Non-linear effects in scalar-tensor gravity

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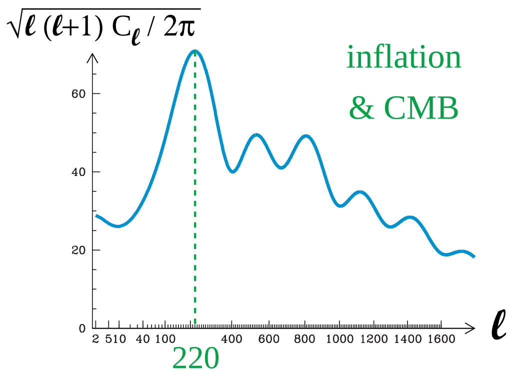
The most natural theories of gravity include
a scalar field φ besides the metric $g_{\mu\nu}$

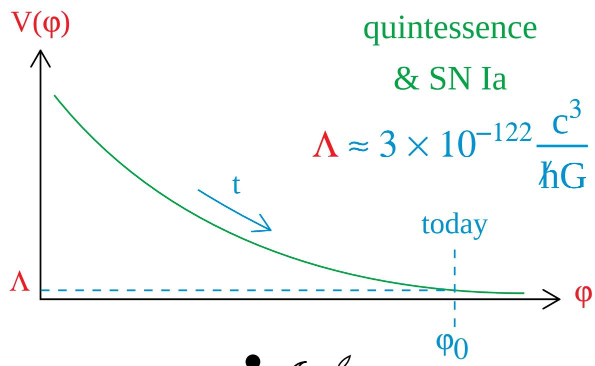
• Mathematically **consistent field theories** (no ghost, no adynamical field)

• **Motivated** by superstrings
– dilaton in the graviton supermultiplet
– moduli after dimensional reduction

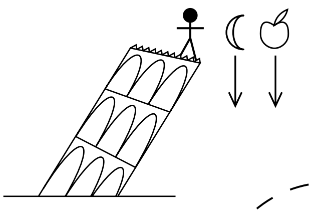
$$g_{mn} = \left(\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\nu & \varphi \end{array}\right)$$

• Scalar fields play a crucial role in modern **cosmology**
(potential $V(\varphi)$ ≈ negative pressure ⇒ accelerated expansion phases of the universe)

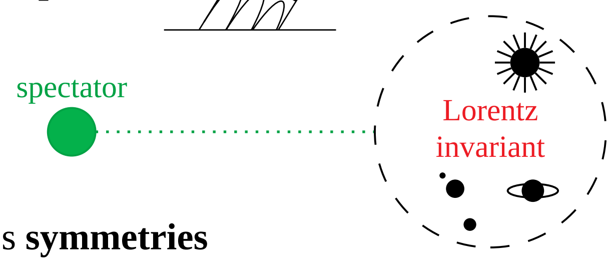




• Only consistent massless field theories able to satisfy the **weak equivalence principle**



• Only known theories satisfying "**extended Lorentz invariance**"

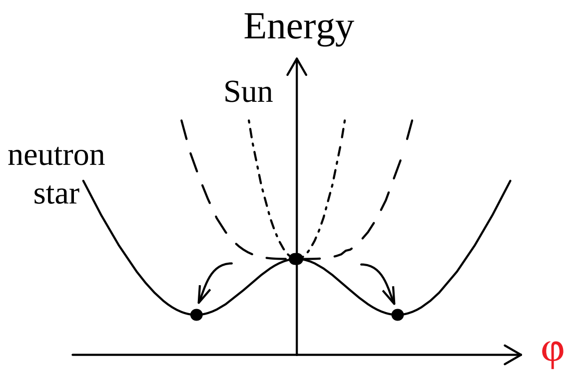


• Preserve most of general relativity's **symmetries**
(explain the key role of β^{PPN} and γ^{PPN})

• Useful as **contrasting alternatives** to general relativity
(simple, but general enough ⇒ many possible deviations)

Outline of the talk

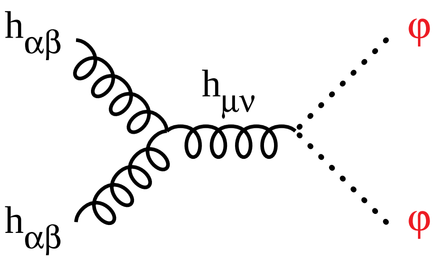
1• Nonperturbative effects in the strong-field regime



2• Highly non-linear effects caused by a scalar–Gauss-Bonnet coupling

$$W(\varphi)\left(R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2\right)$$

3• Instabilities caused by ghosts not manifest at the linear order



Tensor – scalar theories

spin 2 ↓ (R) spin 0 ↓ (φ)

$$S = \frac{c^3}{4\pi G} \int \sqrt{-g} \left\{ \frac{R}{4} - \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi) \right\}$$

$$+ S_{\text{matter}} \left[\text{matter} ; \tilde{g}_{\mu\nu} \equiv A^2(\varphi)\, g_{\mu\nu} \right]$$

↑ physical metric

Solar-system constraints

- "PPN" formalism to study weak-field gravity (order Newton $\times \frac{1}{c^2}$)

  [Eddington, Schiff, Baierlin, Nordtvedt, Will]

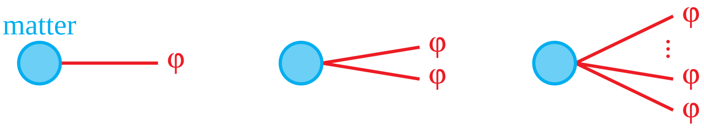
$$\begin{cases} -g_{00} = 1 - 2\dfrac{Gm}{rc^2} + 2\beta^{PPN}\left(\dfrac{Gm}{rc^2}\right)^2 + \dots \\ g_{ij} = \delta_{ij}\left[1 + 2\gamma^{PPN}\dfrac{Gm}{rc^2} + \dots\right] \end{cases}$$

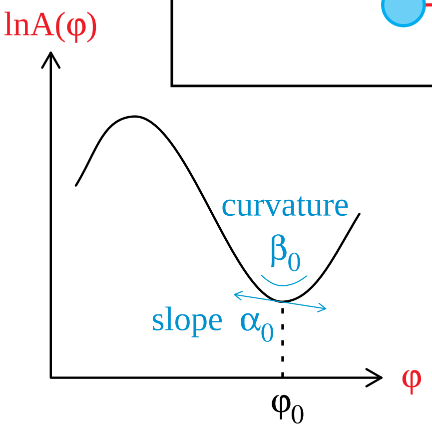
- In scalar-tensor gravity

If $V''(\varphi) = m_\varphi^2 \gg (\text{A.U.})^{-2} \Rightarrow \varphi$ negligible

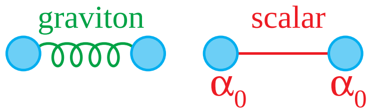
If $V''(\varphi) = m_\varphi^2 \ll (\text{A.U.})^{-2} \Rightarrow$ matter-scalar coupling function $A(\varphi)$ strongly constrained

$$\ln A(\varphi) = \alpha_0 (\varphi - \varphi_0) + \frac{1}{2}\beta_0 (\varphi - \varphi_0)^2 + \dots$$



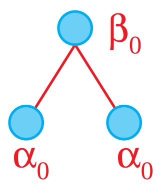


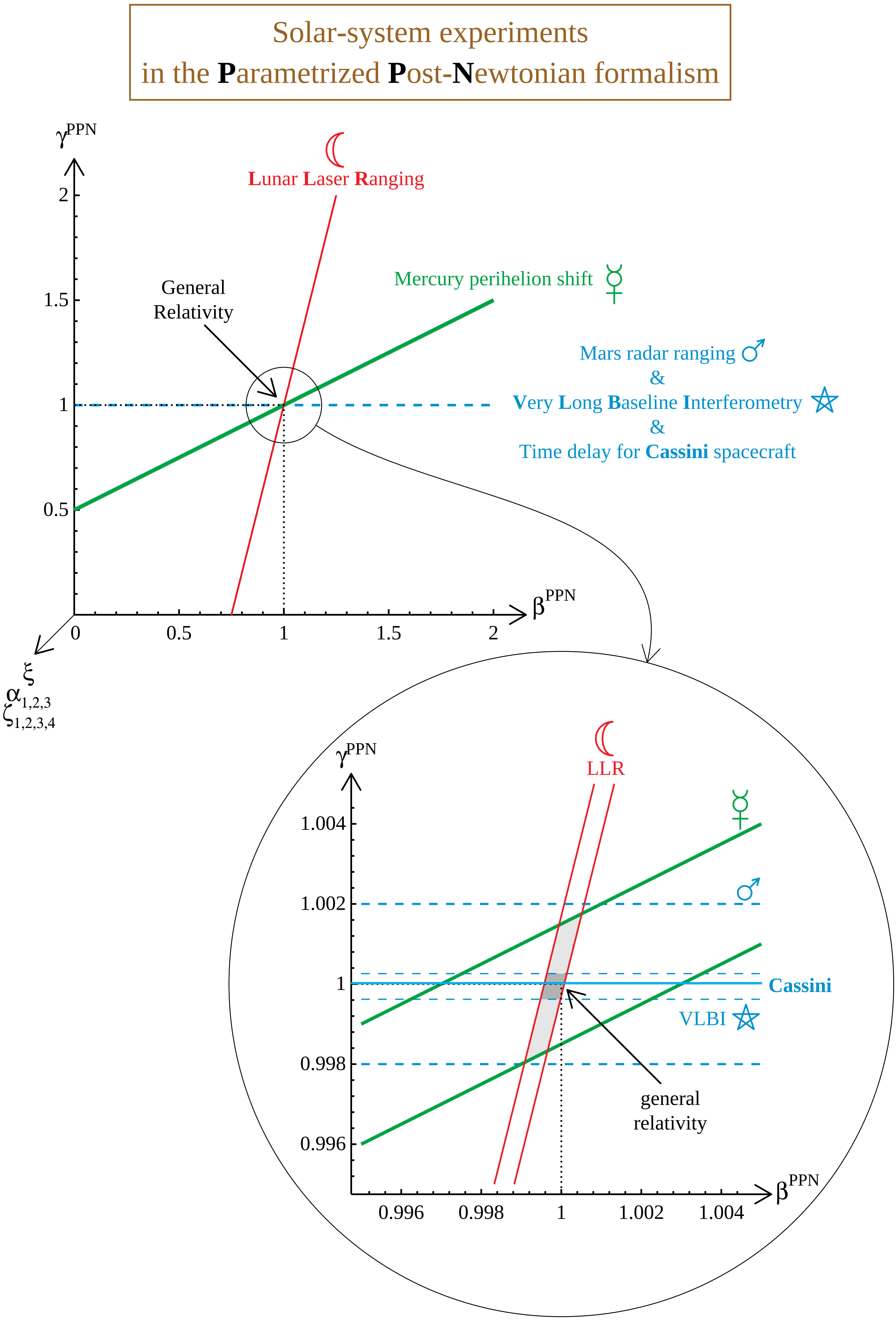
$$G_{\text{eff}} = G\,(1 + \alpha_0^2)$$



$$\gamma^{PPN} - 1 \propto \alpha_0^2$$

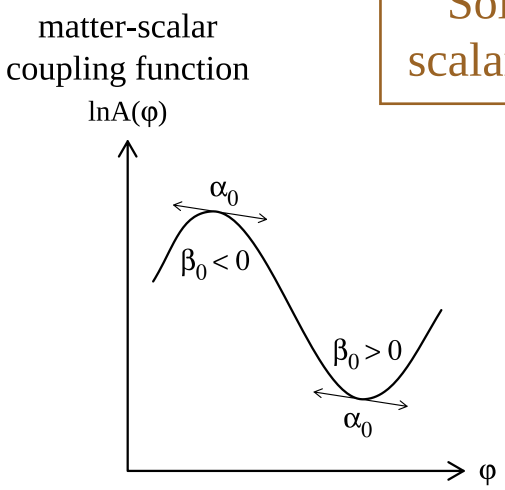
$$\beta^{PPN} - 1 \propto \alpha_0^2 \beta_0$$

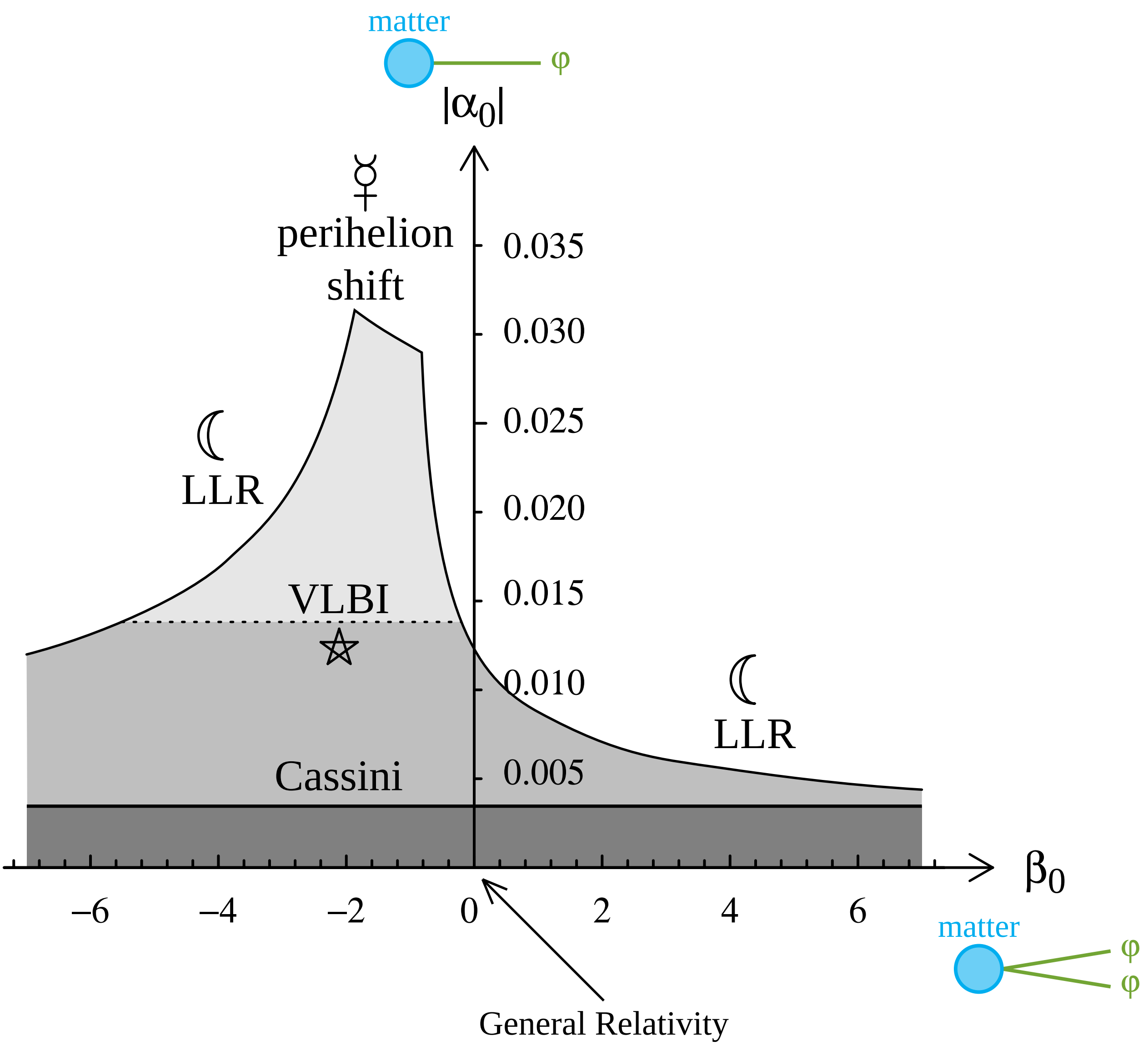




Solar-system experiments
in the Parametrized Post-Newtonian formalism
Lunar Laser Ranging
Mercury perihelion shift
General Relativity
Mars radar ranging
&
Very Long Baseline Interferometry
&
Time delay for Cassini spacecraft
γPPN
βPPN
2
1.5
1
0.5
0
0.5
1
1.5
2
ξ
α1,2,3
ζ1,2,3,4
LLR
1.004
1.002
1
0.998
0.996
Cassini
VLBI
general
relativity
0.996
0.998
1
1.002
1.004

Solar-system constraints on scalar-tensor theories of gravity





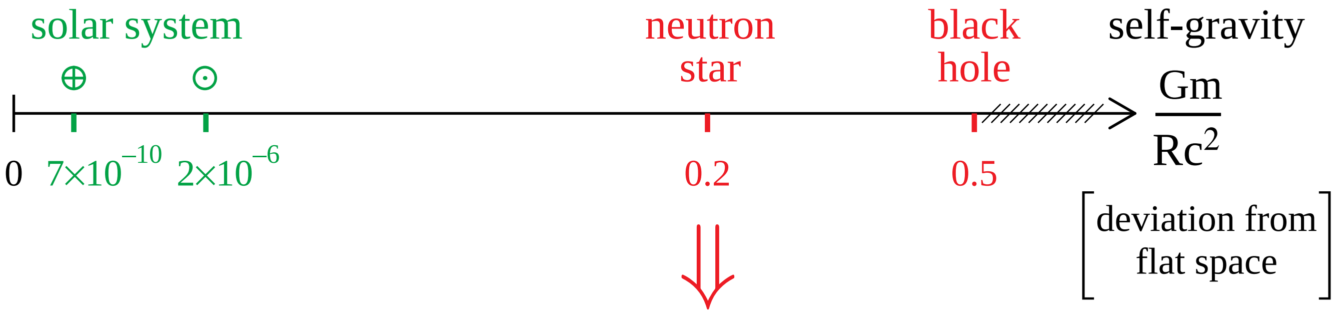
Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \frac{1}{2\,\omega_{BD} + 3}$

Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

Weak-field experiments

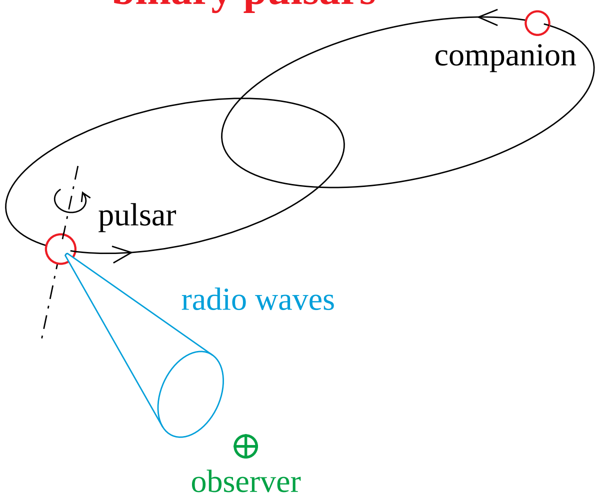
$$\begin{cases} -g_{00} = 1 - 2\,\dfrac{Gm}{rc^2} + 2\,\beta^{PPN}\left(\dfrac{Gm}{rc^2}\right)^2 + \dots \\ g_{ij} = \delta_{ij}\left[\,1 + 2\,\gamma^{PPN}\,\dfrac{Gm}{rc^2} + \dots\right] \end{cases}$$

Strong-field tests ?

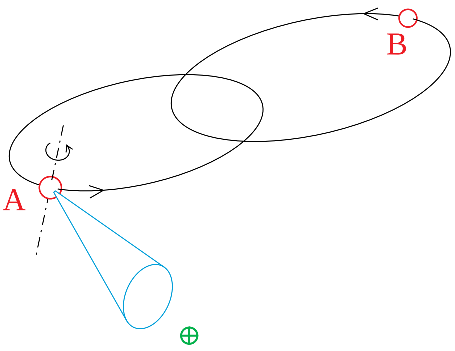


binary pulsars

moving clock giving information about this strong-gravity region

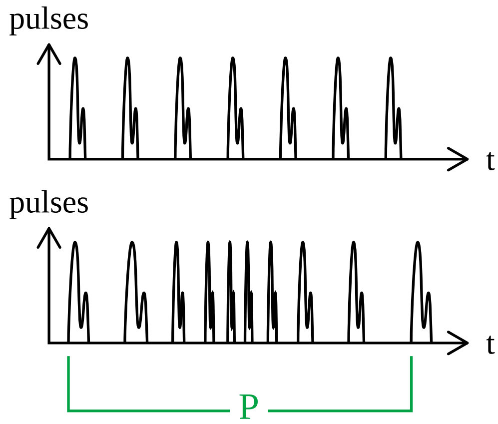


Binary-pulsar tests



pulsar = (very stable) clock

binary pulsar = moving clock



- Time of flight across orbit $\propto \dfrac{\text{size of orbit}}{c}$ (“Roemer time delay”)

– orbital period P
– eccentricity e
– periastron angular position ω
– projected semimajor axis x
– . . .

} “Keplerian” parameters

- Redshift $\propto \dfrac{G\, m_B}{r_{AB}\, c^2}$ + second order Doppler effect $\propto \dfrac{\vec{v}_A^{\,2}}{2\, c^2}$ (“Einstein time delay”)

– parameter γ_{Timing}

- Time evolution of Keplerian parameters

– periastron advance $\dot{\omega}$ (order $\frac{1}{c^2}$)

– gravitational radiation damping $\dot{P}$ (order $\frac{1}{c^5}$)

} “post-Keplerian” observables
[PSR B1913+16 • Hulse & Taylor 1974]

3 observables	–	2 unknown masses m_A, m_B	=	1 test

Plot the three curves [strips]

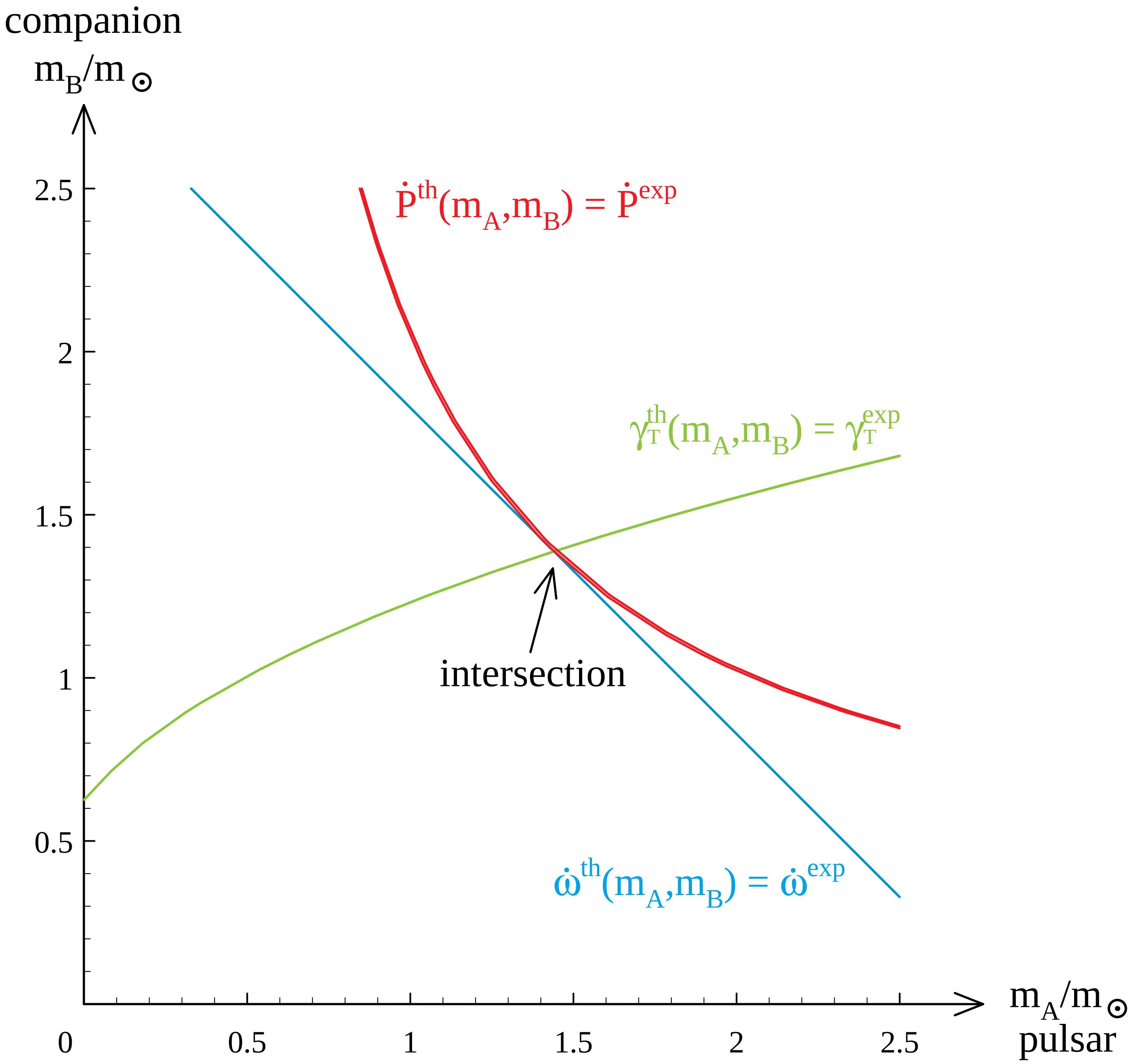
$$\gamma_{\text{Timing}}^{\text{theory}}(m_A, m_B) = \gamma_{\text{Timing}}^{\text{observed}}$$

$$\dot{\omega}^{\text{theory}}(m_A, m_B) = \dot{\omega}^{\text{observed}}$$

$$\dot{P}^{\text{theory}}(m_A, m_B) = \dot{P}^{\text{observed}}$$

} “ γ_T - $\dot{\omega}$ - $\dot{P}$ test ”

PSR B1913+16
in general relativity



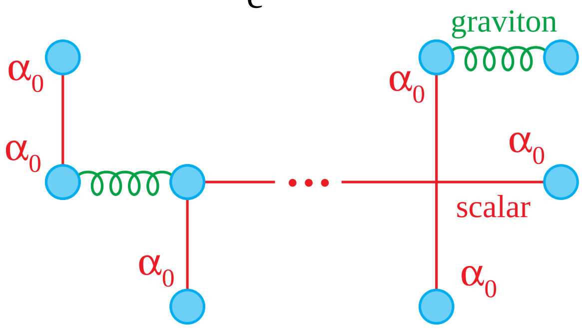
$$\dot{\omega} = 4.22661°/\mathrm{yr}$$
$$\gamma_T = 4.294\ \mathrm{ms}$$
$$\dot{P} = -2.421 \times 10^{-12}$$

$$\overset{GR}{\Longrightarrow}$$

$$m_A = 1.4408\ m_\odot$$
$$m_B = 1.3873\ m_\odot$$

Deviations from general relativity due to the scalar field

- At any order in $\frac{1}{c^n}$, the deviations involve at least two α_0 factors:



= small deviations!

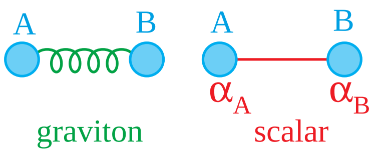
- But **nonperturbative** strong-field effects may occur:

$$\text{deviations} = \underset{<10^{-5}}{\alpha_0^2} \times \underbrace{\left[a_0 + a_1 \frac{Gm}{Rc^2} + a_2 \left(\frac{Gm}{Rc^2} \right)^2 + \ \ldots \right]}_{\text{LARGE for } \frac{Gm}{Rc^2} \approx 0.2 \ ?}$$

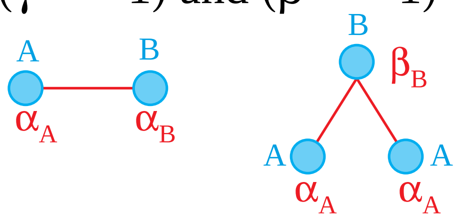
Strong-field effects

$$G_{AB}^{\text{eff}} = G \left(1 + \alpha_A \alpha_B \right)$$

depends on internal structure of bodies A & B



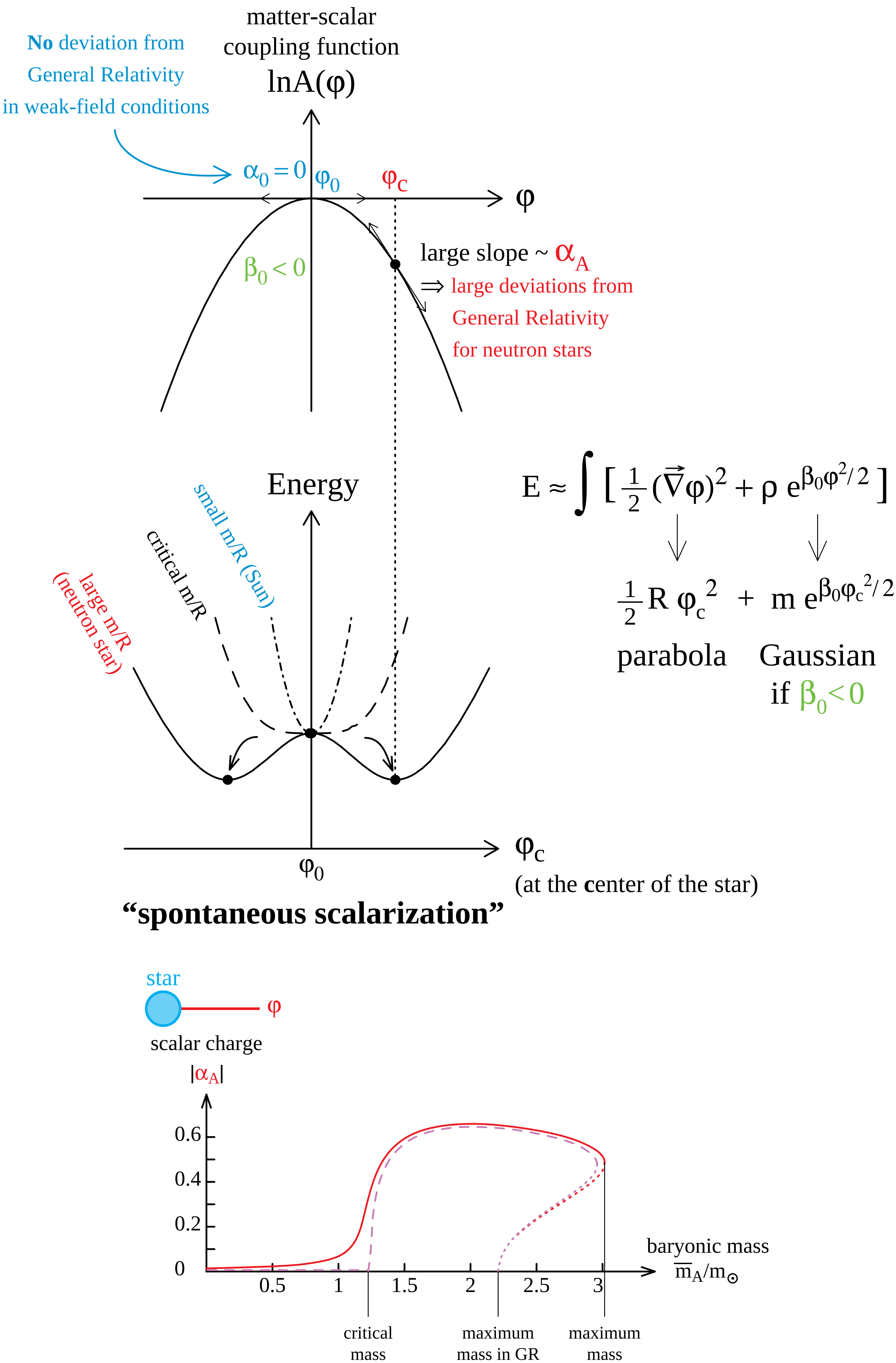
similarly for $(\gamma^{\text{PPN}} - 1)$ and $(\beta^{\text{PPN}} - 1)$ $\Rightarrow$ all post-Newtonian effects



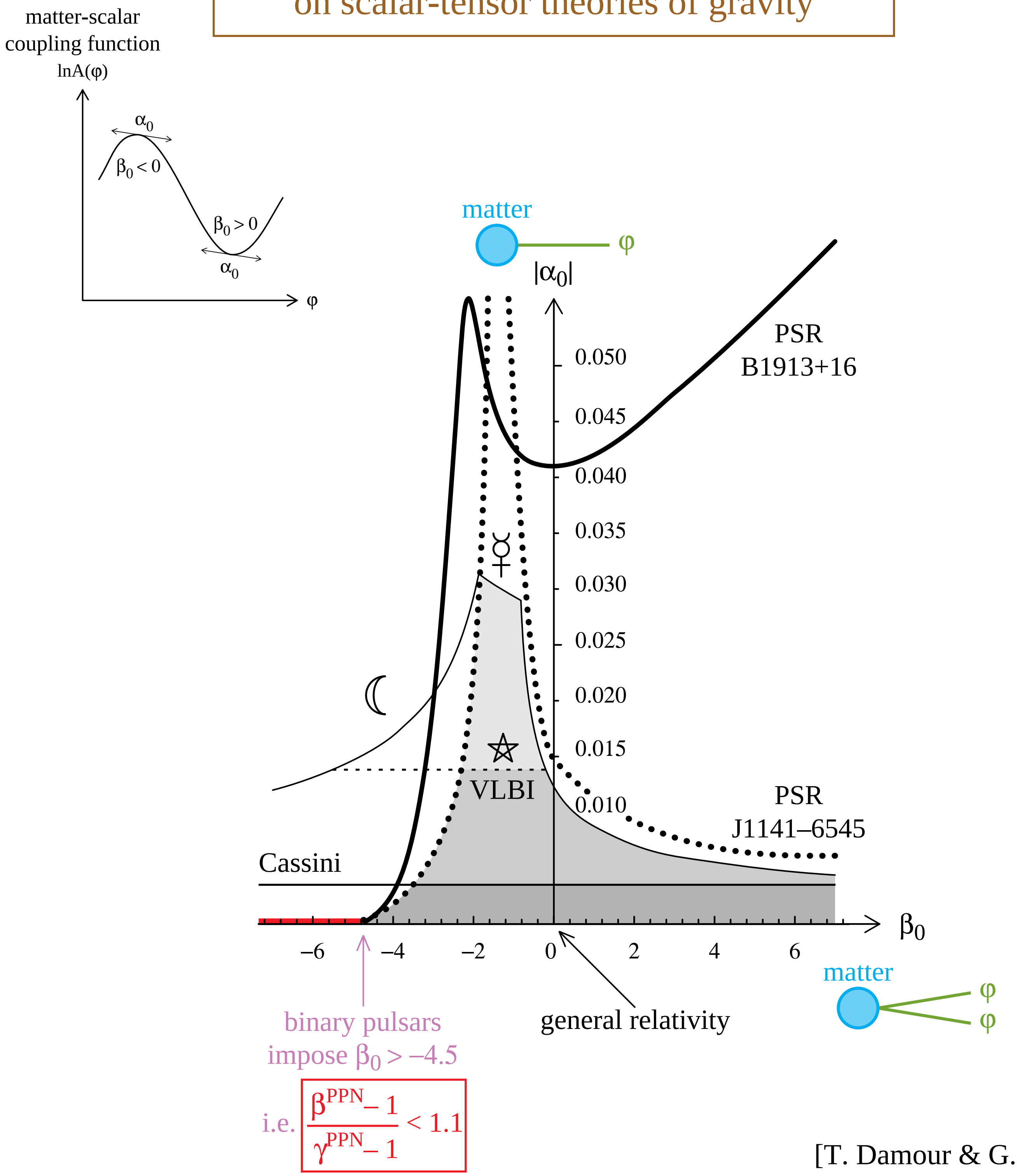
$$\text{Energy flux} = \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \quad \text{spin 2}$$

$$+ \frac{\text{Monopole}}{c} \left(0 + \frac{1}{c^2} \right)^2 + \frac{\text{Dipole}}{c^3} + \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \quad \text{spin 0}$$

Dipole $\propto (\alpha_A - \alpha_B)^2$


matter-scalar
coupling function
lnA(φ)
No deviation from
General Relativity
in weak-field conditions
α0 = 0
φ0
φc
φ
β0 < 0
large slope ~ αA
⇒ large deviations from
General Relativity
for neutron stars
Energy
small m/R (Sun)
critical m/R
large m/R
(neutron star)
E ≈ ∫ [1/2 (∇φ)^2 + ρ e^(β0φ^2/2)]
1/2 R φc^2 + m e^(β0φc^2/2)
parabola
Gaussian
if β0<0
φc
(at the center of the star)
φ0
“spontaneous scalarization”
star
φ
scalar charge
|αA|
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3
baryonic mass
mA/m⊙
critical
mass
maximum
mass in GR
maximum
mass

Solar-system & binary-pulsar constraints on scalar-tensor theories of gravity



[T. Damour & G.E-F]

Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \frac{1}{2\,\omega_{BD} + 3}$

Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

More general tensor – scalar theories

$$S = \frac{c^3}{4\pi G}\int \sqrt{-g}\left\{\frac{R}{4}-\frac{1}{2}\left(\partial_\mu\varphi\right)^2 - V(\varphi)\right\} + S_{\rm matter}\left[\text{matter}\,;\tilde{g}_{\mu\nu}\equiv A^2(\varphi)g_{\mu\nu}\right]$$

spin 2 — spin 0 — physical metric

k-essence — σ-model — quintessence

$$S = \frac{c^3}{4\pi G}\int \sqrt{-g}\left\{\frac{R}{4}-\frac{1}{2}F\left(g^{\mu\nu}\gamma_{ab}(\varphi^c)\,\partial_\mu\varphi^a\partial_\nu\varphi^b\right) - V(\varphi^a)\right\}$$
$$-\hbar\int\sqrt{-g}\,W(\varphi^a)\left(R^2_{\mu\nu\rho\sigma}-4R^2_{\mu\nu}+R^2\right)$$
$$+\,S_{\rm matter}\left[\text{matter}\,;\tilde{g}_{\mu\nu}\equiv A^2(\varphi^a)g_{\mu\nu}\right]$$

Gauss-Bonnet

extended quintessence

N.B.:

- $f(R) \Longleftrightarrow$ an extra scalar field [Teyssandier & Tourrenc 1983]
- $f(R, \Box R, \ldots, \Box^n R) \Longleftrightarrow n+1$ extra scalar fields [Gottlöber *et al.* 1990; Wands 1994]
- $f(R_{\mu\nu})$ and/or $f(R_{\mu\nu\rho\sigma}) \Longleftrightarrow$ an extra massive spin-2 **ghost** [Stelle 1977; Hindawi *et al.* 1996; Tomboulis 1996]

Example of a pure scalar–Gauss-Bonnet coupling

$$S = \frac{c^3}{4\pi G}\int \sqrt{-g}\left\{\frac{R}{4}-\frac{1}{2}\left(\partial_\mu\varphi\right)^2 - 0\right\}$$
$$-\hbar\int\sqrt{-g}\,W(\varphi)\left(R^2_{\mu\nu\rho\sigma}-4R^2_{\mu\nu}+R^2\right)$$
$$+S_{\rm matter}\left[\text{matter}\,;\tilde{g}_{\mu\nu}\equiv 1\times g_{\mu\nu}\right]$$

Experimental constraints on W(φ) ?

- Solar system (& binary pulsars)

$$\Box\varphi = \frac{3\ell_0^2}{r^6}\left(\frac{2GM_\odot}{c^2}\right)^2\left[W_0' + W_0''\varphi + O(\varphi^2)\right]$$

$$\left(\ell_0^2 = \frac{16\pi G\hbar}{c^3}\right)$$

light deflection $\quad \Delta\theta_{\star} = \frac{4GM_\odot}{r_0c^2} + \frac{1536}{35}\left(\frac{GM_\odot}{r_0c^2}\right)^3\left(\frac{\ell_0}{r_0}\right)^4 W_0'^2$

perihelion shift $\quad \Delta\theta_{☿} = \frac{6\pi GM_\odot}{pc^2} + 192\pi\left(\frac{GM_\odot}{pc^2}\right)^2\left(\frac{\ell_0}{p}\right)^4 W_0'^2$

OK if $|W_0'|$ small enough

- Reconstruction of $W(\varphi)$ from cosmological observation of $D_L(z)$

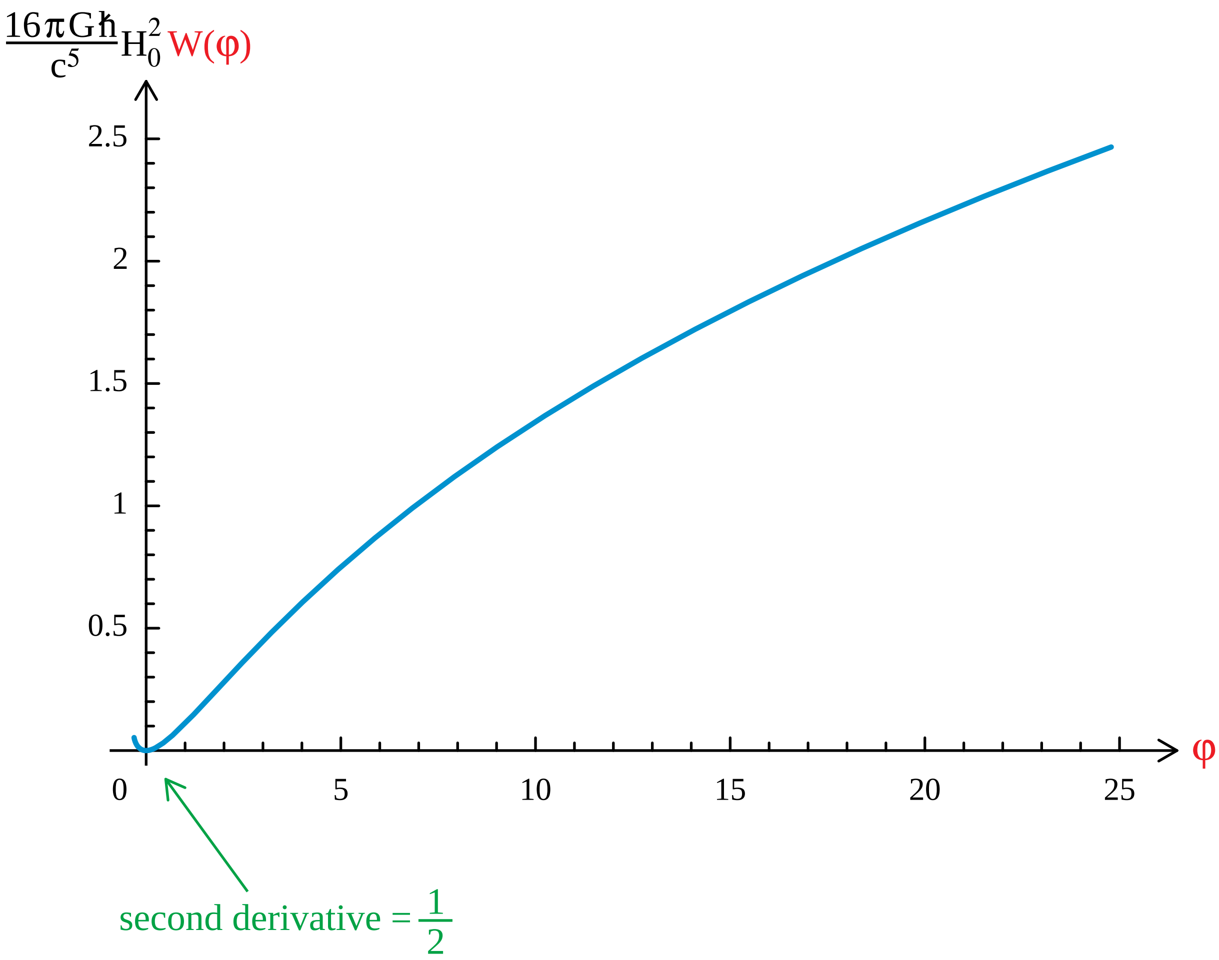
[fit $W(\varphi)$ to reproduce $D_L(z)$, i.e. present accelerated expansion]

– Can always be done without any problem of negative energy
[contrary to fits of the matter–scalar coupling function $A(\varphi)$]

– ∃ attraction mechanism towards a minimum of $W(\varphi)$

$\Rightarrow$ $|W_0'|$ small is expected.

Reconstruction of the scalar–Gauss-Bonnet coupling function $W(\varphi)$ [for $V(\varphi) = 0$ and $A(\varphi) = 1$]



Conclusion : $dW(\varphi)/d\varphi = 0$ possible

but $d^2W(\varphi)/d\varphi^2 \approx 7 \times 10^{119}$

(cf. $\Lambda \approx 3 \times 10^{-122} c^3/\hbar G$)

[G.E-F & E. Semboloni]

Experimental constraints on W(φ) ? (continued)

- Solar system again

If $|W_0''\varphi| \gg |W_0'|$, we cannot neglect it in

$$\Box\varphi = \frac{3\ell_0^2}{r^6}\left(\frac{2GM_\odot}{c^2}\right)^2\left[W_0' + W_0''\varphi + \cancel{O(\varphi^2)}\right]$$

assume parabolic $W(\varphi)$

$$\Rightarrow \quad \varphi = \frac{W_0'}{W_0''}\sum_{n\geq 1}\frac{1}{(3\times 4)(7\times 8)\cdots(4n-1)(4n)}\left(\frac{12\ell_0^2 G^2 M_\odot^2 W_0''}{r^4 c^4}\right)^n$$

$$\simeq \frac{W_0'}{W_0''}\left[\begin{matrix}\cosh\\ \cos\end{matrix}\underbrace{\left(\frac{GM_\odot\ell_0}{r^2c^2}\sqrt{3|W_0''|}\right)}_{\sim 10^8} - 1\right] \quad \begin{matrix}\text{if } W_0''>0\\ \text{if } W_0''<0\end{matrix}$$

if $'' \gg 1$

- $\varphi \longrightarrow 0$ for $r \longrightarrow \infty$

$\Rightarrow$ theory $\simeq$ G.R. for $r > 4\times 10^{14}$ m

(farther than solar system + comet cloud)

- In the solar system, ∃ highly nonlinear corrections in $\dfrac{1}{r^{4n}}$

- $\varphi \longrightarrow 0$ for $W_0' \longrightarrow 0$

$\Rightarrow$ no nonperturbative effect (like spontaneous scalarization)

• Solar system tests

$$ds^2 = -\left(1 + \sum_n \frac{\beta_n}{r^n}\right) c^2 dt^2 + \left(1 + \sum_n \frac{\alpha_n}{r^n}\right) dr^2 + r^2 d\Omega^2$$

light deflection $\Delta\theta_{\star} = \sum_n 2^{n-1} \frac{\Gamma\left(\frac{n+1}{2}\right)^2}{\Gamma(n+1)} \frac{\alpha_n - n\beta_n}{r_0^n} + O(\alpha_n, \beta_n)^2$

perturbative

perihelion shift $\Delta\theta_{☿} = \frac{6\pi G M_\odot}{pc^2} - \sum_n \frac{n(n-1)\beta_n c^2}{2GM_\odot p^{n-1}} \pi + O(\alpha_n, \beta_n)^2$

$\Rightarrow |W_0'| < 10^{-2\times 10^{11}}$!!!

if we take the $W_0'' > 0$ given by the cosmological reconstruction.

Hyperfine tuning, which cannot last for more than a fraction of a second.

$\Rightarrow$ The model $A(\varphi) = 1,\ V(\varphi) = 0,\ W(\varphi) \neq 0$ is already ruled out

*

* *

• N.B.: If $W_0'' < 0$, then

$$\varphi \simeq \frac{W_0'}{W_0''} \left[\cos\left(\frac{GM_\odot \ell_0}{r^2 c^2} \sqrt{3|W_0''|} \right) - 1 \right]$$

and it suffices to have $|\ell_0^2 W_0'| \ll r^2$

to get negligible effects in the solar system,

even if $|W_0''| \sim 10^{120}$.

$\Rightarrow$ Not so trivial that a R^2 term in the Lagrangian must have larger effects on small scales than on large ones.

Instabilities caused by ghosts ($E_{\text{kinetic}} \le 0$)

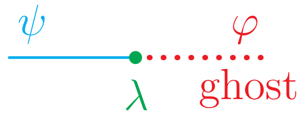
Simplest model (cf. two coupled harmonic oscillators)

$$\mathcal{L} = -(\partial_\mu \psi)^2 + (\partial_\mu \varphi)^2 - 2\lambda\psi\varphi$$

$$\Rightarrow \begin{cases} \Box\psi = \lambda\varphi \\ -\Box\varphi = \lambda\psi \end{cases} \quad \Rightarrow \begin{cases} (\omega^2 - k^2)\psi = \lambda\varphi \\ -(\omega^2 - k^2)\varphi = \lambda\psi \end{cases} \quad \Rightarrow \quad (\omega^2 - k^2)^2 = -\lambda^2$$

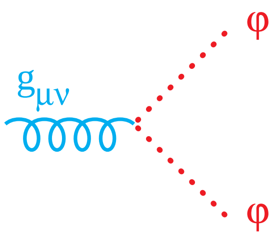
$\Rightarrow \omega = \sqrt{k^2 \pm i\lambda}$ complex

$\Rightarrow e^{-i\omega t}$ involves an exponentially growing mode

Instability is manifest at linear order, cf. 

Gravity with a scalar ghost

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\,(R + 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) + S_{\text{matter}}[\text{matter}; g_{\mu\nu}]$$



Cosmological background

$H \equiv \dot{a}/a\,;$ scalar background denoted Φ

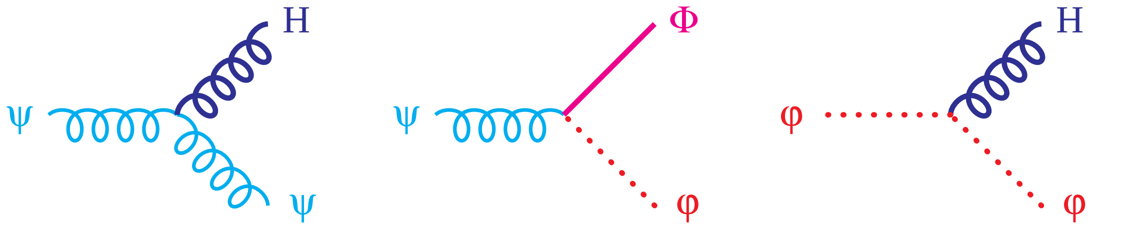
$$\begin{cases} \frac{3}{2}H^2 = 4\pi G\rho - \frac{1}{2}\dot{\Phi}^2 & \\ -3\,\frac{\ddot{a}}{a} = 4\pi G\rho - 2\dot{\Phi}^2 & \\ \ddot{\Phi} + 3H\dot{\Phi} = 0 & \Rightarrow \quad \dot{\Phi}^2 \propto a^{-6} \\ \dot{\rho} + 3H\rho = 0 & \Rightarrow \quad \rho \propto a^{-3} \end{cases}$$

$\ddot{a} > 0$ is possible (“unstable” background)

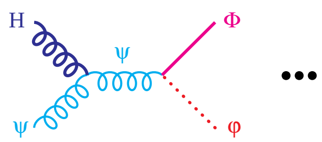
Linear perturbations

$$ds^2 \equiv -(1+2\psi)dt^2 + a^2(1-2\psi)d\mathbf{x}^2\,; \qquad \text{scalar field } \Phi + \varphi$$

$$\dot\psi + H\psi = -\dot\Phi\varphi \quad ; \qquad \dot\Phi\,(\dot\varphi + 3H\varphi) = \left(+\frac{k^2}{a^2} + \dot\Phi^2\right)\psi$$



$$\Rightarrow \begin{cases} \ddot\psi + \left(\frac{k^2}{a^2} - H^2 + 2\dot\Phi^2\right)\psi &= +7H\dot\Phi\varphi \\ \dot\Phi\left[\ddot\varphi + \left(\frac{k^2}{a^2} - 9H^2 + 4\dot\Phi^2\right)\varphi\right] &= -H\left(+3\frac{k^2}{a^2} + 7\dot\Phi^2\right)\psi \end{cases}$$



$$\Rightarrow \quad \omega \approx \sqrt{\frac{k^2}{a^2} - H^2 + \dot\Phi^2} - 2iH$$

Damped oscillators in expanding universe (friction term $H > 0$).
No instability due to ghost at this linear order (even if $V \neq O$).

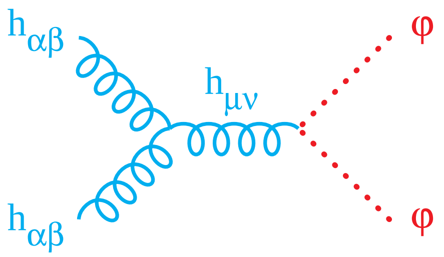
Higher-order perturbations

$$\begin{cases} R_{\mu\nu} &= -2\partial_\mu\varphi\partial_\nu\varphi \quad (+\text{ matter}) \\ -\Box\varphi &= 0 \qquad \Rightarrow \quad \exists \text{ planes waves of any frequency} \end{cases}$$

$\Rightarrow$ Equation for $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ in harmonic gauge has the form:

$$(\omega^2 - \mathbf{k}^2)h_{\mu\nu} = k_\mu k_\nu \left(\frac{1}{2}\,h^2_{\alpha\beta} - 4\varphi^2\right) + \mathcal{O}(h^3)$$

The energies can compensate exactly each other in the r.h.s.



$\Rightarrow$ Spontaneous creation of plane waves of any frequency (gravitons and scalars on mass shell)

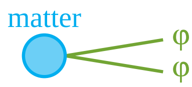
$\Rightarrow$ Unstable vacuum (once created, do not annihilate)

Conclusions

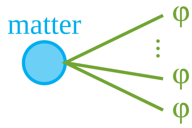
- Scalar–tensor theories are the best motivated alternatives to general relativity.

- Solar-system tests constrain the first derivative of the scalar–matter coupling function $A(\varphi)$.

- Binary-pulsar data constrain the second derivative of $A(\varphi)$, because of nonperturbative strong-field effects.



- Cosmological observations [of $D_L(z)$ and $\delta_m(z)$] give access to $A(\varphi)$ and/or the potential $V(\varphi)$ on a finite interval of φ.



- Scalar–Gauss-Bonnet coupling $W(\varphi)$ strongly constrained by combination of solar-system & cosmological data, because of highly nonlinear effects.

- Instabilities caused by ghost degrees of freedom may not manifest at the linear order.