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# Large-Scale Structure Observations

## Lecture 2

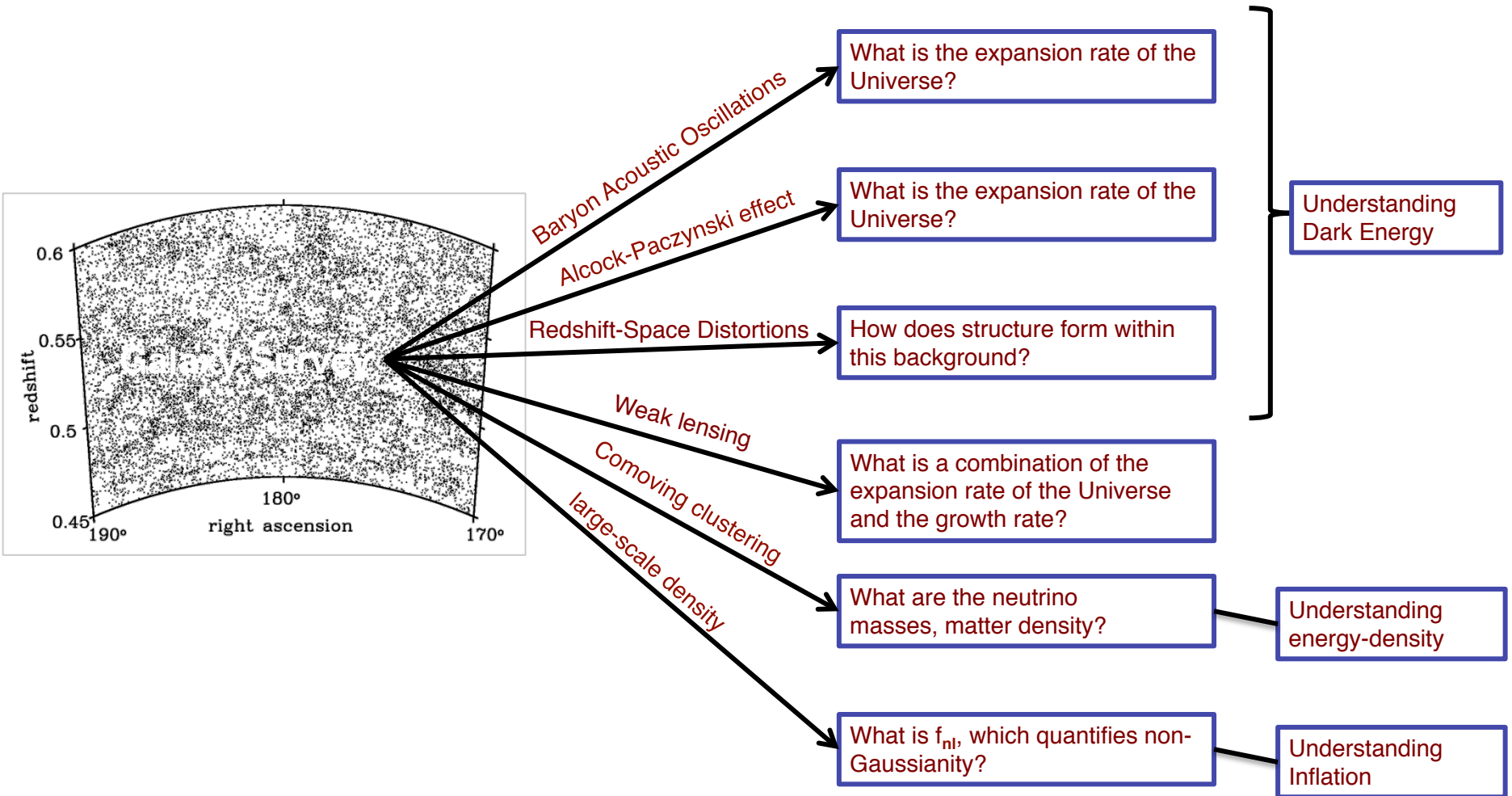
Will Percival



Science & Technology  
Facilities Council



# Cosmology from surveys



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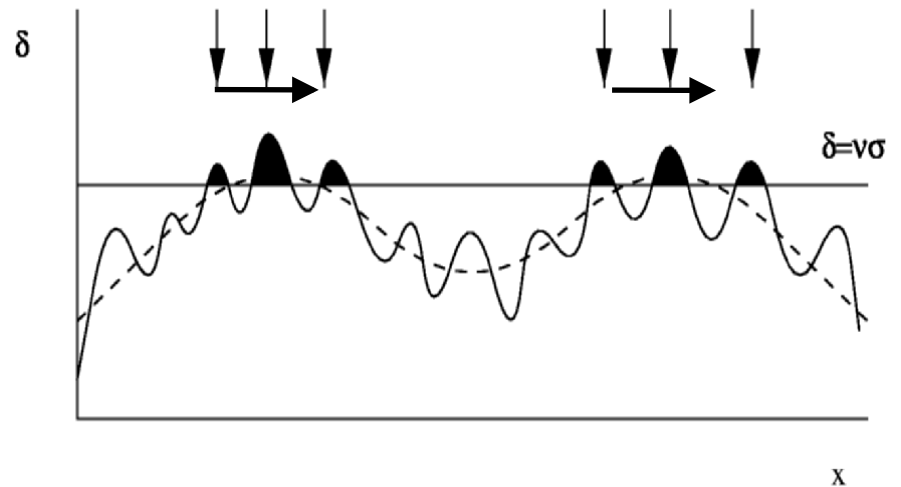
# Redshift-space distortions

# Comoving velocities

Locally, galaxies act as test particles in the flow of matter

On large-scales, the distribution of galaxy velocities is unbiased if galaxies fully sample the velocity field

expect a small peak velocity-bias due to motion of peaks in Gaussian random fields differing from that of the mass



# Redshift-Space Distortions

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When making a 3D map of the Universe the radial distance is usually obtained from a redshift assuming Hubble's law; this differs from the real-space because of its peculiar velocity:

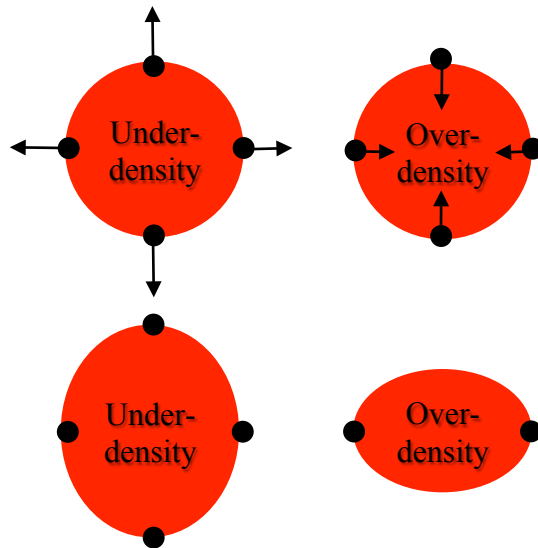
$$\vec{s}(r) = \vec{r} - v_r(r) \frac{\vec{r}}{r}$$

Where  $\mathbf{s}$  and  $\mathbf{r}$  are positions in redshift- and real-space and  $v_r$  is the peculiar velocity in the radial direction

# Two key regimes of interest

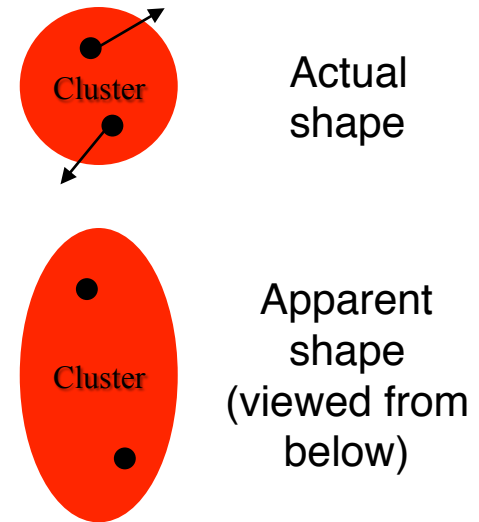
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linear flow



Power is enhanced  
on large-scales

non-linear  
structure



Actual  
shape

Apparent  
shape  
(viewed from  
below)

Power is suppressed  
on small-scales

# Fingers-of-God clearly visible in maps

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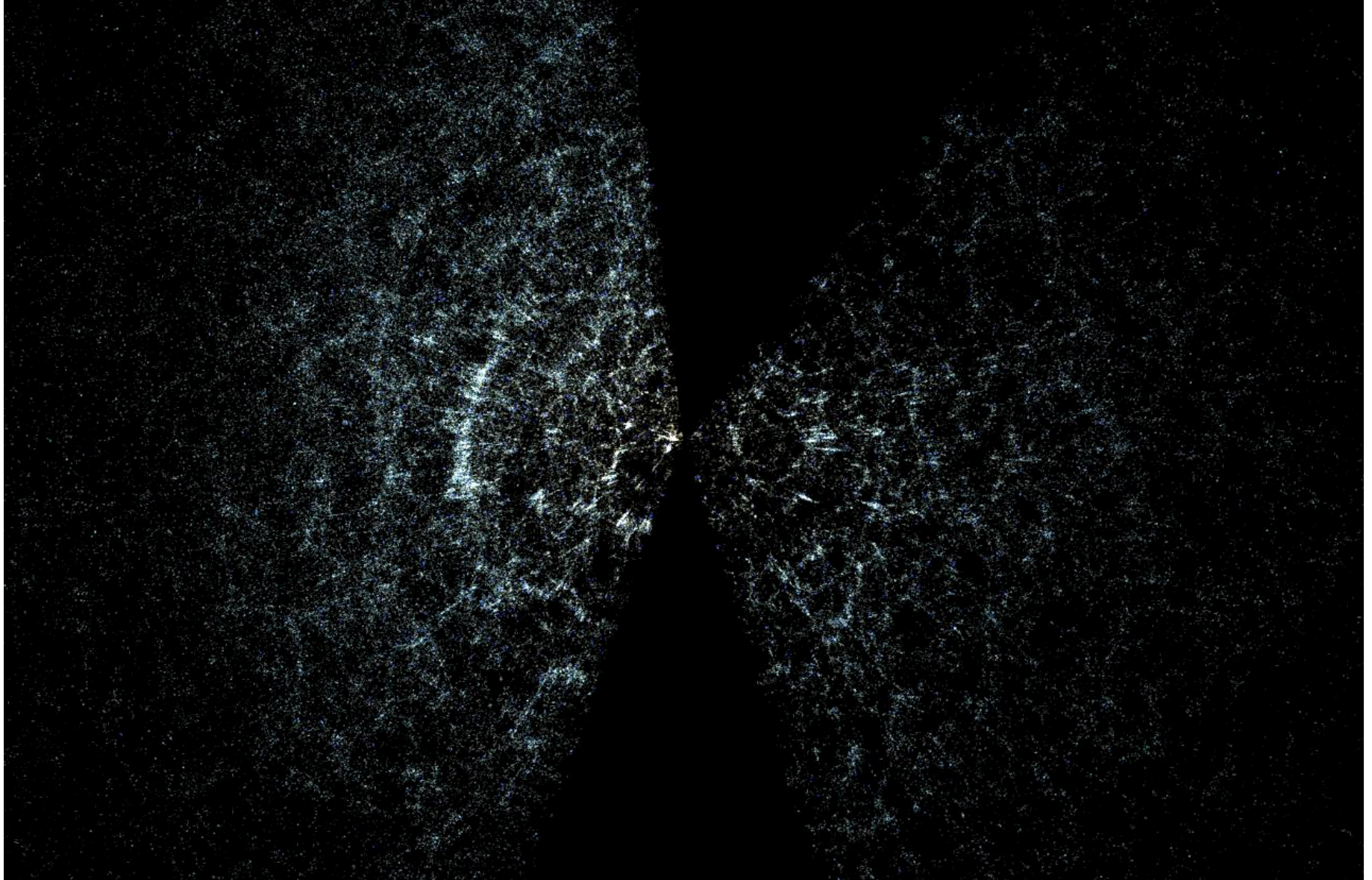


Image of SDSS, from U. Chicago

# Linear plane-parallel redshift-space

Transition from real to redshift space, with peculiar velocity  $\mathbf{v}$  in units of the Hubble flow

$$\mathbf{s} = \mathbf{r} + v_{\text{los}} \hat{\mathbf{r}}_{\text{los}}$$

Jacobian for transformation

$$\frac{d^3 s}{d^3 r} = \left(1 + \frac{v_{\text{los}}}{r_{\text{los}}}\right)^2 \left(1 + \frac{dv_{\text{los}}}{dr_{\text{los}}}\right)$$

Conservation of galaxy number

$$n^r(\mathbf{r}) d^3 r = n^s(\mathbf{s}) d^3 s \quad 1 + \delta_g^s = (1 + \delta_g^r) \frac{d^3 r}{d^3 s} \frac{\bar{n}^r(\mathbf{r})}{\bar{n}^s(\mathbf{s})}$$

Trick to understand velocity field derivative

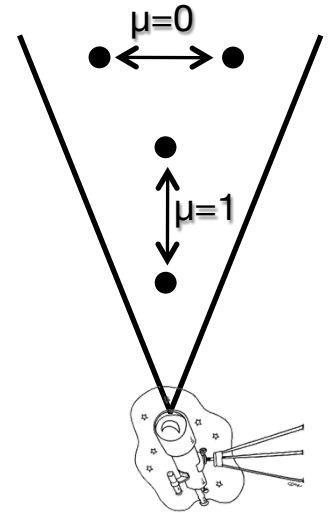
$$\frac{\partial v_{\text{los}}}{\partial r_{\text{los}}} = \left(\frac{\partial}{\partial r_{\text{los}}}\right)^2 \nabla^{-2} \theta = \left(\frac{k_{\text{los}}}{k}\right)^2 \theta = \mu^2 \theta, \quad \theta = \nabla \cdot \mathbf{v}$$

Gives to first order

$$\delta_g^s = \delta_g^r - \mu^2 \theta$$

$$\mu = \cos(\alpha)$$

$$\theta = \nabla \cdot \mathbf{u}$$





# what do linear z-space distortions measure?

linear scales,

$$\delta_g^s(\mu) = \delta_g + \mu^2 \theta$$

$$P_g^s(\mu) = \langle |\delta_g + \mu^2 \theta|^2 \rangle$$

$$= P_{gg} + 2\mu^2 P_{g\theta} + \mu^4 P_{\theta\theta}$$

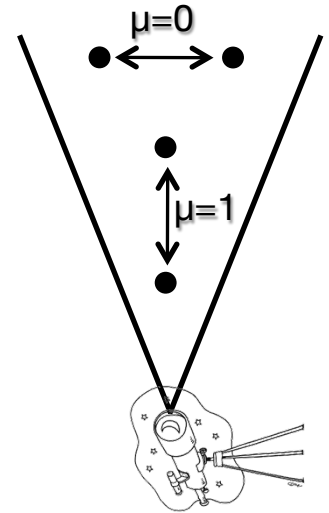
Galaxy-galaxy power

Velocity-velocity power

Galaxy-velocity divergence cross power

$$\mu = \cos(\alpha)$$

$$\theta = \nabla \cdot \mathbf{u}$$



In linear regime

$$\delta_g = b\delta(\text{mass}), \quad \theta = -f\delta(\text{mass}), \quad f \equiv \frac{d \ln G}{d \ln a}$$

Linear growth rate

So, the simplest model for the power spectrum is

$$P_g^s(k, \mu) = [b + \mu^2 f]^2 P_{\text{mass}}(k)$$

# Modeling redshift space distortions

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Include model for both “regimes”

$$P_g^s(k, \mu) = [P_{gg}(k) + 2\mu^2 P_{g\theta}(k) + \mu^4 P_{\theta\theta}(k)] F(k, \mu^2)$$

Note that non-linear model is not necessarily more accurate than the linear one. If we assume linear bias

$$P_g^s(k, \mu) = P_m^r(k) [b^2 + 2\mu^2 fb + \mu^4 f^2] F(k, \mu^2)$$

On small scales, galaxies lose all knowledge of initial position. If pairwise velocity dispersion has an exponential distribution (superposition of Gaussians), then we get this damping term for the power spectrum.

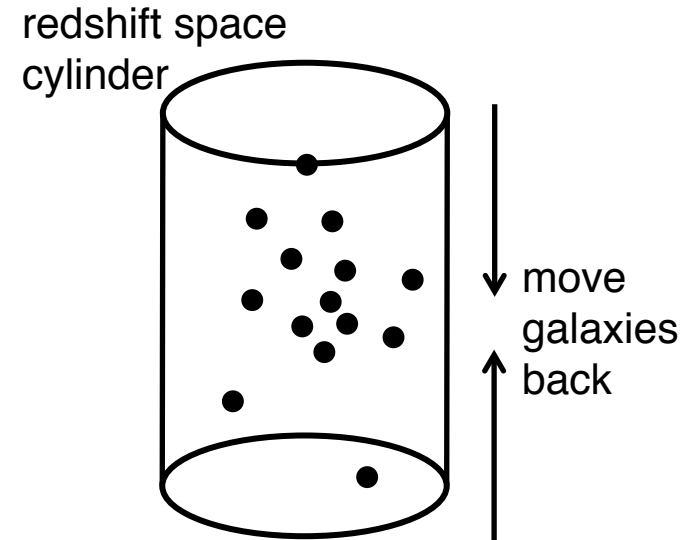
$$F(k, \mu^2) = (1 + k^2 \mu^2 \sigma_p^2 / 2)^{-1}$$

# Modeling redshift space distortions

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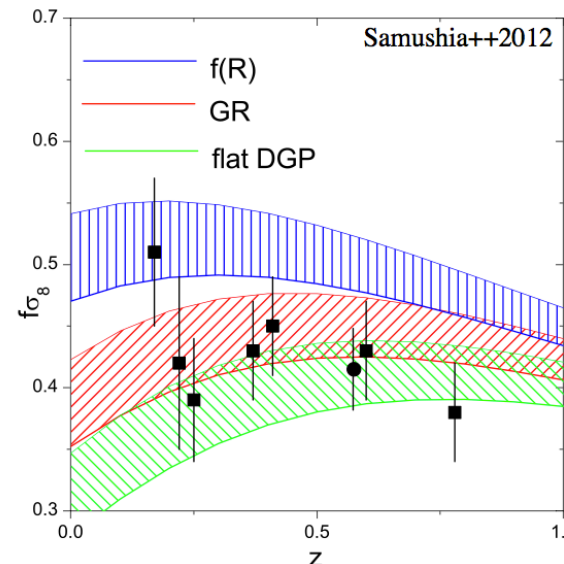
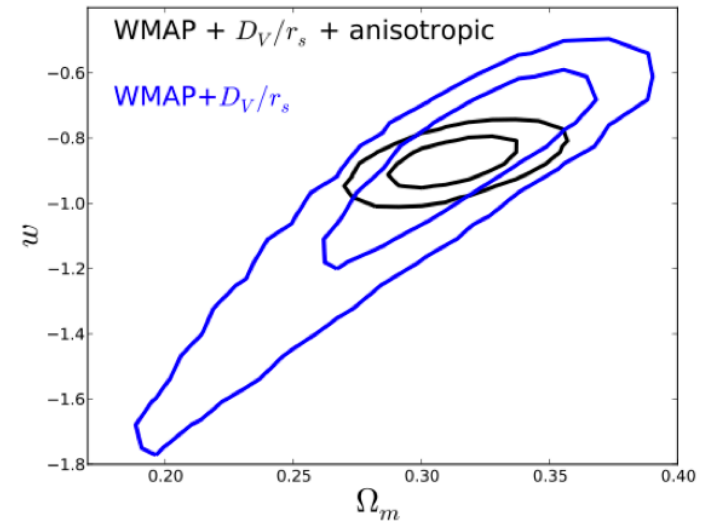
Alternative for the data is to try to “correct” the data by “collapsing the clusters”

- Velocity dispersion of the Luminous Red Galaxies (LRGs) shifts them along the line of sight by  $\sim 9 h^{-1}\text{Mpc}$ , and the distribution of intrahalo velocities has long tails.
- Use an asymmetric “friends-of-friends” (FOF) finder to match galaxies in the same clusters, and collapse to spherical profile
- Parameters of FOF calculated by matching simulations



# Cosmology improved with RSD

- Anisotropic clustering allows huge improvement on  $w$ !
- $w = -0.95 \pm 0.25$  (WMAP +  $D_V(0.57)/r_s$ )
- $w = -0.88 \pm 0.055$  (WMAP + anisotropic)
- Provides a number of GR tests



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# Anisotropic statistics

# Legendre moments – power spectrum

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Remember that, in linear theory and the plane-parallel limit, we have an angular dependence

$$P_g^s(k, \mu) = P_{gg}(k) + 2\mu^2 P_{g\theta}(k) + \mu^4 P_{\theta\theta}(k)$$

Then we can consider the orthogonal Legendre multipoles of P

$$P_\ell^s(k) \equiv \frac{2\ell + 1}{2} \int_{-1}^{+1} d\mu P_g^s(k, \mu) L_\ell(\mu)$$

where

$$L_0(\mu) = 1$$

$$L_2(\mu) = (3\mu^2 - 1)/2$$

$$L_4(\mu) = (35\mu^4 - 30\mu^2 + 3)/8$$

Such that

$$\begin{pmatrix} P_0^s(k) \\ P_2^s(k) \\ P_4^s(k) \end{pmatrix} = \begin{pmatrix} 1 & 2/3 & 1/5 \\ 0 & 4/3 & 4/7 \\ 0 & 0 & 8/35 \end{pmatrix} \begin{pmatrix} P_{gg}(k) \\ P_{g\theta}(k) \\ P_{\theta\theta}(k) \end{pmatrix}$$

$$P^s(k) = P_0^s(k)L_0(\mu) + P_2^s(k)L_2(\mu) + P_4^s(k)L_4(\mu)$$

# Legendre moments – power spectrum

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These can then be manipulated leading to cosmological information

From the monopole, quadrupole and hexadecapole we obtain:

$$\begin{pmatrix} P_{gg}(k) \\ P_{g\theta}(k) \\ P_{\theta\theta}(k) \end{pmatrix} = \begin{pmatrix} 1 & -1/2 & 3/8 \\ 0 & 3/4 & -15/8 \\ 0 & 0 & 35/8 \end{pmatrix} \begin{pmatrix} P_0^s(k) \\ P_2^s(k) \\ P_4^s(k) \end{pmatrix}$$

The ratio of quadrupole to monopole gives:

$$\frac{P_2^s(k)}{P_0^s(k)} = \frac{\frac{4}{3}bf + \frac{4}{7}f^2}{b^2 + \frac{2}{3}fb + \frac{1}{5}f^2}$$

A more complicated formula can be used to eliminate bias:

$$P_{\theta\theta}(k) = \frac{7}{48} \left[ 5(7P_0^s + P_2^s) - \sqrt{35} [35(P_0^s)^2 + 10P_0^s P_2^s - 7(P_2^s)^2]^{1/2} \right]$$

# Legendre moments – correlation function

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The correlation function can similarly be decomposed into Legendre moments

$$\xi_\ell^s(r) = \frac{(2\ell + 1)}{2} \int_{-1}^{+1} d\mu \xi^s(r, \mu) L_\ell(\mu)$$

$$\xi^s(r, \mu) = \sum_{\ell \text{ even}} L_\ell(\mu) \xi_\ell^s(r)$$

The first three even moments  $\xi_0$ ,  $\xi_2$ ,  $\xi_4$  allow the full linear theory to be recovered

$$\xi_0^s(r) = (b^2 + \frac{2}{3}bf + \frac{1}{5}f^2)\xi^r(r)$$

$$\xi_2^s(r) = (\frac{4}{3}bf + \frac{4}{7}f^2)[\xi^r(r) - \bar{\xi}^r(r)]$$

$$\xi_4^s(r) = \frac{8}{35}f^2[\xi^r(r) + \frac{5}{2}\bar{\xi}^r(r) - \frac{7}{2}\bar{\bar{\xi}}^r(r)]$$

$$\bar{\xi}^r(r) \equiv 3r^{-3} \int_0^r \xi^r(r')r'^2 dr' \quad \bar{\bar{\xi}}^r(r) \equiv 5r^{-5} \int_0^r \xi^r(r')r'^4 dr'$$



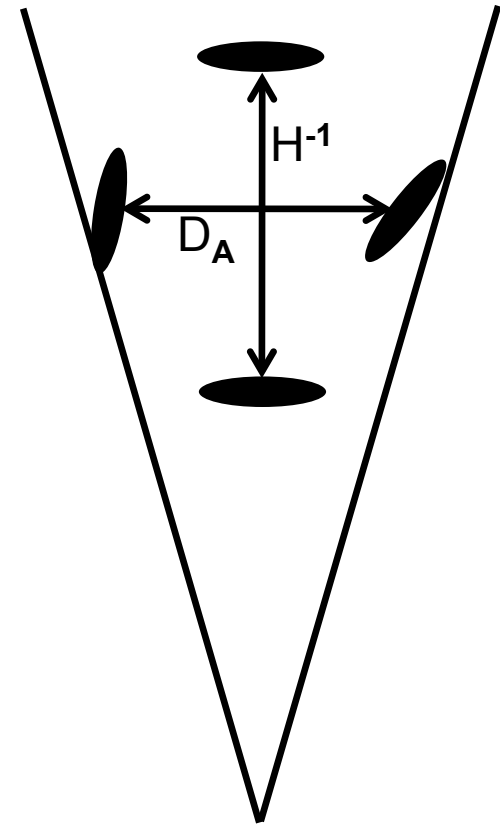
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# The Alcock-Paczynski Effect

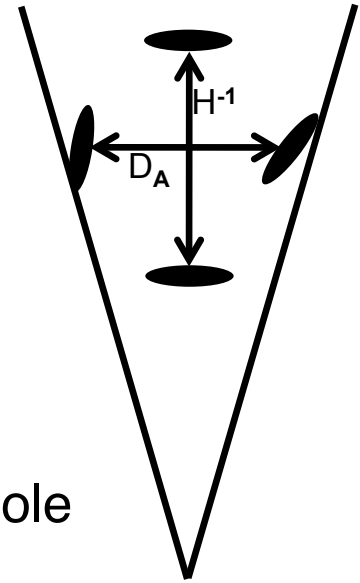
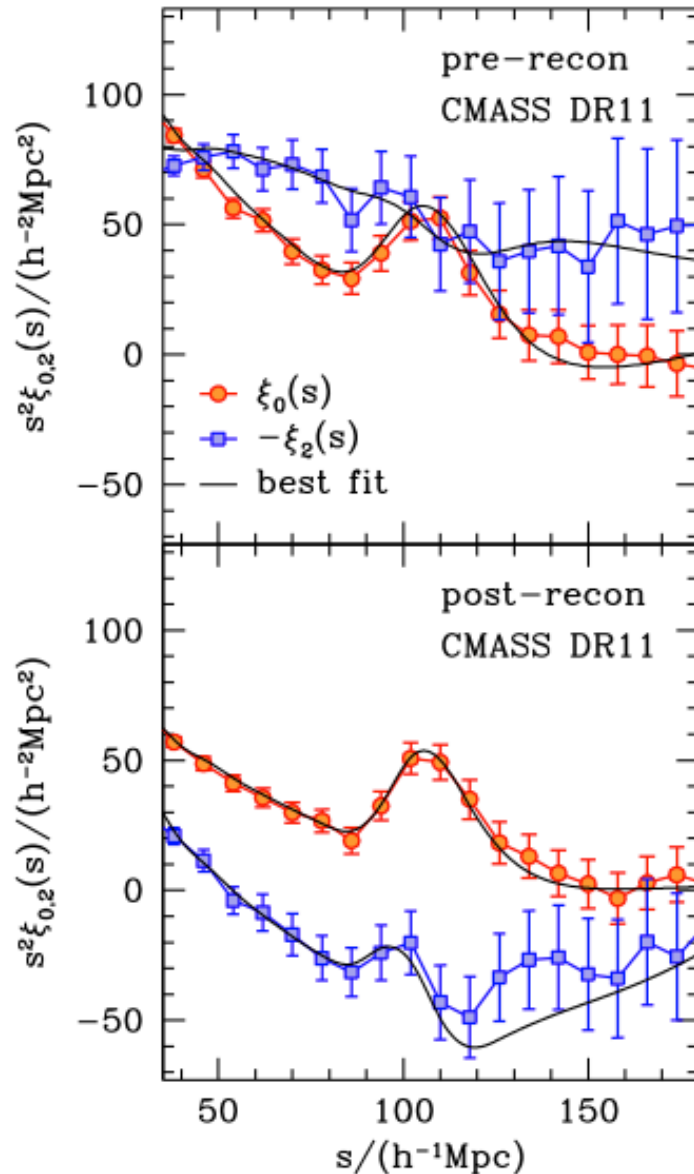
# The Alcock-Paczynski Effect

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- If the Universe is isotropic, clustering is same radial & tangential
- Stretching at a single redshift slice (for galaxies expanding with Universe) depends on
  - $H^{-1}(z)$  (radial)
  - $D_A(z)$  (angular)
- Analyze with wrong model  $\rightarrow$  see anisotropy
- AP effect measures  $F = D_A(z)H(z)$
- RSD limits test to scales where can be modeled



# Fitting BAO along and across line-of-sight



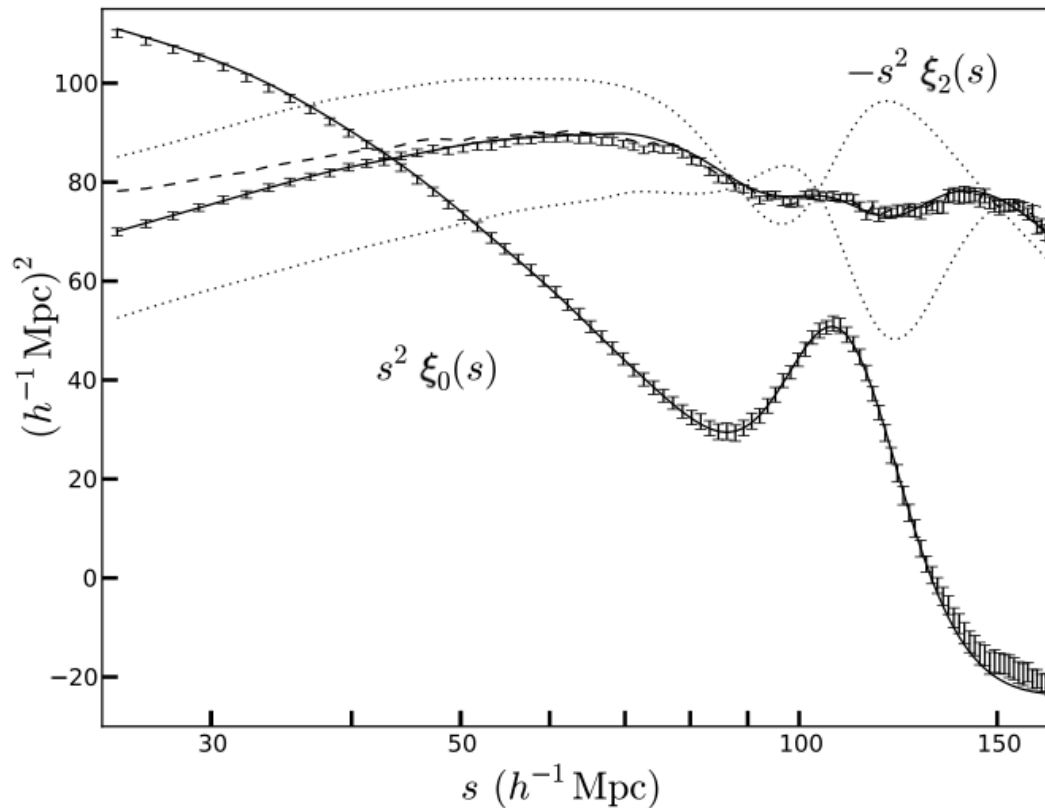
monopole

$$\xi_2^s(r) = \frac{3}{2} \int_{-1}^{+1} d\mu \xi^s(r, \mu)$$

quadrupole

$$\xi_2^s(r) = \frac{5}{2} \int_{-1}^{+1} d\mu \xi^s(r, \mu) \frac{3\mu^2 - 1}{2}$$

# AP effect on monopole & quadrupole



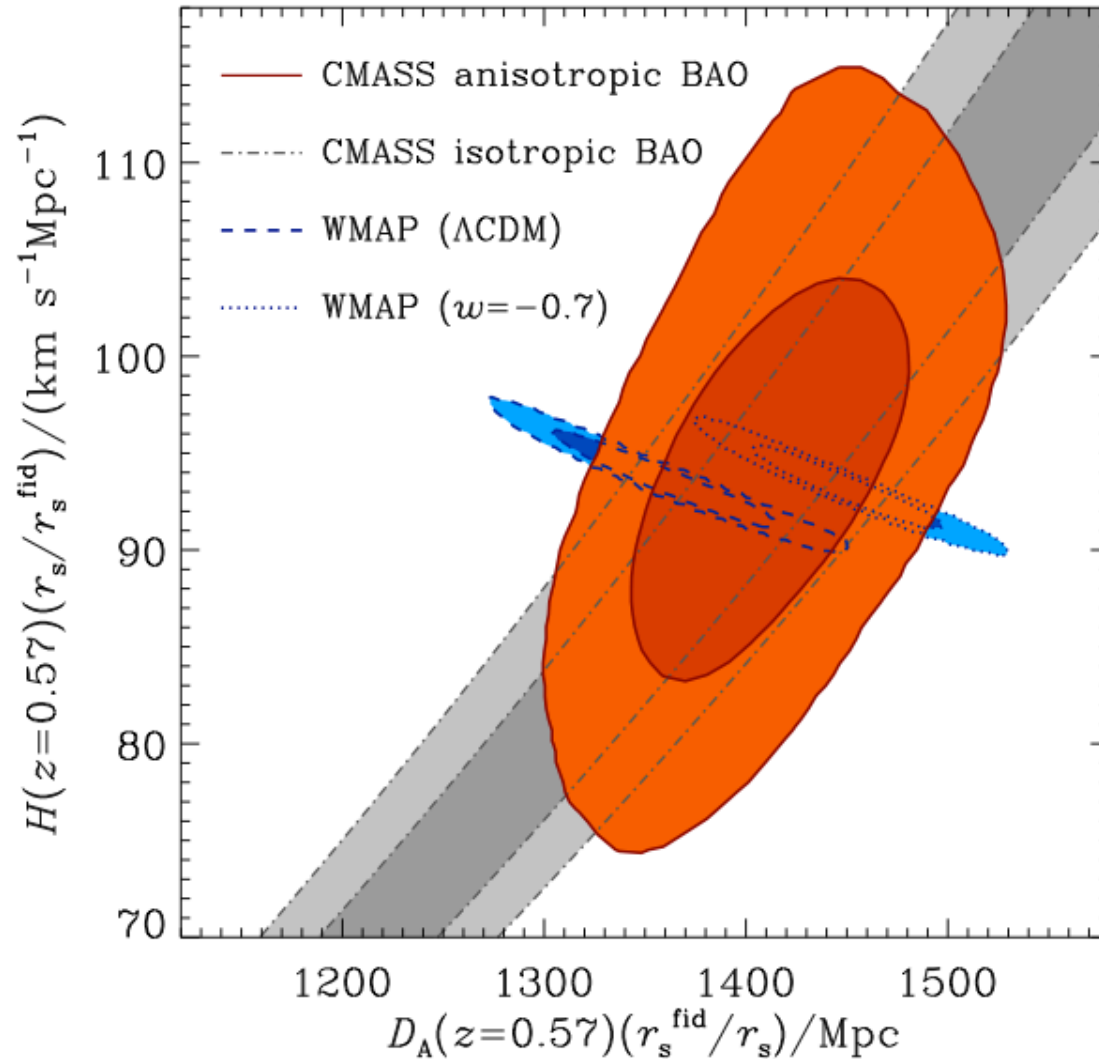
Varying  $F$  by 10%,  
while keeping  
peak position in  
monopole fixed

- AP moves  $\xi(r)$  in scale (left-right).
- Movement of BAO “bump” is clear.
- Shape of  $\xi(r)$  close to power law, so AP is very similar to amplitude shift (as RSD).
- Allows measurements of  $F$  &  $f\sigma_8$  to be separated

# Anisotropic BAO fits to BOSS data

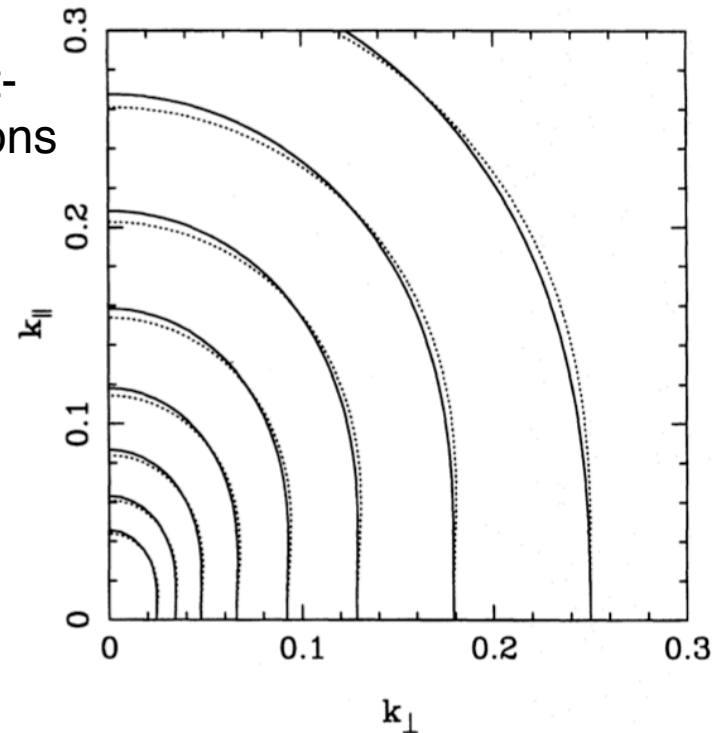
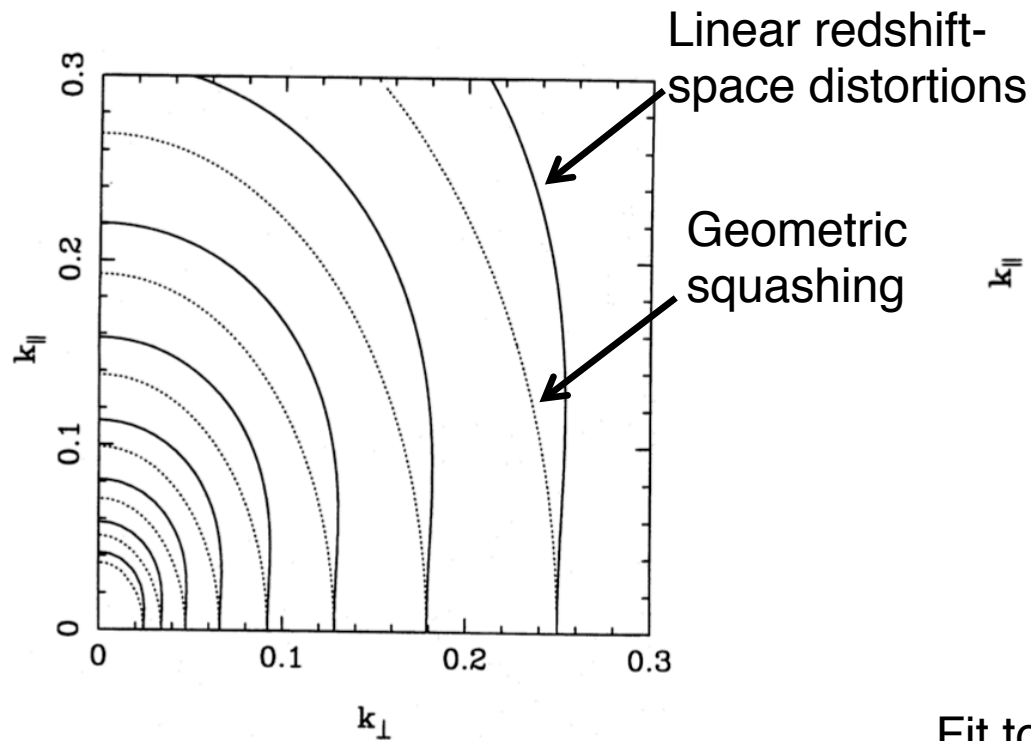
	$D_A(z)(r_s^{\text{fid}}/r_s)$	$H(z)(r_s/r_s^{\text{fid}})$	$\rho_{D_A H}$
<b>Before Reconstruction</b>			
$(\xi_0(s), \xi_2(s))$	$1367 \pm 44$	$86.6 \pm 6.2$	<b>0.65</b>
$(\xi_{\perp}(s), \xi_{\parallel}(s))$	$1379 \pm 42$	$88.3 \pm 5.1$	<b>0.52</b>
<b>After Reconstruction</b>			
$(\xi_0(s), \xi_2(s))$	$1424 \pm 43$	$95.4 \pm 7.5$	<b>0.63</b>
$(\xi_{\perp}(s), \xi_{\parallel}(s))$	$1386 \pm 36$	$90.6 \pm 6.7$	<b>0.50</b>
Consensus	$1408 \pm 45$	$92.9 \pm 7.8$	<b>0.55</b>

# Anisotropic BAO measurements vs CMB



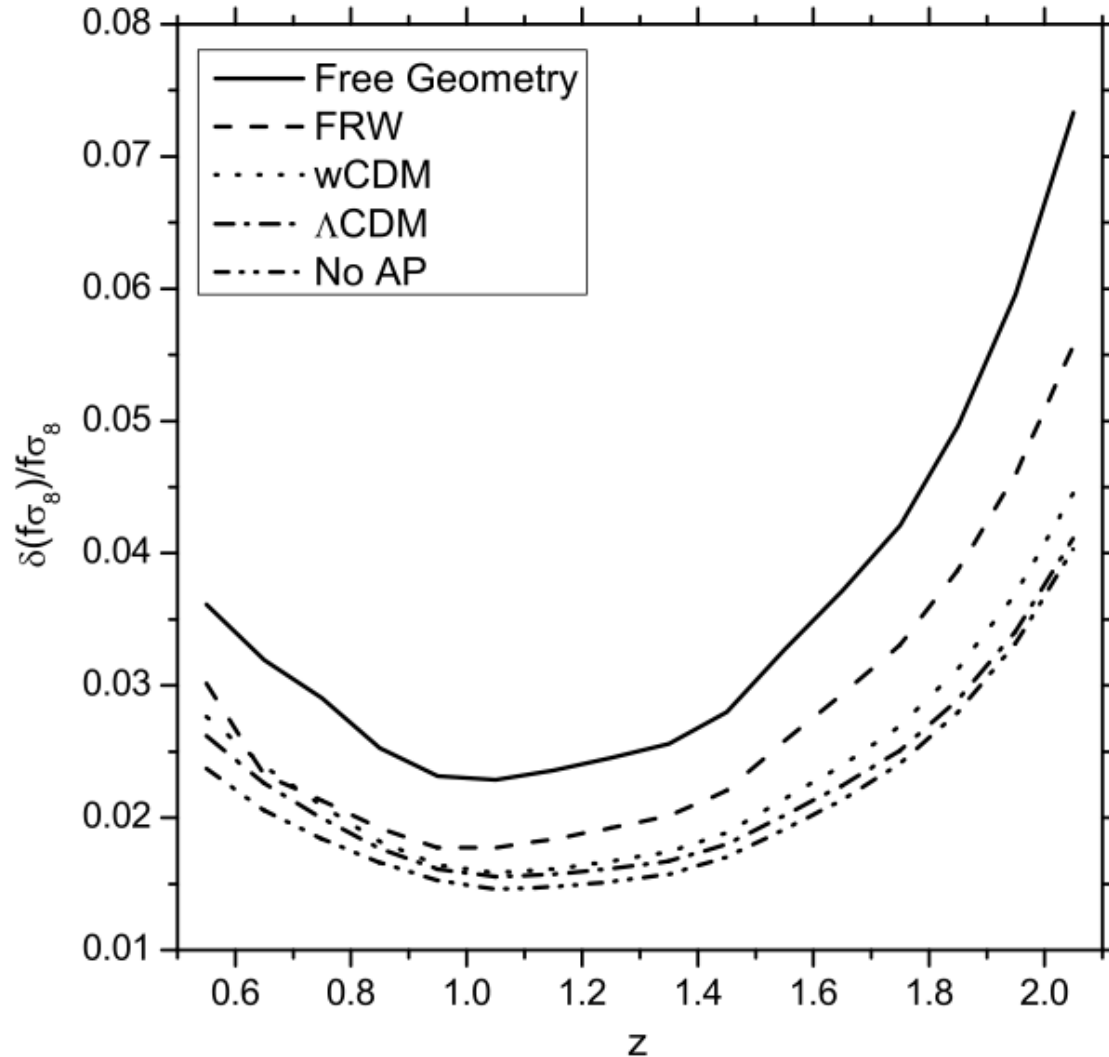
# RSD and AP amplitude shift strongly correlated

We should allow for the coupling between the redshift-space distortions and the geometrical squashing caused by getting the geometry wrong. Effects are not perfectly degenerate



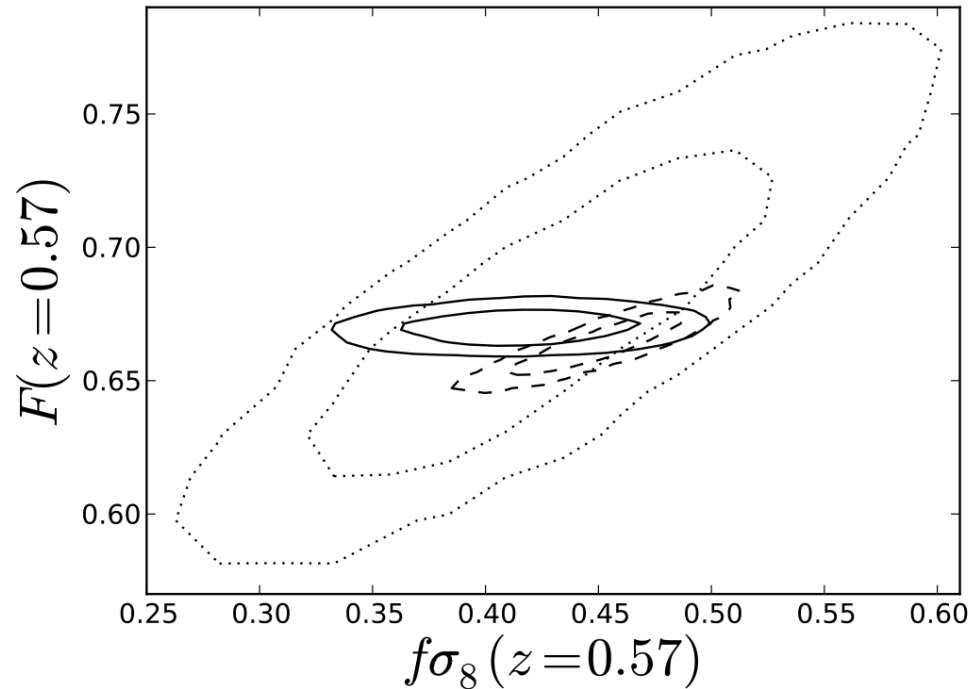
Fit to redshift-space distortions cannot mimic geometric squashing

# Degradation of RSD measurements by AP effect





# BOSS AP & RSD measurement degeneracy

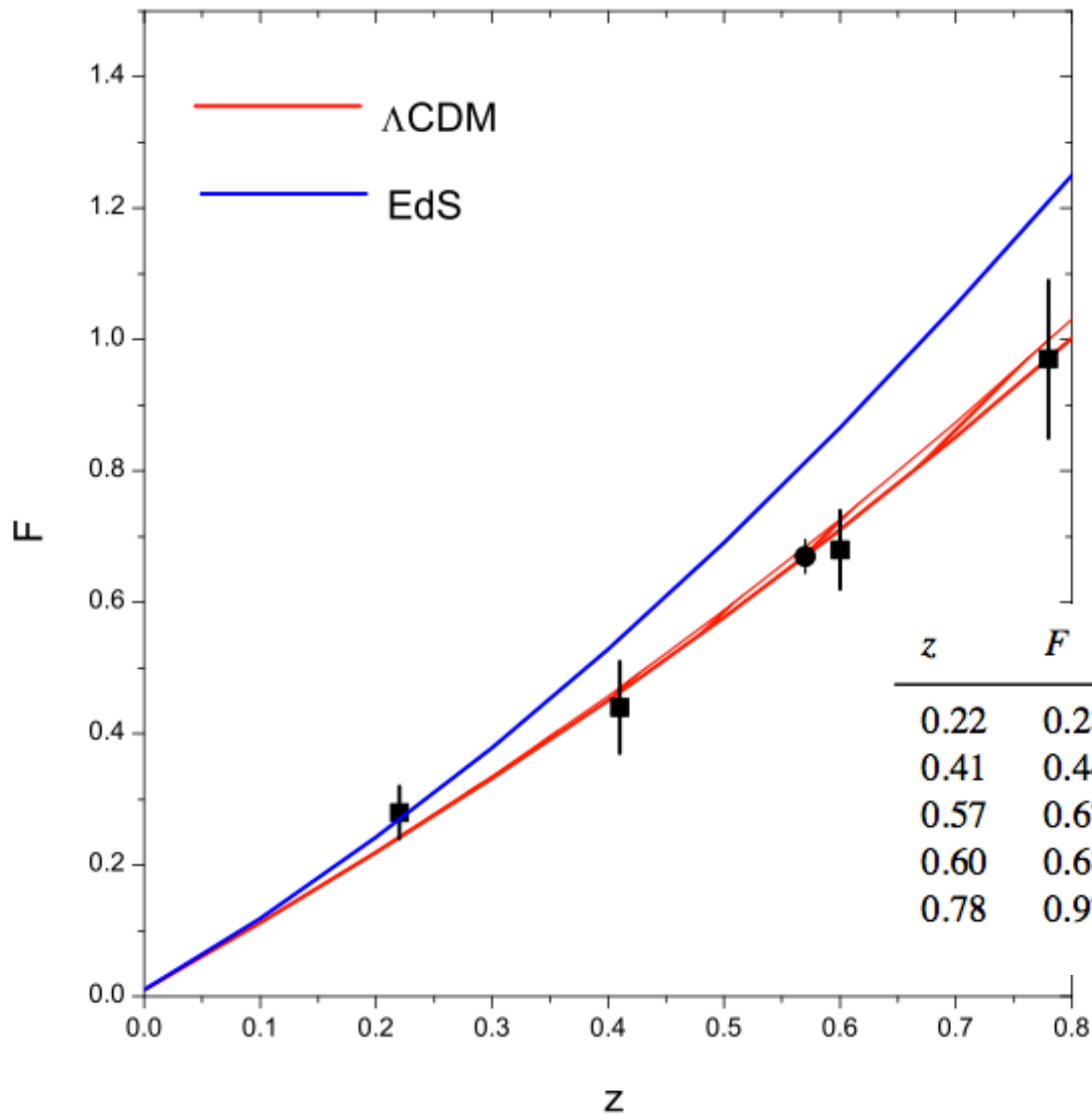


Dotted: free growth, geometry,  $\Lambda$ CDM prior on large-scale linear  $P(k)$  shape at  $z=0.57$

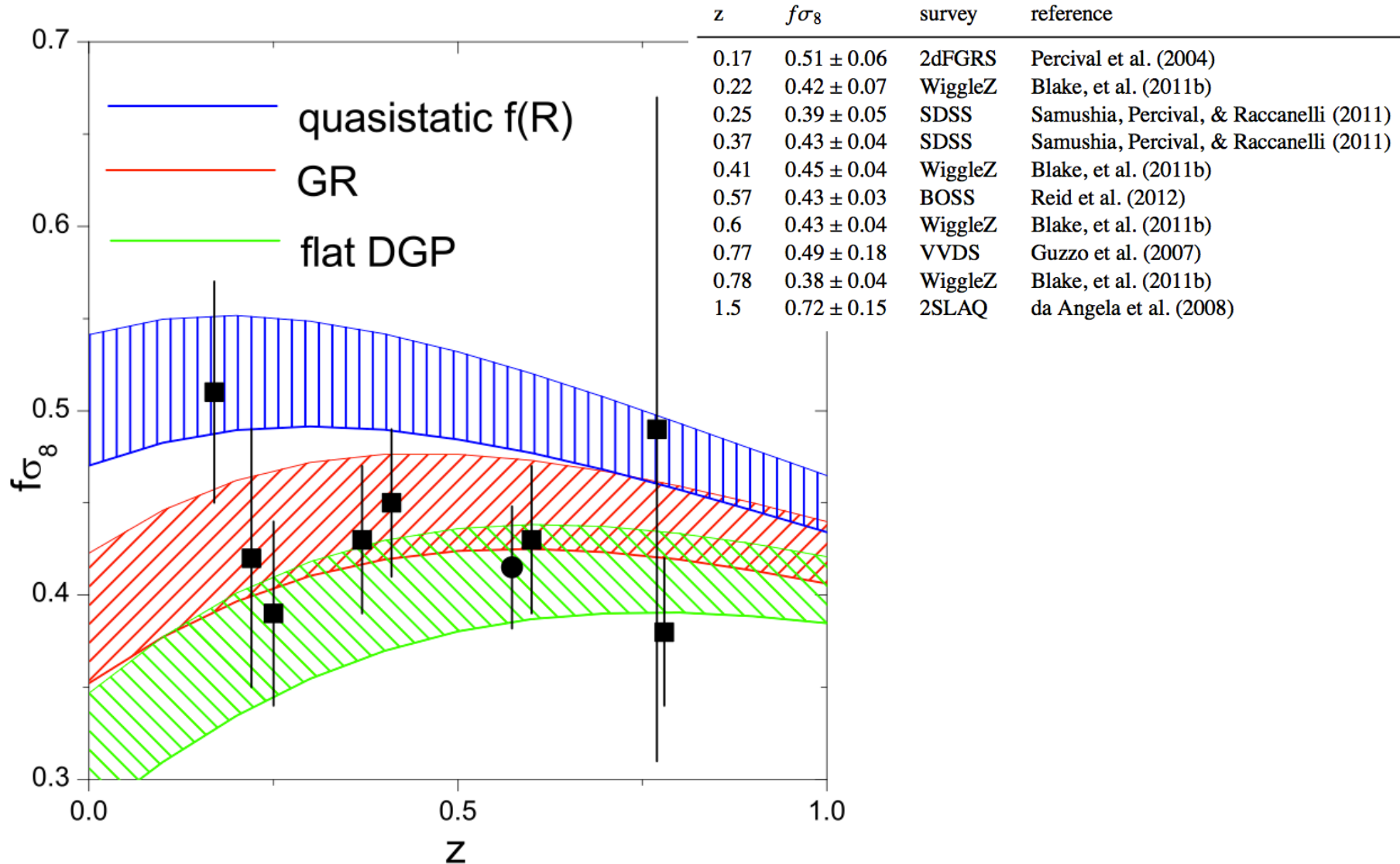
Solid:  $F$  forced to match  $\Lambda$ CDM model

Dashed: WMAP  $\Lambda$ CDM+GR prediction

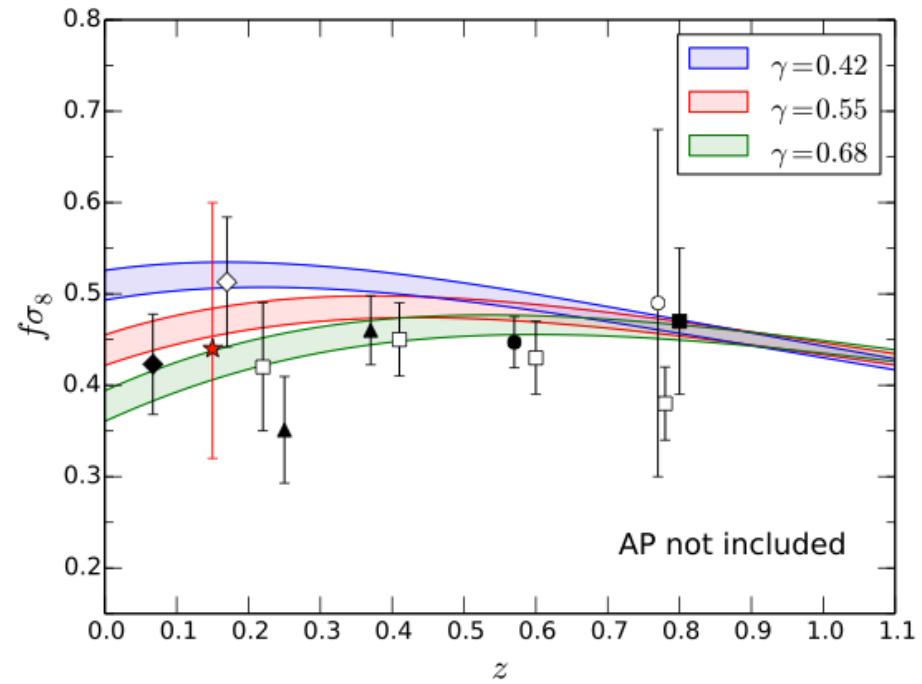
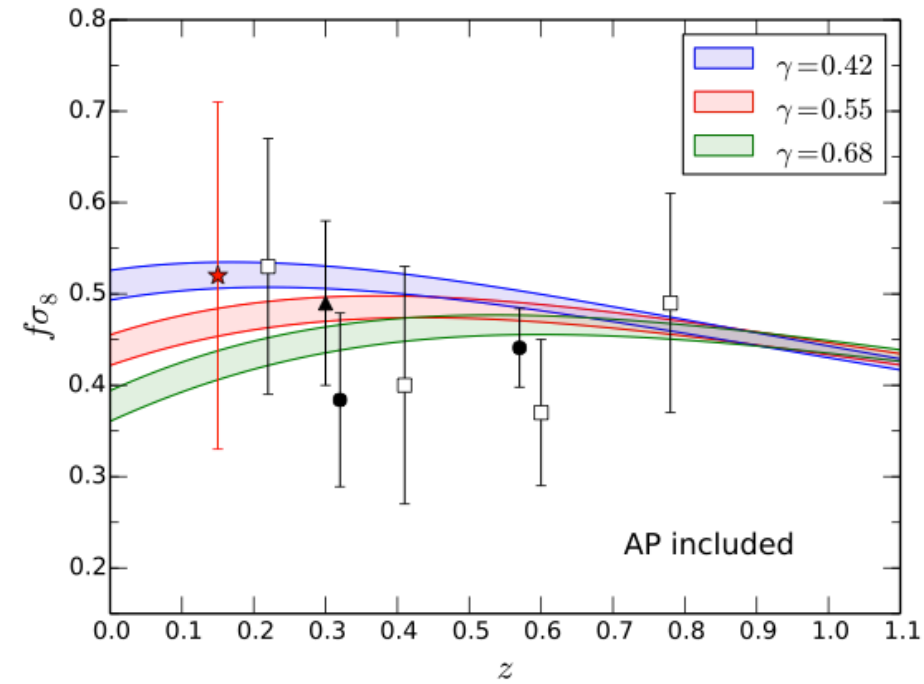
# BOSS F measurements in context



# BOSS RSD measurements in context



# The effect of AP uncertainty on RSD



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# Primordial non-Gaussianity

# Measuring primordial non-Gaussianity: $f_{NL}$ $g_{NL}$

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$\delta$  is sourced from a potential field  $\Phi$ , whose form might not be Gaussian  $\nabla^2 \Phi(\mathbf{x}) = 4\pi G \delta(\mathbf{x})$

$$\Phi(\mathbf{x}) \sim \phi(\mathbf{x}) + f_{NL} \phi^2(\mathbf{x}) + \dots$$

skewness  $\sim f_{NL}$   
kurtosis  $\sim f_{NL}^2$   
...

$\phi$  is a Gaussian field. the non-linear terms in  $\Phi$  make  $\Phi$  non-Gaussian. This map completely specifies  $\Phi$  statistics.

Salopek and Bond 1990;  
Gangui, Lucchin, Matarrese, Mollerach 1994;  
Komatsu and Spergel 2001

$$\Phi(\mathbf{x}) \sim \phi(\mathbf{x}) + g_{NL} \phi^3(\mathbf{x}) + \dots$$

skewness  $\sim 0$   
kurtosis  $\sim g_{NL}$   
...

$f_{NL}$  is not the only option for local potential fluctuations ... you can go even further down this route ...

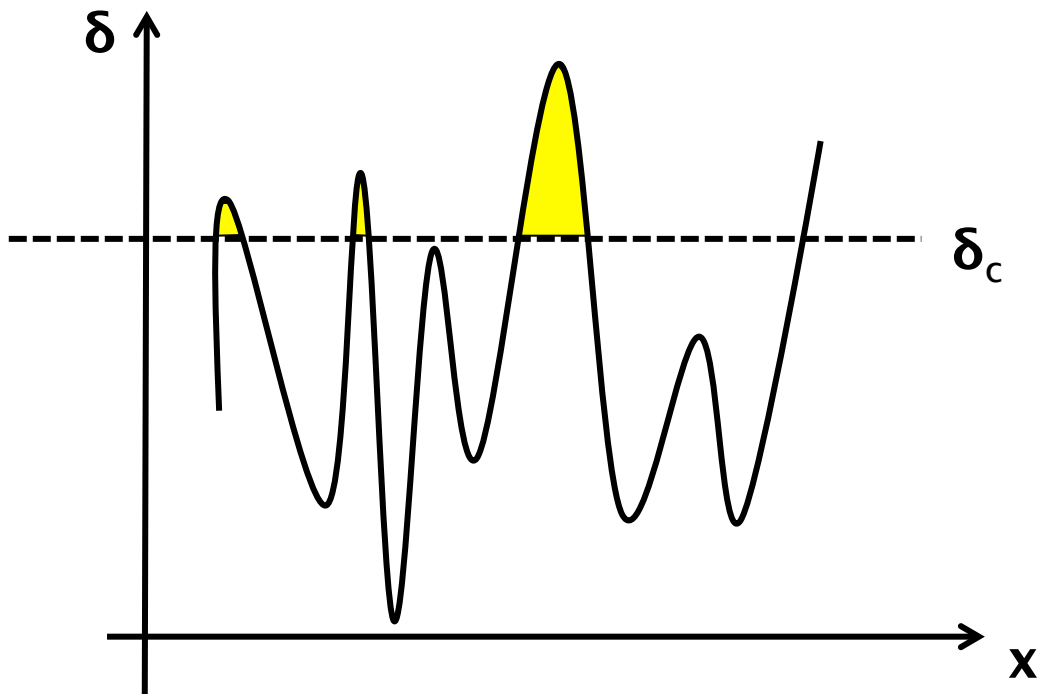
Okamoto and Hu 2002;  
Enqvist and Nurmi 2005

Non-local models introduce non-trivial higher order correlations in  $\Phi$

# Measuring non-Gaussianity: halo abundance

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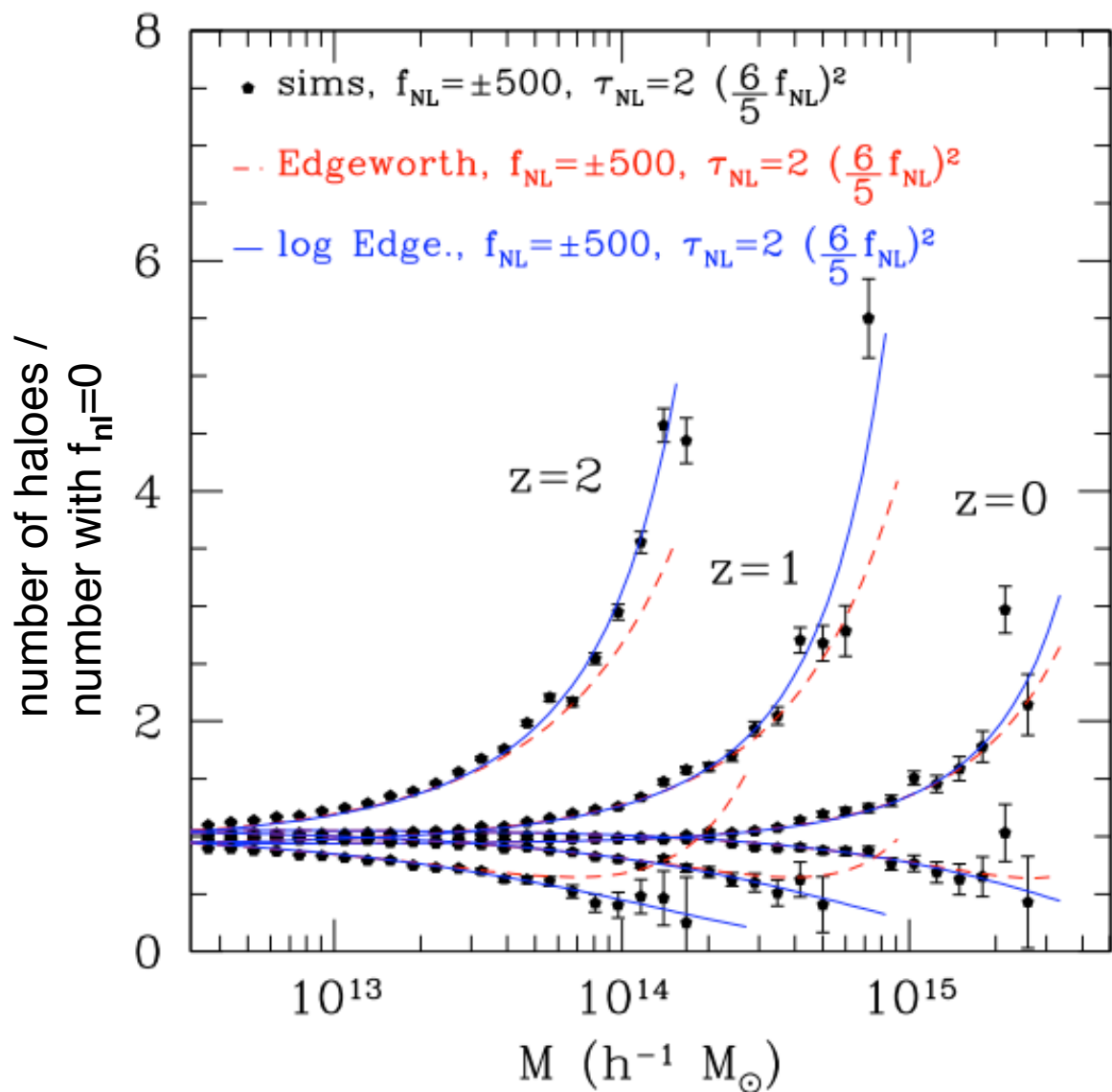
Dark matter halos form in the peaks of the density field



Non-Gaussianity changes the number density of the peaks

This in turn affects the halo mass function

# Measuring non-Gaussianity: halo abundance



Largest effect is seen  
at highest masses

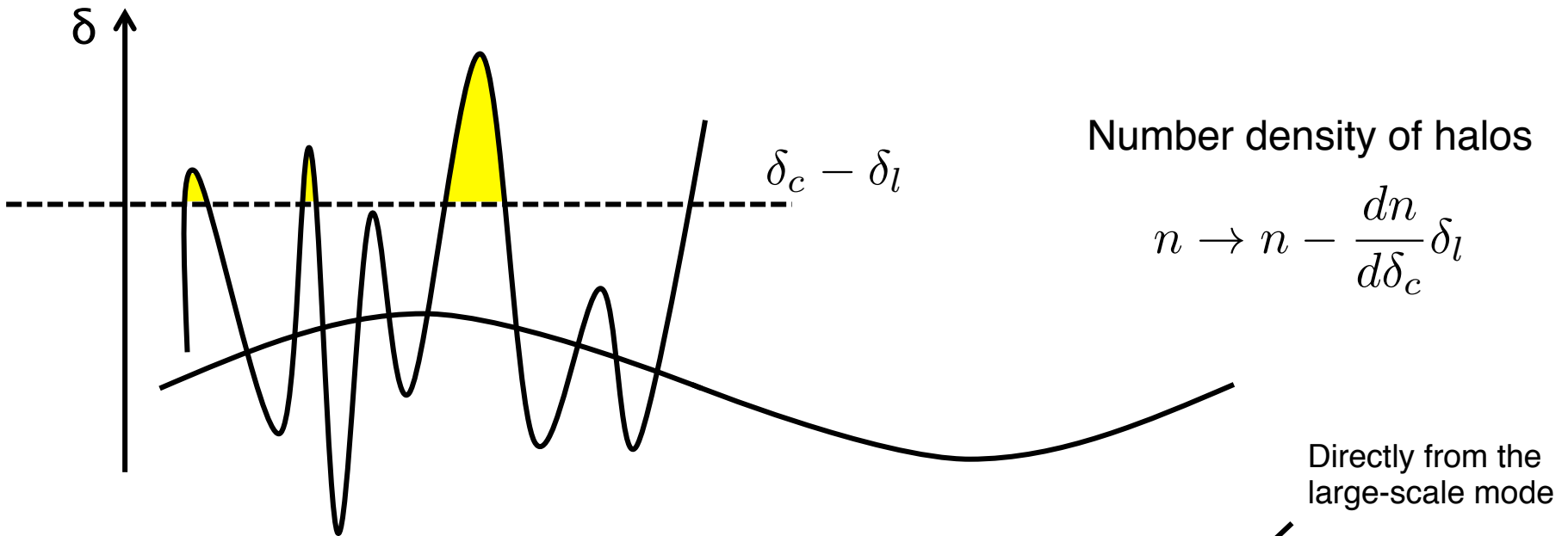
Insensitive to shape  
of bispectrum

But difficult to  
observe – relies on  
cluster masses being  
precisely known



# Peak-background split bias model

Halo formation much easier with additional long-wavelength fluctuation

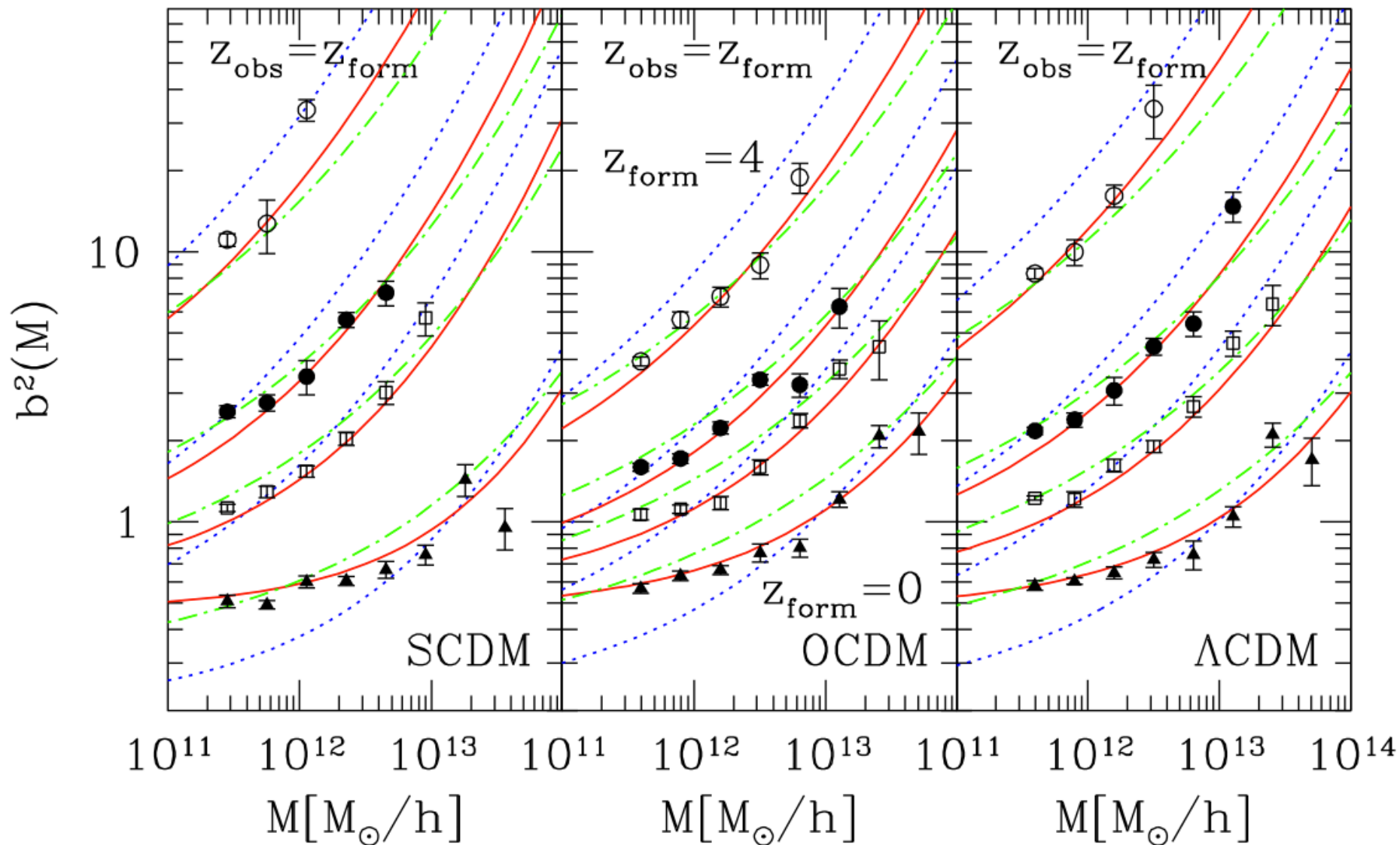


Leads to a revised density  $(1 + \delta_{\text{new}}) = \left(1 + \frac{\Delta n}{n}\right) (1 + \delta_l)$

To first order, this leads to a bias

$$\delta_{\text{new}} = \left( \delta_l + \frac{\Delta n}{n} \right) \quad b = \frac{\delta_{\text{new}}}{\delta_l} = 1 + \frac{\Delta n}{n \delta_l} = 1 - \frac{d \ln n}{d \delta_c}$$

# Peak-background split galaxy bias model



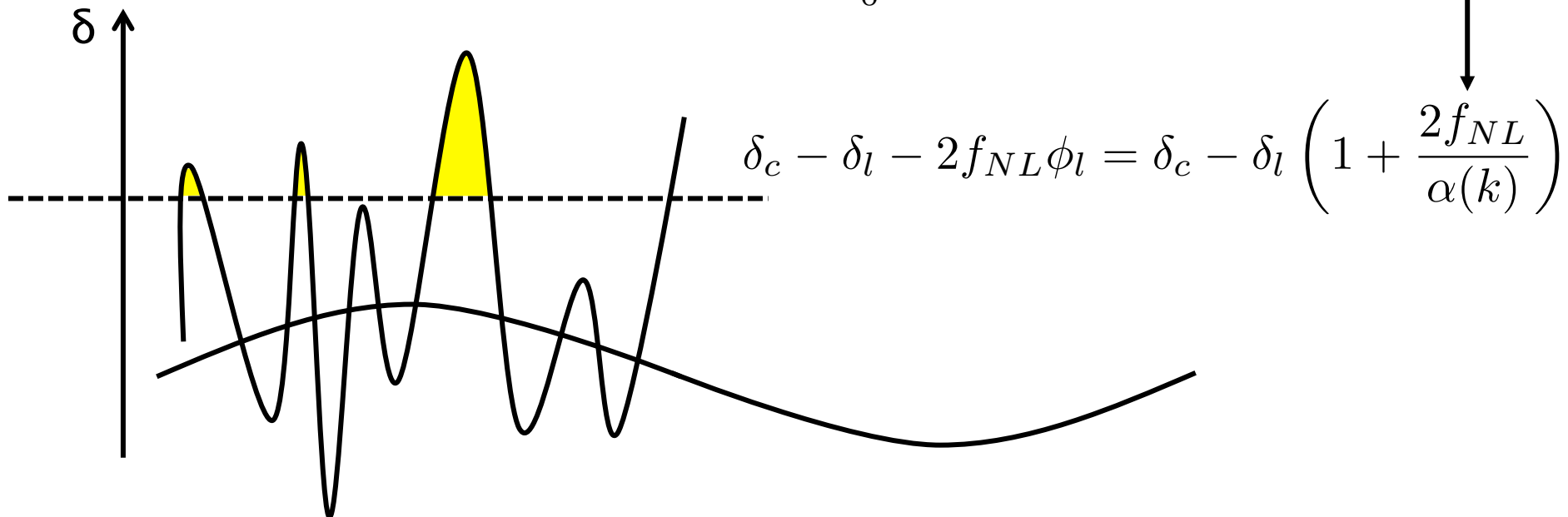
# This is altered by $f_{NL}$ signal

Now split non-Gaussian potential into long and short wavelength components

$$\Phi(\mathbf{x}) = \phi_l + \underbrace{f_{NL}\phi_l^2}_{\text{small}} + \underbrace{(1 + 2f_{NL}\phi_l)}_{\text{small}}\phi_s + f_{NL}\phi_s^2 + \text{cnst}$$

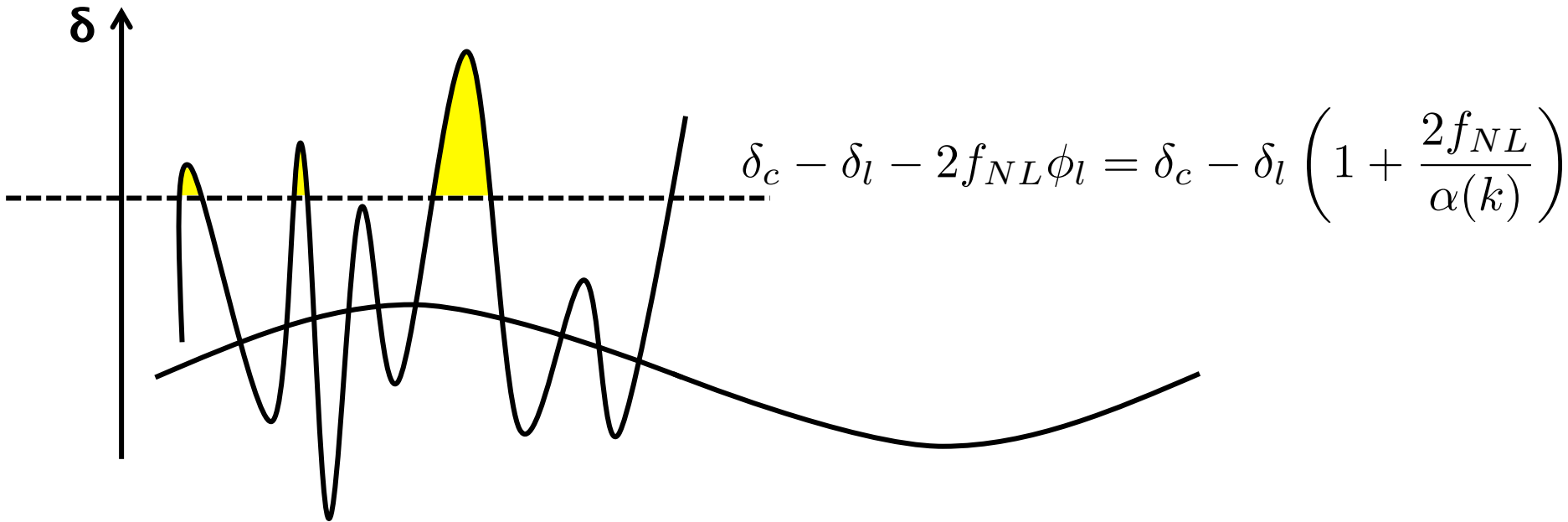
Link between potential and overdensity field shows how changing long wavelength potential component changes “critical density”

$$\delta_l(k) = \alpha(k)\Phi(k) \quad \alpha(k) = \frac{2c^2 k^2 T(k) D(z)}{3\Omega_m H_0^2}$$



# Peak-background split for non-Gaussianity

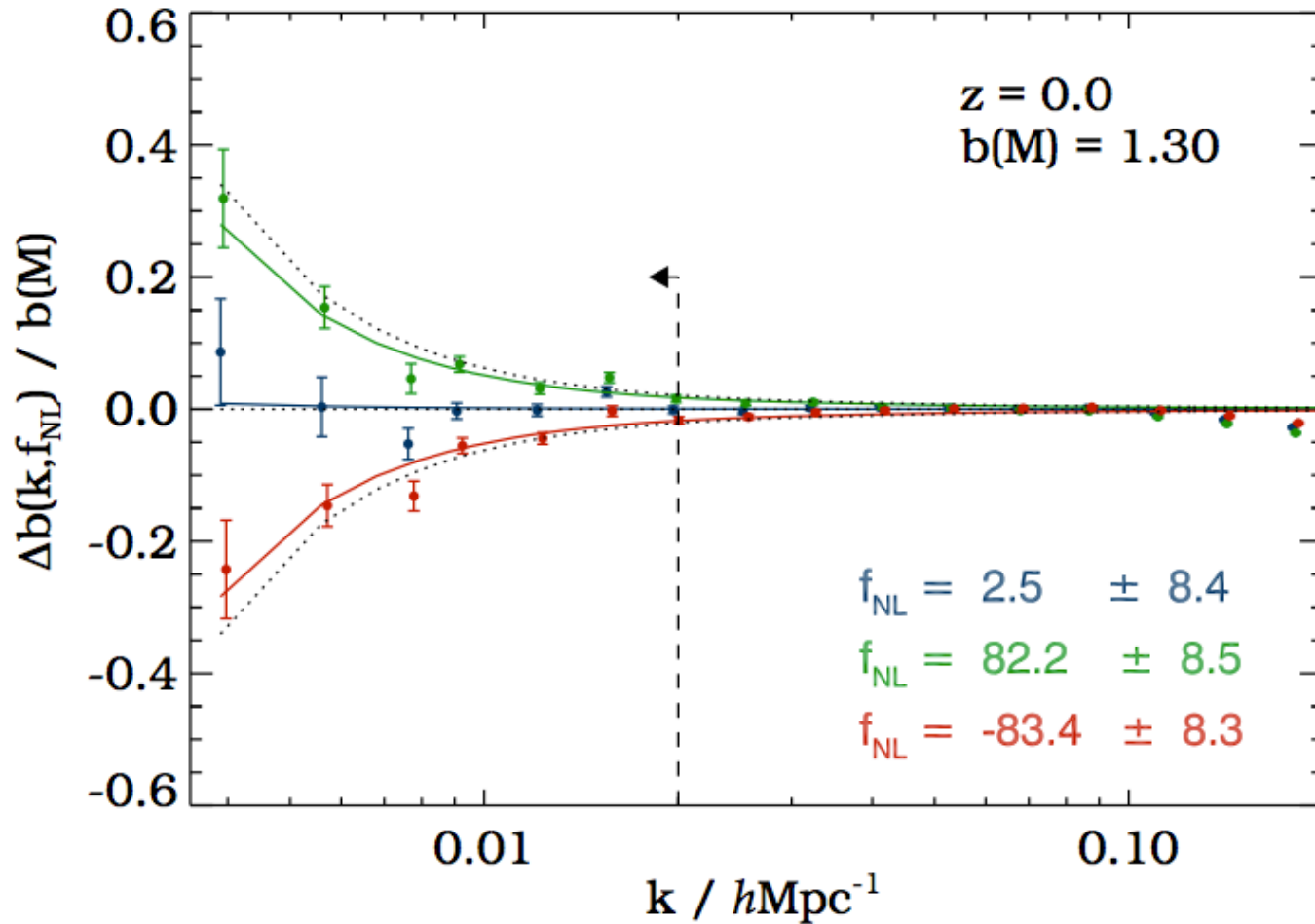
Halo formation much easier with additional long-wavelength fluctuation



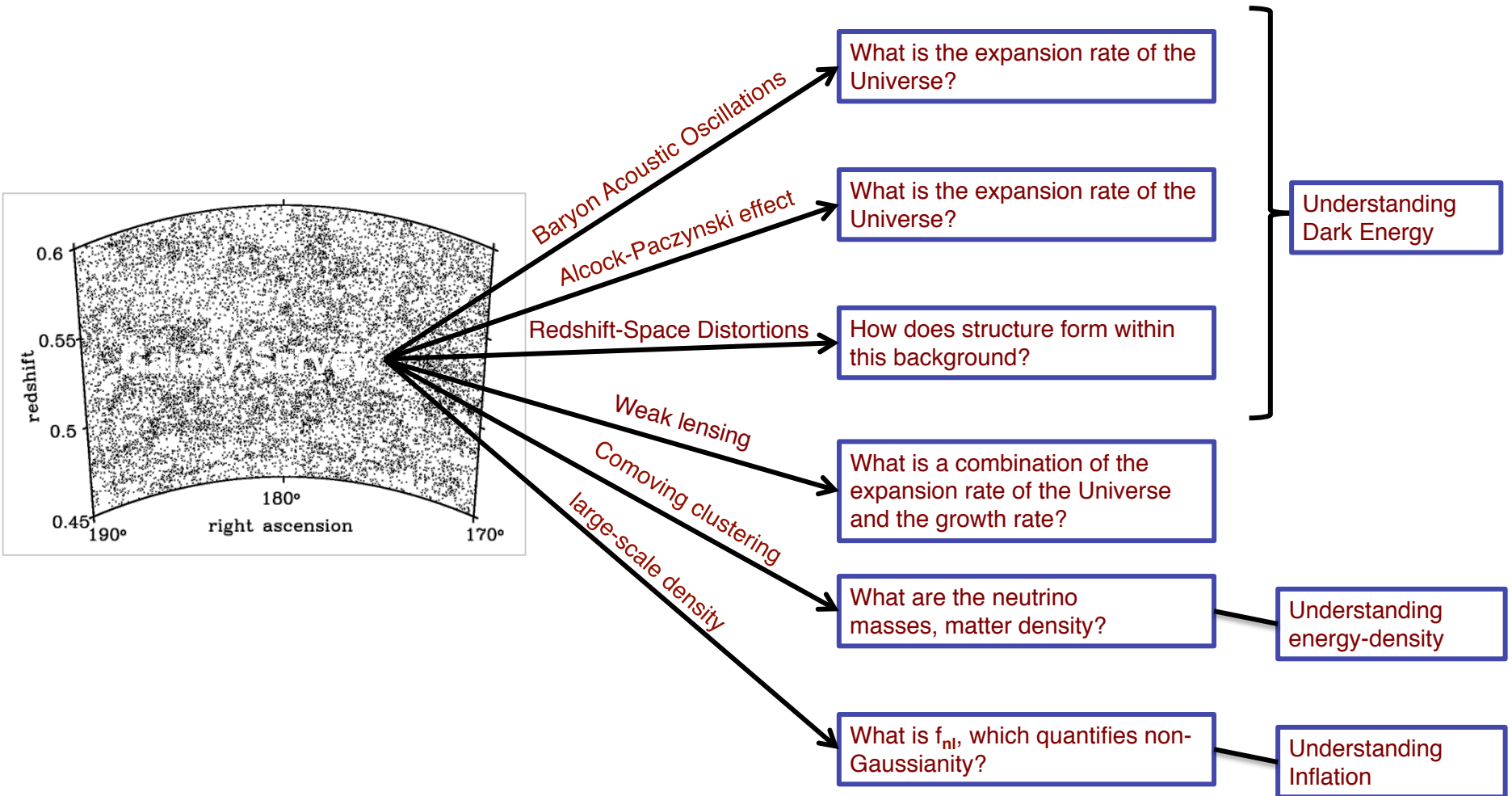
Number density of halos  $n \rightarrow n - \left( 1 + \frac{2f_{NL}}{\alpha(k)} \right) \frac{dn}{d\delta_c} \delta_l$

Leads to a revised bias  $b = 1 - \left( 1 + \frac{2f_{NL}}{\alpha(k)} \right) \frac{d \ln n}{d\delta_c}$

# $K^2$ dependence in simulations



# Cosmology from surveys



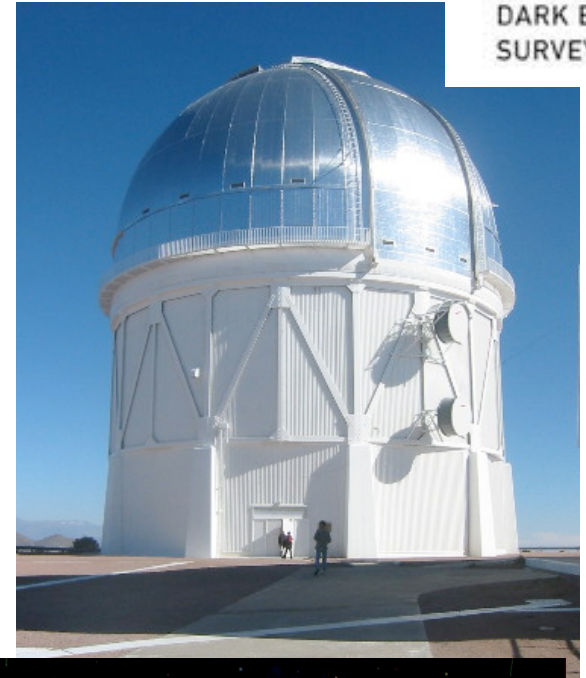
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Future surveys: next 4-6 years

# Dark Energy Survey (DES)



DARK ENERGY  
SURVEY



- New wide-field camera on the 4m Blanco telescope
- Survey underway, with first year of data in hand
- $\Omega = 5,000\text{deg}^2$
- multi-colour optical imaging (g,r,i,z) with link to IR data from VISTA hemisphere survey
- 300,000,000 galaxies
- Aim is to constrain dark energy using 4 probes  
LSS/BAO, weak lensing, supernovae  
cluster number density
- Redshifts based on photometry  
weak radial measurements  
weak redshift-space distortions
- See also: Pan-STARRS, VST-VISTA,  
SkyMapper





# eBOSS / SDSS-IV

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- The new cosmology project with SDSS
- Use the Sloan telescope and MOS to observe to higher redshift
- Basic parameters
  - $\Omega = 1,500\text{deg}^2 - 7,500\text{deg}^2$
  - $\sim 1,000,000$  galaxies (direct BAO)
  - $\sim 60,000$  quasars (BAO from Ly- $\alpha$  forest)
- Distance measurements
  - 0.9% at  $z=0.8$  (LRGs)
  - 1.8% at  $z=0.9$  (ELGs)
  - 2.0% at  $z=1.5$  (QSOs)
  - 1.1% at  $z=2.5$  (Ly- $\alpha$  forest, inc. BOSS)
- Survey has just started, lasting 4--6 years
- Received \$10M from Sloan foundation and significant funding from partners



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Future surveys: > 4 years

# MOS on 4m-telescope

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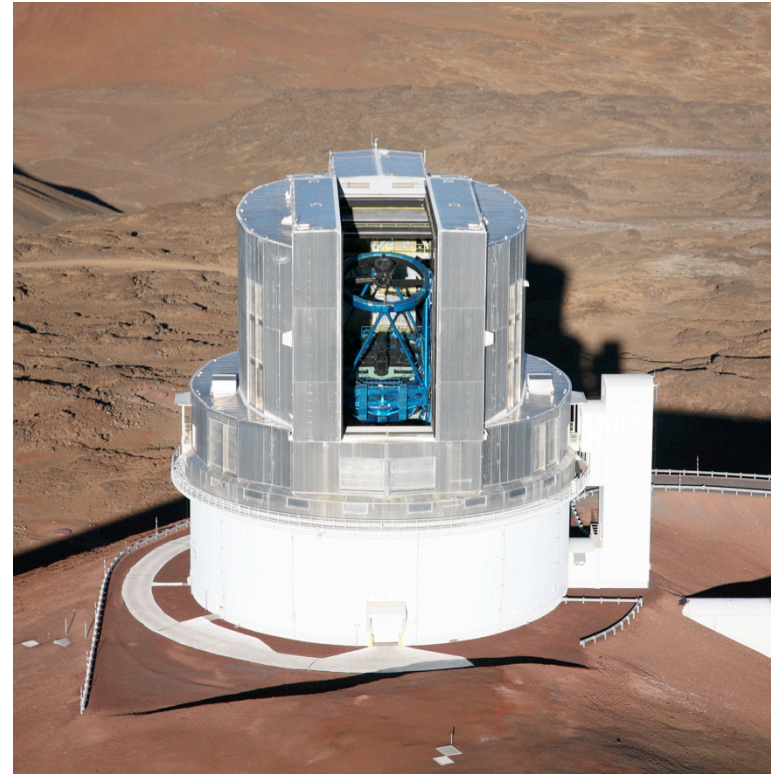
- New fibre-fed spectroscopes proposed for 4m telescopes
  - Mayall (BigBOSS) } DESI
  - Blanco (DESPEC) }
  - WHT (WEAVE)
  - VISTA (4MOST)
- Various stages of planning & funding
  - DESI has just passed DOE CD-1, 2019 start
  - 4MOST chosen by ESO, 2020 start?
  - WEAVE, 2018 start
- All capable of observing
  - $\Omega = 5\text{--}14,000\text{deg}^2$
  - 2--40,000,000 galaxies (direct BAO)
  - 1--600,000 quasars (BAO from Ly- $\alpha$  forest)
  - Cosmic variance limited to  $z \sim 1.4$



# MOS on 10m-telescope

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- New fibre-fed spectroscopes proposed for 10m telescopes
  - Hobby-Eberly (HETDEX)
  - Subaru (PFS)
- Different baseline strategies
- HETDEX
  - 420deg<sup>2</sup> Ly-alpha emitters
  - 800,000 galaxies  $1.9 < z < 3.5$
  - Greig, Komatsu & Wyithe, 2012, arXiv:12120977
- PFS
  - 1400deg<sup>2</sup> ELGs
  - 3,000,000 galaxies  $0.6 < z < 2.4$
  - Ellis et al., 2012, arXiv:1206.0737



# What is Euclid?

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ESA Medium-Class mission  
M2 slot in the Cosmic Visions Programme  
Due for launch 2020

## Scientific Objectives

To understand the origins of the Universe's accelerated expansion  
Using at least 2 independent complementary probes

## Geometry of the Universe

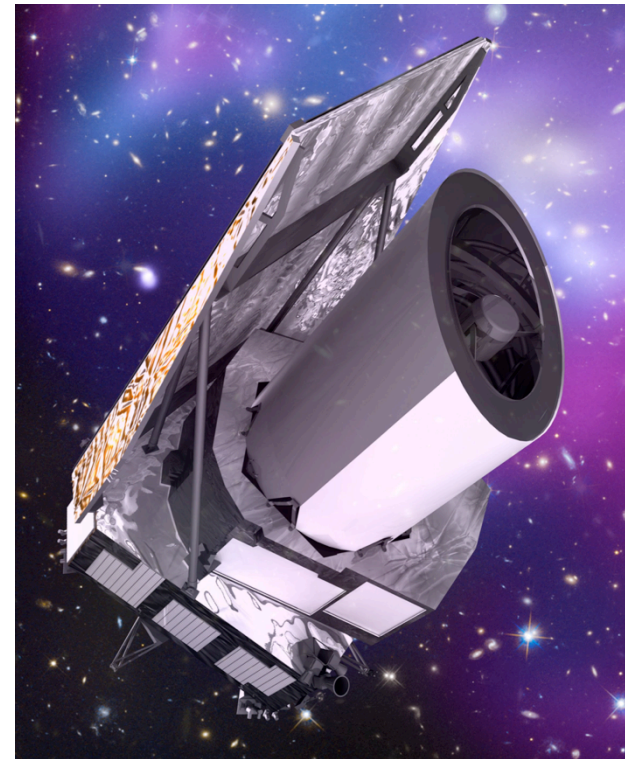
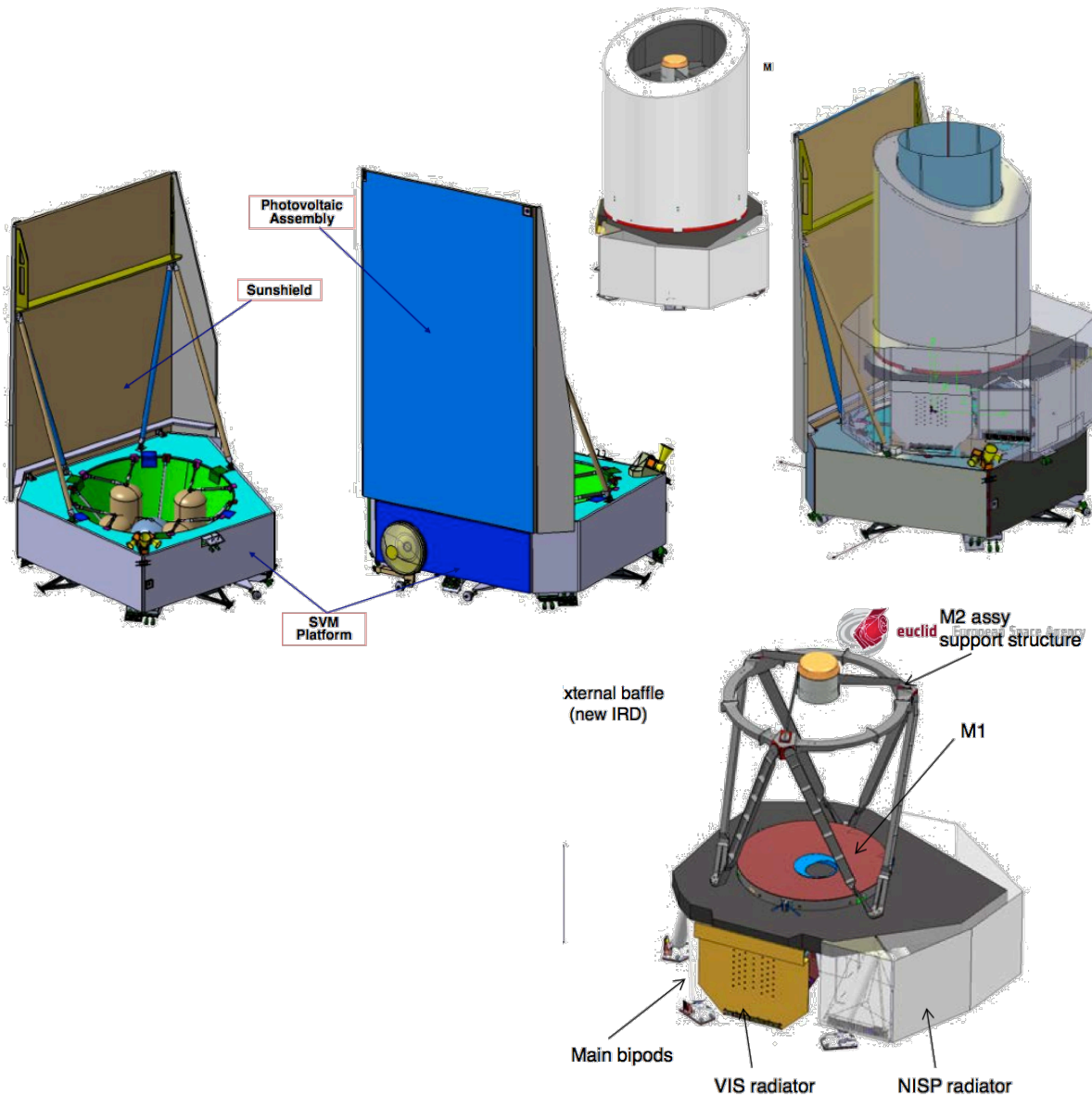
Weak Lensing (WL), Galaxy Clustering (BAO)

## Cosmic history of structure formation

WL, Redshift-Space Distortions (RSD), Clusters of Galaxies (CL)

Using space-based observations to control residual systematic errors to an unprecedented low level

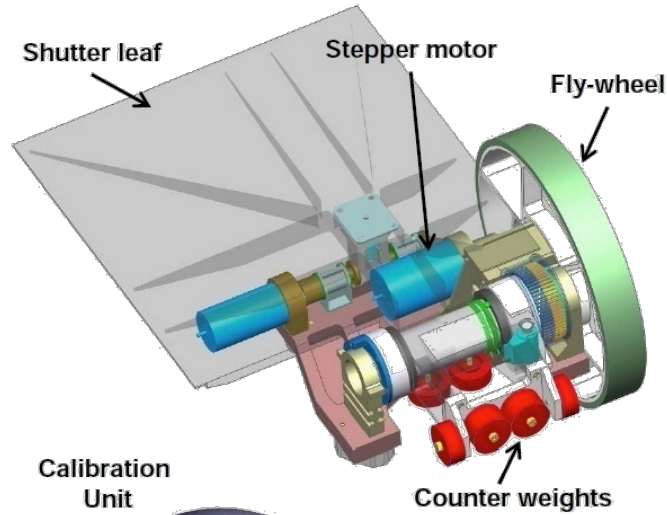
# The spacecraft & telescope



# The VIS instrument

Courtesy: S. Pottinger, R. Cole, M. Cropper and the VIS team

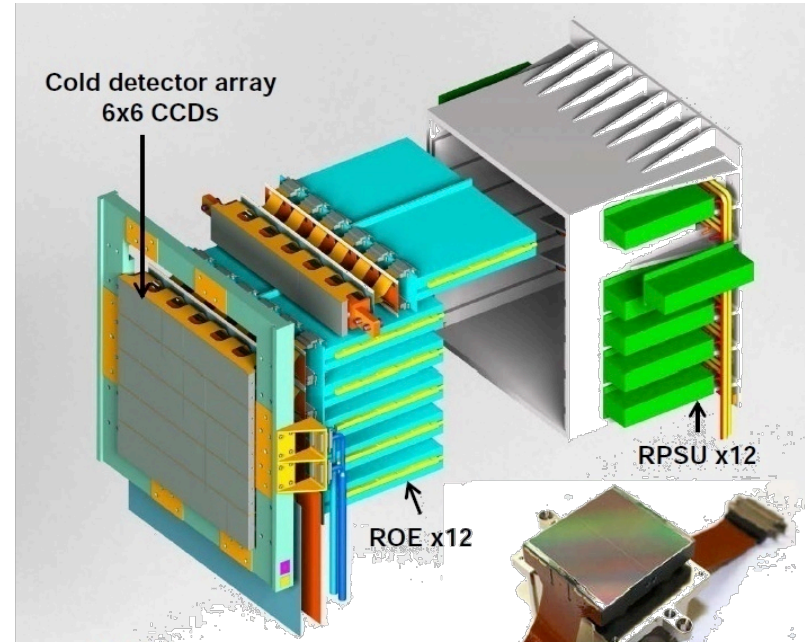
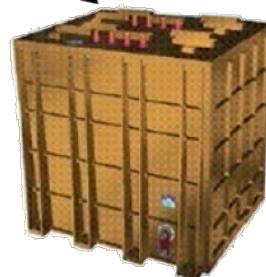
- 36 4kx4k CCDs with  $12\mu\text{m}$  pixels
- 0.1 arcsec/pixel on sky
- 550-900nm (wide band channel)
- Lim. mag: AB 24.5 ;  $10\sigma$  pt source
- Data volume: 520 Gbit/day



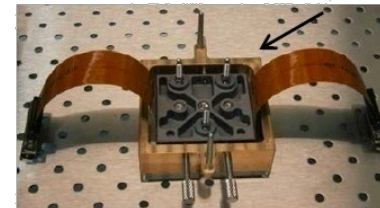
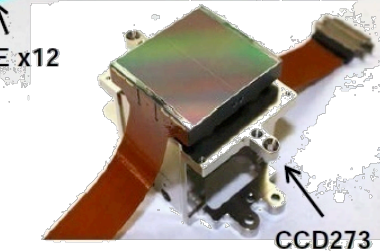
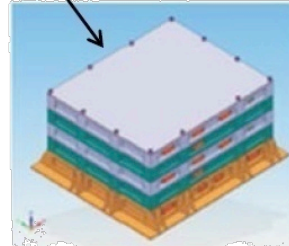
Calibration Unit



PMCU



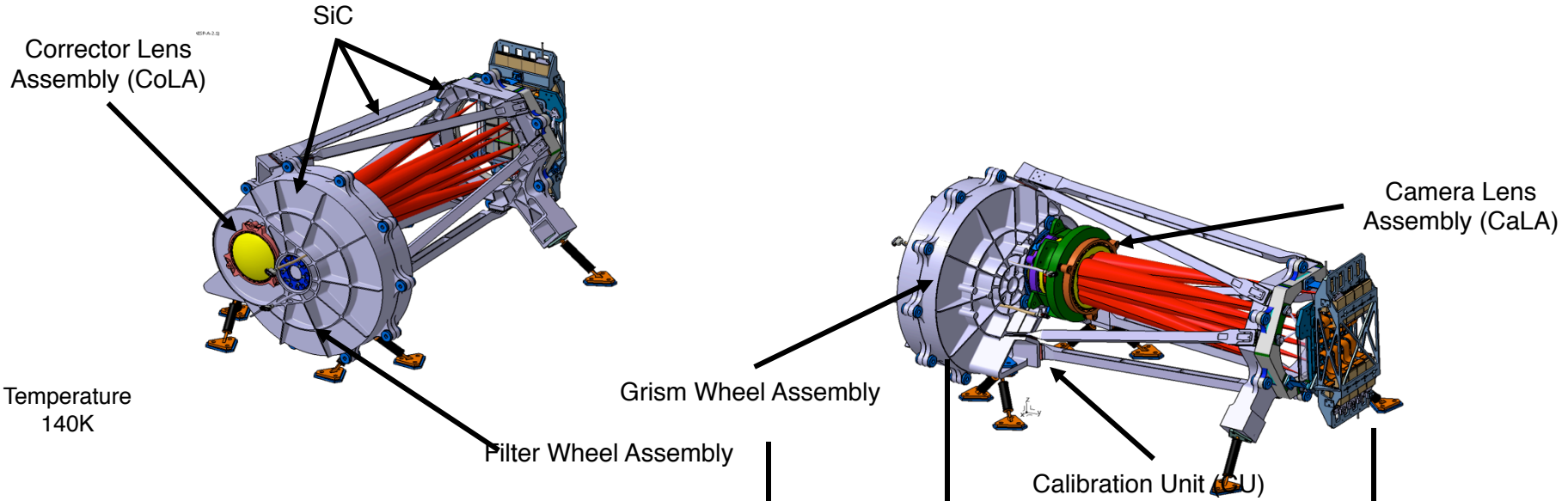
CDPU



# The NISP Instrument

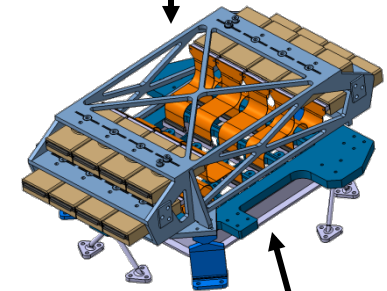
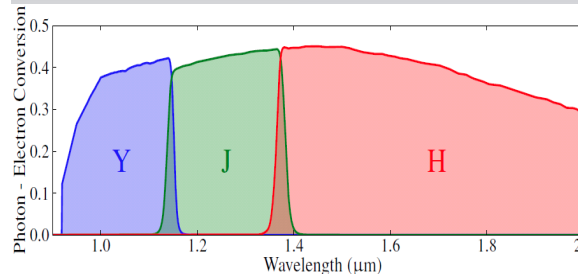
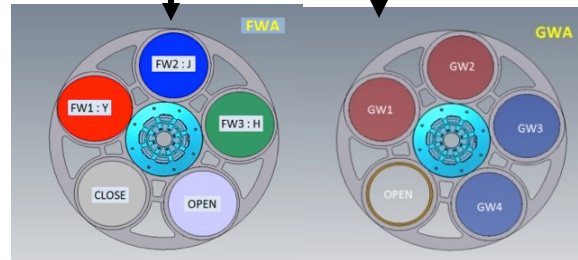
Courtesy: T. Maciaszek and the NISP team

## Structure Assembly & Thermal Control



Temperature  
140K

- 16 2kx2k H2GR NIR detectors
- 0.3 arcsec/pixel
- 3 NIR filters: H,J,K
- 4 Grisms (1 <B>; 3 <R> )
- Lim. mag: AB 24.0 ;  $5\sigma$  pt source
- Data volume: 180 Gbit/day



16 H2RG DETECTORS  
(provided by ESA/NASA)



# Euclid targets

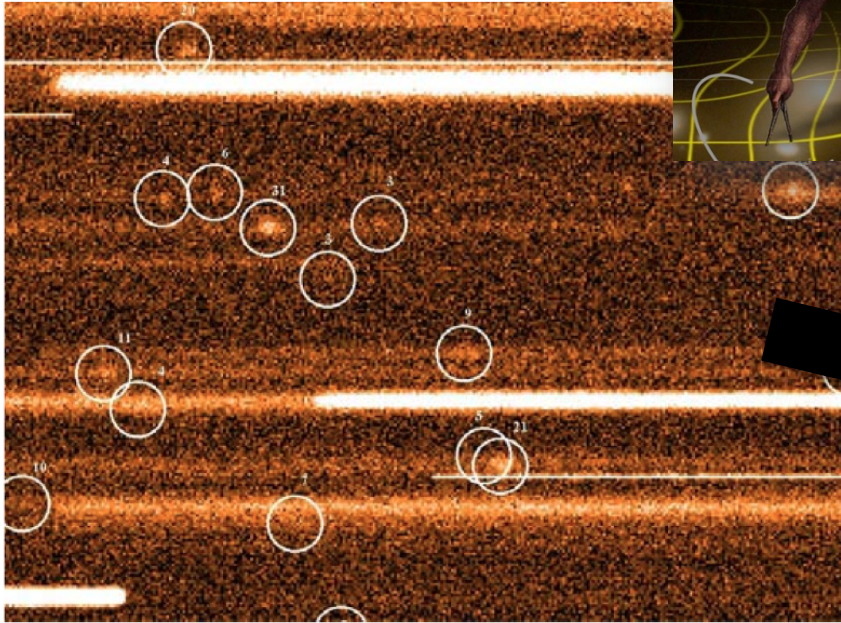
**Photo-z:** Ground based Photometry and Spectroscopy

SURVEYS In ~6 years					
	Area (deg <sup>2</sup> )	Description			
Wide Survey	<b>15,000 deg<sup>2</sup></b>	Step and stare with 4 dither pointings per step.			
Deep Survey	<b>40 deg<sup>2</sup></b>	In at least 2 patches of > 10 deg <sup>2</sup> 2 magnitudes deeper than wide survey			
PAYLOAD					
Telescope	1.2 m Korsch, 3 mirror anastigmat, f=24.5 m				
Instrument	VIS	NISP			
Field-of-View	0.787×0.709 deg <sup>2</sup>	0.763×0.722 deg <sup>2</sup>			
Capability	Visual Imaging	NIR Imaging Photometry			NIR Spectroscopy
Wavelength range	550– 900 nm	Y (920-1146nm),	J (1146-1372 nm)	H (1372-2000nm)	1100-2000 nm
Sensitivity	24.5 mag 10σ extended source	24 mag 5σ point source	24 mag 5σ point source	24 mag 5σ point source	3 10 <sup>-16</sup> erg cm <sup>-2</sup> s <sup>-1</sup> 3.5σ unresolved line flux
	<b>Shapes + Photo-z of <math>n = 1.5 \times 10^9</math> galaxies</b>			<b>z of <math>n = 2.6 \times 10^7</math> galaxies</b>	

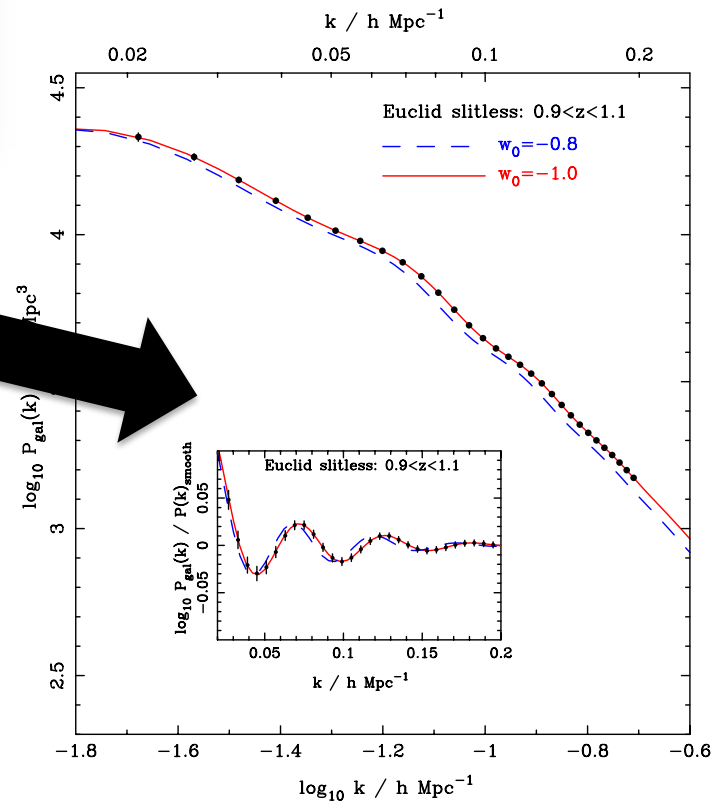
**Possibility of other surveys:** SN and/or m-lens surveys, Milky Way?

# Euclid redshifts

Redshifts will be from slitless spectroscopy, mainly picking up H- $\alpha$  line



Long way from raw spectra to cosmology



# Recent revision in Euclid strategy

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Main motivations:

Evolving landscape: new surveys covering  $z < 1$  (e.g. eBOSS, DESI)

Revised estimates for density of H-alpha emitters

Instrument performance updates

Last chance to modify hardware/strategy to improve science and robustness

Systematic effects with slitless technique intrinsically hard to control

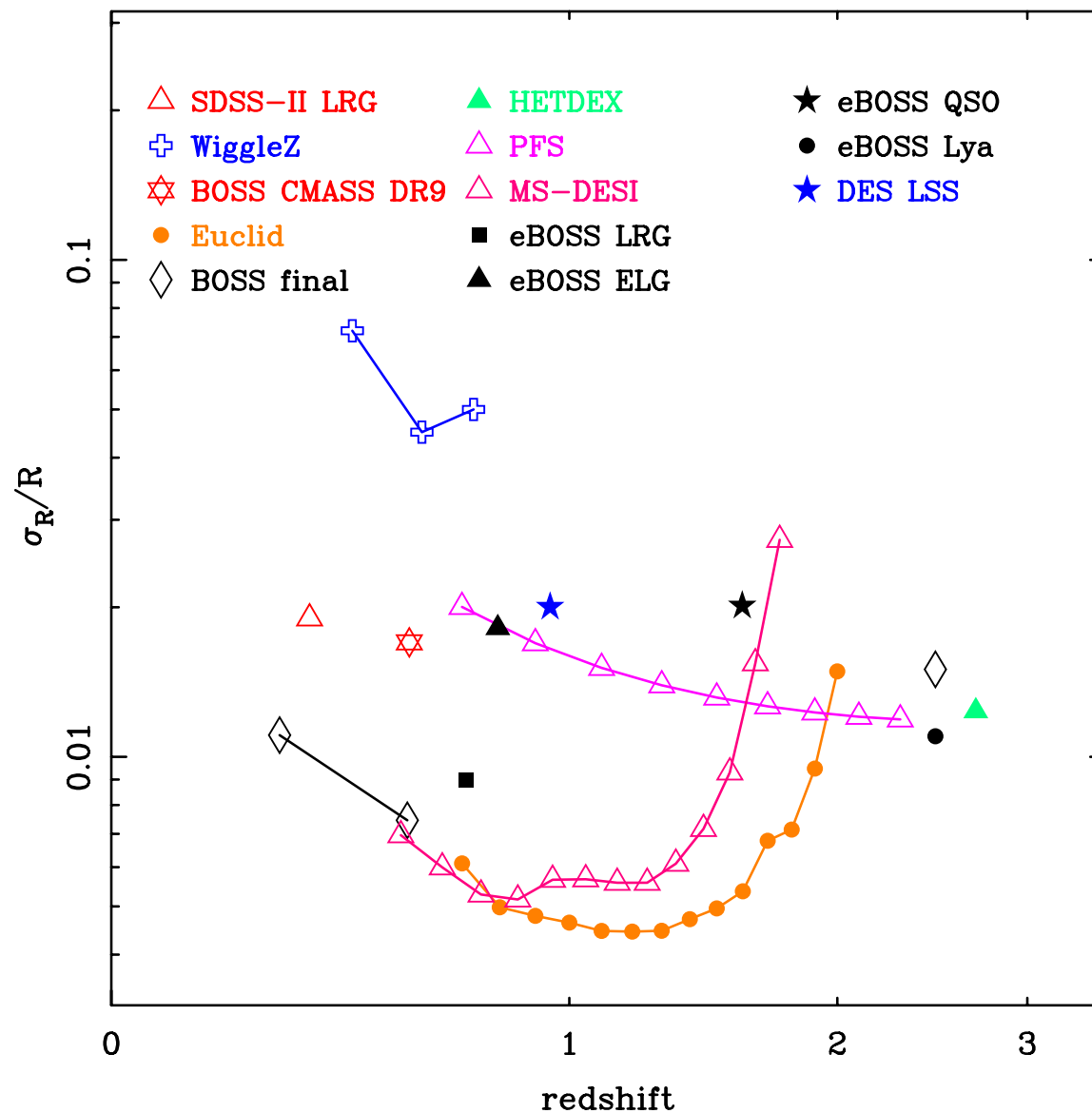
Outcome:

switch to strategy using grisms with single wavelength coverage

go deeper ( $2 \times 10^{-16}$  erg cm<sup>-2</sup> s<sup>-1</sup>), but “only” over  $0.9 < z < 1.8$

“expected yield” of 26M redshifts

# BAO measurements for future surveys



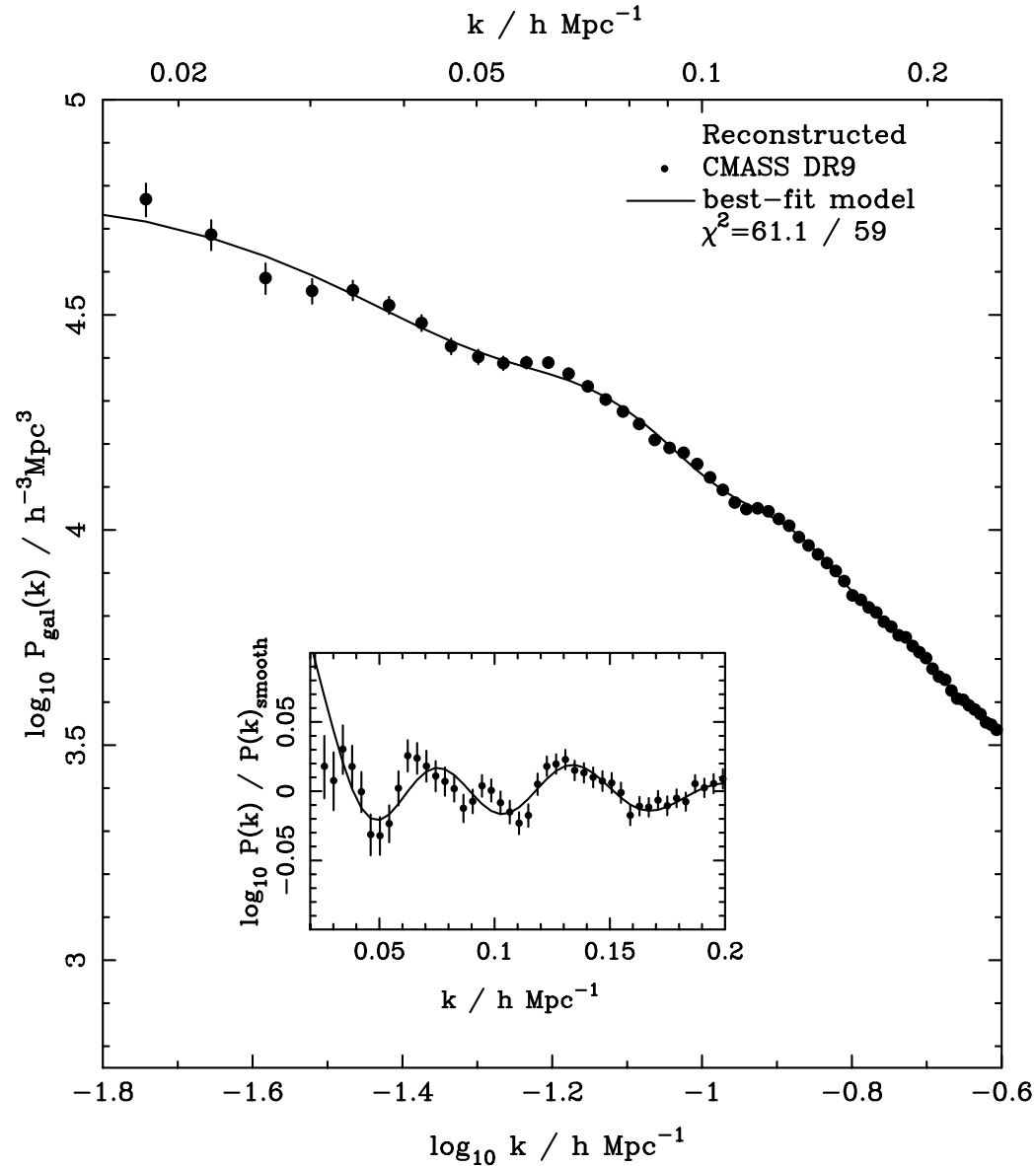
using the code of Seo & Eisenstein 2007, arXiv:0701079

# BOSS CMASS DR9 galaxy clustering

BOSS CMASS  
galaxies at  $z \sim 0.57$

Total effective  
volume

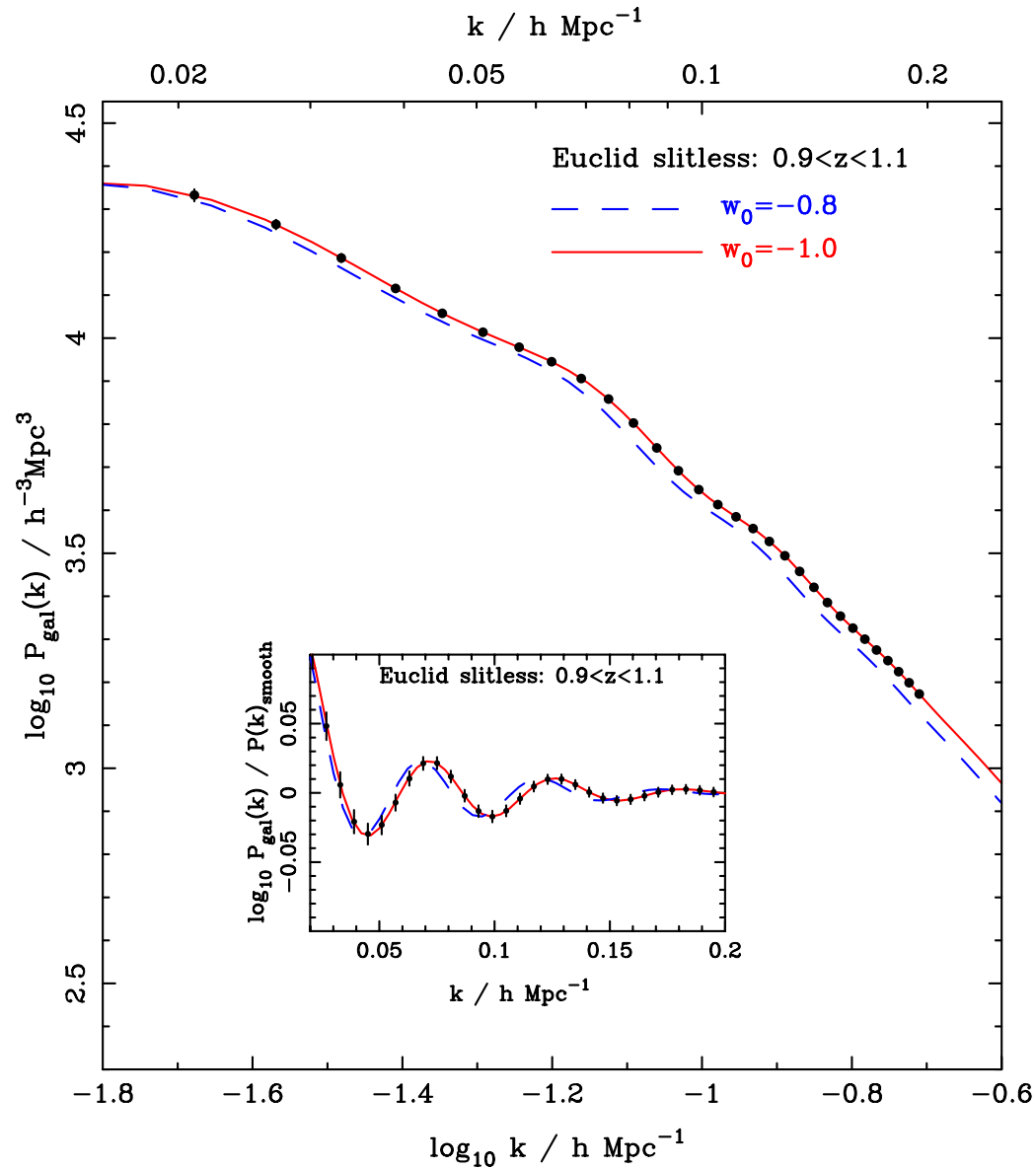
$$V_{\text{eff}} = 2.2 \text{ Gpc}^3$$



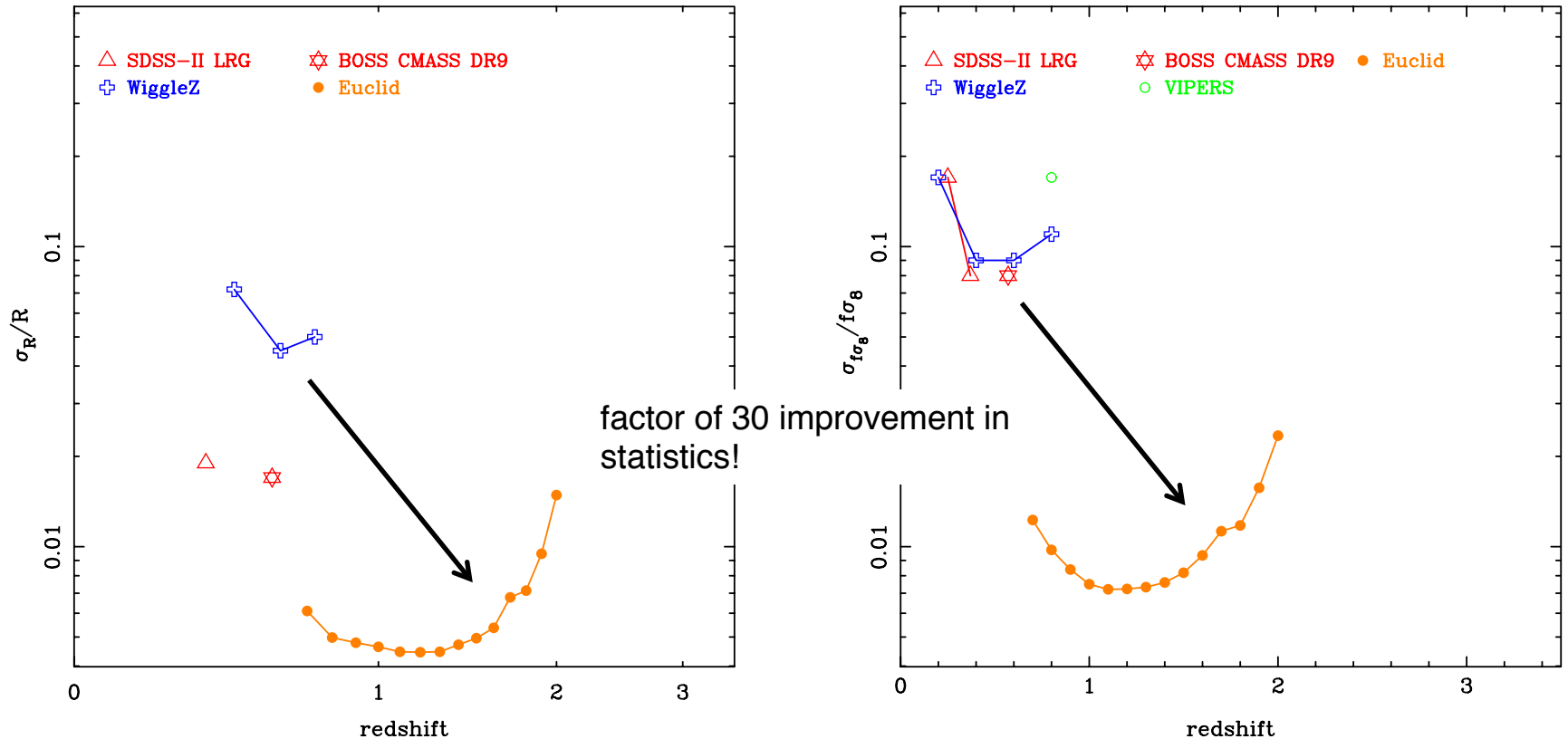
# Predicted Euclid galaxy clustering

Redshift slice  
 $0.9 < z < 1.1$

Total effective  
volume (of Euclid)  
 $V_{\text{eff}} = 57.4 \text{ Gpc}^3$



# Improvement in precision



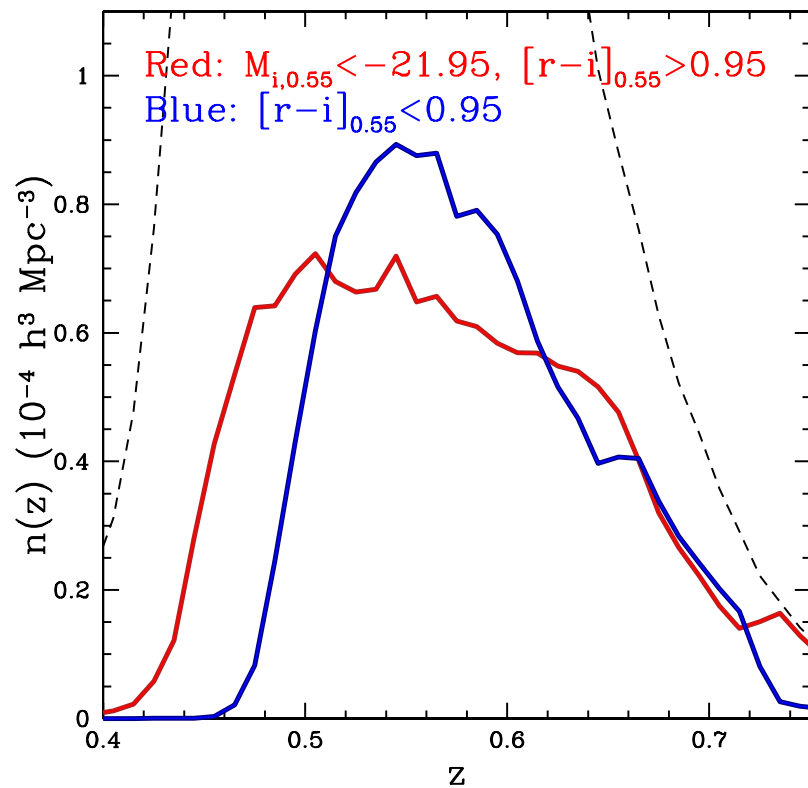
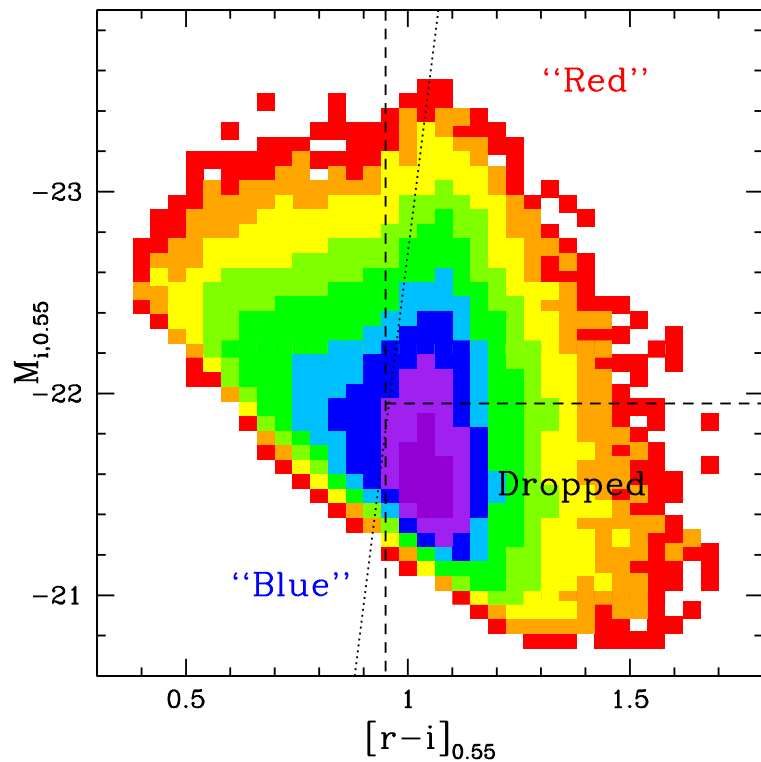
getting here relies on the low level of systematics in  
BAO measurements...

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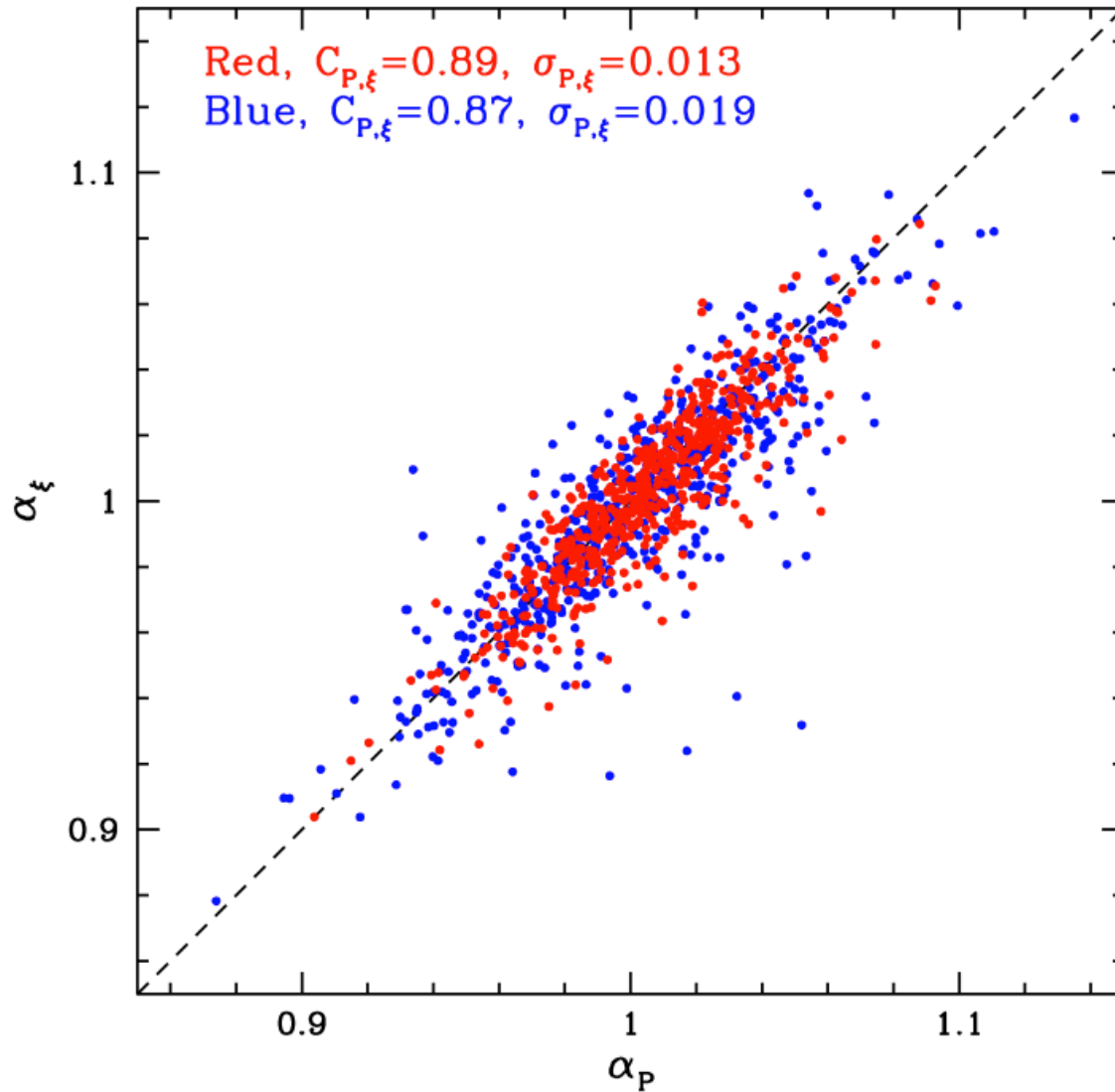
# Testing with subsamples



# Testing with blue / red subsamples



# Testing with blue / red subsamples



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Getting the likelihood right

# Getting the likelihood calculation 100% correct

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The Likelihood under the standard assumption of a set of data drawn from a multi-variate Gaussian distribution is given by

$$\mathcal{L}(\mathbf{x}|\mathbf{p}, \Psi^t) = \frac{|\Psi^t|}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \chi^2(\mathbf{x}, \mathbf{p}, \Psi^t) \right],$$

where  $\chi^2(\mathbf{x}, \mathbf{p}, \Psi^t) \equiv \sum_{ij} [x_i^d - x_i(\mathbf{p})] \Psi_{ij}^t [x_j^d - x_j(\mathbf{p})]$ .

now suppose that the covariance matrix (size  $n_b \times n_b$ ) has been calculated from  $n_s$  simulations

$$\mu_i = \frac{1}{n_s} \sum_s x_i^s \quad C_{ij} = \frac{1}{n_s - 1} \sum_s (x_i^s - \mu_i)(x_j^s - \mu_j)$$

then an unbiased estimator of the inverse covariance matrix is

$$\Psi = \frac{n_s - n_b - 2}{n_s - 1} C^{-1}$$

# Errors in the covariance matrix

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Simply providing an unbiased estimator of the inverse covariance matrix is not enough

The inverse covariance matrix also has its own error

$$\langle \Delta \Psi_{ij} \Delta \Psi_{i'j'} \rangle = A \Psi_{ij} \Psi_{i'j'} + B (\Psi_{ii'} \Psi_{jj'} + \Psi_{ij'} \Psi_{ji'}),$$

$$A = \frac{2}{(n_s - n_b - 1)(n_s - n_b - 4)}$$

$$B = \frac{(n_s - n_b - 2)}{(n_s - n_b - 1)(n_s - n_b - 4)}$$

Strictly, we should form a joint likelihood

$$\mathcal{L}(\mathbf{x}, \Psi | \mathbf{p}, \Psi^t) = \mathcal{L}(\mathbf{x} | \mathbf{p}, \Psi) \mathcal{L}(\Psi | \Psi^t),$$

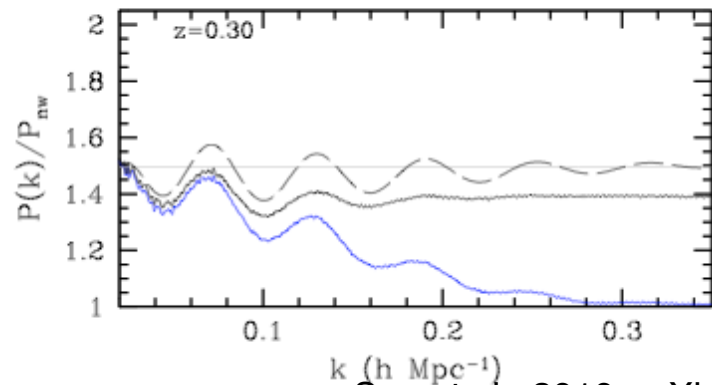
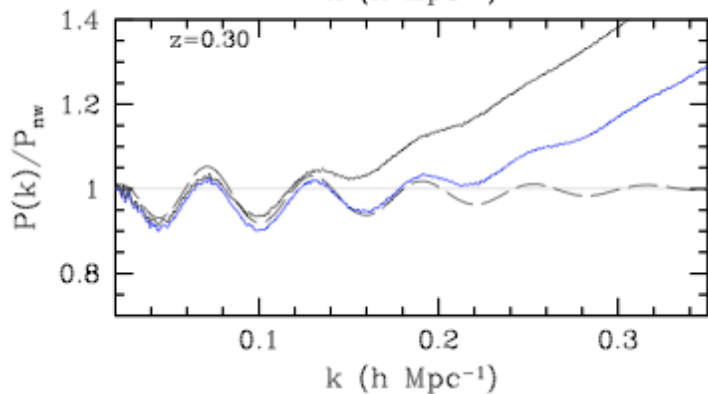
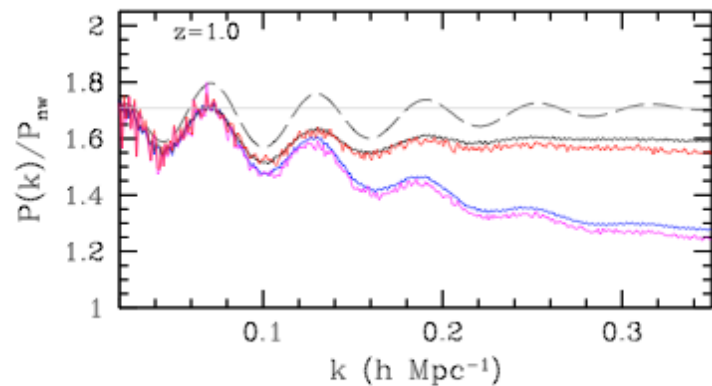
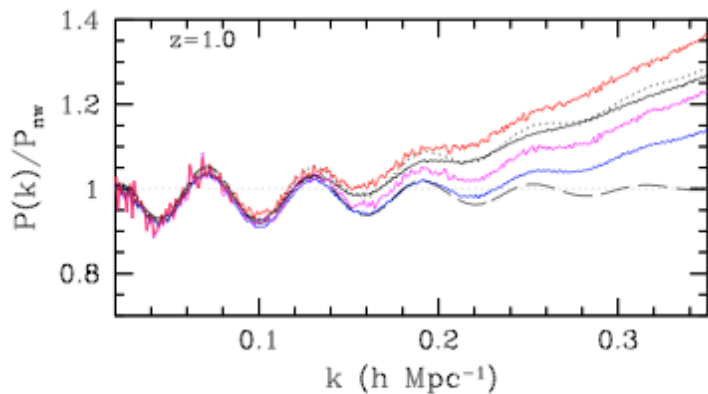
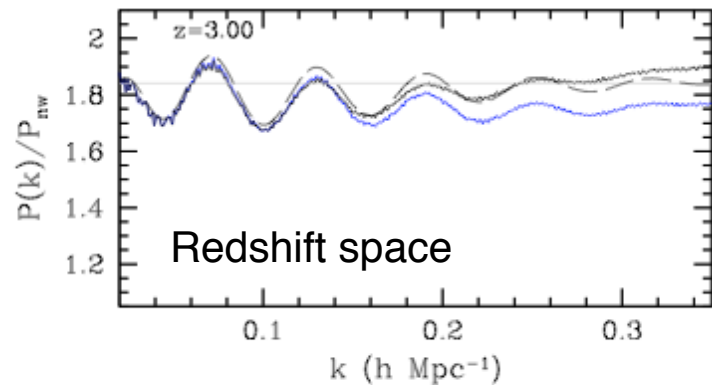
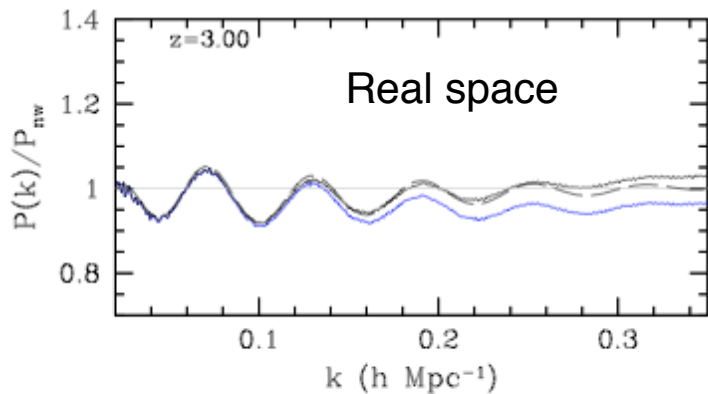
If we don't, this leads to an additional error on the  $n_p$  parameters being fitted

$$\langle p_\alpha p_\beta \rangle |_{s.o.} = B(n_b - n_p) F_{\alpha\beta}^{-1},$$

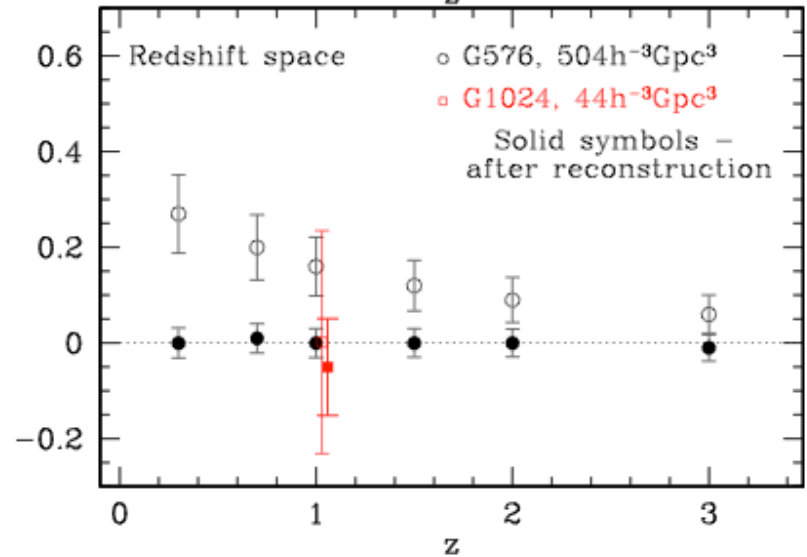
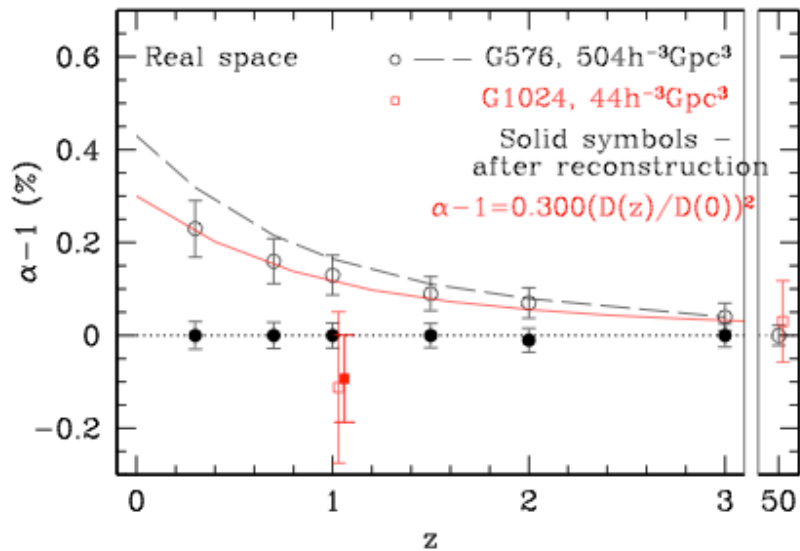
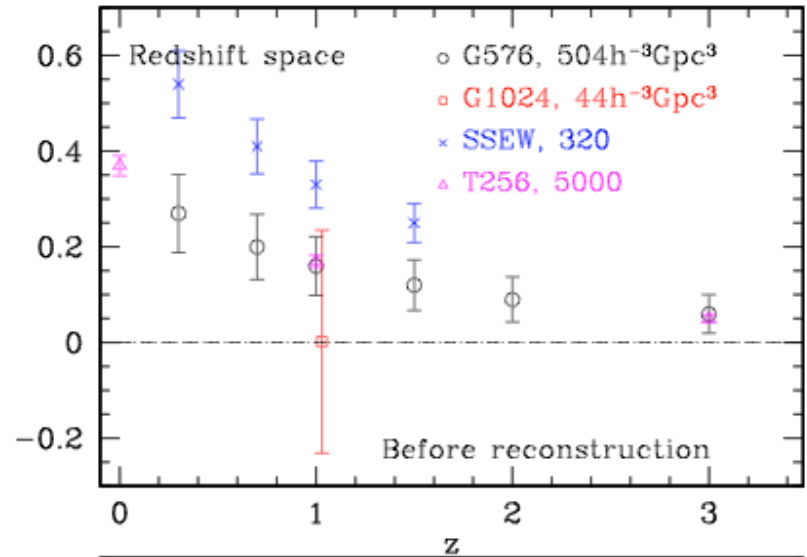
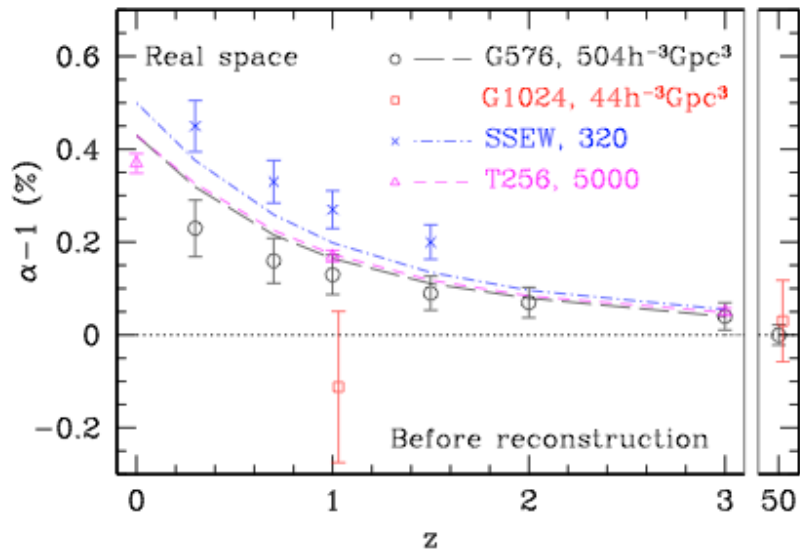
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Getting the model right

# BAO from simulations



# BAO from simulations





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What will you be showing in 15 years time?

# At the same time as my PhD ...

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Photo: U. Montan

**Saul Perlmutter**



Photo: U. Montan

**Brian P. Schmidt**



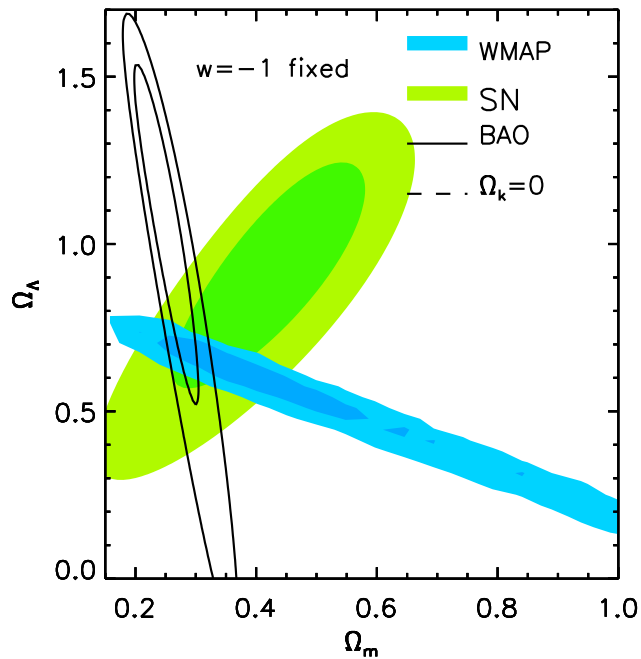
Photo: U. Montan

**Adam G. Riess**

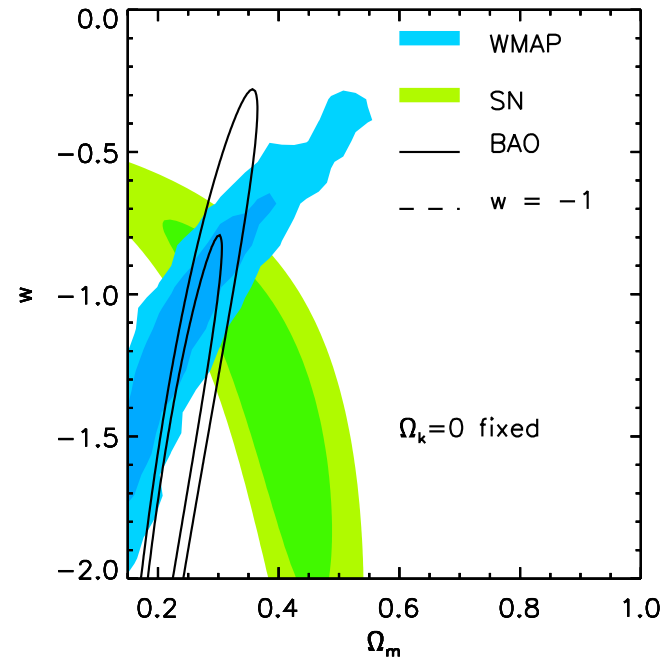
The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*.

# SDSS-II LRG BAO vs other data

$\Lambda$ CDM models with curvature



flat wCDM models

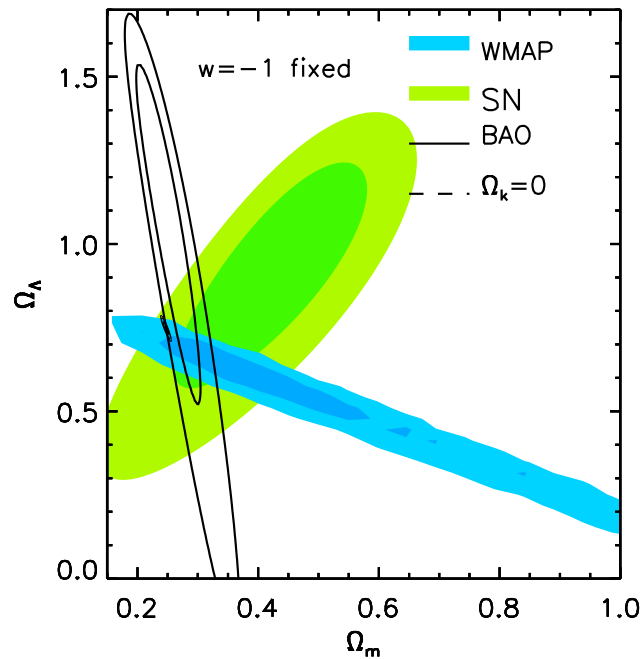


- Union supernovae
- WMAP 5year

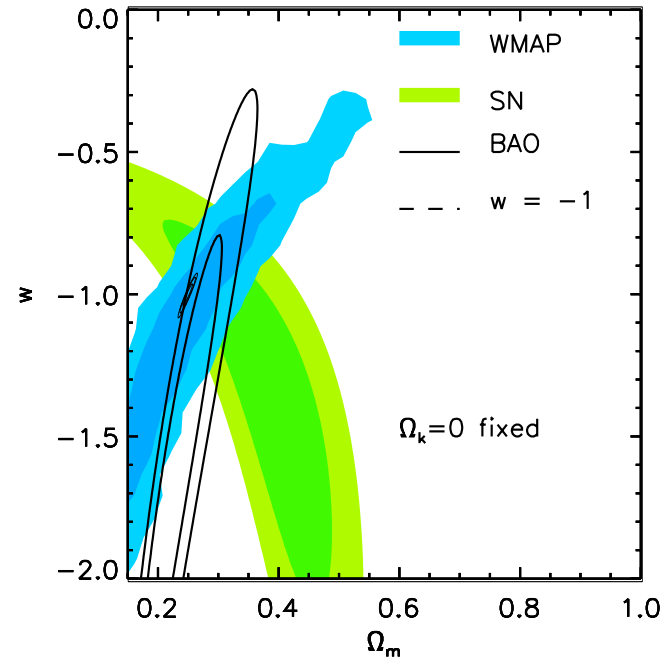
— SDSS-II BAO Constraint on  $r_s(z_d)/D_V(0.2)$  &  $r_s(z_d)/D_V(0.35)$

# Euclid BAO predictions

$\Lambda$ CDM models with curvature



flat wCDM models



■ Union supernovae

■ WMAP 5year

— SDSS-II BAO Constraint on  $r_s(z_d)/D_V(0.2)$  &  $r_s(z_d)/D_V(0.35)$

# Cosmology from surveys

