Cosmic strings: from theoretical motivations to cosmological signatures

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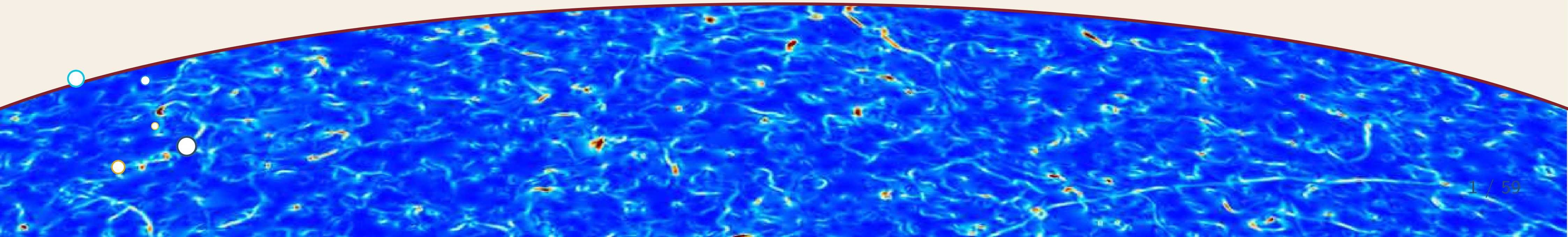

Outline

Theoretical aspects

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Theoretical aspects

- Original motivations: topological defects
- Formation of topological defects
- Abelian Higgs strings
- Strings of various types and origins
- Dynamics of infinitely thin strings
- The simplest case: Nambu–Goto strings
- Temporal gauge
- String dynamics
- Intercommutation of Abelian Higgs strings

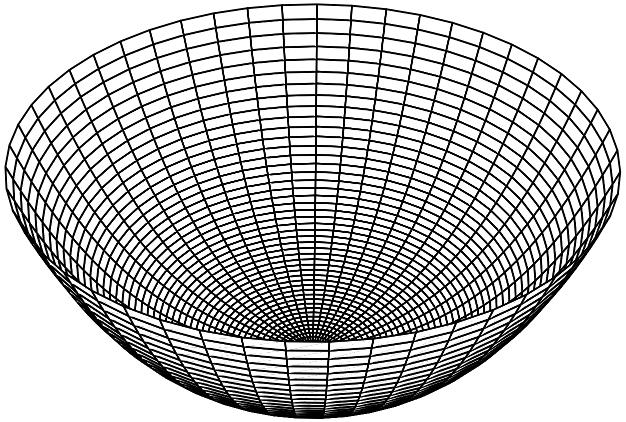
Original motivations: topological defects

- Phase transitions in the early universe
  - Triggered by the spontaneous breakdown of the fundamental interactions [Kirzhnits:1972,Kobsarev:1974, Kibble:1976]
- Example: Abelian Higgs model and $U(1)$ symmetry

$$\mathcal{L}_\mathrm{h} = \frac{1}{2}\left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) - \frac{1}{4} H_{\mu\nu} H^{\mu\nu} - V(\Phi),$$

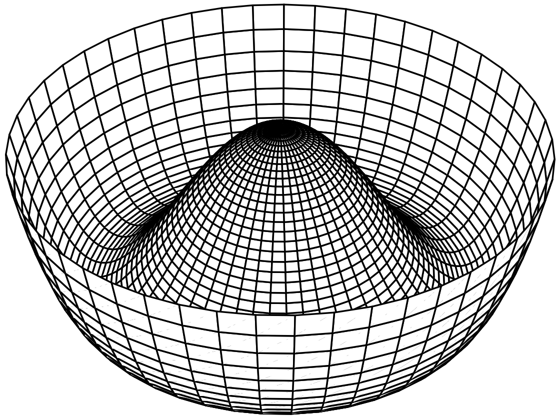
$$D_\mu = \partial_\mu + i g B_\mu, \quad V(\Phi) = \frac{\lambda}{8}\left(|\Phi|^2 - \eta_\mathrm{v}^2\right)^2 + \mathcal{O}\left(\Theta^2 |\Phi|^2\right)$$

$V\left[\Re(\Phi), \Im(\Phi)\right]$ at $\Theta > \Theta_\mathrm{c}$



$\Phi_0 = 0,\ \delta\Phi_0 = 0$

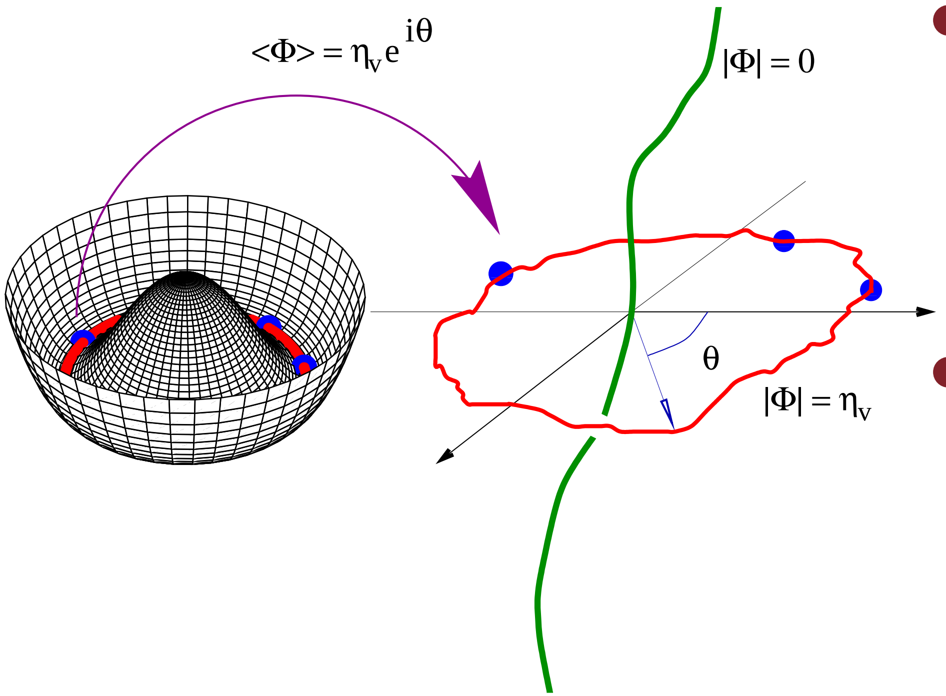
$V\left[\Re(\Phi), \Im(\Phi)\right]$ at $\Theta < \Theta_\mathrm{c}$



$\Phi_0 = \eta_\mathrm{v} e^{i\theta},\ \delta\Phi_0 \neq 0$

Formation of topological defects

- Kibble–Zurek mechanism: $\ell_c < d_h$



- Conserved topological charge

$$\oint \frac{d\theta(s)}{ds} ds = 2\pi n$$

- Invariance group $\mathcal{M}$ of the vacuum

For $\mathcal{G} \to \mathcal{H}$ and $\forall g \in \mathcal{G}$, one has

$$\mathcal{M} \equiv \{g\Phi_0 / \Phi_0 \in \mathcal{H}\} \sim \mathcal{G}/\mathcal{H}$$

- Homotopy groups and defects

$$\pi_0(\mathcal{M}) \nsim \{I\} \implies \exists \, \text{domain walls}$$
$$\pi_1(\mathcal{M}) \nsim \{I\} \implies \exists \, \text{cosmic strings}$$
$$\pi_2(\mathcal{M}) \nsim \{I\} \implies \exists \, \text{monopoles}$$

Abelian Higgs strings

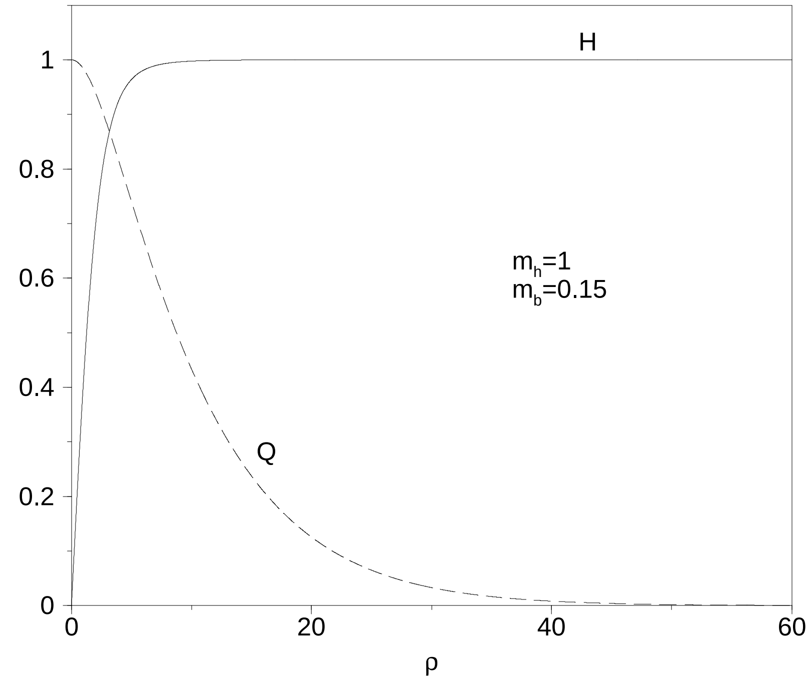
- Field profiles within a Nielsen-Olesen vortex

$$\Phi = \eta_{\rm v} H(\varrho) \mathrm{e}^{in\theta},$$

$$B_\mu = \frac{Q(\varrho) - n}{g} \delta_{\mu\theta}$$

with

$$m_{\rm h} = \sqrt{\lambda}\eta_{\rm v}, \quad m_{\rm b} \equiv g\eta_{\rm v}$$



- Stress tensor integrated over transverse directions

$$T^{tt} = -T^{zz} = \frac{\lambda \eta_{\rm v}^4}{2}\left[(\partial_\varrho H)^2 + \frac{Q^2 H^2}{\varrho^2} + \frac{(H^2-1)^2}{4} + \frac{\lambda}{g^2}\frac{(\partial_\varrho Q)^2}{\varrho^2}\right]$$

$$\Longrightarrow \text{energy density} = \text{string tension} \Leftrightarrow U = T = \mathcal{O}\left(\frac{m_{\rm h}^2}{m_{\rm b}^2}\right)\eta_{\rm v}^2$$

Strings of various types and origins

- Topological defects
  - Global strings [Davis:1985, Durrer:1998rw, Yamaguchi:1999yp]
  - Non-Abelian strings [Vilenkin:1984rt, Dvali:1993qp, Spergel:1996ai, Bucher:1998mh, McGraw:1998]
  - K- and DBI-strings [Babichev:2006cy, Babichev:2007tn, Sarangi:2007mj]
  - Current-carrying strings [Witten:1984eb, Davis:1988ip,Carter:1989dp, Peter:1992dw, Peter:1992ta]
- Line-like energy density distributions
  - Semi-local strings: energetically favoured for $m_\mathrm{b} > m_\mathrm{h}$ [Vachaspati:1991, Hindmarsh:1991jq, Achucarro:1999it]
  - Cosmic superstrings: bound states made of p F-strings and q $D1$-brane [Witten:1985fp,Copeland:2009ga, Sakellariadou:2008ie, Polchinski:2004ia, Davis:2008dj]
  - Nambu–Goto strings: Lorentz invariant two-dimensional worldsheet [Goto:1971ce,Nambu:1974]
- Carter strings [Carter:1989xk, Carter:1992vb, Carter:1994zs, Carter:2000wv]
  - Infinitely thin strings with an internal structure: $U \neq T$
  - Two-dimensional models of current carrying strings

Dynamics of infinitely thin strings

- String = two-dimensional worldsheet located at: $x^\mu = X^\mu(\xi^a)$
  - ✦ Induced metric on the string: $\gamma_{ab} = g_{\mu\nu} \dfrac{\partial X^\mu}{\partial \xi^a} \dfrac{\partial X^\nu}{\partial \xi^b}$
- Carter's covariant formalism
  - ✦ First fundamental form: projector onto the string worldsheet

$$q^{\mu\nu} \equiv \gamma^{ab} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b} \Longrightarrow \begin{cases} \perp^\mu{}_\nu \equiv g^\mu{}_\nu - q^\mu{}_\nu, \\ \bar{\nabla}_\mu \equiv q^\alpha{}_\mu \nabla_\alpha \\ K_{\mu\nu}{}^\rho \equiv q^\alpha{}_\nu \bar{\nabla}_\mu \, q^\rho{}_\alpha \end{cases}$$

  - ✦ Stress tensor in its eigenvector basis: $u^2 = -1$, $v^2 = 1$, $u^\alpha v_\alpha = 0$

$$\bar{T}^{\mu\nu} = U u^\mu u^\nu - T v^\mu v^\nu = (U - T) u^\mu u^\nu - T q^{\mu\nu},$$
$$q^{\mu\nu} = -u^\mu u^\nu + v^\mu v^\nu$$

  - ✦ For a barotropic equation of state $U = U(T)$

$$\bar{\nabla}_\rho \bar{T}^{\rho\sigma} = 0 \Longrightarrow K^\rho = \perp^\rho{}_\sigma \left(\frac{U}{T} - 1 \right) u^\alpha \bar{\nabla}_\alpha u^\sigma$$

The simplest case: Nambu–Goto strings

- Lorentz invariance $\Longrightarrow U = T \Longrightarrow K^\mu = 0$
  - ✦ Equations of motion

$$K^\mu = \frac{1}{\sqrt{-\gamma}}\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \Gamma^\mu_{\nu\rho}\gamma^{ab}\partial_a X^\nu \partial_b X^\rho = 0$$

  - ✦ Can also be directly obtained from: $\mathcal{S} = -U\int \mathrm{d}^2\xi\sqrt{-\gamma}$

- In Friedmann–Lemaître background: $\tau \equiv \xi^0$ and $\sigma \equiv \xi^2$
  - ✦ Transverse gauge: $g_{\mu\nu}\dfrac{\partial X^\mu}{\partial\tau}\dfrac{\partial X^\nu}{\partial\sigma} = \dot{X}^\mu \acute{X}_\mu = 0$
  - ✦ Equation of motion: $\varepsilon \equiv \sqrt{-\dfrac{\acute{X}^2}{\dot{X}^2}}$

$$\ddot{X}^\mu + \left(\frac{\dot\varepsilon}{\varepsilon} + \frac{2}{a}\frac{\mathrm{d}a}{\mathrm{d}X^0}\dot{X}^0\right)\dot{X}^\mu - \frac{1}{\varepsilon}\left(\frac{\acute{X}^\mu}{\varepsilon}\right)' - \frac{2}{a}\frac{\mathrm{d}a}{\mathrm{d}X^0}\frac{\acute{X}^0}{\varepsilon}\frac{\acute{X}^\mu}{\varepsilon} + \delta^\mu_0\frac{2}{a}\frac{\mathrm{d}a}{\mathrm{d}X^0}\dot{X}^2 = 0$$

Temporal gauge

- Gauge fixing complete by identifying τ with the background time

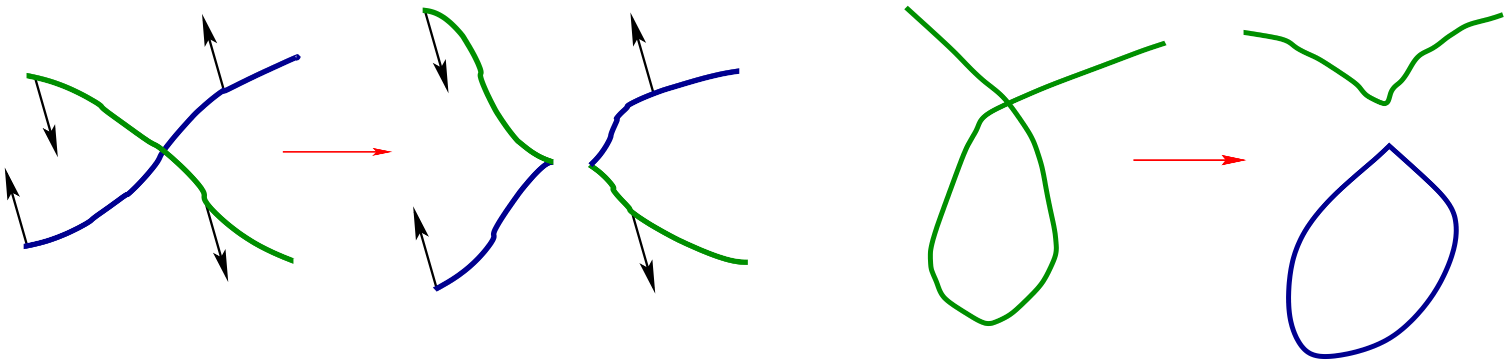
$$\tau = X^0 = \eta \implies \begin{cases} \dot{\vec{X}} \cdot \acute{\vec{X}} = 0, \quad \varepsilon = \sqrt{\dfrac{\acute{\vec{X}}^2}{1 - \dot{\vec{X}}^2}}, \quad \dot{\varepsilon} + 2\mathcal{H}\varepsilon \dot{\vec{X}}^2 = 0, \\ \ddot{\vec{X}} + 2\mathcal{H}\left(1 - \dot{\vec{X}}^2\right) - \dfrac{1}{\varepsilon}\left(\dfrac{\acute{\vec{X}}}{\varepsilon}\right)' = 0 \end{cases}$$

- Bennet–Bouchet equivalent equations [Bouchet:1988,Bennett:1989,Bennett:1990]
  - ✦ Lightcone-like coordinates: $u = \int \varepsilon \mathrm{d}\sigma - \tau$ and $v = \int \varepsilon \mathrm{d}\sigma + \tau$
  - ✦ Left and right movers: $\vec{p}(\tau, u) \equiv \dfrac{\acute{\vec{X}}}{\varepsilon} - \dot{\vec{X}}$ and $\vec{q}(\tau, v) \equiv \dfrac{\acute{\vec{X}}}{\varepsilon} + \dot{\vec{X}}$

$$\frac{\partial \vec{p}}{\partial \tau} = -\mathcal{H}\left[\vec{q} - \vec{p}\,(\vec{p}\cdot\vec{q})\right], \quad \frac{\partial \vec{q}}{\partial \tau} = -\mathcal{H}\left[\vec{p} - \vec{q}\,(\vec{p}\cdot\vec{q})\right], \quad \frac{\dot{\varepsilon}}{\varepsilon} = -\mathcal{H}\left(1 - \vec{p}\cdot\vec{q}\right)$$

String dynamics

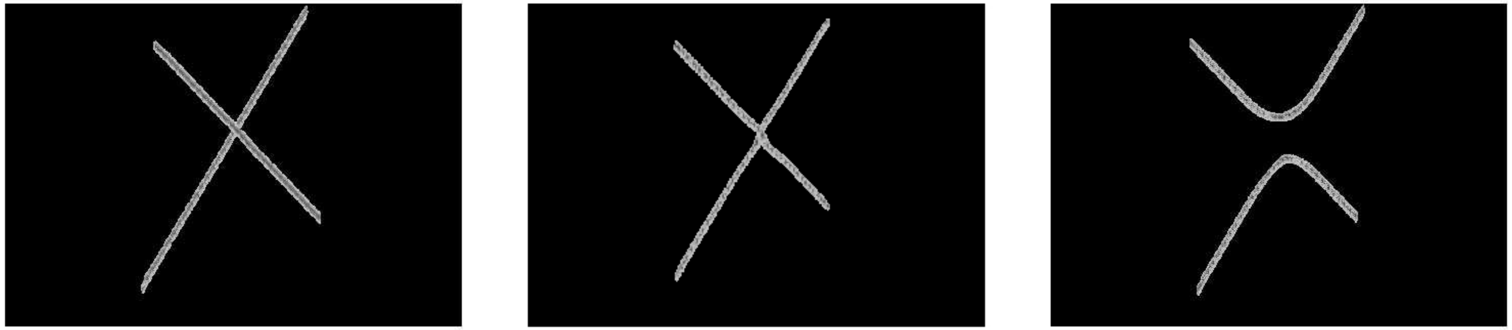
- Propagation of left- right-moving waves
  - In Minkowski ($\mathcal{H} = 0$): $\vec{X}(\tau, \sigma) = \frac{1}{2}\left[\vec{p}(\sigma + \tau) + \vec{q}(\sigma - \tau)\right]$
  - In FLRW spacetime: damped and interactions on Hubble scales
- Interaction between strings is microphysics dependent
  - Abelian Higgs strings with $m_\mathrm{h} \simeq m_\mathrm{b}$: $P \simeq 1$ and formation of 2 kinks



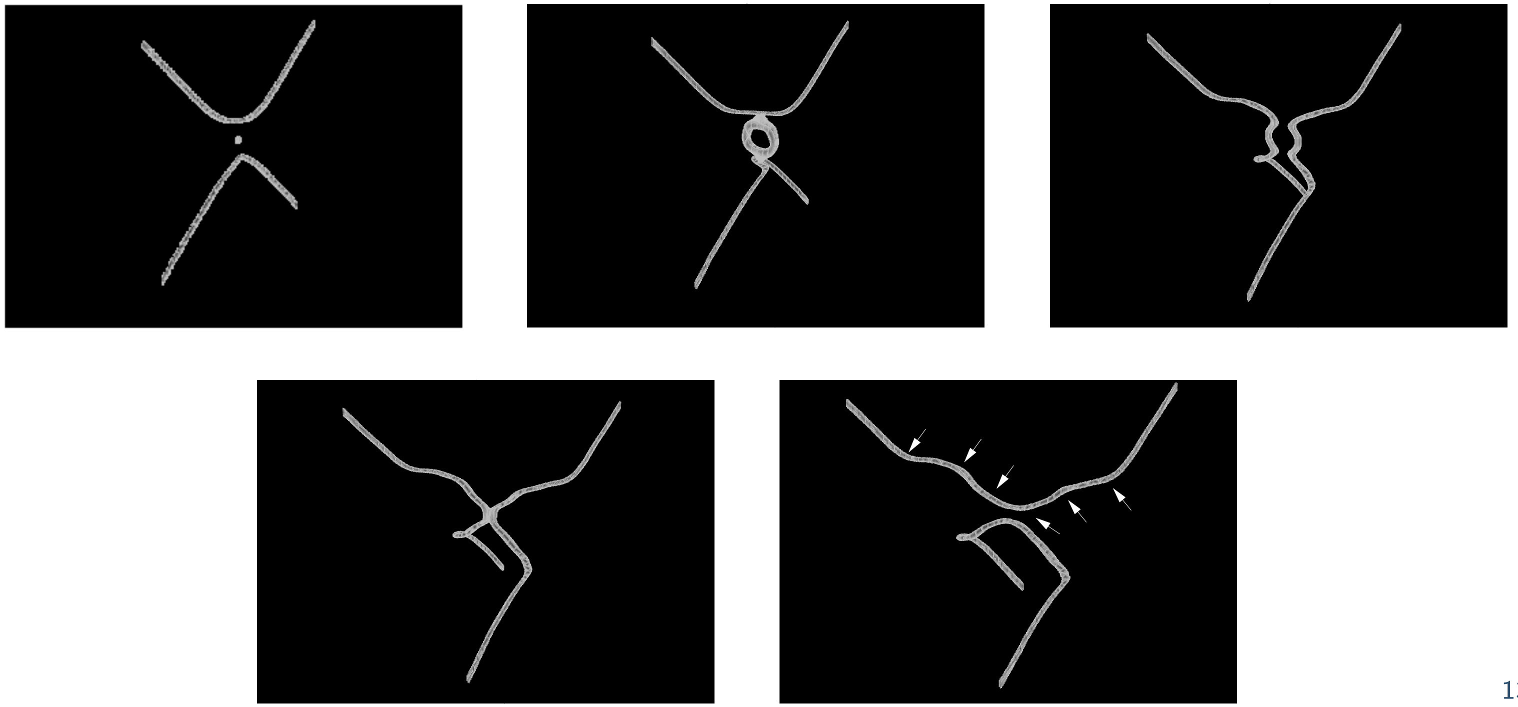
  - Cosmic superstrings: $P \ll 1$ (presence of extra-dimensions)
  - (p, q)-strings: charge conservation $\Longrightarrow$ Y-junctions, kinematic constraints and kinks proliferation [Copeland:2007nv, Bevis:2009az, Binetruy:2010bq, Steer:2013nea]

Intercommutation of Abelian Higgs strings

- Standard case



- Multiple reconnections possible with $m_{\rm h} \gg m_{\rm b}$ and $v \simeq 1$ [Verbiest:2011kv]



Nambu–Goto simulations

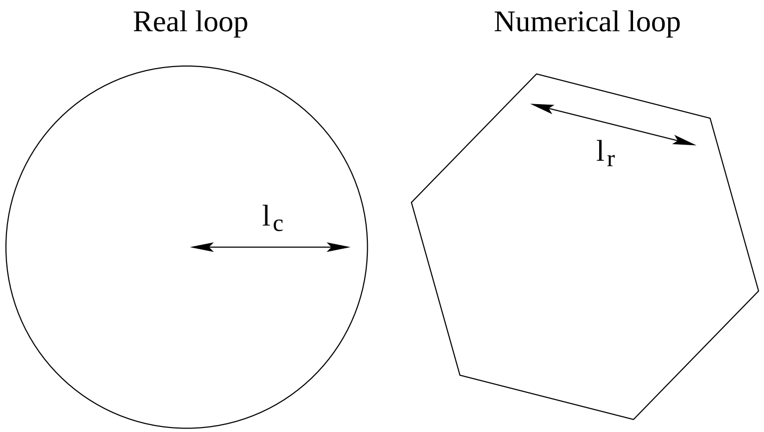
Outline

Nambu–Goto simulations

- A small matter era run (movie)
- Cosmological attractor
- Scaling of the energy density
- Relaxation towards scaling
- Loop distribution in scaling

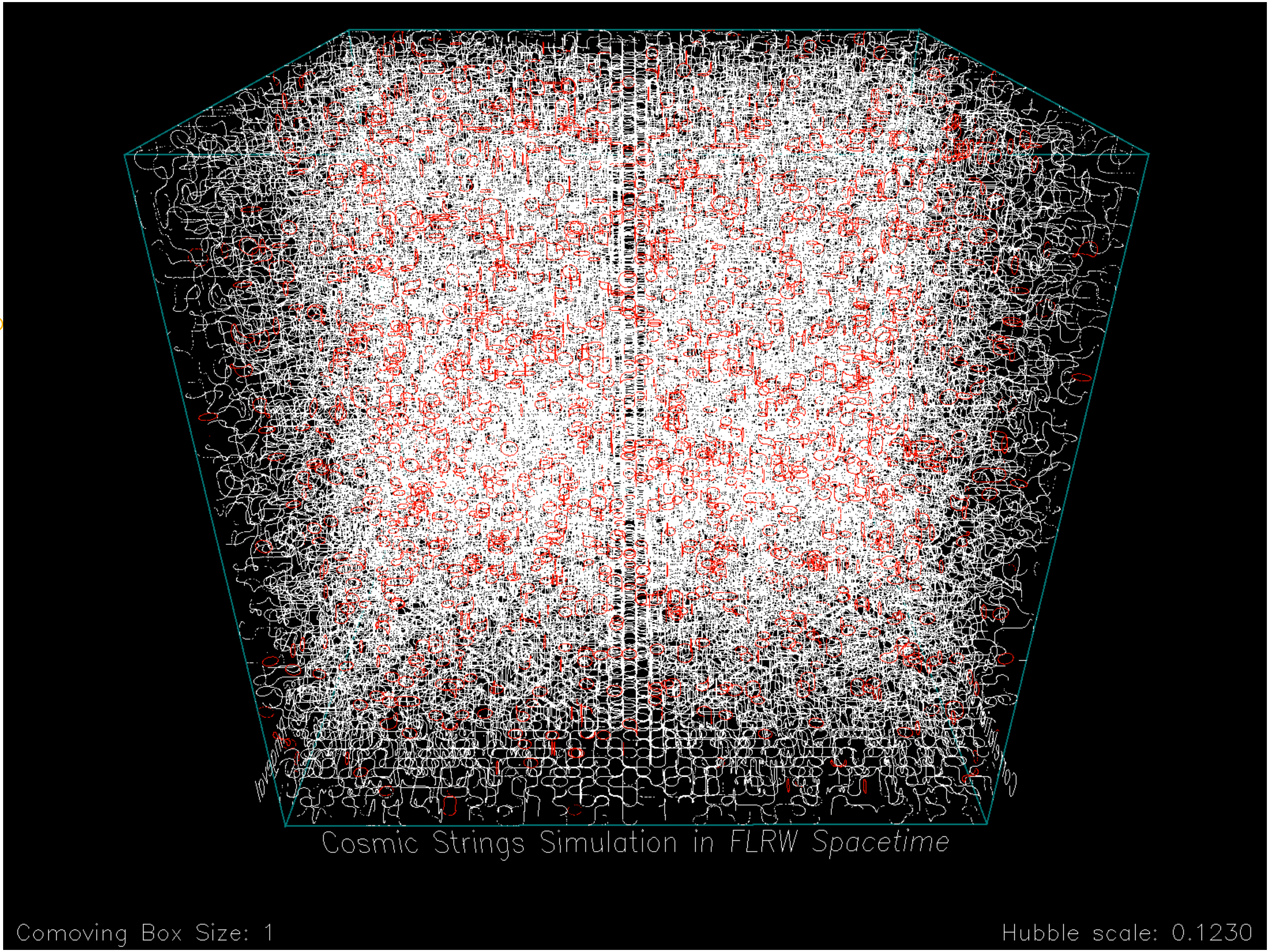
Nambu–Goto simulations

- Goal: getting realistic statistics of string networks in FLRW
  - Only one parameter : U
  - Nambu–Goto networks are already complex: non-linear and non-local properties
- Method: solve numerically the string evolution in FLRW
  - From some representative initial conditions [Vachaspati:1984]
  - IC are mostly irrelevant due to the existence of a cosmological attractor [Bennett:1989,Allen:1990,Albrecht:1989,Ringeval:2005kr,Vanchurin:2005pa]
- Numerical parameters



$$\textit{Comoving box size} = 1$$
$$\textit{Initial correlation length } \ell_c = 1/100$$
$$\textit{Initial resolution length } \ell_r = 1/2000$$

A small matter era run (movie)



Cosmological attractor

- Long strings ($\ell > d_\mathrm{h}$) rapidly reach a scaling evolution
  - ✦ Energy density evolves as radiation/matter ($\propto 1/d_\mathrm{h}^2$) instead of naively expected $\rho \propto 1/a^2$

$$\rho_\infty \frac{d_\mathrm{h}^2}{U}\bigg|_\mathrm{mat} = 28.4 \pm 0.9 \qquad \rho_\infty \frac{d_\mathrm{h}^2}{U}\bigg|_\mathrm{rad} = 37.8 \pm 1.7$$

  - ✦ Kibble mechanism: formation of loops that transfer some energy to sub-horizon length scales
- A similar mechanism happens to loops themselves due to their self-intersections

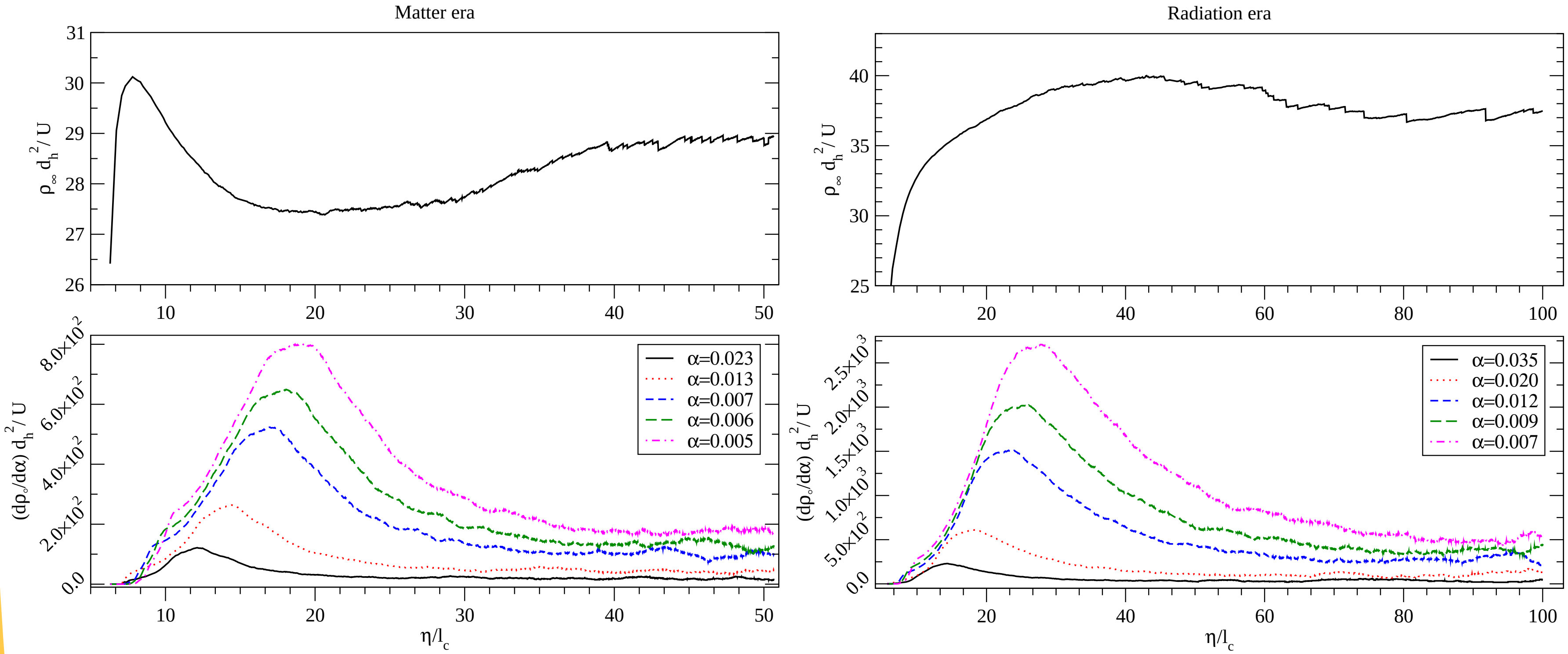
$$\text{With } \alpha \equiv \frac{\ell}{d_\mathrm{h}}, \qquad \frac{\mathrm{d}\rho_\circ}{\mathrm{d}\alpha} = \mathcal{S}(\alpha)\frac{U}{d_\mathrm{h}^2} \quad \Longrightarrow \quad \frac{\mathrm{d}n}{\mathrm{d}\alpha} = \frac{\mathcal{S}(\alpha)}{\alpha\, d_\mathrm{h}^3}$$

  - ✦ The scaling function $\mathcal{S}(\alpha)$ can be determined from simulations
  - ✦ But only for $\alpha_\mathrm{c} < \alpha < 1$ where α_c involves physical effects not accounted in the Nambu–Goto model

Scaling of the energy density

- Scaling of the energy densities for loops and long strings

  [Ringeval:2005kr,Blanco-Pillado:2013qja]

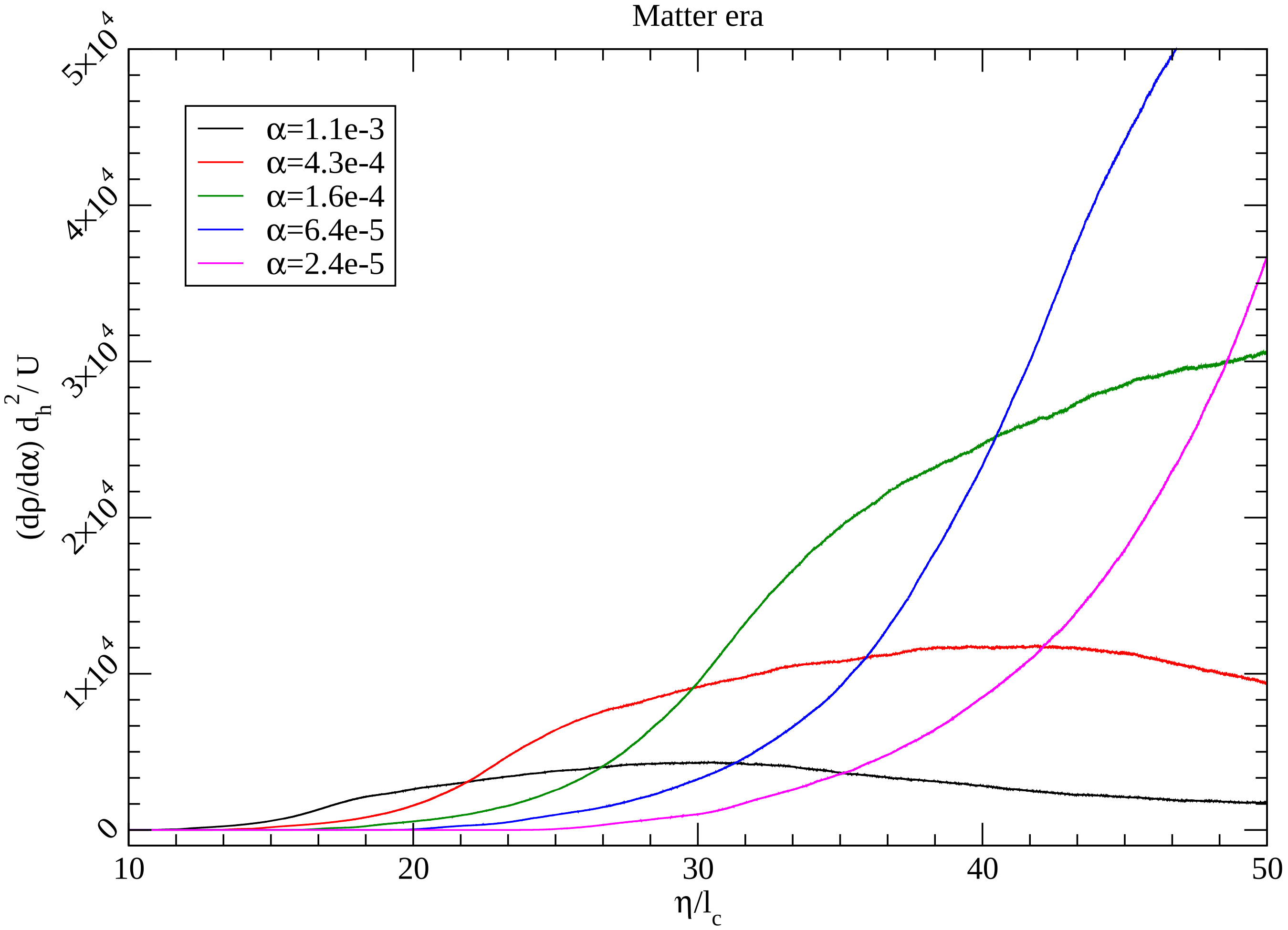


$$\rho_\infty \frac{d_\mathrm{h}^2}{U}\bigg|_\mathrm{mat} = 28.4 \pm 0.9 \qquad \rho_\infty \frac{d_\mathrm{h}^2}{U}\bigg|_\mathrm{rad} = 37.8 \pm 1.7$$

$$\mathrm{d}\rho_\circ \times \frac{d_\mathrm{h}^2}{U} = \mathcal{S}(\alpha) \quad \text{(time independent)}$$

Relaxation towards scaling

- Transient effects last longer for smaller loops

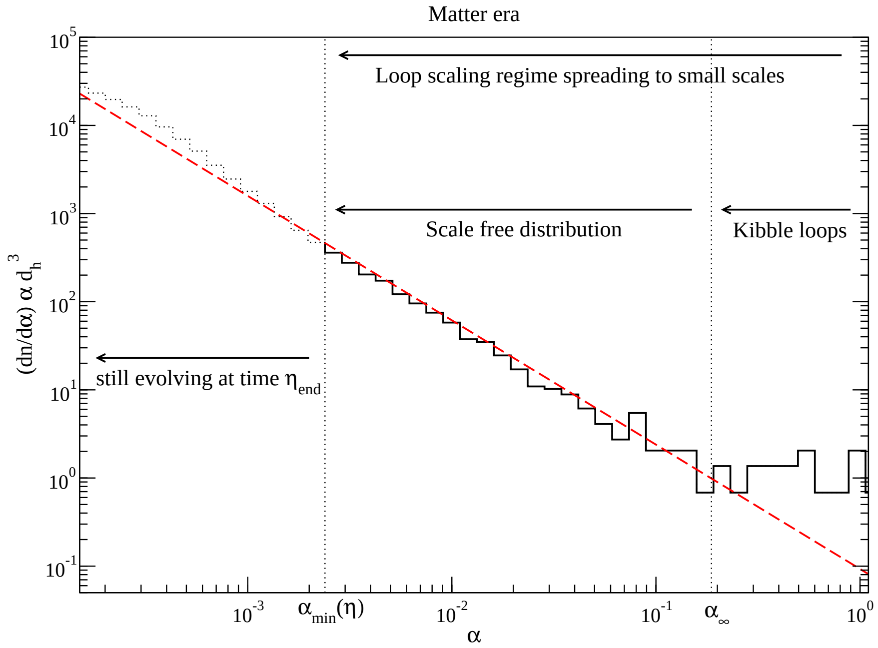


- NG simulations do not incorporate GW $\Rightarrow$ do not describe $\alpha < \alpha_c$

Loop distribution in scaling

- By the end of the run

Scaling parts



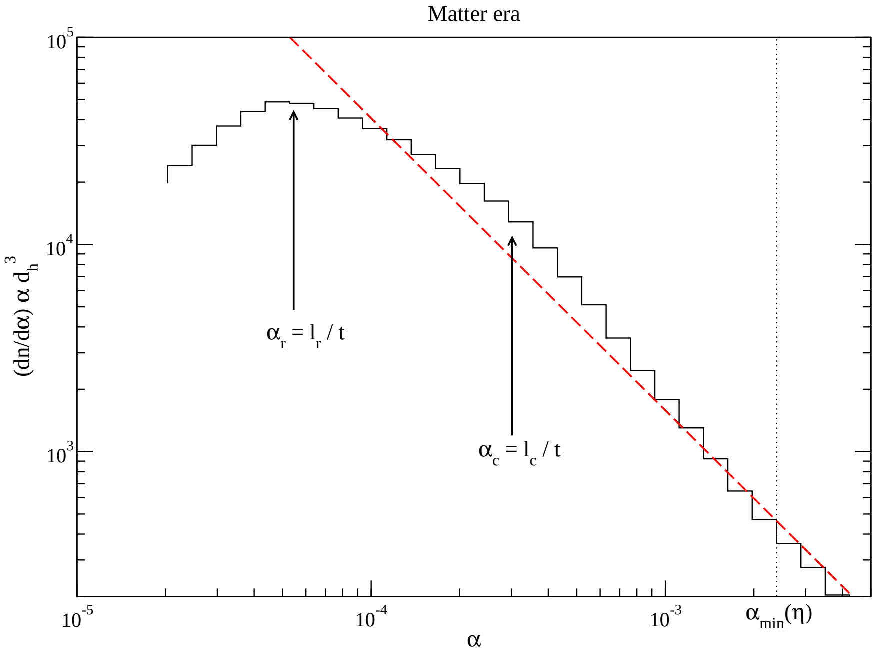
- Scaling form $\mathcal{S}(\alpha) = \frac{C_\circ}{\alpha^p}$ with

$$\left\{\begin{array}{lcl} p & \underset{\text{mat}}{=} & 1.41\,^{+0.08}_{-0.07} \\ C_\circ & \underset{\text{mat}}{=} & 0.09\,^{-0.03}_{+0.03} \end{array}\right. \quad \text{and} \quad \left\{\begin{array}{lcl} p & \underset{\text{rad}}{=} & 1.60\,^{+0.21}_{-0.15} \\ C_\circ & \underset{\text{rad}}{=} & 0.21\,^{-0.12}_{+0.13} \end{array}\right.$$

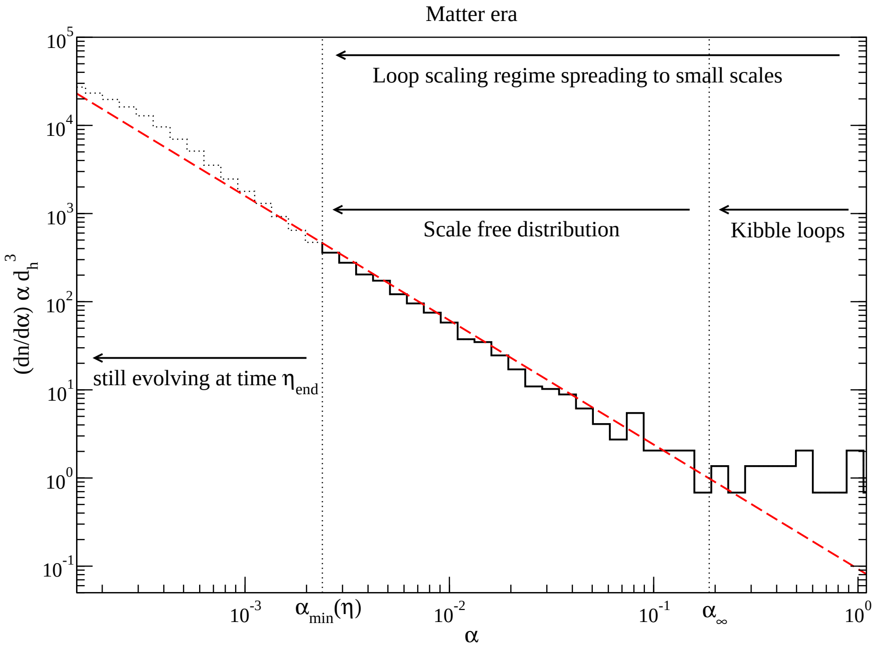
Loop distribution in scaling

- By the end of the run

Non-scaling parts



Scaling parts



- Scaling form $\mathcal{S}(\alpha) = \dfrac{C_\circ}{\alpha^p}$ with

$$\left\{ \begin{array}{ccc} p & \underset{\text{mat}}{=} & 1.41\,^{+0.08}_{-0.07} \\ C_\circ & \underset{\text{mat}}{=} & 0.09\,^{-0.03}_{+0.03} \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{ccc} p & \underset{\text{rad}}{=} & 1.60\,^{+0.21}_{-0.15} \\ C_\circ & \underset{\text{rad}}{=} & 0.21\,^{-0.12}_{+0.13} \end{array} \right.$$

Analytical models

Outline

Analytical models

- Polchinsky-Rocha model
- Inclusion of gravitational backreaction
- Cosmological loop distribution
- Relaxation effects are accounted
- Some numerical values
- Extension to vortons

Polchinsky-Rocha model

- No fragmentation, no reconnection, loops from long string only
[Polchinski:2006ee,Dubath:2007mf,Rocha:2007ni]
  - ✦ Predicts a power law scaling function

$$\mathcal{S}(\alpha) \propto \alpha^{2\chi-2} \implies p = 2(1-\chi)$$

  - ✦ Parameter χ can be inferred from the long string scaling

- From Martins & Shellard simulations, they independently found

$$p_{\mathrm{mat}} \simeq 1.5, \qquad p_{\mathrm{rad}} \simeq 1.8$$

- In the PR model χ is related to two-point functions [Hindmarsh:2008dw]

$$\left\langle \acute{X}^A(\sigma)\acute{X}^B(\sigma')\right\rangle = \frac{1}{2}\delta^{AB}T(\sigma-\sigma') \qquad T(\sigma) \simeq \bar{t}^2 - c_1\left(\frac{\sigma}{\hat{\xi}}\right)^{2\chi}$$

- Agreement with simulations suggests that all neglected effects mostly renormalise $C_\circ$

Including loop's gravitational radiation

- Boltzmann equation + PR production function
  - PR loop production function (from string shape correlations)

$$t^5 \mathcal{P}(\ell, t) = c \left(\frac{\ell}{t}\right)^{2\chi - 3}$$

  - In an expanding universe

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) = a^3 \mathcal{P}(\ell, t)$$

  - A loop shrinks due to G.W. emission ($\gamma \equiv \ell/t$) [Allen:1992]

$$\frac{\mathrm{d}\ell}{\mathrm{d}t} = -\gamma_{\mathrm{d}} \simeq 100 G U$$

- Evolution equation [Rocha:2007ni]

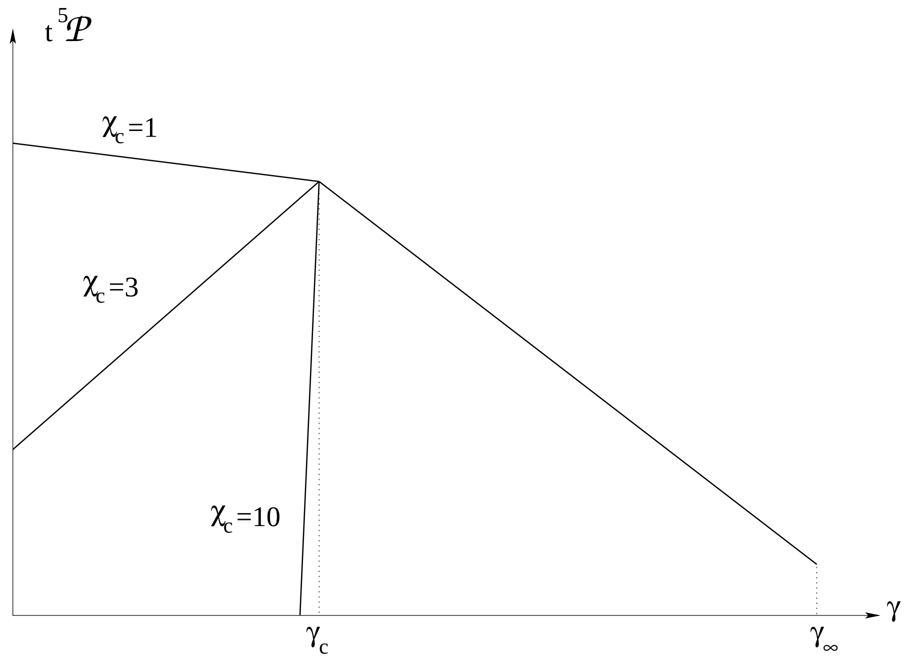
$$\frac{\partial}{\partial t}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) - \gamma_{\mathrm{d}} \frac{\partial}{\partial \ell}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) = a^3 \mathcal{P}(\ell, t)$$

Inclusion of gravitational backreaction

- PR model + GW emission + GW backreaction [Lorenz:2010sm]
  - Allows us to extrapolate numerical simulations to small ℓ
  - Boltzmann equation ($\gamma_\mathrm{d} = \Gamma G U$)

$$\frac{\partial}{\partial t}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) - \gamma_\mathrm{d} \frac{\partial}{\partial \ell}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) = a^3 \mathcal{P}(\ell, t).$$

  - Postulated piecewise scaling loop production function



$$t^5 \mathcal{P}\left(\gamma = \frac{\ell}{t}, t\right) \propto \gamma^{2\chi - 3}$$

$$\gamma_\mathrm{c} \ll \gamma_\mathrm{d} \ll \gamma_\infty \lesssim 1$$

Cosmological loop distribution

- Can be completely solved analytically (see arXiv.1006.0931)
- From any initial loop distribution $\mathcal{N}_{\mathrm{ini}}(\ell)$, one gets $\mathcal{F}(\gamma,t) \equiv \dfrac{\partial n}{\partial \ell}(\gamma,t)$

$$
\begin{aligned}
t^4 \mathcal{F}(\gamma \geq \gamma_{\mathrm{c}}, t) = {}& \left(\frac{t}{t_{\mathrm{ini}}}\right)^4 \left(\frac{a_{\mathrm{ini}}}{a}\right)^3 t_{\mathrm{ini}}^4 \mathcal{N}_{\mathrm{ini}}\left\{\left[\gamma + \gamma_{\mathrm{d}}\left(1 - \frac{t_{\mathrm{ini}}}{t}\right)\right] t\right\} + C(\gamma+\gamma_{\mathrm{d}})^{2\chi-3} f\left(\frac{\gamma_{\mathrm{d}}}{\gamma+\gamma_{\mathrm{d}}}\right) \\
& - C(\gamma+\gamma_{\mathrm{d}})^{2\chi-3}\left(\frac{t}{t_{\mathrm{ini}}}\right)^{2\chi+1}\left(\frac{a_{\mathrm{ini}}}{a}\right)^3 f\left(\frac{\gamma_{\mathrm{d}}}{\gamma+\gamma_{\mathrm{d}}}\frac{t_{\mathrm{ini}}}{t}\right),
\end{aligned}
$$

$$
\begin{aligned}
t^4 \mathcal{F}(\gamma_{\mathcal{T}} \leq \gamma < \gamma_{\mathrm{c}}, t) = {}& \left(\frac{t}{t_{\mathrm{ini}}}\right)^4 \left(\frac{a_{\mathrm{ini}}}{a}\right)^3 t_{\mathrm{ini}}^4 \mathcal{N}_{\mathrm{ini}}\left\{\left[\gamma + \gamma_{\mathrm{d}}\left(1 - \frac{t_{\mathrm{ini}}}{t}\right)\right] t\right\} + C_{\mathrm{c}}(\gamma+\gamma_{\mathrm{d}})^{2\chi_{\mathrm{c}}-3} f_{\mathrm{c}}\left(\frac{\gamma_{\mathrm{d}}}{\gamma+\gamma_{\mathrm{d}}}\right) \\
& - C(\gamma+\gamma_{\mathrm{d}})^{2\chi-3}\left(\frac{t}{t_{\mathrm{ini}}}\right)^{2\chi+1}\left(\frac{a_{\mathrm{ini}}}{a}\right)^3 f\left(\frac{\gamma_{\mathrm{d}}}{\gamma+\gamma_{\mathrm{d}}}\frac{t_{\mathrm{ini}}}{t}\right) \\
& + K\left(\frac{\gamma_{\mathrm{c}}+\gamma_{\mathrm{d}}}{\gamma+\gamma_{\mathrm{d}}}\right)^4 \left[\frac{a\left(\dfrac{\gamma+\gamma_{\mathrm{d}}}{\gamma_{\mathrm{c}}+\gamma_{\mathrm{d}}}t\right)}{a(t)}\right]^3,
\end{aligned}
$$

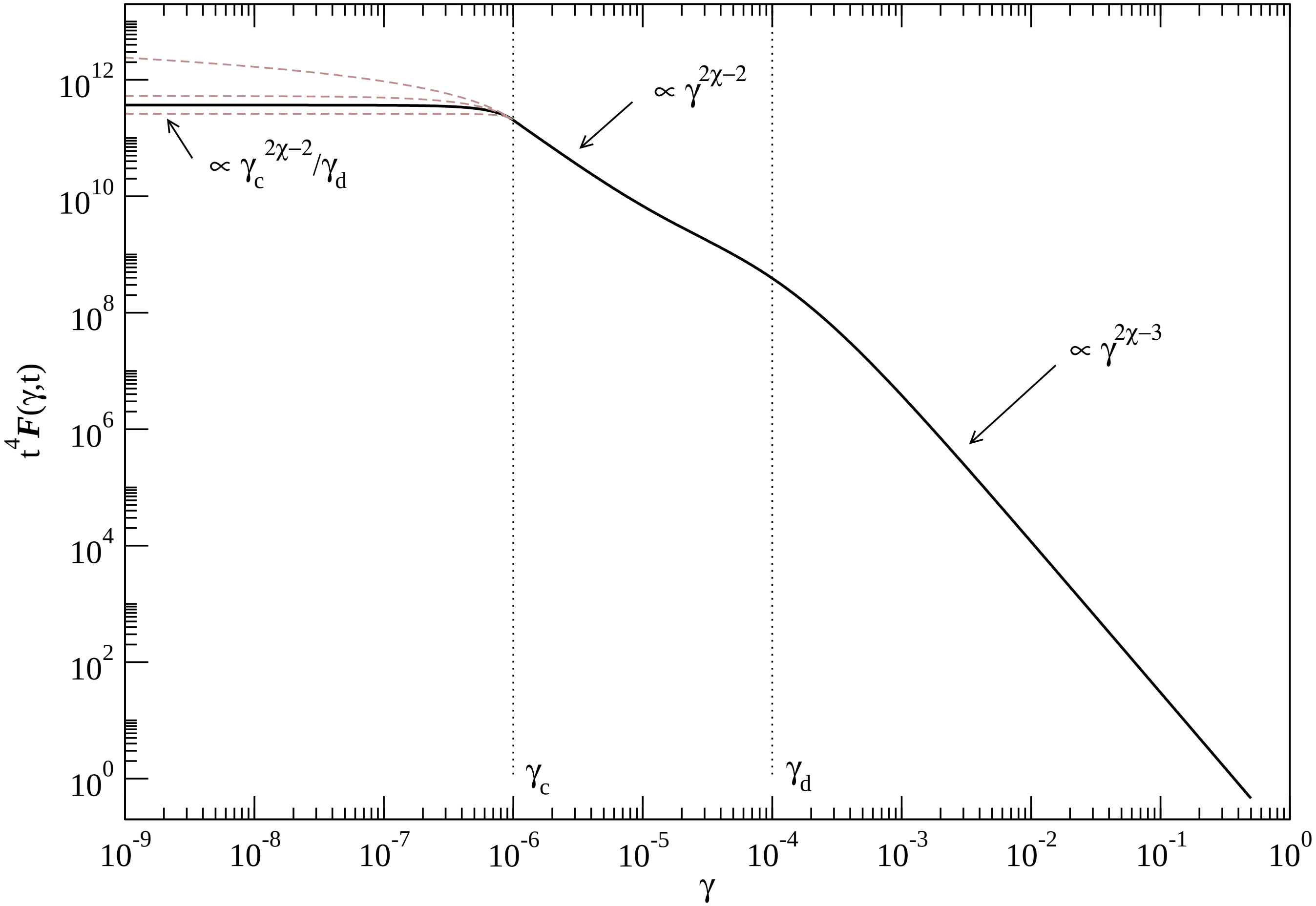
$$
t^4 \mathcal{F}(0 < \gamma < \gamma_{\mathcal{T}}, t) = \left(\frac{t}{t_{\mathrm{ini}}}\right)^4 \left(\frac{a_{\mathrm{ini}}}{a}\right)^3 t_{\mathrm{ini}}^4 \mathcal{N}_{\mathrm{ini}}\left\{\left[\gamma + \gamma_{\mathrm{d}}\left(1 - \frac{t_{\mathrm{ini}}}{t}\right)\right] t\right\} + C_{\mathrm{c}}(\gamma+\gamma_{\mathrm{d}})^{2\chi_{\mathrm{c}}-3} f_{\mathrm{c}}\left(\frac{\gamma_{\mathrm{d}}}{\gamma+\gamma_{\mathrm{d}}}\right)
$$

$$
\gamma_{\mathcal{T}}(t) \equiv (\gamma_{\mathrm{c}}+\gamma_{\mathrm{d}})\frac{t_{\mathrm{ini}}}{t} - \gamma_{\mathrm{d}}, \qquad \mu \equiv 3\nu - 2\chi - 1
$$

$$
f(x) \equiv {}_2\mathrm{F}_1(3-2\chi, \mu; \mu+1; x) \qquad f_{\mathrm{c}}(x) \equiv {}_2\mathrm{F}_1(3-2\chi_{\mathrm{c}}, \mu_{\mathrm{c}}; \mu_{\mathrm{c}}+1; x)
$$

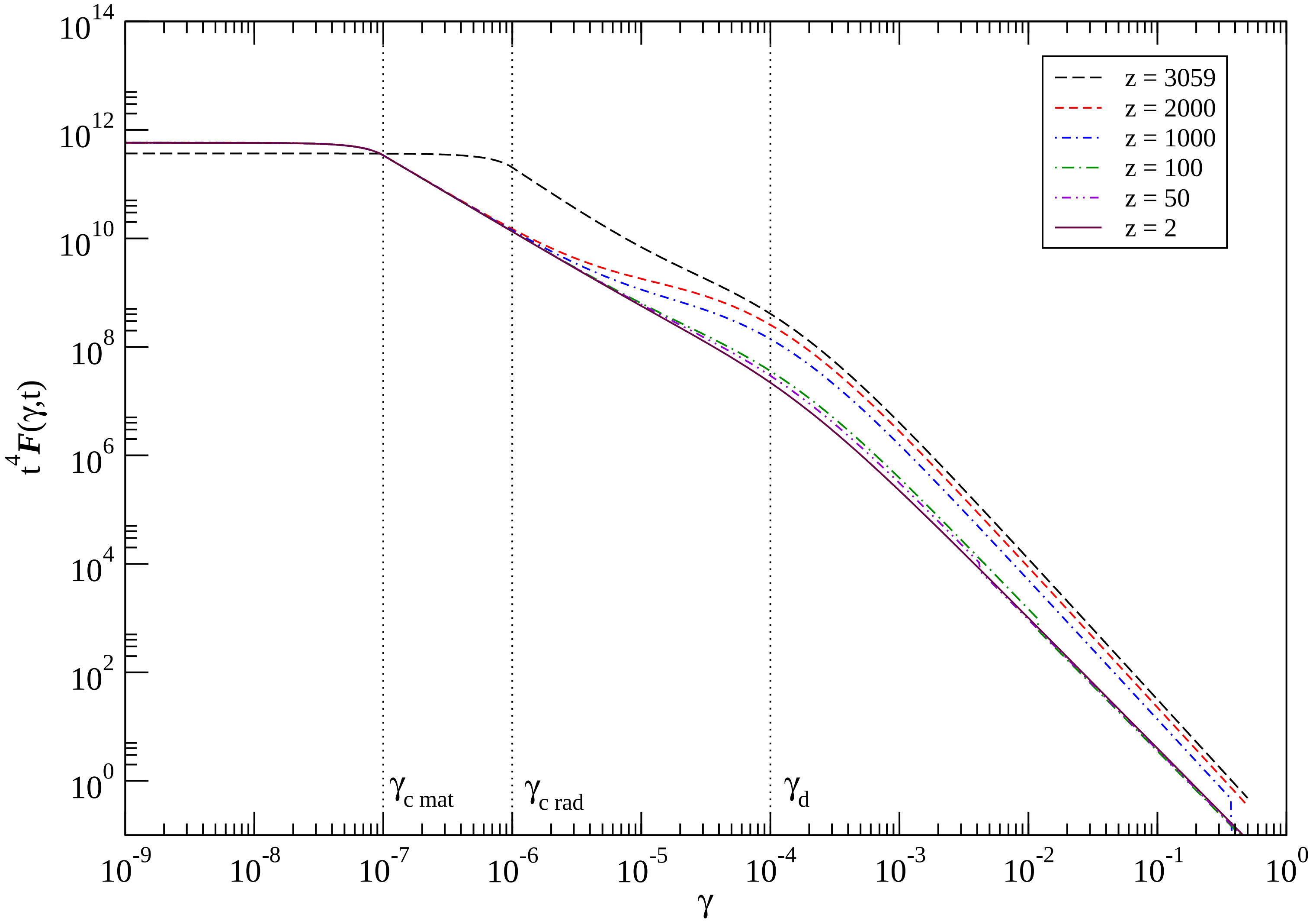
Cosmological loop distribution

- Can be completely solved analytically (see arXiv.1006.0931)
- Scaling attractor does not depend on $\mathcal{N}_{\mathrm{ini}}$ nor on GW backreaction details



Relaxation effects are accounted

- Example: transition radiation–matter



Some numerical values

- Density parameter of cosmic string loops (assuming $\gamma_c \ll \gamma_d \ll 1$)

$$\left.\begin{array}{l} \rho_\circ = \dfrac{U}{t^2}\displaystyle\int_0^3 t^4 \mathcal{F}(\gamma, t)\gamma \,\mathrm{d}\gamma \\ C \equiv C_\circ (1-\nu)^{3-p}, \quad \chi = 1 - \dfrac{p}{2} \end{array}\right\} \Longrightarrow \Omega_\circ = \frac{3\pi^2 C}{(1-\chi)\sin(2\pi\chi)} \frac{GU}{\gamma_\mathrm{d}{}^{1-2\chi}}$$

  ✦ With NG typical values and $\gamma_\mathrm{d} \simeq 100 GU$ $(\gamma_\mathrm{d} t_0 < 380\,\mathrm{kpc})$

$$\Omega_\circ \simeq 0.10 (GU)^{0.59} < 10^{-5} \quad \text{(with current CMB bounds on } GU\text{)}$$

- Number density of cosmic strings loops in a box of size L (today)

$$t^3 n_L = \int_0^{L/t} t^4 \mathcal{F}(\gamma, t)\,\mathrm{d}\gamma \simeq \frac{C}{\gamma_\mathrm{d} \gamma_\mathrm{c}{}^{1-2\chi}}$$

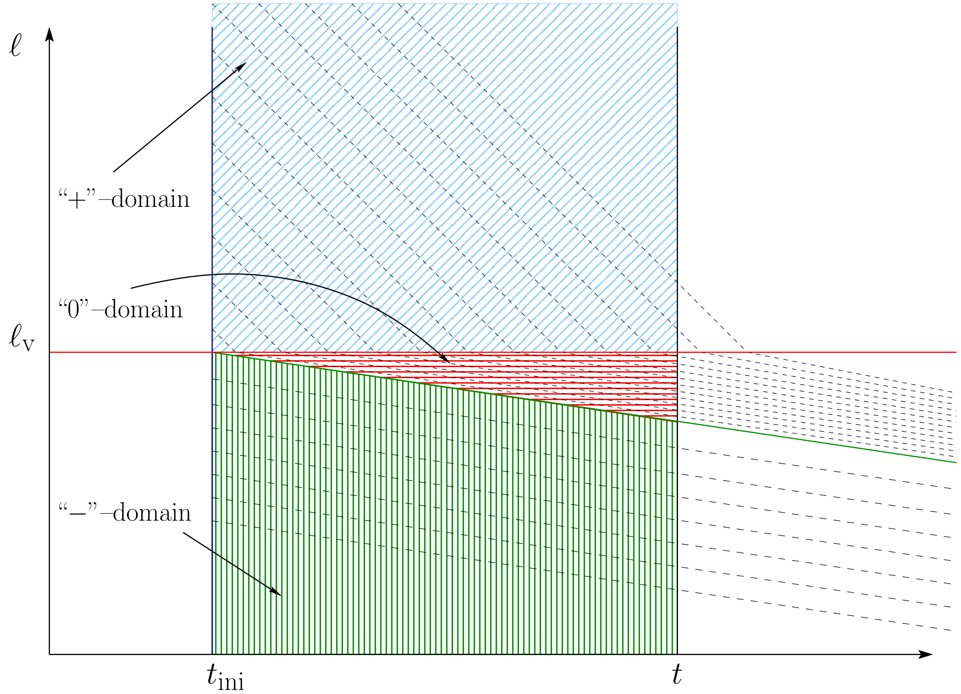
  ✦ From PR model [Polchinski:2007rg]: $\gamma_\mathrm{c} \simeq 10(GU)^{1+2\chi}$ $(\gamma_\mathrm{c} t_0 < 8\,\mathrm{pc})$

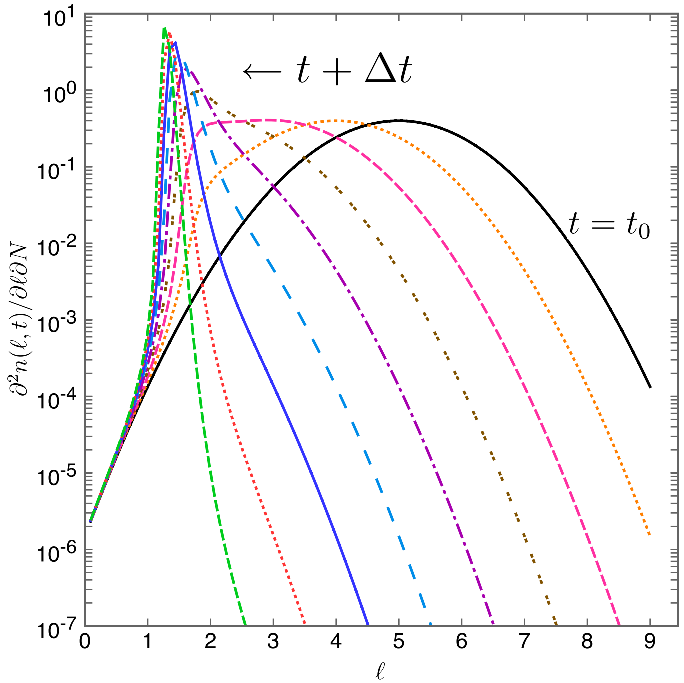
$$t^3 n_L \simeq 6.1 \times 10^{-5} (GU)^{-1.65} > 5.5 \times 10^{-6}\,\mathrm{Mpc}^{-3}$$

Extension to vortons

- Boltzmann equation for current carrying loops [Peter:2013jj]: $n(\ell, t, N)$

$$\frac{\partial}{\partial t}\left[a^3 \mathcal{J}(\ell,t)\frac{\partial^2 n}{\partial\ell\partial N}\right] - \left[\gamma_{\mathrm{d}}\Theta\left(\ell - \frac{N}{\sqrt{U}}\right) + \gamma_{\mathrm{v}}\Theta\left(\frac{N}{\sqrt{U}} - \ell\right)\right]\frac{\partial}{\partial \ell}\left[a^3 \mathcal{J}(\ell,t)\frac{\partial^2 n}{\partial\ell\partial N}\right]$$
$$= a^3 \mathcal{J}(\ell,t)\mathcal{P}(\ell,t)\delta\left(N - \sqrt{\frac{\ell}{\lambda}}\right)$$





- Again exactly solvable for any $\mathcal{N}_{\mathrm{ini}}(\ell)$ (see arXiv:1302.0953)

Cosmological signatures

Outline

Cosmological signatures

Power spectrum of string induced anisotropies
String-induced CMB distorsions
Small angles and flat sky limit
Systematics from loops not in scaling
String effects since last scattering
Basic non-Gaussian estimators
Observable string correlators
Bispectrum of string induced CMB anisotropies
Bispectrum comes from expansion
Isoscele triangle configurations
Compatible with small angle simulated maps

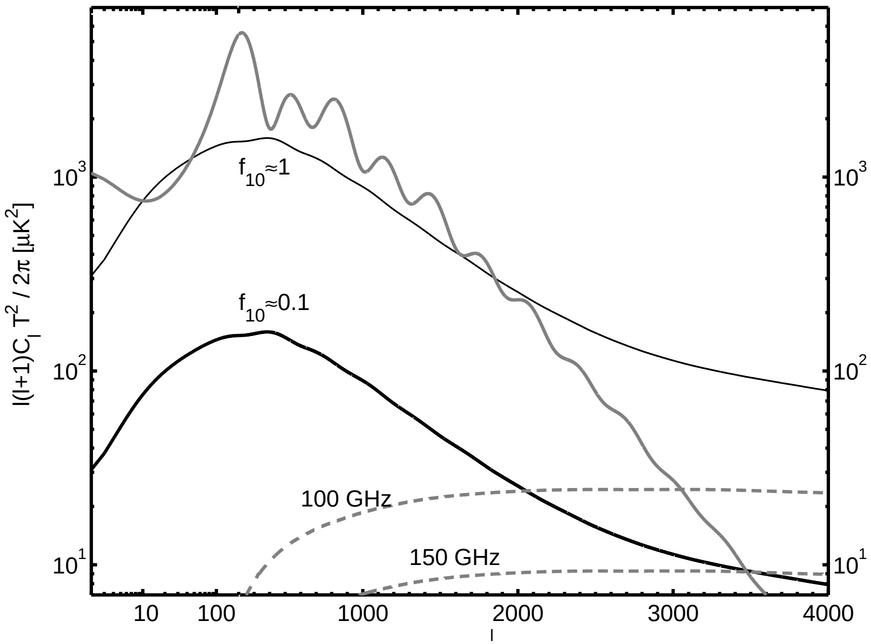
Power spectrum of string induced anisotropies

- External source of cosmological perturbations: $\mathcal{D}\mathcal{X} = \mathcal{S}$ [Durrer:1997ep, Bevis:2006mj,Urrestilla:2007sf,]

$$\langle \mathcal{X}^\dagger(\eta_0,k)\mathcal{X}(\eta_0,k)\rangle \propto \iint^{\eta_0} G_k^\dagger(\eta')G_k(\eta)\langle \mathcal{S}^\dagger(\eta',k)\mathcal{S}(\eta,k)\rangle \mathrm{d}\eta\mathrm{d}\eta'$$

- Abelian string simulations [Bevis:2010gj]

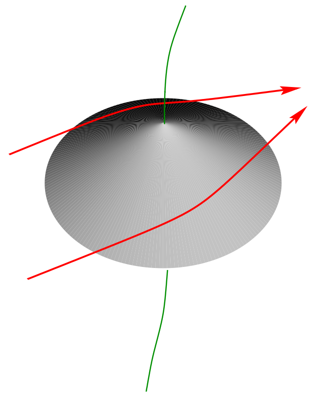
$$\langle \mathcal{S}^\dagger \mathcal{S}\rangle \propto \frac{1}{\sqrt{\eta\eta'}} f_{\mu\nu\rho\sigma}\left(k\sqrt{\eta\eta'}, \frac{\eta}{\eta'}\right)$$



- Planck + WP + ACTSPT + BICEP2 [Lizarraga:2014xza]
- Fraction (at $\ell = 10$) $\leq 2\%$
- Tension $GU \leq 3 \times 10^{-7}$
- Dominate at $\ell > 3000$?

String-induced CMB distorsions

- Nambu–Goto strings ($U = T$): no static gravitational effects
- Do have General Relativity effects on light and thus on CMB (Gott-Kaiser-Stebbins)



- ISW from Nambu–Goto stress tensor + Einstein equations: [Hindmarsh 94, Stebbins 95]

$$\Theta(\hat{n}) \equiv \frac{\delta T}{T_{\mathrm{CMB}}} = -4GU \int_{\boldsymbol{X} \cap \boldsymbol{x}_\gamma} \left[\boldsymbol{u}(\hat{n}) \cdot \frac{\boldsymbol{X}_\perp}{\boldsymbol{X}_\perp^2}\right] \left(1 + \hat{n} \cdot \dot{\boldsymbol{X}}\right) \mathrm{d}\sigma$$

$$\boldsymbol{u} = \dot{\boldsymbol{X}} - \frac{(\hat{n} \cdot \boldsymbol{X}') \cdot \boldsymbol{X}'}{1 + \hat{n} \cdot \dot{\boldsymbol{X}}} \qquad \boldsymbol{X}_\perp \equiv X\hat{n} - \boldsymbol{X}$$

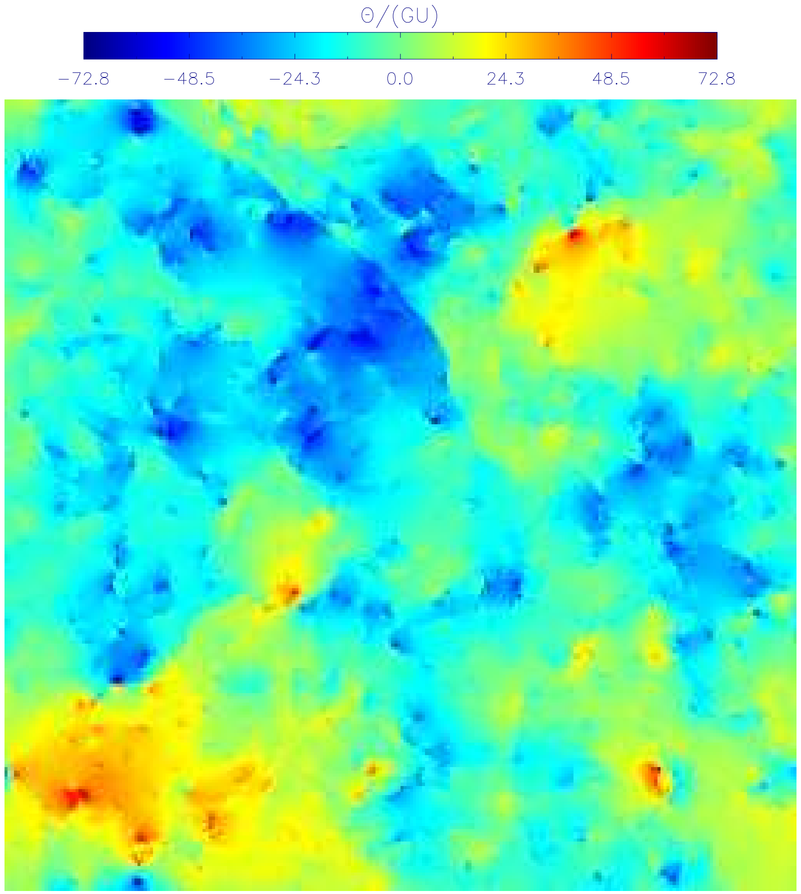
- At small angular scales, in 2D transverse Fourier space ($\boldsymbol{k} \cdot \hat{n} \simeq 0$):

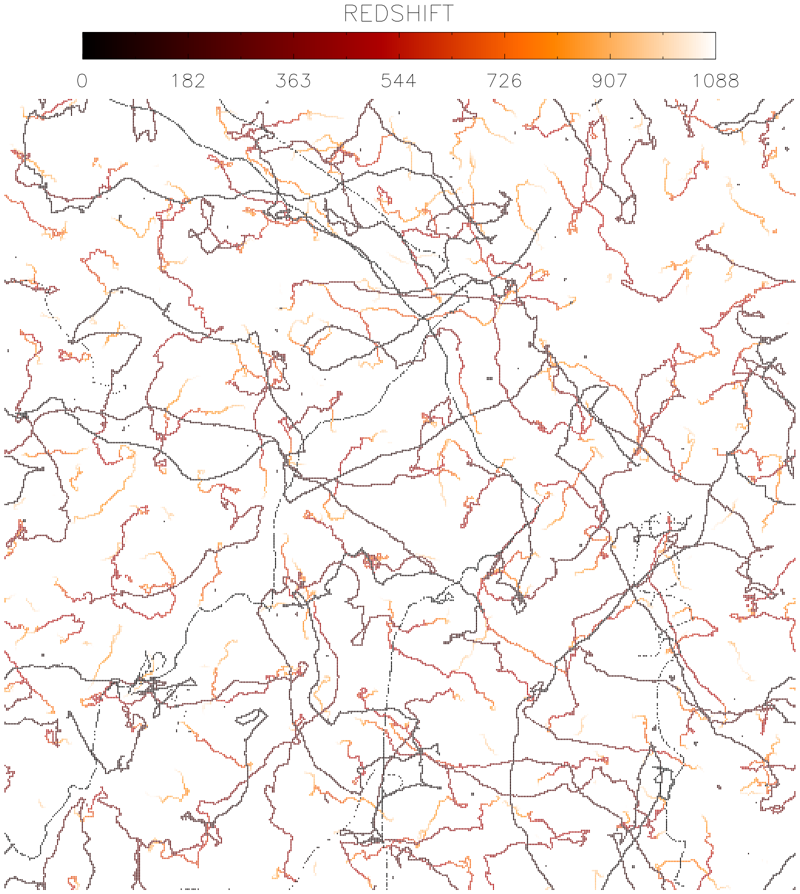
$$\Theta \simeq \frac{8\pi i\, GU}{\boldsymbol{l}^2} \int_{\boldsymbol{X} \cap \boldsymbol{x}_\gamma} (\boldsymbol{u} \cdot \boldsymbol{l})\, \mathrm{e}^{-i\,\boldsymbol{l} \cdot \boldsymbol{X}}\, \mathrm{d}\sigma$$

Small angles and flat sky limit

- Statistics: 1000 independant maps on a $7.2°$ field of view
- Temperature anisotropies from long strings and loops in scaling

[Fraisse:2007nu]

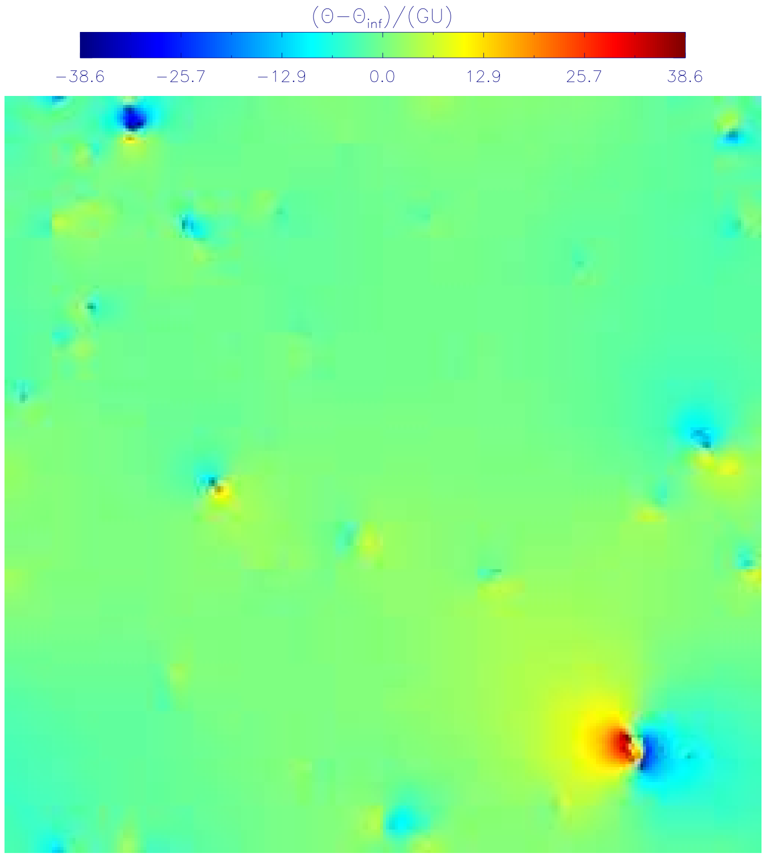




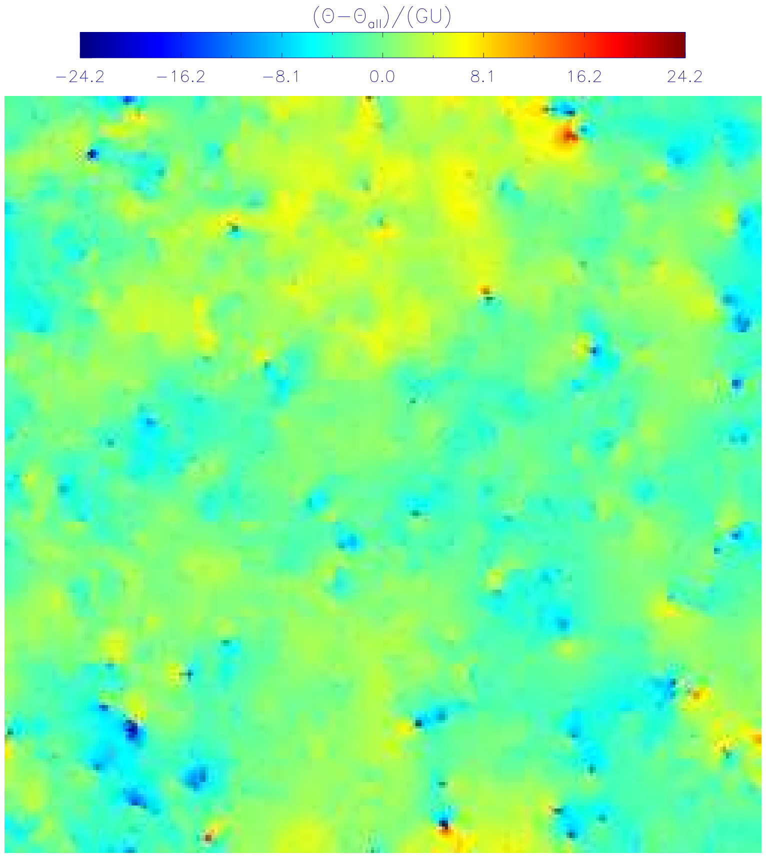
Systematics from loops not in scaling

- Maps with no loops or with all loops

no loops



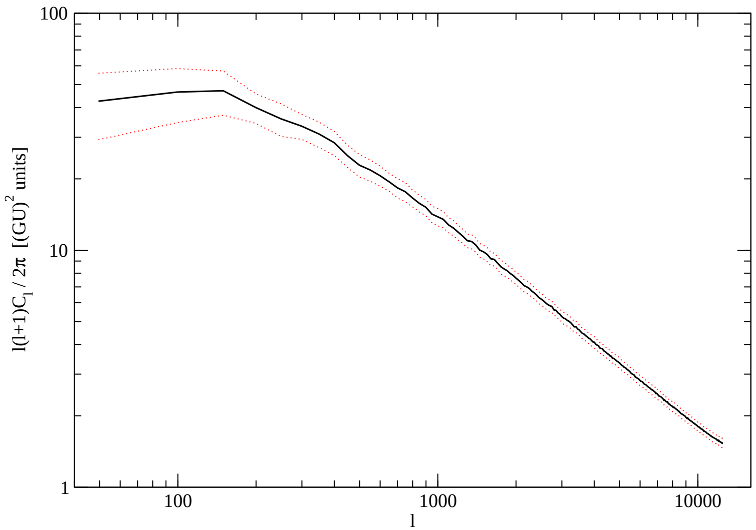
all structures, including IC effects

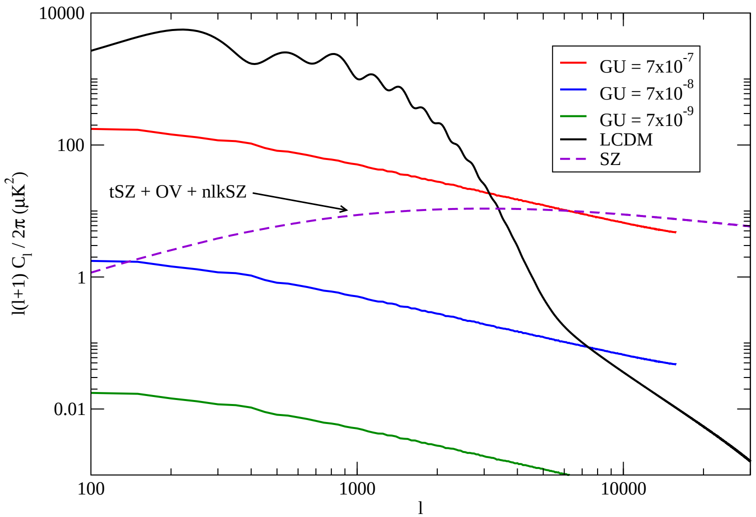


- Mostly renormalize the amplitude by at most a few percents

String effects since last scattering

- ISW from strings explains the power spectrum at the large multipoles





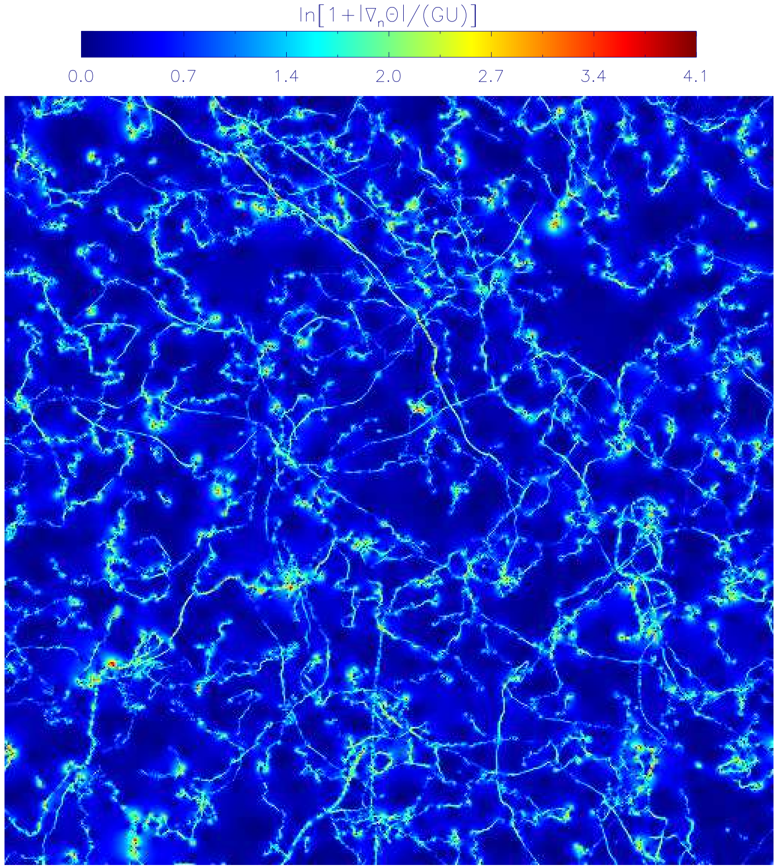
- Amplitude at $\ell = 1000$: $\ell(\ell+1)\,C_\ell/(2\pi) \simeq 14\,(GU)^2$
  - Compatible with Abelian Higgs power spectrum
- Variance: $\sigma^2 \simeq (150.7 \pm 18)\,(GU)^2$
- Power law behaviour at small scales

$$\ell(\ell+1)\,C_\ell \underset{\ell \gg 1}{\propto} \ell^{-p} \quad \text{with} \quad p = 0.889^{+0.001}_{-0.090}$$

Basic non-Gaussian estimators

- Gradient magnitude

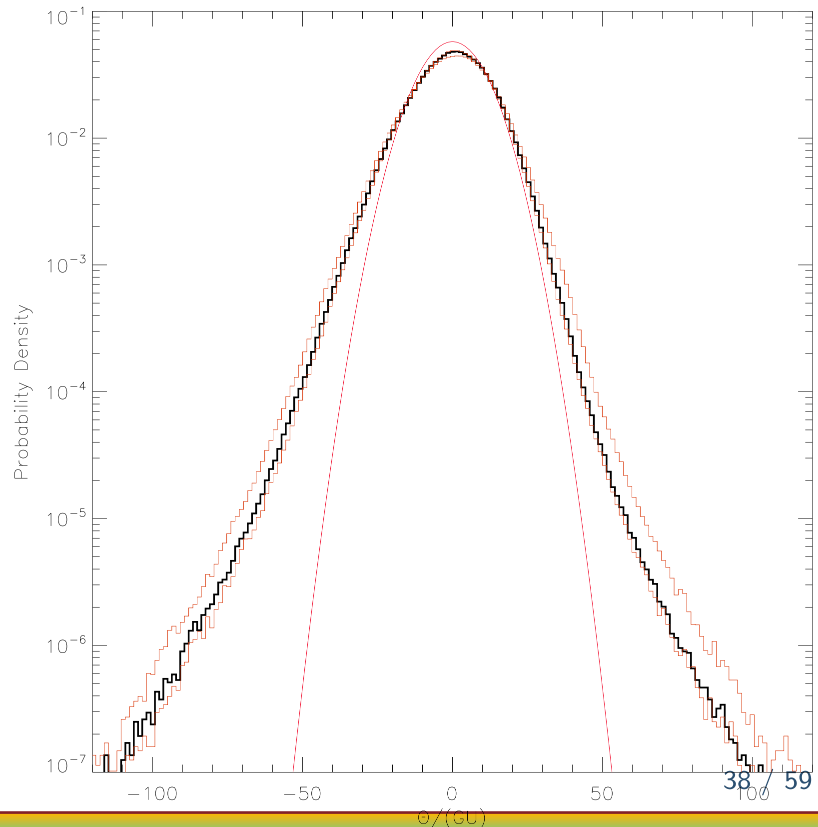
$$|\nabla\Theta| \equiv \sqrt{\left(\frac{\mathrm{d}\Theta}{\mathrm{d}\alpha}\right)^2 + \left(\frac{\mathrm{d}\Theta}{\mathrm{d}\beta}\right)^2}$$



- One-point functions

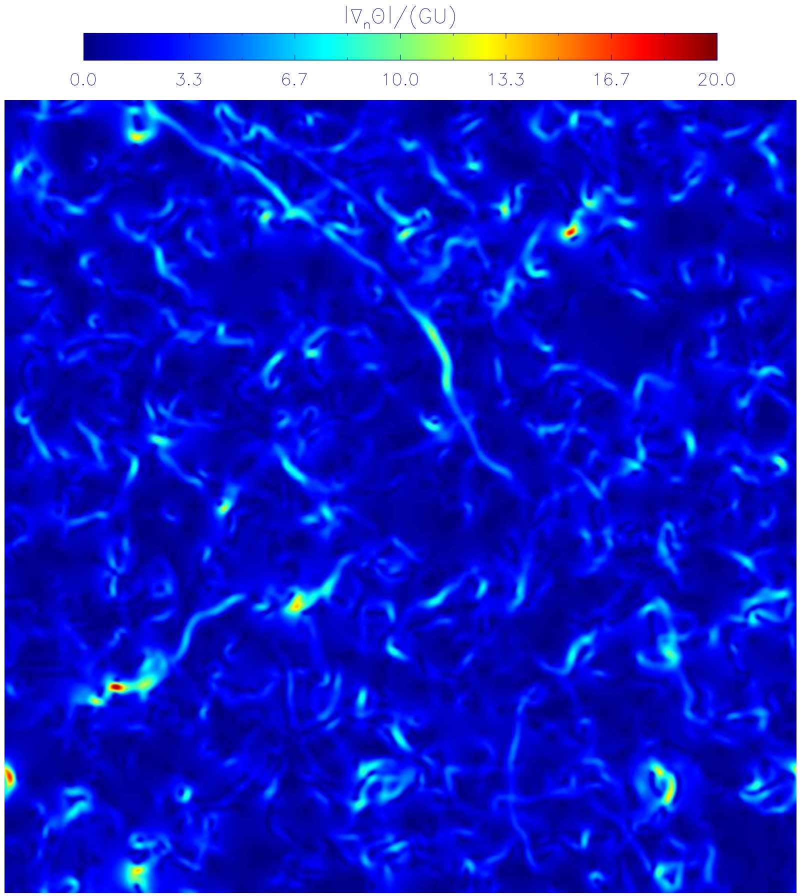
$$g_1 \equiv \left\langle \frac{\overline{(\Theta - \bar{\Theta})^3}}{\sigma^3} \right\rangle \simeq -0.22 \pm 0.12$$

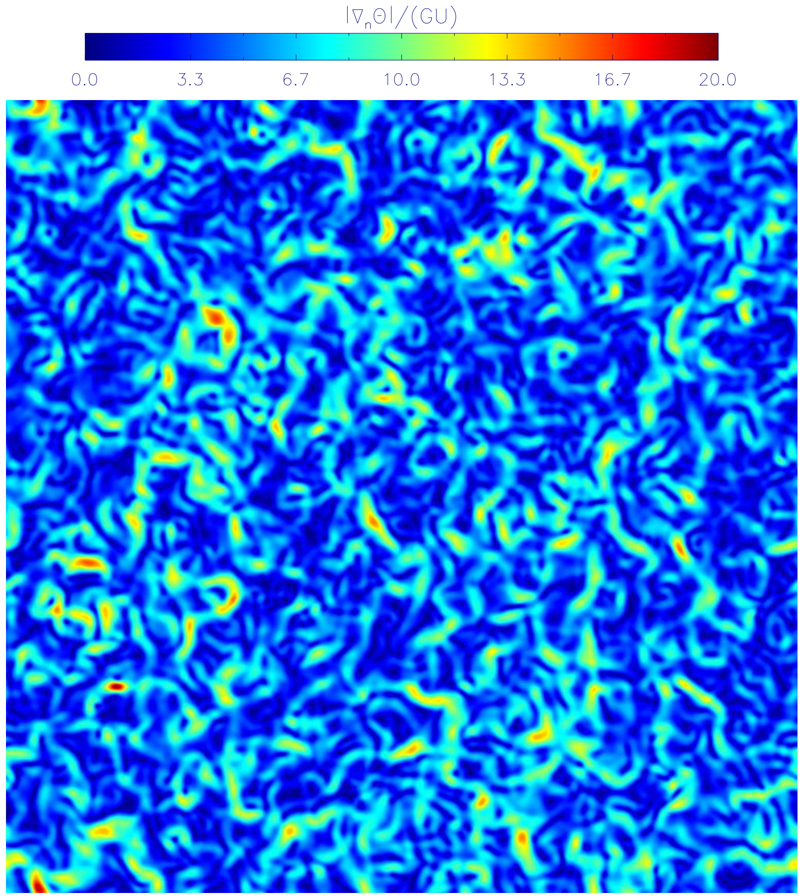
$$g_2 \equiv \left\langle \frac{\overline{(\Theta - \bar{\Theta})^4}}{\sigma^4} \right\rangle - 3 \simeq 0.69 \pm 0.29.$$



Not enough for detection?

- Experimental beam damps the signal: PLANCK $217\,\mathrm{GHz}$
  - ✦ One-point function is nearly Gaussian, up to the rare events.
  - ✦ Gradient magnitude is sensitive to all: inf + SZ + stgs





Three-point function of the CMB anisotropies

- Non-vanishing skewness $\Rightarrow$ 3-pts function $\neq 0$

$$\langle \hat{\Theta}_{\boldsymbol{k}_1} \hat{\Theta}_{\boldsymbol{k}_2} \hat{\Theta}_{\boldsymbol{k}_3} \rangle = B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)(2\pi)^2 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$$

- From ISW, can be evaluated analytically at small angle

[Hindmarsh:2009qk,Ringeval:2010ca]

  - ✦ Calculation easier in the light cone gauge (instead of temporal)

$$\tau = X^0 + X^3 \implies \boldsymbol{u} = \dot{\boldsymbol{X}}$$

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = i\epsilon^3 \frac{1}{\mathcal{A}} \frac{k_{1_A} k_{2_B} k_{3_C}}{k_1^2 k_2^2 k_3^2} \int \mathrm{d}\sigma_1 \mathrm{d}\sigma_2 \mathrm{d}\sigma_3 \left\langle \dot{X}_1^A \dot{X}_2^B \dot{X}_3^C e^{i\delta^{ab} \boldsymbol{k}_a \cdot \boldsymbol{X}_b} \right\rangle$$

with $\dot{X}_a^A = \dot{X}^A(\sigma_a)$, $a, b \in \{1, 2, 3\}$, $\epsilon = 8\pi G U$

- Assuming $\dot{\boldsymbol{X}}$ and $\acute{\boldsymbol{X}}$ are Gaussian random variables

$$\left\langle C^{ABC} e^{iD} \right\rangle = i \left\langle C^{ABC} D \right\rangle e^{-\langle D^2 \rangle/2}$$

Observable string correlators

- Expand everything in terms of two-point correlators

$$\langle C^{ABC} D \rangle = \frac{1}{4}\delta^{AB}\left[k_1^C \Pi(\sigma_{13}) + k_2^C \Pi(\sigma_{23})\right] V(\sigma_{12}) + \circlearrowleft$$

$$\langle D^2 \rangle = -\frac{1}{2}[\boldsymbol{k}_1 \cdot \boldsymbol{k}_3 \Gamma(\sigma_{13}) + \boldsymbol{k}_2 \cdot \boldsymbol{k}_3 \Gamma(\sigma_{23}) + \boldsymbol{k}_1 \cdot \boldsymbol{k}_2 \Gamma(\sigma_{12})]$$

$$\Gamma(\sigma - \sigma') \equiv \left\langle [\boldsymbol{X}(\sigma) - \boldsymbol{X}(\sigma')]^2 \right\rangle = \int_{\sigma'}^{\sigma} \mathrm{d}\sigma_1 \int_{\sigma'}^{\sigma} \mathrm{d}\sigma_2 T(\sigma_1 - \sigma_2)$$

$$\Pi(\sigma - \sigma') \equiv \left\langle [\boldsymbol{X}(\sigma) - \boldsymbol{X}(\sigma'))] \cdot \dot{\boldsymbol{X}}(\sigma') \right\rangle = \int_{\sigma'}^{\sigma} \mathrm{d}\sigma_1 M(\sigma_1 - \sigma')$$

- Depend on three functions

$$\left\langle \dot{X}^A(\sigma)\dot{X}^B(\sigma') \right\rangle = \frac{1}{2}\delta^{AB} V(\sigma - \sigma')$$

$$\left\langle \dot{X}^A(\sigma)\acute{X}^B(\sigma') \right\rangle = \frac{1}{2}\delta^{AB} M(\sigma - \sigma')$$

$$\left\langle \acute{X}^A(\sigma)\acute{X}^B(\sigma') \right\rangle = \frac{1}{2}\delta^{AB} T(\sigma - \sigma')$$

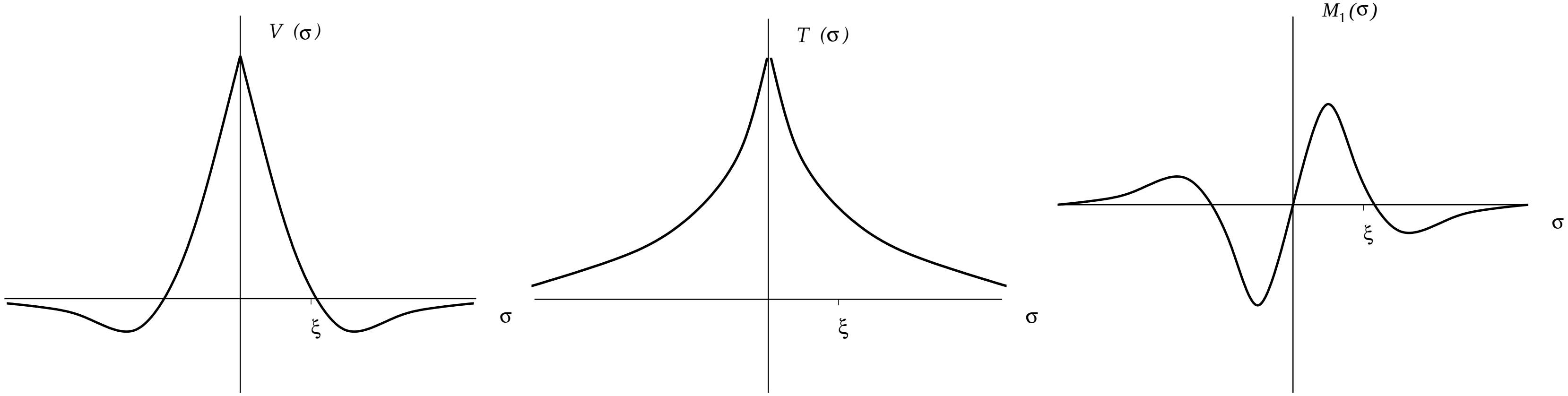
Bispectrum of string induced CMB anisotropies

- Integration can be done at large wavenumbers: $\kappa_{ab} \equiv \boldsymbol{k}_a \cdot \boldsymbol{k}_b \gg 1$

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\epsilon^3 \pi c_0 \frac{\bar{v}^2}{\bar{t}^4} \frac{L\hat{\xi}}{\mathcal{A}} \frac{1}{\hat{\xi}^2} \frac{1}{k_1^2 k_2^2 k_3^2} \left[\frac{k_1^4 \kappa_{23} + k_2^4 \kappa_{31} + k_3^4 \kappa_{12}}{\left(\kappa_{23}\kappa_{31} + \kappa_{12}\kappa_{31} + \kappa_{12}\kappa_{23}\right)^{3/2}} \right]$$

- Sensitive to the (averaged projected) small scales $\sigma \to 0$ [Hindmarsh:1995]

$$V(\sigma) \sim \bar{v}^2, \qquad \Gamma(\sigma) \sim \bar{t}^2 \sigma^2, \qquad \Pi(\sigma) \sim \frac{1}{2}\frac{c_0}{\hat{\xi}}\sigma^2 \quad [\hat{\xi} \equiv \Gamma'(\infty)]$$



Bispectrum comes from expansion

- Proportional to $c_0 \equiv \hat{\xi} \left\langle \ddot{\boldsymbol{X}} \cdot \dot{\boldsymbol{X}} \right\rangle \neq 0$?
  - Light cone gauge + FLRW + $\dot{\boldsymbol{X}}$, $\acute{\boldsymbol{X}}$ Gaussian random variables

$$\left\langle \ddot{\boldsymbol{X}} \cdot \dot{\boldsymbol{X}} \right\rangle = \bar{\mathcal{H}} \left(\left\langle \dot{\boldsymbol{X}}^2 \right\rangle \left\langle \acute{\boldsymbol{X}}^2 \right\rangle - \left\langle \dot{\boldsymbol{X}} \cdot \acute{\boldsymbol{X}} \right\rangle^2 \right) = \bar{\mathcal{H}} \bar{v}^2 \bar{t}^2$$

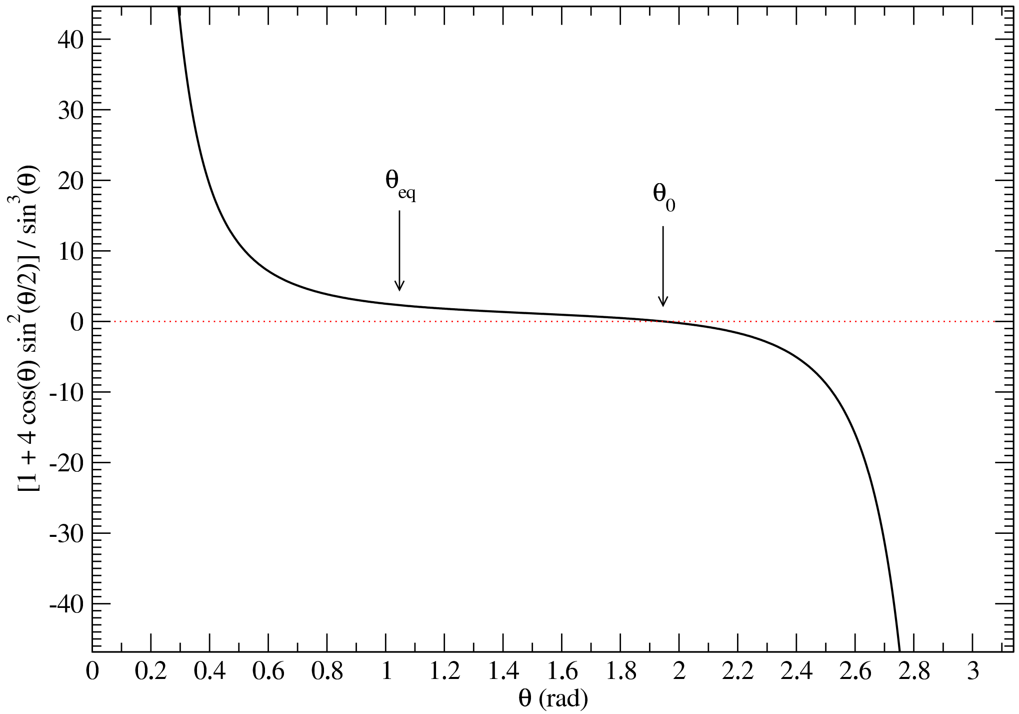
  - For $\bar{\mathcal{H}} > 0 \Rightarrow c_0 > 0$: breaking of time reversal invariance
- String bispectrum exists only in an expanding universe
  - Gives a negative skewness by integration
  - Decays as a power law at small scales
  - This is the CMB temperature bispectrum (what you see!)
    - As opposed to primordial ($f_{\rm NL}$)

Isoscele triangle configurations

- Wavenumbers such that $k_1 = k_2 = k$ and $k_3 = 2k\sin(\theta/2)$

$$B_{\ell\ell\theta}(k,\theta) = -\epsilon^3 \pi c_0 \frac{\bar{v}^2}{\bar{t}^4} \frac{L\hat{\xi}}{\mathcal{A}} \frac{1}{\hat{\xi}^2 k^6} \frac{1+4\cos\theta\sin^2(\theta/2)}{\sin^3\theta}$$

- Amplified on elongated triangles; $\pm$ at $\theta_0 = 2\arccos\dfrac{\sqrt{3-\sqrt{3}}}{2}$

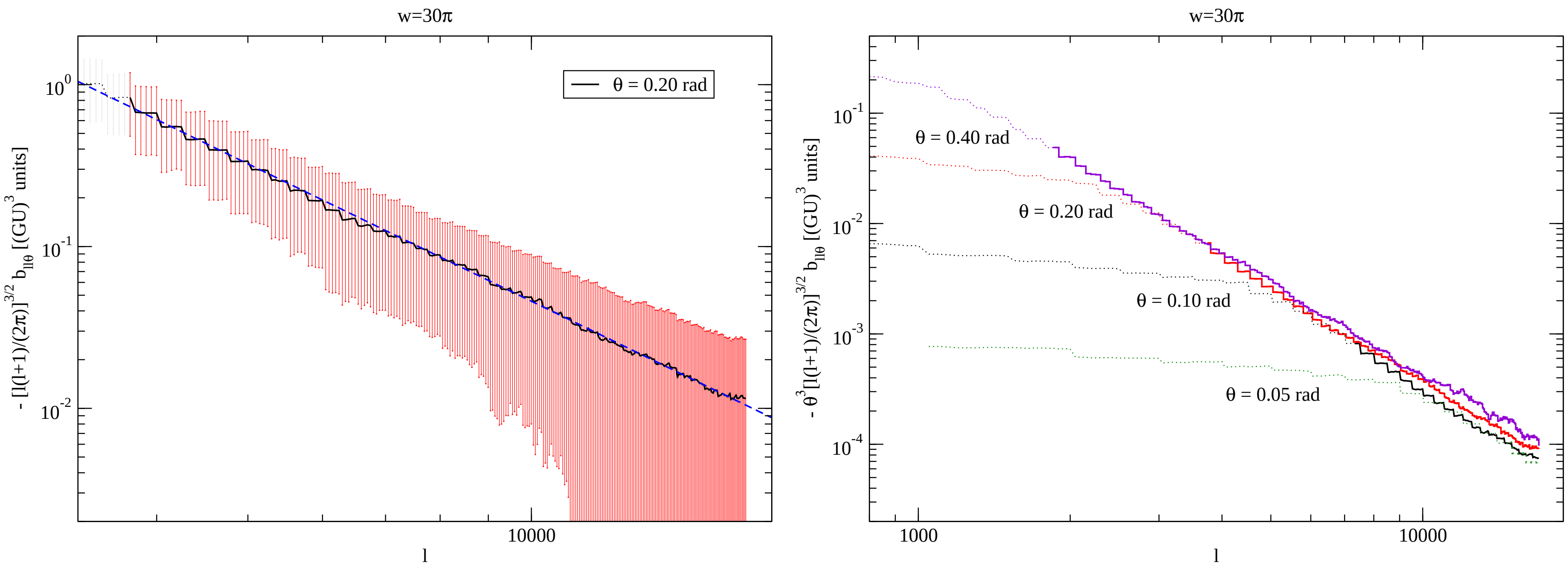


Compatible with small angle simulated maps

- Estimator: $\Theta_u(\boldsymbol{x}) \equiv \int \frac{\mathrm{d}\boldsymbol{l}}{(2\pi)^2} \hat{\Theta}_{\boldsymbol{l}} W_u(l)\, e^{-i\boldsymbol{l}\cdot\boldsymbol{x}}$

$$B_{k_1k_2k_3} = \frac{\left\langle \int \Theta_{k_1}(\boldsymbol{x})\Theta_{k_2}(\boldsymbol{x})\Theta_{k_3}(\boldsymbol{x})\mathrm{d}\boldsymbol{x} \right\rangle}{\int \frac{\mathrm{d}\boldsymbol{p}\mathrm{d}\boldsymbol{q}}{(2\pi)^4} W_{k_1}(p)\, W_{k_2}(q)\, W_{k_3}(|\boldsymbol{p}+\boldsymbol{q}|)}$$

- Power-law and dependency in θ recovered



String non-Gaussianities with Planck

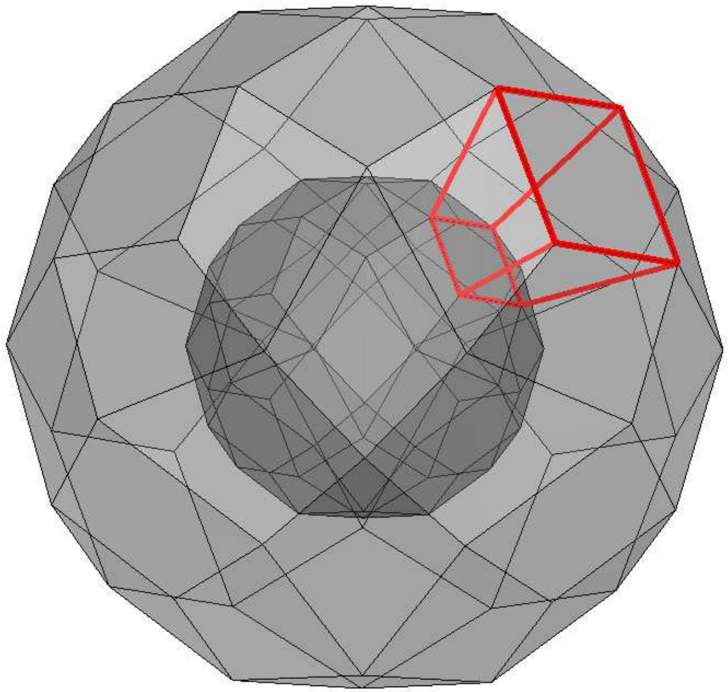
Outline

String non-Gaussianities with Planck

- Filling the transparent universe with strings
- Massively parallel ray tracing method
- After a million of cpu-hours
- Comparison between flat and full sky
- Non-Gaussian searches for cosmic strings

Filling the transparent universe with strings

- Searching for string NG with Planck requires full sky ⇒ simulations
  - Each simulation is a box of initial resolution 2000^3 (movie box)
  - Have to be stacked to fill 13 billion light years (HEALpix)



- This can be done with 3072 CS runs
- In which we propagate the CMB. . .

Massively parallel ray tracing method

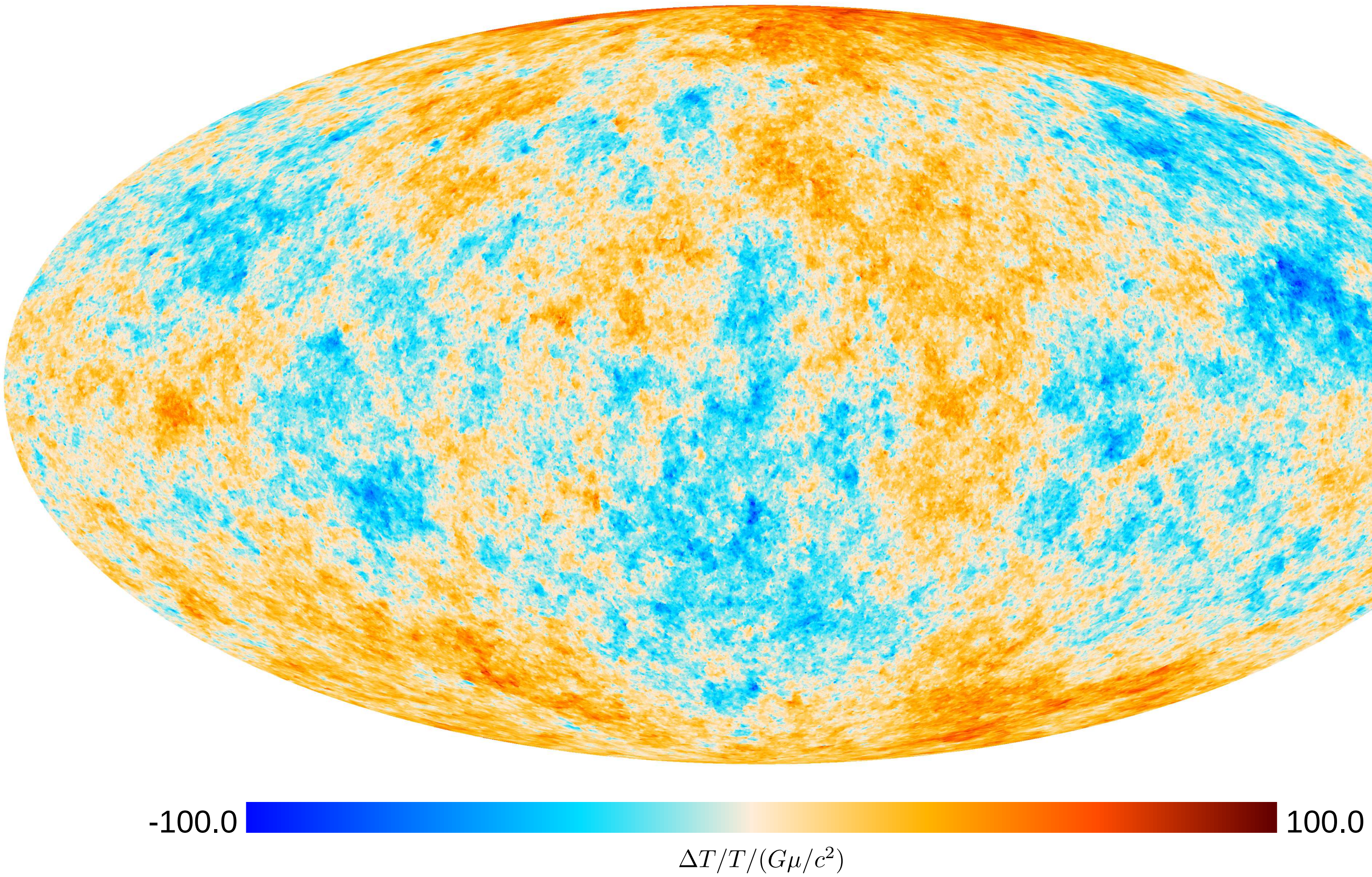
- Sky pixelized with $200\,000\,000$ lines of sight (4 times Planck maps)
  - Each direction receives cumulative contributions from all CS
  - Account for roughly 10^{17} iterations
- Parallelization implementation
  - `MPI` over the 3072 boxes + reduction
  - `OpenMP` over the $200\,000\,000$ pixels
  - `Vectorization` of the most inner loop (string segments)
- Code development performed on the `CP3-cosmo` cluster (100 cores)
- Reasonable computing time demands a 100 TeraFlops computer :-/
  - The Planck collaboration has a few... (thanks to J. Borrill) :)

512 nodes / 12K cores runs at NERSC

- National Energy Research Scientific Computing Center (Berkeley U.S.)
- The "Hopper" Cray XE6 machine (world rank 8 in Nov 2011)
  - ✦ More than 6000 nodes with Dual processor 24 cores
  - ✦ 3D Cray Gemini: Maximum injection bandwidth per node 20 GB/s

After a million of cpu-hours

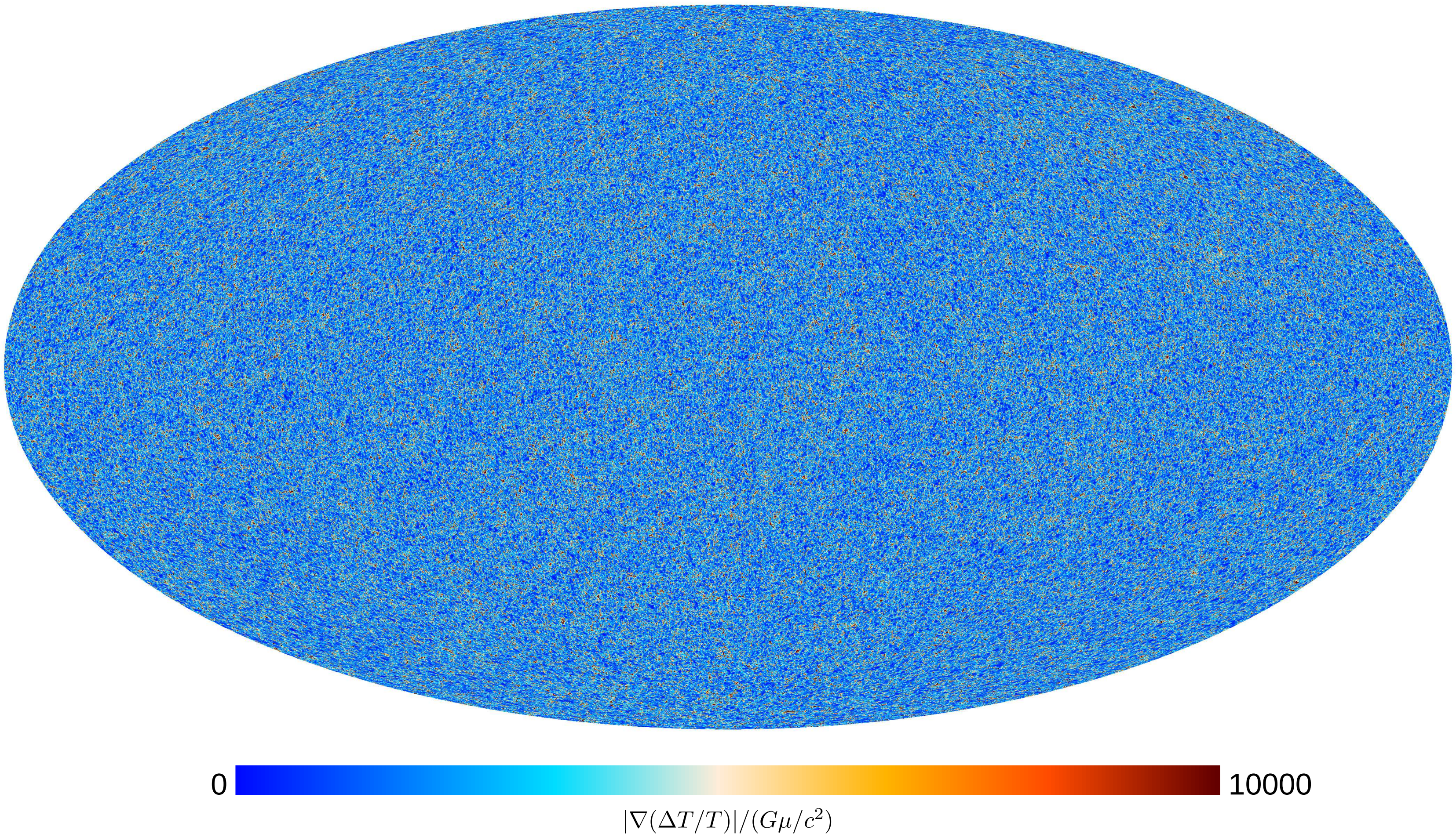
- Full sky synthetic string map of 2×10^8 pixels [Ringeval:2012tk, Ade:2013xla]
- Temperature anisotropies



- $\times 4$ for tests and string challenges

After a million of cpu-hours

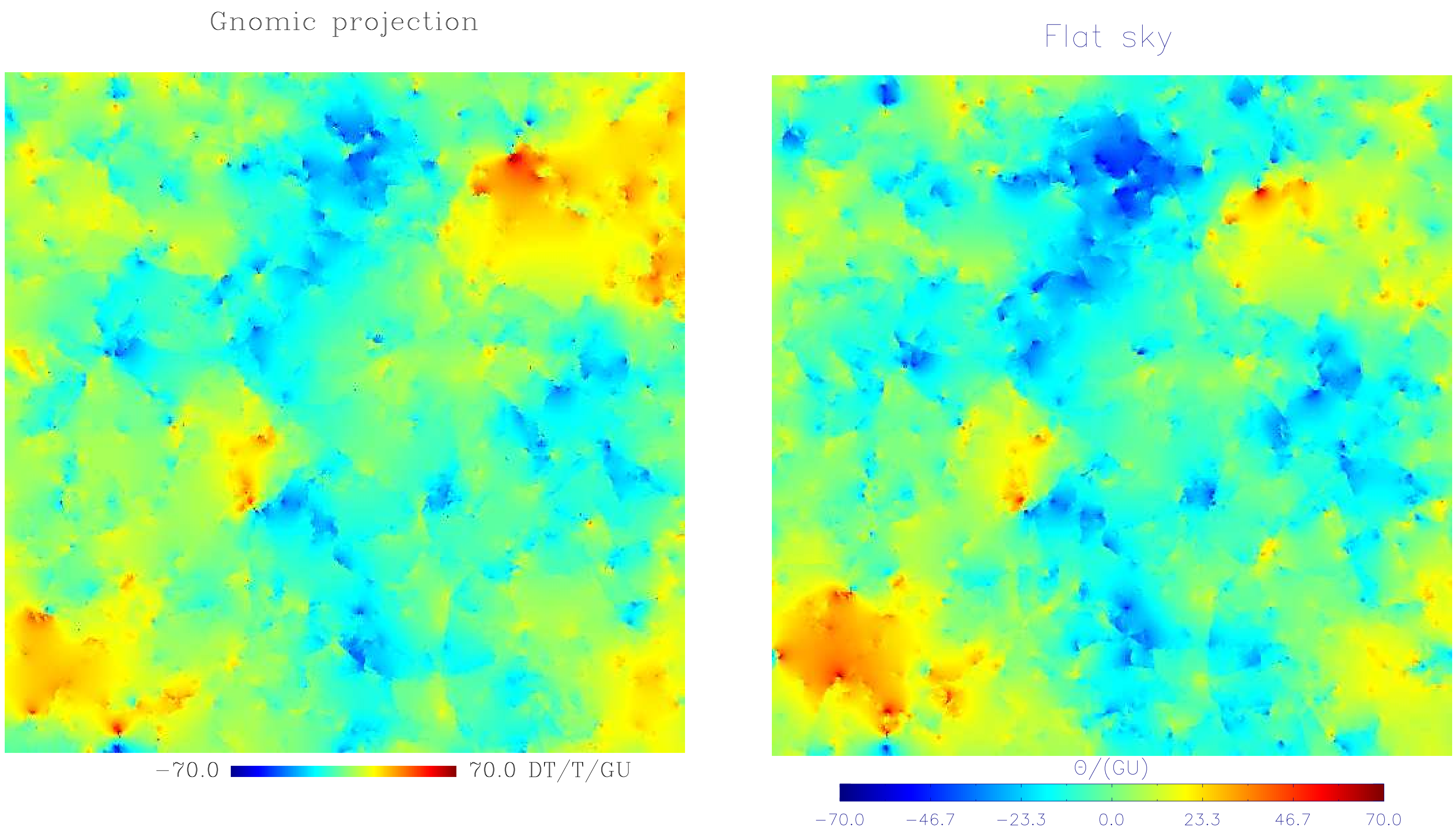
- Full sky synthetic string map of 2×10^8 pixels [Ringeval:2012tk, Ade:2013xla]
- Gradient magnitude



- $\times 4$ for tests and string challenges

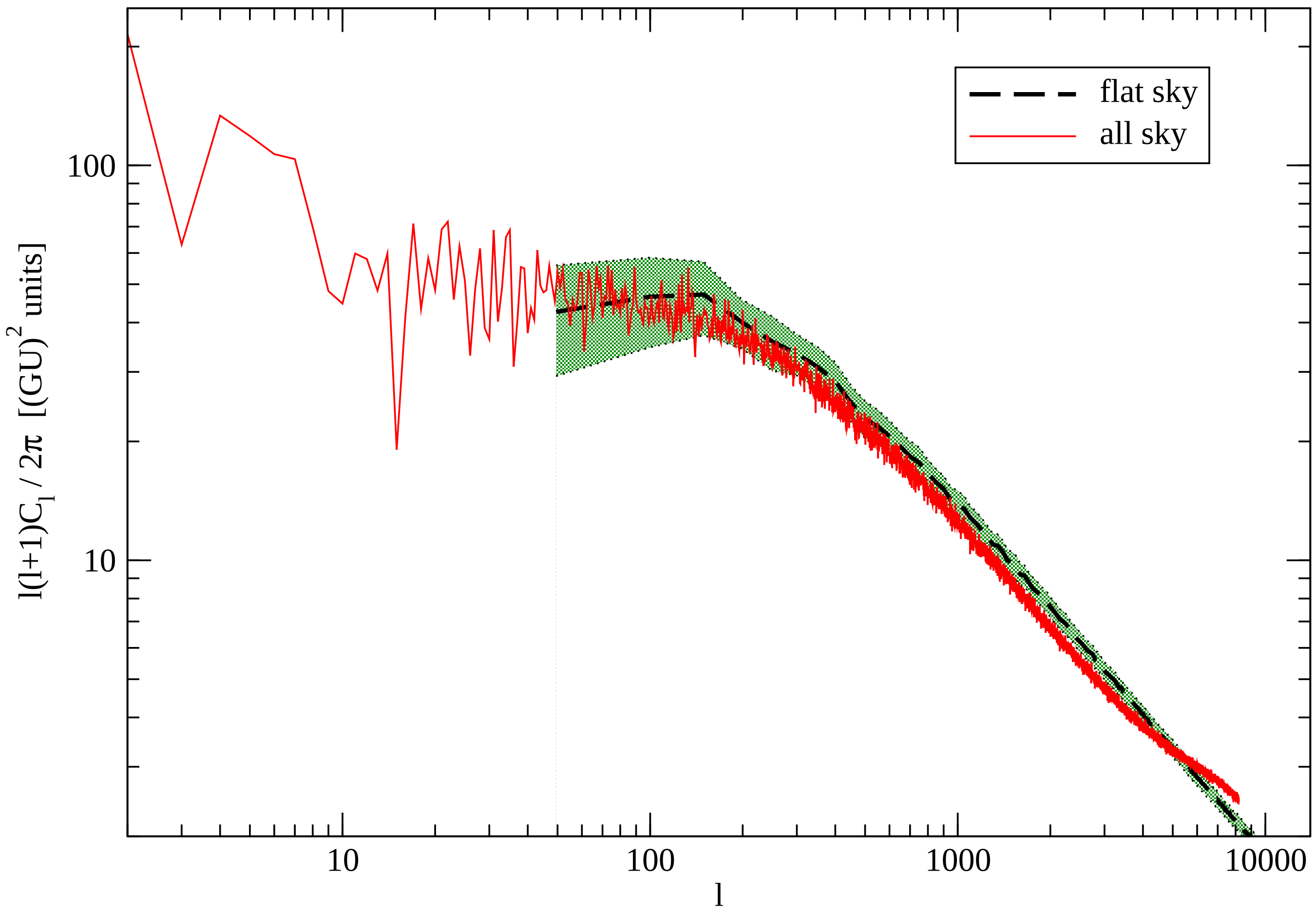
Comparison between flat and full sky

- Small spherical distorsions on the edges and smoother temperature contrasts



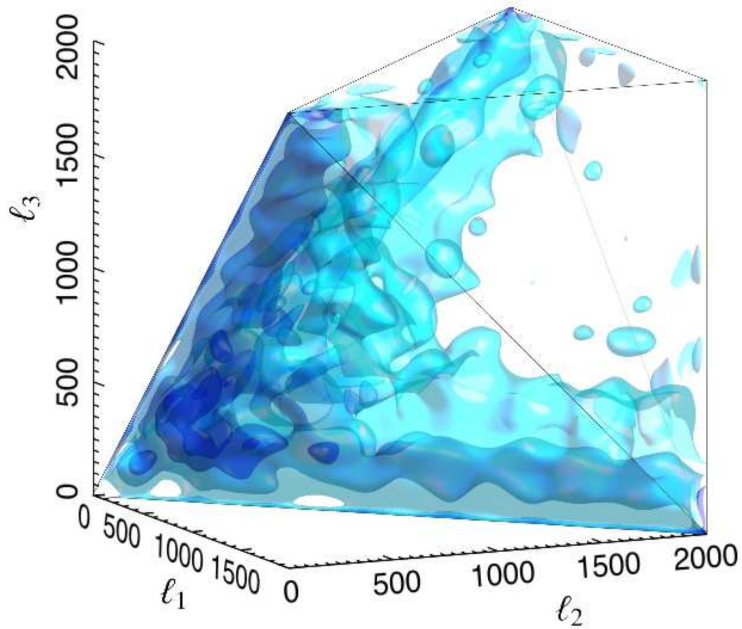
Comparison between flat and full sky

- CMB angular power spectrum match



Non-Gaussian searches for cosmic strings

- Full sky bispectrum [Ade:2013xla]



✦ Different methods used
  - Modal bispectrum
  - Wavelets
  - Minkowski functionals

- Planck constraints on cosmic strings non-Gaussianities

$$f_{\rm NL}^{\rm strg} = 0.30 \pm 0.21 \Rightarrow GU < 8.8 \times 10^{-7}$$
$$\text{Real space} \Rightarrow GU < 7.8 \times 10^{-7}$$

- Very robust (ISW only) but slightly weaker than power spectrum bounds $GU < 1.3 \times 10^{-7} \to 3.2 \times 10^{-7}$

Perspectives and conclusion

Outline

Perspectives and conclusion

Four-point function of the CMB anistropies
Loops and CMB trispectrum
Example: kite quadrilaterals
Conclusion

Four-point function of the CMB anistropies

- Same method as for the bispectrum with new features [Hindmarsh:2009es]

$$\left\langle \hat{\Theta}_{\boldsymbol{k}_1} \hat{\Theta}_{\boldsymbol{k}_2} \hat{\Theta}_{\boldsymbol{k}_3} \hat{\Theta}_{\boldsymbol{k}_4} \right\rangle = T(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)(2\pi)^2 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)$$

$$T_{1234} = \frac{\epsilon^4}{\mathcal{A}} \frac{k_{1A} k_{2B} k_{3C} k_{4D}}{k_1^2 k_2^2 k_3^2 k_4^2} \int \mathrm{d}\sigma_1 \mathrm{d}\sigma_2 \mathrm{d}\sigma_3 \mathrm{d}\sigma_4 \left\langle \dot{X}_1^A \dot{X}_2^B \dot{X}_3^C \dot{X}_4^D e^{i\delta^{ab} \boldsymbol{k}_a \cdot \boldsymbol{X}_b} \right\rangle$$

- Flat directions: sensitive to higher order in the correlators

$$\text{Polchinski–Rocha model} \Rightarrow T(\sigma) \simeq \bar{t}^2 - c_1 \left(\frac{\sigma}{\hat{\xi}}\right)^{2\chi}$$

- Trispectrum sensitives to the string microstructure!
  - ✦ $0 < \chi < 1$, $c_1 > 0$
  - ✦ NG: power-law exponent of the loop distribution
  - ✦ Other strings: related to the mean square velocity

Loops and CMB trispectrum

- CMB trispectrum from strings is sensitive to $\left\langle \acute{X}^A(\sigma)\acute{X}^B(\sigma')\right\rangle$

$$T_\infty(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \simeq \epsilon^4 \frac{\bar{v}^4}{\bar{t}^2}\frac{L\hat{\xi}}{\mathcal{A}}\left(c_1\hat{\xi}^2\right)^{-1/(2\chi+2)} f(\chi) g(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)$$

$$f(\chi) = \frac{\pi}{\chi+1}\Gamma\left(\frac{1}{2\chi+2}\right)\left[4(2\chi+1)(\chi+1)\right]^{1/(2\chi+2)}$$

- Geometrical factor scales as k^ρ: $\rho = 6 + 1/(1+\chi)$

$$g(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) = \frac{\kappa_{12}\kappa_{34}+\kappa_{13}\kappa_{24}+\kappa_{14}\kappa_{23}}{k_1^2k_2^2k_3^2k_4^2}\left[Y^2\right]^{-1/(2\chi+2)}$$

$$Y^2(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \equiv -\kappa_{12}\left(k_3^2k_4^2-\kappa_{34}^2\right)^{\chi+1} + \circlearrowleft,$$

- This is a consistency relation for loops production mechanism

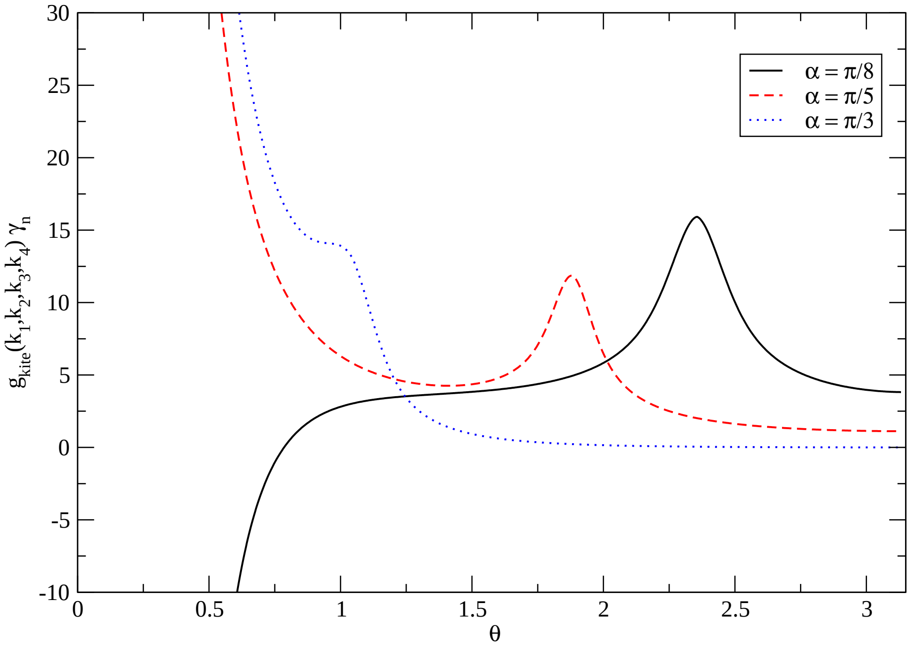
Example: kite quadrilaterals

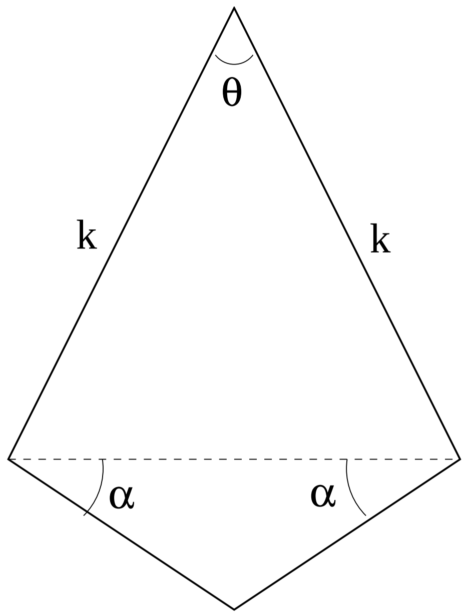
- Geometrical factor for kites: boost on elongated

$$g(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = \frac{\cos^2(\alpha)\left[1 - 2\cos(2\alpha)\cos(\theta)\right]}{\sin^2(\theta/2)} \frac{1}{k^{\rho} y^{2/(2+2\chi)}(\theta, \alpha)}$$

$$\rho = 6 + \frac{1}{1+\chi}$$

- Bump for parallelograms at $\theta = \pi - 2\alpha$ $(Y^2 = 0)$





Conclusion

- Currently: no string non-Gaussianities $\Longrightarrow$ $GU < 7.8 \times 10^{-7}$
- Future improvements
  - Searching for string induced trispectrum $\Longrightarrow$ window on their nature (trispectrum)
  - Next Planck data release + polarization + small scales experiments (BB [Seljak 06])
- Other observables than CMB: signal $\propto (GU)^{2,3,4}$
  - Galaxy surveys
  - GW direct detection (strongly depent on the loop distribution)
  - 21 cm
- Detecting strings would give a lower bound on the energy scale of inflation!