Cosmological and astrophysical applications of (possible) vacuum quantum corrections

Ilya L. Shapiro

Departamento de Física, Universidade Federal de Juiz de Fora, MG, Brazil

Based on collaborations with

$\boxed{\text{P. Letelier}}$ (UNICAMP), D. Rodrigues, J. Fabris, A. Toribio (UFES)

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Contents:

- Effective Action of vacuum.
- Quantum corrections to vacuum action for massive fields.
- Renormalization Group for ρ_Λ and G.
- Interpretation of μ in case of galaxies.
- Do we have a chance for an alternative concordance model?.

Effective Action of vacuum for QFEXT / gravity.

Independent on whether gravity should be quantized or not, we know that the matter fields should.

It is reasonable to ask whether the quantum effects of matter fields are capable to produce significant effects on the astrophysical or cosmological scale.

At quantum level the dynamics of gravity is governed by the Effective Action of vacuum $\Gamma[g_{\mu\nu}]$.

$$e^{i\Gamma[g_{\mu\nu}]} = \int d\Phi e^{iS[\Phi,\, g_{\mu\nu}]}\,, \quad \Phi = \{\text{matter fields}\}\,.$$

In case of renormalizable theory

$$S[\Phi,\, g_{\mu\nu}] = S_{vac}[g_{\mu\nu}] + S_m[\Phi,\, g_{\mu\nu}] \quad \Longrightarrow \quad \Gamma[g_{\mu\nu}] = S_{vac}[g_{\mu\nu}] + \bar{\Gamma}[g_{\mu\nu}]\,.$$

In case of renormalizable theory

$$S_{vac} \,=\, S_{EH} \,+\, S_{HD}\,, \quad S_{EH} \,=\, -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R+2\Lambda)\,,$$

and S_{HD} includes higher derivative terms.

$$S_{HD} \,=\, \int d^4x\sqrt{-g}\,\left\{a_1C^2 + a_2E + a_3\Box R + a_4R^2\right\}\,.$$

Here

$$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3\,R^2$$

is the square of the Weyl tensor and

$$E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4\,R_{\alpha\beta}R^{\alpha\beta} + R^2$$

the integrand of the Gauss-Bonnet topological invariant.

The main problem is to evaluate $\bar{\Gamma}[g_{\mu\nu}]$, at least at 1-loop.

Massive fields are more complicated and interesting.

And especially if we are interested in the low-energy effects, decause one has to account for the decoupling phenomenon.

High energy QED: $$-\frac{e^2}{4}F_{\mu\nu}F^{\mu\nu}+\frac{e^4}{3(4\pi)^2}\,F_{\mu\nu}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,F^{\mu\nu}\,.$$

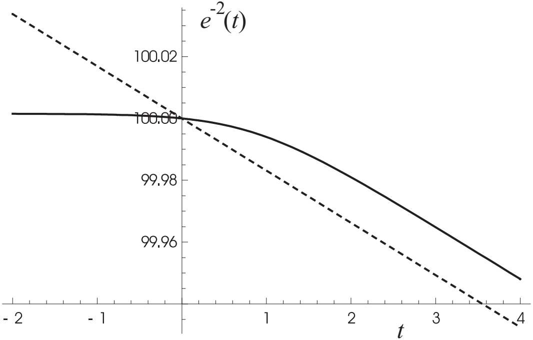
At high energy limit we meet a standard (MS) β-function and at low energies there is quadratic decoupling.

UV limit $$p^2\gg m^2\;\Longrightarrow\;\beta_e^{1\;UV}=\frac{4\,e^3}{3\,(4\pi)^2}+\mathcal{O}\Big(\frac{m^2}{p^2}\Big)\,.$$

IR limit $$p^2\ll m^2\;\Longrightarrow\;\beta_e^{1\;IR}=\frac{e^3}{(4\pi)^2}\cdot\frac{4\,p^2}{15\,m^2}\;+\;\mathcal{O}\Big(\frac{p^4}{m^4}\Big)\,.$$

Appelquist and Carazzone decoupling theorem (PRD, 1977).

General expression interpolates between UV and IR.



These plots show the effective electron charge as a function of $\log(\mu/\mu_0)$ in the case of the MS-scheme, and for the momentum-subtraction scheme, with $\ln(p/\mu_0)$.

An interesting high-energy effect is a small apparent shift of the initial value of the effective charge.

Similar results can be obtained for gravity.

E.g., for a massive scalar field (*Gorbar & I.Sh., JHEP, 2003*).

$$\beta_1 \,=\, -\,\frac{1}{(4\pi)^2}\,\left(\,\frac{1}{18a^2}\,-\,\frac{1}{180}\,-\,\frac{a^2-4}{6a^4}\,A\right)\,.$$

Then

$$\beta_1^{UV} = -\frac{1}{(4\pi)^2}\frac{1}{120} + \mathcal{O}\left(\frac{m^2}{p^2}\right) = \beta_1^{\overline{MS}} + \mathcal{O}\left(\frac{m^2}{p^2}\right),$$

$$\beta_1^{IR} \,=\, -\,\frac{1}{1680\,(4\pi)^2}\,\cdot\,\frac{p^2}{m^2}\,+\,\mathcal{O}\left(\frac{p^4}{m^4}\right)\,,$$

This is the Appelquist & Carazzone Theorem for gravity

However, in the momentum-subtraction scheme $\beta_G = \beta_\Lambda = 0\,.$

In the gravitational sector we meet Appelquist and Carazzone - like decoupling, but only in the higher derivative sectors. In the perturbative approach, with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we do not see running for the cosmological and inverse Newton constants.

Why do we get $\beta_\Lambda = \beta_{1/G} = 0$ **?**

Momentum subtraction running corresponds to the insertion of, e.g., $\ln(\Box/\mu^2)$ **formfactors into effective action.**

Say, in QED: $$-\frac{e^2}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^4}{3(4\pi)^2}\,F_{\mu\nu}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,F^{\mu\nu}\,.$$

Similarly, one can insert formfactors into

$$C_{\mu\nu\alpha\beta}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,C_{\mu\nu\alpha\beta}\,.$$

However, such insertion is impossible for Λ and for $1/G$, because $\Box\Lambda \equiv 0$ and $\Box R$ is a full derivative.

Further discussion:

Ed. Gorbar & I.Sh., JHEP (2003); J. Solà & I.Sh., PLB (2009).

Is it true that physical $\beta_\Lambda = \beta_{1/G} = 0$?

Probably not. Perhaps the linearized gravity approach is simply not an appropriate tool for the CC and Einstein terms.

Let us use the covariance arguments. $\Gamma[g_{\mu\nu}]$ can not include odd terms in metric derivatives. In the cosmological setting this means no $\mathcal{O}(H)$ and also no $\mathcal{O}(H^3)$ terms, etc. Hence

$$\rho_\Lambda(H) = \frac{\Lambda(H)}{16\pi G(H)} = \rho_\Lambda(H_0) + C\left(H^2 - H_0^2\right).$$

Then the conservation law for $G(H;\nu)$ gives

$$G(H;\nu) = \frac{G_0}{1+\nu\,\ln\left(H^2/H_0^2\right)}, \quad \textbf{where} \quad G(H_0) = G_0 = \frac{1}{M_P^2}.$$

Here we used the identification

$$\mu \sim H \qquad \textbf{in the cosmological setting.}$$

A small note on the Cosmological Constant (CC) Problem.

The main relation is $\rho_\Lambda^{obs} = \rho_\Lambda^{vac}(\mu_c) + \rho_\Lambda^{ind}(\mu_c)\,.$

ρ_Λ^{obs} **which is likely observed in SN-Ia, LSS, CMB, etc is**

$$\rho_\Lambda^{obs}(\mu_c) \;\approx\; 0.7\,\rho_c^0 \;\propto\; 10^{-47}\,GeV^4\,.$$

The CC Problem is that the magnitudes of $\rho_\Lambda^{vac}(\mu_c)$ **and** $\rho_\Lambda^{ind}(\mu_c)$ **are a huge 55 orders of magnitude greater than the sum!**

Obviously, these two huge terms do cancel.
"Why they cancel so nicely" is the CC Problem (Weinberg, 1989).

We assume a phenomenological position and don't try solving CC problems. Instead we consider whether CC may vary due to IR quantum effects, e.g., the ones of matter fields.

The same $\rho_\Lambda(\mu)$ immediately follows from the assumption of the Appelquist and Carazzone - like decoupling for CC.

A.Babic, B.Guberina, R.Horvat, H.Štefančić, PRD 65 (2002); I.Sh., J.Solà, JHEP 02 (2002).

We know that for a single particle

$$\beta_\Lambda^{MS}(m) \sim m^4 ,$$

hence the quadratic decoupling gives

$$\beta_\Lambda^{IR}(m) = \frac{\mu^2}{m^2}\, \beta_\Lambda^{MS}(m) \sim \mu^2 m^2 .$$

The total beta-function will be given by algebraic sum

$$\beta_\Lambda^{IR} = \sum k_i \mu^2 m_i^2 = \sigma M^2 \mu^2 \propto \frac{3\nu}{8\pi}\, M_P^2\, H^2 .$$

This leads to the same result in the cosmological setting,

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi}\, M_p^2 \left(H^2 - H_0^2\right) .$$

One can also obtain the same $G(\mu)$ in a different way.

I.Sh., J. Solà, JHEP (2002); C. Farina, I.Sh. et al, PRD (2011).

Consider $\overline{\mathrm{MS}}$-based renormalization group equation for $G(\mu)$:

$$\mu\frac{dG^{-1}}{d\mu} = \sum_{particles} A_{ij}\, m_i\, m_j = 2\nu\, M_P^2\,, \qquad G^{-1}(\mu_0) = G_0^{-1} = M_P^2\,.$$

Here the coefficients A_{ij} depend on the coupling constants, m_i are masses of all particles. In particular, at one loop,

$$\sum_{particles} A_{ij}\, m_i\, m_j = \sum_{fermions} \frac{m_f^2}{3(4\pi)^2} - \sum_{scalars} \frac{m_s^2}{(4\pi)^2}\Big(\xi_s - \frac{1}{6}\Big)\,.$$

One can rewrite it as

$$\mu\frac{d(G/G_0)}{d\mu} = -\,2\nu\,(G/G_0)^2\,, \quad \Longrightarrow \quad G(\mu) = \frac{G_0}{1+\nu\,\ln\left(\mu^2/\mu_0^2\right)}\,. \quad (*)$$

It is the same formula which results from covariance and/or from AC-like quadratic decoupling for the CC plus conservation law. All in all, (*) seems to a unique possible form of a relevant $G(\mu)$.

All in all, it is not a surprise that the eq.

$$G(\mu) = \frac{G_0}{1 + \nu \ln \left(\mu^2 / \mu_0^2 \right)} \,.$$

emerges in different approaches to renorm. group in gravity:

- **Higher derivative quantum gravity.**
A. Salam and J. Strathdee, PRD (1978);
E.S. Fradkin and A. Tseytlin, NPB (1982).

- **Quantum gravity with (hipothetic) UV-stable fixed point.**
A. Bonanno and M. Reuter, PRD (2002).

- **Semiclassical gravity.**
B.L. Nelson and P. Panangaden, PRD (1982).

So, we arrived at the two relations:

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi} M_p^2 \left(\mu^2 - \mu_0^2\right) \qquad (1)$$

and $$G(\mu) = \frac{G_0}{1 + \nu \ln\left(\mu^2/\mu_0^2\right)} \,. \qquad (2)$$

Remember the standard identification

$\mu \sim H$ **in the cosmological setting.**

A. Babic, B. Guberina, R. Horvat, H. Štefančić, PRD (2005).

Cosmological models based on the assumption of the standard AC-like decoupling for the cosmological constant:

Models with (1) and energy matter-vacuum exchange:
I.Sh., J.Solà, Nucl.Phys. (PS), IRGA-2003;
I.Sh., J.Solà, C.España-Bonet, P.Ruiz-Lapuente, PLB (2003).

- **Models with (1), (2) and without matter-vacuum exchange:**

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

- **Models with constant $G \equiv G_0$ and permitted energy exchange between vacuum and matter sectors.**

For the equation of state $P = \alpha\rho$ the solution is analytical,

$$\rho(z;\nu) = \rho_0\,(1+z)^r\,,$$

$$\rho^\Lambda(z;\nu) = \rho_0^\Lambda + \frac{\nu}{1-\nu}\,[\,\rho(z;\nu) - \rho_0\,]\,,$$

The limits from density perturbations / LSS data $|\nu| < 10^{-6}$.

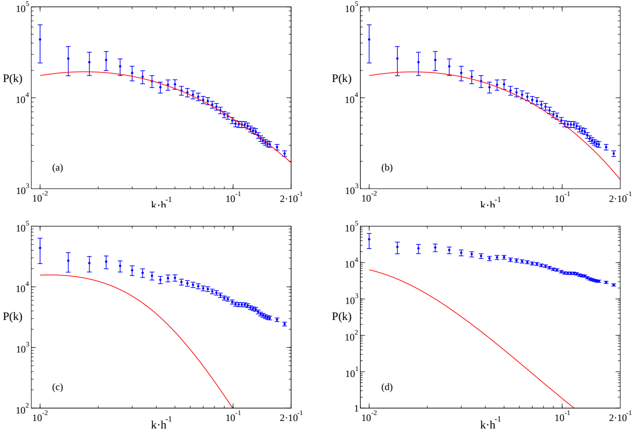
Analog models:
Opher & Pelinson, PRD (2004); Wang & Meng, Cl.Q.Gr. 22 (2005).

Direct analysis of cosmic perturbations:
J. Fabris, I.Sh., J. Solà, JCAP 0702 (2007).

Given the Harrison-Zeldovich initial spectrum, the power spectrum today can be obtained by integrating the eqs. for perturbations.

Initial data based on $w(z)$ from *J.M. Bardeen et al, Astr.J. (1986).*

Results of numerical analysis:



The ν-dependent power spectrum vs the LSS data from the 2dfFGRS. The ordinate axis represents $P(k) = |\delta_m(k)|^2$ where $\delta_m(k)$ is the solution at $z = 0$. In all cases
$(\Omega_B^0, \Omega_{DM}^0, \Omega_\Lambda^0) = 0.04, 0.21, 0.75)$ **&** $\nu = 10^{-8},\ 10^{-6},\ 10^{-4},\ 10^{-3}$.

● ● $G = G(H)$, **no energy exchange between vacuum and matter.**

$$\rho_\Lambda(H) \,=\, \rho_\Lambda(H_0) \,+\, \frac{3\nu}{8\pi}\, M_p^2 \left(H^2 - H_0^2\right).$$

$$G(H;\nu) = \frac{G_0}{1+\nu\,\ln\left(H^2/H_0^2\right)}\,, \quad \textbf{where} \quad G(H_0) = \frac{1}{M_P^2}\,.$$

I.Sh., J.Solà, H.Štefančić, JCAP (2005).
J.Grande, J.Solà, J.Fabris & I.Sh., Cl. Q. Grav. 27 (2010) .

An important general result is: In the models with variable Λ and G in which matter is covariantly conserved, the solutions of perturbation equations *do not* depend on the wavenumber k. As a consequence we meet relatively weak modifications of the spectrum compared to ΛCDM.

The bound $\nu < 10^{-3}$ comes just from the modification of $H(z)$.
R. Opher & A. Pelinson, astro-ph/0703779.
J.Grande, R.Opher, A.Pelinson, J.Solà, JCAP 0712 (2007)
The same restriction comes also from the BBN.

Can we apply the running $G(\mu)$ to other physical problems?

In the renormalization group framework the relation

$$G(\mu) = \frac{G_0}{1 + \nu \,\ln\left(\mu^2/\mu_0^2\right)}\,, \quad \textbf{where} \quad \mu = H$$

in the cosmological setting.

What could be an interpretation of μ in astrophysics?

Consider the rotation curves of galaxies. The simplest assumption is $\mu \propto 1/r$.

Applications for the point-like model of galaxy:

J.T.Goldman, J.Perez-Mercader, F.Cooper & M.M.Nieto, PLB (1992).
O. Bertolami, J.M. Mourao & J. Perez-Mercader, PLB 311 (1993).
M. Reuter & H. Weyer, PRD **70** *(2004); JCAP* **0412** *(2004).*
I.Sh., J.Solà, H.Štefančić, JCAP (2005).

We can safely restrict the consideration by a weakly varying G,

$$G = G_0 + \delta G = G_0(1+\kappa)\,, \qquad |\kappa| \ll 1\,.$$

We already know that the appropriate value of the parameter ν is small, the same should be with $\kappa = \delta G/G_0$.

In order to link the metric in the variable G case with the standard one, perform a conformal transformation

$$\bar{g}_{\mu\nu} = \frac{G_0}{G}\, g_{\mu\nu} = (1-\kappa) g_{\mu\nu}\,.$$

Up to the higher orders in κ, the metric $\bar{g}_{\mu\nu}$ satisfies usual Einstein equations with constant G_0.

The nonrelativistic limits of the two metrics

$$g_{00} = -1 - \frac{2\Phi}{c^2} \quad \text{and} \quad \bar{g}_{00} = -1 - \frac{2\Phi_{\text{Newt}}}{c^2} \quad ,$$

where Φ_{Newt} is the usual Newton potential and Φ is a potential of the modifies gravitational theory.

We have

$$g_{00} \;=\; -1 - \frac{2\Phi}{c^2} \;=\; (1+\kappa)\bar{g}_{00}$$

$$=\; (1+\kappa)(-1 - \frac{2\Phi_{\text{Newt}}}{c^2}) \approx\; -1 \;-\; \frac{2\Phi_{\text{Newt}}}{c^2} \;-\; \kappa$$

and, hence,

$$\Phi = \Phi_{\mathrm{Newt}} + \frac{c^2}{2}\,\kappa = \Phi_{\text{Newt}} + \frac{c^2\,\delta G}{2\,G_0}\,.$$

For the nonrelativistic limit of the modified gravitational force we obtain, therefore,

$$-\Phi^{,i} \;=\; -\Phi^{,i}_{\text{Newt}} \;-\; \frac{c^2\,G^{,i}}{2\,G_0}\,,$$

where we used the relation $G^{,i} = (\delta G)^{,i}$.

The last formula $\quad -\Phi^{,i} = -\Phi^{,i}_{\text{Newt}} - \dfrac{c^2\, G^{,i}}{2\, G_0}$ **is very instructive.**

● **Quantum correction comes with the factor of $c^2 \Longrightarrow$ can make real effect at the typical galaxy scale.**

E.g., for a point-like model of galaxy and $\mu \propto 1/r$ it is sufficient to have $\nu \approx 10^{-6}$ to provide flat rotation curves.

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

●● $\mu \propto 1/r$ **is, obviously, not a really good choice for a non-point-like model of the galaxy.**

The reason is that this identification produces the "quantum-gravitational" force even if there is no mass at all !!

What would be the "right" identification of μ ?

Let us come back to QFT, which offers a good hint:
μ **must be** $\sim$ **energy of the external gravitational line in the Feynman diagram in the almost-Newtonian regime.**

The phenomenologically good choice is

$$\frac{\mu}{\mu_0} = \left(\frac{\Phi_{\text{Newt}}}{\Phi_0}\right)^{\alpha},$$

where α **is a phenomenological parameter We have found that** α **is generally growing with the mass of the galaxy.**

D. Rodrigues, P. Letelier & I.Sh., JCAP (2010).

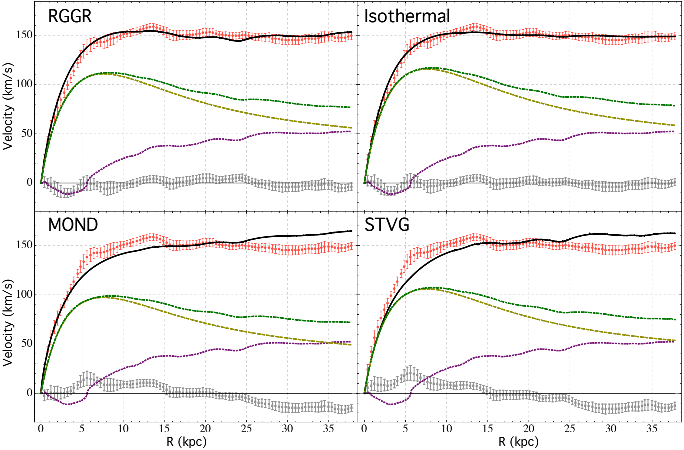
QFT viewpoint: α **reflects** $\mu \sim \Phi_{\text{Newt}}$ **is not an ultimate choice.**

With greater mass of the galaxy the "error" in identification becomes greater too, hence we need a greater α **to correct this.**
α **must be very small at the scale of the Solar system.**

Regular scale-setting procedure gives the same result:
S. Domazet & H. Štefančić, PLB (2011).

Last, but not least, the astro-ph application is impressively successful

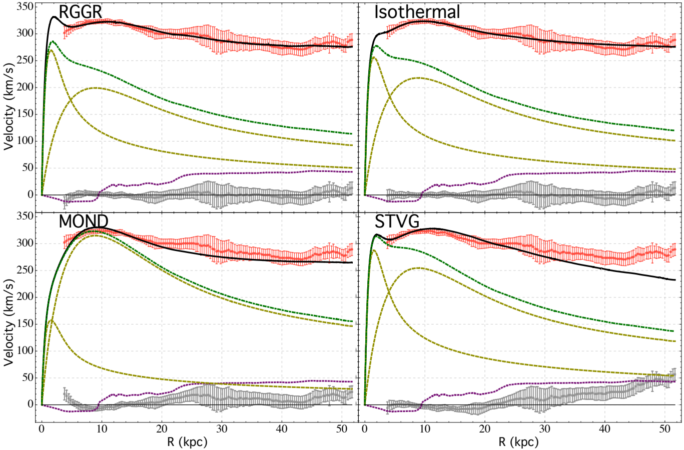
D. Rodrigues, P. Letelier & I.Sh., JCAP (2010). (9 samples)



Rotation curve of the spiral galaxy *NGC 3198.* $\alpha\nu = 1.7 \times 10^{-7}$.
[Collaboration THINGS (2008)].

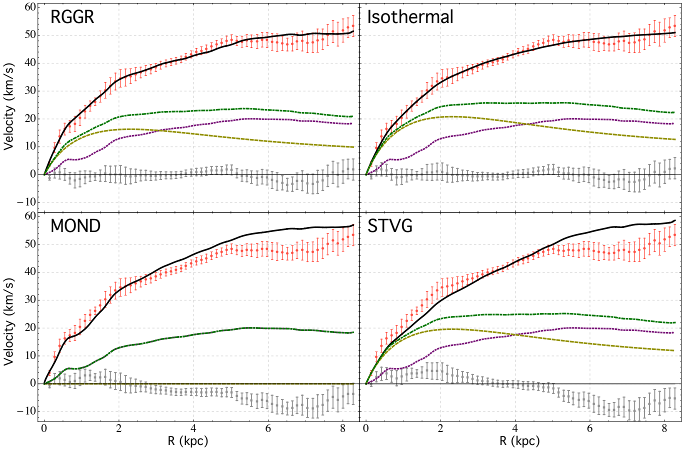
One more example, this time with descendent rotation curve.

$\alpha\nu = 6.7 \times 10^{-7}$.



Rotation curve of the galaxy *NGC 2841*. RGGR is based on hypothetical covariant quantum corrections without DM.

One more example: low-surface brightness galaxy with ascendent rotation curve. $\alpha\nu = 0.2 \times 10^{-7}$.



Rotation curve of the galaxy *DDO 154*. RGGR is based on hypothetical covariant quantum corrections without DM.

What about the Solar System?

C. Farina, W. Kort-Kamp, S. Mauro & I.Sh., PRD 83 (2011).

We used the dynamics of the Laplace-Runge-Lenz vector in the $G(\mu) = G_0/(1 + \mu \log(\mu/\mu_0))$ - **corrected Newton gravity.**

Upper bound for the Solar System: $\alpha\nu \leq 10^{-17}$.

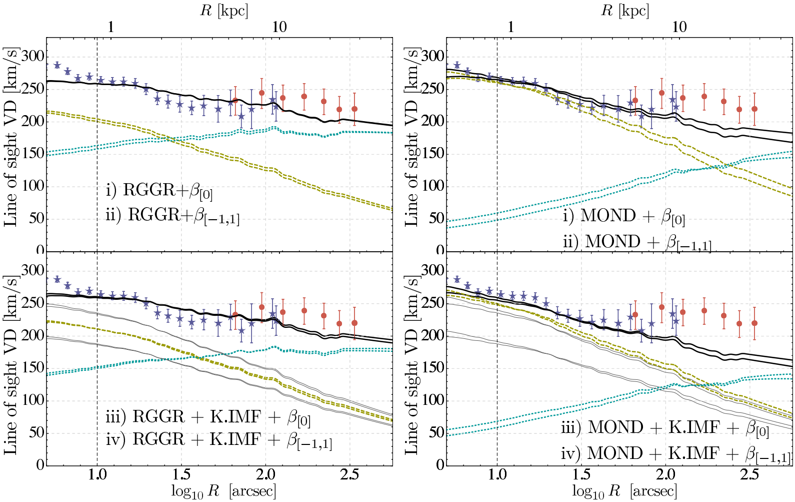
One of the works now on track: extending the galaxies sample.

P. Louzada, D. Rodrigues, J. Fabris, ..., in work: **50+ disk galaxies.**

Davi Rodrigues, JCAP (2012, in print): **elliptical galaxies.**

The general tendency which we observe so far is greater α **needed to for larger mass of the astrophysical object: from Solar System (upper bound) to biggest tested galaxies.**

Very new example. *Davi Rodrigues, JCAP, to appear.*



Rotation curve of the giant elliptic galaxy NGC 4374: RGGR vs MOND. $\alpha\nu = 17 \times 10^{-7}$. **Special thanks to PN.S. Collaboration.**

It looks like we do not need CDM to explain the rotation curves of the galaxies. However, does it really mean that we can really go on with one less dark component?

Maybe not, but it is worthwhile to check it. It is well known that the main requests for the DM come from the fitting of the LSS, CMB, BAO, lenthing etc.

However there is certain hope to relpace, e.g., ΛCDM by a ΛWDM (e.g. sterile neutrino) with much smaller Ω_{DM}.

The idea to trade $0.04, 0.23, 0.73 \quad \Longrightarrow \quad$ 0.04, 0.0x, 0.9(1-x)

Such a new concordance model would have less relevant coincidence problem, and in general such a possibility is interesting to verify.

First move:

J. Fabris, A. Toribio & I.Sh., Testing DM warmness and quantity via the RRG model. arXiv:1105.2275; PRD (2012).

We are using “our” Reduced Relativistic Gas model.

The Reduced Relativistic Gas model is a Simple cosmological model with relativistic gas.

G. de Berredo-Peixoto, I.Sh., F. Sobreira, Mod.Ph.Lett. A (2005); J. Fabris, I.Sh., F.Sobreira, JCAP (2009).

The model describes ideal gas of massive relativistic particles with all of them have the same kinetic energy.

The Equation of State (EOS) of such gas is

$$P = \frac{\rho}{3}\left[1 - \left(\frac{mc^2}{\varepsilon}\right)\right]^2 = \frac{\rho}{3}\left(1 - \frac{\rho_d^2}{\rho^2}\right).$$

In this formula ε is the kinetic energy of the individual particle, $\varepsilon = mc^2/\sqrt{1-\beta^2}$. Furthermore, $\rho_d = \rho_{d0}^2(1+z)^3$ is the mass (static energy) density. One can use one or another form of the equation of state (1), depending on the situation.

The deviation from Maxwell or relativistic Fermi-Dirac distribution is less than 2.5%. **The nice thing is that one can solve the Friedmann equation in this model analytically.**

The model was successfully used to impose an upper bound to the warmness of DM from LSS data, providing the same results as more complicated models.

J. Fabris, I.Sh., F.Sobreira, JCAP (2009).

So, why it is "our" and not just our model?

Because we were not first. The same EOS has been used by A.D. Sakharov in 1965. to predict the oscillations in the CMB spectrum for the first time!!

A.D. Sakharov, Soviet Physics JETP, 49 (1965) 345.

In the recent paper

J. Fabris, A. Toribio & I.Sh., Testing DM warmness and quantity via the RRG model. arXiv:1105.2275 [astro-ph.CO]; PRD-2012

we have used RRG without quantum effects to fit
Supernova type Ia (Union2 sample), $H(z)$**, CMB (**R **factor), BAO, LSS (2dfGRS data)**
In this way we confirm that Λ**CDM is the most favored model.**

However, for the LSS data alone we met the possibility of an alternative model with a small quantity of a WDM.

This output is potentially relevant due to the fact the LSS is the test which is not affected by the possible quantum RG running in the low-energy gravitational action.

Such a model almost has no issue with the coincidence problem, because $\Omega_\Lambda^0 \simeq 0.95$ **.**

Conclusions

- **The low-energy quantum corrections to the GR action can not be calculated within known QFT methods.**

- **At the same time the arguments based on covariance, dimension and quadratic decoupling indicate to the same, unique form of such quantum corrections, such that we have only one free parameter $\alpha\nu$. Then, $\nu = 0$ means no relevant quantum effects.**

- **The question of relevant quantum vacuum effects in IR reduce to existing-nonexisting paradigm.**

- **In the positive case we arrive at the cosmological and astrophysical model with potentially testable predictions, with a (small) chance for an alternative cosmic concordance model.**

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