Slow-roll inflation at the era of precision cosmology

Christophe Ringeval

Centre for Cosmology, Particle Physics and Phenomenology

Institute of Mathematics and Physics

Louvain University, Louvain-la-Neuve, Belgium

Cargese, 09/2014

Outline

Introduction

Outline

Introduction

The ΛCDM model of cosmology

- Homogeneous + isotropic Friedmann–Lemaître scenario

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j, \quad H(t) = \frac{\mathrm{d}\ln a}{\mathrm{d}t}$$

  - Gravitation: $G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa^2 T_{\mu\nu}$
  - Contains: cold dark matter, baryons, photons

$$\rho_{\mathrm{mat}} = (\Omega_{\mathrm{dm}} + \Omega_{\mathrm{b}})\frac{\rho_{\mathrm{cri}}}{a^3}, \quad \rho_{\mathrm{rad}} = \Omega_{\mathrm{rad}}\frac{\rho_{\mathrm{cri}}}{a^4}, \quad \rho_{\mathrm{cri}} = 3\kappa^{-2}{H_0}^2$$

- Plus linear perturbations: origin of CMB and galaxies
  - Need some initial conditions

$$\langle X^*(\boldsymbol{k}, t_{\mathrm{ini}})X(\boldsymbol{k}', t_{\mathrm{ini}})\rangle = (2\pi)^3 P_X(k)\delta(\boldsymbol{k} - \boldsymbol{k}')$$

  - A priori as many $P_X(k)$ as species are required!

The flatness problem

- Evolution of the curvature
  - Friedmann-Lemaître equations for a perfect fluid

$$\left.\begin{aligned} T_{\mu\nu} &= (\rho + P)u_\mu u_\nu - P g_{\mu\nu} \\ \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j &= \frac{\mathrm{d}r^2}{1-\mathcal{K}^2 r^2} + r^2\mathrm{d}\Omega^2 \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} H^2 &= \kappa^2\frac{\rho}{3} - \frac{\mathcal{K}}{a^2} \\ \frac{\ddot{a}}{a} &= -\frac{\kappa^2}{6}(\rho + 3P) \end{aligned}\right.$$

  - Curvature density parameter: $\Omega_{\mathrm{K}} \equiv -\dfrac{\mathcal{K}}{a^2 H^2}$

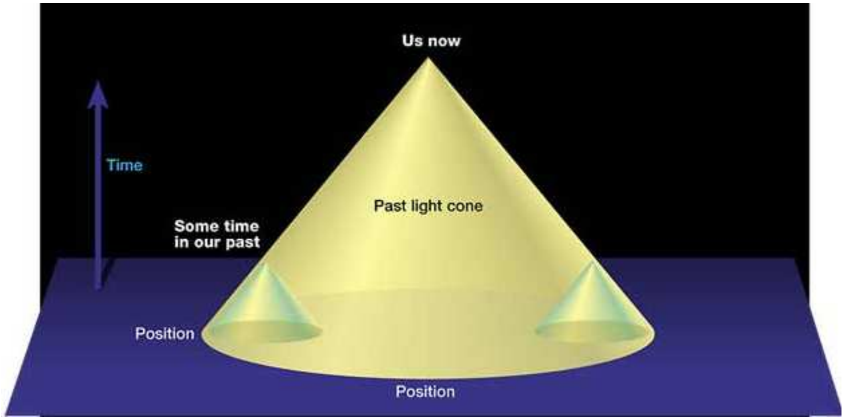
$$\omega \equiv \frac{\Omega_{\mathrm{K}}}{1-\Omega_{\mathrm{K}}} \quad \Rightarrow \quad \frac{\mathrm{d}\ln\omega}{\mathrm{d}\ln a} = 1 + 3\frac{P}{\rho}$$

  - For a constant equation of state $P = w\rho \Rightarrow \omega \propto a^{1+3w}$

- Flatness is instable during matter $(w=0)$ and radiation $(w=1/3)$ eras

Unadressed questions within ΛCDM

- We see causally disconnected regions from the past at any time
  - Distance to the particle horizon: $d_\mathrm{h} = a(t)\int_0^t \frac{\mathrm{d}t'}{a(t')} = a(\eta)\eta \propto t$
  - $(\eta_0/\eta_\mathrm{CMB})^3 \simeq 10^5$ causally disconnected patches: CMB?



- Acausal initial conditions for structure formation

$$\lambda \propto a(t) \propto t^{2/(3+3w)} \Rightarrow \lambda_\mathrm{ini} > d_\mathrm{h}(t_\mathrm{ini})$$

- Monopole problem: $\pi_2(G/H) \neq 1$ for $U(1) \subset H$

The inflationary paradigm

- Proposed in the 80's to solve these issues

[Grishchuk ,Starobinsky, Sato, Guth, Linde, Albrecht, Steinhardt, Sasaki, Mukhanov]

- Flatness, horizon and monopole problems solved for $w < -1/3$

inflation = accelerated expansion of the scale factor

✦ Quasi de Sitter: $w \simeq -1 \Rightarrow H$ is constant $\Rightarrow a(t) \propto \mathrm{e}^{Ht}$

$$\frac{d_{\mathrm{h}}(t_{\mathrm{end}})}{d_{\mathrm{h}}(t_{\mathrm{ini}})} \simeq \mathrm{e}^{H\Delta t} > \frac{\eta_0}{\eta_{\mathrm{Pl}}} \simeq 10^{28} \Leftarrow N = H\Delta t \gtrsim 60$$

- Isotropy: Bianchi smoothed out during inflation ($\to$ FLRW)

$$H^2 = \kappa^2 \frac{\rho}{3} - f(a_x, a_y, a_z), \quad f \lesssim \frac{1}{(a_x a_y a_z)^{2/3}}$$

- Structure formation from quantum fluctuations

Motivations from current observations

- Planck 2013 measurements in favour of inflation
  - Flatness ($\Omega_{\mathrm{K}} = 0$) is instable during decelerated expansion

$$\Omega_{\mathrm{K}} = 1 - \Omega_{\mathrm{dm}} - \Omega_{\mathrm{b}} - \Omega_{\Lambda} - \Omega_{\mathrm{rad}} = 0.000^{+0.0066}_{-0.0067} \quad \texttt{(PLANCK+WP+BAO)}$$

  - Adiabatic initial conditions: isocurvature modes are constrained

$$\forall X \quad P_X(k) = P(k)$$

  - Quasi scale invariance

$$k^3 P(k) = A \left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1} \Rightarrow n_{\mathrm{S}} = 0.9619 \pm 0.0073$$

  - Dark energy?
  - Gaussianity of the CMB anisotropies

$$f_{\mathrm{NL}}^{\mathrm{loc}} = 2.7 \pm 5.8, \quad f_{\mathrm{NL}}^{\mathrm{eq}} = -42 \pm 75, \quad f_{\mathrm{NL}}^{\mathrm{ortho}} = -25 \pm 39$$

- The simplest framework: single-field inflation
  - Makes extra-predictions: $f_{\mathrm{NL}}^{\mathrm{loc}} = \mathcal{O}(n_{\mathrm{S}} - 1)$ and $\exists r > 0$

Slow-roll inflation

Outline

Basic theoretical assumptions

- Dynamics given by ($\kappa^2 = 1/M_{\mathrm{P}}^2$)

$$S = \int \mathrm{d}x^4 \sqrt{-g} \left[\frac{1}{2\kappa^2} R + \mathcal{L}(\phi) \right] \quad \text{with} \quad \mathcal{L}(\phi) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

- Can be used to describe:
  - ✦ Minimally coupled scalar field to General Relativity
  - ✦ Scalar-tensor theory of gravitation in the Einstein frame

    the graviton' scalar partner is also the inflaton (HI, RPI1,...)

- Everything can be consistently solved in the slow-roll approximation
  - ✦ Background evolution $\phi(t)$ (attractor)
  - ✦ Linear perturbations for the field-metric system $\zeta(t, \boldsymbol{x})$, $\delta\phi(t, \boldsymbol{x})$

- Inclusion of the reheating era at the background level
  - ✦ A new parameter R_{rad}

Self-gravitating scalar field

- Stress tensor for a homogeneous scalar field in a flat FLRW metric

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{|g|}} \frac{\delta S_\phi}{\delta g^{\mu\nu}} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} (\nabla \phi)^2 - V(\phi) \right]$$

$$\phi(x^\mu) = \phi(t) \Rightarrow \begin{cases} \rho = \dfrac{1}{2}\dot{\phi}^2 + V(\phi) \\ P = \dfrac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases}$$

  ✦ Potential dominated regime: $P \simeq -\rho \Rightarrow w \simeq -1 \Rightarrow \ddot{a} > 0$

- Friedmann-Lemaître equations: $\delta S / \delta g^{\mu\nu} = 0$

$$3H^2 = \kappa^2 \left(\frac{1}{2}\dot{\phi}^2 + V \right), \qquad \dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2$$

- Klein-Gordon equation: $\delta S / \delta \phi = 0$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

Decoupling field and space-time evolution

- Time measured in e-fold: $N \equiv \ln a$
- Deviations from de-Sitter measured by Hubble flow hierarchy [Schwarz 01]

$$\epsilon_0 = \frac{H_{\mathrm{ini}}}{H}, \qquad \epsilon_{i+1} = \frac{\ln|\epsilon_i|}{\mathrm{d}N}$$

- Friedmann-Lemaître equations in e-fold time (with $M_{\mathrm{P}}^2 = 1$)

$$\begin{cases} H^2 = \dfrac{1}{3}\left(\dfrac{1}{2}\dot{\phi}^2 + V\right) \\ \dfrac{\ddot{a}}{a} = -\dfrac{1}{3}\left(\dot{\phi}^2 - V\right) \end{cases} \Rightarrow \begin{cases} H^2 = \dfrac{V}{3 - \dfrac{1}{2}\left(\dfrac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2} \\ -\dfrac{\mathrm{d}\ln H}{\mathrm{d}N} = \dfrac{1}{2}\left(\dfrac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2 \end{cases} \Leftrightarrow \begin{cases} H^2 = \dfrac{V}{3 - \epsilon_1} \\ \epsilon_1 = \dfrac{1}{2}\left(\dfrac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2 \end{cases}$$

- Klein-Gordon equation in e-folds: relativistic kinematics with friction

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}N^2} + \left(3 + \frac{\mathrm{d}\ln H}{\mathrm{d}N}\right)\frac{\mathrm{d}\phi}{\mathrm{d}N} + \frac{V_{,\phi}}{H^2} = 0 \quad \Rightarrow \quad \frac{1}{3-\epsilon_1}\frac{\mathrm{d}^2\phi}{\mathrm{d}N^2} + \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{\mathrm{d}\ln V}{\mathrm{d}\phi}$$

Background evolution

- The friction term ensures the existence of a "terminal velocity"

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{\mathrm{d}\ln V}{\mathrm{d}\phi} \quad \Rightarrow \quad \epsilon_1 \simeq \frac{1}{2}\left(\frac{\mathrm{d}\ln V}{\mathrm{d}\phi}\right)^2, \quad \epsilon_2 \simeq 2\left[\left(\frac{V_{,\phi}}{V}\right)^2 - \frac{V_{,\phi\phi}}{V}\right]\ldots$$

  - ✦ As for a "sky diver" it does not depend on the initial conditions
  - ✦ Inflation occurs for $\epsilon_1 < 1 \Leftarrow \ln[V(\phi)]$ should be flat enough

$$\epsilon_1 = -\frac{\ln H}{\mathrm{d}N} = -\frac{\dot{H}}{H^2} = 1 - \frac{1}{H^2}\frac{\ddot{a}}{a}$$

- Deviations from terminal velocity behaviour are encoded in ϵ_2

$$\epsilon_2 = \frac{\mathrm{d}\ln\epsilon_1}{\mathrm{d}N} \Rightarrow \frac{\mathrm{d}^2\phi}{\mathrm{d}N^2} = \frac{\epsilon_2}{2}\frac{\mathrm{d}\phi}{\mathrm{d}N}$$

  - ✦ Klein-Gordon equation also reads

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{3-\epsilon_1}{3-\epsilon_1+\dfrac{\epsilon_2}{2}}\frac{\mathrm{d}\ln V}{\mathrm{d}\phi}$$

Slow-roll approximation

- Assume that all $\epsilon_i = \mathcal{O}(\epsilon)$ and $\epsilon_i < 1$
  - The trajectory can be solved for N (taking $N_{\mathrm{ini}} = 0$)

$$N = \mathcal{I}(\phi_{\mathrm{ini}}) - \mathcal{I}(\phi) \quad \text{with} \quad \mathcal{I}(\phi) \equiv \int^{\phi} \frac{V(\psi)}{V_{,\psi}(\psi)} \mathrm{d}\psi$$

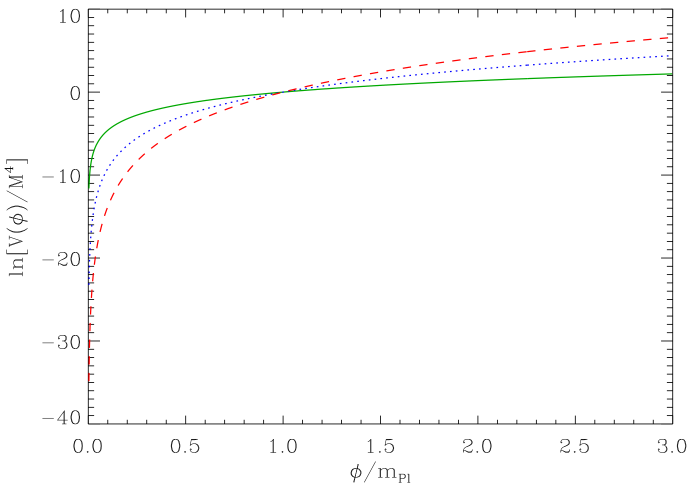
  - In terms of the field values at the end of inflation

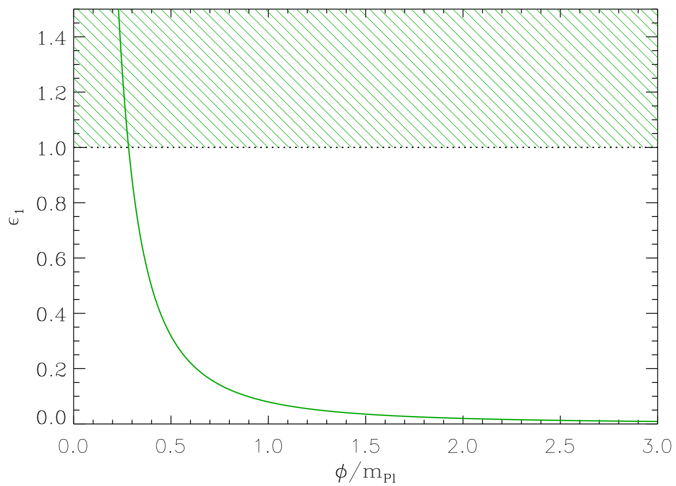
$$N - N_{\mathrm{end}} = \mathcal{I}(\phi_{\mathrm{end}}) - \mathcal{I}(\phi)$$

- The end of inflation
  - Inflation naturally ends when $\epsilon_1 > 1$: ϕ_{end} is solution of the algebraic equation $\epsilon_1(\phi_{\mathrm{end}}) = 1$
  - Or, there is another mechanism ending inflation (tachyonic instability) and ϕ_{end} is a model parameter that must be specified
- The reheating stage: everything after N_{end} till radiation domination

Example: large field inflation

- LFI potential has two-parameters: $V(\phi) = M^4\phi^p$
  - ✦ M^4 do not affect the background evolution (see KG)





- Field trajectory

$$\mathcal{I}(\phi) = \int^{\phi} \frac{M^4\psi^p}{M^4 p \psi^{p-1}}\,\mathrm{d}\psi = \frac{\phi^2}{2p} \quad \Rightarrow \quad N - N_{\rm end} = \frac{1}{2p}\left(\phi_{\rm end}^2 - \phi^2\right)$$

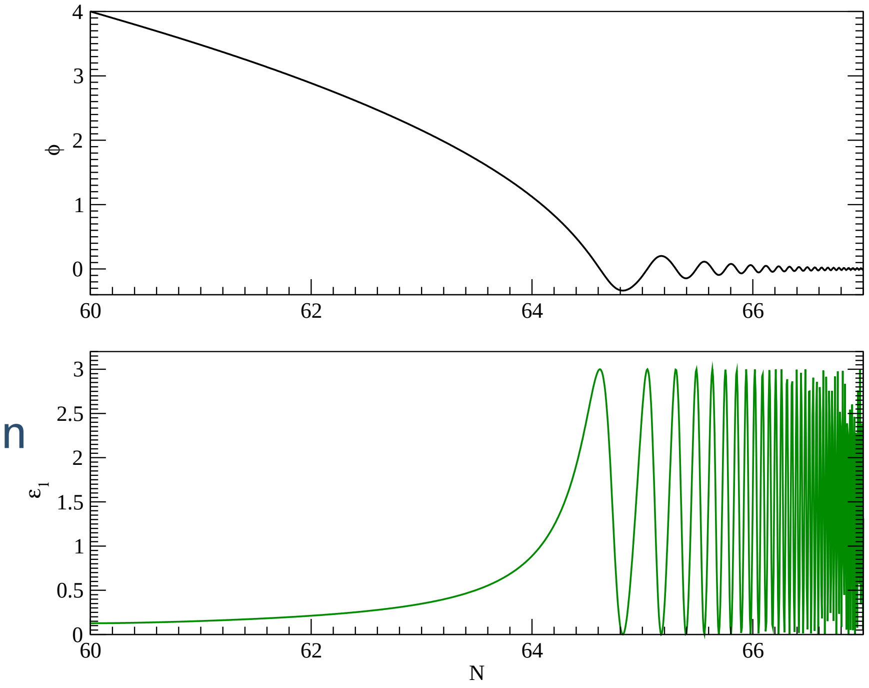
- End of inflation at $\epsilon_1(\phi_{\rm end}) \simeq \dfrac{p^2}{2\phi_{\rm end}^2} = 1$

$$\phi_{\rm end} \simeq \frac{p}{\sqrt{2}} \quad \Rightarrow \quad \phi(N) = \sqrt{2p(N_{\rm end} - N) + \frac{p^2}{2}} \qquad (\phi > 1)$$

Reheating: from inflation to radiation

- Example $V = M^4\phi^2$ (LFI_2)
  - ✦ Harmonic oscillator ($\omega \gg H$)

$$\left\langle \frac{1}{2}\phi_{,t}^2 \right\rangle = \langle V \rangle \Rightarrow \begin{cases} \langle\rho\rangle = \langle\phi_{,t}^2\rangle \\ \langle P \rangle = 0 \end{cases}$$



  - ✦ Coherent oscillations: $\phi \to$ radiation (non pert. decay)
  - ✦ Last $\Delta N_{\mathrm{reh}} \equiv N_{\mathrm{reh}} - N_{\mathrm{end}}$ e-folds

- Total energy density at the end of reheating ρ_{reh}

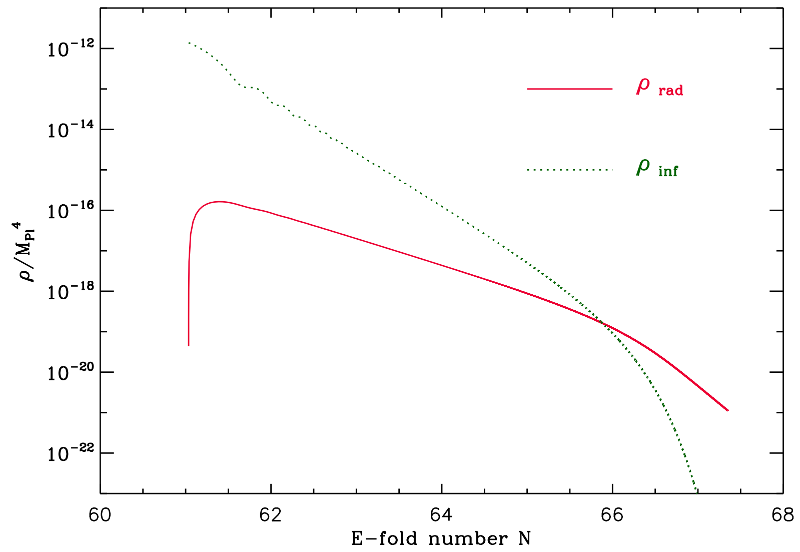
$$\begin{cases} \dot{\rho} = -3H(P+\rho) \\ \bar{w}_{\mathrm{reh}} \equiv \dfrac{1}{\Delta N_{\mathrm{reh}}} \displaystyle\int_{N_{\mathrm{end}}}^{N_{\mathrm{reh}}} \frac{P(N)}{\rho(N)} \mathrm{d}N \end{cases} \Rightarrow \quad \ln\left(\frac{\rho_{\mathrm{reh}}}{\rho_{\mathrm{end}}}\right) = -3(1+\bar{w}_{\mathrm{reh}})\Delta N_{\mathrm{reh}}$$

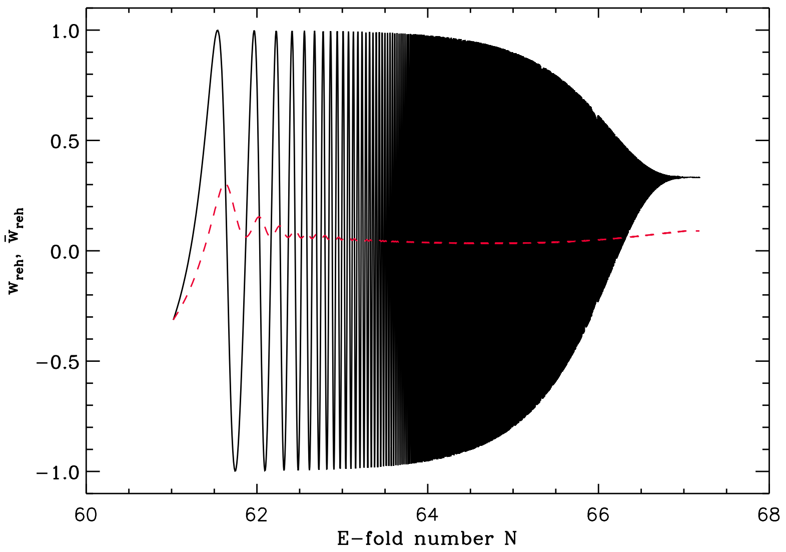
A phenomenological example

- Inflaton decay rate Γ [Turner 83]

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_{,\phi} = 0, \qquad \frac{\mathrm{d}\rho_{\mathrm{rad}}}{\mathrm{d}N} + 4\rho_{\mathrm{rad}} = \frac{\Gamma}{H}\rho_\phi$$

- Inflaton energy is converted into radiation fluid





- At the end of reheating $\rho_{\mathrm{reh}} = \rho_\phi(N_{\mathrm{reh}}) + \rho_{\mathrm{rad}}(N_{\mathrm{reh}}) \simeq \rho_{\mathrm{rad}}(N_{\mathrm{reh}})$

Redshift at which reheating ends

- At $N = N_{\mathrm{reh}}$ the Universe is radiation dominated
  - ✦ If thermalized, and no extra entropy production: $a_{\mathrm{reh}}^3 s_{\mathrm{reh}} = a_0^3 s_0$

$$\begin{cases} s_{\mathrm{reh}} = q_{\mathrm{reh}} \dfrac{2\pi^2}{45} T_{\mathrm{reh}}^3 \\ \rho_{\mathrm{reh}} = g_{\mathrm{reh}} \dfrac{\pi^2}{30} T_{\mathrm{reh}}^4 \end{cases} \Rightarrow \quad \frac{a_0}{a_{\mathrm{reh}}} = \left(\frac{q_{\mathrm{reh}}^{1/3} g_0^{1/4}}{q_0^{1/3} g_{\mathrm{reh}}^{1/4}} \right) \frac{\rho_{\mathrm{reh}}^{1/4}}{\rho_\gamma^{1/4}}$$

$$\text{or} \quad 1 + z_{\mathrm{reh}} = \left(\frac{\rho_{\mathrm{reh}}}{\tilde{\rho}_\gamma} \right)^{1/4}$$

- Depends on ρ_{reh} and $\tilde{\rho}_\gamma \equiv \mathcal{Q}_{\mathrm{reh}} \rho_\gamma$
  - ✦ Energy density of radiation today: $\rho_\gamma = 3 \dfrac{H_0^2}{M_{\mathrm{P}}^2} \Omega_{\mathrm{rad}}$ (CMB photons)
  - ✦ Change in the number of entropy and energy relativistic degrees of freedom (small effect compared to $\rho_{\mathrm{reh}}/\rho_\gamma$)

$$\mathcal{Q}_{\mathrm{reh}} \equiv \frac{g_{\mathrm{reh}}}{g_0} \left(\frac{q_0}{q_{\mathrm{reh}}} \right)^{1/4}$$

Redshift at which inflation ends

- Depends on how the reheating proceeds

$$1 + z_{\mathrm{end}} = \frac{a_0}{a_{\mathrm{end}}} = \frac{a_{\mathrm{reh}}}{a_{\mathrm{end}}}(1 + z_{\mathrm{reh}}) = \frac{a_{\mathrm{reh}}}{a_{\mathrm{end}}}\left(\frac{\rho_{\mathrm{reh}}}{\tilde{\rho}_\gamma}\right)^{1/4} = \frac{1}{R_{\mathrm{rad}}}\left(\frac{\rho_{\mathrm{end}}}{\tilde{\rho}_\gamma}\right)^{1/4}$$

  - The reheating parameter $R_{\mathrm{rad}} \equiv \frac{a_{\mathrm{end}}}{a_{\mathrm{reh}}}\left(\frac{\rho_{\mathrm{end}}}{\rho_{\mathrm{reh}}}\right)^{1/4}$
  - Encodes any deviations from a radiation-like or instantaneous reheating $R_{\mathrm{rad}} = 1$

- R_{rad} can be expressed in terms of $(\rho_{\mathrm{reh}}, \bar{w}_{\mathrm{reh}})$ or $(\Delta N_{\mathrm{reh}}, \bar{w}_{\mathrm{reh}})$

$$\ln R_{\mathrm{rad}} = \frac{\Delta N_{\mathrm{reh}}}{4}(3\bar{w}_{\mathrm{reh}} - 1) = \frac{1 - 3\bar{w}_{\mathrm{reh}}}{12(1 + \bar{w}_{\mathrm{reh}})} \ln\left(\frac{\rho_{\mathrm{reh}}}{\rho_{\mathrm{end}}}\right)$$

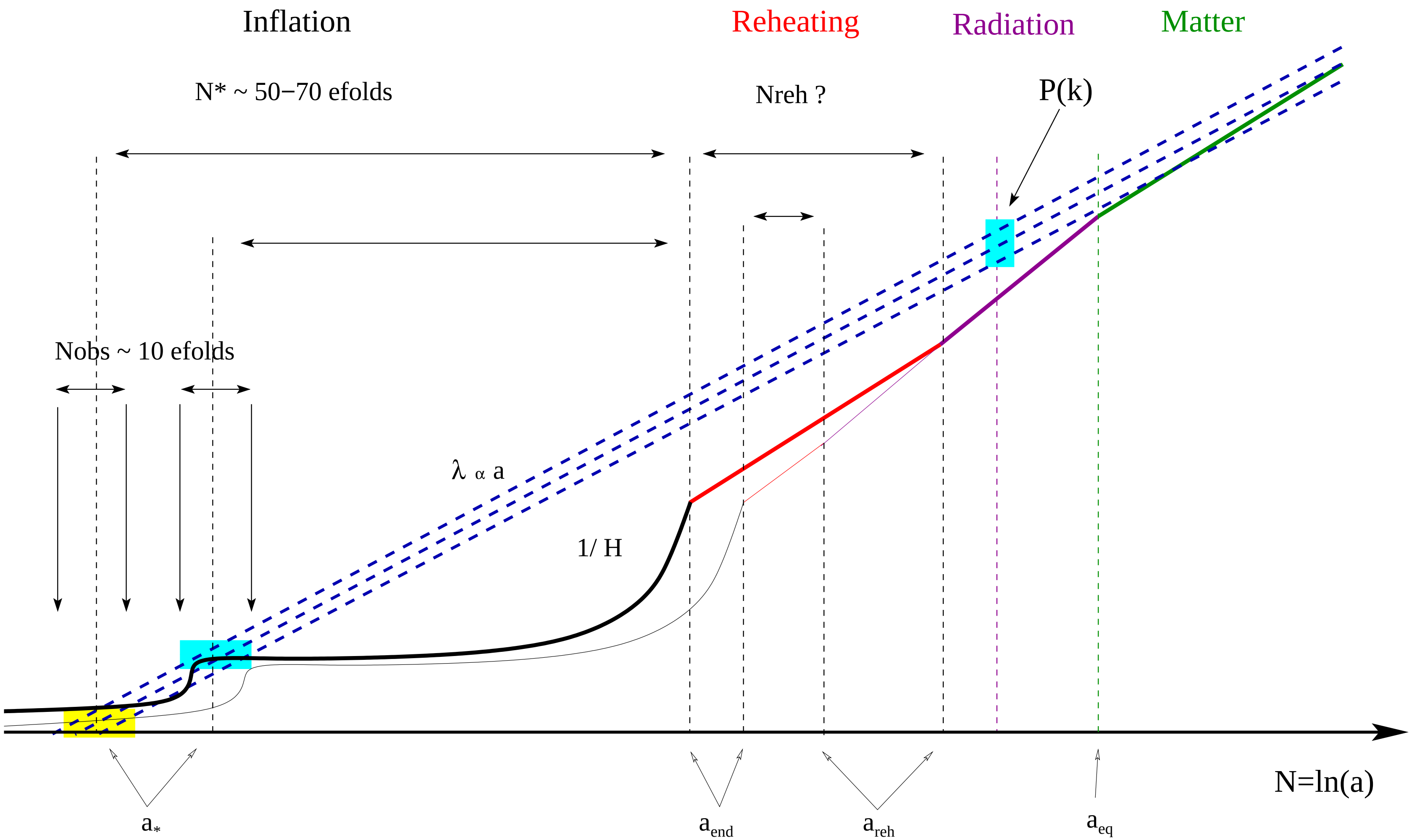
- A fixed inflationary parameters, z_{end} can still be affected by R_{rad}

Primordial power spectra

Outline

Primordial power spectra

Cosmological perturbations of inflationary origin



- Primordial power spectra for tensor and scalar perturbations are generated during inflation from quantum fluctuations $T = H/(2\pi)$

A toy example: test scalar fields in de Sitter

- Test = field fluctuations only ($m \ll H_{\mathrm{inf}}$): $\varphi(t,\boldsymbol{x}) = \phi(t) + \delta\phi(t,\boldsymbol{x})$
- Homogeneous part ($N \equiv \ln a$ "e-folds number")

$$\phi_{,tt} + 3H_{\mathrm{inf}}\phi_{,t} + m^2\phi = 0 \Rightarrow \phi(N) = \phi_0 e^{-Nm^2/(3H_{\mathrm{inf}}^2)} \to 0$$

- Fluctuations in Fourier space: $\mu \equiv a\delta\phi_{\boldsymbol{k}}$ ($aH_{\mathrm{inf}} = -1/\eta$)

$$\delta\phi_{\boldsymbol{k},tt} + 3H_{\mathrm{inf}}\delta\phi_{\boldsymbol{k},t} + \left(k^2+m^2\right)\delta\phi_{\boldsymbol{k}} = 0 \Rightarrow \mu'' + \left(m^2 + k^2 - \frac{2}{\eta^2}\right)\mu = 0$$

- Free field quantization: positive energy waves for $k\eta \gg 1$

$$\mu = e^{i(\nu+1/2)\pi/2}\sqrt{\frac{\pi}{4k}}\sqrt{k\eta}H_\nu^1(k\eta), \quad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H_{\mathrm{inf}}^2}}$$

- Power spectra after Hubble exit: $\mathcal{P}_{\delta\phi} = \lim_{k\eta\ll 1}\frac{k^3}{2\pi^2}\left|\frac{\mu}{a}\right|^2$

Scale invariant primordial power spectrum

- For light fields $m \ll H_{\text{inf}}$

$$\mathcal{P}_{\delta\phi} \simeq \frac{H_{\text{inf}}^2}{4\pi^2}\left(\frac{k}{aH_{\text{inf}}}\right)^{2m^2/(3H_{\text{inf}}^2)} = \frac{H_{\text{inf}}^2}{4\pi^2} + \dots$$

- Does not depend on k (scale invariant) and Gaussian
- Could explain the amplitude of CMB anisotropies $\delta T/T \simeq 10^{-5}$ for $H_{\text{inf}} \simeq 10^{-5} M_{\text{P}}$ (GUT scale)
- But test scalar fields cannot induce gravity perturbations, by definition
- Gravity perturbations must be included!
  - ✦ However, this is the right result for primordial gravity waves (up to a polarization factor)
- And ultra-light test scalar field could explain dark energy

Relic vacuum energy density from inflation

- Field variance in physical space after N e-folds

$$\langle \delta\phi^2 \rangle = \int_{a_i H_{\mathrm{inf}}}^{a H_{\mathrm{inf}}} \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} \left|\delta\phi_{\boldsymbol{k}}\right|^2 = \frac{3H_{\mathrm{inf}}^4}{8\pi^2 m^2}\left[1 - e^{-N(2m^2)/(3H_{\mathrm{inf}}^2)}\right] \rightarrow \frac{3H_{\mathrm{inf}}^4}{8\pi^2 m^2}$$

- Energy density expectation value (does not depend on m)

$$\langle V(\phi)\rangle = \frac{1}{2} m^2 \langle \delta\phi^2 \rangle = \frac{3H_{\mathrm{inf}}^4}{16\pi^2}$$

- Universal, does not even depend on V (for test fields)

$$P(\delta\phi | H_{\mathrm{inf}}) \propto \exp\left[-\frac{8\pi^2}{3H_{\mathrm{inf}}^4} V(\delta\phi)\right] \Rightarrow \langle V \rangle \simeq \frac{3H_{\mathrm{inf}}^4}{8\pi^2}$$

- This is dark energy provided: $H_{\mathrm{inf}} = (\Omega_{\Lambda})^{1/4} \sqrt{4\pi H_0 M_{\mathrm{P}}}$

$$H_{\mathrm{inf}} \simeq 6 \times 10^{-3}\,\mathrm{eV}, \quad \rho_{\mathrm{inf}}^{1/4} = (3 M_{\mathrm{P}}^2 H_{\mathrm{inf}}^2)^{1/4} \simeq 5\,\mathrm{TeV}$$

Linear perturbations during inflation

- Perturbed FLRW metric (longitudinal gauge): $\delta\phi, \Phi, \Psi, h_{ij}$

$$\mathrm{d}s^2 = a^2(1+2\Phi)\mathrm{d}\eta^2 - a^2\left[(1-2\Psi)\delta_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j$$

- Perturbed Einstein + Klein-Gordon equations: $\delta G_{\mu\nu} = \kappa^2 \delta T_{\mu\nu}$
  - ✦ Equations of motion ($' = \partial_\eta$)

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \Delta h_{ij} = 0 \quad \Phi = \Psi, \quad \zeta' = \frac{2aH}{\phi'^2}\Delta\Psi$$

$$\text{where} \quad \zeta \equiv \Psi - \frac{\mathcal{H}}{\mathcal{H}' - \mathcal{H}^2}\left(\Psi' + \mathcal{H}\Phi\right) = \Psi + H\frac{\delta\phi}{\dot{\phi}}$$

  - ✦ Comoving curvature perturbation ζ and h are conserved on large scales $\Delta \sim k^2$ (single-field only!)

- Primordial power spectra can be evaluated anytime after Hubble exit

$$\mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2}\left|\zeta\right|^2, \qquad \mathcal{P}_h(k) = \frac{2k^3}{\pi^2}\left|h\right|^2 \quad \leftarrow 2 \text{ polarizations}$$

Scalar and tensor modes evolution

- Parametric oscillators

$$\left.\begin{aligned} \mu_{\mathrm{T}} &\equiv a h \\ \mu_{\mathrm{S}} &\equiv a\sqrt{2}\phi_{,N}\zeta \end{aligned}\right\} \Rightarrow \mu_{\mathrm{TS}}'' + \left[k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}\right]\mu_{\mathrm{TS}} = 0$$

- Can be recast in terms of Hubble flow functions $\epsilon_i(\eta)$
  - ✦ Using $f' = aH\, f_{,N}\ldots$

$$\frac{\nu^2(\eta) - 1/4}{\eta^2} \equiv \frac{\left(a\sqrt{\epsilon_1}\right)''}{\left(a\sqrt{\epsilon_1}\right)} = \mathcal{H}^2\left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 + \frac{1}{4}\epsilon_2^2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{2}\epsilon_2\epsilon_3\right)$$

  - ✦ Expanding the conformal time in terms of ϵ_i

$$\eta = \int \frac{\mathrm{d}t}{a} = \int \frac{1}{a^2}\frac{\mathrm{d}a}{H} = -\frac{1}{aH} + \int \frac{1}{a}\frac{\mathrm{d}H^{-1}}{\mathrm{d}a}\mathrm{d}a = -\frac{1}{\mathcal{H}} + \int \frac{\epsilon_1}{a^2 H}\mathrm{d}a$$

$$= -\frac{1}{\mathcal{H}} - \frac{1}{a}\frac{\epsilon_1}{H} + \int \frac{1}{a}\frac{\mathrm{d}\left(\epsilon_1 H^{-1}\right)}{\mathrm{d}a}\mathrm{d}a = -\frac{1+\epsilon_1}{\mathcal{H}} + \int \frac{1}{a^2 H}\epsilon_1\left(\epsilon_1 + \epsilon_2\right)\mathrm{d}a$$

Slow-roll expansion for the perturbations

- Within the slow-roll approximation $\epsilon_i < 1$ and $\epsilon_i = \mathcal{O}(\epsilon)$
  - ✦ Consistent expansion at first order in slow-roll

$$\mathcal{H} = -\frac{1+\epsilon_1}{\eta} + \mathcal{O}(\epsilon^2) \Rightarrow \nu^2(\eta) = \frac{9}{4} + 3\epsilon_1(\eta) + \frac{3}{2}\epsilon_2(\eta) + \mathcal{O}(\epsilon^2)$$

  - ✦ Expanding Hubble flow functions around a particular time $\eta_\diamond$ $(N_\diamond)$

$$\left.\begin{aligned} \epsilon_i(N) &= \epsilon_i(N_\diamond) + (N - N_\diamond)\left.\frac{\mathrm{d}\epsilon_i}{\mathrm{d}N}\right|_{N_\diamond} + \dots \\ N - N_\diamond &= -(1+\epsilon_{1\diamond})\ln\left(\frac{\eta}{\eta_\diamond}\right) + \mathcal{O}(\epsilon^2) \end{aligned}\right\} \Rightarrow \epsilon_i(N) = \epsilon_i(N_\diamond) + \mathcal{O}(\epsilon^2)$$

- At first order (only) in slow-roll $\nu(\eta) = \nu_\diamond + \mathcal{O}(\epsilon^2)$ is constant

$$\nu_\diamond = \frac{9}{4} + 3\epsilon_{1\diamond} + \frac{3}{2}\epsilon_{2\diamond} \quad \Rightarrow \left\{ \begin{aligned} &\mu_S'' + \left(k^2 - \frac{\nu_\diamond^2 - 1/4}{\eta^2}\right)\mu_S = 0 \\ &\text{this is a Bessel equation} \end{aligned}\right.$$

Quantum initial conditions

- Canonical quantization of μ_{S} (and μ_{T}) + Bunch-Davies vacuum

$$\hat{\mu}(\eta, \mathbf{x}) = \int \frac{\mathrm{d}^3\mathbf{k}}{\sqrt{2k}} \left[c_{\mathbf{k}}(\eta_{\mathrm{ini}})\xi_k(\eta)\mathrm{e}^{i\mathbf{k}\mathbf{x}} + c_{\mathbf{k}}^{\dagger}(\eta_{\mathrm{ini}})\xi_k^*(\eta)\mathrm{e}^{-i\mathbf{k}\mathbf{x}} \right]$$

$$\xi_k(\eta) \underset{k\eta_{\mathrm{ini}}\to\infty}{\longrightarrow} 1 \times \mathrm{e}^{-ik(\eta-\eta_{\mathrm{ini}})} + 0 \times \mathrm{e}^{+ik(\eta-\eta_{\mathrm{ini}})}$$

- For each mode, we set the equivalent classical initial conditions

$$\mu_{\mathrm{TS}}(\eta_{\mathrm{ini}}) = \kappa\sqrt{2}\frac{1}{\sqrt{2k}}, \quad \mu'_{\mathrm{TS}}(\eta_{\mathrm{ini}}) = -i\kappa\sqrt{2}\sqrt{\frac{k}{2}}$$

- The solution is uniquely determined and depends on $\eta_\diamond$
  - ✦ $\eta_\diamond$ should be chosen for each mode k around Hubble exit: $k\eta_\diamond = -1$. Other choices are possible, for instance $k = a(\eta_\diamond)H(\eta_\diamond)$

- The power spectra are obtained in the super-Hubble limit: $k\eta \to 0$

Scalar primordial power spectrum

- In the super-Hubble limit $k\eta \to 0$ one gets a time-independent expression $(C \equiv \gamma + \ln 2 - 2)$

$$\mathcal{P}_\zeta(\eta_\diamond) = \frac{H_\diamond^2}{8\pi^2 M_{\mathrm{P}}^2 \epsilon_{1\diamond}} \left[1 - 2(C+1)\epsilon_{1\diamond} - C\epsilon_{2\diamond} + \mathcal{O}(\epsilon^2)\right]$$

- Dependency in k is hidden in the definition of $\eta_\diamond \equiv -1/k$
- Can be made explicit with a pivot expansion around k_*
  - ✦ For instance $k_* = 0.05\,\mathrm{Mpc}^{-1} \Rightarrow \eta_* = -1/k_*$
  - ✦ All $f_\diamond$ quantities can be slow-roll expanded around η_*

$$H_\diamond = H_* + (N_\diamond - N_*) \left.\frac{\mathrm{d}H}{\mathrm{d}N}\right|_{N_*} + \cdots = H_* \left(1 - \epsilon_{1*} \ln \frac{\eta_*}{\eta_\diamond}\right) + \mathcal{O}(\epsilon^2)$$

$$\epsilon_{1\diamond} = \epsilon_{1*} + \epsilon_{1*}\epsilon_{2*} \ln \frac{\eta_*}{\eta_\diamond} + \mathcal{O}(\epsilon^3)$$

- Pivot expanded scalar power spectrum

$$\mathcal{P}_\zeta(k) = \frac{H_*^2}{8\pi^2 M_{\mathrm{P}}^2 \epsilon_{1*}} \left[1 - 2(C+1)\epsilon_{1*} - C\epsilon_{2*} - (2\epsilon_{1*} + \epsilon_{2*}) \ln\left(\frac{k}{k_*}\right)\right]$$

Primordial power spectrum

- At second order, after pivot expansion, one gets

$$
\begin{aligned}
\mathcal{P}_\zeta = \frac{H_*^2}{8\pi^2 M_{\rm P}^2 \epsilon_{1*}} \Bigg\{ & 1 - 2(1+C)\epsilon_{1*} - C\epsilon_{2*} + \left(\frac{\pi^2}{2} - 3 + 2C + 2C^2\right)\epsilon_{1*}^2 \\
& + \left(\frac{7\pi^2}{12} - 6 - C + C^2\right)\epsilon_{1*}\epsilon_{2*} + \left(\frac{\pi^2}{8} - 1 + \frac{C^2}{2}\right)\epsilon_{2*}^2 + \left(\frac{\pi^2}{24} - \frac{C^2}{2}\right)\epsilon_{2*}\epsilon_{3*} \\
& + \Big[-2\epsilon_{1*} - \epsilon_{2*} + (2+4C)\epsilon_{1*}^2 + (-1+2C)\epsilon_{1*}\epsilon_{2*} + C\epsilon_{2*}^2 - C\epsilon_{2*}\epsilon_{3*}\Big] \ln\left(\frac{k}{k_*}\right) \\
& + \left[2\epsilon_{1*}^2 + \epsilon_{1*}\epsilon_{2*} + \frac{1}{2}\epsilon_{2*}^2 - \frac{1}{2}\epsilon_{2*}\epsilon_{3*}\right] \ln^2\left(\frac{k}{k_*}\right) \Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_h = \frac{2H_*^2}{\pi^2 M_{\rm P}^2} \Bigg\{ & 1 - 2(1+C)\epsilon_{1*} + \left[-3 + \frac{\pi^2}{2} + 2C + 2C^2\right]\epsilon_{1*}^2 + \left[-2 + \frac{\pi^2}{12} - 2C - C^2\right]\epsilon_{1*}\epsilon_{2*} \\
& + \Big[-2\epsilon_{1*} + (2+4C)\epsilon_{1*}^2 + (-2-2C)\epsilon_{1*}\epsilon_{2*}\Big] \ln\left(\frac{k}{k_*}\right) + \left(2\epsilon_{1*}^2 - \epsilon_{1*}\epsilon_{1*}\right) \ln^2\left(\frac{k}{k_*}\right) \Bigg\}
\end{aligned}
$$

Power law parameters

- Amplitude and spectral indices: $n_{\mathrm{T}} \equiv \left.\frac{\mathrm{d}\ln \mathcal{P}_h}{\mathrm{d}\ln k}\right|_{k_*}$, $n_{\mathrm{S}} - 1 \equiv \left.\frac{\mathrm{d}\ln \mathcal{P}_\zeta}{\mathrm{d}\ln k}\right|_{k_*}$

$$P_* = \mathcal{P}_\zeta(k_*), \quad n_{\mathrm{T}} = -2\epsilon_{1*} - 2\epsilon_{1*}^2 - 2(1+C)\epsilon_{1*}\epsilon_{2*} + \mathcal{O}\left(\epsilon^3\right)$$

$$n_{\mathrm{S}} = 1 - (2\epsilon_{1*} + \epsilon_{2*}) - 2\epsilon_{1*}^2 - (3+2C)\epsilon_{1*}\epsilon_{2*} - C\epsilon_{2*}\epsilon_{3*} + \mathcal{O}\left(\epsilon^3\right)$$

- Running of the spectral index: $\alpha \equiv \left.\frac{\mathrm{d}^2 \ln \mathcal{P}}{\mathrm{d}(\ln k)^2}\right|_{k_*}$

$$\alpha_{\mathrm{S}} = -2\epsilon_{1*}\epsilon_{2*} - \epsilon_{2*}\epsilon_{3*} + \mathcal{O}\left(\epsilon^3\right), \qquad \alpha_{\mathrm{T}} = -2\epsilon_{1*}\epsilon_{2*} + \mathcal{O}\left(\epsilon^3\right)$$

- Tensor-to-scalar ratio $r \equiv \frac{\mathcal{P}_\zeta(k_*)}{\mathcal{P}_h(k_*)} = 16\epsilon_{1*}(1 + C\epsilon_{2*}) + \mathcal{O}\left(\epsilon^3\right)$

- Running of the running: $\beta \equiv \left.\frac{\mathrm{d}^3 \ln \mathcal{P}}{\mathrm{d}(\ln k)^3}\right|_{k_*} = \mathcal{O}\left(\epsilon^3\right)$

$$\beta_{\mathrm{T}} = -2\epsilon_{1*}\epsilon_{2*}\left(\epsilon_{2*} + \epsilon_{3*}\right) + \mathcal{O}\left(\epsilon^4\right)$$

Observable quantities during inflation

- All quantities entering $\mathcal{P}(k)$ are evaluated at η_* such that $k_*\eta_* = -1$
  - Hubble flow functions: $\epsilon_{i*} = \epsilon_i(\phi_*)$ where $\eta(\phi_*) = -1/k_*$

- At leading order in slow-roll: $k_* = a_* H_* = e^{N_* - N_{\mathrm{end}}} \dfrac{a_{\mathrm{end}}}{a_0} a_0 H_*$

$$\frac{k_*}{a_0} = \frac{e^{\Delta N_*}}{1+z_{\mathrm{end}}} H_* = e^{\Delta N_*} R_{\mathrm{rad}} \left(\frac{\rho_{\mathrm{end}}}{\tilde{\rho}_\gamma}\right)^{-1/4} \left(\frac{H_*}{\sqrt{\epsilon_{1*}}}\right) \sqrt{\epsilon_{1*}}$$

- This is a non-trivial integral equation: $\rho_{\mathrm{end}}(\phi_*)$ through M^4
  - FL equation: $\rho_{\mathrm{end}} = 3H^2_{\mathrm{end}} = \dfrac{3V_{\mathrm{end}}}{3-\epsilon_{1\mathrm{end}}} = 3\epsilon_{1*} \dfrac{H^2_*}{\epsilon_{1*}} \dfrac{V_{\mathrm{end}}}{V_*} \dfrac{3-\epsilon_{1*}}{3-\epsilon_{1\mathrm{end}}}$
  - Defining $N_0 \equiv \ln\left(\dfrac{k_*}{a_0}\dfrac{1}{\tilde{\rho}_\gamma^{1/4}}\right)$ (number of e-folds of deceleration)

$$\Delta N_* = -\ln R_{\mathrm{rad}} + N_0 + \frac{1}{4}\ln\left(\frac{3}{\epsilon_{1*}}\frac{V_{\mathrm{end}}}{V_*}\frac{3-\epsilon_{1*}}{3-\epsilon_{1\mathrm{end}}}\right) - \frac{1}{4}\ln\left(\frac{H^2_*}{\epsilon_{1*}}\right)$$

Solving for the time of pivot crossing

- Depends on: model + how inflation ends + reheating + data

$$-\left[\int_{\phi_{\mathrm{end}}}^{\phi_*}\frac{V(\psi)}{V'(\psi)}\mathrm{d}\psi\right] = \ln R_{\mathrm{rad}} - N_0 + \frac{1}{4}\ln(8\pi^2 P_*) - \frac{1}{4}\ln\left\{\frac{9}{\epsilon_1(\phi_*)[3-\epsilon_1(\phi_{\mathrm{end}})]}\frac{V(\phi_{\mathrm{end}})}{V(\phi_*)}\right\}$$

- The rescaled reheating parameter: $\ln R_{\mathrm{reh}} \equiv \ln R_{\mathrm{rad}} + \frac{1}{4}\ln\rho_{\mathrm{end}}$

$$-\left[\int_{\phi_{\mathrm{end}}}^{\phi_*}\frac{V(\psi)}{V'(\psi)}\mathrm{d}\psi\right] = \ln R_{\mathrm{reh}} - N_0 - \frac{1}{2}\ln\left\{\frac{9}{3-\epsilon_1(\phi_{\mathrm{end}})}\frac{V(\phi_{\mathrm{end}})}{V(\phi_*)}\right\}$$

- Assuming $-1/3 < \bar{w}_{\mathrm{reh}} < 1$ and $\rho_{\mathrm{nuc}} \equiv (10\,\mathrm{MeV})^4 < \rho_{\mathrm{reh}} < \rho_{\mathrm{end}} < 1$

$$-46 < \ln R_{\mathrm{reh}} < 15 + \frac{1}{3}\ln\rho_{\mathrm{end}}$$

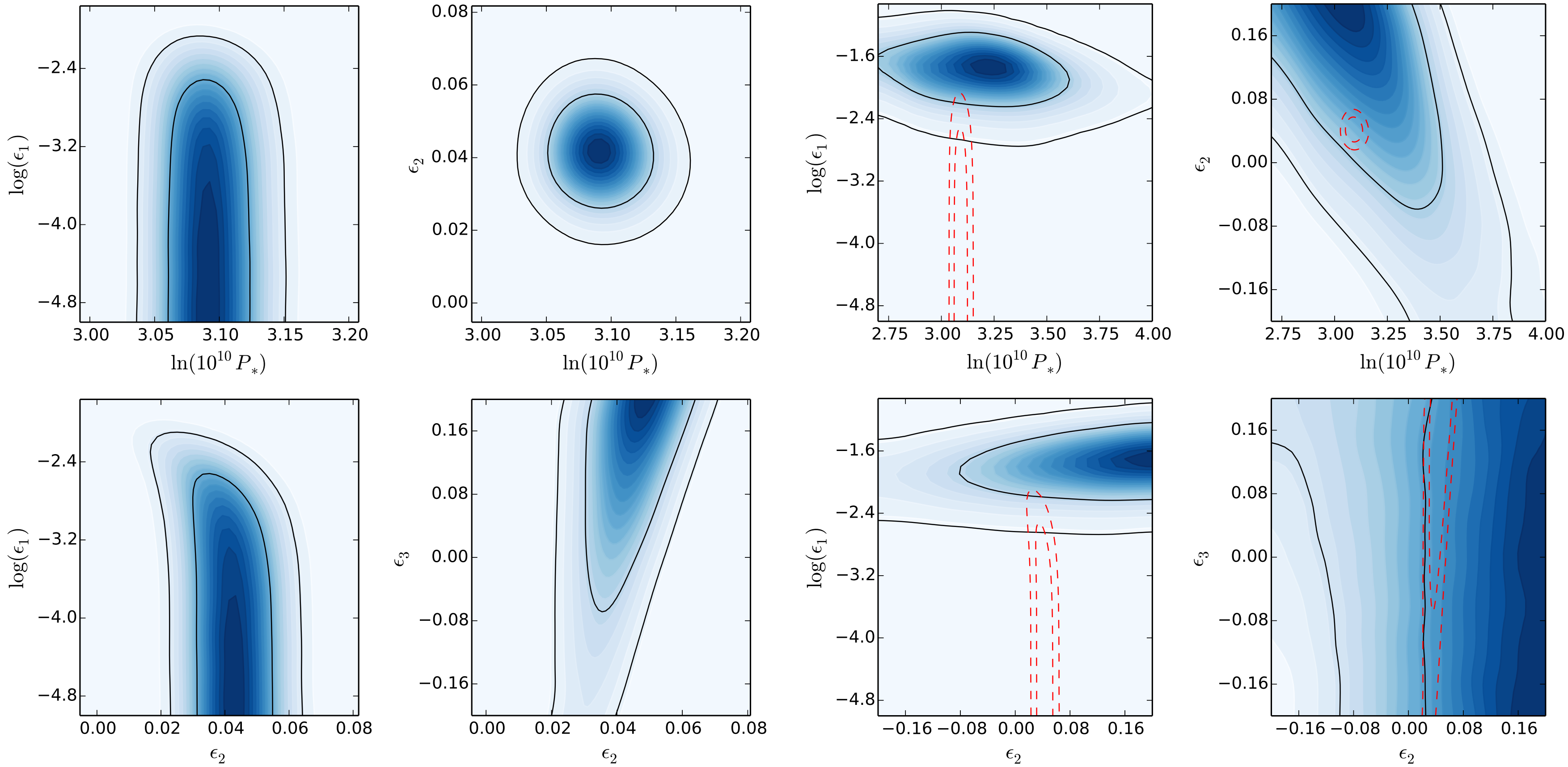
Comparison with observations

Outline

Comparison with observations

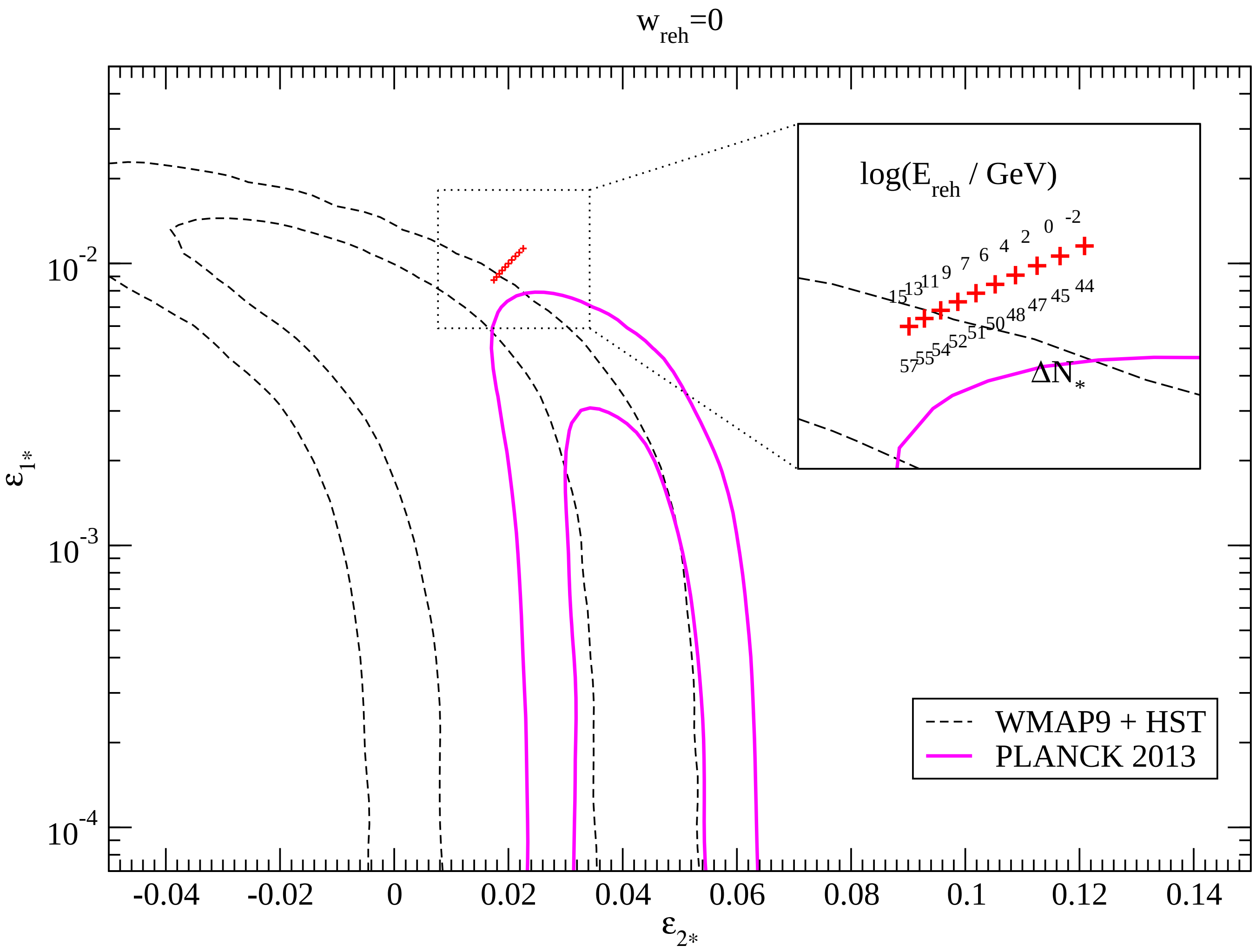
Constraints on the slow-roll parameters

- From the slow-roll expanded expression of $\mathcal{P}_\zeta(k)$ and $\mathcal{P}_h(k)$
  - Constraints on ϵ_{i*} and P_* (or H_*^2/ϵ_{1*})
  - Example from Planck 2013 and BICEP2



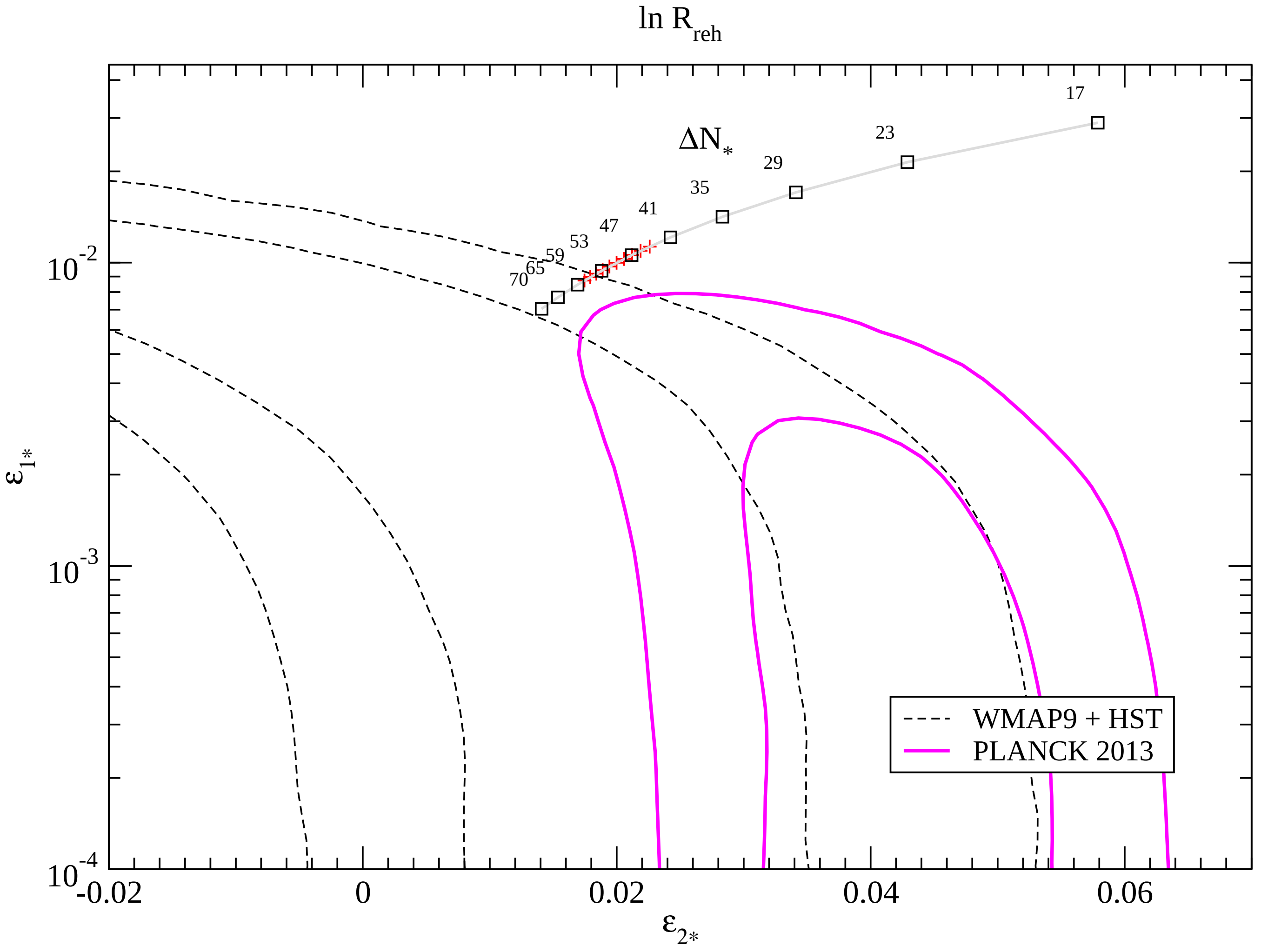
Comparison with model predictions

- Can only be done from the input of $R_{\rm reh}$, or $R_{\rm rad}$, or $(\bar{w}_{\rm reh}, \rho_{\rm reh})$
  - One can scan various reheating histories: ΔN_* is not arbitrary!
  - Example: LFI_2 with $\bar{w}_{\rm reh} = 0$ and $\rho_{\rm nuc} < \rho_{\rm reh} < \rho_{\rm end}$



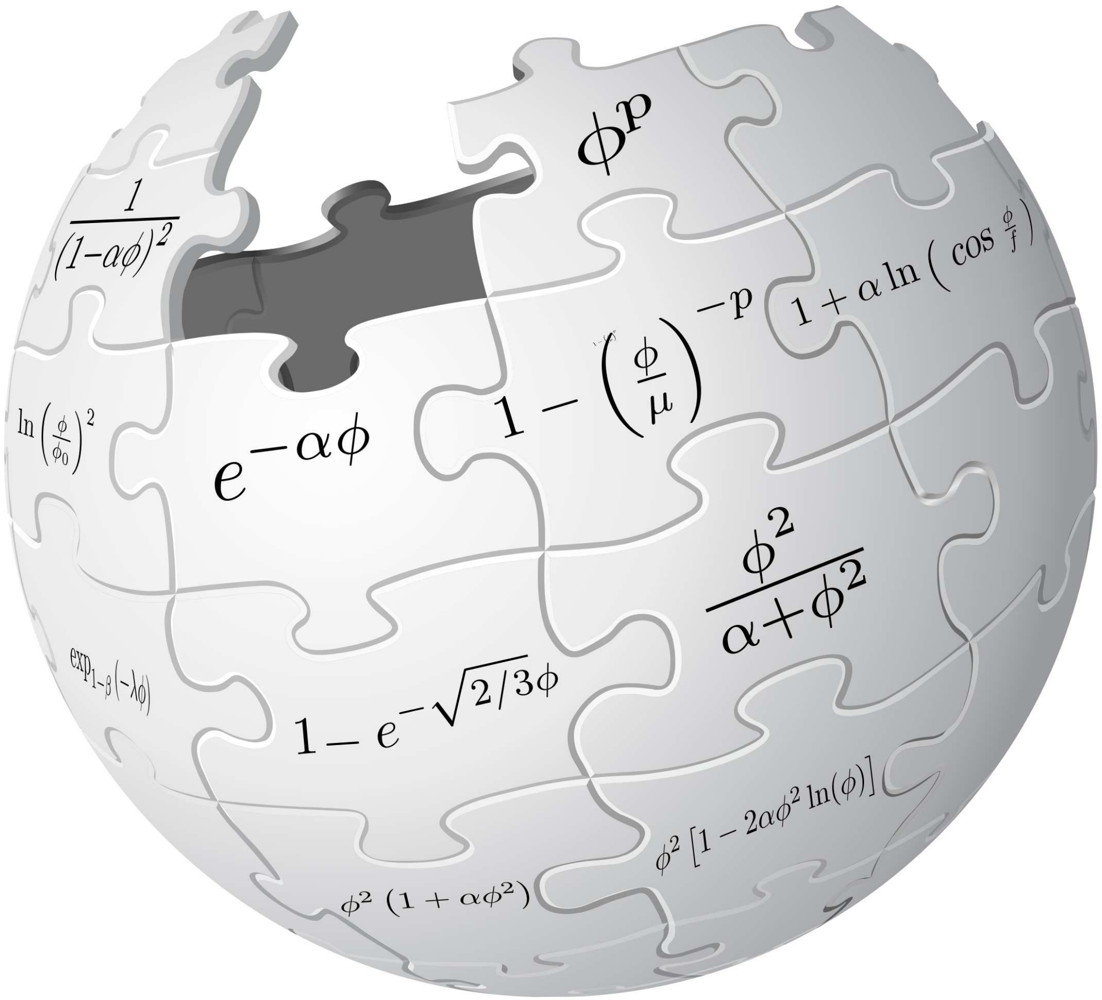
Most generic reheating parametrization

- In the abscence of any information on the reheating, one should use R_{reh} (or R_{rad})
- Same example: LFI_2 without assuming $\bar{w}_{\mathrm{reh}} = 0$



The Encyclopædia

- With J. Martin and V. Vennin



http://arxiv.org/abs/1303.3787
http://cp3.irmp.ucl.ac.be/~ringeval/aspic.html

Purpose

- Quasi-exhaustive analysis to derive reheating consistent observable predictions for all slow-roll single-field inflationary models
- Comes with a public code (`ASPIC`)
- Currently supports more than 50 motivated classes of potential

Name	Parameters	Sub-models	$V(\phi)$
HI	0	1	$M^4\left(1-e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)$
RCHI	1	1	$M^4\left(1-2e^{-\sqrt{2/3}\phi/M_{\rm Pl}}+\frac{A_{\rm I}}{16\pi^2}\frac{\phi}{\sqrt{6}M_{\rm Pl}}\right)$
LFI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^p$
MLFI	1	1	$M^4\frac{\phi^2}{M_{\rm Pl}^2}\left[1+\alpha\frac{\phi^2}{M_{\rm Pl}^2}\right]$
RCMI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^2\left[1-2\alpha\frac{\phi^2}{M_{\rm Pl}^2}\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
RCQI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^4\left[1-\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
NI	1	1	$M^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$
ESI	1	1	$M^4\left(1-e^{-q\phi/M_{\rm Pl}}\right)$
PLI	1	1	$M^4e^{-\alpha\phi/M_{\rm Pl}}$
KMII	1	2	$M^4\left(1-\alpha\frac{\phi}{M_{\rm Pl}}e^{-\phi/M_{\rm Pl}}\right)$
HF1I	1	1	$M^4\left(1+A_1\frac{\phi}{M_{\rm Pl}}\right)^2\left[1-\frac{2}{3}\left(\frac{A_1}{1+A_1\phi/M_{\rm Pl}}\right)^2\right]$
CWI	1	1	$M^4\left[1+\alpha\left(\frac{\phi}{Q}\right)^4\ln\left(\frac{\phi}{Q}\right)\right]$
LI	1	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
RpI	1	3	$M^4e^{-2\sqrt{2/3}\phi/M_{\rm Pl}}\left\lvert e^{\sqrt{2/3}\phi/M_{\rm Pl}}-1\right\rvert^{2p/(2p-1)}$
DWI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-1\right]^2$
MHI	1	1	$M^4\left[1-\mathrm{sech}\left(\frac{\phi}{\mu}\right)\right]$
RGI	1	1	$M^4\frac{(\phi/M_{\rm Pl})^2}{\alpha+(\phi/M_{\rm Pl})^2}$
MSSMI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\left(\frac{\phi}{\phi_0}\right)^6+\frac{1}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
RIPI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\left(\frac{\phi}{\phi_0}\right)^3+\frac{1}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
AI	1	1	$M^4\left[1-\frac{2}{\pi}\arctan\left(\frac{\phi}{\mu}\right)\right]$
CNAI	1	1	$M^4\left[3-\left(3+\alpha^2\right)\tanh^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)\right]$
CNBI	1	1	$M^4\left[\left(3-\alpha^2\right)\tan^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)-3\right]$
OSTI	1	1	$-M^4\left(\frac{\phi}{\phi_0}\right)^2\ln\left[\left(\frac{\phi}{\phi_0}\right)^2\right]$
WRI	1	1	$M^4\ln\left(\frac{\phi}{\phi_0}\right)^2$
SFI	2	1	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^p\right]$

– 15 –

II	2	1	$M^4\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)^{-\beta}-M^4\frac{\beta^2}{6}\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)^{-\beta-2}$
KMIII	2	1	$M^4\left[1-\alpha\frac{\phi}{M_{\rm Pl}}\exp\left(-\beta\frac{\phi}{M_{\rm Pl}}\right)\right]$
LMI	2	2	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^\alpha\exp\left[-\beta(\phi/M_{\rm Pl})^\gamma\right]$
TWI	2	1	$M^4\left[1-A\left(\frac{\phi}{\phi_0}\right)^2e^{-\phi/\phi_0}\right]$
GMSSMI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^6+\frac{\alpha}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
GRIPI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^3+\frac{\alpha}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
BSUSYBI	2	1	$M^4\left(e^{\sqrt{6}\frac{\phi}{M_{\rm Pl}}}+e^{\sqrt{6}\gamma\frac{\phi}{M_{\rm Pl}}}\right)$
TI	2	3	$M^4\left(1+\cos\frac{\phi}{\mu}+\alpha\sin^2\frac{\phi}{\mu}\right)$
BEI	2	1	$M^4\exp_{1-\beta}\left(-\lambda\frac{\phi}{M_{\rm Pl}}\right)$
PSNI	2	1	$M^4\left[1+\alpha\ln\left(\cos\frac{\phi}{f}\right)\right]$
NCKI	2	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)+\beta\left(\frac{\phi}{M_{\rm Pl}}\right)^2\right]$
CSI	2	1	$\frac{M^4}{\left(1-\alpha\frac{\phi}{M_{\rm Pl}}\right)^2}$
OI	2	1	$M^4\left(\frac{\phi}{\phi_0}\right)^4\left[\left(\ln\frac{\phi}{\phi_0}\right)^2-\alpha\right]$
CNCI	2	1	$M^4\left[\left(3+\alpha^2\right)\coth^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)-3\right]$
SBI	2	2	$M^4\left\{1+\left[-\alpha+\beta\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]\left(\frac{\phi}{M_{\rm Pl}}\right)^4\right\}$
SSBI	2	6	$M^4\left[1+\alpha\left(\frac{\phi}{M_{\rm Pl}}\right)^2+\beta\left(\frac{\phi}{M_{\rm Pl}}\right)^4\right]$
IMI	2	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^{-p}$
BI	2	2	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^{-p}\right]$
RMI	3	4	$M^4\left[1-\frac{c}{2}\left(-\frac{1}{2}+\ln\frac{\phi}{\phi_0}\right)\frac{\phi^2}{M_{\rm Pl}^2}\right]$
VHI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^p\right]$
DSI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^{-p}\right]$
GMLFI	3	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^p\left[1+\alpha\left(\frac{\phi}{M_{\rm Pl}}\right)^q\right]$
LPI	3	3	$M^4\left(\frac{\phi}{\phi_0}\right)^p\left(\ln\frac{\phi}{\phi_0}\right)^q$
CNDI	3	3	$\frac{M^4}{\left\{1+\beta\cos\left[\alpha\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)\right]\right\}^2}$

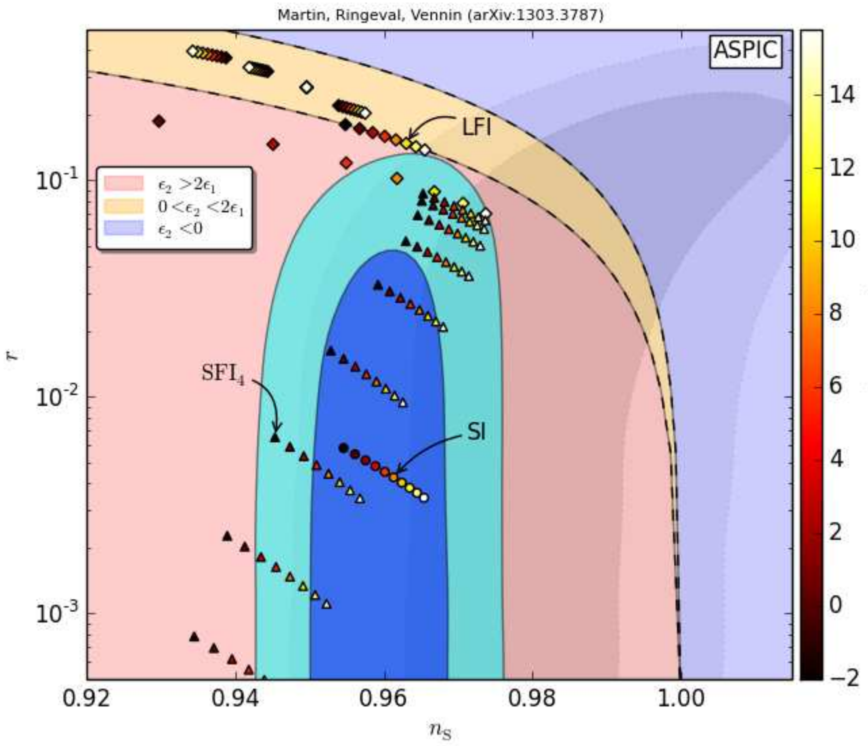
– 16 –

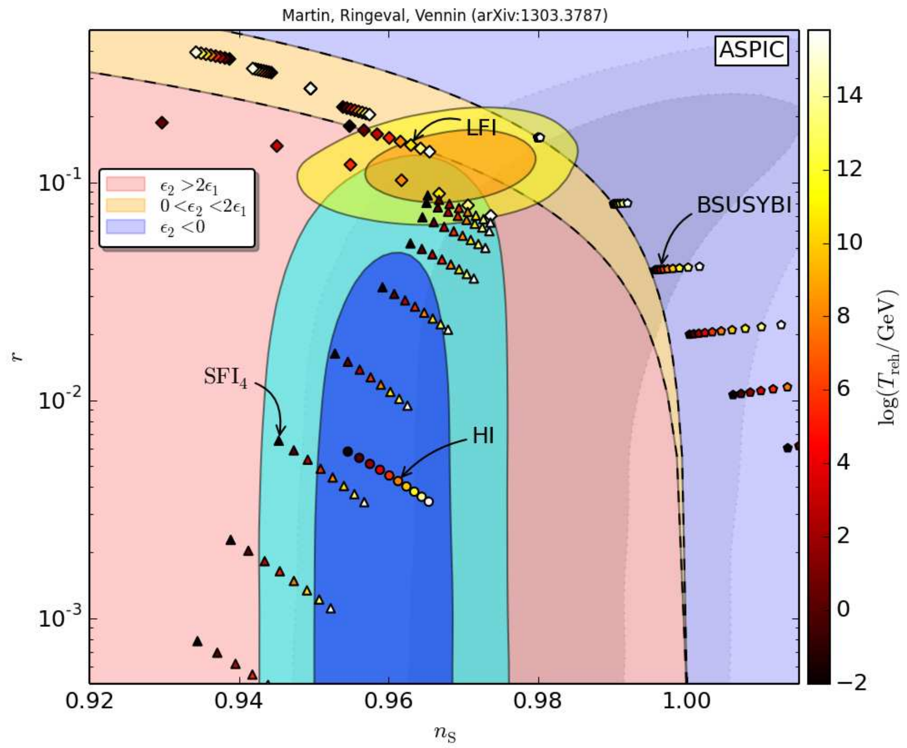
Model predictions with `ASPIC`

- For all *Encyclopædia Inflationaris* models

  $$\text{potential parameters} + \text{reheating} \longrightarrow \epsilon_{i*} \longrightarrow n_{\mathrm{S}},\, r,\, \alpha_{\mathrm{S}} \ldots \text{(with consistency relations)}$$

- Easy to check for which reheating history a model is compatible with the data

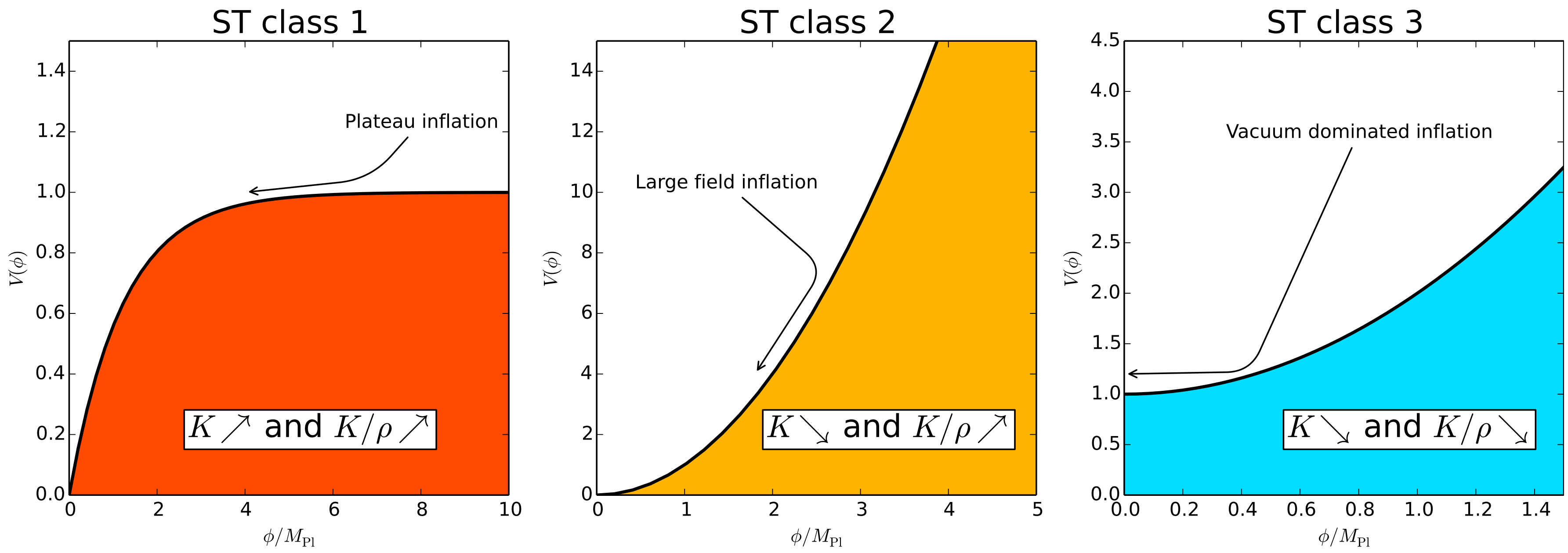




Schwarz Terrero-Escalante classification

- Based on the relative energy evolution at the pivot scale (ϕ_*)

$$K = \frac{1}{2}\dot{\phi}^2 \qquad \rho = K + V \qquad P = K - V \simeq -\rho$$



- In terms of slow-roll parameters

$$\mathrm{ST1:}\ \epsilon_{2*} > 2\epsilon_{1*}, \quad \mathrm{ST2:}\ 0 < \epsilon_{2*} < 2\epsilon_{1*}, \quad \mathrm{ST3:}\ \epsilon_{2*} < 0$$

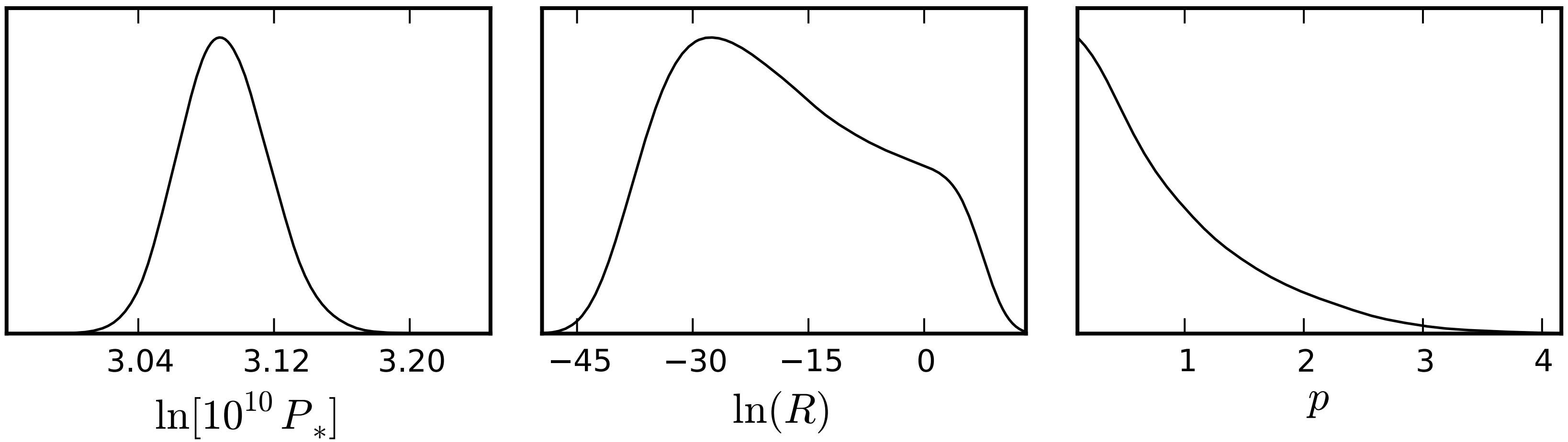
- This is not exactly the color of $\mathcal{P}_\zeta$: $n_{\mathrm{S}} - 1 = -2\epsilon_{1*} - \epsilon_{2*} + \mathcal{O}(\epsilon^2)$

Using the slow-roll approximation as a proxy

- To constrain the fundamental inflationary parameters: $\boldsymbol{\theta}_{\mathrm{inf}}$

$$(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}) \longrightarrow \texttt{ASPIC} \longrightarrow \epsilon_{i*} \longrightarrow \begin{cases} \mathcal{P}_\zeta(k) \\ \mathcal{P}_h(k) \end{cases} \longrightarrow \texttt{CAMB} \longleftrightarrow \text{CMB data}$$

- Example: Planck 2013 data analysis with LFI

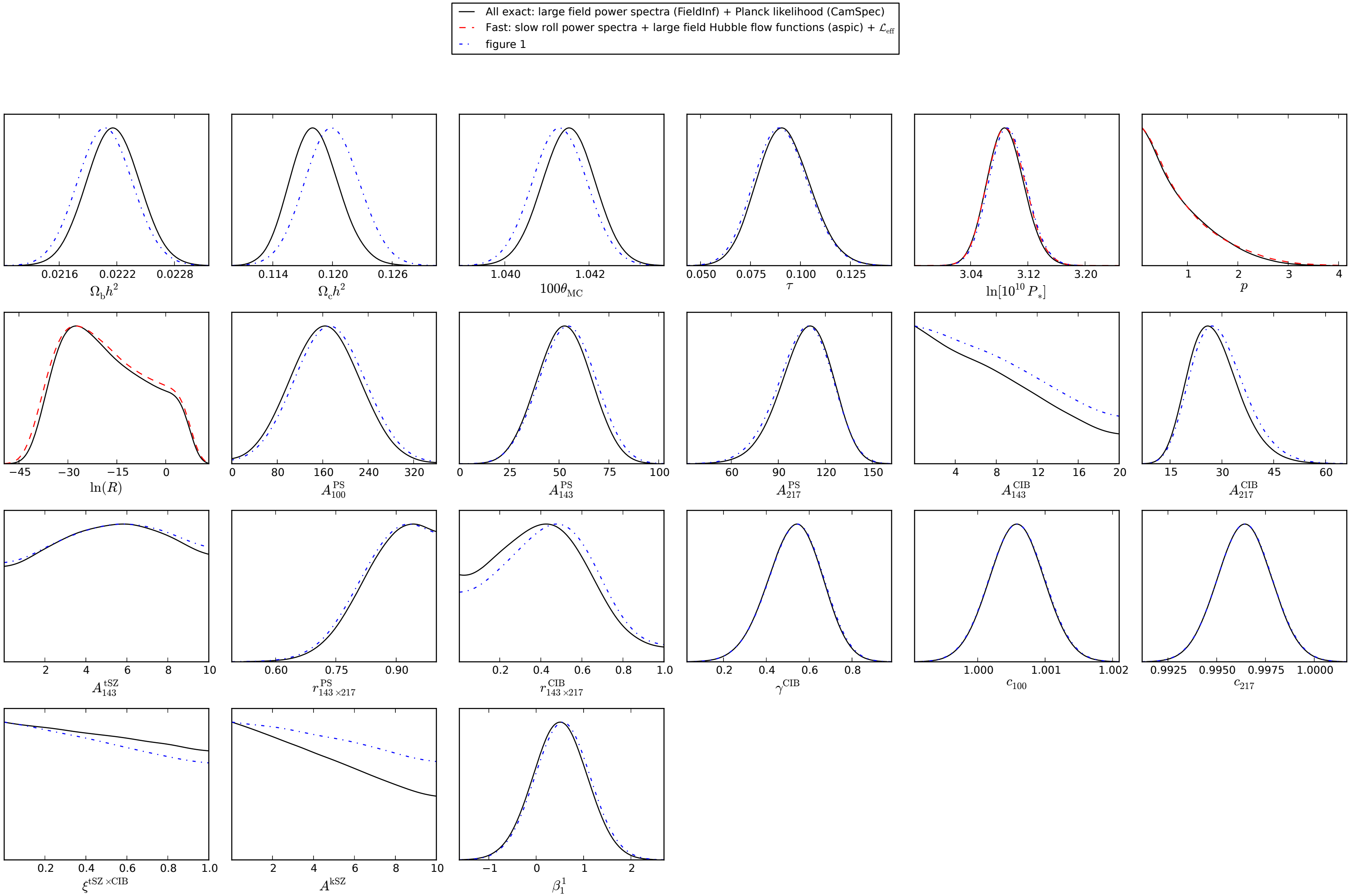


- Confidence intervals are on the relevant parameters (95% CL)

$$p < 2.3, \qquad -37 < \ln R_{\mathrm{reh}} < 6$$

Accuracy of the slow-roll approximation

- First order quantities marginalized over second order



Bayesian model comparison

- Bayesian evidence
  - For each model $\mathcal{M}$, marginalisation over all parameters

$$\mathcal{E}(D|\mathcal{M}) = \int \mathrm{d}\boldsymbol{\theta}\mathcal{L}(\boldsymbol{\theta})\pi(\boldsymbol{\theta}|\mathcal{M})$$

  - Gives the posterior probability of $\mathcal{M}$ to explain the data D

$$p(\mathcal{M}|D) = \frac{\mathcal{E}(D|\mathcal{M})\pi(\mathcal{M})}{p(D)} \quad \textit{where} \quad p(D) = \sum_i \mathcal{E}(\mathcal{M}_i|D)\pi(\mathcal{M}_i)$$

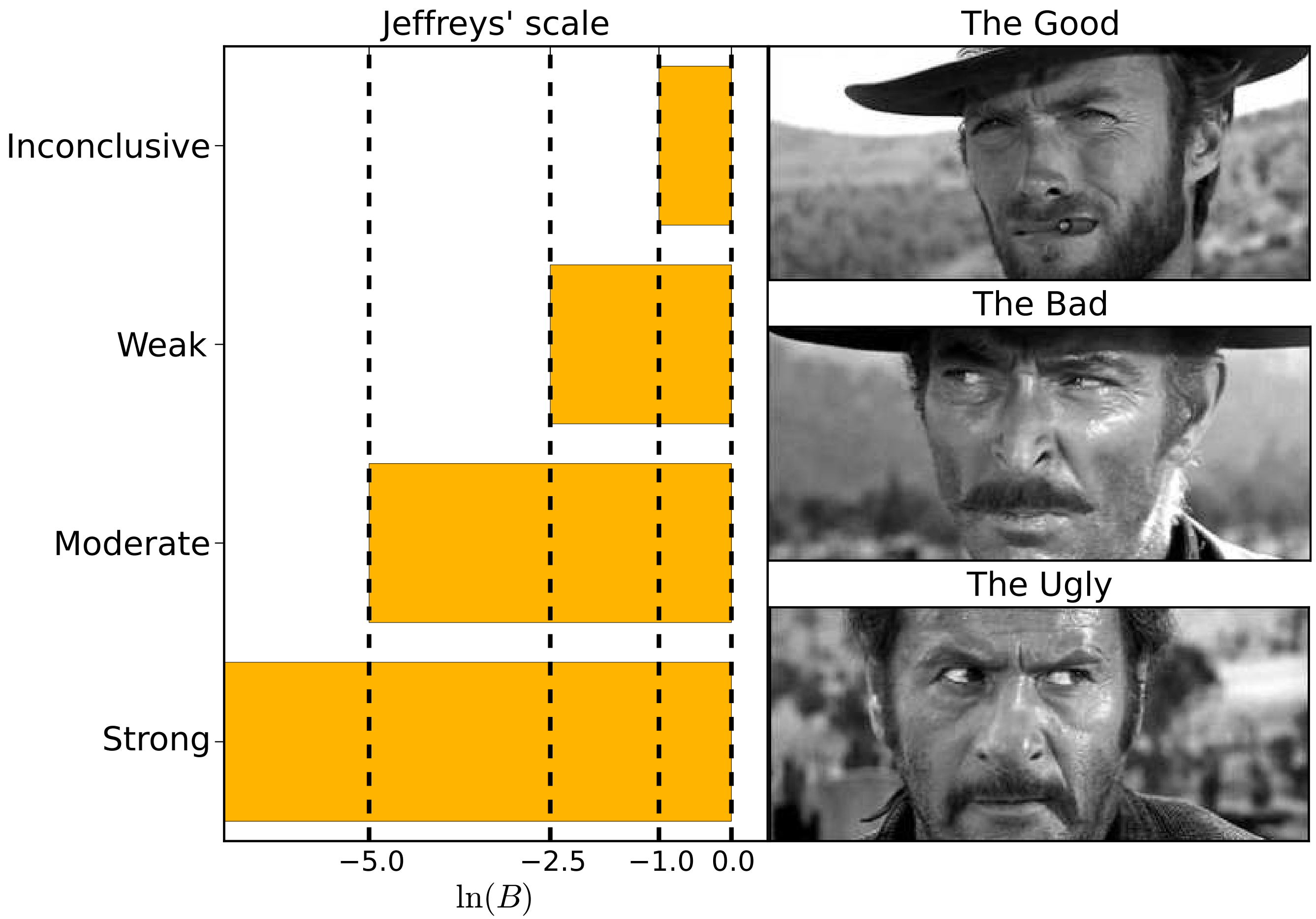
- Bayes' factor
  - Gives the posterior odds between $\mathcal{M}$ and a reference model $\mathcal{M}_0$

$$\frac{p(\mathcal{M}|D)}{p(\mathcal{M}_0|D)} = B\frac{\pi(\mathcal{M})}{\pi(\mathcal{M}_0)} \Rightarrow B = \frac{\mathcal{E}(D|\mathcal{M})}{\mathcal{E}(D|\mathcal{M}_0)}$$

  - Measure of how much the prior information has been updated

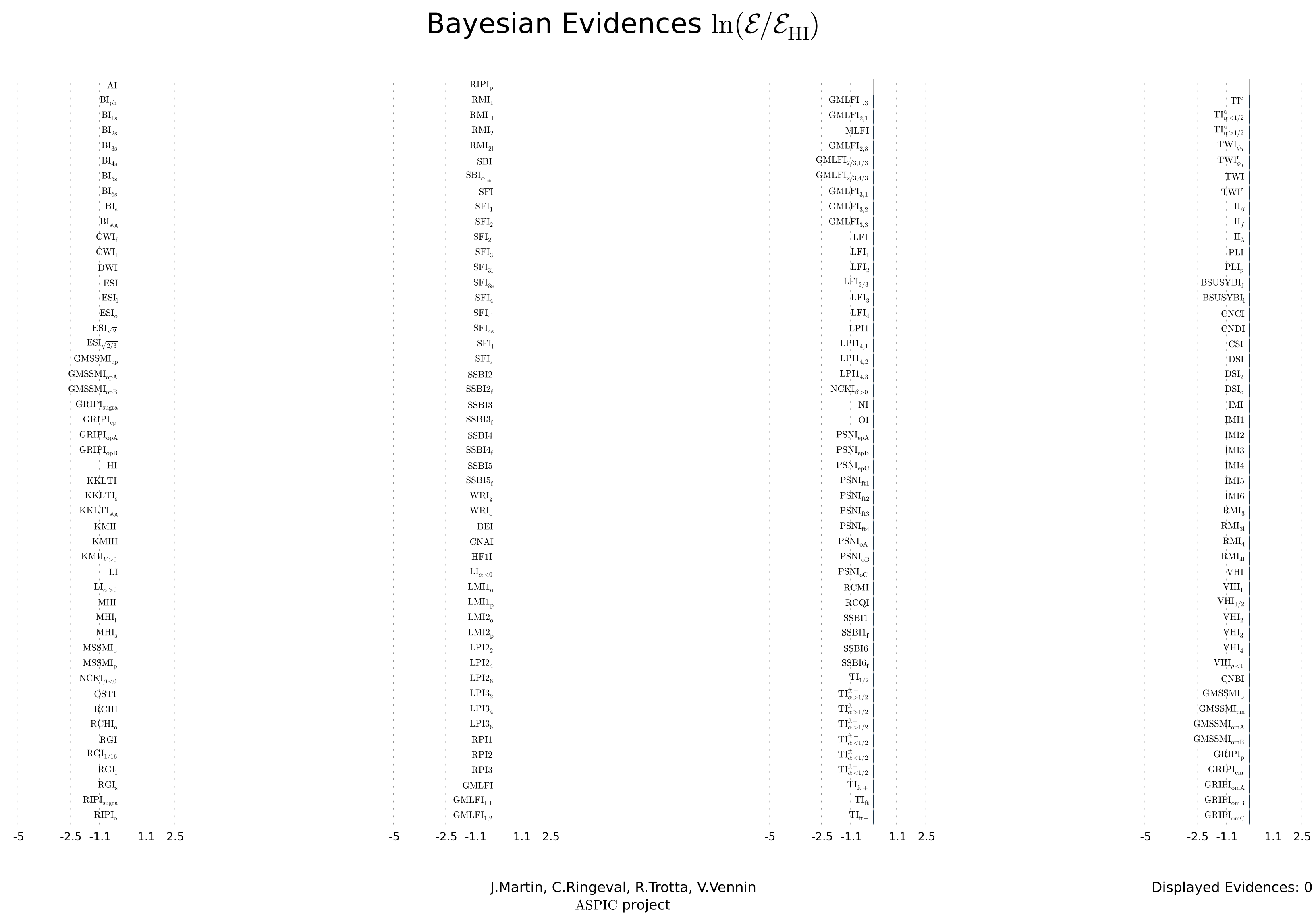
Jeffreys' scale

- Strength of evidence of $\mathcal{M}$ compared to $\mathcal{M}_0$



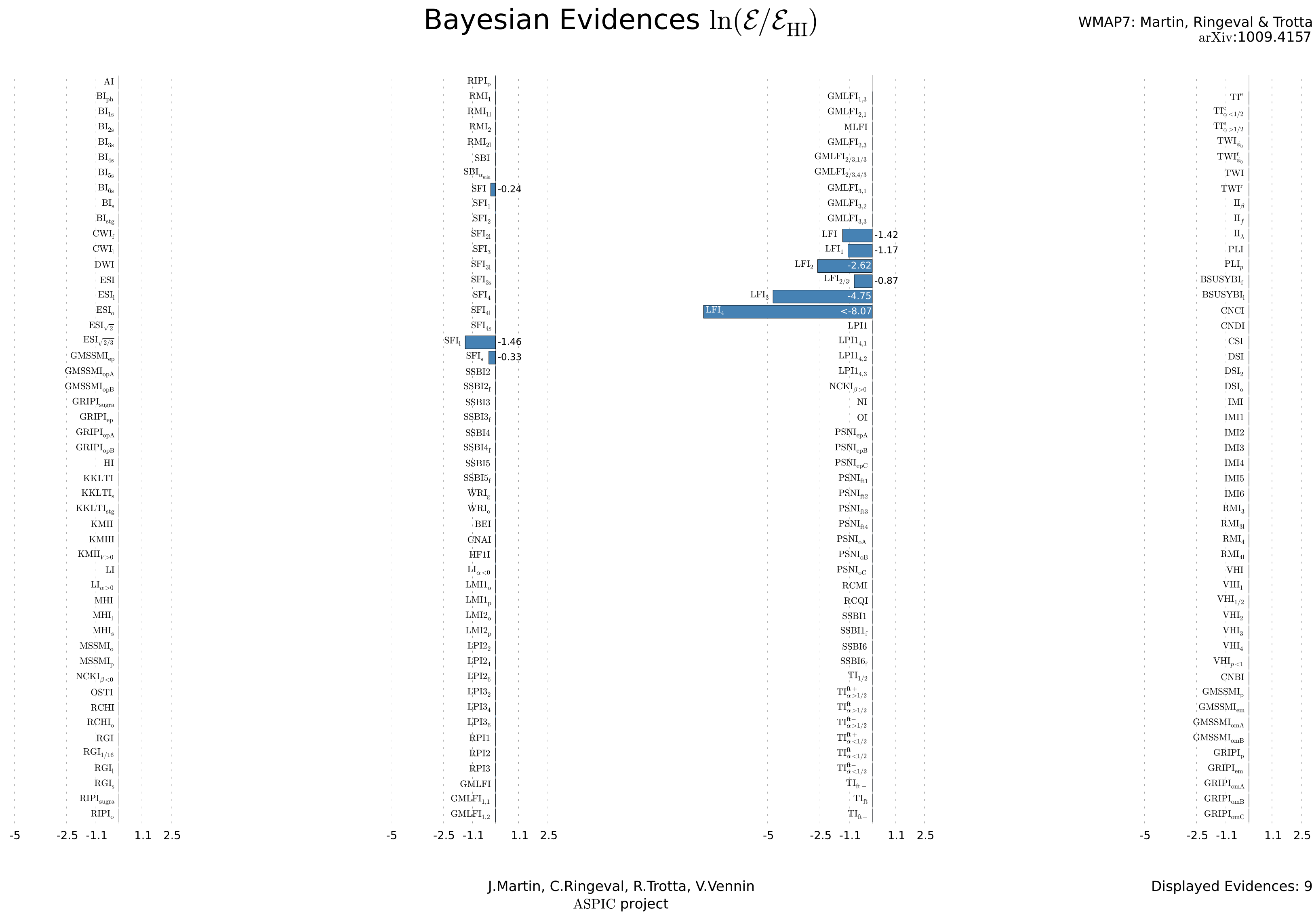
- ASPIC allows to fastly do that for all the *Encyclopædia Inflationaris* models

Bayes factor for hundred of models



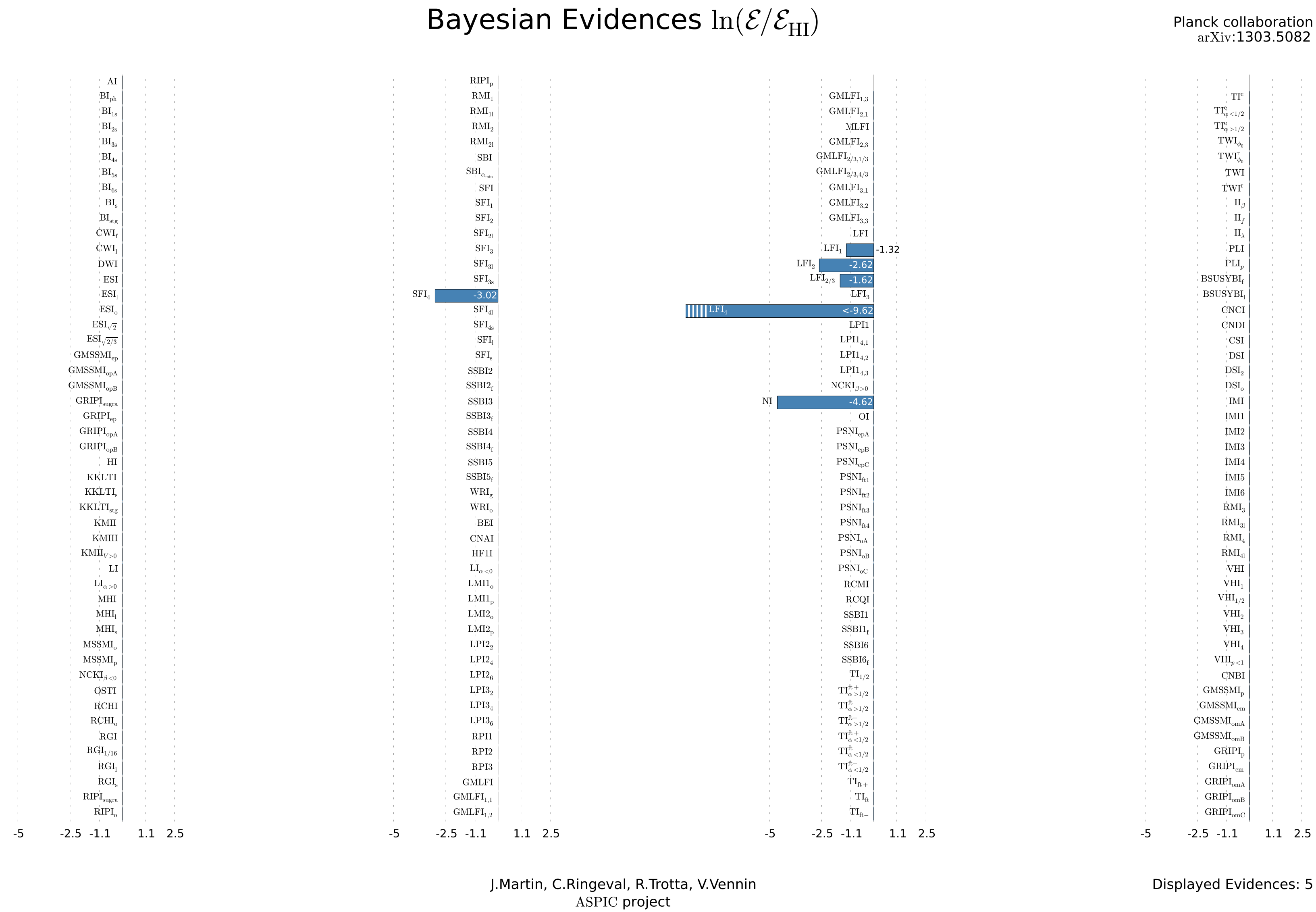
Bayes factor for hundred of models

WMAP7



Bayes factor for hundred of models

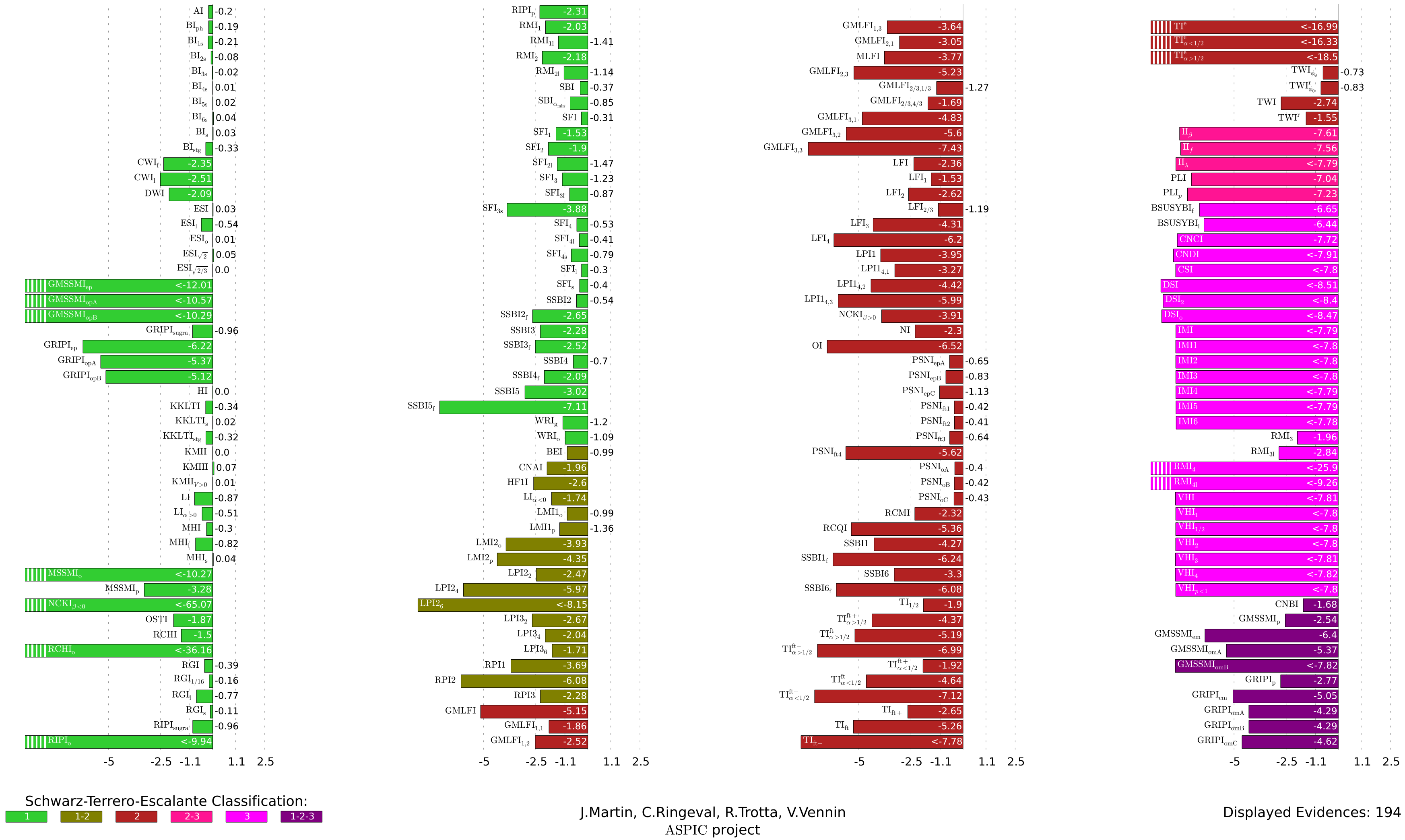
PLANCK



Bayes factor for hundred of models

PLANCK

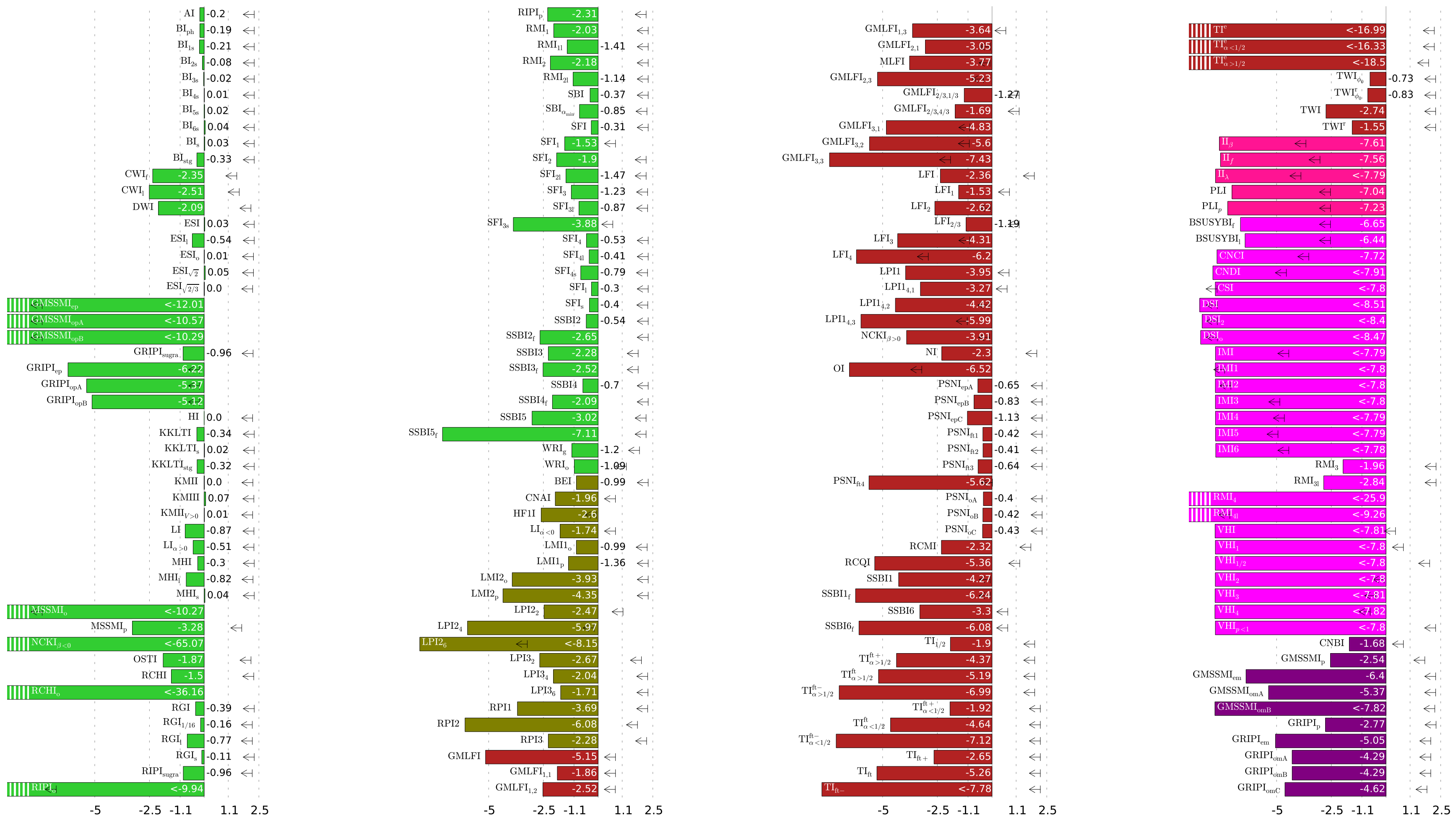
Bayesian Evidences $\ln(\mathcal{E}/\mathcal{E}_{\mathrm{HI}})$



Bayes factor for hundred of models

PLANCK

Bayesian Evidences $\ln(\mathcal{E}/\mathcal{E}_{\mathrm{HI}})$ and $\ln(\mathcal{L}_{\max}/\mathcal{E}_{\mathrm{HI}})$



Schwarz-Terrero-Escalante Classification: 1 | 1-2 | 2 | 2-3 | 3 | 1-2-3

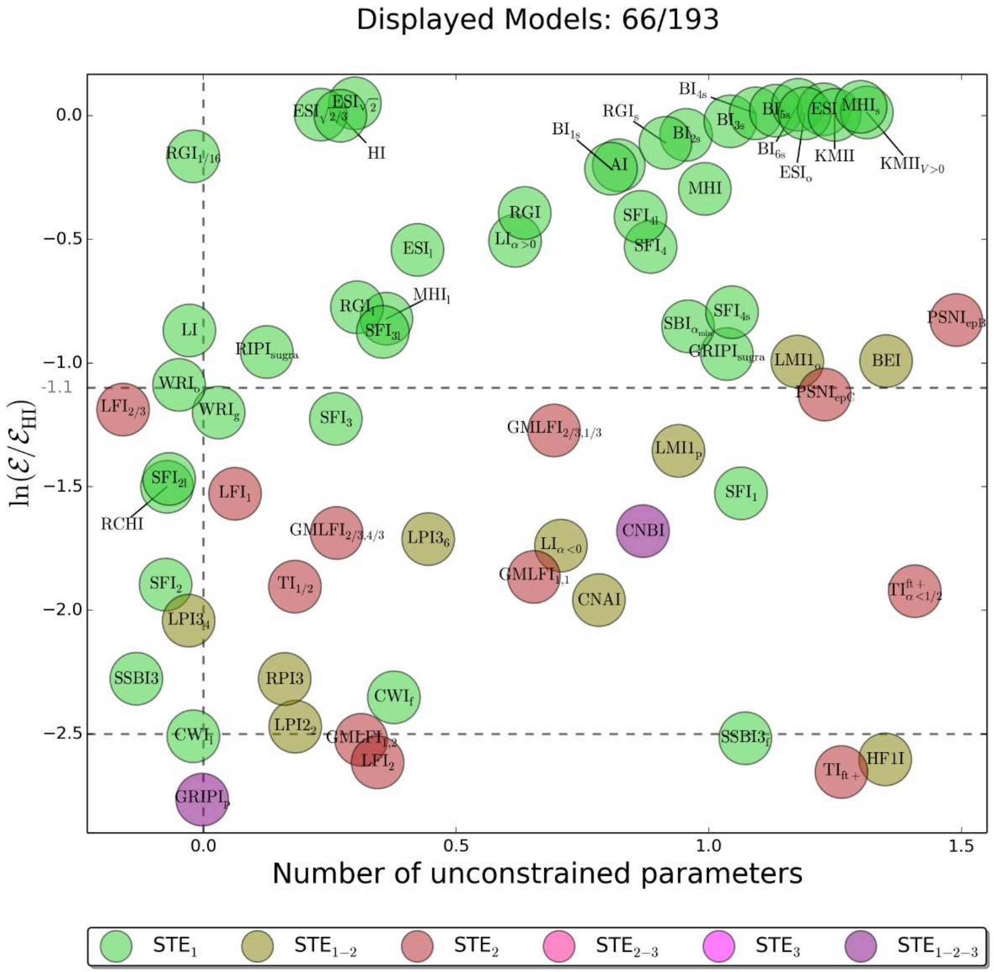
J.Martin, C.Ringeval, R.Trotta, V.Vennin
ASPIC project

Displayed Evidences: 194

Narrowing down the simplest with complexity

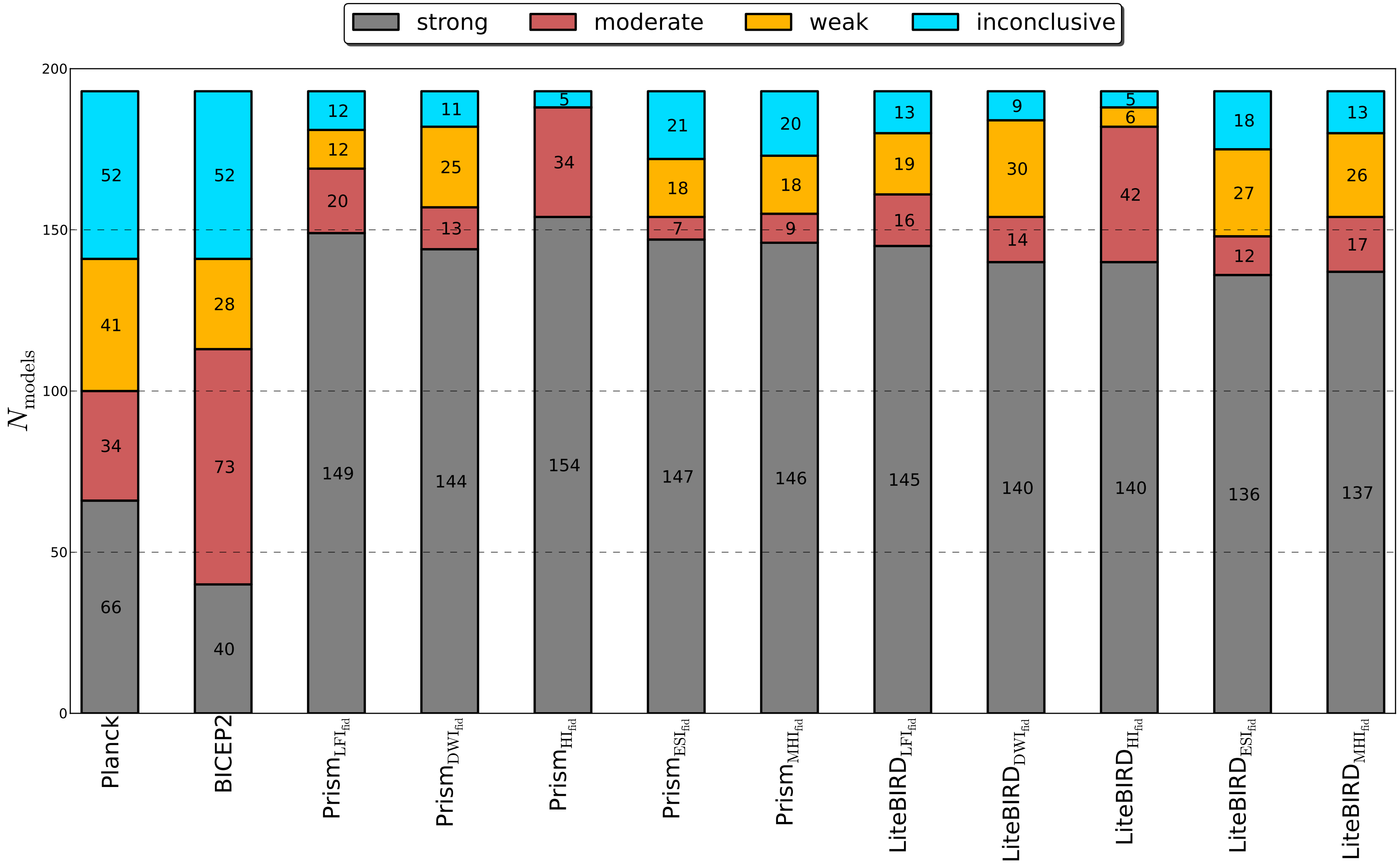
- Bayesian complexity $\simeq$ the number of constrained parameters

$$\mathcal{C} = \langle -2\ln\mathcal{L}\rangle + 2\ln\mathcal{L}_{\max} \quad \Rightarrow \quad N_{\mathrm{unconstrained}} = N_{\mathrm{param}} - \mathcal{C}$$



Data constraining power

- Comparison between PLANCK and future CMB experiments



Using the `ASPIC` library

Outline

Using the `ASPIC` library

- Automated installation
- Importing the library
- More options
- A toy program with LFI

Automated installation

- Released as a GNU software (license GPLv3)
  - Requirements: an Unix-like system (Linux, Mac,. . .) + fortran 08 compiler
  - Download the source code at: `http://cp3.irmp.ucl.ac.be/~ringeval/aspic.html`
  - Unpack the archive, configure, compile and install in `PREFIX`

```
tar -zxvf ./aspic-0.3.1.tar.gz
cd aspic-0.3.1/
./configure --prefix=/home...
make
make install
```

- Within `PREFIX`, standard Unix tree
  - Library in `lib/`
  - Include files in `include/` and documentation in `man/`

Importing the library

- Into your source code
  - Import everything or particular modules and functions

```
include 'aspic.h'
```

```
use lfisr, only : lfi_epsilon_one, lfi_epsilon_two
use srflow, only : scalar_spectral_index
use srflow, only : tensor_to_scalar_ratio
```

- Link your code toy.f90 to the (already) installed library
  - ASPIC is in the library path of your system

```
gfortran -c toy.f90
gfortran toy.o -o toy -laspic
```

  - ASPIC installed in PREFIX=/home/...

```
gfortran -I/home/.../include/aspic -c toy.f90
gfortran toy.o -o toy -L/home.../lib -laspic
```

More options

- Exhaustive documentation
  - ✦ `http://cp3.irmp.ucl.ac.be/~ringeval/man/libaspic.html`
  - ✦ Installed on your system: `man libaspic` and `man aspic_???`

liblpi(3) Module convention liblpi(3)

NAME

lpi1 lpi2 lpi3 - the logarithmic potential inf ation modules

SYNOPSIS

Physical potential	V(phi) = Mˆ4 xˆp log(x)ˆq
Routine units	*real(kp)* :: **x** = phi/phi0
Parameters	*real(kp)* :: **p,q,phi0**

DESCRIPTION

The *lpi1* module is used for the logarithmic inf ation at large f eld values, namely in the region for which 'x > 1'. In this regime, inf ation proceeds at decreasing f eld values and naturally ends at 'xend' returned by the function **lpi1_x_endinf**(p,q,phi0).

The *lpi2* module is used for the logarithmic inf ation at intermediate f eld values, namely in the region for which 'xVmax < x < 1'. In this regime, inf ation proceeds at increasing f eld values and naturally ends at 'xend' returned by the function **lpi2_x_endinf**(p,q,phi0).

Finally, the *lpi3* module is used for the logarithmic inf ation at small f eld values, namely in the region for which '0 < x < xVmax'. In this regime, inf ation proceeds at decreasing f eld values and naturally ends at 'xend' returned by the function **lpi3_x_endinf**(p,q,phi0).

Shared functions can be found in a module named **lpicommon**. The value of 'xvMax' is returned by **lpi_x_potmax**(p,q,phi0) accessible through

use lmicommon, only : lmi_x_potmax

AUTHORS

Jerome Martin, Christophe Ringeval, Vincent Vennin

- Checkout the `README` file for more options and troubleshootings

A toy program with LFI

```
program toy
  use infprec, only : kp
  use lfisr, only : lfi_epsilon_one, lfi_epsilon_two
  use lfisr, only : lfi_epsilon_three, lfi_x_endinf
  use lfireheat, only : lfi_x_rreh, lfi_x_star
  use srflow, only : scalar_spectral_index, tensor_to_scalar_ratio
  use cosmopar, only : lnMpinGeV, PowerAmpScalar
  implicit none

  real(kp) :: lnR
  real(kp), dimension(3) :: eps

  real(kp) :: DeltaN
  real(kp) :: p, xstar, xend
  real(kp) :: ns, r

  real(kp) :: ErehGeV, wreh,lnRhoReh

  p=2

!radiation-like reheating
  lnR = 0._kp

  xend = lfi_x_endinf(p)
  xstar = lfi_x_rreh(p,lnR,DeltaN)

  print *,'xend= xstar= DeltaN= ',xend,xstar,DeltaN

  eps(1) = lfi_epsilon_one(xstar,p)
  eps(2) = lfi_epsilon_two(xstar,p)
  eps(3) = lfi_epsilon_three(xstar,p)

  ns = scalar_spectral_index(eps)
  r = tensor_to_scalar_ratio(eps)

  print *,'ns= r= ',ns,r

  read(*,*)

!matter like reheating at Ereh=10^8 GeV
  ErehGeV = 1e8
  wreh = 0

  lnRhoReh = 4._kp*(log(ErehGev)-lnMpinGev)

  xstar = lfi_x_star(p,wreh,lnRhoReh,PowerAmpScalar,DeltaN)

  print *,'xend= xstar= DeltaN= ',xend,xstar,DeltaN

  eps(1) = lfi_epsilon_one(xstar,p)
  eps(2) = lfi_epsilon_two(xstar,p)
  eps(3) = lfi_epsilon_three(xstar,p)

  ns = scalar_spectral_index(eps)
  r = tensor_to_scalar_ratio(eps)

  print *,'ns= r= ',ns,r

end program toy
```

```
FC=gfortran

FCFLAGS=-g
LFLAGS=-L/home/chris/usr/lib -laspic

INCLUDE=-I/home/chris/usr/include/aspic

default: toy

%.o: %.f90
	$(FC) $(FCFLAGS) $(INCLUDE) -c $<

toy: toy.o
	$(FC) $(FCFLAGS) toy.o -o $@ $(LFLAGS)


clean:
	rm toy *.o *.mod
```