Strong Gravitational Lensing: 
Effects of Substructure

Léon Koopmans 
(Kapteyn Astronomical Institute)
Outline

Lecture #1 - Theory:
Flux/Surface-brightness anomalies.

Lecture #2 - Observations
Examples of anomalies due to luminous and dark substructures.

Lecture #3 - Modeling
How to use anomalies to infer properties about substructure(s).
Before continuing: I will discuss these two phenomena

**Flux-ratio anomaly**
Compact sources
Only their flux can be measured

**Perturbations to a smooth lens mass model**

**Surface-brightness anomaly**
Extended sources
Image surface-brightness can be measured

Thursday, September 20, 12
Observed (compact) sources: CLASS

Compact (AGN-source) gravitational lenses discovered in the Cosmic Lens All Sky Survey (Browne et al. 2003)

Some of these lenses are claimed to be anomalous (Dalal & Kochanek 2002)
Observed (extended) sources: SLACS
Gravitational lensing maps points in the source plane on to (multiple) points on the images plane through light-rays that follow geodesics. Surface brightness is conserved. In terms of Fermat’s principle: light-rays follow stationary (min/max/saddle) paths in their travel time, including both geometric/Shapiro delays.

Studies of the lens or source require one to invert this mapping and derive both the source brightness distribution and the lens potential.
Typical Multiple-image Configurations

What these lines are will be explained later

Fold images

Cusp images

Critical Curves

Caustics
Image Distortions

The lens equation that relates the image positions to a (unknown) source position is given - in dimensionless form - by (see lectures by Wambsganss/Bacon):

\[ \vec{y} = \vec{x} - \vec{\alpha}(\vec{x}) \]

where the reduced deflection angle is related to the 2D lens potential (the latter is derived from the 2D Poisson equation applied to the surface density of the lens)

\[ \vec{\alpha}(\vec{x}) = \vec{\nabla}\psi(\vec{x}) \]

In case of a small perturbation one can modify this relation as follows and then linearize/Taylor-expand all subsequent derivations (if needed).

\[ \vec{\alpha}(\vec{x}) + \delta \vec{\alpha}(\vec{x}) = \vec{\nabla}(\psi(\vec{x}) + \delta\psi(\vec{x})) \]

In these lectures, we study the theory, observational effects and modeling of small (in mass, not in effect!) perturbations to the lens potential
Image Distortions

Since surface brightness is conserved in gravitational lensing (Liouville’s theorem) one finds for an unperturbed (smooth) lens:

\[ S_x(\vec{x}) = S_y(\vec{y}) = S_y(\vec{x} - \vec{\alpha}(\vec{x})) \]

where \( S_y \) is the surface brightness distribution of the source and \( S_x \) that of the lensed images.

Imagine now that the source is a Dirac delta function with flux \( F_0 \):

\[ S_y(\vec{y}) = F_0 \times \delta(\vec{y} - \vec{y}_0) \]

Let’s choose for simplicity the coordinates such that \( y_0 = (0,0) \). In that case

\[ S_x(\vec{x}) = F_0 \times \delta(\vec{x} - \vec{\alpha}(\vec{x})) \]
Image Distortions

Since the lens equation can have multiple solutions, multiple (distorted) images will appear in the image plane.

Since the images are infinitesimally small, the sum of the fluxes of the images are:

\[ F_{\text{img}} = F_0 \times \int \int \delta(\vec{x} - \vec{\alpha}(\vec{x})) d\vec{x} = F_0 \times \sum_i \frac{1}{|A(\vec{x}_i)|} \]

The sum is made over all images \( i \) and the matrix (see also lectures Wambganss)

\[ A = \frac{\partial (\vec{x} - \vec{\alpha}(\vec{x}))}{\partial \vec{x}} = \begin{pmatrix} 1-\psi_{11} & -\psi_{12} \\ -\psi_{21} & 1-\psi_{22} \end{pmatrix} \]

This Jacobian matrix indicates the distortion of an infinitesimally small source when projected on the image plane and causes the integrated flux of the image to be modified (either decrease or increase: (de)magnified).
Image Distortions

This Jacobian matrix can be related (Sach 1961) to the mass of lens (and field) that a distorts a ray-bundle while passing through the lens (traveling on geodetics).

Using the dimensionless Poisson equation in 2D, we can define the dimensionless surface density (i.e. convergence) in critical units using $\psi$,

$$\kappa = \frac{1}{2} (\psi_{11} + \psi_{22}) \quad \text{or} \quad \nabla^2 \psi = 2\kappa$$

In addition we can define two shear components as function of the potential as well

$$\gamma_1 = \frac{1}{2} (\psi_{11} - \psi_{22}) \quad \text{and} \quad \gamma_2 = \psi_{12} = \psi_{21}$$
The Jacobian matrix $A$, of the lens equation, tells us how a ray-bundle is (de)magnified and distorted by the surface-mass density (convergence) inside the ray-bundle and the shear coming from mass outside the bundle, as rays move along geodesics.

\[
A = \begin{pmatrix}
1 - \kappa - \gamma_1 & -\gamma_2 \\
-\gamma_2 & 1 - \kappa + \gamma_1
\end{pmatrix}
\]

\[
\kappa(\vec{x}) = \frac{\Sigma(\vec{x})}{\Sigma_{\text{crit}}} \\
\gamma(\vec{x}) = \gamma_1(\vec{x}) + i\gamma_2(\vec{x})
\]

\[
\mu(\vec{x}) = \frac{1}{\det[A(\vec{x})]}
\]

(de)magnification of an infinitesimally small source. The sign of $\mu$ is the parity.
Imagine now that the lens plane contains a small perturbation $\delta \psi$ to a smooth lens potential $\psi$.

This causes light-rays that an observer sees to come from a different position in the source plane compared to the smooth-only lens model. Hence a different surface brightness is seen and a different source shape compared to the smooth model.
Flux-ratio Anomalies

A difference in the flux-ratio between compact (point-like) images from that expected from a smooth lens model.
Perturbations to the Lens Potential

A small perturbation to the lens leads to a change to the magnification matrix:

$$\delta A' = \begin{pmatrix} -\delta \kappa - \delta \gamma_1 & -\delta \gamma_2 \\ -\delta \gamma_2 & -\delta \kappa + \delta \gamma_2 \end{pmatrix}$$

Before adding, we transform the coordinates (locally to the image) such that $A$ becomes diagonal (no loss of generality), with the eigenvalues of $A$ on the diagonal,

$$A' = \begin{pmatrix} 1 - \kappa - \gamma & 0 \\ 0 & 1 - \kappa + \gamma \end{pmatrix} \quad \gamma^2 = \gamma_1^2 + \gamma_2^2$$

Now we (i) add $A'$ and $\delta A$, (ii) evaluating again their eigenvalues, (iii) Taylor-expand to first order in the perturbation, assuming the perturbations are small, and (iv) transform this to the frame ("") of eigenvectors.
Perturbations to the Lens Potential

We then get:

\[ A'' \approx \begin{pmatrix} 1 - \kappa - \delta \kappa - \gamma - \delta \gamma_1 & 0 \\ 0 & 1 - \kappa - \delta \kappa + \gamma + \delta \gamma_2 \end{pmatrix} \]

The diagonal shows the eigenvalues of the perturbed matrix, in its locally transformed eigenvector frame.

Now note that the magnification is \( \mu = 1/\det(A'') \) or

\[ \mu = \frac{1}{(1 - \kappa - \delta \kappa - \gamma - \delta \gamma_1)(1 - \kappa - \delta \kappa + \gamma + \delta \gamma_2)} \]

Let us examine where this magnification is most affected by the perturbation. For that we need to digress a little into catastrophe theory.
Caustics are due the folding of a single wavefront moving in curved space-time, caused by a mass distribution (say a lens).

Multiple images are equivalent to seeing the same but folded and distorted wavefront multiple times (but delayed!). When this happens cusps, folds, etc. can form.

Kayser & Refsdal 1983
Catastrophe Theory 101

In gravitational lensing theory we have:

Critical curves: All closed curves with $\det(A) = 0$ (i.e. $\mu=\infty$).
Caustics: Maps of critical curves on to the source plane.

(related to image multiplicity)

The Fermat-potential related to these critical curves/caustic have generic forms that have special properties:
Catastrophe Theory:
Description of Images Near Critical Curves

Lens equation: \[
\vec{y} = \vec{x} - \vec{\alpha}(\vec{x})
\]

Deflection Angle: \[
\vec{\alpha}(\vec{x}) = \nabla \psi(\vec{x})
\]

Fermat Potential: \[
\phi = \frac{1}{2}(\vec{x} - \vec{y})^2 - \psi(\vec{x})
\]

Images form at extrema: \[
\nabla \phi = 0
\]

See also lecture by Bacon describing Fermat's principle
Fold Images

Fermat potential:

\[ \phi \approx y_1 x_1 + y_2 x_2 - \frac{1}{2} x_1^2 - \frac{1}{3} x_2^3 \]

Lensed Images:

\[ \frac{\partial \phi}{\partial x_1} = y_1 - x_1 = 0 \quad x_1 = y_1 \]
\[ \frac{\partial \phi}{\partial x_2} = y_2 - x_2^2 = 0 \quad x_2 = \pm \sqrt{y_2} \]
Fold Images

$$\frac{\partial^2 \phi}{\partial x_1^2} = -1$$
$$\frac{\partial^2 \phi}{\partial x_2^2} = -2x_2$$
$$\frac{\partial^2 \phi}{\partial x_2 \partial x_1} = 0$$

Magnification of the lensed images become

$$\mu = [(1 - \phi_{,11})(1 - \phi_{,22}) - \phi_{,12}^2]^{-1} = \frac{1}{2x_2} \quad \text{or} \quad \mu = \frac{\pm 1}{\sqrt{y_2}}$$

Fold relations

$$\mu_{\text{tot}} = \sum_{i=A,B} \mu_i = 0$$
Cusp Images

\[ y_1 < 0 \quad \text{A} \quad \text{B} \quad y_1 > 0 \]

\[ \phi \approx y_1 x_1 + y_2 x_2 - \frac{1}{2} x_1^2 - \frac{1}{2} y_1 x_2^2 - \frac{1}{4} x_2^4 \]

\[ \frac{\partial \phi}{\partial x_1} = y_1 - x_1 = 0 \quad x_1 = y_1 \]
\[ \frac{\partial \phi}{\partial x_2} = y_2 - y_1 x_2 - x_2^3 = 0 \]

Let's assume \( y_2 = 0 \) then \( x_2^3 - y_1 x_2 = 0 \) \( \Rightarrow x_2 = 0 \) \( x_2 = \pm \sqrt{y_1} \)
Cusp Images

\[ \frac{\partial^2 \phi}{\partial x_1^2} = -1 \quad \frac{\partial^2 \phi}{\partial x_2^2} = -y_1 - 3x_2^2 \quad \frac{\partial^2 \phi}{\partial x_2 \partial x_1} = 0 \]

\[ \mu = \frac{1}{y_1 - 3x_2^2} \quad \text{with} \quad x_1 = y_1 \quad \text{and} \quad \mu = 0 \]

Leads to a parabolic c.c. \[ x_2 = \pm \sqrt{x_1/3} \]

\[ \mu(x_2 = 0) = \frac{1}{y_1} \quad \mu(x_2 = \pm \sqrt{y_1}) = \frac{-1}{2y_1} \]

Cusp relations \[ \mu_{\text{tot}} = \sum_{i=A,B,C} \mu_i = 0 \]
Flux-ratio anomalies near folds/cusps

The sum of (parity-signed) magnifications is zero

Fold relation

\[ \mu_{\text{tot}} = \sum_{i=A,B} \mu_i = 0 \]

Cusp relation

\[ \mu_{\text{tot}} = \sum_{i=A,B,C} \mu_i = 0 \]

(e.g. Blandford 1989; Mao & Schneider 1998)
Perturbations to a SIE

Magnifications are very large near critical curves, and formally infinite on the critical curve. In fact one of the eigenvalues of matrix $A$ goes to zero on the critical curve.

Let us take a simple example for the SIE. In that case, on the critical curve,

$$\kappa_{\text{SIE}} = \gamma_{\text{SIE}} = 1/2$$

The matrix $A''$ then becomes

$$A'' \approx \begin{pmatrix} -\delta \kappa - \delta \gamma_1 & 0 \\ 0 & 1 -\delta \kappa + \delta \gamma_2 \end{pmatrix} \approx \begin{pmatrix} -\delta \kappa - \delta \gamma_1 & 0 \\ 0 & 1 \end{pmatrix}$$

Whereas w/o the perturbation the magnification was infinite (or very large), it can now become very different due to a shift in the critical curve. The images themselves will also shift, but by far less.

In short: a small perturbation can have a major impact on highly magnified images near critical curves. We will now examine this in greater detail.
Perturbations to the Lens Potential

Effects on the critical curve/caustic

Xu et al. (2012)
Perturbations of Catastrophes

Perturbations cause the critical curves to change and the caustics to transform as well, sometimes developing other types of catastrophes such as swallowtails and butterflies. This can cause a major change in the flux-ratio between cusp/fold images.

Xu et al. (2012)

Petters et al. (2001)
Cusp Relation

Cusp/fold magnification relation:
Measure of deviations from a perfectly smooth lens

Cusp relation

Bradac et al. 2004
Cusp Relation

Expected for Perturbed Lens

Observed Lenses

Expected for Smooth Lens

Xu et al. 2009
Surface Brightness Anomalies

A difference in the surface brightness between extended images from that expected from a smooth lens model.
Surface Brightness Anomalies

To solve for (i) the source brightness distribution and (ii) the potential, using

$$S_x(\vec{x}) = S_y(\vec{y})$$

Conservation of source surface brightness

with

$$\vec{y} = \vec{x} - \vec{\alpha}(\vec{x})$$

The usual lens equation

Koopmans (2005)
Surface Brightness Anomalies

Conservation of surface brightness

\[ S_x(\vec{x}) = S_y(\vec{y}) \]

\[ \vec{y} = \vec{x} - \vec{a}(\vec{x}) \]

If

\[ \psi \rightarrow \psi + \delta \psi \]

\[ S_x \rightarrow S_x + \delta S_x \]

then

\[ S_y + \left( \frac{\partial S_y}{\partial \vec{y}} \right) \cdot \delta \vec{y} \approx S_x + \delta S_x \]

\[ \left( \frac{\partial S_y}{\partial \vec{y}} \right) \cdot \delta \vec{y} \approx \delta S_x \]

\[ \delta S_x \approx -\vec{\nabla}_y S_y \cdot \vec{\nabla}_x \delta \psi \]
Surface Brightness Anomalies

Unperturbed smooth image  Perturbed smooth image

Can this small deviation from the smooth model be reconstructed from the image on the right? Yes under specific conditions!
Surface Brightness Anomalies

Inoue & Chiba 2005
Surface Brightness Anomalies

The first step is solving for $S(x)$, given a smooth model. This is simple and leads to a linear equation (e.g. Warren & Dye 2003)

$$S_x(x) = S_y(y) = S_y(x, \psi(x)) \quad \text{Functional Form}$$

or equivalently (see next lectures)

$$L(\psi)\mathbf{s} = \mathbf{d} \quad \text{Algebraic Form}$$

(e.g. Warren & Dye 2003; Koopmans 2005; Suyu et al. 2006/8; Brewer & Lewis 2005; Vegetti & Koopmans 2009)
Surface Brightness Anomalies

In algebraic form this linearized equation reads (see next lectures):

\[
B[L(\vec{\psi}) \mid - D_s(\vec{s}) D_x] \left( \begin{array}{c} \delta \vec{s} \\ \delta \vec{\psi} \end{array} \right) = \vec{d} - B L(\vec{\psi}) \vec{s} = \delta \vec{d}.
\]

Gauss-Newton equation that can be solved iteratively

This linear algebraic equation can be solved using a Bayesian penalty function for the residuals and standard Cholesky/gradient methods

(Koopmans et al 2005; Vegetti & Koopmans 2008)
Simulations

Strong image distortion

Simulation of lens system: SIE + SIS

SIE: $10^{11}$ solar mass

SIS substructure of $10^8$ solar mass

Koopmans 2005
Simulations

Potential Correction

Koopmans 2005

Reconstruction: SIS substructure of $\sim 10^8$ solar mass
Summary

Flux-ratio anomalies:

- Caused by perturbations to the magnification of a point-like image where information on A is lost (only 1/det(A) is known).
- BUT, how do we know this is not caused by a difference in the smooth model?
- Near folds/cusps, catastrophe theory predicts that the sum of magnifications (including parity) adds to zero, INDEPENDENT of the global smooth model.
- The latter thus says that the potential must be locally perturbed.
- Since the sum of magnifications does not obey this relation in some observed lensed, we call them flux-ratio (better magnification-sum) anomalies.

Surface-brightness anomalies:

- Caused by perturbations to the surface brightness of an extended image, where information (apart from rotation) is retained. In principle $\delta\psi$ can be recovered.
- BUT, how do we know this is not caused by a difference in the smooth model?
- There are multiple extended images, i.e. maps of the source, hence a change in the source occurs in all lensed images. The local nature of a perturbation is not seen in the other images, hence we know it’s due to the potential.
Observations/ Examples
Outline

Flux-ratio anomalies -
- Evidence for CDM substructure
- Be careful when assuming an anomaly is due to mass even in the radio/MIR
- Propagation effects?

Luminous Substructures
- Also substructure, but observable
- High-mass end of the mass-function

Dark Substructures
- Surface-brightness anomalies
- Two detections with mass estimates
- L.O.S. contamination
Flux-ratio Anomalies:

Evidence for CDM substructure or are there also other effects that can cause them?
“Aquarius” Simulation

Formation of a Galaxy (MW equivalent)
Fewer satellites are seen in observations compared with simulations (e.g. Moore et al. 1999). Springel et al. 1999

The “Missing Satellite Problem”

Springel et al. 1999
Substructure Mass Fraction

Substructure mass fraction is large outside, but decreases inward due to dynamical effects.
Anomalous Fold & Cusp Systems

Do anomalous flux-ratios between merging fold/cusp images indicate the presence of mass substructure? (e.g. Mao & Schneider 1998; Dalal & Kochanek 2002).

Fluxes are different

Middle image is fainter than the outer two images
Anomalous Fold & Cusp Systems

Dalal & Kochanek 2002

B1555+375

Fluxes are different

We analyzed the lenses MG 0414+0534 (Hewitt et al. 1992), B0712+472 (Jackson et al. 1998), PG 1115+080 (Weymann et al. 1980), B1422+231 (Patnaik et al. 1992), B1608+656 (Fassnacht et al. 1996), B1933+503 (Sykes et al. 1998), and B2045+265 (Fassnacht et al. 1999). Of these seven four-image lenses, six show anomalous flux ratios that might be due to the effects of substructure.
Anomalous Fold & Cusp Systems

Some issues

MG0414+0534

B0712+472

B1422+231

B1608+656

B1933+503

B2045+265

PG1115+080 not shown
Anomalous Fold & Cusp Systems

Anomalous in terms of the cusp-relation

Keeton et al. 2003
Anomalous Fold & Cusp Systems

Based on 7 quad lenses from CLASS++ and PG1115+080.

\[ f_{\text{sat}} = 0.6-7\% \ (90\% \text{CL}) \]

Dalal & Kochanek 2002
Kochanek & Dalal 2004
A gallery of issues with flux-ratio anomalies in the radio, optical, MIR

- Micro-lensing in the radio and MIR
- Extrinsic variability
- Scattering due to the ionized IGM
- Edge-one disks/disk-lenses
- Luminous satellites [also substructure!]
Flux-ratio anomalies in the mid-infrared

Chiba et al. 2008

The MIR is dominated by the dust-torus and less affected by dust, scattering or microlensing, but still sensitive to substructure.
Flux-ratio anomalies in the mid-infrared

Microlensing can affect some compact quasars. In the MIR most emission comes from a few-pc dust-torus, smearing out this effect. However, still anomalies of ~10% can be expected.

Stalevski et al. 2012
Flux-ratio anomalies and extrinsic variability

### TABLE 2

**Significance and Estimates of Extrinsic Variability**

<table>
<thead>
<tr>
<th>Systems</th>
<th>(a\text{ext}) (%)</th>
<th>(b\text{ext}) (%)</th>
<th>(c\text{ext}) (%)</th>
<th>$\chi^2$/dof</th>
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<tbody>
<tr>
<td>B0128+437</td>
<td>2.9</td>
<td>1.9</td>
<td>2.5</td>
<td>3.3</td>
</tr>
<tr>
<td>B0712+472</td>
<td>4.8</td>
<td>4.2</td>
<td>4.8</td>
<td>6.2</td>
</tr>
<tr>
<td>B1359+154</td>
<td>1.0</td>
<td>4.6</td>
<td>...</td>
<td>2.8</td>
</tr>
<tr>
<td>B1422+231</td>
<td>...</td>
<td>0.6</td>
<td>0.9</td>
<td>3.7</td>
</tr>
<tr>
<td>B1555+375</td>
<td>3.3</td>
<td>4.2</td>
<td>3.0</td>
<td>5.3</td>
</tr>
<tr>
<td>B2045+265</td>
<td>6.1</td>
<td>7.0</td>
<td>7.2</td>
<td>17.1</td>
</tr>
</tbody>
</table>

**Note.**—Estimated rms levels of extrinsic variability in images A, B, and C. The reduced values of $\chi^2$ are given to indicate the significance of the presence of extrinsic variability in the combined set of images. *Ellipses:* Estimated variance was smaller than zero (see §3.2 for more details).
Radio Microlensing in B1600+434?

Do we really understand radio source to the level we should? i.e. what about scintillation and microlensing

Radio Microlensing in B1600+434?

A test carried out in K&dB 2000 was to place one of the best studied nearby relativistic jets on to a caustic network, 3C120, scaled to $z=1.59$ and see what happens.

Simulated 3.6-cm lightcurve of 3C120 at $z=1.59$.

- (1) $1.4-M_o$ IMF ($\alpha=0.2; \gamma=0.2$)
- (2) Size scale factor: 1.0 [S$_{250}$=1 mJy]
- (3) Speed scale factor: 1.0 [$\beta_{app}=$3.2 h$^{-1}$]
- (4) Constant surface brightness components
- (5) $H_o=65$ km/s/Mpc, $\Omega_m=1$, $\Omega_k=0$
Radio Microlensing in B1600+434?

Simulated Light curves

Observed Light curves

Level and duration of fluctuations

<table>
<thead>
<tr>
<th>LC</th>
<th>$\alpha_r$</th>
<th>$\alpha_v$</th>
<th>rms (%)</th>
<th>$t_{typ}$ (d)</th>
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<tbody>
<tr>
<td>a</td>
<td>5.7</td>
<td>1</td>
<td>4.5</td>
<td>~50</td>
</tr>
<tr>
<td>b</td>
<td>5.7</td>
<td>3</td>
<td>5.4</td>
<td>~15</td>
</tr>
<tr>
<td>c</td>
<td>3.7</td>
<td>1</td>
<td>2.6</td>
<td>~35</td>
</tr>
<tr>
<td>d</td>
<td>3.7</td>
<td>3</td>
<td>2.9</td>
<td>~10</td>
</tr>
<tr>
<td>e</td>
<td>5.7</td>
<td>1</td>
<td>6.4</td>
<td>~50</td>
</tr>
<tr>
<td>f</td>
<td>5.7</td>
<td>3</td>
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<td>~20</td>
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<tr>
<td>g</td>
<td>3.7</td>
<td>1</td>
<td>3.3</td>
<td>~40</td>
</tr>
<tr>
<td>h</td>
<td>3.7</td>
<td>3</td>
<td>3.4</td>
<td>~15</td>
</tr>
</tbody>
</table>
Flux-ration anomalies and differential scattering

CLASS 0128+437
Anomalous Fold & Cusp Systems

One of the systems mentioned in D&K 2002 shows a clear anomaly. While making this lecture, I came across an old lens model made in 2006 to explain this anomaly with an edge-on disk. Low and behold!

Keck-AO observations 2012
Luminous Satellites
Astrometric Anomalies Due to Luminous Substructure

A1 and A2 should be mirror image. They are not using a smooth mass model.

Trotter et al. 2000
A visible luminous satellite largely causes this anomaly. They can be accounted for in the macro-model.

Model-fit improves by $\Delta \chi^2 = 100$ when object X is included in the mass model.

Kochanek & Dalal 2004
Astrometric Anomalies Due to Luminous Substructure

MG2016+112

Koopmans & Treu 2002

More et al. 2009
Luminous Substructure as Proxy for Dark Substructure

CLASS (radio) lenses seem to have too many nearby bright secondaries compared to e.g. COSMOS [or GOODS].

(see also Nierenberg et al. 2011)

This is not yet explained, but could explain some of the anomalies observed in this sample.

Jackson et al. 2010
Luminous Substructure on an arc

A luminous substructure

\[ z_{\text{lens}} = 0.422 \]
\[ z_{\text{source}} = 2.00 \]

Lin et al. 2009
Vegetti, Czoske, LVEK. 2010
Luminous Substructure on an arc

$$M_{\text{sub}} = (2.75 \pm 0.04) \times 10^{10} M_\odot \ (\sigma \sim 100 \ \text{km/s})$$
inside its tidal radius of $$r_t = 0.68 \ \text{arcsec}$$

$$(M/L)_B = (17.2 \pm 8.5) \ M/L_\odot$$

The potential reconstruction recovers the substructure in mass
Dark Structures?
The Double Einstein Ring

Vegetti, LVEK, et al. 2010

Data minus Galaxy

Best Lens Image Model

Residuals (data-model)

Best Source Model
The Double Einstein Ring

Vegetti, LVEK, et al. 2010

This overdensity seems robust
Adding a tidally truncated SIS (Pseudo-Jaffe) model to the model finds a substructure at the convergence over density with a mass of $3 \times 10^9$ solar mass (16-$\sigma$ CL).

Vegetti et al. 2010
Double Einstein Ring: Substructure Mass Fraction?

The detection of such a high mass substructure implies a high substructure mass fraction of 2.2% \([+2.1\% / -1.3\%; 68\% \text{ CL}]\)

Some lenses (e.g. 0414, 2016, 2045) have nearby luminous companions. Do lenses or galaxies have too many of these in their vicinity?

Vegetti & LVEK 2009; Vegetti et al. 2010

Dalal & Kochanek 2002; Xu et al. 2009
M/L ratio of the substructure is large compared to MW satellites, but this might not be unexpected near a massive elliptical (e.g. stronger feedback)

\[(M/L)_v > 120 \, M/L_{\odot,\text{sun}} \text{ (3-sigma) inside 0.3 kpc (} r_{\text{tidal}} = 1.1 \text{ kpc)}\]

Vegetti et al. 2010
Detecting Lower-Mass Substructures

Different approaches to find even low mass substructures:

• More source structure  >  higher amplitude of SB anomalies  - HST UV/B
• Higher S/N observations >  better measurement of the SB anomalies - HST IR
• Higher spatial resolution >  more constraints on the SB anomalies     - Ground-based AO
Gravitational detection of a low-mass dark satellite galaxy at cosmological distance

S. Vegetti\textsuperscript{1}, D. J. Lagattuta\textsuperscript{2}, J. P. McKeen\textsuperscript{3}, M. W. Auger\textsuperscript{4}, C. D. Fassnacht\textsuperscript{2} & L. V. E. Koopmans\textsuperscript{5}

The mass function of dwarf satellite galaxies that are observed around Local Group galaxies differs substantially from simulations\textsuperscript{1–5} based on cold dark matter: the simulations predict many more dwarf galaxies than are seen. The Local Group, however, may be anomalous in this regard\textsuperscript{6,7}. A massive dark satellite in an early-type lens galaxy at a redshift of 0.222 was recently found\textsuperscript{8} using a method based on gravitational lensing\textsuperscript{9,10}, suggesting that the mass fraction contained in substructure could be higher than is predicted from simulations. The lack of very low-mass detections, however, prohibited any constraint on their mass function. Here we report the presence of a $(1.9 \pm 0.1) \times 10^5 M_\odot$ dark satellite galaxy in the Einstein ring system JVAS B1938+666 (ref. 11) at a redshift of 0.881, where $M_\odot$ denotes the solar mass. This satellite galaxy has a mass similar to that of the Sagittarius\textsuperscript{12} galaxy, which is a satellite of the Milky Way. We determine the logarithmic slope of the mass function for substructure beyond the local Universe to be $1.1^{+0.6}_{-0.4}$, with an average mass fraction of $3.3^{+3.6}_{-1.8}$ per cent, by combining data on both of these recently discovered galaxies. Our results are consistent with the predictions from cold dark matter simulations\textsuperscript{13–15} at the 95 per cent confidence level, and therefore agree with the view that galaxies formed hierarchically in a Universe composed of cold dark matter.

The gravitational lens system JVAS B1938+666 (ref. 11) has a bright infrared background galaxy at redshift $z = 2.059$ (ref. 16), which is gravitationally lensed into an almost complete Einstein ring of diameter radius, $r$. The best-fitting model was then fixed and further refined using local potential corrections defined on a regular grid, which are translated into surface density corrections using the Laplace operator. We found for both the 1.6- and the 2.2-\micron adaptive optics data sets that there was a significant positive density correction, which indicated the presence of a mass substructure (Fig. 1 and Supplementary Information). Directly from the pixelated potential correction, we measured a substructure mass of $\sim 1.7 \times 10^8 M_\odot$ inside a projected radius of 600 pc around the density peak.

As an independent test, we repeated the analysis of the 2.2-\micron data set, which had the highest-significance positive density correction, with different models of the point spread function, different data reduction techniques, different rotations of the lensed images, different models for the lens galaxy surface brightness subtraction and different resolutions for the reconstructed source. We also analysed an independent data set taken at 1.6\micron with the Near Infrared Camera and Multi-Object Spectrograph on board NASA’s Hubble Space Telescope. In total, we tested fourteen different models and three different data sets that all independently led to the detection of a positive density correction at the same spatial position, although with varying levels of significance (Supplementary Information). Differential extinction across the gravitational arc could also produce a surface brightness anomaly. However, the colour of the arc was found to be consistent around and at the location of substructure, ruling out the possibility that dust affected our results.
Detecting Lower-Mass Substructures

A smooth mass model shows residuals at a significant level in the AO data; the HST data is of too low quality to assess this effect.

Keck-AO

HST

Lagattuta et al. 2012
Detection Lower-mass Substructure

A smooth mass model shows residuals at a significant level in the AO data; the HST data is of too low quality to assess this effect.
Detecting Lower-Mass Substructures

A grid-based mass model shows a significant detection of a mass substructure near the upper arc image.

Vegetti et al. 2012, Nature
A full Bayesian analysis, using a Pseude-Jaffe mass model for the substructure shows its impact on the smooth-model parameters

$$M_{0.6\text{kpc,sub}} = (1.1\pm0.1)\times10^8\,M_\odot\,(12-\sigma\,\text{CL})$$

[or $\sigma_{\text{SIS,sub}} \approx 16\,\text{km/s}$.]

A perturbation of $<0.01$ on the main galaxy indicates the extreme level of sensitivity to perturbations of this strong-lensing methodology

Vegetti et al. 2012, Nature
Detecting Lower-Mass Substructures

Substructure as a parametric model

\[ M_{sub} = (1.9 \pm 0.01) \times 10^8 M_\odot \]
\[ r_t = 440 \text{pc} \]
\[ \sigma_v \sim 16 \text{ km s}^{-1} \]
\[ L_V \leq 5 \times 10^8 L_\odot \]

The host galaxy has an isothermal profile

Vegetti et al. 2012, Nature
Constraints on the mass-function by combining the DR & B1938+666 Results

\[ P(\alpha, f \mid \{n_s, m\}, p) = \frac{\mathcal{L}(\{n_s, m\} \mid \alpha, f, p) P(\alpha, f \mid p)}{P(\{n_s, m\} \mid p)} \]

Within the inner 5 kpc

\[ f = 3.33^{+3.64}_{-1.81} \%
\]
\[ \alpha = 1.06^{+0.56}_{-0.44} \]
\[ f = 1.21^{+0.6}_{-0.6} \%
\]
\[ \alpha = 1.87^{+0.08}_{-0.04} \]
\[ f_{CDM} \sim 0.1\% \]
How many lenses are needed to quantify the substructure mass function?

Several hundred (~200) lenses with extended rings/arcs are needed (comparable to the DR) to quantify $f_{\text{sub}}$ (to $<<1\%$) and the mass-function slope.

New instruments (e.g. Herschel/ALMA, Euclid/JDEM, LSST, SKA, etc) can provide these numbers of new strong galaxy-scale lenses in the next 5-10 years.

Vegetti & Koopmans 2009
Constraints on the mass-function by combining the DR & B1938+666 Results

Also our results give consistently more mass substructure toward lens galaxies (as D&K02) compared to simulations.

Why?

Are CDM simulations wrong or is there something else?
Contamination by L.O.S. objects?

Contamination along the l.o.s. could be substantial. But the jury is still out on this!

Xu et al.
Contamination if there can help!

Boost of substructures!
A few (~10) lenses are enough to pin down $f_{\text{sub}}$!