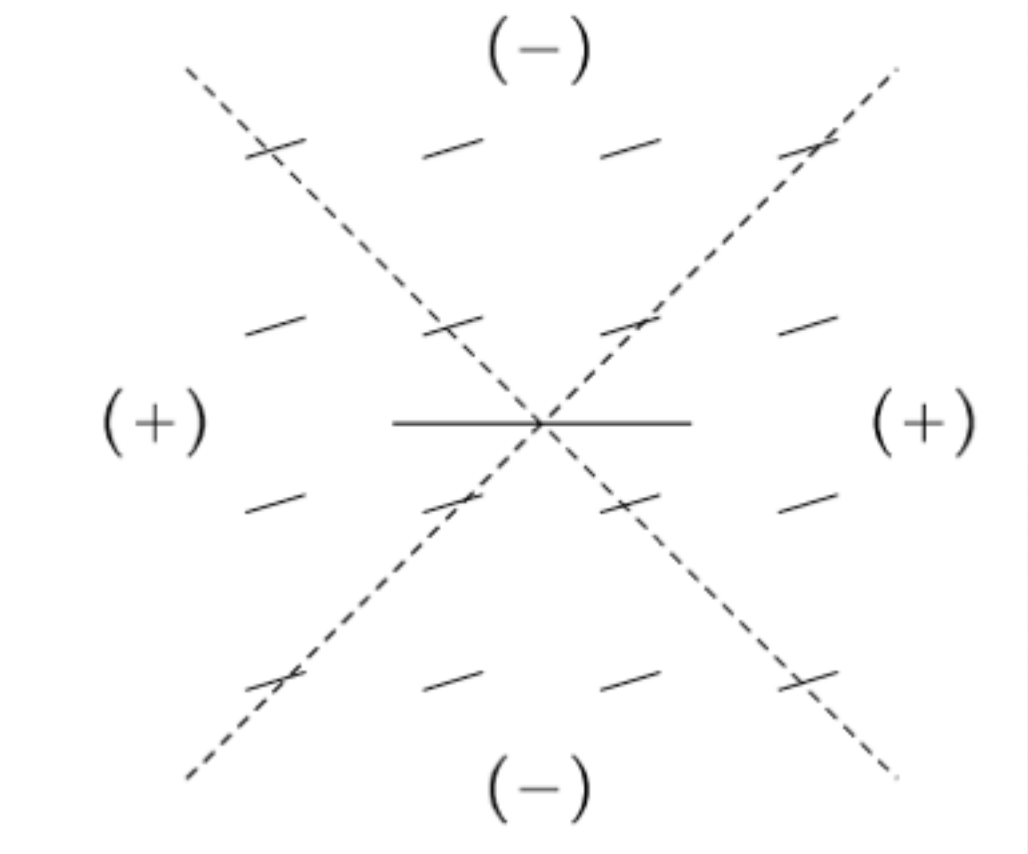


Fig. 3. Schematic of a lens galaxy. The tangential shear is measured in regions A and B, the cross shear is measured in regions C and D. The cross shear is subtracted from the tangential shear to correct for systematic contributions to the shear.

Shapes of DM halos



Contribution from PSF residual:

$$\hat{\gamma}_T = -\hat{\gamma}[\cos(2\phi)\cos(2\theta) + \sin(2\phi)\sin(2\theta)] = -\hat{\gamma}\cos[2(\theta - \phi)]$$

Shapes of DM halos

$$f_{\mathrm{obs}} = \frac{\gamma_- + \hat{\gamma}\cos(2\phi)}{\gamma_+ - \hat{\gamma}\cos(2\phi)}$$ with $$\langle\cos(2\phi)\rangle = \frac{\alpha\hat{\gamma}}{2\gamma}$$

As shown in Hoekstra et al. (2004) for a large ensemble of lenses we expect that

$$f_{\mathrm{obs}} = \frac{\gamma_- + \hat{\gamma}^2\alpha/2}{\gamma_+ - \hat{\gamma}^2\alpha/2}$$

Residual shear from incomplete PSF correction is limited to small radii and small. However, cosmic shear also aligns lenses and sources...

Shapes of DM halos

The contribution from PSF anisotropy and cosmic shear can be reduced by considering:

$$f_{\mathrm{mm}}^{\mathrm{corr}}(r) = \frac{\gamma_{t,\mathrm{B}}(r) + \gamma_{\times,\mathrm{C-D}}(r)}{\gamma_{t,\mathrm{A}}(r) - \gamma_{\times,\mathrm{C-D}}(r)}$$

A better approach is to assume that:

$$\Delta\Sigma_{\mathrm{model}}(r) = \Delta\Sigma_{\mathrm{iso}}(r)[1 + 2 f e_g \cos(2\Delta\theta)]$$

mass profile *lens ellipticity*

Shapes of DM halos

Mandelbaum et al. (2006) have shown that the azimuthally varying part is given by:

$$f\Delta\Sigma_{\rm iso}(r) = \frac{\sum_i w_i \Delta\Sigma_i e_{g,i}\cos(2\Delta\theta_i)}{2\sum_i w_i e_{g,i}^2 \cos^2(2\Delta\theta_i)}$$

The systematics contribution is

$$f_{45}\Delta\Sigma_{\rm iso}(r) = \frac{\sum_i w_i \Delta\Sigma_{i,45} e_{g,i}\cos(2\Delta\theta_i + \pi/2)}{2\sum_i w_i e_{g,i}^2 \cos^2(2\Delta\theta_i + \pi/2)}$$

Shapes of DM halos

It is useful to assess the signal-to-noise we expect to obtain for the shear anisotropy measurement compared to the signal-to-noise of the tangential shear itself. For this purpose, we write Equation (6) in its most basic form:

$$\Delta\Sigma_{\rm model}(r) = \Delta\Sigma_{\rm iso}(r)[1 + \bar{f}\cos(2\Delta\theta)], \qquad (11)$$

which has the following solution for the anisotropic part:

$$\bar{f}\Delta\Sigma_{\rm iso} = \frac{\sum_i w_i \Delta\Sigma_i \cos(2\Delta\theta_i)}{\sum_i w_i \cos^2(2\Delta\theta_i)}. \qquad (12)$$

Shapes of DM halos

If the halo is perfectly aligned with the lens then $f = e_h/2$ and the anisotropic signal is lower by this factor compared to the isotropic signal.

Evaluating the other contributions leads to:

$$(\mathrm{S/N})_{\mathrm{ani}} = \frac{0.15}{\sqrt{2}} \left(\frac{e_h}{0.3} \right) (\mathrm{S/N})_{\mathrm{iso}} .$$

This is a difficult measurement!

Shapes of DM halos

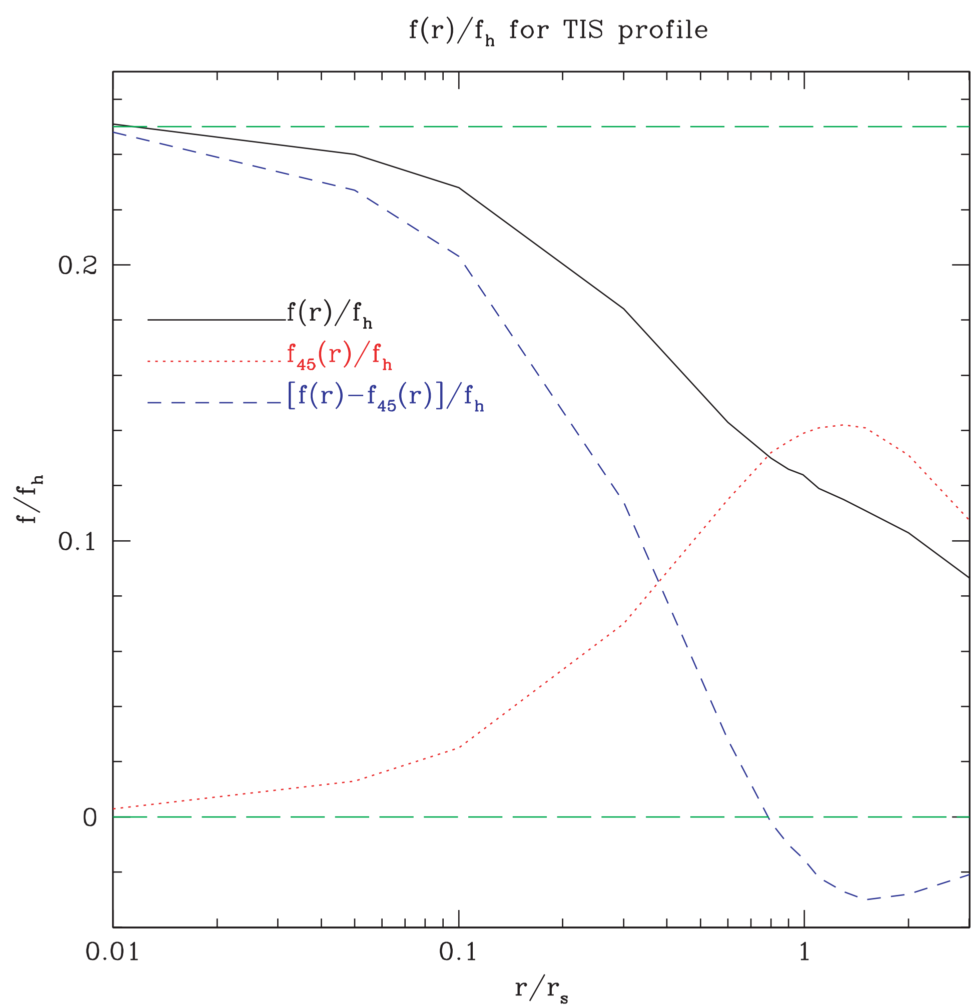


Figure 1. Plot of predicted $f(r)/f_{\rm h}$ and $f_{45}(r)/f_{\rm h}$ for an elliptical TIS density profile dark matter halo; the maximum radius shown, $3r_{\rm s}$, is larger than the scales used in this paper. Horizontal lines indicate the SIS predictions $f/f_{\rm h} = 0.25$ and $f_{45}/f_{\rm h} = 0$.

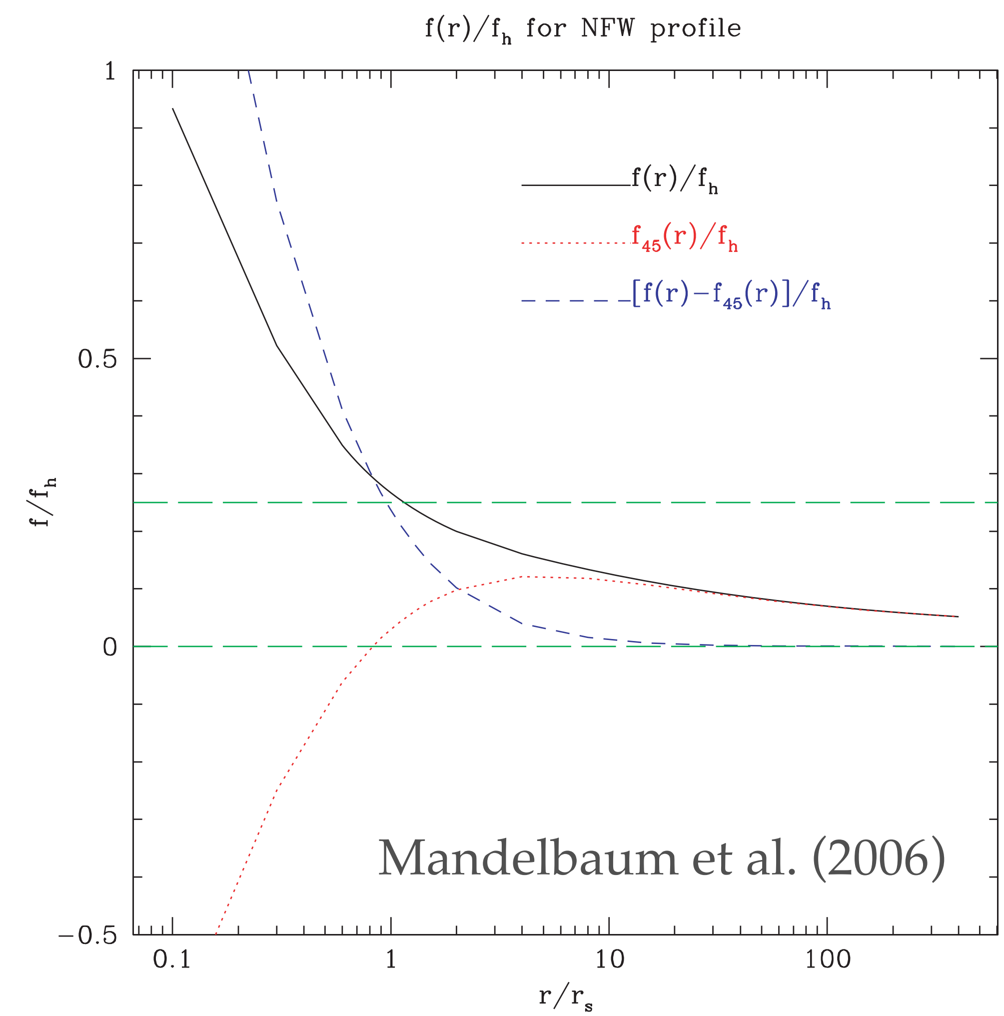
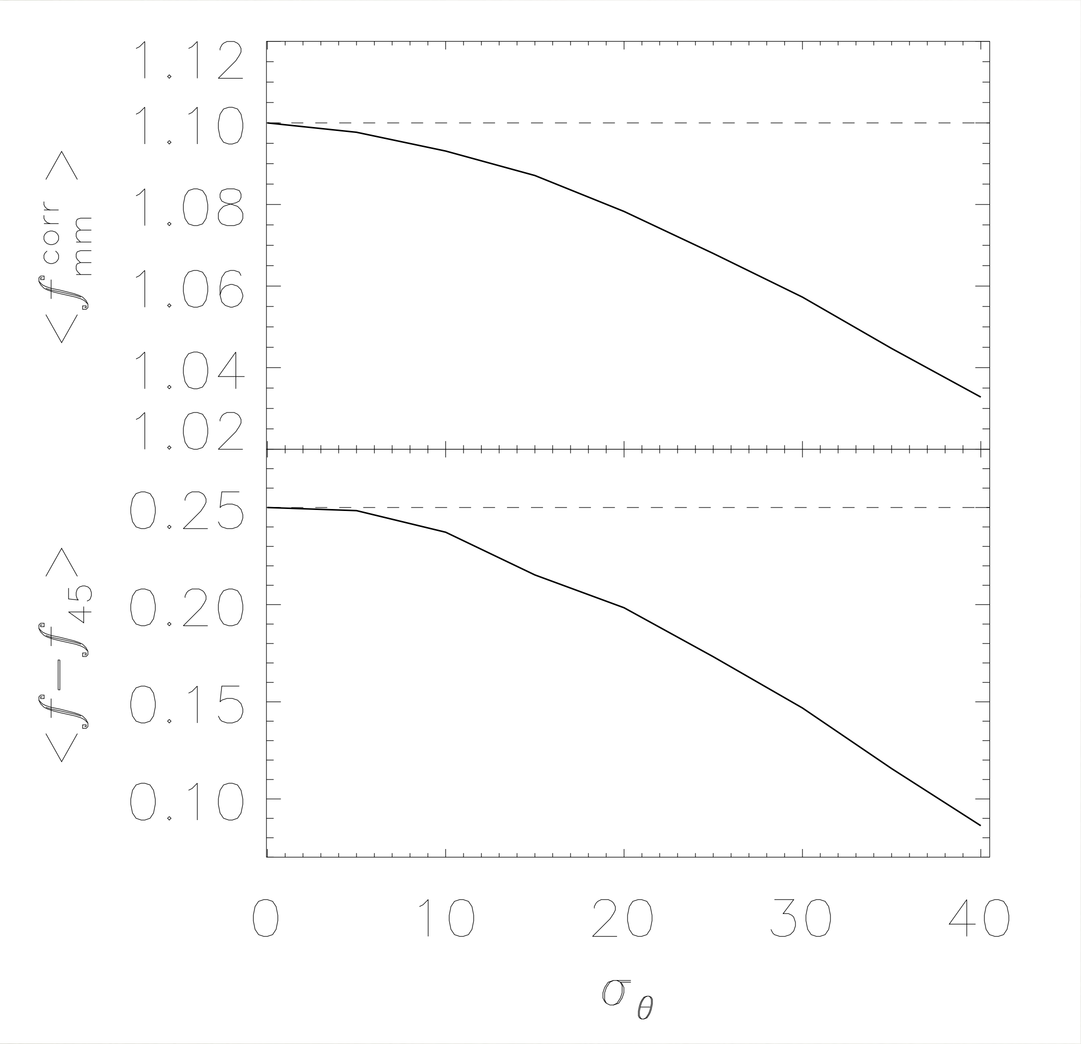


Figure 2. Plot of predicted $f(r)/f_{\rm h}$ and $f_{45}(r)/f_{\rm h}$ for an elliptical NFW lensity profile dark matter halo. The horizontal lines indicate the SIS prelictions $f/f_{\rm h} = 0.25$ and $f_{45}/f_{\rm h} = 0$.

Ellipticity signal only appreciable on small scales. This is where flexion measurements can help.

Shapes of DM halos



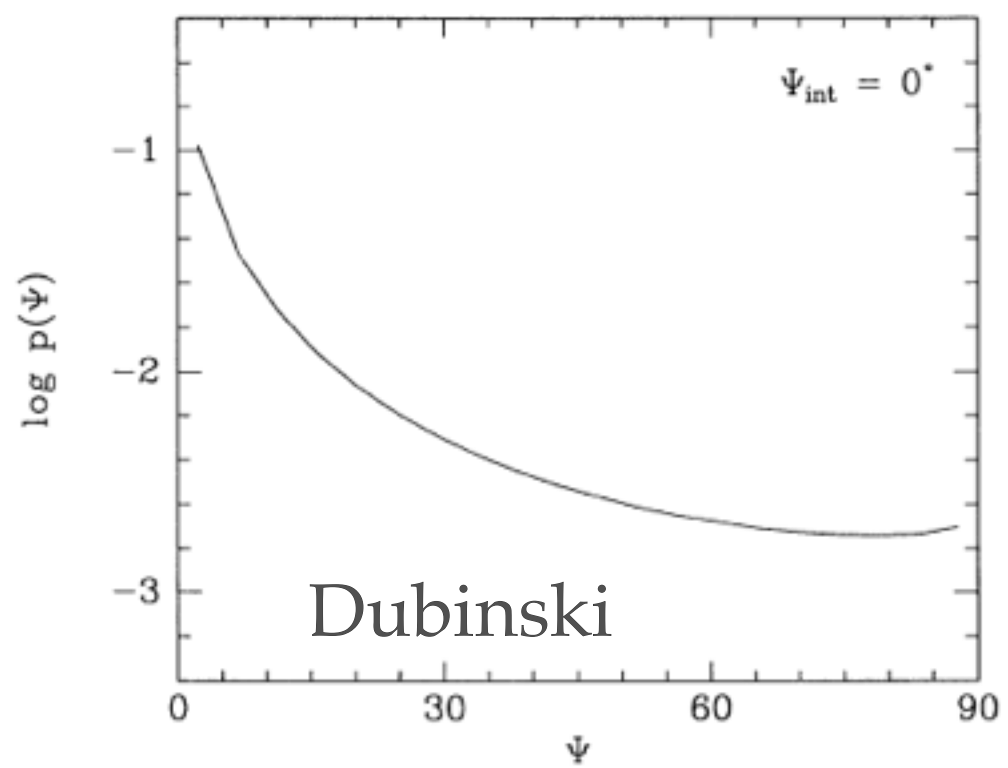
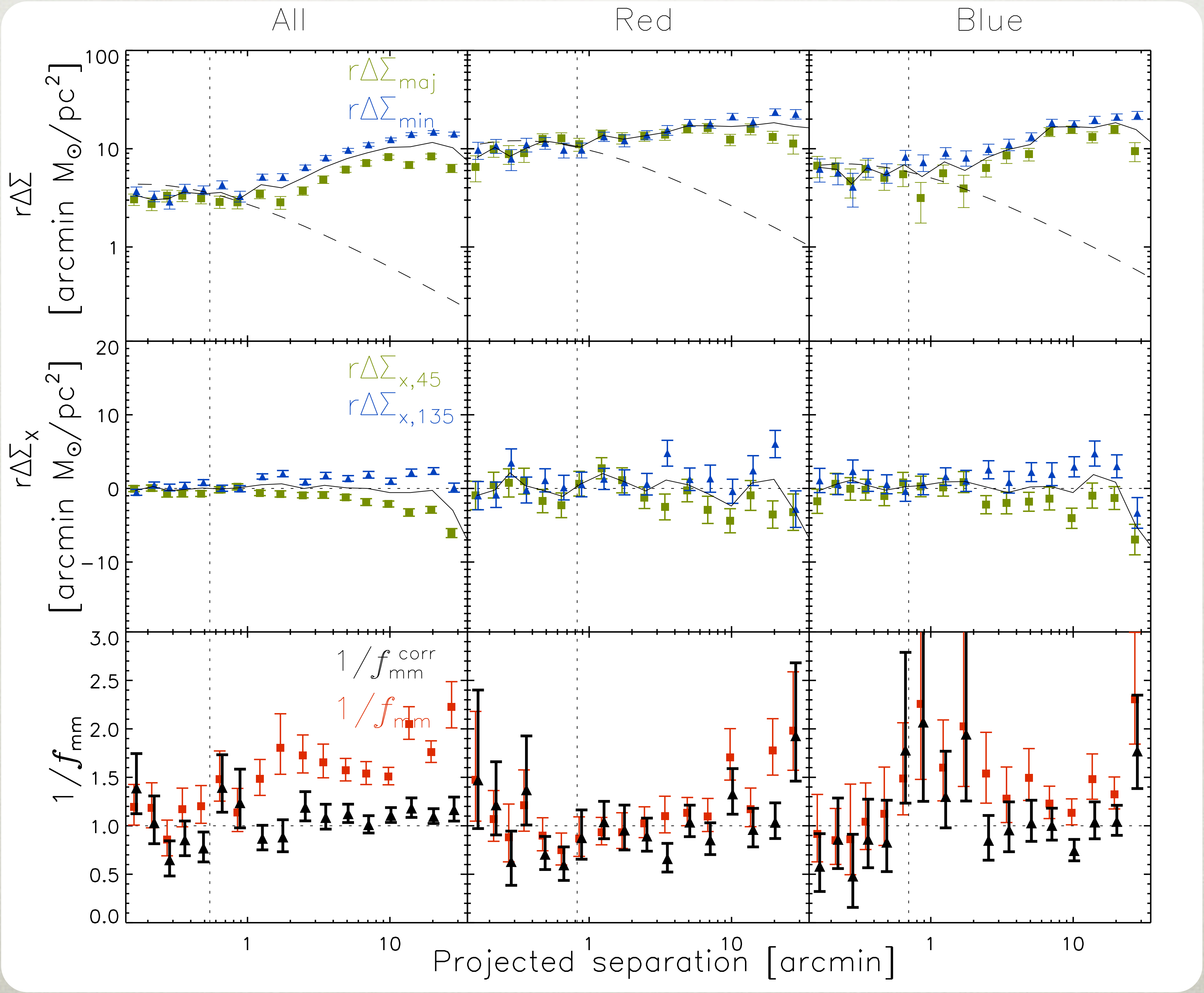


FIG. 7.—The distribution of kinematic misalignment angle, Ψ for the intrinsic distribution of shapes with $\mu_{\epsilon 1} = 0.64$ and $\sigma_{\epsilon 1} = 0.13$ (Franx et al. [1991] sample). A total of 17% of galaxies are misaligned by more than 30° despite an intrinsic alignment about the minor axis.

The signal is lowered even further if the halo is misaligned with the light distribution.

Shapes of DM halos



van Uitert et al. (2012)

Shapes of DM halos

van Uitert et al. (2012)

Sample	α	$\langle f_{\rm eff} \rangle$	$\langle f - f_{45} \rangle$	$f_{\rm h}$(SIE)	$f_{\rm h}$(NFW)
All	0.0	$1.3 \pm 0.6 \times 10^{-3}$	0.19 ± 0.10	0.47 ± 0.37	$0.96^{+0.83}_{-0.80}$
All	0.5	$1.1 \pm 0.7 \times 10^{-3}$	$0.21^{+0.11}_{-0.10}$	0.57 ± 0.40	$1.19^{+0.89}_{-0.85}$
All	1.0	$0.8 \pm 0.8 \times 10^{-3}$	0.23 ± 0.12	0.70 ± 0.46	$1.50^{+1.03}_{-1.01}$
All	1.5	$0.6 \pm 1.0 \times 10^{-3}$	0.26 ± 0.15	0.83 ± 0.55	$1.80^{+1.23}_{-1.19}$
All	2.0	$0.4 \pm 1.2 \times 10^{-3}$	0.29 ± 0.17	0.97 ± 0.65	$2.12^{+1.45}_{-1.42}$
Red	0.0	$11.9 \pm 1.8 \times 10^{-3}$	0.13 ± 0.15	0.00 ± 0.58	$-0.19^{+1.09}_{-1.08}$
Red	0.5	$11.3 \pm 2.1 \times 10^{-3}$	0.19 ± 0.16	0.05 ± 0.60	$-0.14^{+1.12}_{-1.10}$
Red	1.0	$9.3 \pm 2.5 \times 10^{-3}$	0.28 ± 0.18	0.25 ± 0.70	$0.20^{+1.34}_{-1.31}$
Red	1.5	$7.2 \pm 3.1 \times 10^{-3}$	0.40 ± 0.22	0.61 ± 0.86	$0.87^{+1.67}_{-1.63}$
Red	2.0	$5.2 \pm 4.0 \times 10^{-3}$	0.54 ± 0.27	1.09 ± 1.07	$1.82^{+2.12}_{-2.08}$
Blue	0.0	$1.5 \pm 1.4 \times 10^{-3}$	$-0.16^{+0.18}_{-0.19}$	-0.56 ± 0.68	$-1.24^{+1.62}_{-1.65}$
Blue	0.5	$2.0 \pm 1.6 \times 10^{-3}$	-0.25 ± 0.19	-0.75 ± 0.70	$-1.62^{+1.69}_{-1.72}$
Blue	1.0	$2.3 \pm 1.9 \times 10^{-3}$	$-0.35^{+0.21}_{-0.22}$	-1.01 ± 0.81	$-2.17^{+1.97}_{-2.03}$
Blue	1.5	$2.5 \pm 2.3 \times 10^{-3}$	-0.45 ± 0.26	-1.24 ± 0.96	$-2.67^{+2.36}_{-2.44}$
Blue	2.0	$2.5 \pm 2.7 \times 10^{-3}$	$-0.53^{+0.31}_{-0.32}$	-1.44 ± 1.17	$-3.06^{+2.85}_{-2.95}$

Much more data are needed!

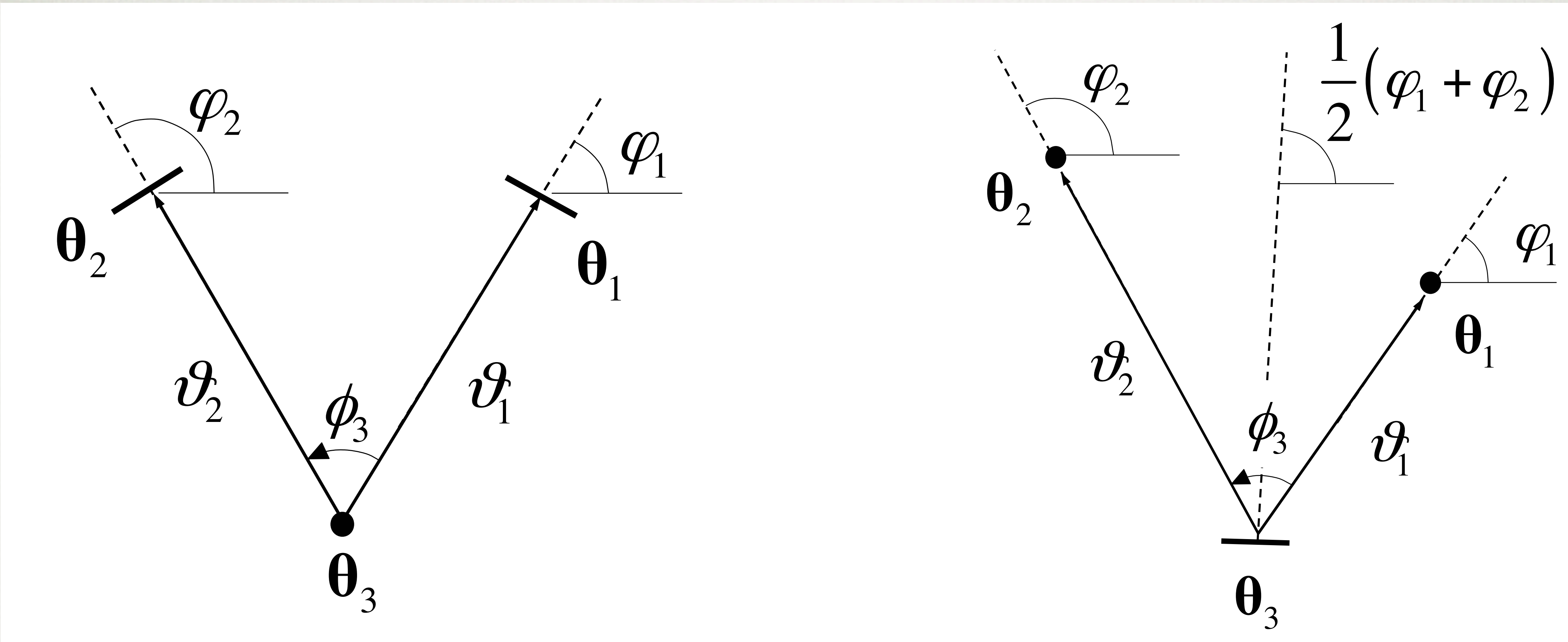
Higher-order statistics

The galaxy-galaxy lensing provides information about the mean density profile around an ensemble of galaxies.

To study environmental differences, we can pre-select lenses, or we can examine these statistically using 3-point statistics:

- lensing signal around a pair of lenses of a given separation
- correlation of lensing signal around a lens (more like GGL)

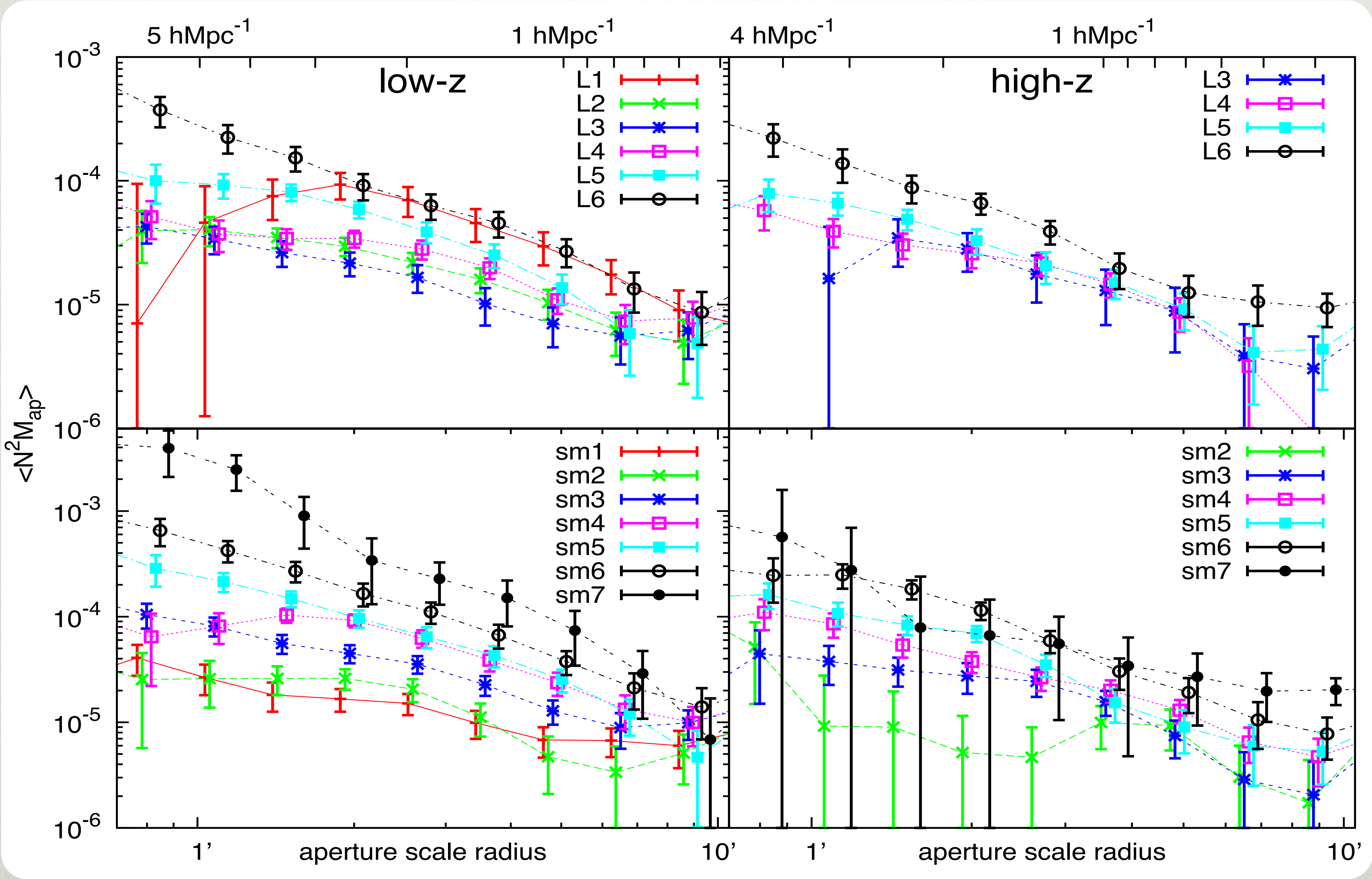
Higher-order statistics



The 3-point signal does not vanish if

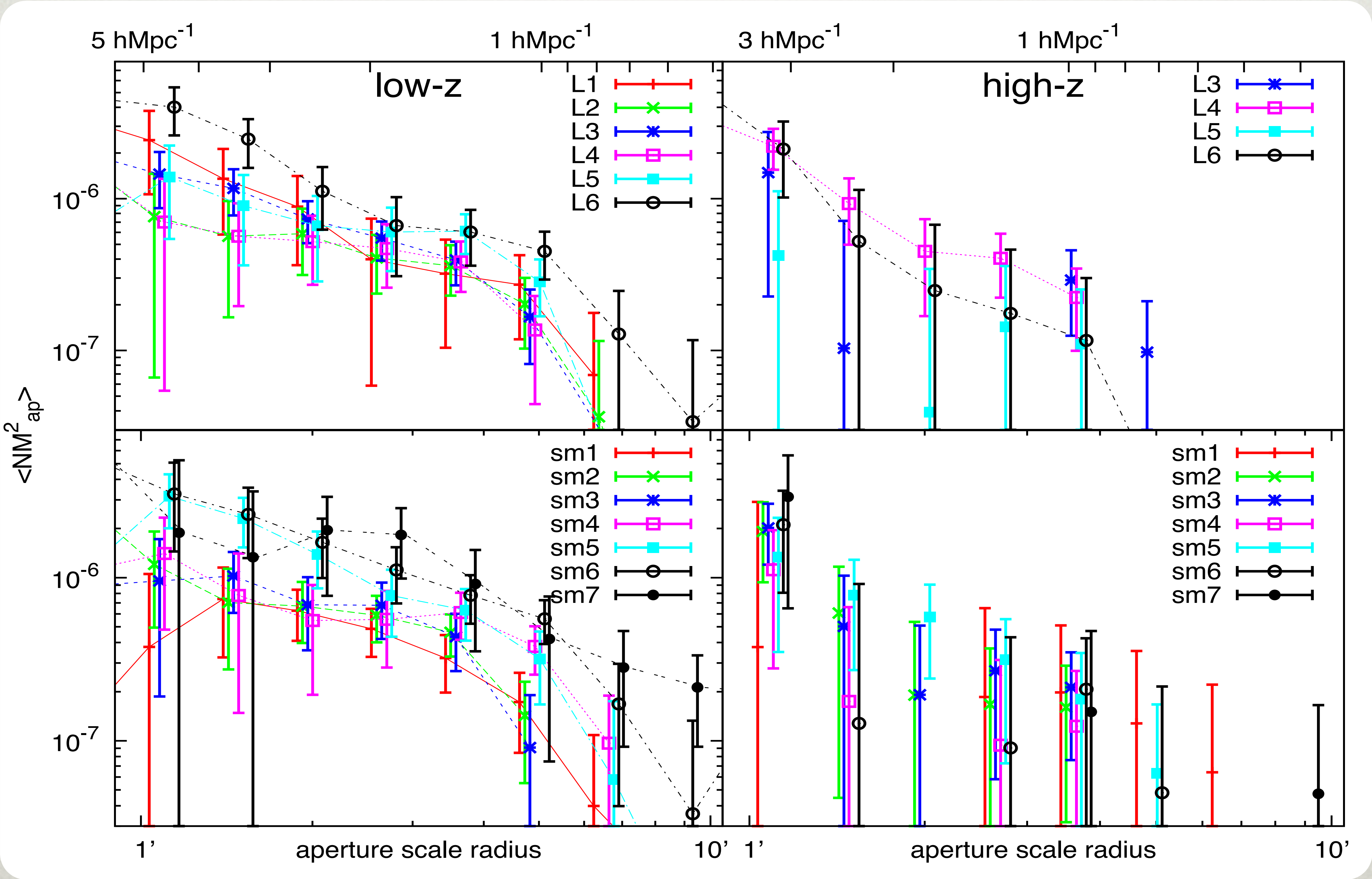
- the surface density around a pair of galaxies differs from that around individual galaxies *(sensitive to environment)*.
- the matter 2-point correlations close to lenses differs from correlations independent of lens positions *(sensitive to halo properties)*.

Higher-order statistics



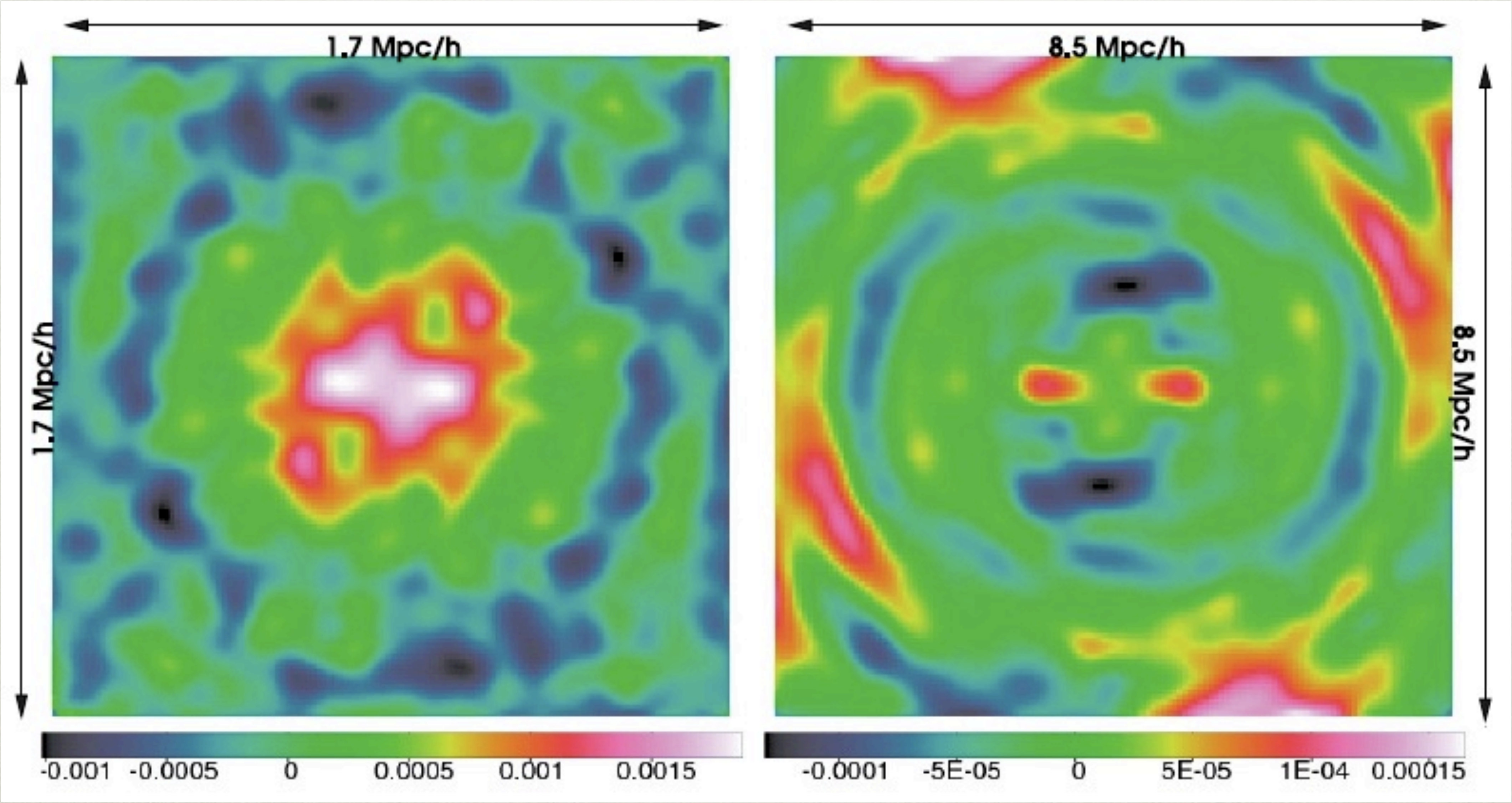
CFHTLenS measurements by Simon et al. (in prep.)

Higher-order statistics



CFHTLenS measurements by Simon et al. (in prep.)

Higher-order statistics



Excess convergence around lenses with different separations from Simon et al. (2008)

Do not waste data

The study of isolated galaxies or limiting the analysis to small scales simplifies the interpretation, but limits the analysis to low density. The sample is not very representative. Can we do better?

Redshift information is "expensive", so why waste it?

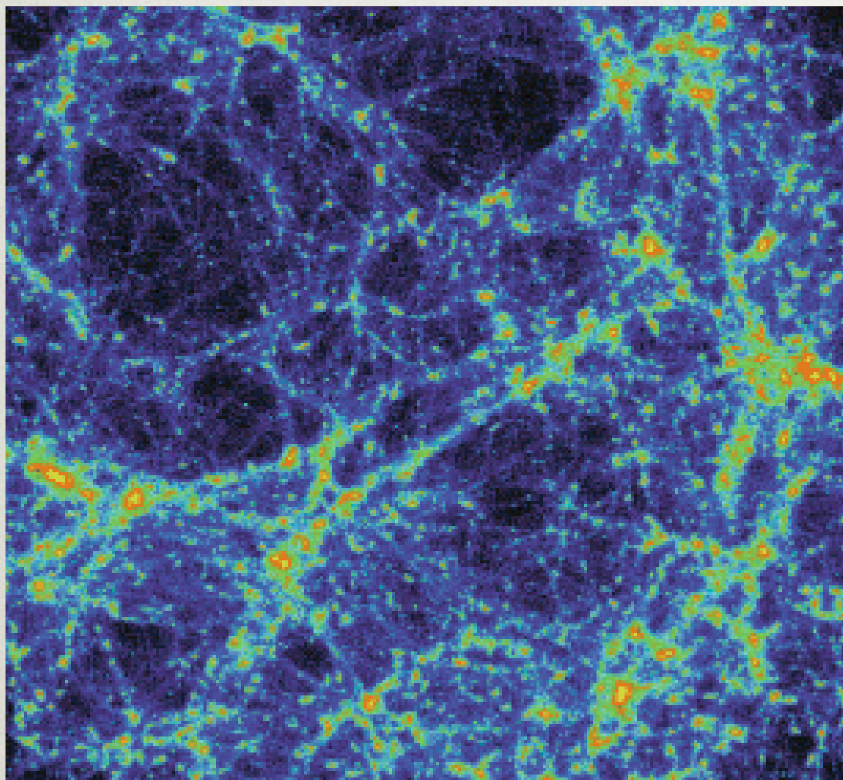
The halo model

Ingredients of the model

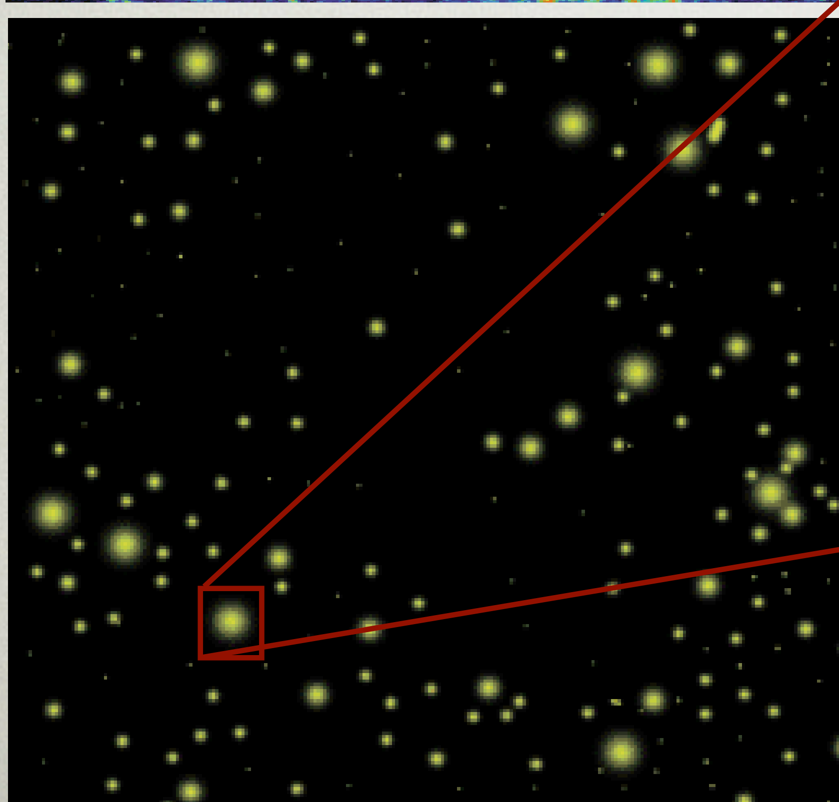
- galaxies are host or satellite
- density profiles for hosts & satellites
- prescription of the clustering of halos
- prescription of the occupation of halos
- every dark matter particle resides in a halo

The Halo-Model

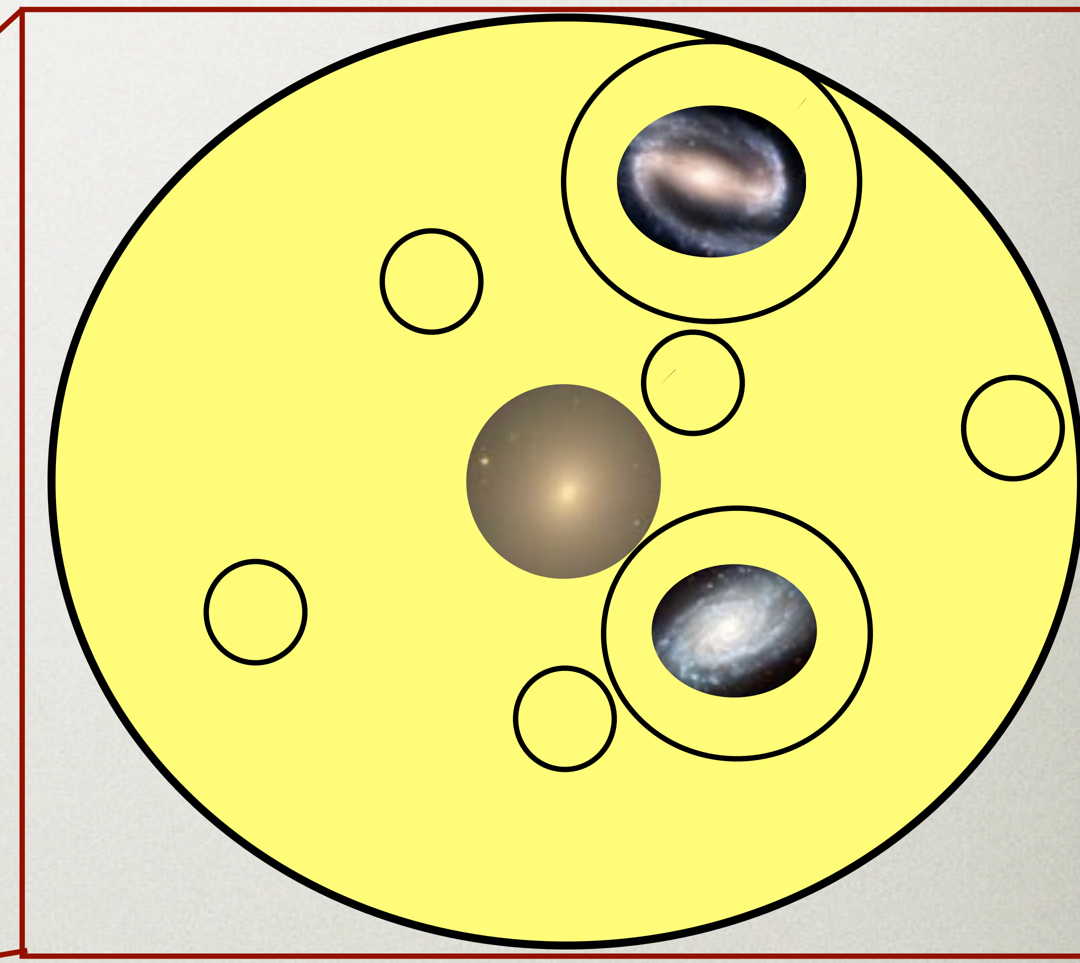
Numerical Simulation view



Halo Model view



$$M = \frac{4\pi}{3}(180\bar{\rho})r^3$$



central and satellite galaxies

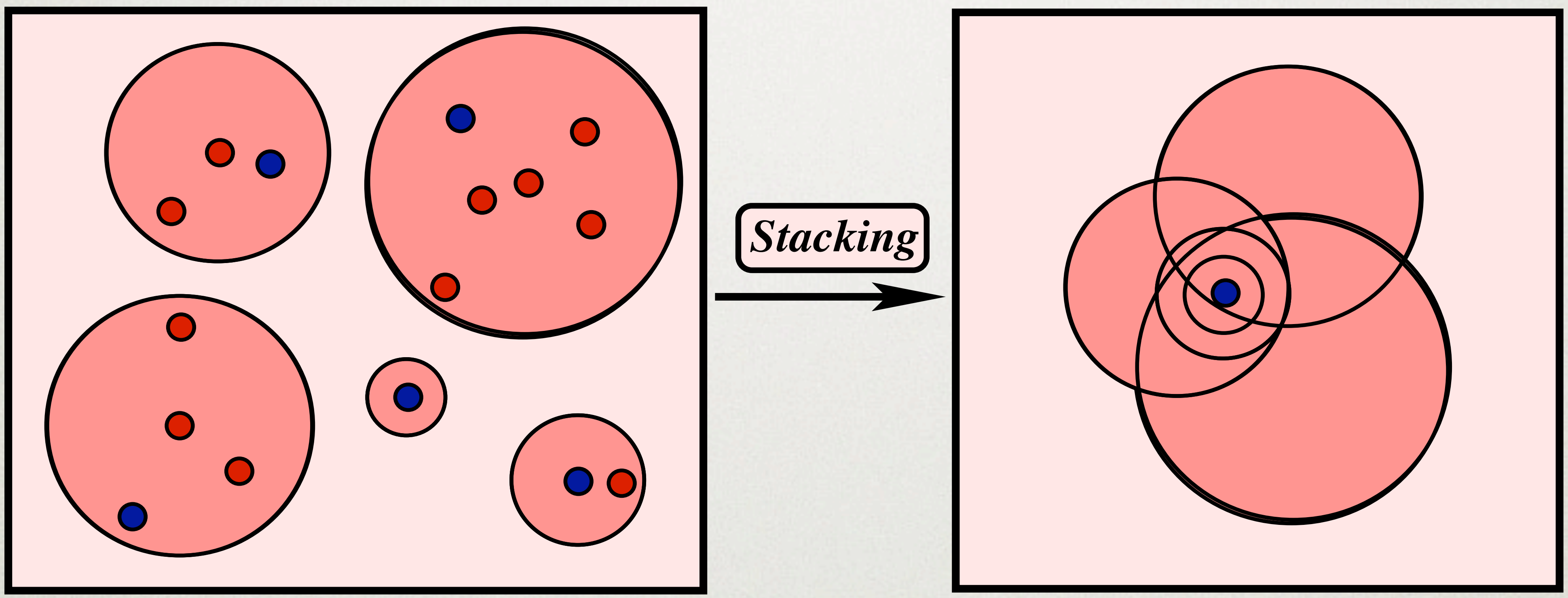
The Halo-Model

The halo model shares some features with the maximum-likelihood approach discussed earlier, but there are differences:

It is statistical in nature:
- *it predicts the radial dependence of the signal*
- *it does not make use of the observed positions of lenses*
- *it naturally can account for central and satellite galaxies.*

A closer look at stacking

Stacking according to an observed galaxy property

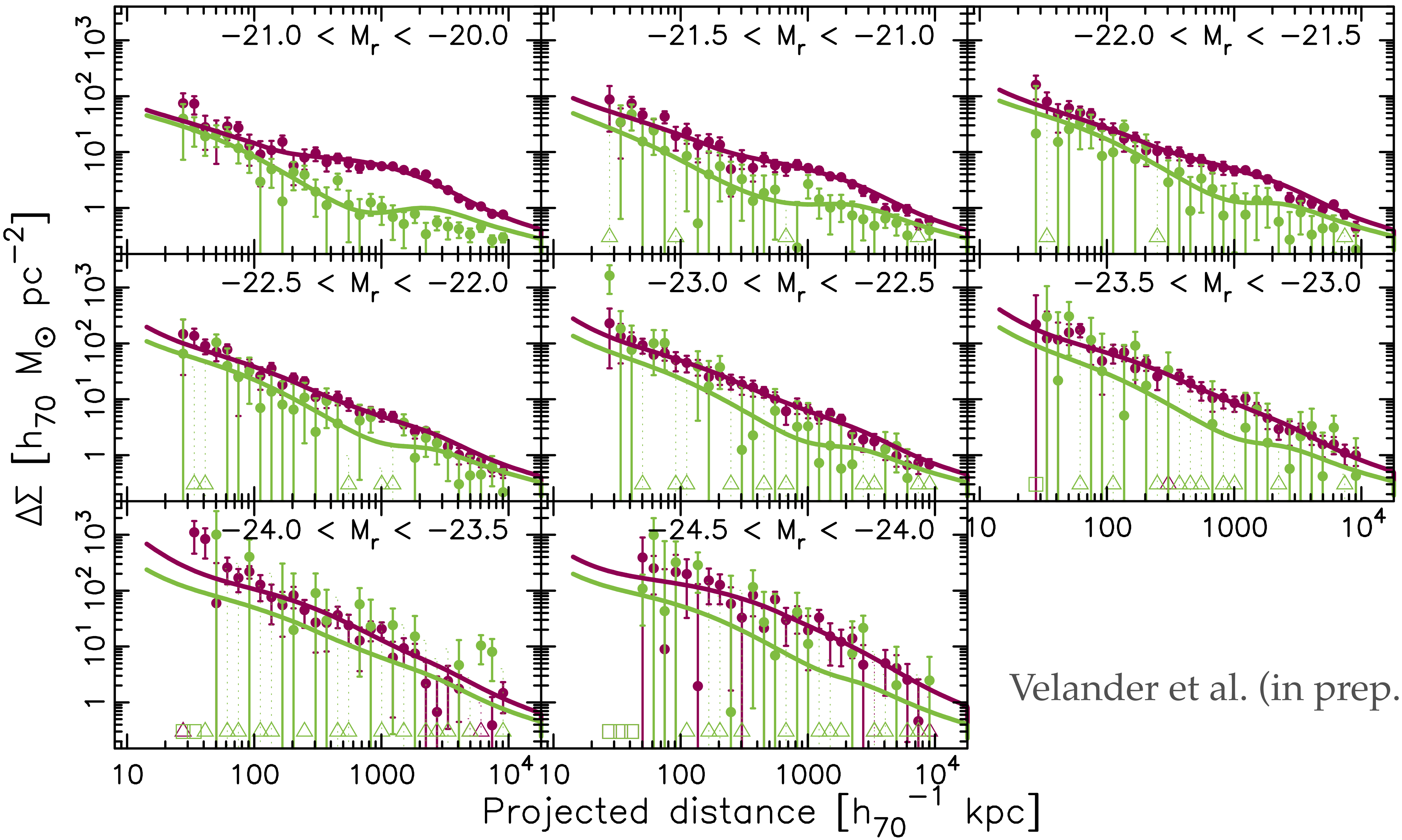


haloes of different masses
central and satellite galaxies

} Mixed together

This complicates the interpretation

The Halo-Model



Velander et al. (in prep.)

Modeling the stacking

$$\Delta\Sigma(R|L) = \int \mathcal{P}^{\rm c}(M|L)\Delta\Sigma^{\rm c}(R|M)\mathrm{d}M$$ *central*

$$+ \int \mathcal{P}^{\rm s}(M|L)\Delta\Sigma^{\rm s}(R|M)\mathrm{d}M$$ *satellite*

$$\Delta\Sigma^{\rm c}(R|M) \longleftrightarrow \rho_{\rm dm}(r|M)$$

Dark matter halo density profile

$$\Delta\Sigma^{\rm s}(R|M) \longleftrightarrow \rho_{\rm dm}(r|M) \otimes n_{\rm s}(r|M)$$

Convolution of the halo density profile and the number density distribution of galaxies

+ contributions from the clustering of lenses (2-halo term)

Modeling the stacking

Bayes' theorem:

$$\mathcal{P}^{\rm c}(M|L)\mathrm{d}M = \frac{\Phi^{\rm c}(L|M)n(M)}{\phi^{\rm c}(L)}\mathrm{d}M$$

$$\mathcal{P}^{\rm s}(M|L)\mathrm{d}M = \frac{\Phi^{\rm s}(L|M)n(M)}{\phi^{\rm s}(L)}\mathrm{d}M$$

where

$$\phi^{\rm c}(L) = \int \Phi^{\rm c}(L|M)n(M)\mathrm{d}M$$

$$\phi^{\rm s}(L) = \int \Phi^{\rm s}(L|M)n(M)\mathrm{d}M$$

with *n(M)* the halo mass function

$\Phi^{\rm c}(L|M)$

$\Phi^{\rm s}(L|M)$

Central and satellite term of the
Conditional Luminosity Function

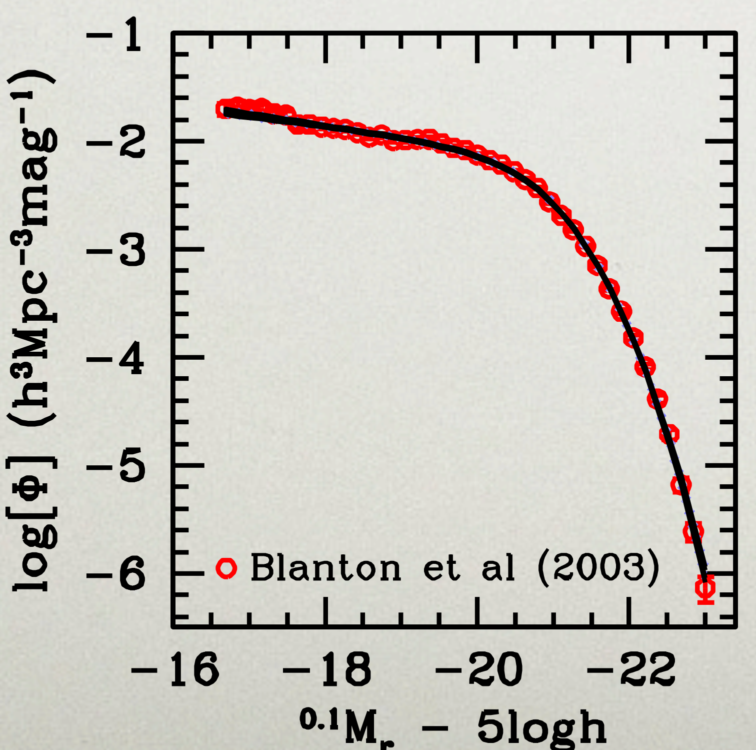
Conditional LF

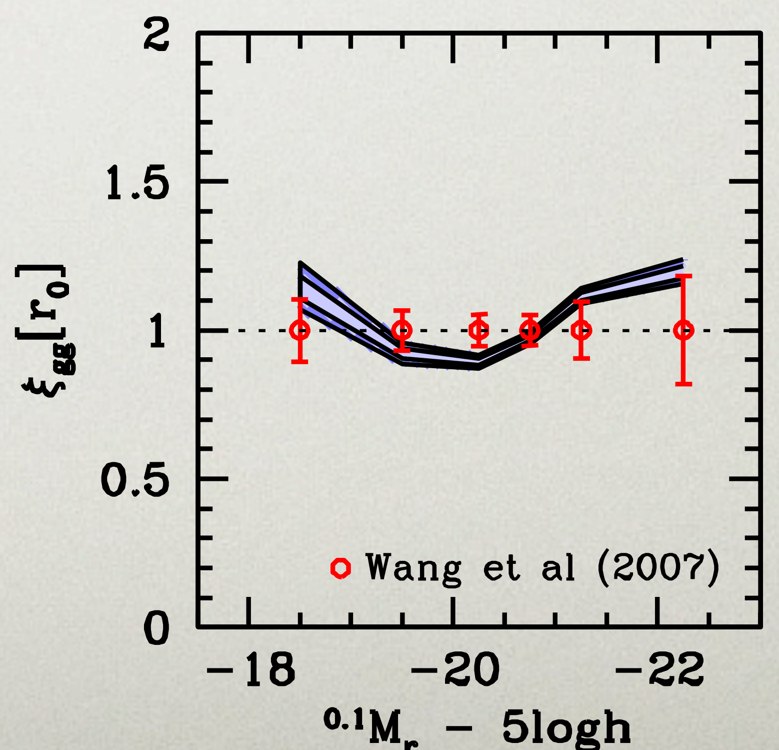
Number of galaxies with luminosity *L* living in a halo of mass *M*

$$\Phi(L|M) = \Phi_{\rm c}(L|M) + \Phi_{\rm s}(L|M)$$

Assume a functional form with parameters constrained by

Luminosity Function





Correlation Length

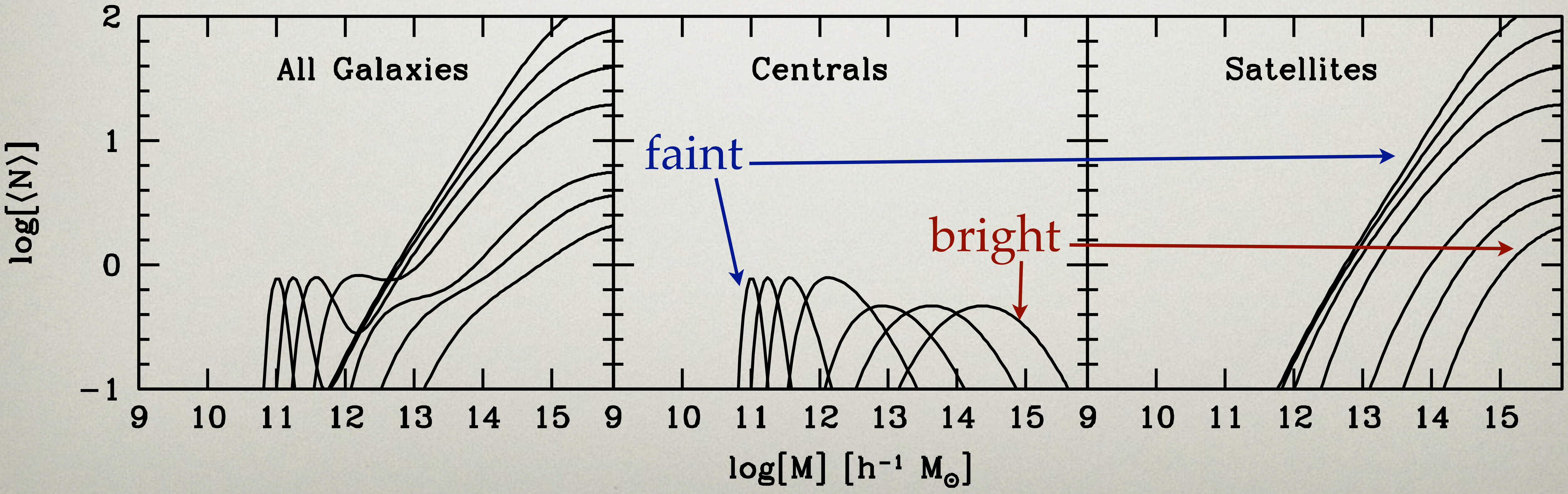
Halo Occupation Distribution

$$\mathcal{P}_{\rm c}(M|L_1, L_2)\,{\rm d}M = \frac{\langle N_{\rm c}\rangle_M(L_1,L_2)}{\overline{n}_{\rm c}(L_1,L_2)}\,n(M){\rm d}M$$

$$\langle N_{\rm c}\rangle_M(L_1, L_2) = \int_{L_1}^{L_2} \Phi_{\rm c}(L|M)\,{\rm d}L$$

$$\mathcal{P}_{\rm s}(M|L_1, L_2)\,{\rm d}M = \frac{\langle N_{\rm s}\rangle_M(L_1,L_2)}{\overline{n}_{\rm s}(L_1,L_2)}\,n(M){\rm d}M$$

$$\langle N_{\rm s}\rangle_M(L_1, L_2) = \int_{L_1}^{L_2} \Phi_{\rm s}(L|M)\,{\rm d}L$$



Modeling the stacking

$$\gamma_t(\theta) = 6\pi^2\left(\frac{H_0}{c}\right)^2\Omega_M\int_0^\infty \mathrm{d}\chi W_1(\chi)\frac{f(\chi)}{a(\chi)} \times \int \mathrm{d}k k P(k,\chi,\theta) J_2(kr(\chi)\theta), \quad (8)$$

with χ the radial distance (in a flat universe, $\chi = a^{-1}\ D_A$ with a the scale factor and D_A the angular diameter distance), $W_1(\chi)$ the normalized radial distribution of the lenses, $f(\chi) = \int_\chi^\infty \mathrm{d}\chi' g(\chi,\chi')W_2(\chi')$, with $W_2(\chi')$ the radial distribution of the sources, and

$$g(\chi,\chi') = \frac{D_l D_{ls}}{D_s a(z_L)}. \quad (9)$$

$P(k)$ is the power spectrum under consideration, and J_2 is the second Bessel function of the first kind. Instead of

Modeling the stacking

$$\gamma_{t,\mathrm{cent}} = \gamma_{t,\mathrm{cent}}^{1h} + \gamma_{t,\mathrm{cent}}^{2h}$$

The calculation of $\gamma_{t,\mathrm{cent}}^{2h}$ requires the power spectrum describing the correlation between the galaxy in the central halo and the dark matter of nearby haloes:

$$P_{\mathrm{cent}}^{2h}(k, M_h, r) = b_g(M_h, r)\frac{P_{\mathrm{NL}}(k)}{(2\pi)^3} \times \int_0^{M_{\mathrm{lim}}} \mathrm{d}\nu f(\nu) b(\nu, r) y_{\mathrm{dm}}(k, M), \tag{16}$$

with $b_g(M_h, r)$ the bias of the central galaxy, $P_{\mathrm{NL}}(k)$ the non-linear power spectrum from Smith et al. (2003), and $y_{\mathrm{dm}}(k, M)$ the radial Fourier transform of the central halo density profile divided by mass:

$$y_{\mathrm{dm}}(k, M) = \frac{1}{M}\int_0^{r_{200}} \mathrm{d}r 4\pi r^2 \rho_{\mathrm{dm}}(r, M)\frac{\sin(kr)}{kr}, \tag{17}$$

Modeling the stacking

$$\gamma_{t,\mathrm{sat}} = \gamma_{t,\mathrm{sat}}^{\mathrm{trunc}} + \gamma_{t,\mathrm{sat}}^{1h} + \gamma_{t,\mathrm{sat}}^{2h}$$

satellites are tidally stripped

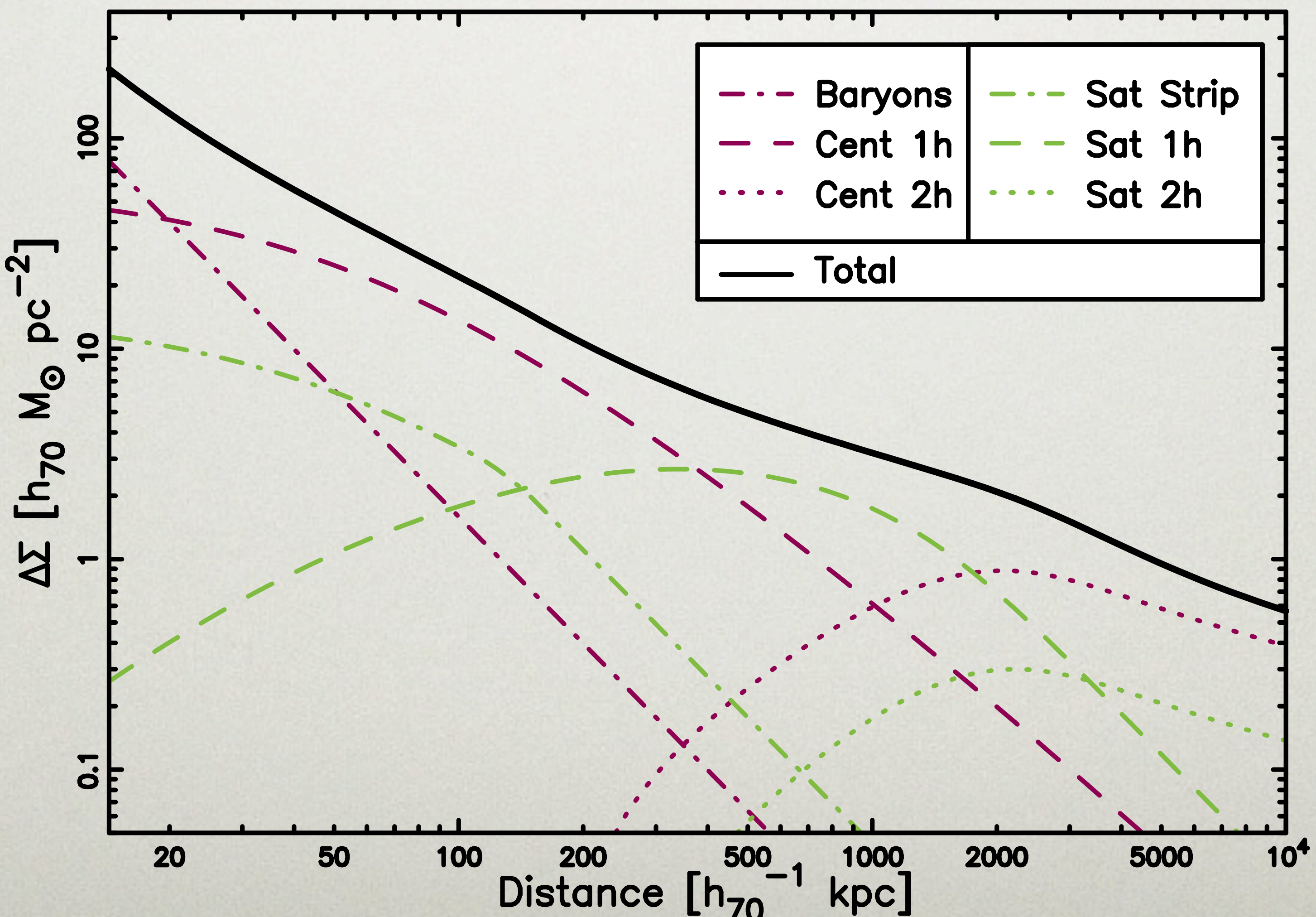
$$P_{\mathrm{sat}}^{1h}(k, M_h) = \frac{1}{(2\pi)^3 \bar{n}} \int \mathrm{d}\nu f(\nu) N_s(M, M_h) \times y_{\mathrm{dm}}(k, M) y_{\mathrm{g}}(k, M),$$

$$P_{\mathrm{sat}}^{2h}(k, M_h, r) = \frac{P_{\mathrm{NL}}(k)}{(2\pi)^3} \int_0^{M_{\mathrm{lim}}} \mathrm{d}\nu f(\nu) b(\nu, r) y_{\mathrm{dm}}(k, M) \times \frac{\bar{\rho}}{\bar{n}} \int \mathrm{d}\nu f(\nu) b(\nu, r) \frac{N_s(M, M_h)}{M} y_{\mathrm{g}}(k, M).$$

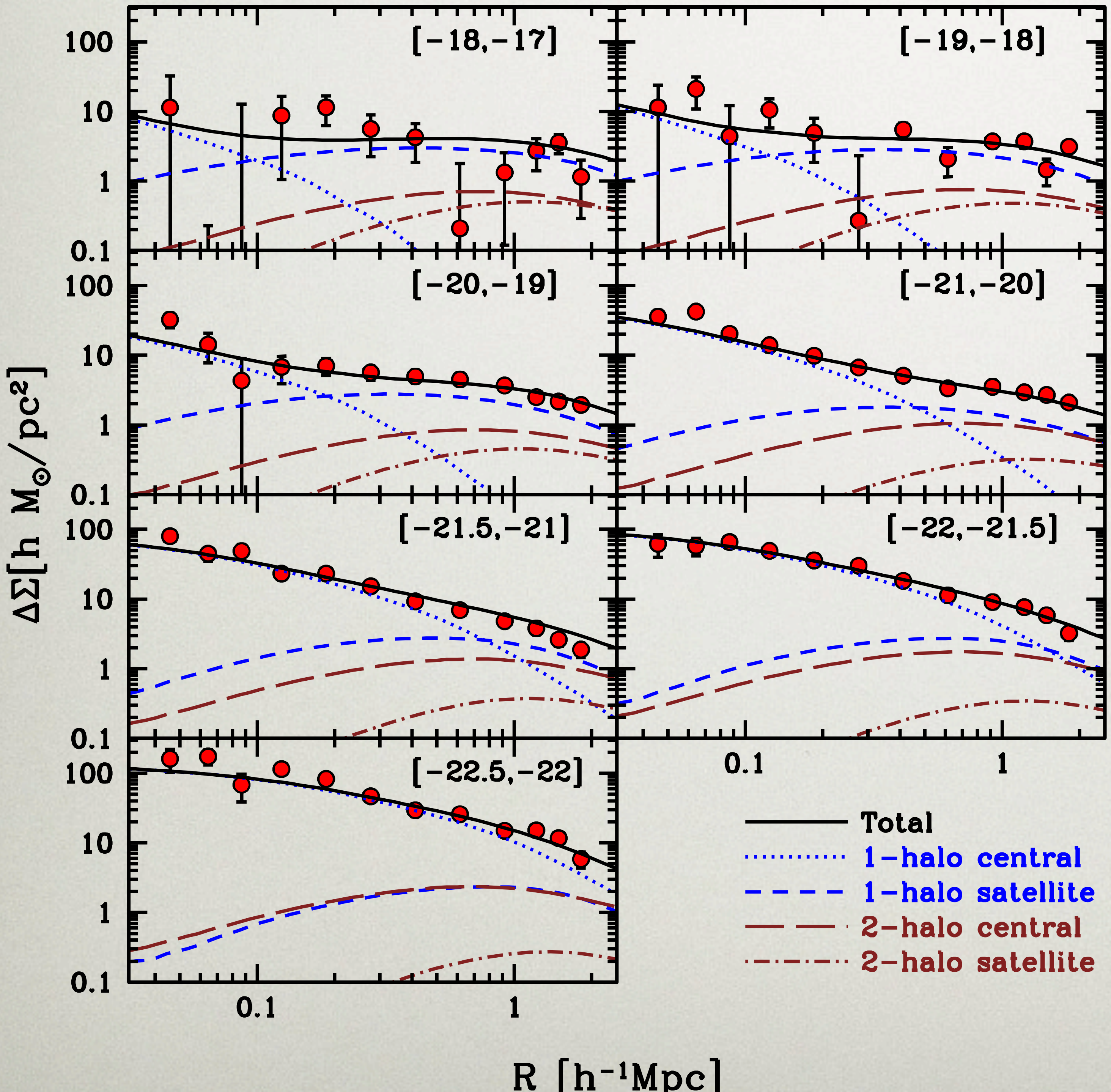
Modeling the stacking

$$\gamma_t = (1-\alpha)\,\gamma_{t,\mathrm{cent}} + \alpha\,\gamma_{t,\mathrm{sat}}, \qquad (21)$$

where α is the fraction of satellites of the sample. The resulting model is compared to the data.



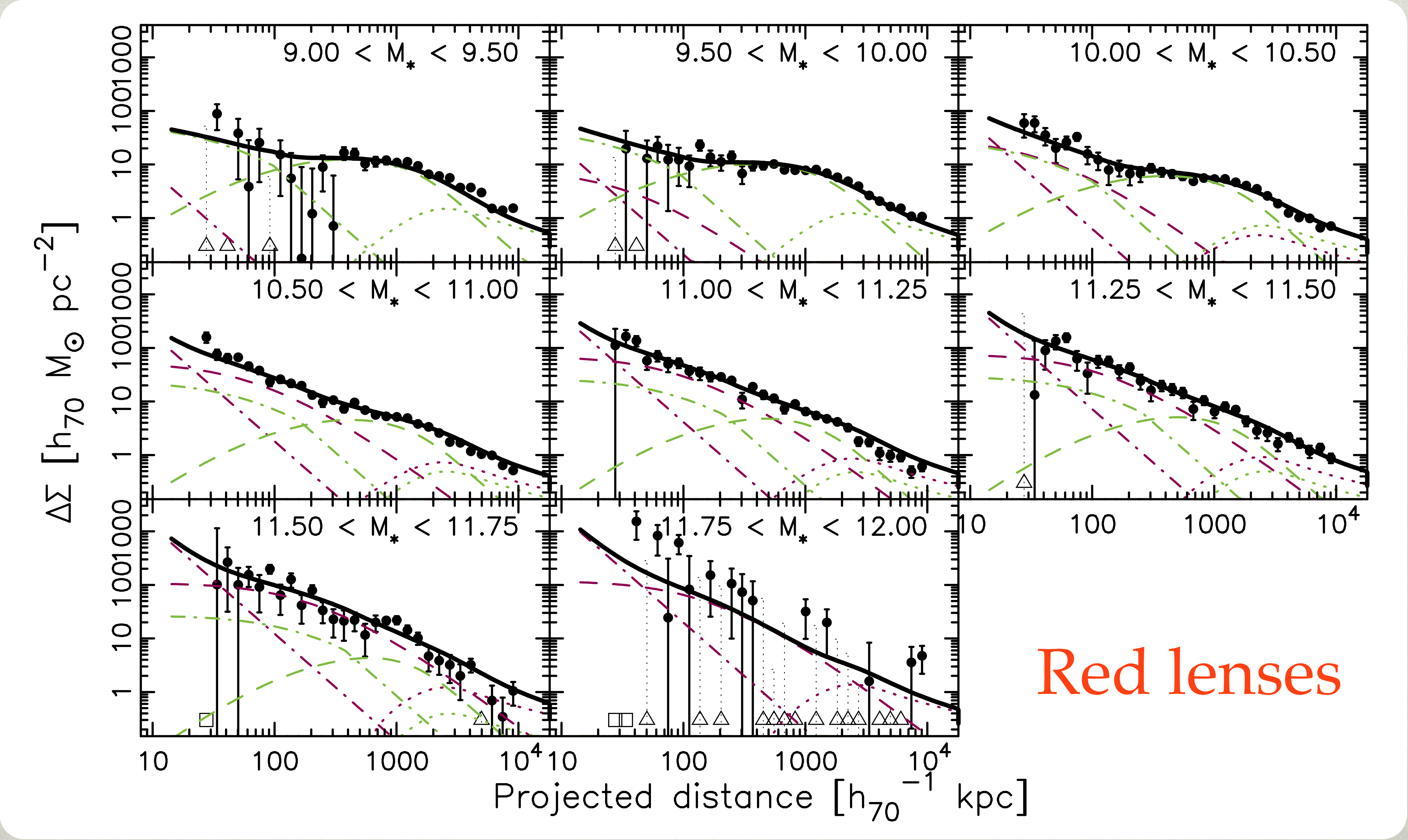
Predicting the signal



No fit!

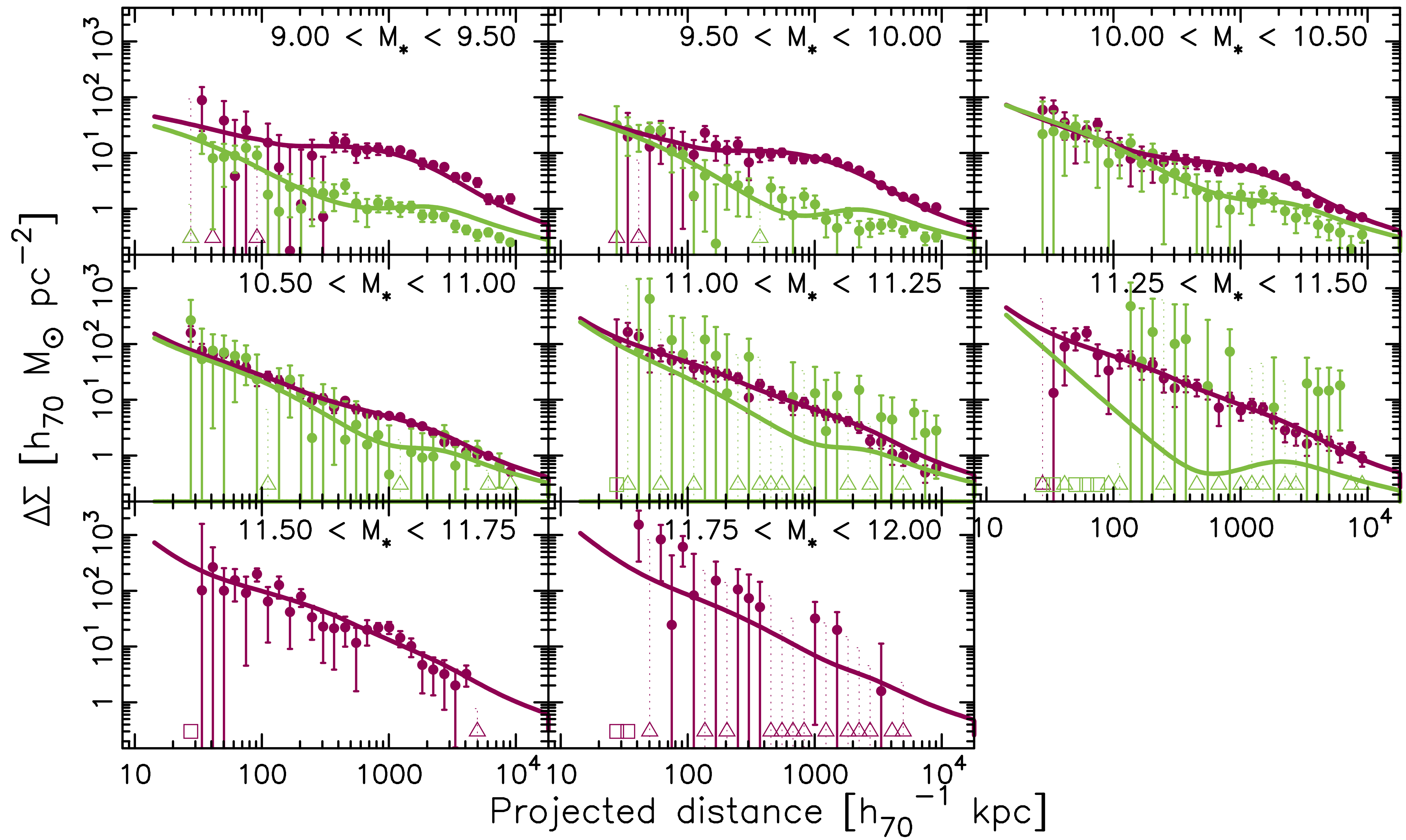
Signal completely predicted by the conditional luminosity function (Cacciato et al.)

CFHTLenS results



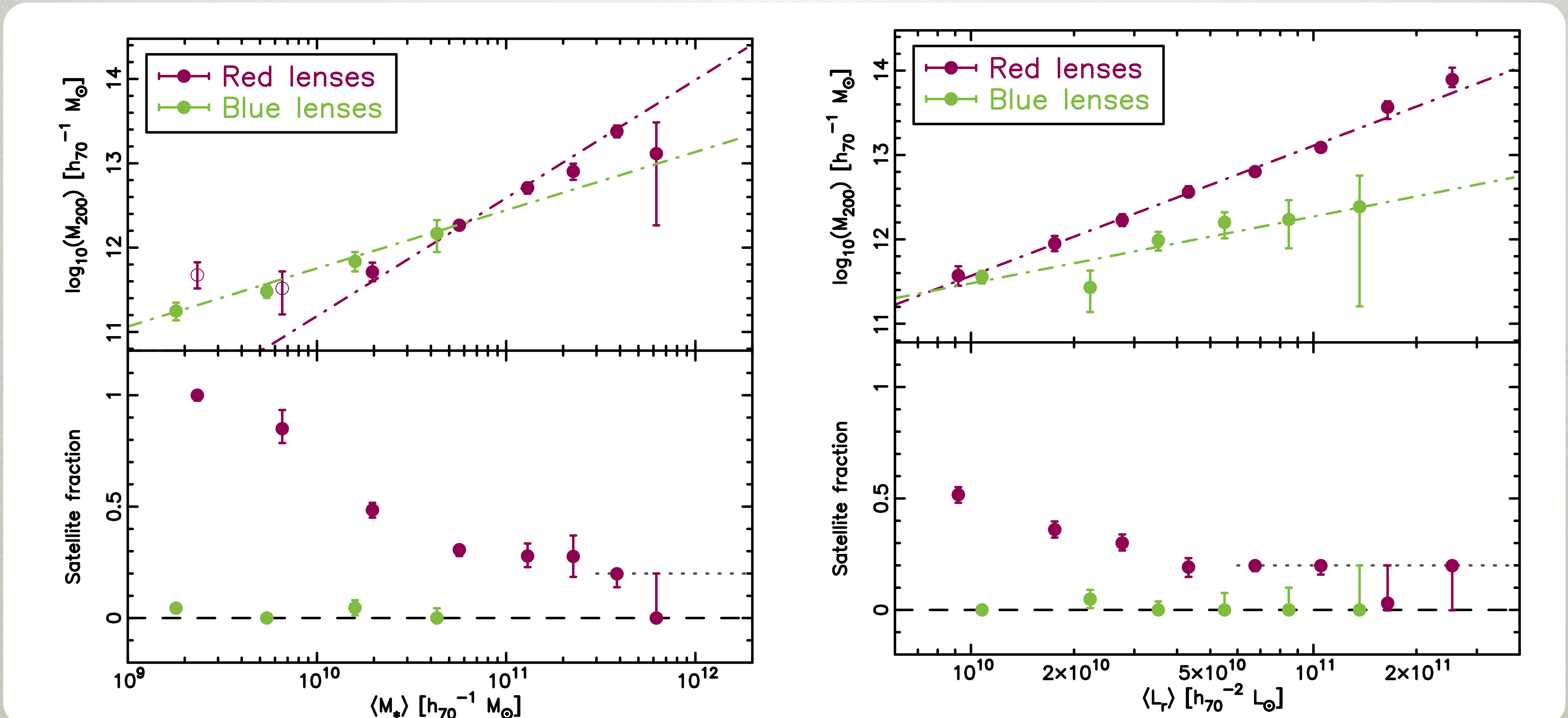
Velander et al. (in prep.)

CFHTLenS results



red & blue galaxies: Velander et al. (in prep.)

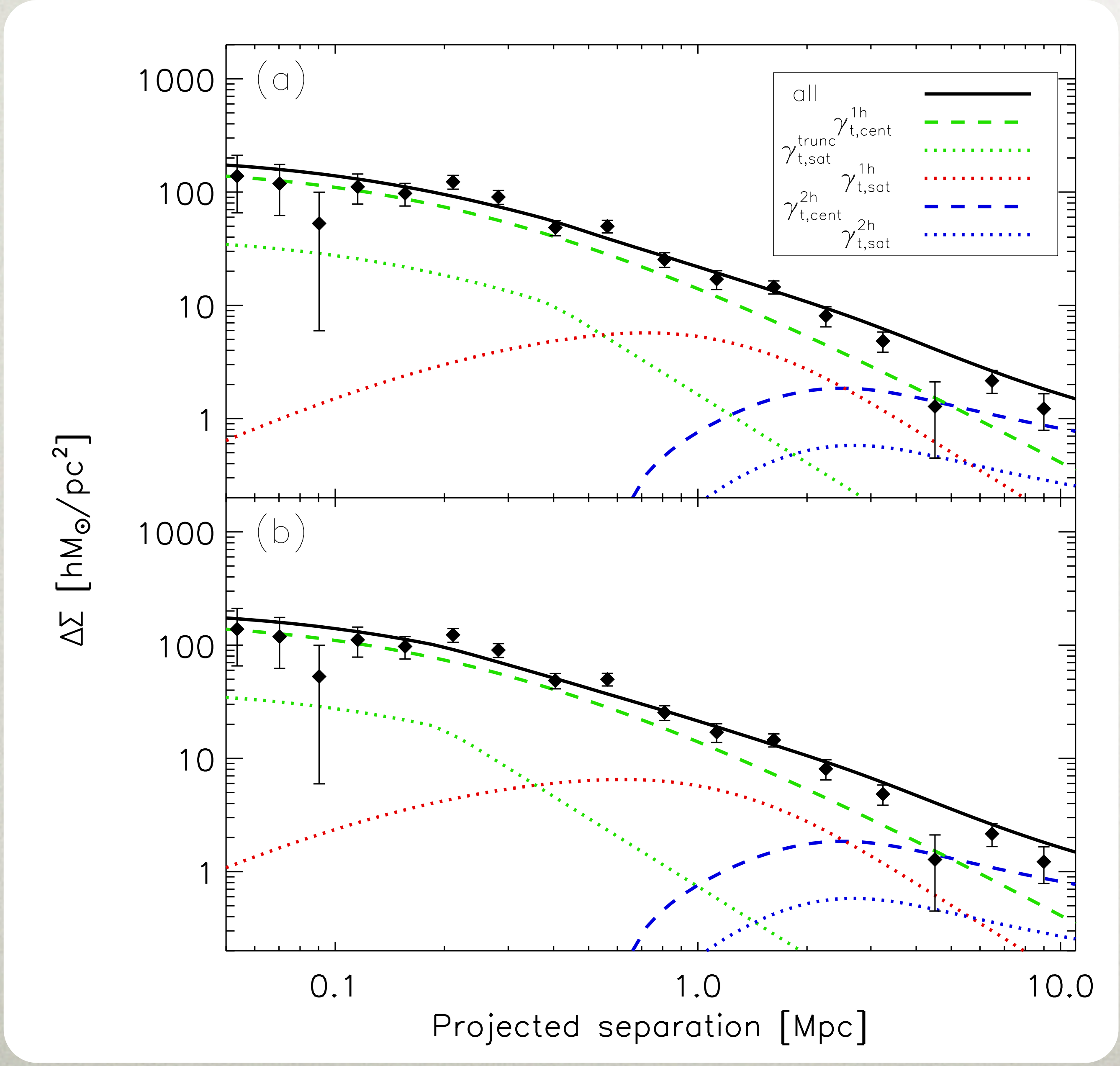
CFHTLenS results



Velander et al. (in prep.)

BETTER MODEL FOR STRIPPED HALOS

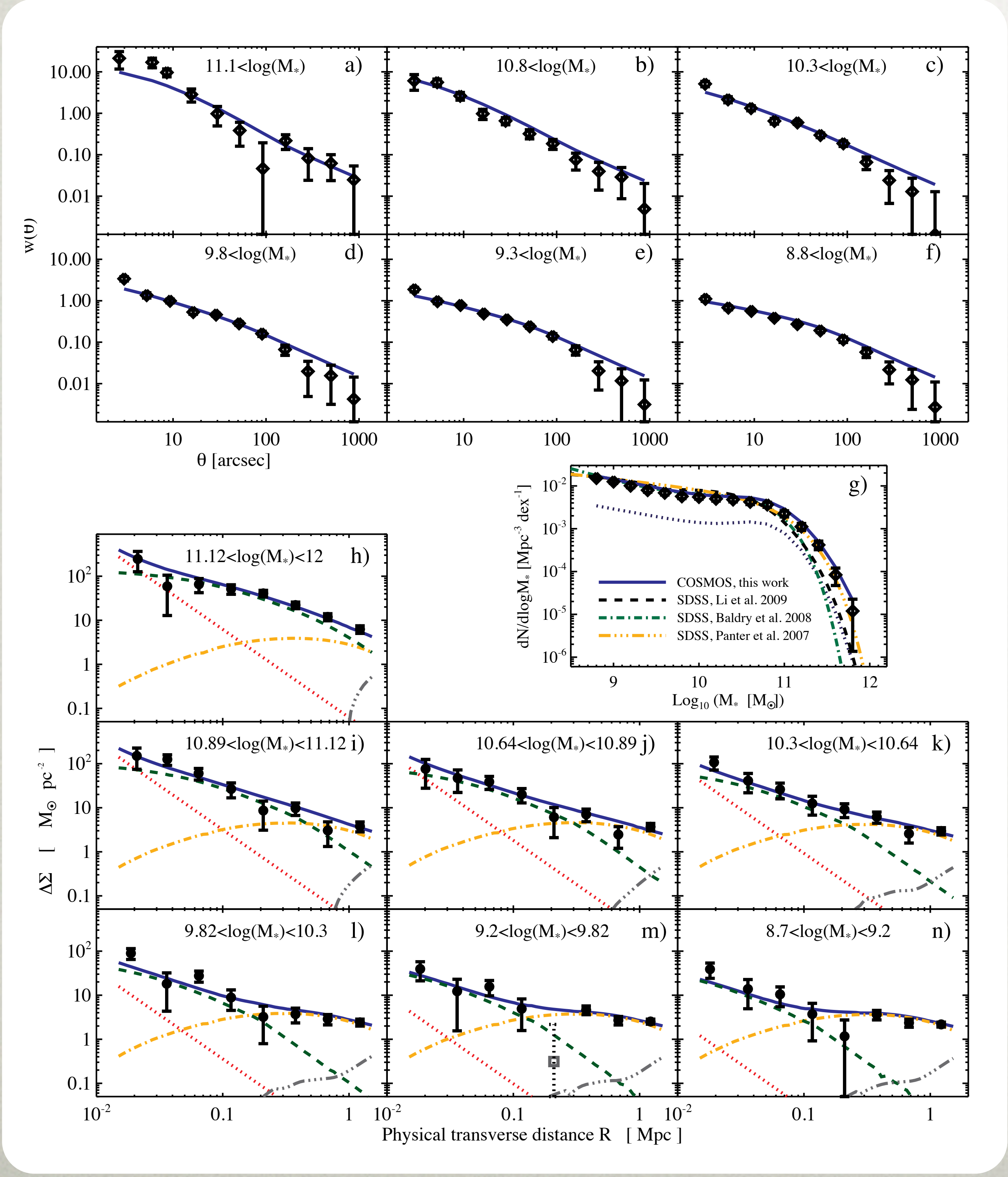
For massive galaxies we cannot model the satellite fraction well because the combined satellite term is very similar to the central signal.



$r_{trunc}=0.4r_{200}$

$r_{trunc}=0.2r_{200}$

COMBINE WITH CLUSTERING



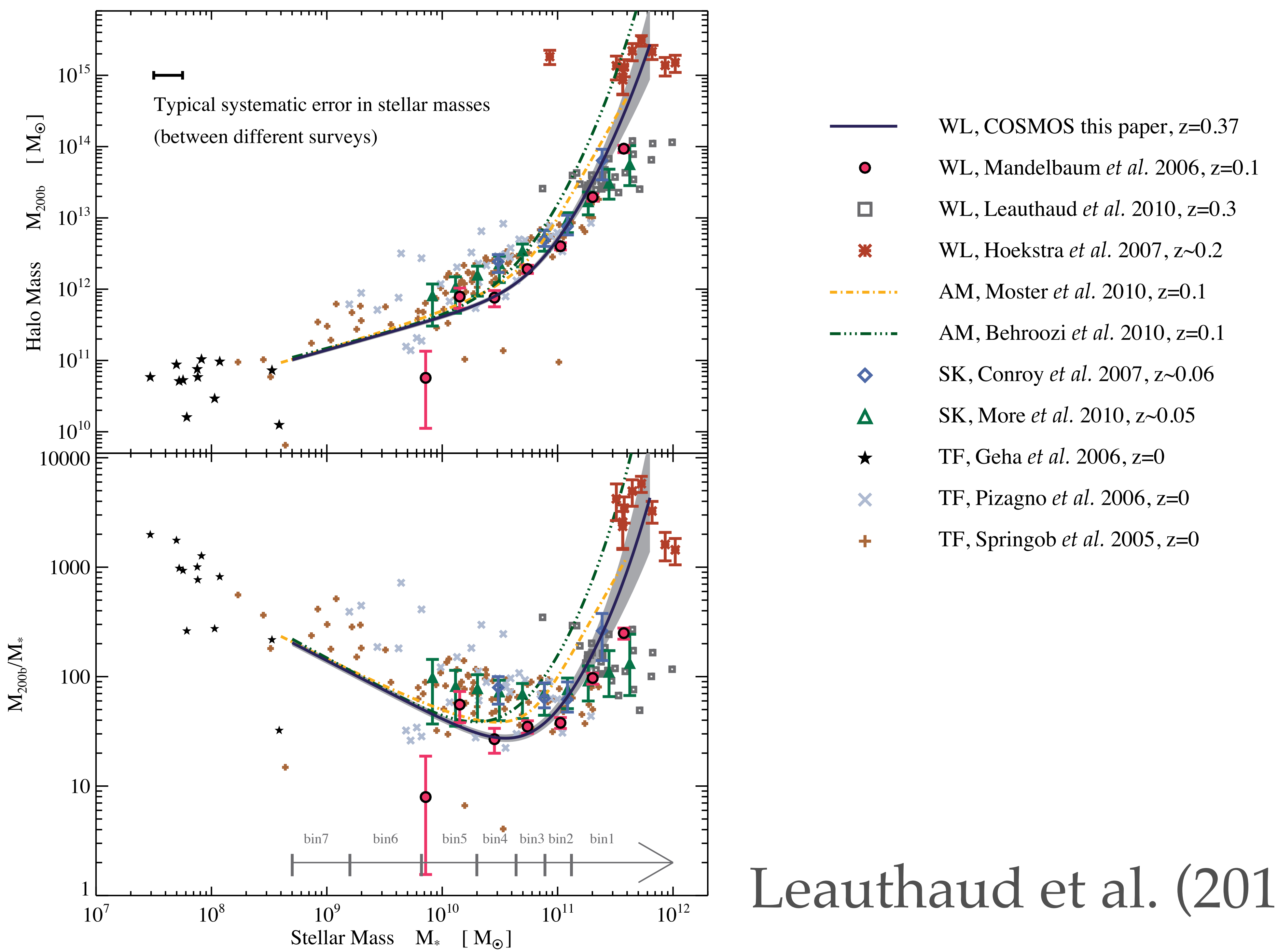
clustering

luminosity function

lensing

Leauthaud et al. (2011)

Combine with clustering



Leauthaud et al. (2011)

The Halo-Model

Open questions:

- better description of satellites

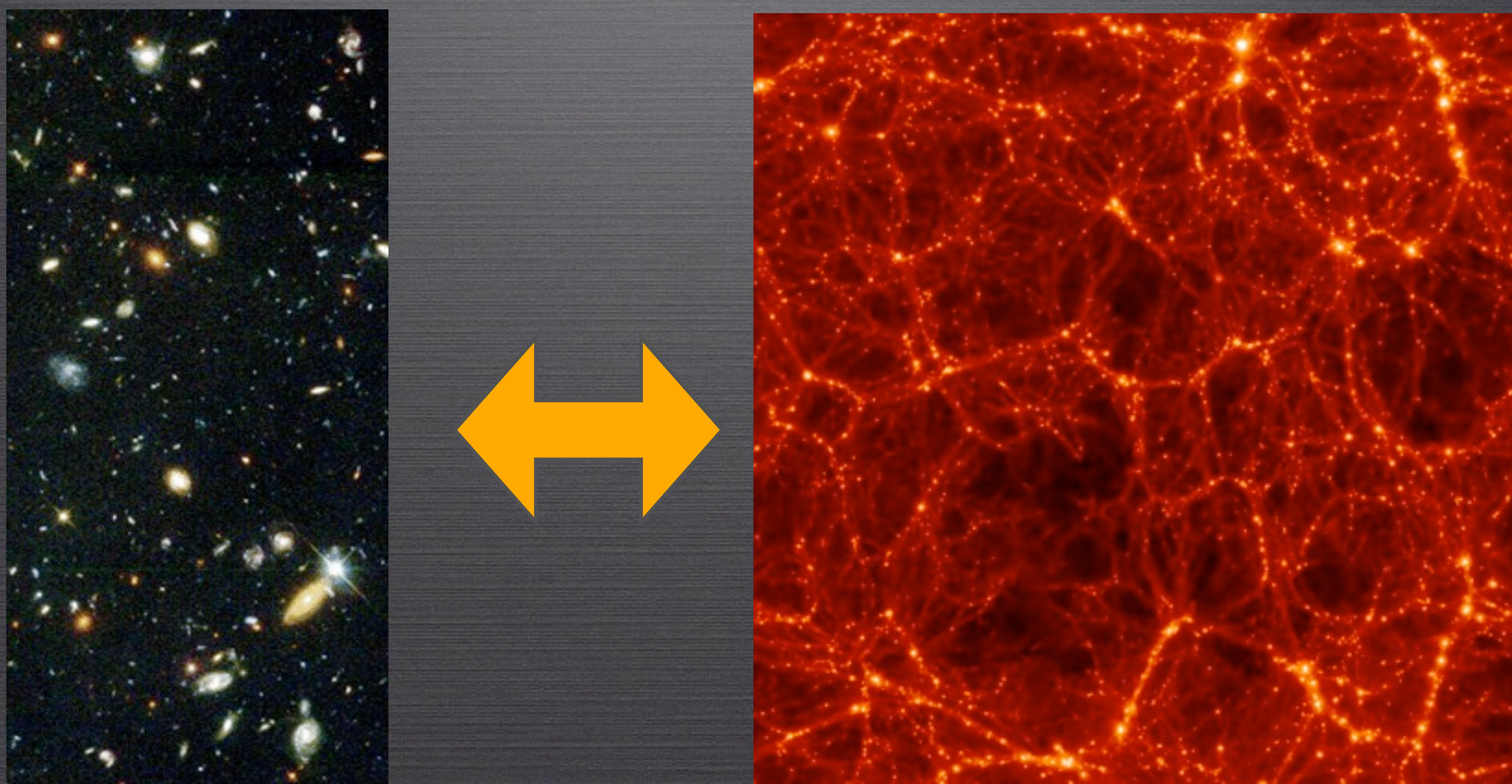
  distribution in host, density profile after tidal stripping

- can we make a 2D version?

  more optimal use of data

Galaxy biasing

Can we statistically relate the clustering of galaxies and dark matter?



Light ≠ Density!

Light ≠ Density!

Galaxy biasing

Cosmic shear measures the clustering of (dark) matter. We can compare this clustering signal to that of galaxies.

$$\delta_{\mathrm{g}}(\mathbf{x}) = \frac{n_{\mathrm{g}}(\mathbf{x}) - \bar{n}_{\mathrm{g}}}{\bar{n}_{\mathrm{g}}} \qquad \text{and} \qquad \delta_{\mathrm{m}}(\mathbf{x}) = \frac{\rho_{\mathrm{m}}(\mathbf{x}) - \bar{\rho}_{\mathrm{m}}}{\bar{\rho}_{\mathrm{m}}}$$

The bias is linear and deterministic if

$$\delta_{\mathrm{g}}(\mathbf{x}) = b_{\mathrm{g}}\delta_{\mathrm{m}}(\mathbf{x})$$

Galaxy biasing

Linear bias is expected to be a good approximation when smoothing the density field on sufficiently large scales:

$$\delta_g^R(x) = f(\delta^R(x)) \longrightarrow \delta_g \approx b_1\delta + \frac{b_2}{2!}\delta^2 + \frac{b_3}{3!}\delta^3$$

In practice on cannot not compare over-densities locally and instead we evaluate the ratio of the power spectra.

$$P_g(k) = b_1^2 P(k) + \ldots$$

Cross-correlation

The bias measures the relative variances of the matter and galaxy field. This only has meaning if the two fields are *correlated*.

The level of correlation can be obtained from the galaxy-mass cross-correlation function:

$$\langle\gamma_t\rangle(\theta) = \frac{3\Omega_m}{4\pi}\left(\frac{H_0}{c}\right)^2 br \int dw \frac{g(w)p_f(w)}{a(w)f_K(w)} \times \int dl\, l P_{3\mathrm{d}}\left[\frac{l}{f_K(w)}; w\right] J_2(l\theta)$$

Galaxy biasing

It is possible to relate a “classical” description of bias to the ingredients of the halo model.

$$b(\delta_{\mathrm{m}})\delta_{\mathrm{m}} \equiv \langle \delta_{\mathrm{g}} | \delta_{\mathrm{m}} \rangle = \int \delta_{\mathrm{g}}\, P(\delta_{\mathrm{g}} | \delta_{\mathrm{m}})\, \mathrm{d}\delta_{\mathrm{g}}$$

mean biasing function

The stochasticity is captured by the random biasing field

$\varepsilon \equiv \delta_{\mathrm{g}} - \langle \delta_{\mathrm{g}} | \delta_{\mathrm{m}} \rangle$ and variance $\langle \varepsilon^2 \rangle = \int \langle \varepsilon^2 | \delta_{\mathrm{m}} \rangle\, P(\delta_{\mathrm{m}})\, \mathrm{d}\delta_{\mathrm{m}}$

Note that $\langle \varepsilon | \delta_{\mathrm{m}} \rangle = 0$.

Galaxy biasing

$$b(M) \equiv \frac{\bar{\rho}_{\rm m}}{\bar{n}_{\rm g}} \frac{\langle N|M\rangle}{M}$$

mean HOD

mean biasing function

Define the moments using $\langle A\rangle \equiv \frac{\int A\, n(M)\, \mathrm{d}M}{\int n(M)\, \mathrm{d}M}$

$$\hat{b} \equiv \frac{\langle b(M) M^2\rangle}{\sigma_M^2}, \qquad \text{and} \qquad \tilde{b}^2 \equiv \frac{\langle b^2(M) M^2\rangle}{\sigma_M^2} \quad \text{where} \quad \sigma_M^2 \equiv \langle M^2\rangle$$

bias is linear if $\tilde{b}/\hat{b} = 1$. This implies *b(M)* is independent of mass and hence $\langle N|M\rangle \propto M$.

Galaxy biasing

We can also define the *halo stochasticity function*

$$\sigma_{\mathrm{b}}^2(M) \equiv \left(\frac{\bar{\rho}_{\mathrm{m}}}{\bar{n}_{\mathrm{g}}}\right)^2 \frac{\langle \varepsilon_N^2 | M \rangle}{\sigma_M^2}$$

Averaging over all halo masses defines the *stochasticity parameter*

$$\sigma_{\mathrm{b}}^2 \equiv \left(\frac{\bar{\rho}_{\mathrm{m}}}{\bar{n}_{\mathrm{g}}}\right)^2 \frac{\langle \varepsilon_N^2 \rangle}{\sigma_M^2}$$

Bias is deterministic if the stochasticity parameter is zero.

Galaxy biasing

The ratio of the variances defines: $b_{\rm var} \equiv \langle \delta_{\rm g}^2 \rangle / \langle \delta_{\rm m}^2 \rangle$

$$b_{\rm var} \equiv \left(\frac{\bar{\rho}_{\rm m}}{\bar{n}_{\rm g}} \right)^2 \frac{\sigma_N^2}{\sigma_M^2} = \left(\frac{\bar{\rho}_{\rm m}}{\bar{n}_{\rm g}} \right)^2 \frac{\langle N^2 \rangle}{\langle M^2 \rangle}$$

$$\langle N^2 \rangle = \left(\frac{\bar{\rho}_{\rm m}}{\bar{n}_{\rm g}} \right)^2 \left[\tilde{b}^2 + \sigma_{\rm b}^2 \right] \sigma_M^2$$

galaxy clustering

$$\langle NM \rangle = \frac{\bar{\rho}_{\rm m}}{\bar{n}_{\rm g}} \, \hat{b} \, \sigma_M^2$$

galaxy lensing

Galaxy biasing

correlation coefficient $r \equiv \dfrac{\langle NM \rangle}{\sigma_N \, \sigma_M}$

$$\hat{b} = b_{\rm var} \, r$$

Hence, the first moment of the mean bias function $b(M)$ is simply the product of the ratio of variances, $b_{\rm var}$, and the linear correlation coefficient, r.

Galaxy biasing

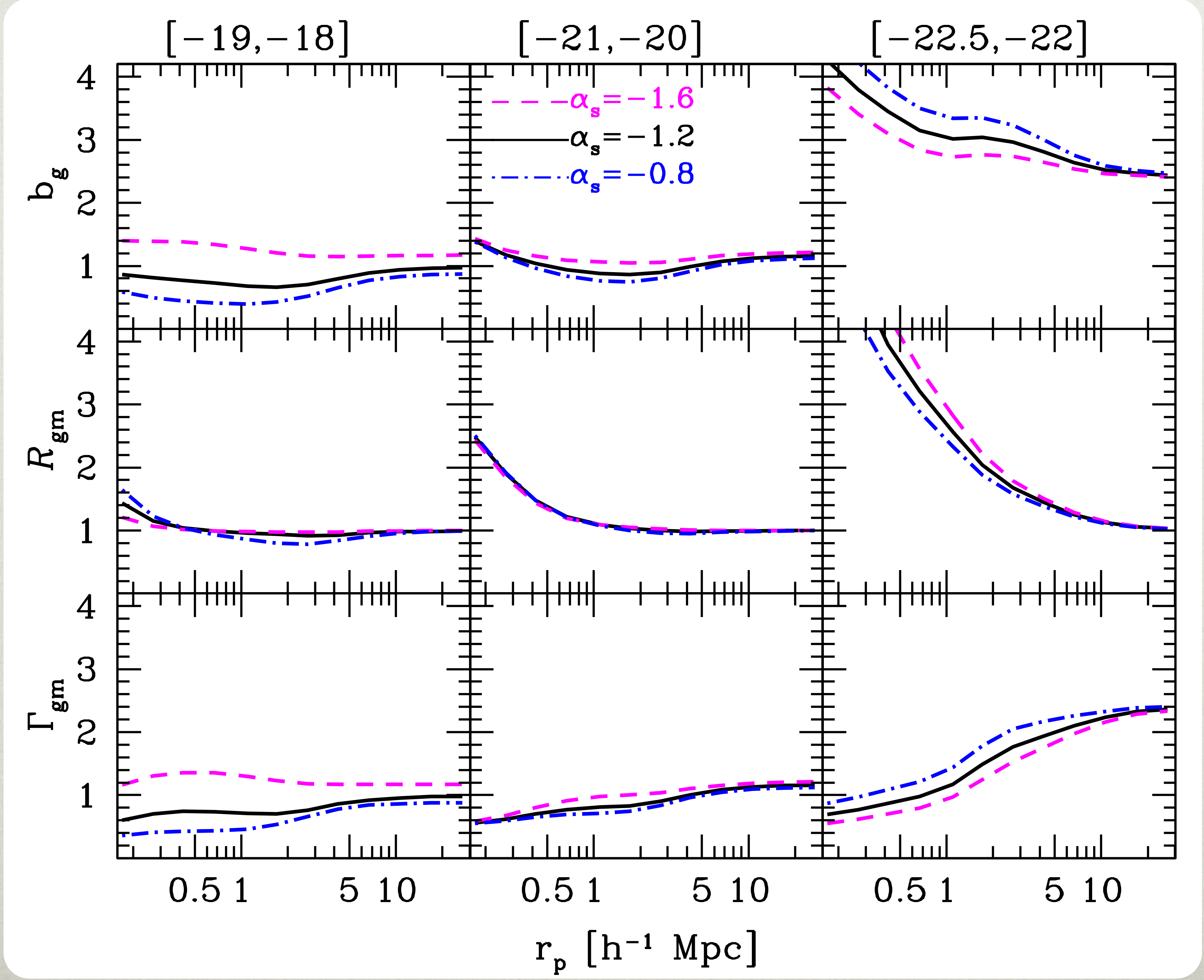
Using these parameters, we can now characterize a few special cases. As already mentioned above, the discrete nature of galaxies does not allow for a bias that is both linear and deterministic. However, the halo occupation statistics can in principle be such that the bias is *linear* and *stochastic*, in which case

$$\begin{aligned} \hat{b} = \tilde{b} = b(M) = 1 \qquad\qquad & b_{\rm var} = (1+\sigma_{\rm b}^2)^{1/2} \\ \sigma_{\rm b} \neq 0 \qquad\qquad & r = (1+\sigma_{\rm b}^2)^{-1/2}\,, \end{aligned} \tag{20}$$

so that $b_{\rm var} > 1$, while $r = 1/b_{\rm var} < 1$. In the case of *non-linear, deterministic* biasing these relations reduce to

$$\begin{aligned} 1 \neq \hat{b} \neq \tilde{b} \neq 1 \qquad\qquad & b_{\rm var} = \tilde{b} \\ \sigma_{\rm b} = 0 \qquad\qquad & r = \hat{b}/\tilde{b} \neq 1 \end{aligned} \tag{21}$$

Galaxy biasing



Cacciato et al. (2012)

How to measure this?

$$\tilde{M}_{\rm ap} = \pi\theta_{\rm ap}^2 \frac{\sum_{i=1}^{N_b} Q(\theta_i) w_i \gamma_{{\rm t},i}}{\sum_{i=1}^{N_b} w_i}, \qquad \tilde{\mathcal{N}} = \frac{1}{\bar{N}} \sum_{i=1}^{N_f} U(\theta_i)$$

We use the filter function suggested by Schneider et al. (1998),

$$U(\phi) = \frac{9}{\pi\theta_{\rm ap}^2}\left[1 - \left(\frac{\phi}{\theta_{\rm ap}}\right)^2\right]\left[\frac{1}{3} - \left(\frac{\phi}{\theta_{\rm ap}}\right)^2\right], \qquad (3)$$

with the corresponding $Q(\phi)$,

$$Q(\phi) = \frac{6}{\pi\theta_{\rm ap}^2}\left(\frac{\phi}{\theta_{\rm ap}}\right)^2\left[1 - \left(\frac{\phi}{\theta_{\rm ap}}\right)^2\right]. \qquad (4)$$

How to measure this?

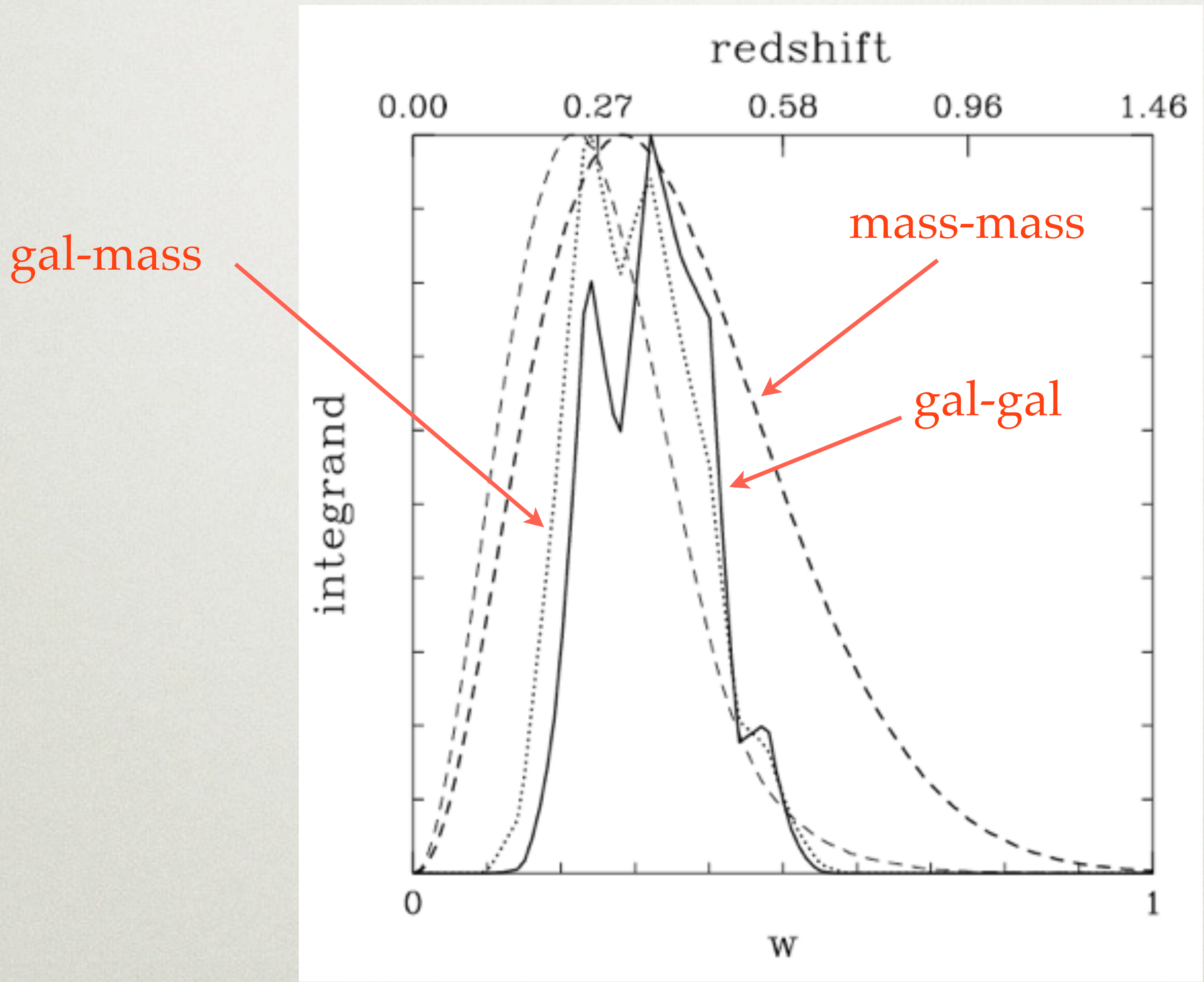
$$b^2 = \frac{9}{4}\left(\frac{H_0}{c}\right)^2 \left[\frac{\int dw h_2(w;\theta_{\rm ap})}{\int dw h_1(w;\theta_{\rm ap})}\right] \Omega_m^2 \times \frac{\langle \mathcal{N}^2(\theta_{\rm ap})\rangle}{\langle M_{\rm ap}^2(\theta_{\rm ap})\rangle}$$

$$= f_1(\theta_{\rm ap}, \Omega_m, \Omega_\Lambda) \times \Omega_m^2 \times \frac{\langle \mathcal{N}^2(\theta_{\rm ap})\rangle}{\langle M_{\rm ap}^2(\theta_{\rm ap})\rangle} \ . \qquad (14)$$

$$r = \frac{\sqrt{\int dw h_1(w;\theta_{\rm ap})}\sqrt{\int dw h_2(w;\theta_{\rm ap})}}{\int dw h_3(w;\theta_{\rm ap})} \times \frac{\langle M_{\rm ap}\mathcal{N}\rangle}{\langle M_{\rm ap}^2\rangle^{1/2}\langle \mathcal{N}^2\rangle^{1/2}}$$

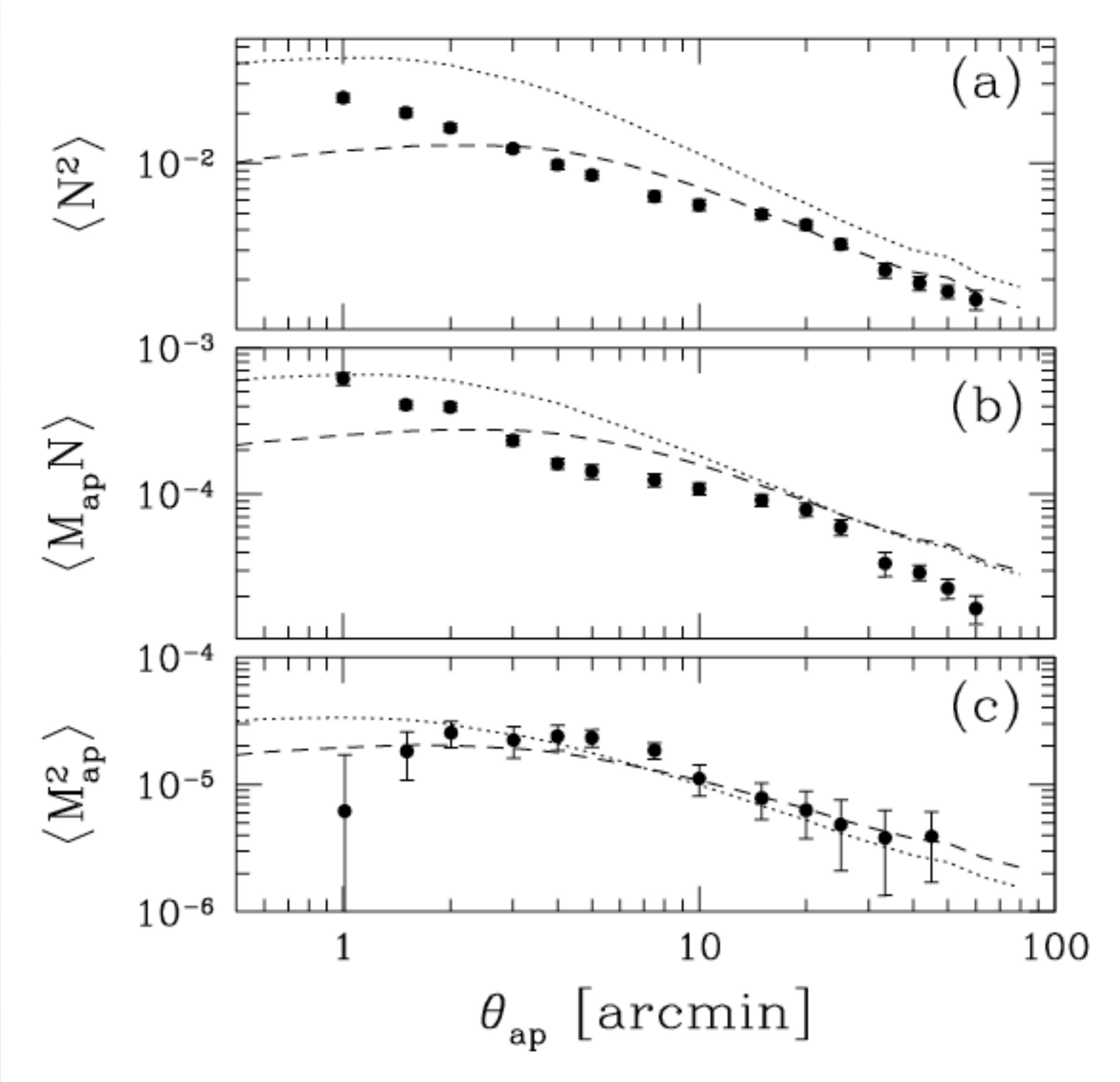
$$= f_2(\theta_{\rm ap}, \Omega_m, \Omega_\Lambda) \times \frac{\langle M_{\rm ap}(\theta_{\rm ap})\mathcal{N}(\theta_{\rm ap})\rangle}{\sqrt{\langle \mathcal{N}^2(\theta_{\rm ap})\rangle\langle M_{\rm ap}^2(\theta_{\rm ap})\rangle}} \ . \qquad (17)$$

How to measure this?

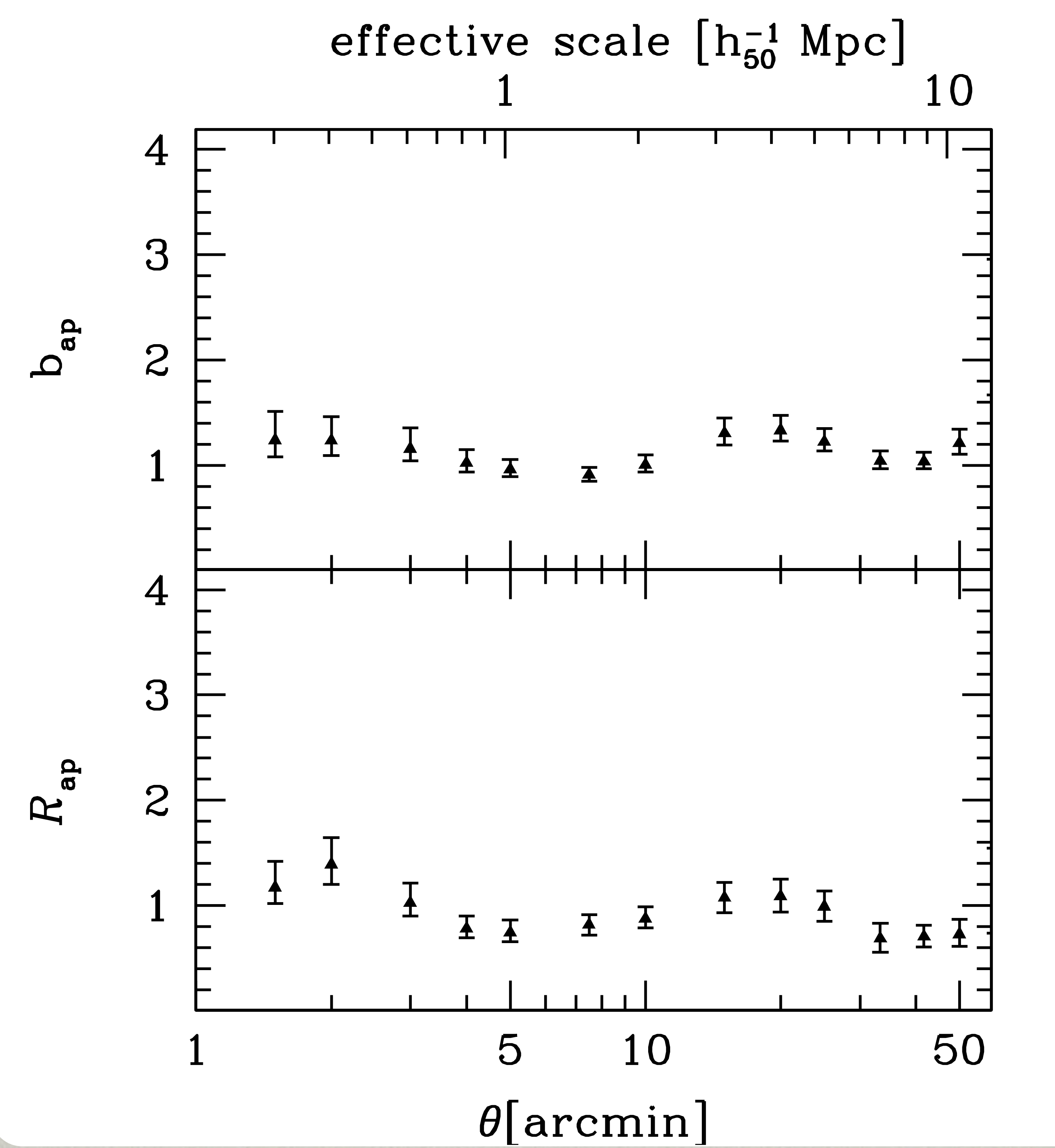


It is important the measurements probe the same redshift range.

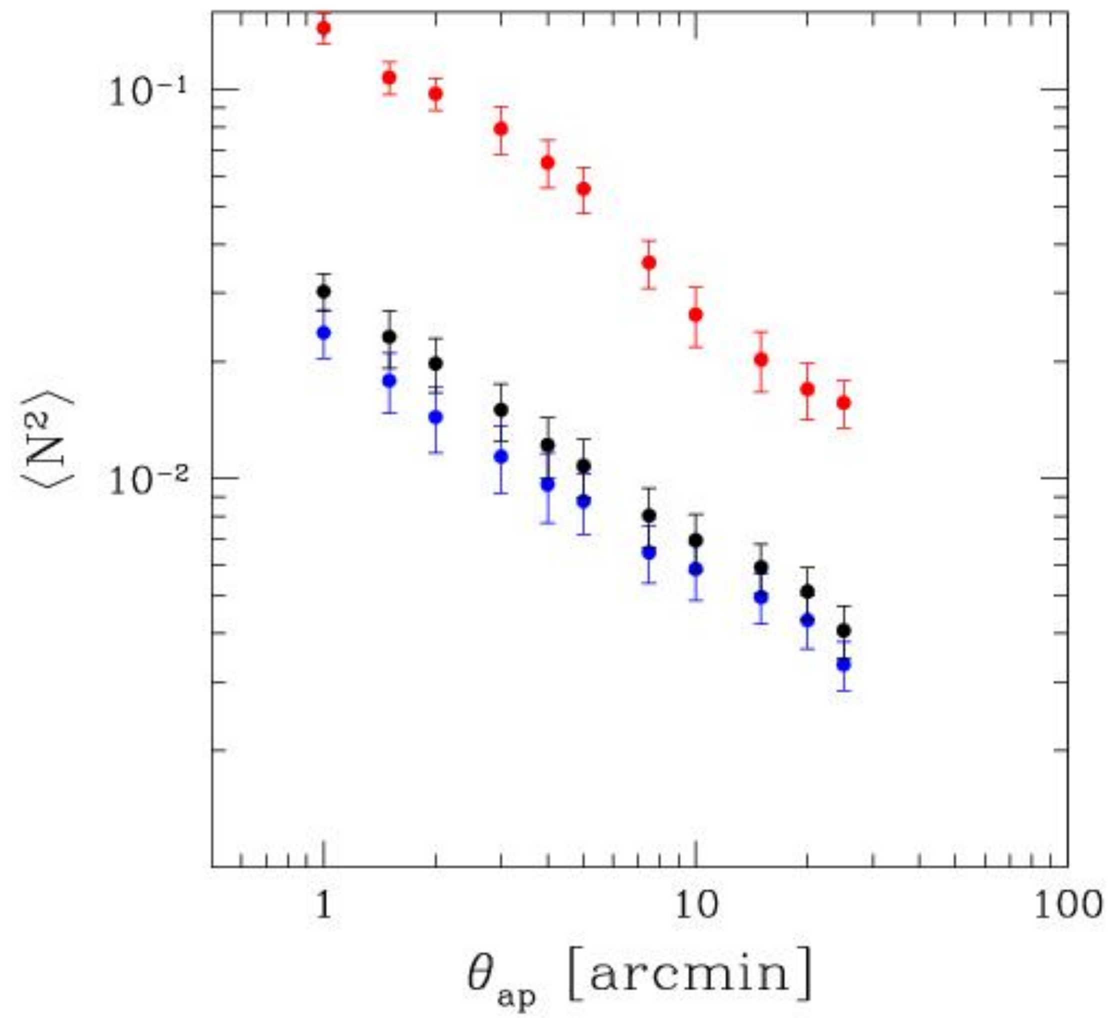
Galaxy biasing



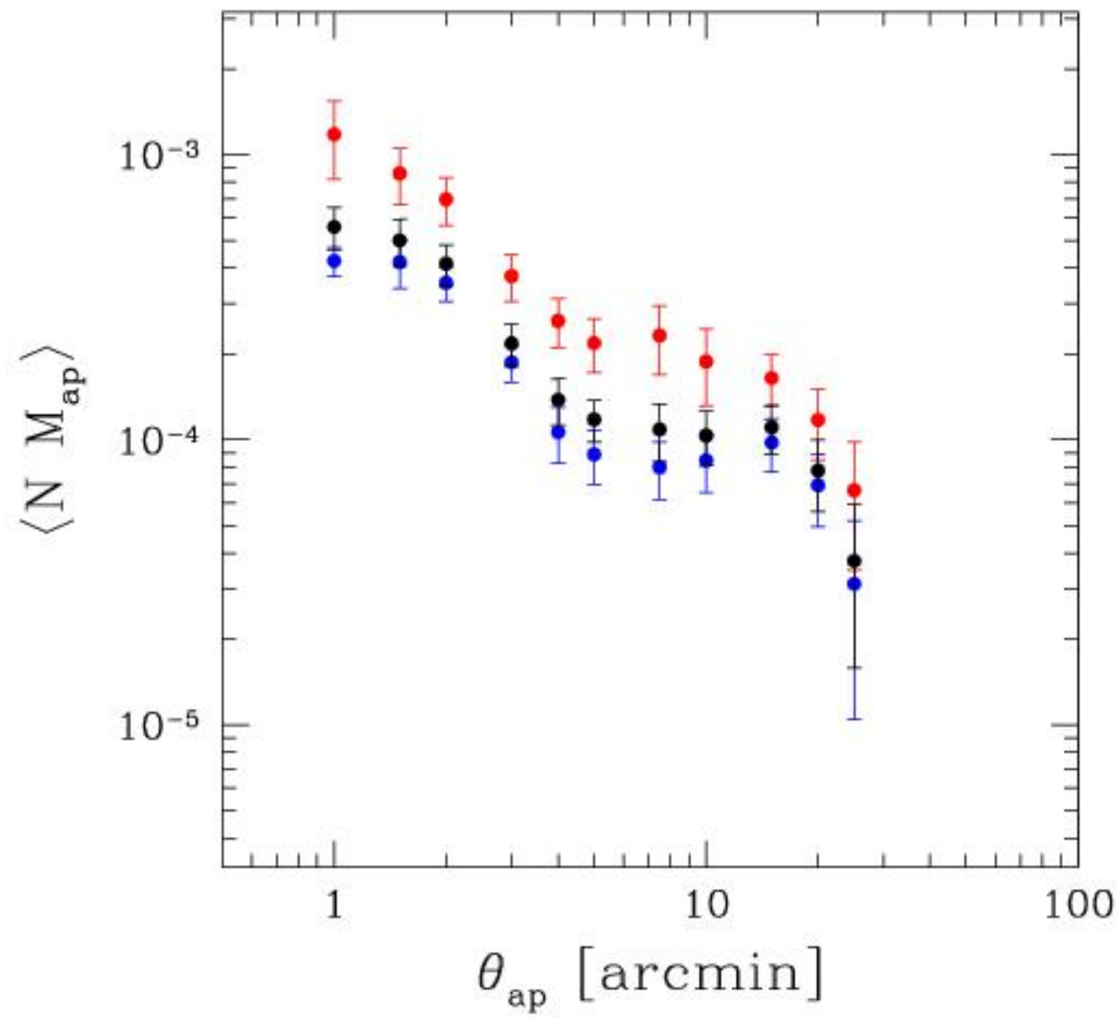
Galaxy biasing



Galaxy biasing

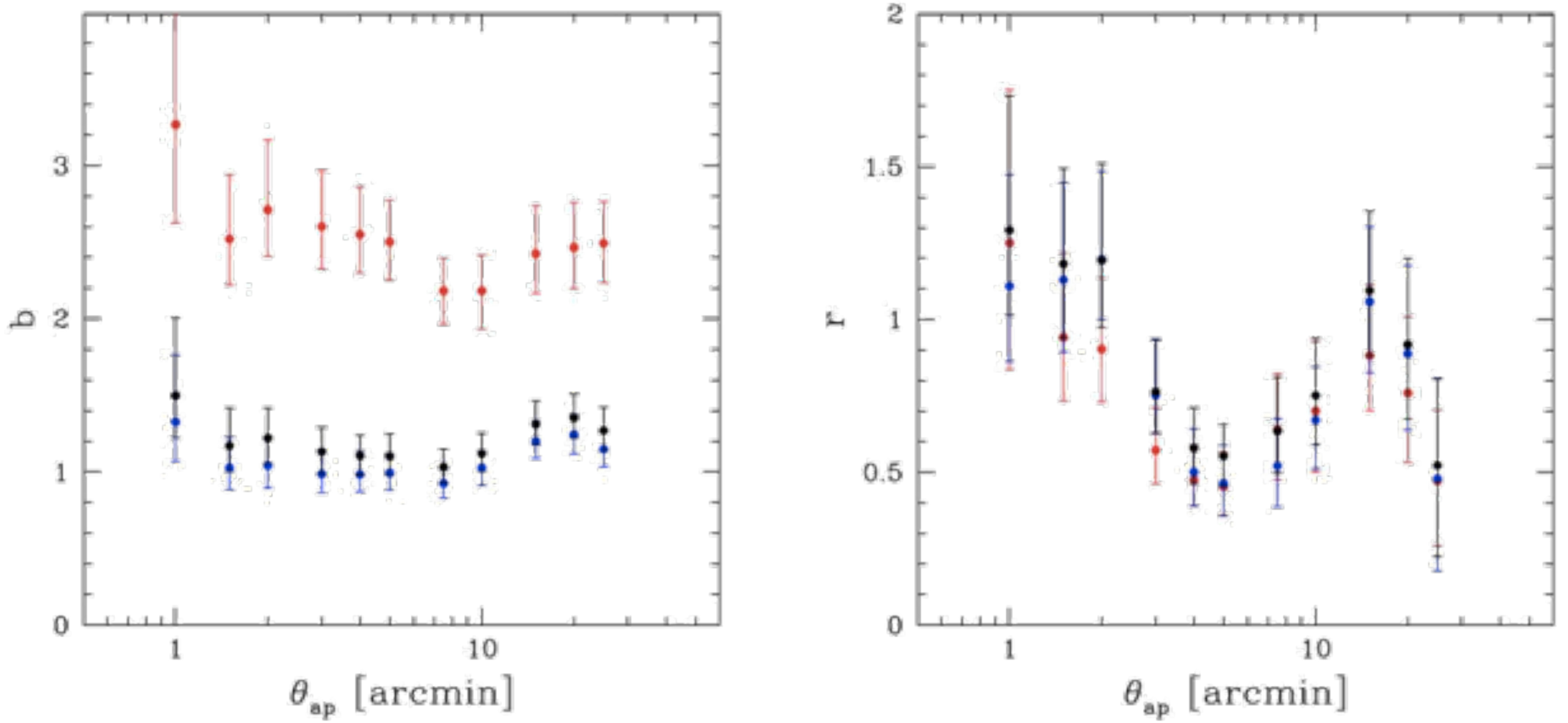


from lensing



Red: 1.5<B-V<2.0 Blue: 0.75<B-V<1.5

Galaxy biasing



Red: 1.5<B-V<2.0 Blue: 0.75<B-V<1.5

Conclusions

Applications of the galaxy-mass correlation function

- tests key predictions of CDM structure formation
- important constraints on models of galaxy formation
- can improve constraints on cosmological parameters

Lots of data are coming!