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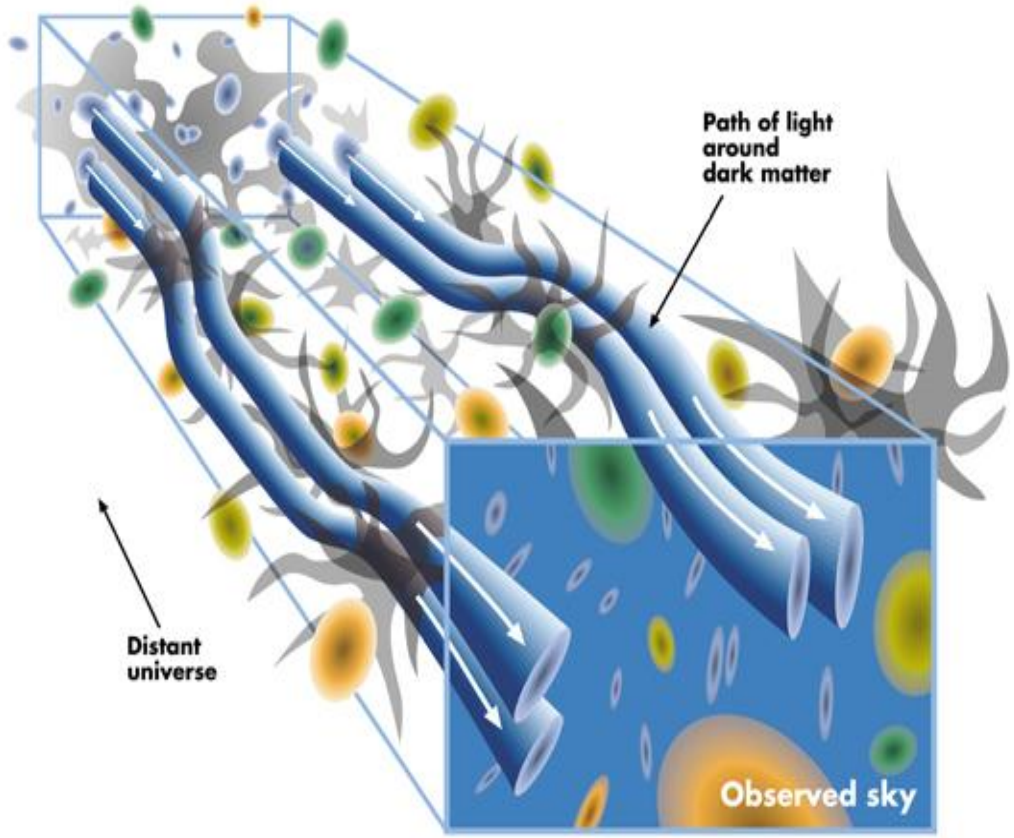
COSMIC SHEAR TOMOGRAPHY AND EFFICIENT DATA COMPRESSION USING COSEBIS

COSMIC SHEAR

- Structures distort light in its path
- The shape of the galaxies are the first observable
- Studying cosmic shear, reveals the cosmological parameters
- Shear Correlation functions are one of the basic products of observations.

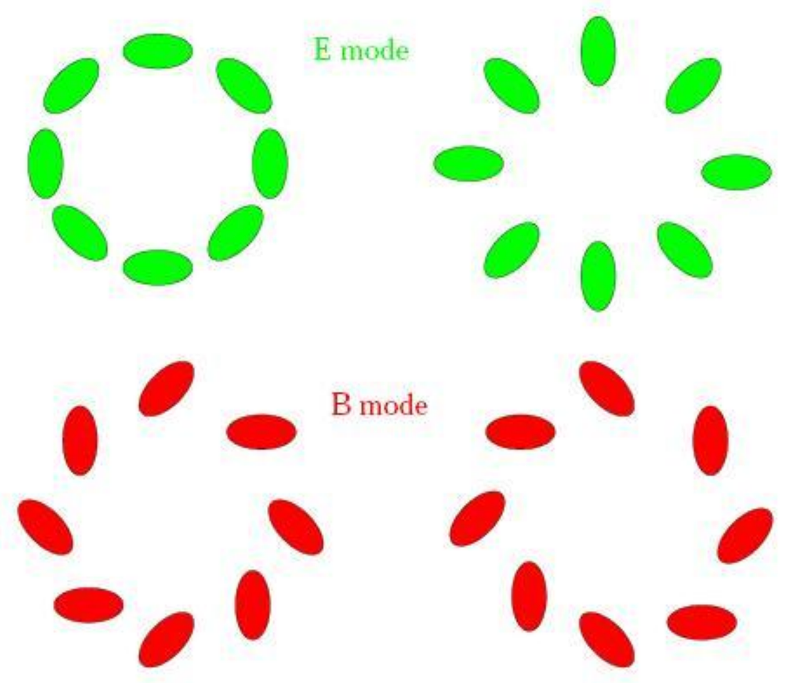


E-/B-MODES

- The E-modes can be generated from lensing

  (The curl free modes)
- The B-modes have non-lensing origin

  (The divergence free modes)

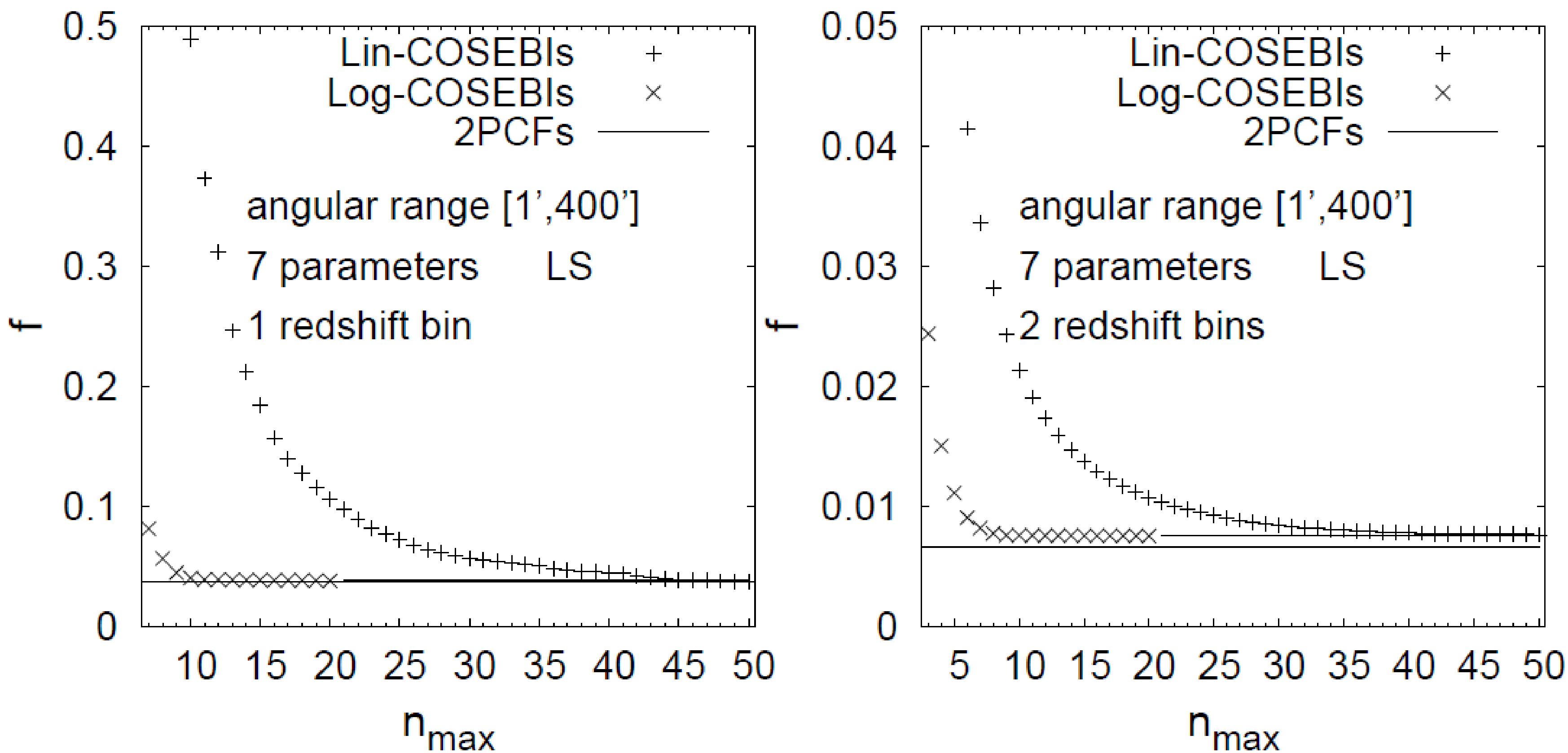


THE COSEBIS

- Complete Orthogonal Sets of E-/B-Integrals
- They are constructed by complete sets of bases for the filter function space
- The linear and logarithmic COSEBIs filters are polynomials
- The linear COSEBIs filter functions are the Legendre Polynomials of the 4th order and higher + two other lower order polynomials.
- The logarithmic COSEBIs are polynomials in $\ln(\theta)$
- A finite number of them is essentially sufficient to get the full information available

INFORMATION LEVEL VS. NUMBER OF COSEBIS MODES

- Fewer log-COSEBIs are needed to reach a saturated value compared to linear ones
- The lin- and log-COSEBIs reach the same saturated value
- The 2PCFs value is always lower



CONCLUSIONS

- COSEBIs is a very compact way of cosmic shear analysis
- A relatively small number of COSEBIs modes is enough to reach the full information level
- Adding tomography greatly increases the information about the parameters up to a certain number of redshift bins
- 7 parameters can be constrained simultaneously with the data from the large surveys of the future
- Most of the cosmic shear information is in small scales
- There is some independent information in large scales
- LS constraints are tighter compared to MS
- LS needs more modes to converge
- Marginalizing over parameters mixes their behavior
- Adding priors helps further tighten the constraints
- Each parameter behaves differently

E/B

$$\left\langle \hat{\kappa}(\boldsymbol{\ell})\hat{\kappa}^*(\boldsymbol{\ell}')\right\rangle = (2\pi)^2\,\delta_{\rm D}(\boldsymbol{\ell}-\boldsymbol{\ell}')\,P_\kappa(\ell)$$

$$\kappa = \kappa^{\rm E} + {\rm i}\kappa^{\rm B}$$

$$\left\langle \hat{\kappa}^{\rm E}(\boldsymbol{\ell})\hat{\kappa}^{\rm E*}(\boldsymbol{\ell}')\right\rangle = (2\pi)^2\,\delta_{\rm D}(\boldsymbol{\ell}-\boldsymbol{\ell}')\,P_{\rm E}(\ell)$$
$$\left\langle \hat{\kappa}^{\rm B}(\boldsymbol{\ell})\hat{\kappa}^{\rm B*}(\boldsymbol{\ell}')\right\rangle = (2\pi)^2\,\delta_{\rm D}(\boldsymbol{\ell}-\boldsymbol{\ell}')\,P_{\rm B}(\ell)$$
$$\left\langle \hat{\kappa}^{\rm E}(\boldsymbol{\ell})\hat{\kappa}^{\rm B*}(\boldsymbol{\ell}')\right\rangle = (2\pi)^2\,\delta_{\rm D}(\boldsymbol{\ell}-\boldsymbol{\ell}')\,P_{\rm EB}(\ell)$$

CONSTRUCTING PRACTICAL STATISTICS II
THE FILTER FUNCTIONS DEFINED ON FINITE INTERVALS

$$\int_0^\infty \mathrm{d}\vartheta\,\vartheta\,T_+(\vartheta)\,\mathrm{J}_0(\ell\vartheta) = \int_0^\infty \mathrm{d}\vartheta\,\vartheta\,T_-(\vartheta)\,\mathrm{J}_4(\ell\vartheta)$$

$$T_+(\vartheta) = T_-(\vartheta) + \int_\vartheta^\infty \mathrm{d}\theta\,\theta\,T_-(\theta)\left(\frac{4}{\theta^2} - \frac{12\vartheta^2}{\theta^4}\right)$$

$$T_-(\vartheta) = T_+(\vartheta) + \int_0^\vartheta \mathrm{d}\theta\,\theta\,T_+(\theta)\left(\frac{4}{\vartheta^2} - \frac{12\theta^2}{\vartheta^4}\right)$$

- There are infinite number of functions satisfying these conditions

$$\int_{\vartheta_{\mathrm{min}}}^{\vartheta_{\mathrm{max}}} \frac{\mathrm{d}\vartheta}{\vartheta}\,T_-(\vartheta) = 0 = \int_{\vartheta_{\mathrm{min}}}^{\vartheta_{\mathrm{max}}} \frac{\mathrm{d}\vartheta}{\vartheta^3}\,T_-(\vartheta)$$

$$\int_{\vartheta_{\mathrm{min}}}^{\vartheta_{\mathrm{max}}} \mathrm{d}\vartheta\,\vartheta\,T_+(\vartheta) = 0 = \int_{\vartheta_{\mathrm{min}}}^{\vartheta_{\mathrm{max}}} \mathrm{d}\vartheta\,\vartheta^3\,T_+(\vartheta)$$

CONSTRUCTING PRACTICAL STATISTICS I

- Any other estimator should be constructed from the correlations.
- The general form:

$$E = \int_0^\infty \mathrm{d}\vartheta \, \vartheta \left[\xi_+(\vartheta) T_+(\vartheta) + \xi_-(\vartheta) T_-(\vartheta)\right]$$

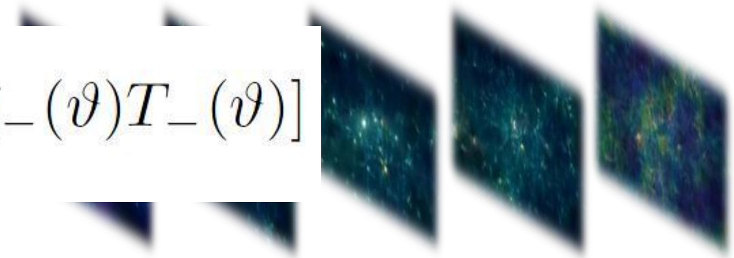
$$B = \int_0^\infty \mathrm{d}\vartheta \, \vartheta \left[\xi_+(\vartheta) T_+(\vartheta) - \xi_-(\vartheta) T_-(\vartheta)\right]$$

$$\xi_+(\vartheta) = \int_0^\infty \frac{\mathrm{d}\ell \, \ell}{2\pi} \mathrm{J}_0(\ell\vartheta) \left[P_\mathrm{E}(\ell) + P_\mathrm{B}(\ell)\right]$$

$$\xi_-(\vartheta) = \int_0^\infty \frac{\mathrm{d}\ell \, \ell}{2\pi} \mathrm{J}_4(\ell\vartheta) \left[P_\mathrm{E}(\ell) - P_\mathrm{B}(\ell)\right]$$

$$\int_0^\infty \mathrm{d}\vartheta \, \vartheta \, T_+(\vartheta) \, \mathrm{J}_0(\ell\vartheta) = \int_0^\infty \mathrm{d}\vartheta \, \vartheta \, T_-(\vartheta) \, \mathrm{J}_4(\ell\vartheta)$$

THE MATHEMATICAL FORM



$$E = \int_0^\infty \mathrm{d}\vartheta \, \vartheta \left[\xi_+(\vartheta) T_+(\vartheta) + \xi_-(\vartheta) T_-(\vartheta)\right]$$

$$E_n^{ij} = \int_0^\infty \frac{\mathrm{d}\ell \, \ell}{2\pi} P_{\mathrm{E}}^{ij}(\ell) \, W_n(\ell)$$

$$W_n(\ell) = \int_{\vartheta_{\min}}^{\vartheta_{\max}} \mathrm{d}\vartheta \, \vartheta \, T_{+n}(\vartheta) \, \mathrm{J}_0(\ell\vartheta)$$

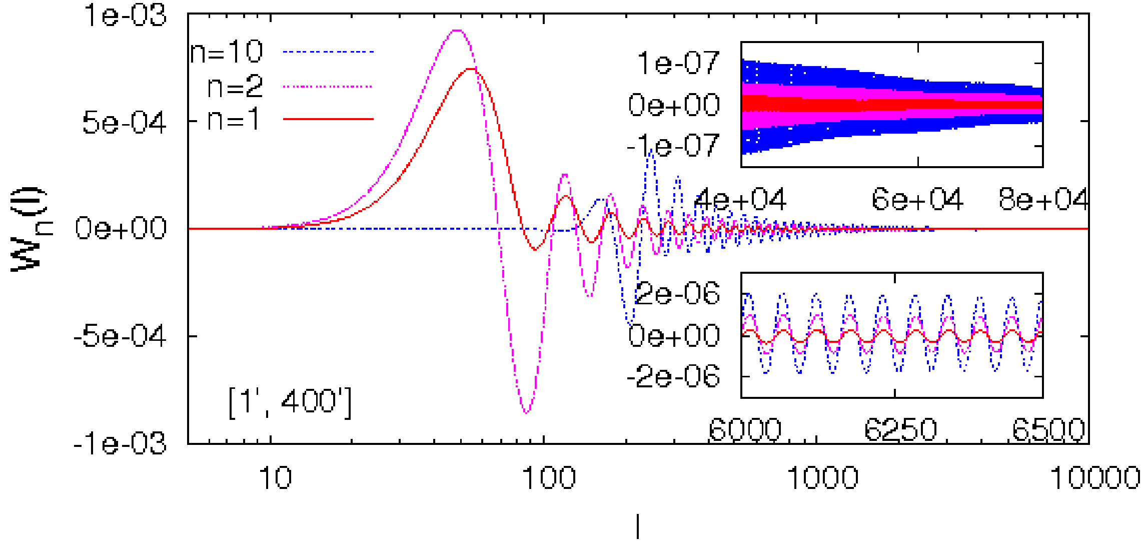
$$P_{\mathrm{E}}^{ij}(\ell) = \frac{9 H_0^4 \Omega_{\mathrm{m}}^2}{4c^4} \int_0^{\chi_{\mathrm{h}}} \mathrm{d}\chi \, \frac{g^j(\chi) g^i(\chi)}{a^2(\chi)} \, P_\delta\left(\frac{\ell}{f_K(\chi)}, \chi\right)$$

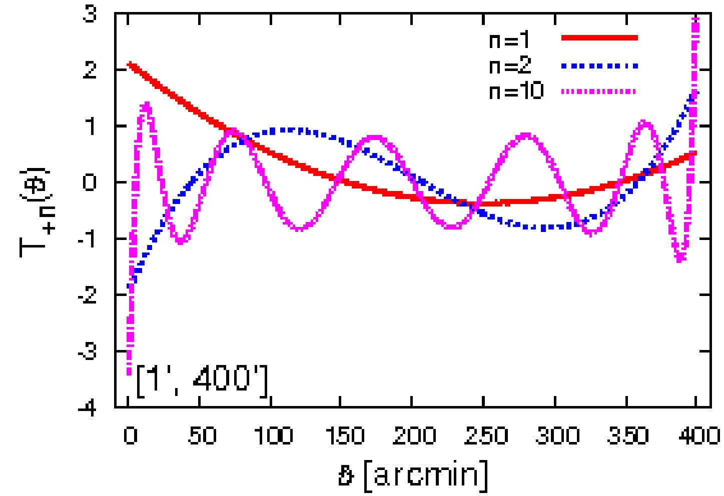
$$g^i(\chi) = \int_\chi^{\chi_{\mathrm{h}}} \mathrm{d}\chi' \, p_\chi^i(\chi') \frac{f_K(\chi' - \chi)}{f_K(\chi')}$$

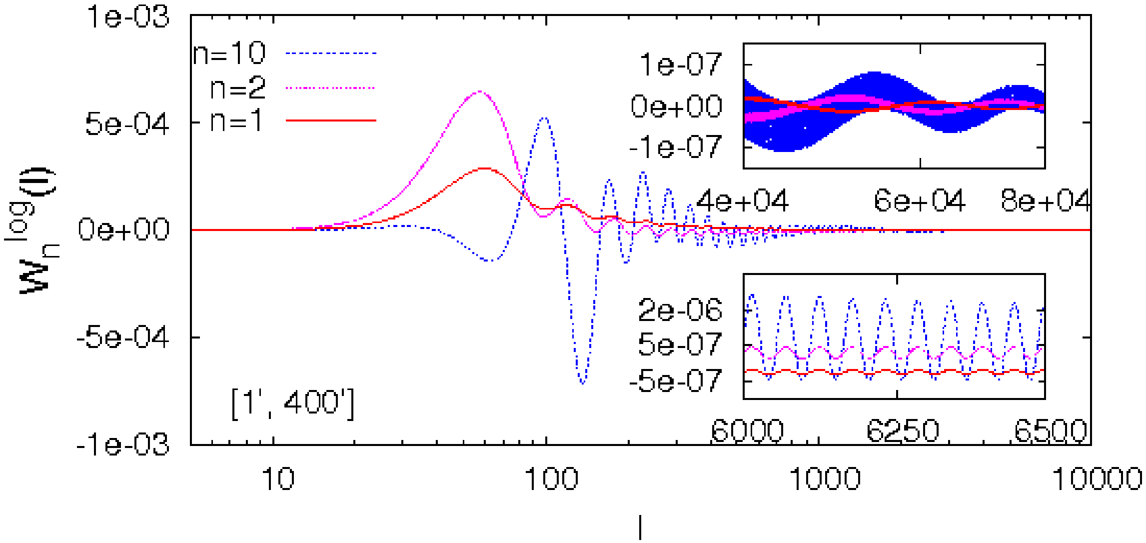
- $i=j$ ➡ Auto-correlation $i \neq j$ ➡ Cross-correlation

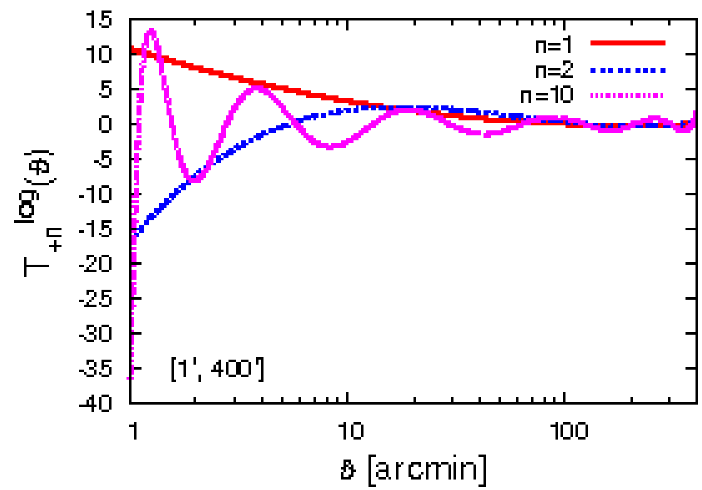
FILTER FUNCTIONS

$$W_n(\ell) = \int_{\vartheta_{\min}}^{\vartheta_{\max}} \mathrm{d}\vartheta\, \vartheta\, T_{+n}(\vartheta)\, \mathrm{J}_0(\ell\vartheta)$$



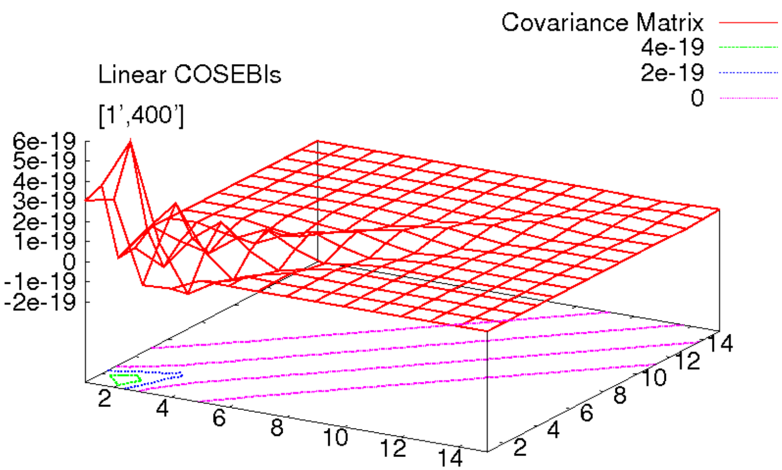


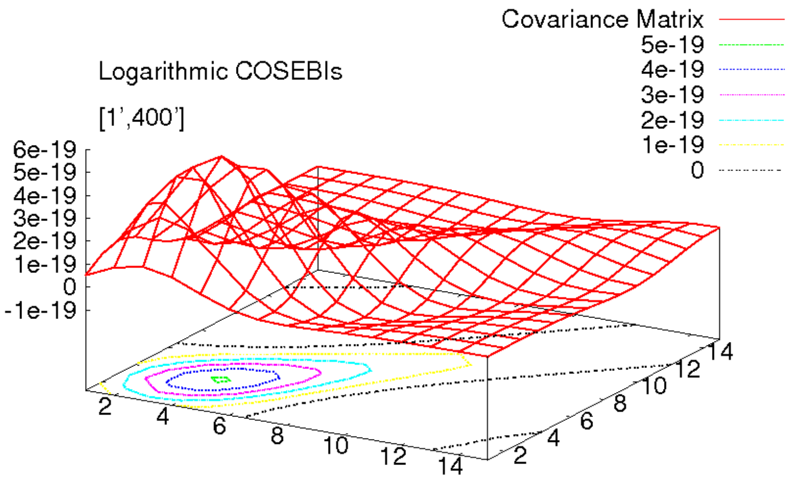




COVARIANCE

- The COSEBIs covariance is a band like matrix

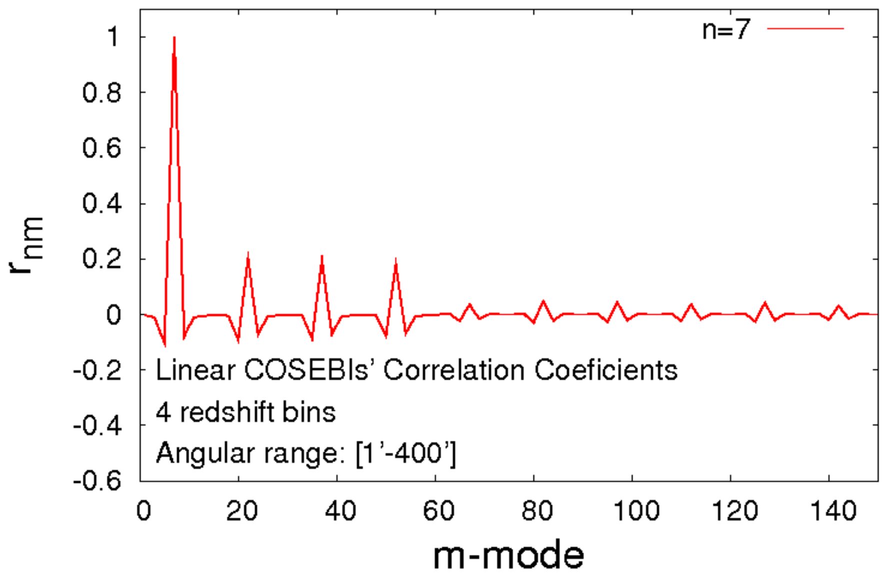


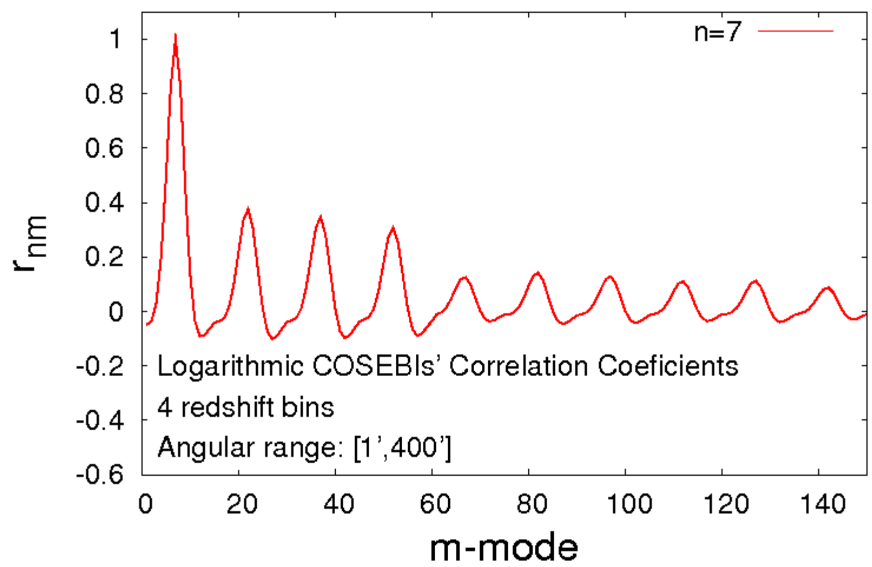


$$C_{mn}^{\mathrm{X}} = \frac{1}{\pi A} \int_0^\infty \mathrm{d}\ell \, \ell \, W_m(\ell) W_n(\ell) \left(P_{\mathrm{X}}(\ell) + \frac{\sigma_\epsilon^2}{2\bar{n}} \right)^2$$

CORRELATION COEFFICIENTS

- Correlation coefficients are the normalized covariance elements
- Each pick corresponds to correlations between the same pair of COSEBIs modes
- 4 redshift bins and 15 COSEBIs modes are used here





STATISTICAL METHOD: FISHER ANALYSIS

- Fisher matrix is related to the log-likelihood function
- Quantifies the shape and size of the confidence regions

$$F_{ij} = \langle \mathfrak{L}_{,ij} \rangle = \frac{1}{2}\mathrm{Tr}[C^{-1} M_{ij}]$$

$$M_{ij} = \mathbf{E}_{,i}\, \mathbf{E}_{,j}^{\mathrm{T}} + \mathbf{E}_{,j}\, \mathbf{E}_{,i}^{\mathrm{T}}$$

- Our figure-of-merit is a measure of the geometric mean of the standard deviations of the parameters

$$f = \left(\frac{1}{\sqrt{\det F}} \right)^{1/n_p}$$

OTHER CRITERIA

- Prior: The parameter covariance of WMAP7 from a Population Monte Carlo (PMC) run by Martin Kilbinger is our prior
- Angular ranges: [1’:20’], [20’:400’],[1’:400’]
- Handling the parameters: fixing or marginalizing.

OLD METHODS FOR E-/B-SEPARATION

APERTURE MASS DISPERSION

- Can be defined as E and B mode estimators
- The aperture has a finite support inside a circle
- The correlations must be measured to very small separations

RING STATISTICS

- The Ring Statistics filters are defined on a finite interval
- The Ring Statistics cleanly separates E- and B-modes
- The signal is low
- The filter functions are not mathematically beautiful

The motivation: Future Surveys

- Better precision
- Larger fields of view
- Deeper Images
- More accurate redshifts

2 POINT CORRELATION FUNCTIONS (2PCFS)

- On average shear is zero on the sky
- We need to go to two point statistics and higher
- The 2PCFs are

$$\xi_{\pm}(\theta) = \langle \gamma_t \gamma_t \rangle (\theta) \pm \langle \gamma_\times \gamma_\times \rangle (\theta)$$

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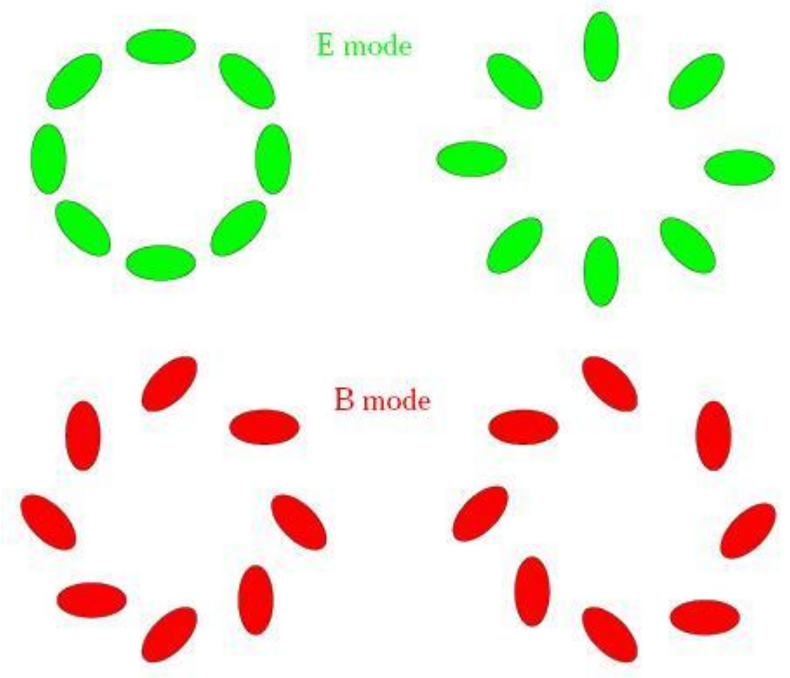

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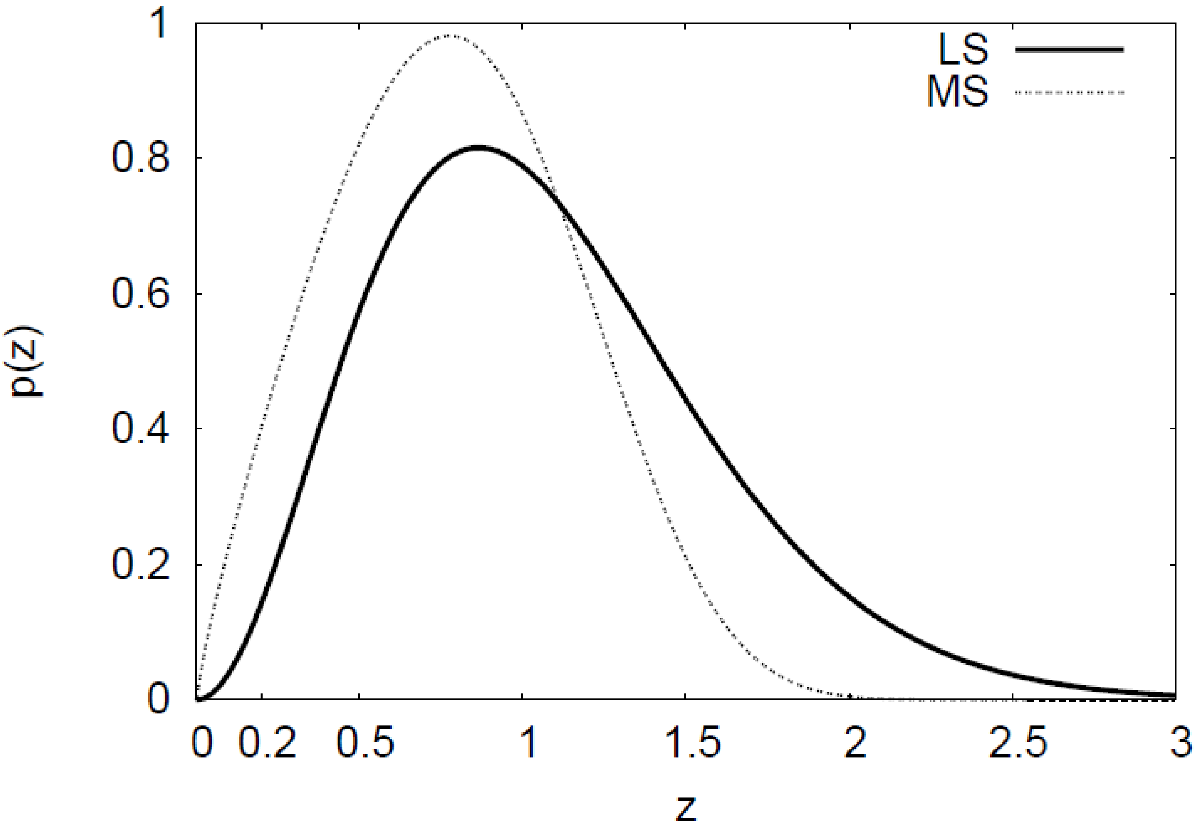
THE 2 SURVEYS: MEDIUM AND LARGE



	z-distribution parameters					survey parameters		
	α	β	z_0	$z_{\min}$	$z_{\max}$	A	σ_ϵ	$\bar{n}$
MS	0.836	3.425	1.171	0.2	1.5	170	0.42	13.3
LS	2.0	1.5	0.71	0.0	2.0	20000	0.3	35

COSMOLOGICAL MODEL

- wCDM model:
- Dark energy with an equation of state parameter w_0
- And cold dark matter
- 7 free parameters in total with fiducial values of

σ_8	Ω_m	Ω_Λ	w_0	n_s	h	Ω_b
0.8	0.27	0.73	-1.0	0.97	0.70	0.045

TOMOGRAPHY

- 3-4 redshift bins are enough to capture all the information
- Priors do not have a big effect on LS for fixed parameters
- Priors flatten the curves and tighten the constraints