Lensing Constraints on Galaxy Cluster Mass and Structure

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Part 1:
Primer on the Physics of Galaxy Clusters

1. Primer on the physics of galaxy clusters
2. Galaxy clusters as cosmological probes
3. Galaxy clusters as gravitational lenses
Overview

• Composition of galaxy clusters
• Origin and properties of intracluster medium
• Spherical collapse model
• Dark matter halo mass function
• Self-similar model
• Mass-observable scaling relations
• X-ray mass measurements
• Self-similar density profiles

Composition of galaxy clusters
the largest collapsed structures in the Universe

• Total masses = $10^{14} - 10^{15} M_\odot$
  - How do you define (and measure) the mass of a cluster?
• Dark matter:
  - 85% of matter in clusters is “missing mass” / dark matter
• Galaxy content:
  - hundreds of luminous ($\geq 10^{10} L_\odot$) galaxies
  - only $\sim 5 - 10\%$ of galaxies are found in clusters
  - 30 – 50% of galaxies in a galaxy groups ($M_{\text{viral}}=2x10^{12} - 10^{14} M_\odot$)
• X-ray emitting atmosphere (intracluster medium):
  - 80% of cluster baryons are in a hot ionized plasma
  - $\sim 2x10^{7} - 10^{8} K$, i.e. $2 - 10$ keV
• Intracluster light:
  - stars that lie in between galaxies
• Distinction between groups and clusters is arbitrary
  - typical cut-off: $M \sim 10^{14} M_\odot$ or an $T_X \sim 2$keV.

Composition of the Universe:

Composition of clusters:

Central cDs and intracluster light

Many clusters, and some galaxy groups, have a dominant elliptical (known as a cD – for “central-Dominant”) near the centre, which is surrounded by an extended diffuse stellar halo. M87, in the Virgo cluster, is an example.
Central cDs and intracluster light

The diffuse light within such clusters used to be thought of as a stellar halo surrounding the central cD, but more recently it has been realised that this light may be highly structured, and pervade the whole cluster, as seen in this image of Virgo.

This has led to it being referred to as intracluster light (ICL).

Global properties

The balance between stars and gas is observed to vary substantially from groups to clusters.

The hot gas fraction rises with system mass, whilst the stellar mass fraction falls. In poor groups, the two are approximately equal.

The total baryon fraction seems to be rather less than the cosmic mean value of 17% (from WMAP).

Whether $f_{\text{baryon}}$ drops in groups is controversial.

What is the origin of the ICM?

– a simple model of cluster formation –

- Galaxies form by baryon cooling within dark halos
- These then cluster into groups which grow into clusters
- Galaxy dark halos merge
- Infalling gas is compressed and shocked at $R_{\text{virial}}$

Virial temperature of the ICM

$T \approx \left( \frac{\sigma}{1000 \, \text{km} \, \text{s}^{-1}} \right)^2 \times 6.2 \, \text{keV}$

- $\sigma =$ velocity dispersion of the cluster (typical speed at which galaxies move)
- Try to derive this expression
- Hint:
  - assume that the specific energy of the galaxies and the ICM are roughly equal
  - why is that a reasonable thing to do?
ICM is an optically thin plasma that radiates through a combination of bremsstrahlung and atomic line emission. The bremsstrahlung emissivity has the form:

$$\epsilon(E) = A \left( \sum_{i} Z_i^2 n_e n_i \right) T^{3/2} g(T, E) e^{-E/kT},$$

where for gas of temperature $T$, the Gaunt factor is a slowly varying function of energy:

$$g(T, E) \propto E^{-0.4}.$$  

Line emission starts to dominate in cooler systems: $T \ll 3$ keV.

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**Defining galaxy cluster mass**

- What do we mean by “mass of a cluster”?  
  - Ideally we want to add up all the mass, meaning:  
    - **all kinds** of matter (luminous and dark) …  
    - that is **inside** the cluster

- What do we mean by “inside the cluster”?  
- Where is the edge of a cluster?

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**Spherical Collapse Model**

Consider a uniform spherical density perturbation – a spherical top hat:

$$\Delta_{\text{virial}} = \frac{\rho_{\text{virial}}}{\rho_{\text{crit}}} = 8 \times 5.5 \times 4 = 178$$

(for EdS Universe)

Virialized structures – meaning they obey the virial theorem

$$R_v = 2 R_{\text{virial}} \Rightarrow \frac{\rho_{\text{crit}}}{\rho_v} = 8$$
Virial radius

• Definition:
  – Radius within which the cluster obeys the virial theorem
  – Radius within which the mean cluster density is: virial over-density x critical density

\[ \langle \rho(< r_{\text{virial}}) \rangle = \Delta_{\text{virial}} \rho_{\text{crit}} \]

Virial mass

We can therefore write this expression for the virial mass of a cluster:

\[ M_{\text{virial}} = \frac{4 \pi r_{\text{virial}}^3}{3} \Delta_{\text{virial}} \rho_{\text{crit}} \]

More generally, we can define an “over-density” mass:

\[ M_\Delta = \frac{4 \pi r_\Delta^3}{3} \Delta \rho_{\text{crit}} \]

Common over-densities include: 200, 500, 2500

Which of these radii is the smallest?: \( r_{\text{virial}} \), \( r_{200} \), \( r_{500} \), \( r_{2500} \)

From this we obtain a convenient scaling between mass and radius:

\[ M_\Delta \propto r_\Delta^3 \]

Virial over-density in \( \Lambda \)CDM

• For a flat universe with cosmological constant:
  (Bryan & Norman, 1998)

\[ \Delta_{\text{virial}} = 18 \pi^2 + 82 \left[ \Omega_m(z) - 1 \right] - 39 \left[ \Omega_\Lambda(z) - 1 \right] \]

\[ \Delta_{\text{virial}}(\text{SCDM}) = 18 \pi^2 = 178 \]

\[ \Delta_{\text{virial}}(\Lambda \text{CDM}) \]

Mass function

Galaxy clusters occupy the exponential tail of the halo mass function
Self similar model

• A simple model to relate galaxy cluster mass to observable quantities
  • Assumes that cluster properties are determined by solely gravitational physics
    – Gravitational collapse of DM halos
    – Infall of galaxies and gas
    – Shock heating of intracluster gas

Apply virial theorem to galaxies orbiting in the cluster potential:

\[ M_{\text{virial}} \propto \sigma_{\text{los}}^2 r_{\text{virial}} \]

In virial equilibrium the specific energy of ICM equals specific energy of galaxies, from which we can show:

\[ T_X \propto \sigma_{\text{los}}^2 \Rightarrow M_{\text{virial}} \propto T_X r_{\text{virial}} \]

Remember that mass is proportional to volume \((r^3)\), to obtain:

\[ M_{\text{virial}} \propto T_X^{3/2} \quad \text{More generally:} \quad M_\Delta \propto T_X^{3/2} \]

Think about: how would this relation change if mass is measured within a radius of fixed size?

Self similar model

X-ray luminosity of a cluster depends on density, size, and \(T_X\):

\[ L_X \propto n_e n_i V \Lambda(T_X) \propto r_{\text{virial}}^3 T_X^{4/2} \]

The mass-temperature relation (previous slide) implies a temperature-size relation:

\[ r_{\text{virial}}^3 \propto T_X^{3/2} \Rightarrow L_X \propto T_X^2 \]

Combining the L-T and M-T relation, we obtain the M-L relation:

\[ M_{\text{virial}} \propto L_X^{3/4} \quad \text{More generally:} \quad M_\Delta \propto L_X^{3/4} \]

But clusters are not self-similar

(on all scales)

Gas cools more quickly at higher density, and gas is heated by AGN in cluster cores

Sanderson, O’Sullivan, Ponman, 2009
Example scaling relation slopes

Self-similar evolution

Putting it all together from previous slides:

\[
M(\leq r) = \frac{4\pi r^3}{3} \rho_{\text{crit}} \Delta
\]

Critical density depends on redshift:

\[
\rho_{\text{crit}}(z) = \frac{3H(z)^2}{8\pi G}
\]

\[
H(z) = H_0 \left[ \Omega_m (1+z)^3 + \Omega_{\Lambda} \right]^{1/2} = H_0 E(z)
\]

Two options regarding over-density:

1. Fixed value, e.g. 200, 500, 2500
2. Virial over-density

\[
\Delta_{\text{virial}}(z) = 18\pi^2 + 82 \left[ \Omega_m(z) - 1 \right] - 39 \left[ \Omega_m(z) - 1 \right]^2
\]

\[
\Omega_m(z) = \frac{\Omega_{\Lambda} (1+z)^3}{E(z)^2}
\]

M-T evolution is consistent with self-similar, but beware selection effects and mass measurement systematics …

Putting it all together from previous slides:

\[
M_\Delta \propto T^{\Delta/2} L_X^{\Delta/2}
\]

\[
L_X \propto T_X^{2/3} L_X
\]

Basic idea: assume the ICM is in hydrostatic equilibrium with a spherical gravitational potential, and infer mass from the pressure gradient.

\[
\frac{dP}{dr} = -\frac{GM(<r)\rho}{r^2} \Rightarrow M(<r) = -\frac{rkT(r) \rho}{G \mu_m p} \left[ \frac{d\ln \rho}{d\ln r} + \frac{d\ln T}{d\ln r} \right]
\]

Figures from Trevor Ponman
Self-similar density profile?
Navarro, Frenk, White, 1997

The density profile of "equilibrium" dark matter halos in DM-only numerical simulations is independent of mass and cosmology.

\[ \rho(r) = \frac{\rho_s}{r_s \sqrt{1 + r_s/r}} \]

\[ \delta_c = \frac{\Delta}{3} \ln(1 + \delta_c) - \frac{\delta_c}{1 + \delta_c} \]

\[ c_s = \frac{\Delta}{r_s} \]

\[ \rho \propto r^{-2} \quad (r = r_s \text{ (the scale radius)}) \]
\[ \rho \propto r^{-1} \quad (r < r_s) \]
\[ \rho \propto r^{-3} \quad (r > r_s) \]

Which of these density profiles has the higher concentration parameter?

Summary

- Galaxy clusters contain:
  - a fair sample of the content of universe
  - DM, galaxies, intracluster stars, hot gas
- Galaxy clusters inhabit the exponential tail of the DM halo mass function
- Galaxy clusters are therefore powerful cosmological tools
- Cluster cosmology would be easy (and boring!) if clusters are self-similar
- Cluster masses can be inferred from X-ray (and optical) observations by assuming relationship between baryons and DM
- Departures from self-similarity \( \rightarrow \) lots of interesting physics

Part 2

Galaxy Clusters as Cosmological Probes
- examples and mass measurement issues –
Overview

- Examples of cluster-based cosmological constraints
  - Dark matter
  - Low density universe
  - Nature of dark matter
  - Dark energy
- Mass measurement issues
  - Critique of cluster mass measurement in state of the art cluster-based DE constraints
The baryon content of galaxy clusters: a challenge to cosmological orthodoxy

Simon D. M. White, Julio F. Navarro, August E. Evrard & Carlos S. Frenk

Baryonic matter constitutes a larger fraction of the total mass of rich galaxy clusters than is predicted by a combination of cosmic nucleosynthesis considerations (light-element formation during the Big Bang) and standard inflationary cosmology. This cannot be accounted for by gravitational and dissipative effects during cluster formation. Either the density of the Universe is less than that required for closure, or there is an error in the standard interpretation of element abundances.

Flat, matter dominated flat universe combined with primordial nucleosynthesis:

\[ \Omega_{\text{baryon}} = \frac{\rho_{\text{baryon}}}{\rho_{\text{crit}}} = 0.0125 \, h^{-2} \]

Inventory of galaxies, gas, plus X-ray/dynamical estimates of total mass, and assumption that cluster content is representative of the universe:

\[ f_{\text{baryon}} = \frac{M_{\text{galaxies}} + M_{\text{gas}}}{M_{\text{total}}} \approx 0.06 \, h^{-1.5} \]

White et al., 1993

Thanks to Nigel Watson, Birmingham & CERN

The bullet in context

Convert cross-section to self-interaction to particle physics units:

\[ 1 \, \text{cm}^2/\text{g} = 1 / (0.931/1.66 \times 10^{-24}) \, \text{cm}^2/\text{GeV} = 1.8 \times 10^{-24} \, \text{cm}^2/\text{GeV} = 1.8 \times 10^{-1} \, \text{pb}/\text{GeV} \]

For illustrative purposes, assume DM candidate has mass comparable with electroweak scale (100GeV):

\[ \sigma_0 \leq 1.8 \times 10^{-7} \, \text{pb} \]

This is huge!! Comparable to proton-proton inelastic cross-section at LHC!!

Direct detection experiments target DM-nucleon cross-sections of \( \sim 10^{-6} - 10^{-2} \, \text{pb} \)

The only direct evidence for the existence of DM comes from astrophysical observations.

Impossible to test DM-DM self-interaction in any other way than via astrophysical observations.

Dark Energy

Growth of structure (see Cristiano’s 3rd lecture)

Relies on scaling relations and unrealistic simulations

Expansion history of universe

\[ f_{\text{gas}} \propto D_L D_A^{0.5} \]

\( f_{\text{gas}} \) assumed not to evolve

But it is a function of mass!

Figure 9. Examples of cluster data used in recent cosmological work. Top: Measured mass functions of clusters at low and high redshifts are compared with predictions of a flat, \( \Omega = 0.3 \), and an open model without dark energy (from Vikhlinin et al. 2004). Bottom: \( D_L \) measurements for relaxed clusters are compared with predictions for a flat, \( \Omega = 0.3, \Lambda = 0.7 \) model (right; from Allen et al. 2009). For purposes of illustration, cosmology-dependent shifter quantities are shown (green and \( f_{\text{gas}} \)). In practice, model predictions are compared with cosmology-independent measurements.

Allen et al., 2011, ARA&A
Reliability of cluster masses

- In principle:
  - Count clusters as function of mass and redshift

- In reality:
  - As a function of a mass-like observable and redshift
  - Use a mass-observable scaling relation to get mass

[Image of graphs and data from Kravtsov et al., 2006; Vikhlinin et al., 2009]

Systematic errors?

Reliability of cluster masses

- Critique of Vikhlinin et al., 2009 –

Cluster samples selected purely on X-ray flux / luminosity
- i.e. no morphological selection

X-ray temperature and gas density measurement methods calibrated on over-cooled simulations
Mass-Y_X relation (predicted by over-cooled simulations to be reliable)
used to convert X-ray observables to mass

Mass-Y_X relation measured for 17 "relaxed" clusters – meaning they look round

Figure 3. Typical examples of X-ray images for the low-mass halo clusters (A85, A2163, and A2257 top to bottom). The left panels show the Chandra images (each panel is 50' x 50'). ROSAT PSPC images (94' x 94') are shown on the right. Yellow circles show detected sources unrelated to the clusters, the general increase of their radius at large off-center distances reflects the degradation of the telescope PSF. The red crosses indicate the cluster substructures that were removed from the profile analysis (Section 3.2). The red crosses mark the location of the adopted cluster centroid (Section 3.2).

Reliability of cluster masses

- Kravtsov et al. simulations
  - DO include baryons, but …
  - DO NOT reproduce observed clusters, because they …
  - DO NOT include AGN feedback

Controlling cluster mass measurement systematics

Adopt a mass observable scaling relation of the form: $M = M_0 X^\alpha$

For simplicity assume there is no intrinsic scatter.

Error on normalization of relation will scale with sample size, N, and error on individual cluster mass measurements, $M_{\text{cluster}}$, like this:

$$\frac{\delta M_0}{M_0} \approx \frac{\delta M_{\text{cluster}}}{M_{\text{cluster}}} \frac{1}{\sqrt{N}}$$

Typical statistical errors on mass measurements are 20%

A sample of 50 – 100 clusters therefore yields a statistical error on normalisation of scaling relation of:

$$\frac{\delta M_0}{M_0} \approx 0.02 - 0.03$$

A useful benchmark goal is to control systematic errors to the same level – this has not yet been achieved / proven.
1. Primer on the physics of galaxy clusters

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Summary of Part 1

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- Galaxy clusters inhabit the exponential tail of the DM halo mass function
- Galaxy clusters are therefore powerful cosmological tools
- Cluster cosmology would be easy (and boring!) if clusters are self-similar
- Cluster masses can be inferred from X-ray (and optical) observations by assuming relationship between baryons and DM
- Departures from self-similarity → lots of interesting physics

Summary of Part 2

- Clusters are long-standing probes of cosmology
- Clusters can constrain dark energy in multiple ways:
  - growth of structure (number counts, mass function, clustering)
  - expansion history ($f_{\text{gas}}$, strong-lensing tomography)
- Main challenges:
  - control systematics in mass measurements
  - calibrate and understand departures from self-similarity
  - constrain evolution of cluster population
  - understand selection biases
- Gravitational lensing is a promising tool …
Part 3

Galaxy Clusters as Gravitational Lenses

Overview

- Strong lensing by clusters
  - Simple mass measurements
  - Parametric mass measurements
  - Substructure of cluster cores
  - Example results (emphasising samples)

- Weak lensing by clusters
  - From raw data to cosmology
  - X-ray/lensing mass comparison
  - $M_{WL}$-observable scaling relations

Lensing by Galaxy Clusters

Cluster Strong Lensing

Einstein radius depends on lensing efficiency and mass distribution

\[ \theta_E = \theta_s + \frac{D_{LS}}{D_{OS}} \alpha(\theta_s) \]

For a singular isothermal sphere of velocity dispersion $\sigma$:

\[
\theta_E \approx \frac{4 \pi \sigma^2 D_{LS}^2}{c^2 D_{OS}^2} \left( \frac{\sigma}{1000 \, \text{km/s}} \right)^2 \left( \frac{D_{LS}/D_{OS}}{0.4} \right)^2 20 \text{arcsec}
\]

\[
R_E = D_{OS} \theta_E = \frac{4 \pi \sigma^2 D_{LS}^2 D_{OS}}{c^2} \left( \frac{\sigma}{1000 \, \text{km/s}} \right)^2 \left( \frac{D}{500 \, \text{Mpc}} \right) 70 \text{kpc}
\]
Simplest mass measurement

\[ M(< R_E) = \Sigma_{\text{crit}} \pi R_E^2 \]

\[ \Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_{\text{DS}}}{D_{\text{OL}}D_{\text{LS}}} \]

Questions

- How reliable is the assumption of circular symmetry?
- Strongly distorted single image or bona fide multiple-imaging?
- How sensitive is the measurement to the source redshift?
- How to combine several multiple image systems?

Circular Symmetry?

Single or multiple images?

If the arc is not multiply-imaged, then the mass estimate is an upper limit:

\[ \Sigma(< R_{\text{arc}}) < \Sigma_{\text{crit}} \]

\[ M(< R_{\text{arc}}) < \pi R_{\text{arc}}^2 \Sigma_{\text{crit}} \]

From Kneib & Natarajan 2011; courtesy of Johan Richard

Adapted from: Kneib & Natarajan, 2011
Sensitivity to redshift?

Calibrating the accuracy of $M(<R_E)$

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**Large Einstein Radii:
A Problem for LCDM(?)**

- Most spectacular SL clusters are $\sim 2\sigma$ outliers when compared with simulations

- Interpretation?
  - Early collapse of cluster cores
  - Modify properties of dark matter?
  - Modify slope of primordial power spectrum?
  - Primordial non-Gaussianity?

- Better to compare the **"full"** distribution of observed and simulated SL clusters

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**Einstein radius distribution**


- Log-normal $\theta_E$ distribution for strong-lensing/X-ray selected clusters

- Observed and theoretical distributions offset by $\sim 1\sigma$

- Future needs:
  - Larger observed sample
  - Better calibration of selection functions
  - Full SL+WL models to improve comparison with simulations
  - More realistic simulations (BCG formation)
Fitting models to SL constraints

• Choose a projected mass distribution
• Calculate deflection field
• Calculate predicted positions of images
• Compare predicted and observed positions
• Change mass distribution
• Decide to stop
• Repeat with different model?
• Choose model (i.e. how complex?)

Parameterising the mass distribution

• One singular isothermal sphere/ellipse
  – Over-produces central images
  – X-ray surface brightness profiles are curved

• One non-singular sphere/ellipse
  – Images reproduced to ~1 – 5 arcsec precision
  – Corresponds to deflection angle of galaxies/groups

• Multiple non-singular cluster/group/galaxy-scale masses
  – Images reproduced to ~0.1 – 0.5 arcsec precision
  – Precision depends on number of constraints

\[ \phi_{\text{total}} = \sum_i \phi_{\text{extended}}^i + \sum_j \phi_{\text{galaxies}}^j \]

Parameterising the mass distribution

Examples from GPS, Kneib, et al. 2005 (5 SL clusters in total)

Examples from Richard, GPS, Kneib, et al., 2010 (20 SL clusters from LoCuSS)

See also: Paraficz et al., 2012 – explicit inclusion of ICM in a SL model
Parameterising the mass distribution

- Extended halos:
  - Navarro, Frenk, White (1997) profile
  - Smoothly truncated pseudo-isothermal elliptical mass distribution (PIEMD)
    Kassiola & Kovner (1993), Kneib et al., (1996), Jean-Paul’s lectures

- Choice depends on what question you ask:
  - Questions about total mass distribution – NFW and PIEMD agree within errors (Richard, GPS, Kneib, et al., 2010)
  - Questions about the distribution of DM – must use NFW (e.g. Sand et al., 2008)

- Typical PIEMD parameter values:
  \[
  \frac{\sigma_0}{300} < \frac{r_{\text{core}}}{100} < 1300 \text{ km/s}, \quad 20 < r_{\text{cut}} < 150 \text{ kpc}, \quad r_{\text{cut}} = 1 \text{ Mpc}
  \]

NFW or PIEMD?

Examples from Richard, GPS, Kneib, et al., 2010 (20 SL clusters from LoCuSS)

Parameterising the mass distribution

- Parameterise galaxy-scale halos as PIEMDs
- Individually optimise parameters (generally just velocity dispersion) of galaxies close to multiple-images
- Scale mass of other galaxies (can be 10s of galaxies) on their luminosity

Fitting models to SL constraints

- Choose a projected mass distribution
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- Choose model (i.e. how complex?)

David and Leon’s lectures
MCMC
Bayesian evidence
Tom’s lecture
Jullo et al., 2007
Smith et al., 2009
Limousin et al., 2012
Non-analytic modeling schemes

- **Basic idea:**
  - Parameterise the mass distribution as a grid of pixels
  - Pixel values are the model parameters – i.e. “non-parametric” is misnomer

- **Advantages:**
  - Greater flexibility helps to explore complicated merging clusters

- **Disadvantages:**
  - Arbitrarily good fits can be achieved – how robust?
  - Known (visible) mass not included explicitly
  - Strong lensing signal is typically sparse – not true for a few spectacular systems
  - Additional assumptions invoked: e.g. smoothest mass distribution, mass positivity

- **Examples:**
  - Bradac et al., 2005; Diego et al. 2005, 2007; Saha and Williams 1997; Coe et al. 2010; …

- **Hybrid analytic+non-analytic schemes also under development:**
  - Jullo & Kneib, 2009; Paraficz et al., 2012

Substructure in cluster cores

X-ray/lensing mass comparison

Richard, GPS, et al., 2010

Richard, GPS, et al., 2010

Richard, GPS, et al., 2010
Recommended further reading

- Constraining the slope of cluster density profiles
  - with radial arcs: Fort et al., Smith et al., 2001;
  - with radial arcs + stellar dynamics: Gavazzi et al., ;
    Sand et al., 2004, 2008;
  - with radial arcs + dynamics + … : Newman et al.,
    – with rare image configurations: Limousin et al.
- Joint fitting of SL + X-ray + SZ data:
  - Morandi et al., …
  - Mahdavi et al., …
- *Will be completed before notes go online!*

Lensing by Galaxy Clusters

Strong- and weak-lensing

Extracting mass from WL data

- Fit an analytic density profile
  - e.g. SIS, CPL, NFW, ...

- Aperture mass densitometry
  - Fahlman et al., 1994; Clowe et al., 1998; Hoekstra 2007; Okabe et al., 2010

- Projected density contrast, $\Delta \Sigma$
  - Mandelbaum et al., 2005, 2010; Johnston et al., 2007; Leauthaud et al., 2010; High et al., 2012
Stacked weak-lensing

Are cluster density profiles curved?

- SIS profile rejected:
  - \(6\sigma (M_{\text{vir}} < 6\times 10^{14} M_\odot/h)\)
  - \(11\sigma (M_{\text{vir}} > 6\times 10^{14} M_\odot/h)\)

Are clusters over-concentrated?

- \(10^{15} M_\odot/h\) clusters:
  - \(<c_{\text{vir}}^2> = 3.48_{-1.15}^{+1.65}\)
  - Inconsistent with \(c=10\) at \(-4\sigma\)

From raw data to cosmology

- Raw data \(\rightarrow\) galaxy shapes
  - See David and Tom’s lectures
- Galaxy shapes \(\rightarrow\) shear signal
  - Redshift distribution of faint galaxies
- Shear signal \(\rightarrow\) \(M_{WL}\)
  - Model choice
- \(M_{WL}\) \(\rightarrow\) cosmological constraints
  - \(M_{WL}\) is itself a “mass proxy”
Basic idea: faint cluster galaxies (and foreground galaxies) can dilute the measured shear signal

\[ \text{Galaxy shapes} \rightarrow \text{shear signal} \]

\[ \text{Redshift distribution of faint galaxies} \]

\[ (\theta_0^*, \theta_0^\text{crit}) \]

\[ \text{High et al., 2012} - 5 \text{ clusters from SPT} \]

\[ \text{Limousin et al., 2007} - \text{A1689} \]

\[ \text{Okabe et al., 2010} - 30 \text{ clusters} \]

Shear signal \( \rightarrow \) \( M_{\text{WL}} \)

Model Choice

Basic idea: triaxiality and substructure cause systematic errors in WL mass measurement of individual clusters

\[ M_{\text{WL}} \rightarrow \text{cosmological constraints} \]

\[ M_{\text{WL}} \text{ is itself a mass proxy} \]

- Accurately calibrated \( M_{\text{WL}} \)-observable scaling relation is not enough …
- \( M_{\text{WL}}-M_{\text{true}} \) relation is also required – see previous slide
- Open questions:
  - How to define mass? 3D, 2D, over-density, fixed radius, …
  - How do alternatives perform relative to realistic hydro simulations? (they are coming!)

\[ \text{Meneghetti et al., 2010} \]

\[ \text{King & Corless 2007} \]

See also: Becker & Kravtsov 2011; Bahe et al., 2012; Corless & King 2007, 2008, 2009
Aside on $M_X$ systematics

$M_X/M_{WL}$ comparison

**CCCP:** Mahdavi et al., 2008
**LoCuSS:** Zhang et al., 2010

$M_{WL}/X$-ray scaling relations

Okabe et al. (2010) – Lowest scatter X-ray observable appears to be $M_{\text{gas}}$ (but the sample is small – so far) – NOT $Y_X$

See also: Mahdavi et al. submitted

$M_{WL}/SZ$ scaling relations

Marrone, GPS, et al., 2012:
- 20% scatter on MWL-Y relation
- BCG ellipticity indicates DM halo orientation?

See also: Hoekstra et al., 2012
Summary

• “Creative tension” between lensing and X-ray approaches is very stimulating
• Both communities making good progress on controlling systematic errors
• Lots of opportunities/work remains to be done
• Lots of data arriving in the next decade and more
• Prospects are strong for cluster cosmology (and learning lots of interesting astrophysics!)