Gravitational lensing

The early history

David Valls-Gabaud

Observatoire de Paris

XI School of cosmology
Cargèse
2012 Sep 17

Philosophy and history of science is as useful to scientists as ornithology is to birds.

Famous anonymous (ca 1970s)

Quand j'ouvre une Histoire des Sciences, j'ai l'impression de lire une sorte de roman. Non que tout m'y paraisse faux, mais tout m'y paraît factice - ni vrai ni faux, mais inauthentique [...]

G. Simon (2008)

When I read a book on the history of science, I feel like reading a novel. Not that all seems false, but rather bogus. Not right, not wrong, just untrue.

The term "*gravitational lens*" appears for the first time in a rather negative context :

[...] it is not permissible to say that the solar gravitational field acts as a lens, for it has no focal length.

Sir Oliver Lodge (1919)

Nature 104, 354

December *Phil. Mag.* (p. 737), the refractivity $\mu-1$, necessary at every point of a gravitational field to produce the Einstein deflection, is the ratio of the energy of a constant-mass particle fallen there from infinity to the energy of the same particle moving with the speed of light; but it is not permissible to say that the solar gravitational field acts like a lens, for it has no focal length. If the sun were backed by a nebula or any luminous area, the light grazing the rim all round would be brought to a focus at a place seventeen times the distance of Neptune, while light from any larger circle would focus still further off in proportion to the area of the circle. So from a uniformly luminous area there would result a focal *line* of constant brightness. The moon is, unfortunately, impotent to make an annular eclipse interesting.

For an extended solar atmosphere to produce the deflection, its density would have to vary with the inverse distance, which seems unlikely; but this is just the way in which an æther tension ought to vary in order to cause gravitation—as Newton knew. The extra æther-tension factor, μ^2-1, would be twice the refractivity.

Possibly the concluding sentence in the *Phil. Mag.* article above referred to is not expressed with sufficient clearness. Permit me to explain my points thus:—

(1) The quasi-elasticity of æther—the property which enables it to transmit light and to effect electrical discharge—is probably due to exceedingly fine-grained constitutional vorticity with high-speed circulation, as argued in my book "The Ether of Space." Consequently it would have facility for gyrostatic action, yielding a perpendicular result to an acting force.

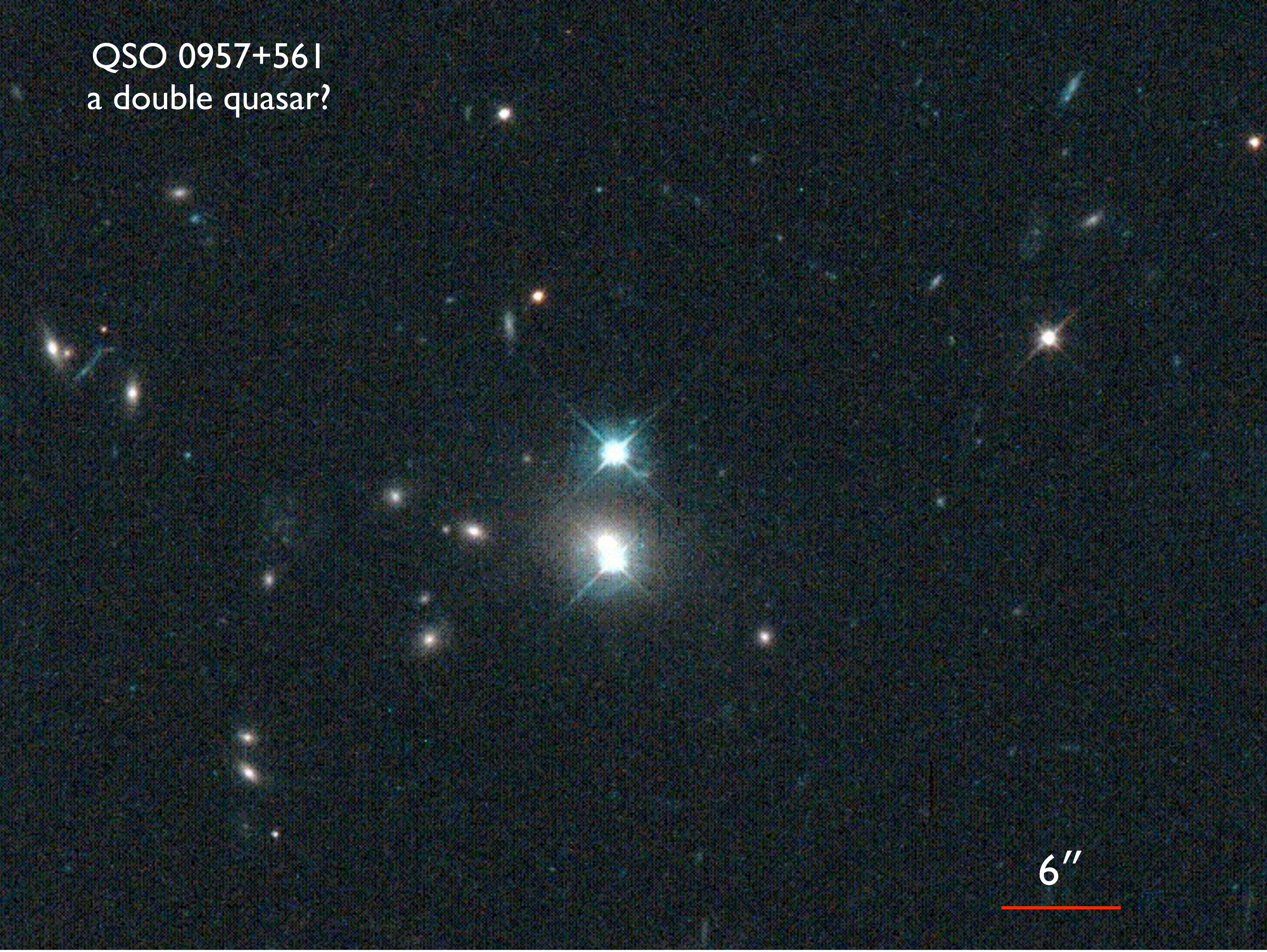
(2) That a gravitational force acting obliquely on light would probably be unable to alter speed, but, through the co-operation of its transverse and longitudinal components, it might be expected to produce an extra dose of deflection—assuming light to be subject to gravitation, as Newton surmised. So that by the time a beam of light coming from infinity had arrived at its nearest point to the sun, it would already have been deflected as much as an ordinary heavy particle would be deflected along its whole course.

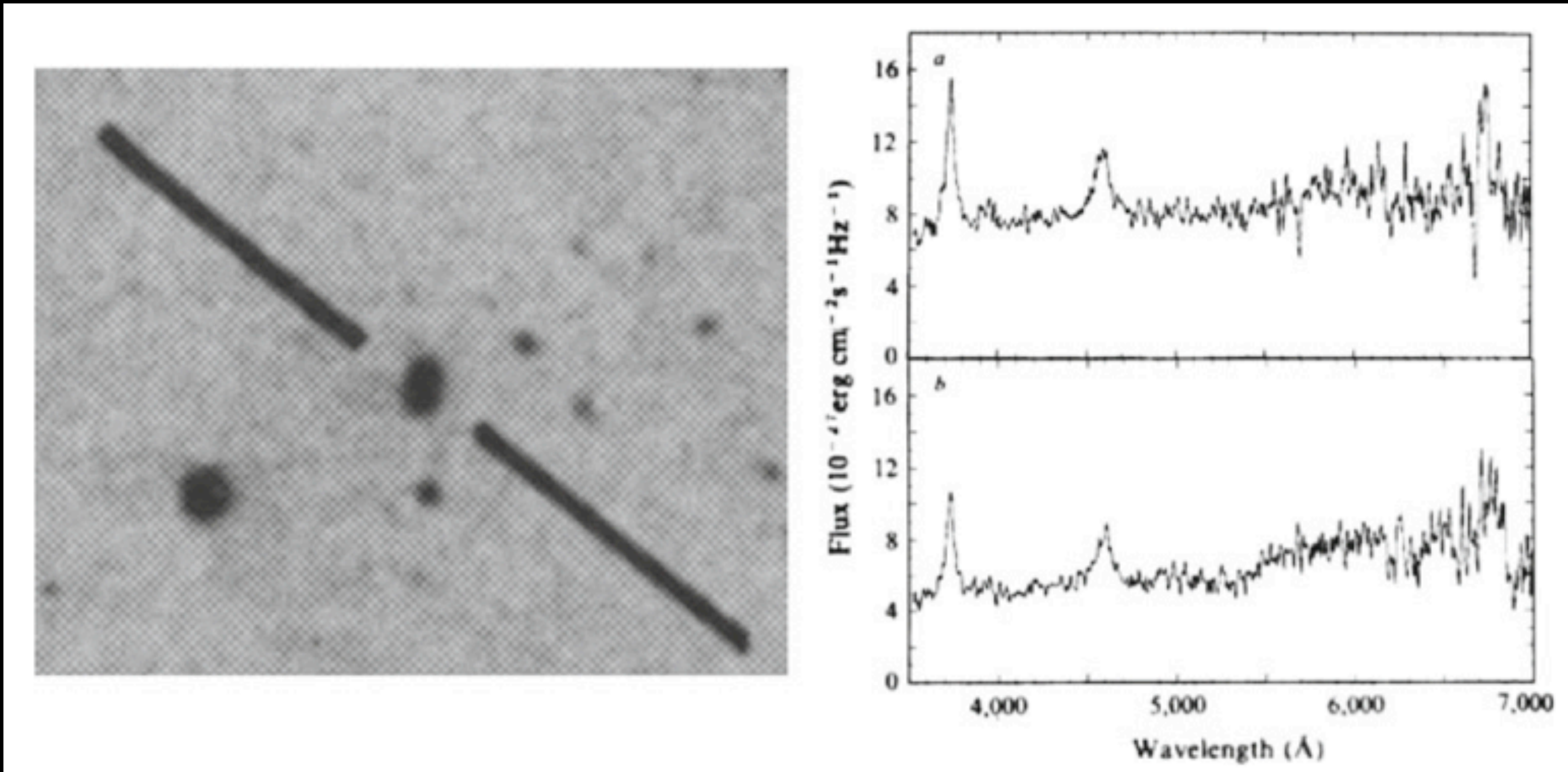
I am aware that these are only suggestions for working out.

Einstein's equations, based on the impossibility of observing motion through æther, seem powerful instruments for extracting results; just as more familiar equations, based on the impossibility of "perpetual motion," have proved themselves effective; but neither set of equations explains, nor attempts to explain, the mechanism of the consequences they deduce. Dynamics have served us so well in the past that it must be still legitimate to try, wherever possible, to apply well-established principles to new phenomena.

OLIVER J. LODGE.

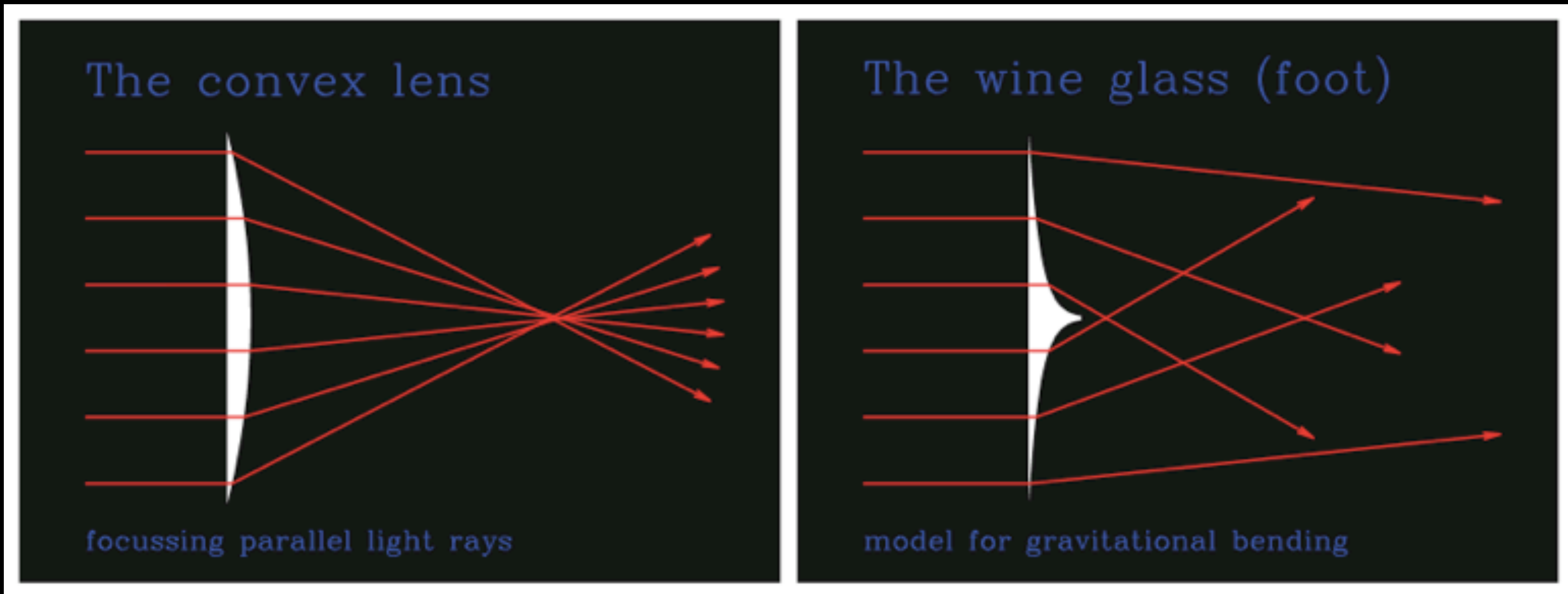
Edgbaston, Birmingham, November 30.


QSO 0957+561
a double quasar?
6″



Walsh, Carswell & Weymann (1979) *Nature* 279, 381

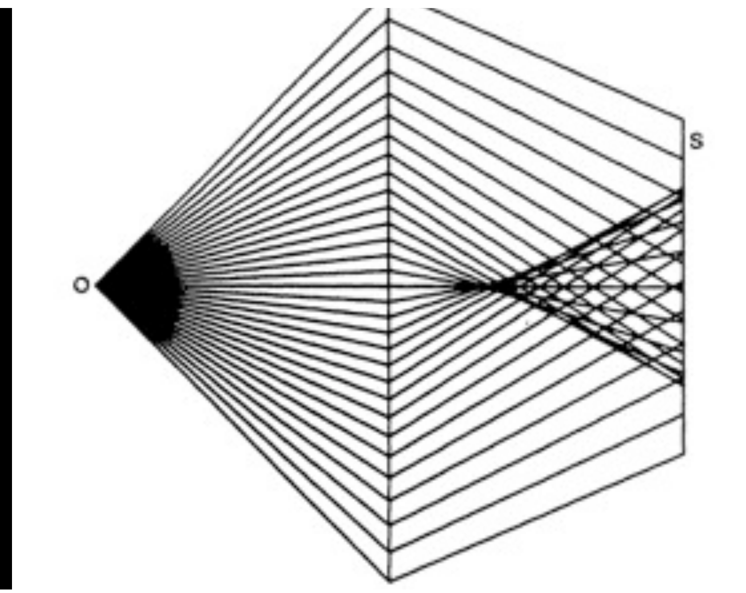
Lodge was of course right !

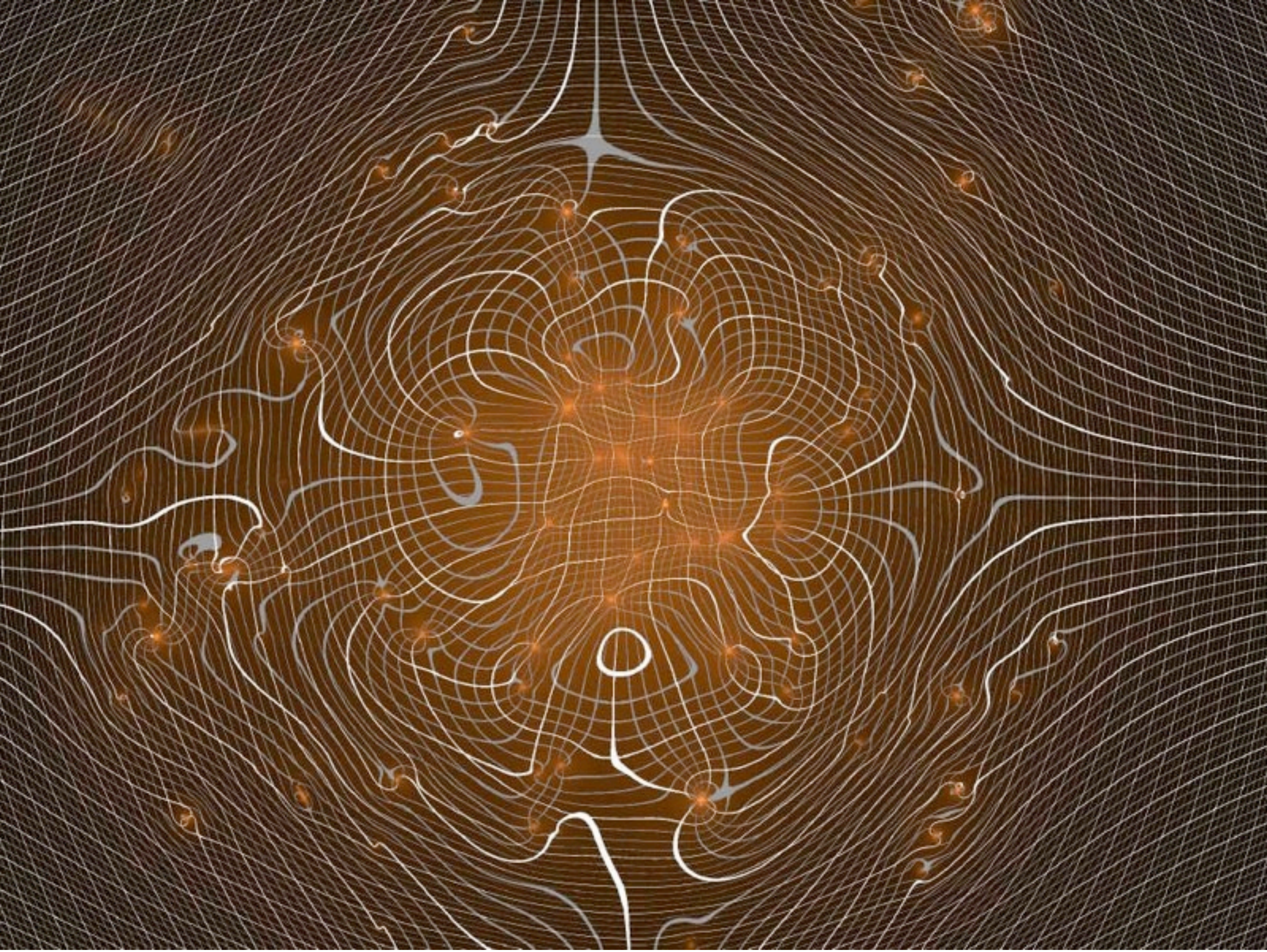


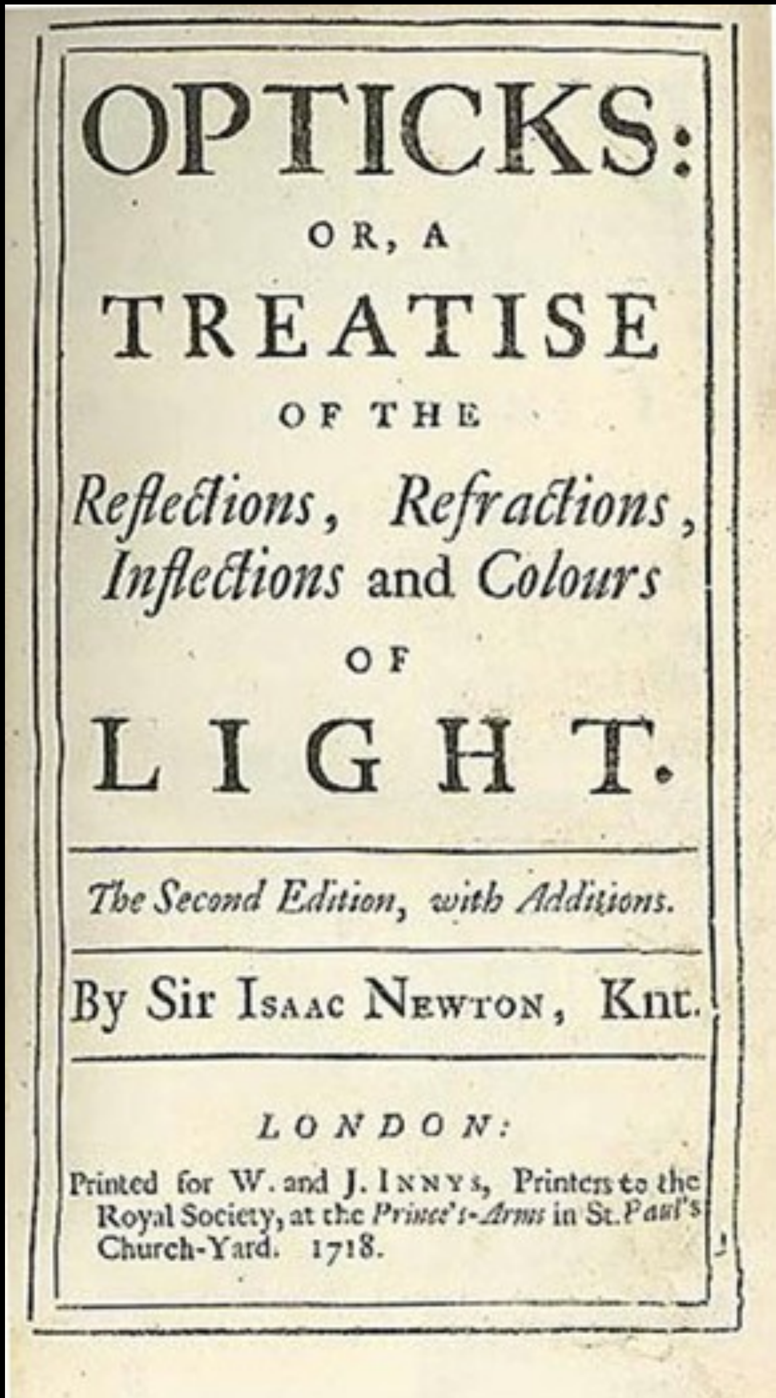
Gravitational lenses have no single focus

strong astigmatism

$\alpha = 4 G M / c^2 R$







OPTICKS:

OR, A

TREATISE

OF THE

Reflections, Refractions, Inflections and *Colours*

OF

LIGHT.

The Second Edition, with Additions.

By Sir ISAAC NEWTON, Knt.

LONDON:

Printed for W. and J. INNYS, Printers to the Royal Society, at the *Prince's-Arms* in St. *Paul's* Church-Yard. 1718.

Was Newton the pioneer ?

Query 1. Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action (*caeteris paribus*) strongest at the least distance?

Opticks, 1704, Book III

but the context is not quite right ...

Last (and very sketchy) observations in *Book II* :

Obs. 10. When the Fringes of the Shadows of the Knives, fell perpendicularly upon a Paper at a great distance from the Knives, they were in the form of Hyperbola's

Newton was performing Grimaldi's *diffraction* experiments

When I made the foregoing Observations, I design'd to repeat most of them with more care and exactness, and to make new ones for determining how the Rays of Ligth are bent in their passage by Bodies, for making Fringes of Colours with the dark lines between them. But I was then interrupted [...]
And since I have not finish'd this part of my Design, I shall conclude with proposing some Queries, in order to a farther search to be made by others.

All the first queries are related to diffraction effects :

Que. 2. [...]and after what manner are they inflected to make those fringes ?

Que. 3. Are not the Rays of Light in passing by the edges and sides of Bodies, bent several times backwards and forwards [..]

Que. 4. Do not the Rays of Light which fall upon Bodies [...] begin to bend before they arrive at the Bodies?

Que. 5. Do not Bodies and Light act mutually upon one another?

First citation claiming that Newton was the precursor :

Place : Joint meeting of the Royal Astronomical Society and the Royal Society, Londres

Date : 6 November 1919

Subject : Discussion of the results obtained by the Sobral and Principe expeditions to measure the gravitational deflection of light during the May 25 1919 eclipse.

THE PRESIDENT. I know call for discussion on this momentous communication. If the results obtained had been only that light was affected by gravitation, it would have been of the greatest importance. Newton did, in fact, suggest this very point in the first query of his 'Optics' and his suggestion would presumably have led to the half-value. [...]

J.J. Thomson (1919)

Studies on the attraction of light by matter

s'Gravesande (1747) *Eléments de physique, Vol. II, p. 113*

Nous avons dit ci-devant que le feu est attiré par les corps; cela est évident dans les rayons de Soleil, qui se fléchissent vers les corps lorsqu'ils passent à une petite distance d'eux. Cela fait connaître que *les rayons* sont poussés par une certaine force vers les corps, et qu'ils *sont attirés par ces corps*.

Voltaire, Clairaut, etc... non gravitational inflections

- All cases deal with *diffraction*, which is described as an *inflexion* of the light rays.

- e.g., Priestley (1772) *The history and present state of discoveries relating to vision, light and colours*

Inflected rays : those rays of light which, on their near approach to the edges of bodies, in passing by them, are bent out of their course, being turned either from the body or towards it. This property of the rays of light is generally termed *diffraction* by foreigners, and Dr. Hooke sometimes called it *deflection*.

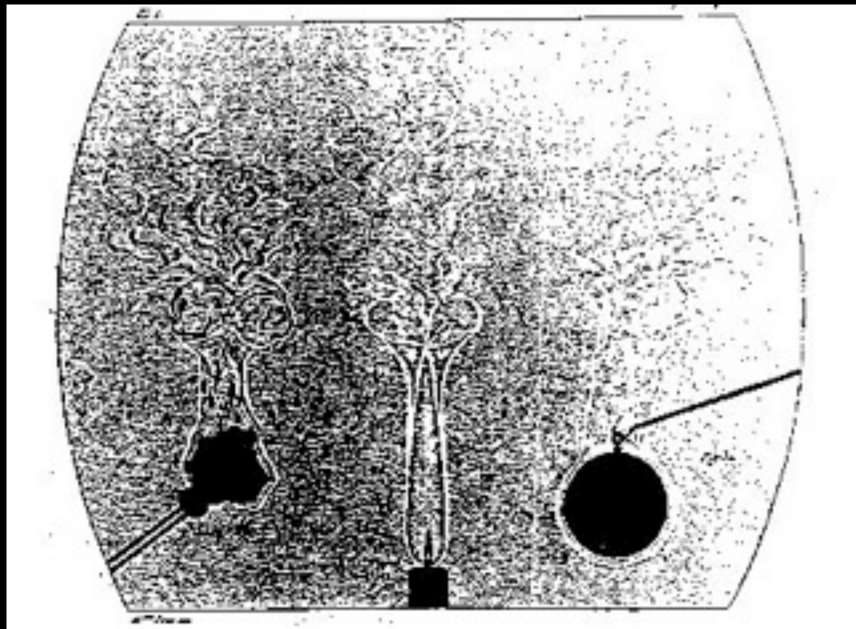
Jean-Paul Marat
(1743-1793)

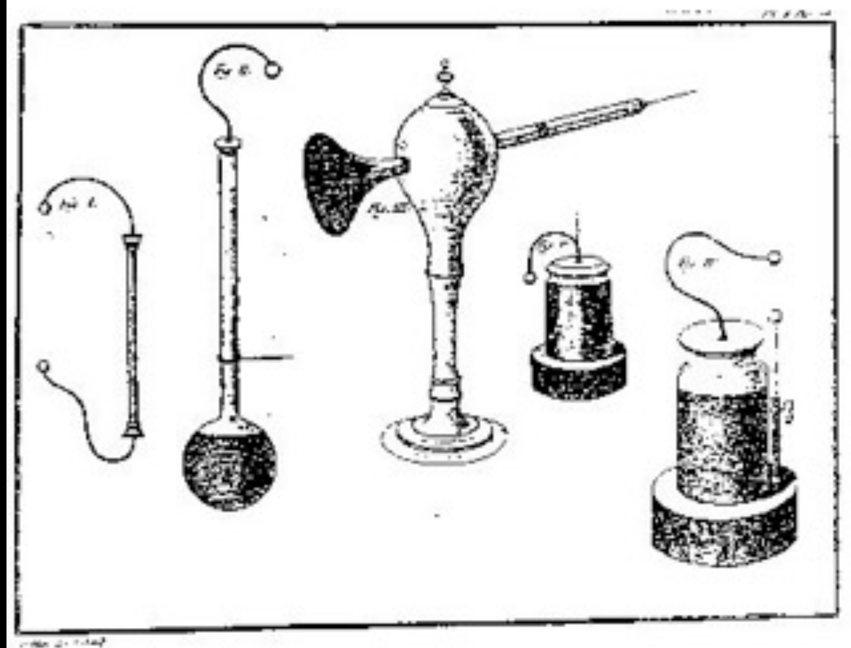
- Physician (M.D. from St. Andrews)
- Paris 1776. Service of the Comte d'Artois.

- Becomes rich enough to build instruments and carry out many experiments on the nature of fire, electricity and optics.

- One of the first translators of Newton's *Opticks* into French

- Excellent relations with B. Franklin, bad ones with Lavoisier ...





Marat (1778)

DÉCOUVERTES

DE M. MARAT,

(*Docteur en Médecine & Médecin des Gardes-du-Corps de* MONSEIGNEUR LE COMTE D'ARTOIS.)

SUR LA LUMIÈRE;

Conſtatées par une ſuite

D'EXPÉRIENCES NOUVELLES

Qui ont été faites un très-grand nombre de fois ſous les yeux de MM. les Commiſſaires de l'Académie des Sciences.

A LONDRES;

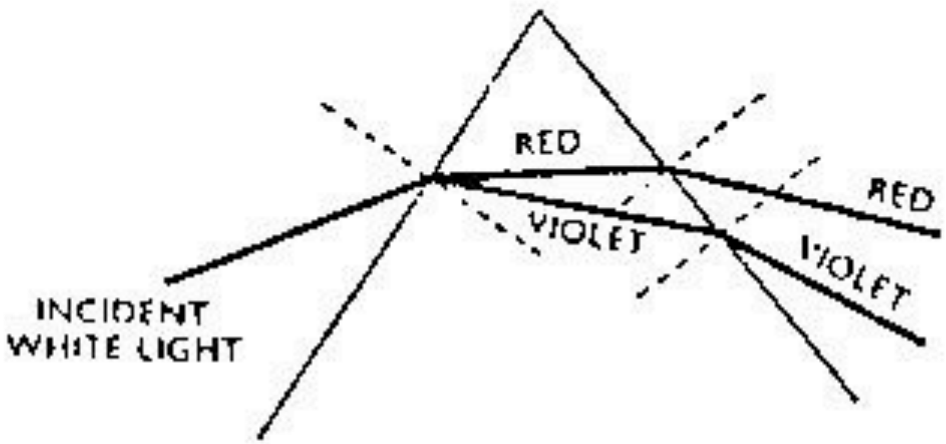
Et ſe trouve à PARIS,

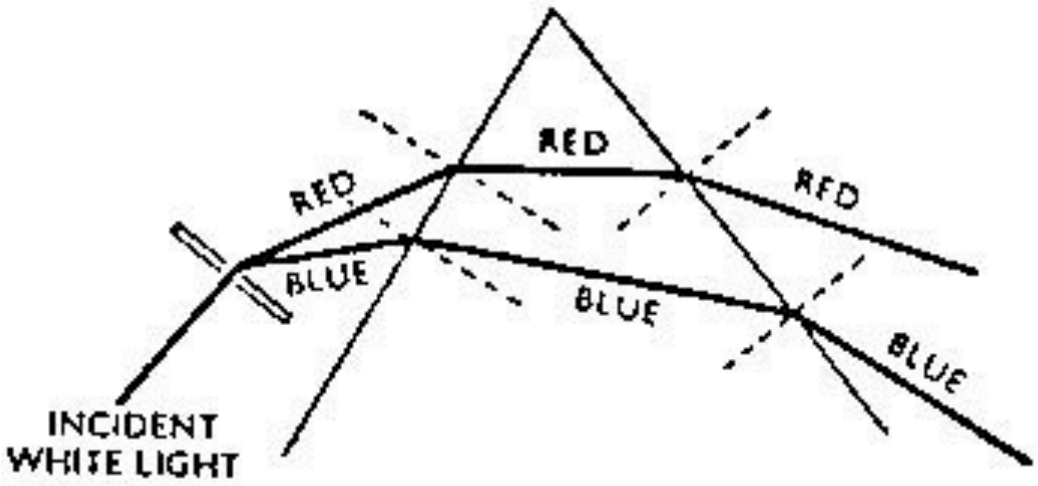
Chez JOMBERT, Fils aîné, rue Dauphine.

M. DCC. LXXX.

Marat (1780)

Critique of Newton's *experimentum crucis*





Newton:
colours produced by refraction

Marat:
colours produced by deviation

Tous les corps connus décomposent la lumière en l'attirant [...] La sphère d'attraction de la lumière [...] dépend de la densité superficielle [...], d'un facteur d'affinité, [...] et en raison du carré inverse de la distance.

Marat (1780)

Books translated into German
will influence Goethe's theory of colours

The first calculation : Cavendish (1804)

- Unpublished manuscript, initially dated 1783-1784 :

To find the bending of a ray of light which passes near the surface of any body by the attraction of that body

Let s be the center of a body and a a point of surface. Let the velocity of body revolving in a circle at a distance as from the body be to the velocity of light as $1{:}u$, then will the sine of half bending of the ray be equal to $1/(1+u^2)$.

rather criptic ...

Assumes (correctly) the speed of light at infinity

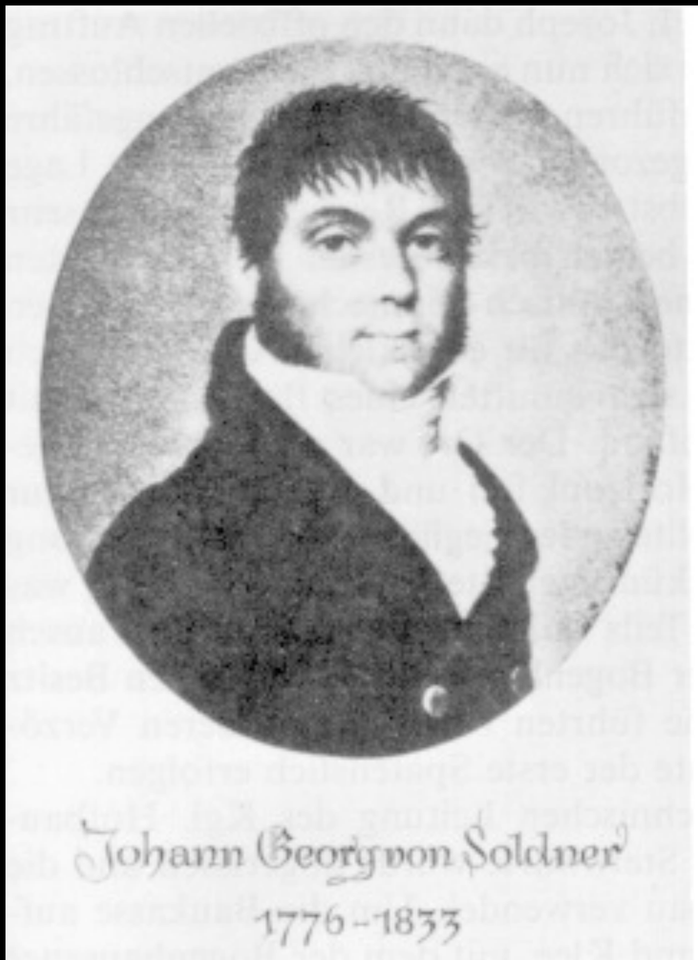
$$c^2 \equiv v^2 |_{\phi=\phi_\infty} = \frac{GM}{(1+e)R}(e^2-1) = GM(e-1)/R$$

$$e = \frac{Rc^2}{GM} + 1 \quad \Longrightarrow \quad \boxed{\sin\delta = \varepsilon/(1+\varepsilon) = 1/(1+u^2)}$$

The net deflection becomes :

$$\Delta\theta \equiv 2\delta \approx 2\sin\delta \approx 2\varepsilon = 2\frac{GM}{Rc^2}$$

The first wrong calculation :   Soldner (1801)



Autodidact
Director of Munich Observatory

Well-known in geodesy

Interested in stellar motions,
the Michell-Laplace objects...

Aſtronomiſches

Jahrbuch

für das Jahr 1804.

nebſt einer Sammlung

der neueſten

in die aſtronomiſchen Wiſſenſchaften

einſchlagenden Abhandlungen, Beobachtungen und Nachrichten.

Mit Genehmhaltung

der Königl. Akademie der Wiſſenſchaften

berechnet und herausgegeben

von

J. E. Bode, Aſtronom und Mitglied der Akademie.

Mit zwey Kupfertafeln.

Berlin, 1801.

Bey dem Verfaſſer, und in Commiſſion bey G. A. Lange in Berlin.

Gedruckt bey C. F. E. Späthen.

von de Lambre beſtimmt; dieſe geben in der Länge 11″ zu viel. Bringt man dieſe Verbeſſerung an; ſo erhält man den Gegenſchein ☍ mit der Sonne zu Prag 1800. den 15. März mittlerer Zeit $10^u\ 20'\ 33''$.

Zu dieſer Zeit war die Länge der ☉	11^s	$25°$	$5'$	$7'',6$
☍	5	25	5	7,6
Geocentriſche Breite	—	—	48	12

Die Sonnenlängen für dieſen und des ♂ Gegenſchein, ſind aus des Hrn. Aſtronom Trieſneckers Tafeln berechnet. (Wien. Ephemeriden 1793.)

Tet

Ueber die Ablenkung eines Lichtſtrals von ſeiner geradlinigen Bewegung, durch die Attraktion eines Weltkörpers, an welchem er nahe vorbei geht.

Von Hrn. *Joh. Soldner.*

Berlin, im März 1801.

Bey dem jetzigen, ſo ſehr vervollkommneten, Zuſtande der praktiſchen Aſtronomie wird es immer nothwendiger, aus der Theorie, das heiſst aus den allgemeinen Eigenſchaften und Wechſelwirkungen der Materie, alle Umſtände zu entwickeln, welche auf den wahren oder mittlern Ort eines Weltkörpers Einfluſs haben können: um aus einer guten Beobachtung den Nutzen ziehen zu können, deſſen ſie an ſich fähig iſt.

Es iſt zwar wahr, daſs man beträchtliche Abweichungen von einer angenommenen Regel ſchon durch Beobachtungen und zufällig gewahr wird: wie es z. B. der Fall mit der Aberration des Lichtes war. Es kann aber Abweichungen geben, die ſo klein ſind, daſs es ſchwer iſt zu entſcheiden, ob es wirkliche Abweichungen, oder Fehler der Beobachtungen ſind. Auch kann es Abweichungen geben, die zwar beträchtlich ſind; aber mit Gröſsen kombinirt, mit deren Ausmitte-

On the deflection of light rays wrt a straight line produced by the attraction of a celestial body which passes by the line of sight.

- Goal: to compute the circumstances that perturb the true (astrometric) positions of the stars

- Difficult to read, confusing notation (a=s/t^2), and a missing factor of two (twice) which compensates

Conceptually wrong !

Unlike Cavendish, Soldner adopts the speed of light at the impact parameter, disregarding the acceleration

$$c^2 \equiv v^2 |_{\phi=0} = GM(1+e)/R$$

To first order the value is the same :

$$e = \left(\frac{Rc^2}{GM}\right) - 1 \quad \Longrightarrow \quad \sin\delta = \varepsilon/(1-\varepsilon)$$

Though the combination of several bodies which a light ray could encounter on its way, would be a larger result, for our observations it is nevertheless unnoticeable. Therefore it is clear that nothing makes it necessary, at least in the present state of practical astronomy, that one should take into account the perturbation of light rays by attracting celestial bodies.

The second published calculation : Einstein (1911)

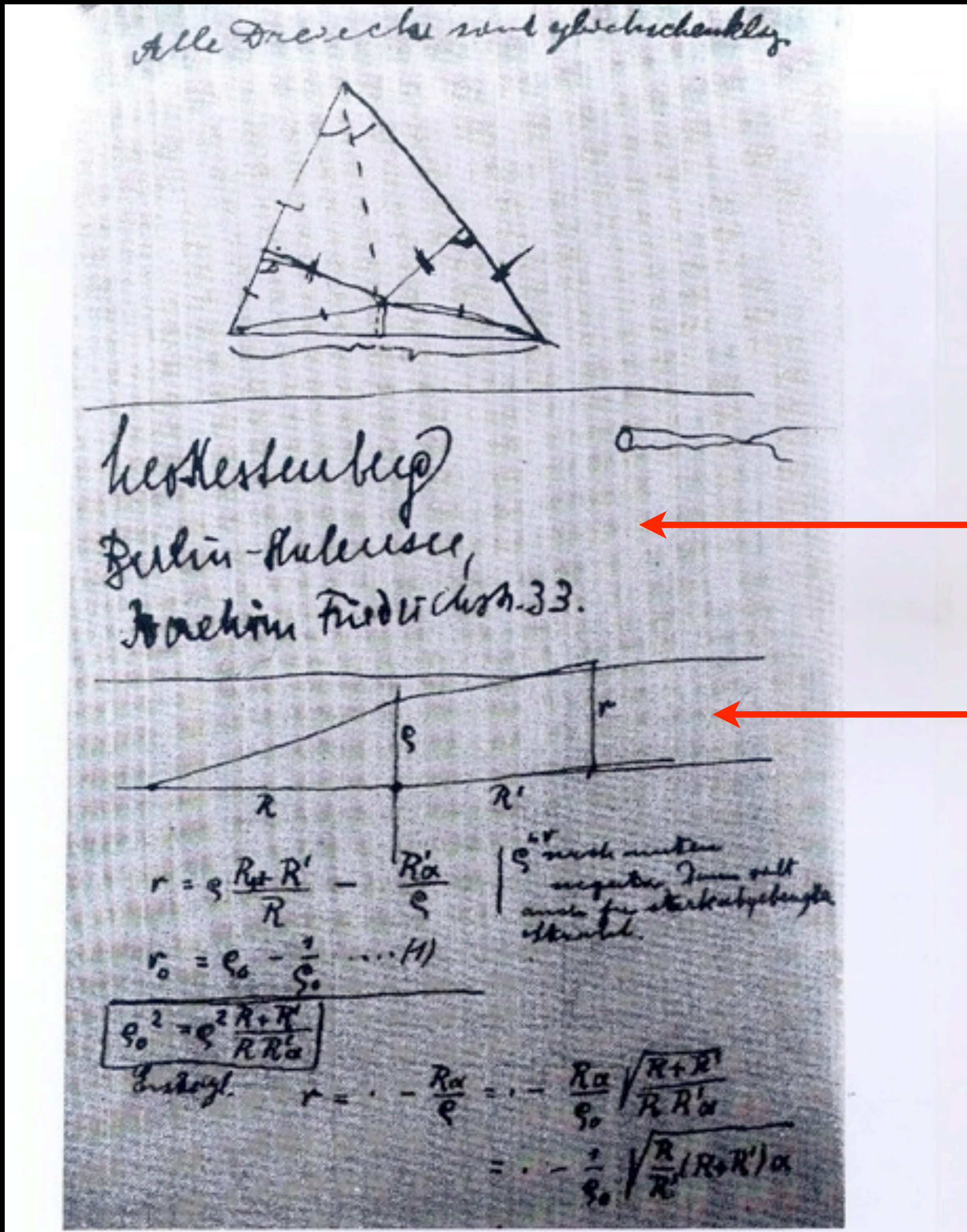
Simple application of the equivalence principle

Prediction : $0.''83$ at the limb of the Sun

Observable during a total solar eclipse

Discussions with Erwin Freundlich to prepare expeditions to observe the effect

Funds the ill-fated (end of Summer 1914) expedition

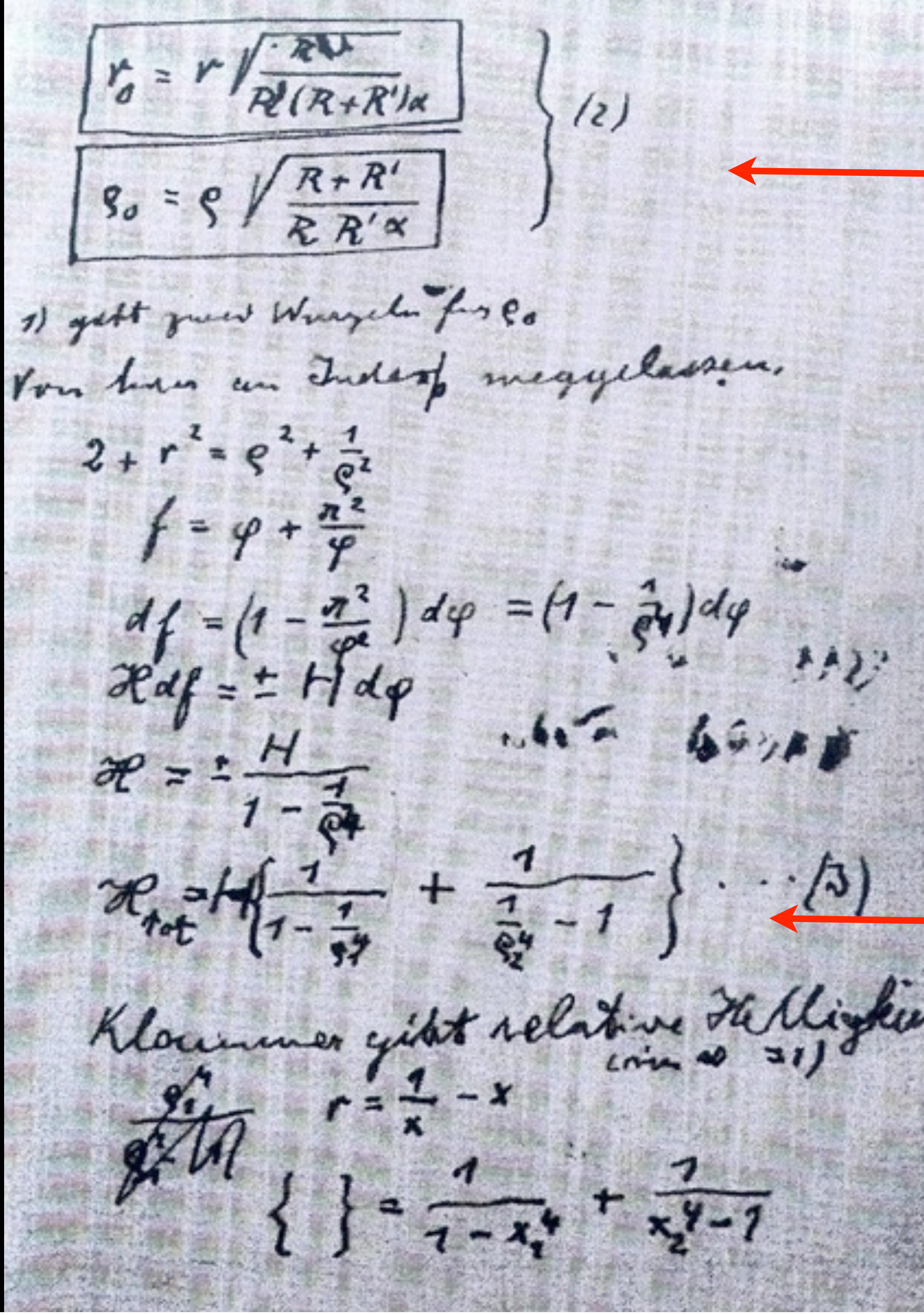


Prague-Berlin long train journeys ....

laundry ...

lens effect ...

Einstein notebooks (1912) Courtesy HUJ



$$\boxed{r_0 = r\sqrt{\frac{R'}{R(R+R')\alpha}}}$$
$$\boxed{\varrho_0 = \varrho\sqrt{\frac{R+R'}{RR'\alpha}}} \quad (2)$$

1) gibt zwei Wurzeln für ϱ_0
Von hier an Index 0 weggelassen.

$$2 + r^2 = \varrho^2 + \frac{1}{\varrho^2}$$
$$f = \varphi + \frac{\pi^2}{\varphi}$$
$$df = \left(1 - \frac{\pi^2}{\varphi^2}\right) d\varphi = \left(1 - \frac{1}{\varrho^4}\right) d\varphi$$
$$\mathcal{H}\, df = \pm H\, d\varphi$$
$$\mathcal{H} = \pm \frac{H}{1 - \frac{1}{\varrho^4}}$$
$$\mathcal{H}_{tot} = H\left\{\frac{1}{1-\frac{1}{\varrho_1^4}} + \frac{1}{\frac{1}{\varrho_2^4} - 1}\right\} \cdots (3)$$

Klammer gibt relative Helligkeit (min ∞ = 1)

$$\cancel{\frac{\varrho_1^4}{\varrho_2^4}} \qquad r = \frac{1}{x} - x$$
$$\{\ \} = \frac{1}{1 - x_1^4} + \frac{1}{x_2^4 - 1}$$

positions of images ...

amplification effect ...

.. and then forgets about it ....

Einstein notebooks (1912) Courtesy HUJ

A speculation : fake double stars

Orest D. Chwolson

Professor at Petrograd

Traité de Physique (15 vols), translated into German, French and Spanish

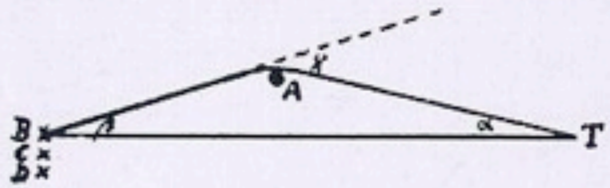
His only work in Astronomy :

Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson.*

Es ist gegenwärtig wohl als höchst wahrscheinlich an- nehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche nes Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese blenkung und γ_0 der Maximumwert an der Oberfläche, so $\gamma_0 \geq \gamma \geq 0$. Die Größe des Winkels ist bei der Sonne $= 1.''7$; es dürften aber wohl Sterne existieren, bei denen gleich mehreren Bogensekunden ist; vielleicht auch noch ehr. Es sei A ein großer Stern (Gigant), T die Erde, ein entfernter Stern; die Winkeldistanz zwischen A und B, n T aus gesehen, sei α, und der Winkel zwischen A und T, n B aus gesehen, sei β. Es ist dann

$$\gamma = \alpha + \beta .$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es nn also α gleich mehreren Bogensekunden sein, und der aximumwert von α wäre etwa gleich γ_0. Man sieht den ern B von der Erde aus an zwei Stellen: direkt in der ichtung TB und außerdem nahe der Oberfläche von A, nalog einem Spiegelbild. Haben wir mehrere Sterne B, C, D, würden die Spiegelbilder umgekehrt gelegen sein wie in einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B, wenn von A aus gerechnet wird (D wäre am nächsten zu A).



Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

Petrograd, 1924 Jan. 28.

O. Chwolson.

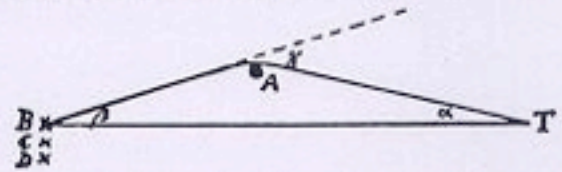
O. Chwolson (1924)

Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson.*

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geq \gamma \geq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1''7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant), T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B, von T aus gesehen, sei α, und der Winkel zwischen A und T, von B aus gesehen, sei β. Es ist dann

$$\gamma = \alpha + \beta\,.$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0. Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A, analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D, so würden die Spiegelbilder umgekehrt gelegen sein wie in einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B, wenn von A aus gerechnet wird (D wäre am nächsten zu A).



Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

O. Chwolson.

Petrograd, 1924 Jan. 28.

Antwort auf eine Bemerkung von *W. Anderson.*

Daß ein Elektronengas einer Substanz mit negativem Brechungsvermögen optisch äquivalent sein müßte, kann bei dem heutigen Stand unserer Kenntnisse nicht zweifelhaft sein, da dasselbe einer Substanz von verschwindend kleiner Eigenfrequenz äquivalent ist.

Aus der Bewegungsgleichung

$$\varepsilon X = \mu\, \mathrm{d}^2x/\mathrm{d}t^2$$

eines Elektrons von der elektrischen Masse ε und der ponderabeln Masse μ folgt nämlich für einen sinusartig pendelnden Prozeß von der Frequenz ν die Gleichung

$$\varepsilon X = -(2\pi\nu)^2\, \mu x\,.$$

Berücksichtigt man, daß εx das »Moment« eines schwingenden Elektrons ist, so erhält man für die Polarisation $p = n\varepsilon x$ eines Elektronengases mit n Elektronen pro Volumeinheit

$$p = -\varepsilon^2 n/[\mu\,(2\pi\nu)^2] \cdot X\,.$$

Hieraus folgt, daß die scheinbare Dielektrizitätskonstante

$$D = 1 + 4\,\pi p/X = 1 - \varepsilon^2\, n/(\pi\mu\,\nu^2)$$

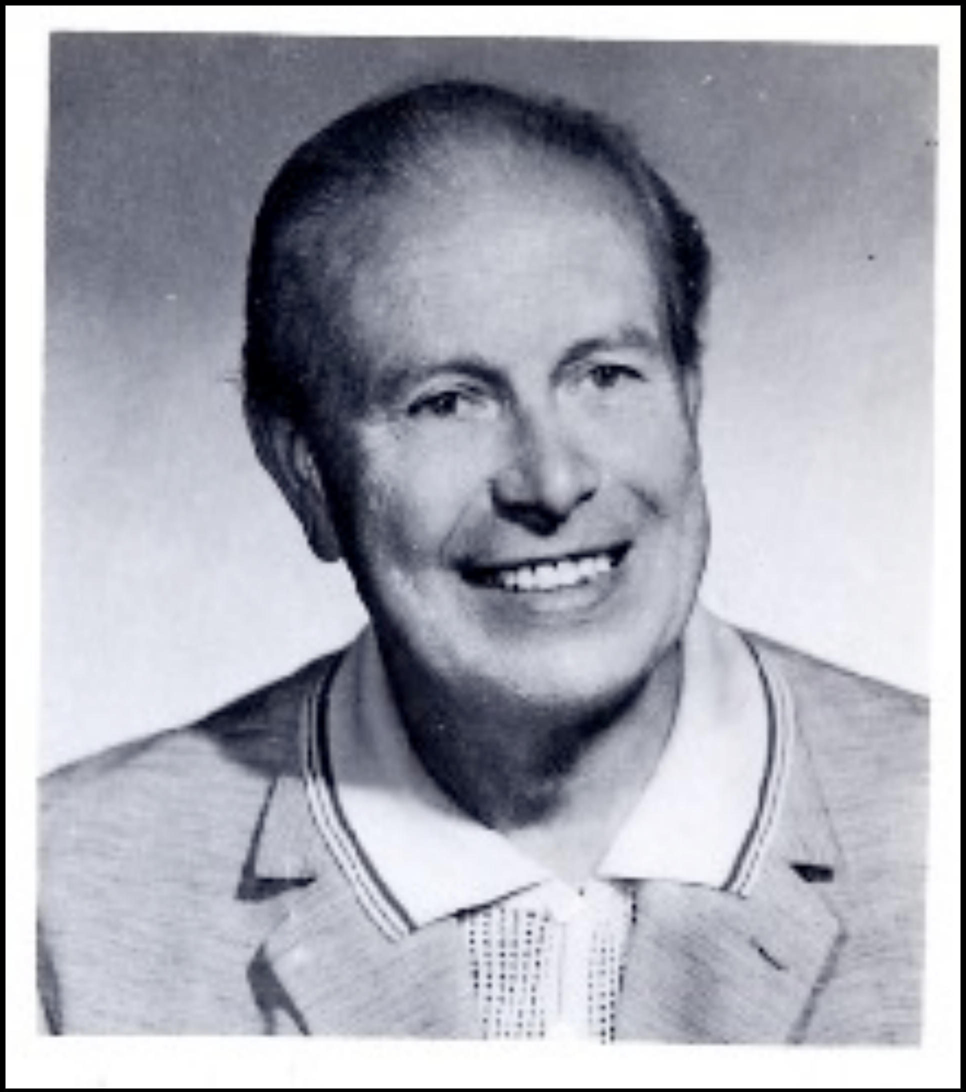
ist. $\sqrt{D}$ ist in diesem Falle der Brechungsexponent, also jedenfalls kleiner als 1. Es erübrigt sich bei dieser Sachlage, auf das Quantitative einzugehen.

Es sei noch bemerkt, daß ein Vergleich des Elektronengases mit einem Metall unstatthaft ist, weil die bei der elementaren Theorie der Metalle zugrundegelegte »Reibungskraft« bei freien Elektronen fehlt; das Verhalten der letzteren ist allein durch die Einwirkung des elektrischen Feldes und durch die Trägheit bedingt.

Berlin, 1924 April 15.

A. Einstein.

The true pioneer : František Link



Specialist of eclipses, he realises that, formally, the gravitational deflection produces effects similar to refraction by an atmosphere

Influenced by Eddington's (1923) *Mathematical theory of Relativity*

ASTROPHYSIQUE. — *Sur les conséquences photométriques de la déviation d'Einstein*. Note de M. F. LINK, présentée par M. Charles Fabry.

Dès le début de la théorie de la relativité on a cherché de vérifier la déviation des rayons lumineux passant normalement au champ de gravitation d'un corps céleste. La déviation ω est

$$\omega = kK\frac{1}{R,a}, \qquad k = 1'',75, \qquad K = \frac{M}{a},$$

où M est la masse et a le rayon du corps déviant, R la distance minima des rayons au centre. La masse du Soleil et son rayon sont pris comme unités.

Or la déviation variable a pour conséquence la modification de l'intensité des rayons. C'est en somme le cas analogue que l'on considère dans la théorie des occultations par les planètes [1] et dans la théorie des éclipses

[1] CH. FABRY, *Comptes rendus*, **187**, 1928, p. 627, 693, 741, et *Journal des Observateurs*, **12**, 1, 1929, p. 1; F. LINK, *Bulletin astronomique*, **9**, IV, 1935, p. 227.

Wednesday, September 19, 2012

de Lune (¹). Seulement la cause et la loi de la déviation sont différentes. Dans le cas de la déviation d'Einstein on trouve, pour le rapport des intensités avant et pendant l'occultation, l'expression

$$\frac{\mathrm{J}}{i} = s = 1 - \mathrm{K}^2 \frac{k^2}{(\alpha_1 + \alpha_2)^2} \frac{\alpha_1^2}{\rho^4},$$

où α_1 et α_2 sont les rayons apparents du corps occultant vus de la Terre et d'un point de l'étoile occultée et ρ la distance apparente des deux étoiles.

Pendant les éclipses de Soleil la valeur de s est très peu inférieure à l'unité. Il y a donc une très faible augmentation d'éclat des étoiles vues au voisinage du Soleil. Au bord même on trouve $s = 1 - 4.10^{-4}$, de façon que la déviation seule reste mesurable.

Les circonstances sont plus favorables lors d'un rapprochement optique de deux étoiles, si nous ne voulons pas parler d'une occultation très peu probable. Pour simplifier supposons le corps occulté tellement loin que l'on peut négliger α_2 par rapport à α_1 et suivons les variations d'éclat avec la distance apparente décroissante. L'éclat sera d'abord sensiblement constant même très près de l'étoile occultée et à peine supérieur à sa valeur normale. Lorsque la distance apparente s'approche de la valeur

$$\boxed{\rho_m = \pm\sqrt{\mathrm{K} k \alpha_1}.}$$

L'intensité commence à augmenter pour atteindre à cette distance la valeur infinie, si l'étoile occultée était rigoureusement ponctuelle. Ce cas étant laissé de côté, il peut cependant se produire une augmentation notable d'éclat. Ceci peut réellement arriver puisque, dans la majorité des cas, $\rho_m > \alpha_1$. Cette distance dépassée, l'éclat diminue très vite et au bord de l'étoile occultante, on trouve en général un très fort affaiblissement d'éclat.

Si donc le corps occultant est très faible ou même invisible, son passage devant une étoile normalement faible peut temporairement augmenter son éclat. Si, au contraire, l'étoile occultante est plus brillante que le corps occulté, le phénomène se traduira par une fluctuation d'éclat qui aura ses origines dans la lumière du corps occulté.

Dans toutes ces considérations où nous négligeons le rôle de la diffraction, la probabilité d'un rapprochement nécessaire n'a que le sens pratique. Théoriquement, les phénomènes décrits sont possibles. On trouve, par exemple, pour un corps comme notre Soleil, placé à la distance de 100 par secs,

$$\alpha_1 = 0'',0043, \qquad \rho_2 = 0'',088.$$

De tels rapprochements optiques seront sans doute rares, sauf dans certaines régions du ciel, en particulier dans les nébuleuses spirales. Nous discuterons ces phénomènes dans un autre Recueil.

Text

This paper is never cited ...
and yet contains the first ever quantitative prediction of the gravitational lens effect

Link (1936) *Comptes Rendus Acad. Sciences*, 202, 917

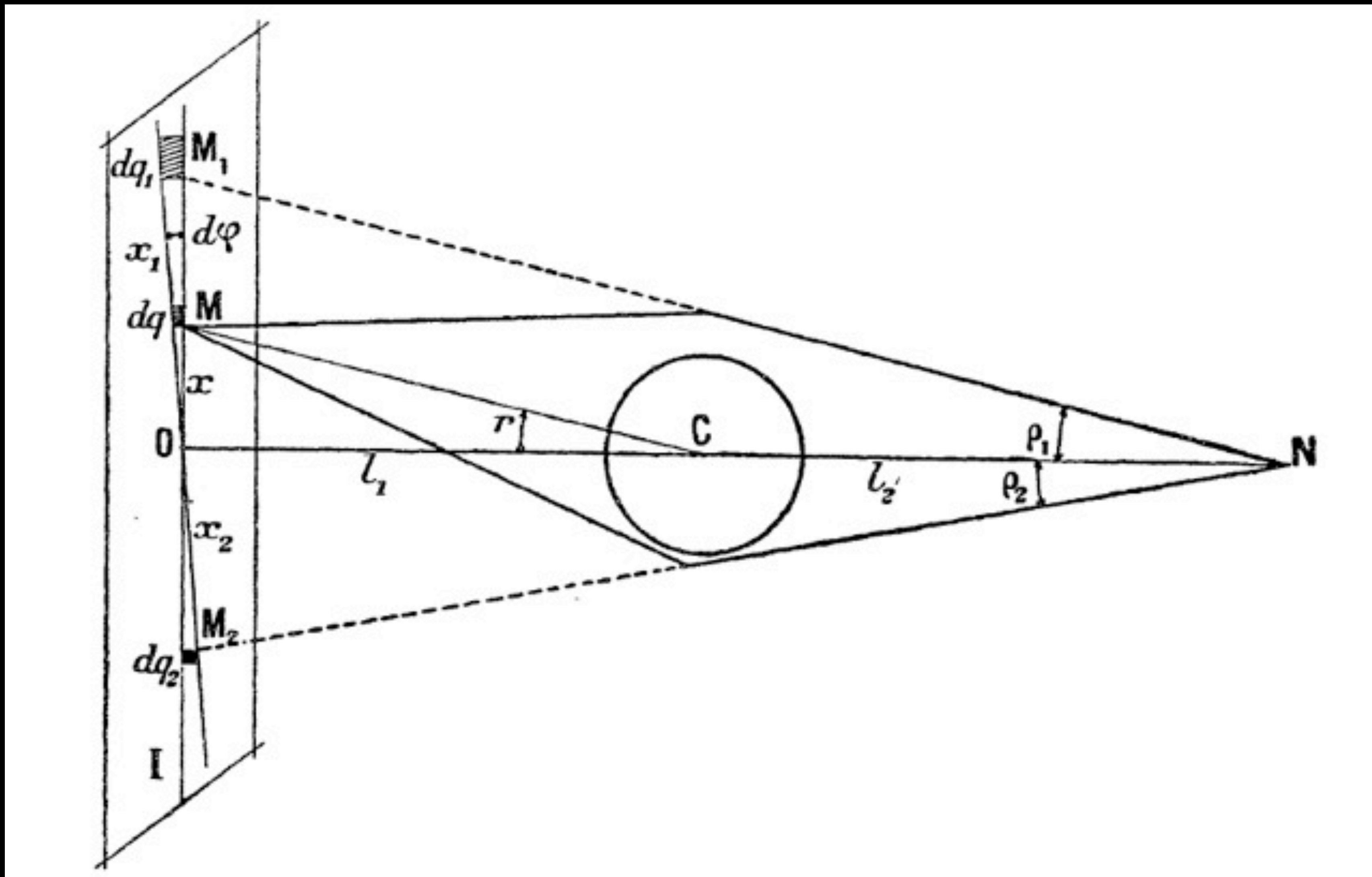
SUR LES CONSÉQUENCES PHOTOMÉTRIQUES DE LA DÉVIATION D'EINSTEIN.

Par M. F. LINK.

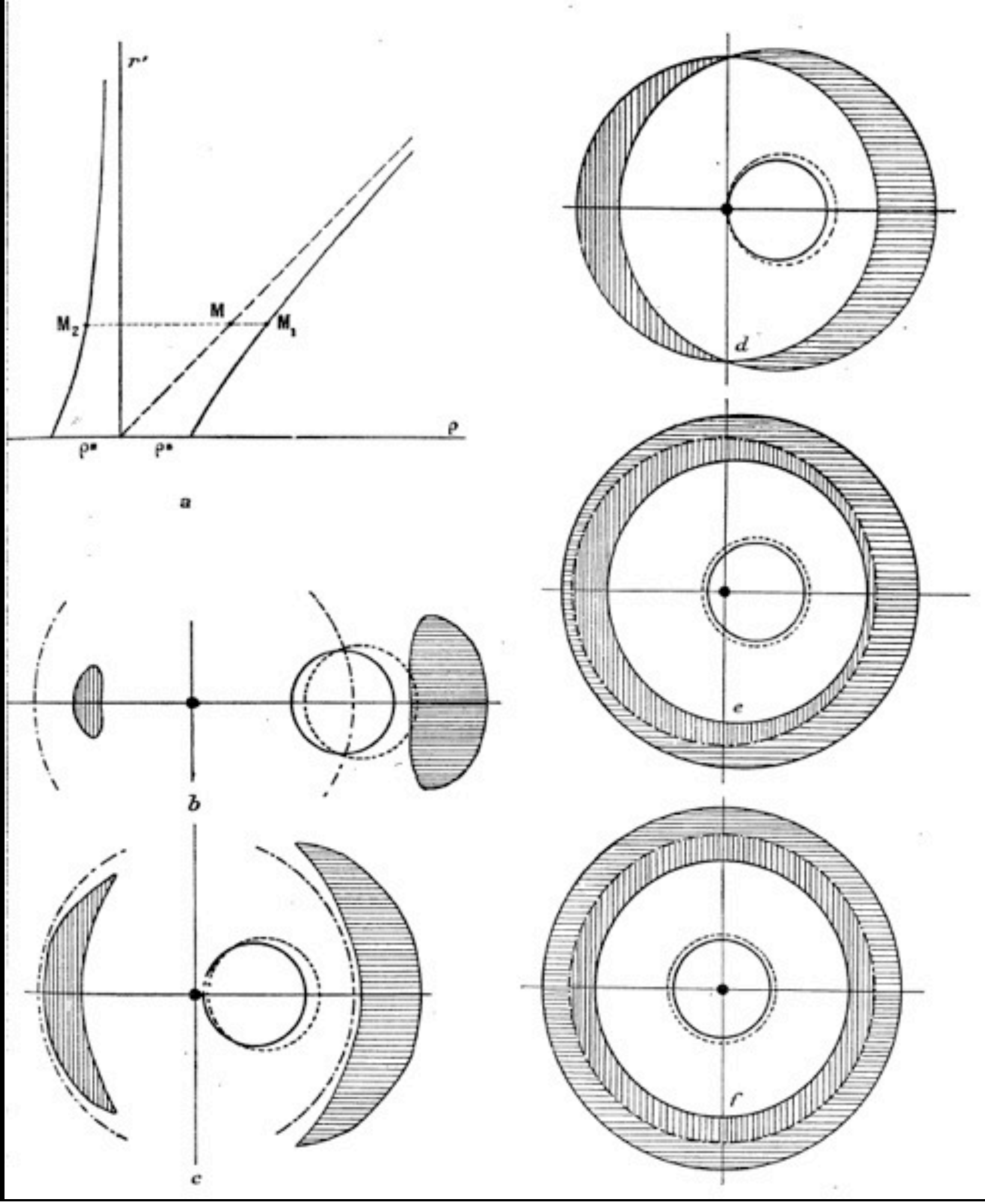
SOMMAIRE. — La déviation d'Einstein des rayons lumineux dans le champ de gravitation a pour conséquence une modification de l'éclairement produit par ces rayons. Le phénomène est discuté et quelques applications aux systèmes stellaires sont données. C'est surtout dans les amas globulaires que les conséquences photométriques de la déviation d'Einstein apparaissent. Les modifications de l'éclairement sont accompagnées des variations de forme des sources lumineuses, dont l'observation paraît plus difficile.

Link (1937)

First gravitational optics diagram ever published



Link (1937)



First calculation of the shapes of the images for extended sources

Link (1937)

- Effects produced by the finite size of the source
- Effets due to occultations
- Invariance of surface brightness
- Formation of arcs, “lentils” (counter-images)
- Numerical examples

all this in 1937 !!

9. **Conclusions.** — La réalité des phénomènes dont nous avons donné un aperçu dépend de la validité de la déviation d'Einstein. Il serait alors très intéressant de chercher systématiquement dans tous les domaines de l'astronomie stellaire des cas favorables à la production de ces effets, non seulement pour la vérification supplémentaire de la théorie, mais aussi pour l'explication de certaines variations d'éclat.

Bibliographie.

1. O. Chwolson, *A. N.* Bd. 221, p. 329.
2. A. Eddington, *Mathematische Grundlagen der Relativitätstheorie.*
3. H. Turner, *Publ. Astr. Soc. Pac.*, vol. 41, feb. 1929.
4. G. P. Kuiper, *Publ. Astr. Soc. Pac.*, vol. 47, dec. 1935.
5. F. Link, *Bull. astr.*, t. VIII, fasc. II, 1933.
6. F. Link, *C. R. Acad. Sc.*, Paris, t. 202, 16 mars 1936. p. 917.

ing to F. Bertola the following lines were ascertained: hydrogen lines from $H\alpha$ to $H9$, further He I (λ 6678, 5876 Å), Fe II (5317, 5170, 5017, 4924 Å), N II and Fe II [14]. D. R. Palmer states that he found some absorption lines including the interstellar H and K lines [8]. N. Richter emphasizes the similarity of the spectrum of the Nova in the region 3900—6600 Å to that of Nova DK Lacertae from 1950 [7]. According to G.

[6] Astr. Circ. USSR 209 (1960).
[7] Circ. IAU 1715.
[8] Circ. IAU 1717.
[9] Circ. IAU 1735.
[10] Circ. IAU 1727.
[11] Circ. IAU 1729.
[12] Circ. IAU 1728.
[13] Circ. IAU 1730.
[14] Circ. IAU 1718.

LA DÉVIATION D'EINSTEIN DANS L'ASTRONOMIE STELLAIRE*)

F. Link, Institut Astronomique de l'Académie Tchécoslovaque des Sciences, Prague

Manuscrit reçu le 24 octobre 1960

On donne une revue des travaux parus sur la déviation d'Einstein qui sont tous d'accord sur ses conséquences dioptriques et photométriques dans l'astronomie stellaire. Il en découle une nécessité de rechercher activement les phénomènes en question et d'examiner en détail la probabilité de leur occurrence.

Эйнштейново отклонение в звездной астрономии. В статье приводится обзор работ об Эйнштейновом отклонении света, совпадающих в вопросах диоптрийных и фотометрических последствий в звездной астрономии. Из этого вытекает необходимость отыскивать эти явления и детально изучать вероятность их появления.

1. *Introduction*

Depuis une quarantaine d'années, la déviation d'Einstein au voisinage du Soleil éclipsé est l'objet d'études et d'efforts incessants sans que sa réalité soit démontrée d'une façon absolument convaincante. Dans l'astronomie stellaire, la déviation d'Einstein a attiré beaucoup moins d'attention. Quelques travaux parus il y a un quart de siècle et leurs citations—d'ailleurs fort incomplètes – au cours des dernières années sont tout ce que l'on peut signaler à ce sujet.

*) Conférence tenue le 17 mars 1961 à l'Institut d'Astrophysique de Paris.

Wednesday, September 19, 2012

PHOTOMETRICAL TABLES FOR EINSTEIN'S DEFLECTION OF THE LIGHT

F. Link, Astronomical Institute of the Czechoslovak Academy of Sciences, Praha

Received February 11, 1967

Abridged theory and numerical tables for photometrical and imaging action of Einstein's deflection are given in order to be used in stellar astronomy.

Фотометрические таблицы отклонения света Эйнштейна. Приводятся сокращенная теория и нумерические таблицы для фотометрических и изображающих последствий отклонения света Эйнштейна в применении к звездной астрономии.

1. Introduction

The photometrical consequences of Einstein's deflection of light have been theoretical known since 1936 (Link 1936), but the observational test has not been performed up to the present time. Nevertheless, many authors have treated this problem again in the last few years bringing several interesting suggestions concerning the phenomena to be observed.

The common defect of the majority of these papers is to be found in different and only approximative treatments of the basic problem, i.e. the computation of the illumination produced by a star whose rays are deflected and intensified in the graviation field of another star lying nearly in the line of sight. As the exact solution of this problem is still lacking we

or the total illumination

$$I = \frac{g^2 + 2\varrho_0^2}{g\sqrt{(g^2 + 4\varrho_0^2)}}\,. \tag{3}$$

The angle ϱ_0 called the critical distance is computed from

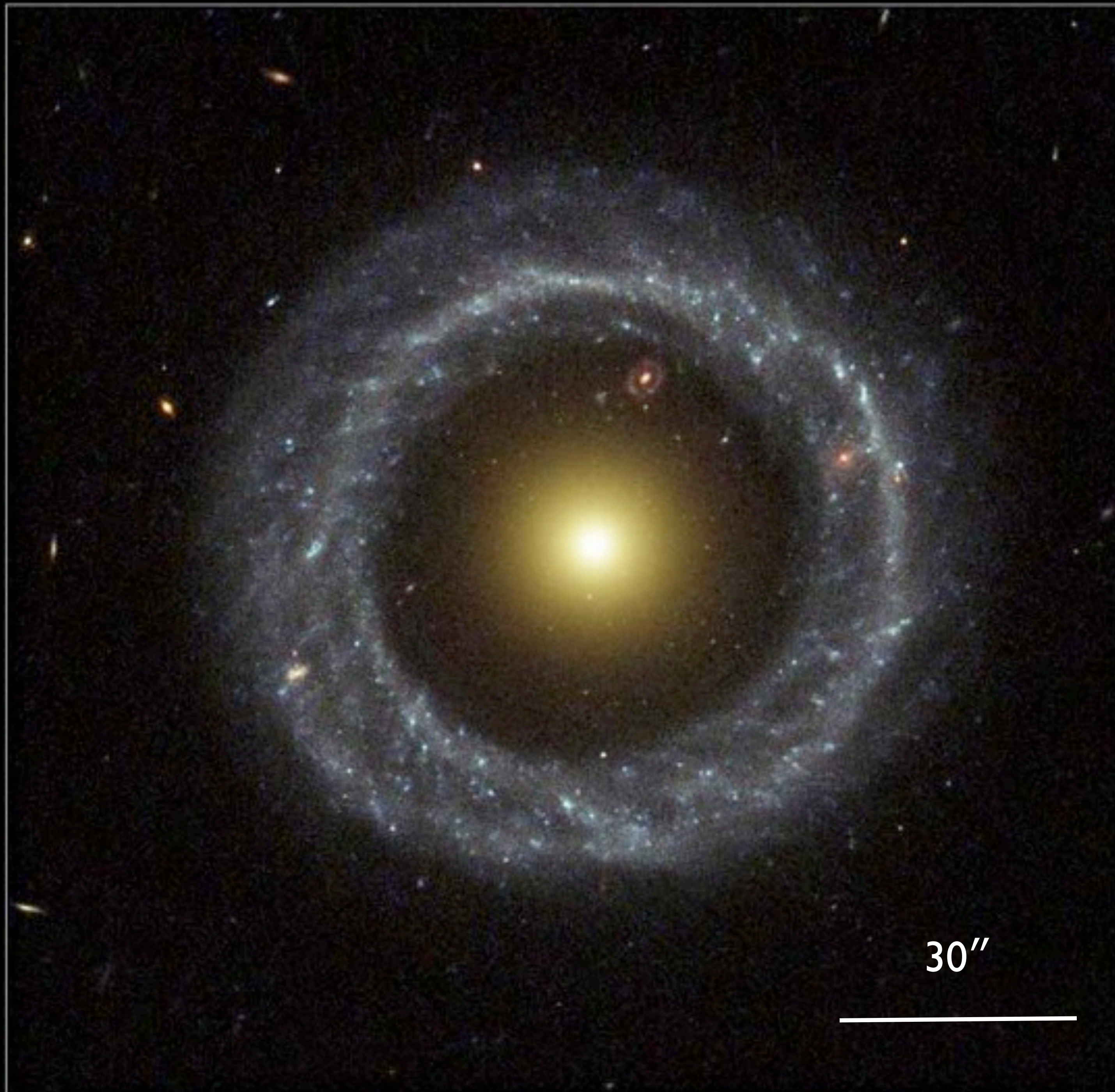
$$\varrho_0 = \sqrt{\left(k\,\frac{m}{m_\odot}\,\frac{a_\odot}{l_2}\,\frac{l_1}{l_1 + l_2}\right)}, \quad k = 1{\cdot}75'' , \tag{4}$$

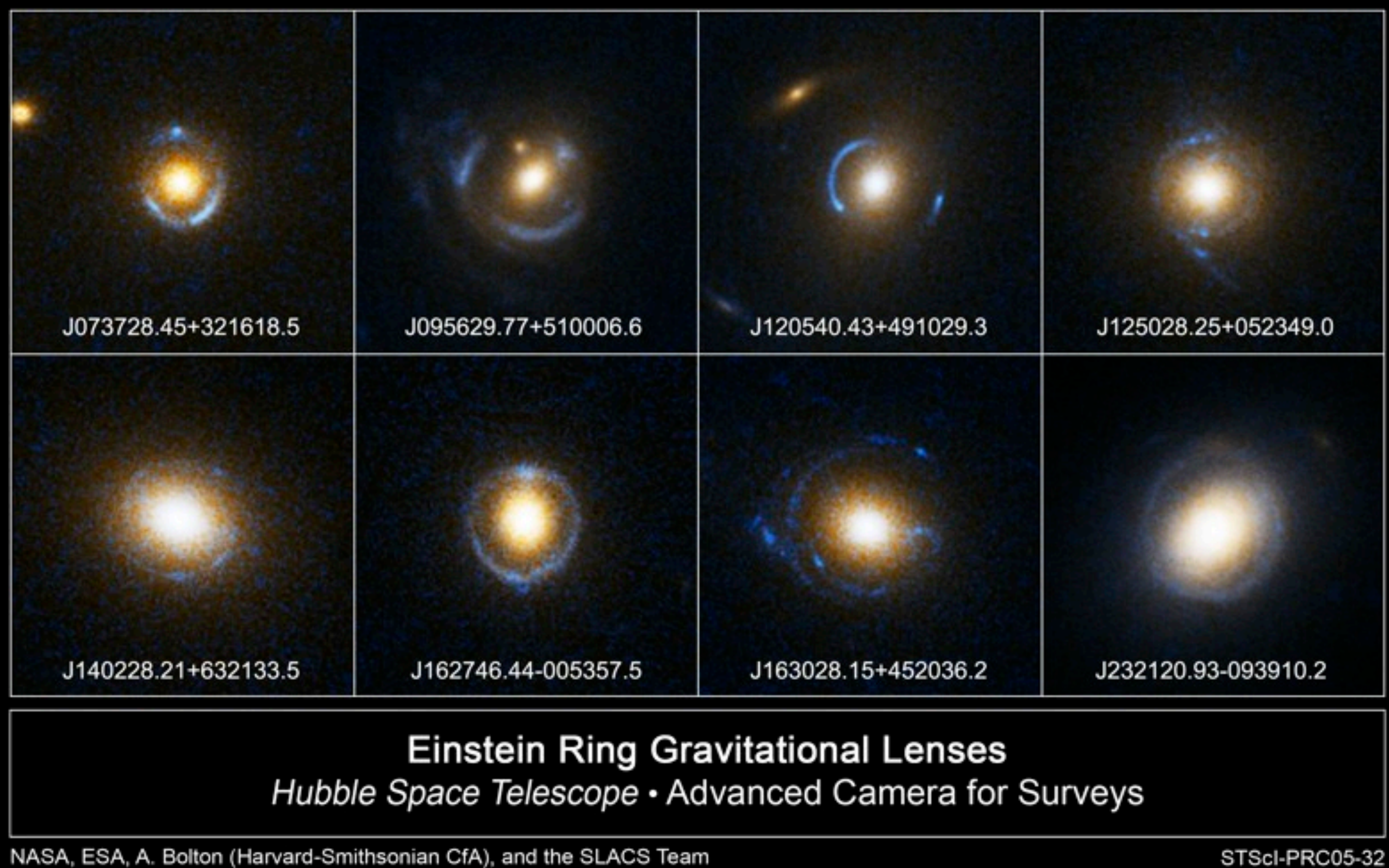
where l_1 and l_2 are the distances in the Fig. 1, m and $m_\odot$ the mass of the deflecting star and of the Sun respectively and $a_\odot$ its radius.

The above formulae can be normalised by the

1. Reproduction of Hoag's object from plate No. 7885. North is at the top and East to the right

Hoag's Object




J073728.45+321618.5
J095629.77+510006.6
J120540.43+491029.3
J125028.25+052349.0
J140228.21+632133.5
J162746.44-005357.5
J163028.15+452036.2
J232120.93-093910.2
Einstein Ring Gravitational Lenses
Hubble Space Telescope • Advanced Camera for Surveys
NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team
STScI-PRC05-32

Meanwhile, over the other side of the pond ...

17 April 1936

- Robert Mandl visits Einstein at Princeton
- Claims that the lens effect can account for :
  - ★ planetary nebulae ;
  - ★ cosmic rays ;
  - ★ the extinction of dinosaurs due to the burst produced during stellar eclipses

2. VI. 36.

2 June 1936

Brief an Herrn Mandl.

$$\xi = \frac{\xi \Delta_0}{\Delta} = \frac{\Delta}{b} + \frac{\Delta - x}{a}$$

Gleichung für Δ

$$\Delta^2 - \underbrace{\frac{x b}{a+b}}_{\xi}\Delta - \underbrace{\frac{\xi_0 \Delta_0 a b}{a+b}}_{A} = 0 \quad (\text{Wurzeln } \Delta_1 (>0) \quad \Delta_2 (<0))$$

für $x = 0$ $\quad \Delta = \sqrt{\dfrac{\xi_0 \Delta_0 a}{1 + \frac{a}{b}}}$ $\quad$ Schwinkel $\dfrac{\Delta}{a} = \sqrt{\dfrac{\xi_0 \Delta_0}{a(1 + \frac{a}{b})}}$

$$\Delta_1 + \Delta_2 = \xi \qquad \Delta_1 \Delta_2 = -A$$

$$\Delta_1 - \Delta_2 = \sqrt{\xi^2 + 4A}$$

Verhältnis der Flächen $\dfrac{d(\Delta_1^2 - \Delta_2^2)}{d(x^2)}$

$$V = \left(1 + \frac{a}{b}\right)^2 \frac{d(\Delta_1^2 - \Delta_2^2)}{d(x)^2} = \frac{d(\Delta_1^2 - \Delta_2^2)}{d(\xi^2)} = \frac{d}{d(\xi^2)} \sqrt{(\xi^2)^2 + 4A\xi^2}$$

$$= \frac{1}{\xi} \frac{2\xi^2 + 2A}{2\sqrt{\xi^2 + 4A}} = \frac{\sqrt{A}}{\xi} \frac{1 + \frac{\xi^2}{2A}}{\sqrt{1 + \frac{\xi^2}{4A}}} \Bigg| = \cancel{\frac{\sqrt{A}}{\xi}}$$

Für grosse V

$$V = \frac{\sqrt{A}}{\xi} = \sqrt{\xi_0 \Delta_0 a \frac{a+b}{b}}$$

Einstein listens politely, carries out the calculations and writes to Mandl :

I have come to the conclusion that the phenomenon in question will, after all, not be observable so that I am no longer in favour of publishing anything about it.

Einstein, 18 April 1936

but Mandl insists ...

DISCUSSION

LENS-LIKE ACTION OF A STAR BY THE DEVIATION OF LIGHT IN THE GRAVITATIONAL FIELD

SOME time ago, R. W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request. This note complies with his wish.

The light coming from a star A traverses the gravitational field of another star B, whose radius is R_o. Let there be an observer at a distance D from B and at a distance x, small compared with D, from the extended central line $\overline{AB}$. According to the general theory of relativity, let α_o be the deviation of the light ray passing the star B at a distance R_o from its center.

For the sake of simplicity, let us assume that $\overline{AB}$ is large, compared with the distance D of the observer from the deviating star B. We also neglect the eclipse (geometrical obscuration) by the star B, which indeed is negligible in all practically important cases. To permit this, D has to be very large compared to the radius R_o of the deviating star.

It follows from the law of deviation that an observer situated exactly on the extension of the central line $\overline{AB}$ will perceive, instead of a point-like star A, a luminius circle of the angular radius β around the center of B, where

$$\beta = \sqrt{\alpha_0 \frac{R_0}{D}}.$$

It should be noted that this angular diameter β does not decrease like $1/D$, but like $1/\sqrt{D}$, as the distance D increases.

Of course, there is no hope of observing this phenomenon directly. First, we shall scarcely ever approach closely enough to such a central line. Second, the angle β will defy the resolving power of our instruments. For, α_o being of the order of magnitude of one second of arc, the angle R_o/D, under which the deviating star B is seen, is much smaller. Therefore the light coming from the luminous circle can not be distinguished by an observer as geometrically different from that coming from the star B, but simply will manifest itself as increased apparent brightness of B.

The same will happen, if the observer is situated at a small distance x from the extended central line $\overline{AB}$. But then the observer will see A as two point-like light-sources, which are deviated from the true geometrical position of A by the angle β, approximately.

The apparent brightness of A will be increased by the lens-like action of the gravitational field of B in the ratio q. This q will be considerably larger than unity only if x is so small that the observed positions of A and B coincide, within the resolving power of our instruments. Simple geometric considerations lead to the expression

$$q = \frac{l}{x} \cdot \frac{1 + \frac{x^2}{2l^2}}{\sqrt{1 + \frac{x^2}{4l^2}}},$$

where

$$l = \sqrt{\alpha_0 D R_0}.$$

Einstein (1936, December)

is a sufficient approximation, since $\frac{x^2}{l^2}$ may be neglected. Even in the most favorable cases the length l is only a few light-seconds, and x must be small compared with this, if an appreciable increase of the apparent brightness of A is to be produced by the lens-like action of B.

Therefore, there is no great chance of observing this phenomenon, even if dazzling by the light of the much nearer star B is disregarded. This apparent amplification of q by the lens-like action of the star B is a most curious effect, not so much for its becoming infinite, with x vanishing, but since with increasing distance D of the observer not only does it not decrease, but even increases proportionally to $\sqrt{D}$.

ALBERT EINSTEIN

INSTITUTE FOR ADVANCED STUDY,

Einstein (1936, December)

Gavriil Tikhov

PhD at Meudon (with Janssen)

In 1937, claims to have had the idea in the summer of 1935, and given talks in January 1936 on the subject

Is eventually sent to Alma-Ata, where he creates the Institute of Astrobotany


G. TIKHOV
L'ENIGME
DES PLANETES

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1937. Volume XVI, № 4

ASTRONOMIE

SUR LA DÉVIATION DES RAYONS LUMINEUX DANS LE CHAMP DE GRAVITATION DES ÉTOILES

Par G. A. TIKHOV, membre correspondant de l'Académie

L'idée d'étudier ce problème m'est venue en été 1935. Après avoir ébauché ses traits principaux j'en ai fait des recherches dans la littérature. Tout ce que j'ai pu y trouver jusqu'en 1935, s'était une note de O. Chwolson «Über eine mögliche Form fiktiver Doppelsterne», parue en 1924 (1). Dans cette note O. Chwolson ne fait point de calculs, mais donne seulement des indications qualitatives sur les phénomènes à attendre.

A la fin de 1935 j'ai déjà résolu la géométrie du problème et j'en ai fait une communication à l'observatoire de Poulkovo le 19 janvier 1936. Je n'ai pas publié immédiatement les résultats de mes recherches parce que la solution ne serait pas suffisamment claire sans étude photométrique du problème, s'est à quoi je me suis mis, autant que les autres travaux urgents me le permettaient.

A présent, quand je rédige cette communication, on trouve déjà dans la littérature un certain nombre de notes concernant ce problème (2, 3, 4, 5, 6).

I. Trajectoire de la lumière dans la théorie classi-

Wednesday, September 19, 2012

Бесплатно.

1938 № 130

ИЗВЕСТИЯ

ГЛАВНОЙ АСТРОНОМИЧЕСКОЙ ОБСЕРВАТОРИИ В ПУЛКОВЕ.

Том XVI, 1.

BULLETIN

DE L'OBSERVATOIRE CENTRAL À POULKOVO.

Vol. XVI, 1.

Sur la déviation des rayons lumineux dans le champ de gravitation des étoiles.

Par G. A. Tikhov.

On a étudié des phénomènes géométriques et photométriques qui devraient se produire parmi les étoiles, si celles-ci faisaient dévier par leur gravitation les rayons lumineux. Le problème a été étudié au point de vue de la gravitation Newtonienne et au point de vue de la théorie de relativité généralisée.

On a donné des valeurs numériques pour quelques étoiles typiques. Il en résulte qu'on pourrait observer dans quelques cas une augmentation considérable d'éclat des étoiles.

L'idée d'étudier ce problème m'est venue en été 1935. Après avoir ébauché ses traits principaux j'ai fait des recherches dans la littérature. Tout ce que j'ai pu

- Calculations within the newtonian framework
- Cites Link (1936)
- Publishes in French (Bull. Obs. Poulkovo, 1937)
- Relativist calculation by A. Bogodorsky, yielding same results as Link
- Cited (sometimes) by Soviet astronomers

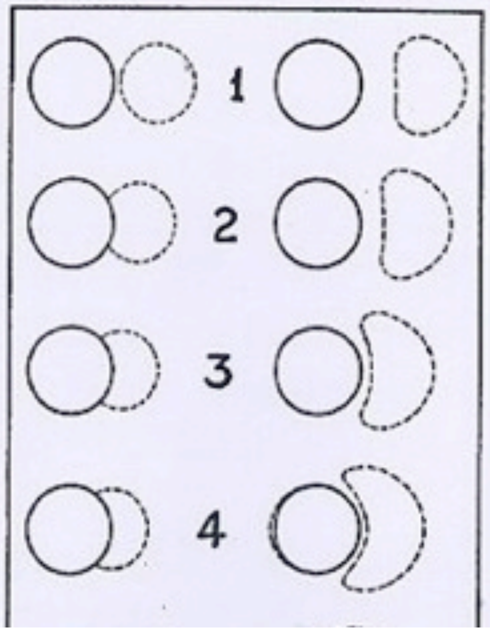
A Relativistic Eclipse

What Might be Seen from a Planet Conveniently Placed Near the Companion of Sirius . . . Perfect Tests of General Relativity that are Unavailable

By HENRY NORRIS RUSSELL, Ph. D.
Chairman of the Department of Astronomy and Director of the Observatory at Princeton University. Research Associate of the Mount Wilson Observatory of the Carnegie Institution of Washington. President of the American Astronomical Society.

IT is a familiar aphorism that the theory of relativity, despite its enormous importance, both in physics and philosophy, may be forgotten in ordinary practical life. There is a good reason. In almost every known case its results agree so closely with those of the older "classical" theories that very accurate observations are required to distinguish between them. Thus, for example, the famous Michelson-Morley experiment requires long series of the most careful measures. The three famous tests of the general theory (which includes gravitation in its scope) are of this sort. The advance of the perihelion of Mercury presents itself in observation as an increase above the much larger, but definitely calculable, amount due to the attractions of the planets. The bending of light rays which pass close to the sun is so small that it can be detected only by observations of stars in the field surrounding the totally eclipsed sun, employing a host of precautions to eliminate sources of error. The slowing of "atomic clocks," which reveals itself as a displacement of spectral lines toward the red, is so minute for the sun that its exact value can hardly yet be determined by observation. In all cases, the results of the best observations agree decisively with the theory of Einstein; but these observations themselves are triumphs of the art of exact measurement. Outside the solar system, the line-shift in the spectrum of a white dwarf star, like the companion of Sirius, is a little larger, but still small enough to demand accurate work to find its value. One related effect, however, is certainly

quences of such an event have just been worked out in detail by Professor Einstein himself.[1] From a point exactly in line, the distant star would appear to be expanded into a ring of light surrounding the other; but, at actual stellar distances, this ring would be too small to be resolved by even the greatest tele-



scopes, and would escape our observation altogether, even if we were watching for it. The focusing effect, however, would concentrate the light of the distant star to a notable degree. For a star which appeared as a mathematical point, with no angular diameter at all, the increase in brightness would be infinite. But any real star must have a finite angular size, however small, and for such a star the increase in brightness, though it might be large, would have a limit.

If a bright star passed in front of a fainter one, its own light would drown out the effect; but if one of the faint red stars, which are really more abundant than any other sort, should get directly between us and a distant super-giant, like Canopus or Rigel, a really conspicuous brightening would occur.

The chances of such an event are, however, negligibly small. The notable increase in brightness happens only when the observer is distant but "a few light-seconds," as Einstein puts it—that is, a million miles or so—from the line through the center of the two stars. The chance that the earth would pass so very

Russell (1937)
Scientific American

Nebulae as Gravitational Lenses

Einstein recently published[1] some calculations concerning a suggestion made by R. W. Mandl, namely, that a star *B* may act as a "gravitational lens" for light coming from another star *A* which lies closely enough on the line of sight behind *B*. As Einstein remarks the chance to observe this effect for stars is extremely small.

Last summer Dr. V. K. Zworykin (to whom the same idea had been suggested by Mr. Mandl) mentioned to me the possibility of an image formation through the action of gravitational fields. As a consequence I made some calculations which show that extragalactic *nebulae* offer a much better chance than *stars* for the observation of gravitational lens effects.

In the first place some of the massive and more concentrated nebulae may be expected to deflect light by as much as half a minute of arc. In the second place nebulae, in contradistinction to stars, possess apparent dimensions which are resolvable to very great distances.

Suppose that a distant globular nebula *A* whose diameter is 2ξ lies at a distance, a, which is great compared with the distance D of a nearby nebula *B* which lies exactly in front of *A*. The image of *A* under these circumstances is a luminous ring whose average apparent radius is $\beta=(\gamma_0 r_0/D)^{\frac{1}{2}}$, where γ_0 is the angle of deflection for light passing at a distance r_0 from *B*. The apparent width of the ring is $\Delta\beta=\xi/a$. The apparent total brightness of this luminous ring is q times greater than the brightness of the direct image of *A*. In our special case $q=2la/\xi D$, with $l=(\gamma_0 r_0 D)^{\frac{1}{2}}$. In actual cases the factor q may be as high as $q=100$, corresponding to an increase in brightness of five magnitudes. The surface brightness remains, of course, unchanged.

The discovery of images of nebulae which are formed

A detailed account of the problems sketched here will appear in *Helvetica Physica Acta*.

F. Zwicky

Norman Bridge Laboratory,
California Institute of Technology,
Pasadena, California,
January 14, 1937.

[1] A. Einstein, Science 84, 506 (1936).
[2] F. Zwicky, Helv. Phys. Acta 6, 124 (1933).
[3] Sinclair Smith, Astrophys. J. 83, 23 (1936).

Zwicky (1937)

- Not a single quantitative estimate
- The ``detailed account'' was never published

The desert of the 1950s

- Few developments, as RG was not trendy :

> *[...] from 1936 [...] to 1961, no courses in General Relativity, not even one single quarter, were given at the University, and the University of Chicago was not atypical.*
>
> Chandrasekhar (1979)

- G. Darwin (1958) : wrong calculation

The wonderful year 63-64

- Idlis & Gridneva (1960) predict weak lensing
- Sachs (1961) optical scalar equations for Riemann space
- Metzner (1963) : *Both Einstein and Darwin have pointed out that the magnitudes involved makes these results more or less irrelevant from the point of view of observational astronomy.*
- Klimov (1963) nevertheless computes the (weak) effects produced by galaxies

SOVIET PHYSICS - DOKLADY VOL. 8, NO. 2 AUGUST, 1963

ASTRONOMY

THE DEFLECTION OF LIGHT RAYS IN THE GRAVITATIONAL FIELDS OF GALAXIES

Yu. G. Klimov

Moscow Engineering Physics Institute

(Presented by Academician A. P. Aleksandrov, August 14, 1962)
Translated from Doklady Akademii Nauk SSSR, Vol. 148, No. 4, pp. 789-792, February, 1963
Original article submitted July 14, 1962

It is known from the general theory of relativity that a ray of light which passes through the gravitational field of any body is deflected. This effect can be observed experimentally at the time of a solar eclipse. In the present paper we consider the possibility of observing the deflection of light rays in the gravitational fields of galaxies.

It is proposed that the sources of light used in these observations should be screened galaxies. Here and in what follows we use the expression "screened galaxy" to mean a galaxy G_1 (Fig. 1) which is located in the solid angle formed by a galaxy G_2 and the point of observation P, and which is hidden from the observer by the galaxy G_2. We shall agree to call screened galaxies which are intersected by the axis SP "axial screened galaxies" and galaxies which the axis SP does not intersect "off-axis galaxies."

Let us assume that the space which surrounds the

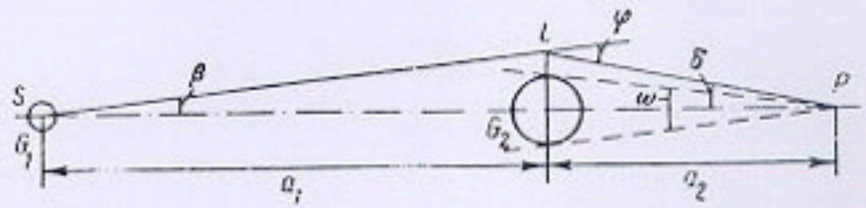


Fig. 1

The condition for the ray SL to arrive at the point P can be written in the form

$$\frac{4\alpha}{a_2(1+k)} = \delta^2, \qquad (3)$$

where

$$k = a_2/a_1;\ a_2 = G_2P;\ a_1 = G_1G_2;\ a_1 = \frac{a_2}{4\alpha/a_2\delta^2 - 1}.$$

If the distance $a_1 \gg a_2$, then the expression (3) can be written in the form

- Liebes (1964) writes a review and considers the lens effect on all scales : stars of the Milky Way, globular clusters, black or unobservable stars, stars in M31, gravitational waves, spikes/flashes, distant galaxies ...

Gravitational Lenses*

SIDNEY LIEBES, JR.
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 16 August 1963)

A stellar gravitational lens has the capacity to intensify by a factor in excess of 1000 the image of another star suitably aligned behind it. This capability, reported in 1936 by A. Einstein in response to a suggestion by R. W. Mandl, appears today not to be generally well known. In the present paper, the imaging properties of the gravitational lens are discussed, and mechanisms whereby lens effects might manifest themselves are evaluated. The relative proper motions of the stars in our galaxy will occasionally give rise to transitory alignments producing time symmetric pulses of light intensity. Typically, for two stars aligned to better than $\sim 10^{-8}$ rad, the image configuration of the more distant, object star will be characterized by an intensification varying inversely with the angular separation of the stars. This inverse angular dependence persists until the disk of the object star, subtending perhaps $\sim 10^{-11}$ rad, passes behind the center of the deflector star. It is estimated, for events involving the $\sim 10^9$ observable stars of the Milky Way, that $\tau_I/A_M \approx 3\times 10^6$ sec, and $\tau_W A_M \approx 5\times 10^7$ sec, where τ_I represents the mean interval between all events which will exhibit at closest alignment an intensity gain for the object star in excess of A_M, and τ_W denotes the temporal width of that pulse characterized by A_M. For the case of a globular cluster of $\sim 3\times 10^5$ deflector stars moving between the earth and the relatively dense object stellar population in the direction of the galactic center, we find $\tau_I/A_M \approx 3\times 10^8$ sec, and $\tau_W A_M \approx 7\times 10^6$ sec. For events involving stars entirely within a single globular cluster of $\sim 10^6$ stars, we find $\tau_I/A_M \approx 8\times 10^7$ sec, and $\tau_W A_M \approx 3\times 10^6$ sec, with A_M restricted, in this case, to be less than ~ 60. Unaccounted for white dwarf and neutron deflector stars are not likely to contribute more than a factor of ~ 2 increase in the frequency of events. No planets of the sun are useful for producing image intensifications. As a result of their gravitational lens action upon the light from other stars, all stars are surrounded by radial spikes of concentrated light intensity. Such spikes might occasionally manifest themselves as luminous beams in cosmic dust or gas clouds. A statistical analysis of the images of distant galaxies might reveal spectral anomalies explicable in terms of red shifted object galaxies intensified by the lens action of nearer deflector galaxies.

- Refsdal (1964) publishes the first microlensing calculation : *It seems safe to conclude that passages observable from the Earth occur rather frequently* [sic]. *The problem is to find where and when the passages take place [...]*
- Zel'dovich (1964) considers the cosmological effects

THE GRAVITATIONAL LENS EFFECT*

Sjur Refsdal

(Communicated by H. Bondi)

(Received 1964 January 27)

Summary

The so-called gravitational lens effect, previously worked out by Tikhov in 1937, is derived in a simple manner. The effect is caused by the gravitational deflection of light from a star S in the gravitational field of another star B, and occurs when S lies far behind B, but close to the line of sight through B. It turns out that a considerable increase in the apparent luminosity of S is possible. A method is given to determine the mass of a star which acts as a gravitational lens. The possibility of observing the effect is discussed.

---

1. *Introduction.*—When a star S lies far behind and close enough to the line of sight through another star B, the light from S to the observer O can, due to the gravitational deflection of light, follow two different paths—both in the plane through S, B and O, and on opposite sides of B—corresponding to two "images", S_1 and S_2 of S (Fig. 1). Chwolson (1924) called attention to this phenomenon, but he did not make any calculations. In 1936 Einstein calculated the light intensity of the two "images", assuming the distance to S to be large compared to the distance to B. He found that the intensity of S_1 and S_2 could be much greater than the normal intensity of S, but concluded that the chance of observing the effect was too small to be of practical interest. Tikhov (1937) calculated the intensities in the general case, but his presentation is not easily followed. In

The ~~forgotten~~ censored chapter of a thesis

- Maria Petrou's PhD thesis deals with several problems on the dynamics of galaxies
- Supervised by D Lynden-Bell (Cambridge)
- Estimates the lensing effects by dark matter compact objects in the halo on stars of the Magellanic Clouds

VII. THE GRAVITATIONAL LENS EFFECT DUE TO THE HALO OBJECTS OF OUR GALAXY.

1. INTRODUCTION.

The most plausible way to confirm the existence of heavy haloes is to detect the one which is supposed to surround our own Galaxy. Indeed, for some time there has been speculation that our Galaxy may possess a dark halo with mass up to 10^{12} $M_{\odot}$. However, we do not know what this halo may be composed of. It can not be gaseous as then it would emit at some wavelength. After the news came out that neutrinos may have a finite mass, there have been suggestions that haloes may consist of neutrinos. The only other possibility is that they may consist of star-like objects. If this last possibility is true, it means that we see the Universe through a screen of dark, maybe compact objects. In this chapter I will investigate what one might expect to see if our Galaxy has a halo made from "Jupiters", white dwarfs or black holes.

Petrou (1981)

Also one might hope to detect stars which if not amplified are not detectable. If $A = 10$, the change in magnitude is 2.5 and a star of 24^{th} magnitude may become detectable. For such minimum amplification we need $k = 1/20$ and from (VII.19) we find

$$p' = \frac{3}{4} \cdot 10^{-8} \tag{VII.28}$$

By considering the undetectable stars too, we have increased the number of sources to 10^{10} and this makes the above probability quite significant.

All these are quite unlikely to be observed but it may be the only way to detect these invisible objects, if after all there are any, around us.

Petrou (1981)

8. CONCLUSIONS.

If we consider only point sources it seems that if our Galaxy possesses a halo of dark compact objects we do not expect to see double images of any source, but we might be able to see transient amplification of extragalactic stars, the duration of which depends upon the kind of halo objects acting as lenses.

If the halo objects are black holes of 2-100 $M_{\odot}$ we may see quasars, HII regions or globular clusters amplified by a factor of about 10. Such an amplification could be detected only in the case of HII regions for which we can predict the luminosity from the metallicity, if known. The amplification may last a few years and eventually disappear due to the proper motions of the halo objects. (An object at distance 50 kpc will take 2400 years to transverse a distance of 1" with velocity 100 km s^{-1}).

- She predicts that *[...] expected number of amplified stars is about 20 and that [...] the variability of the star will be different from other kinds of variability because there will be no change in its colour.*
- Her PhD advisor refuses to let her publish these results (all the other chapters were published)
- DLB would apologise later on

Summary

Epistemology

- Newton was *not* the precursor
- Soldner : unsound calculation, unlike Cavendish
- Link was the pioneer of the subject
- The Einstein mystery : pessimistic for once

Sociology

- Citations : Link *vs* Zwicky
- Fairness : Chwolson-Link-Einstein (CLE) rings
- Maria Petrou : PhD students *vs* supervisors

No light, but rather darkness revealed

Milton (1667)