THE THREE DIMENSIONAL SKELETON

A tool for the extraction and analysis of the large scale structures in the universe

Thierry Sousbie

Centre de Recherche Astrophysique de Lyon (CRAL)
HORIZON project

S. Colombi, H. Courtois, D. Novikov and C. Pichon

Outline

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1. The Three Dimensional Skeleton
   - Introduction
   - Definition
   - Implementation
2. Application to Observational Data : SDSS
   - Making Mock Catalogs: MoLUSC
   - Measuring the skeleton in the SDSS
   - Sensitivity to Biases
   - Results : constraints on Ω_m
3. Other Applications: Computing Quantities Along the Filaments
   - The velocity field
   - The dark matter pressure
4. Summary and Conclusion

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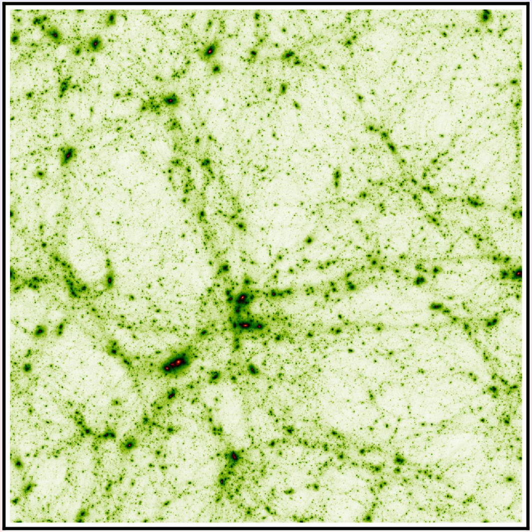
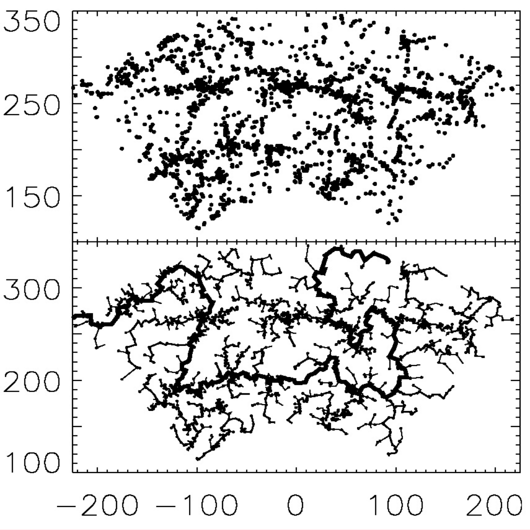


Goal

- Sum-up matter distribution features
- Extract filamentary structure
- Be able to compute quantities along filaments
- Eventually constraint cosmology

Minimal spanning tree



Doroshkevich, A. G. et al. 2001, mnras, 322, 369

Counterparts

- Non local
- No analytic predictions possible
- Definition of a filament ?
- Discreteness

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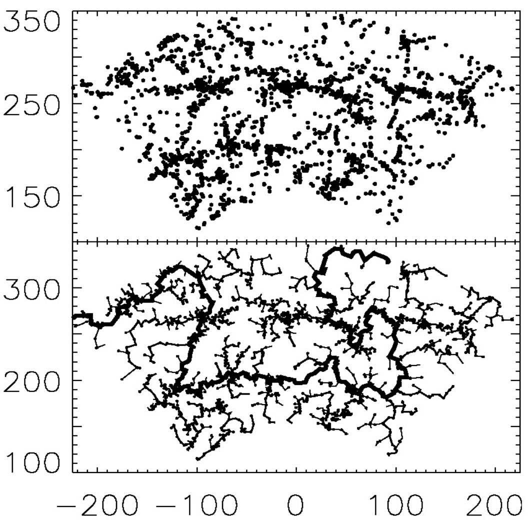

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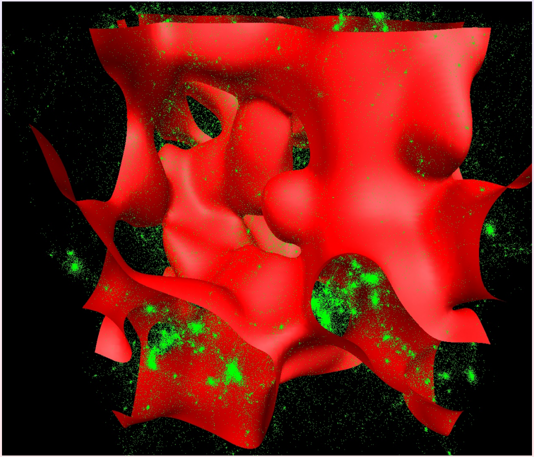

d-1 Minkowski functionals

- Volume
- Surface
- Mean curvature
- Euler characteristic

Counterparts

- Statistical features only
- No local properties
- They are functions of density threshold

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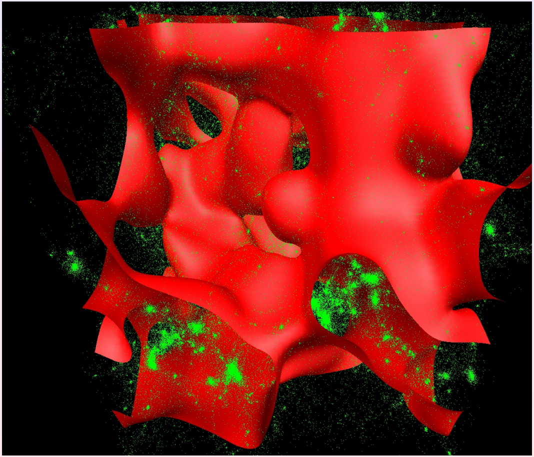

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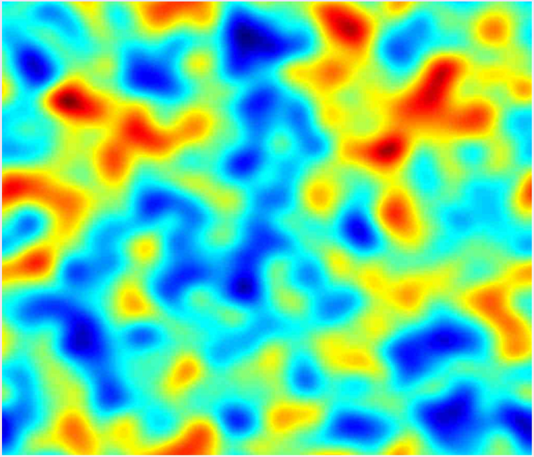
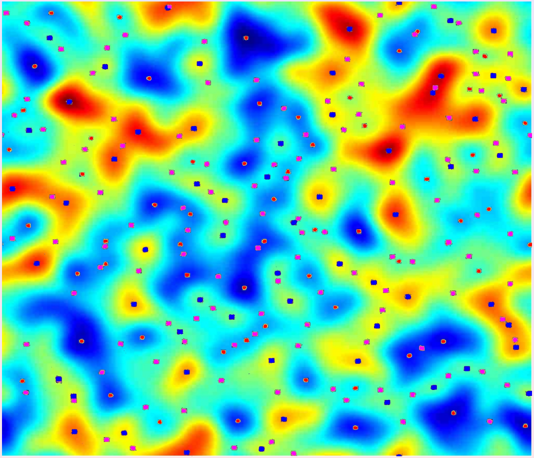


Method

- From a density field
- Compute extrema
- Start from saddle points and follow gradiant up to the maxima

Nice but ...

- This is not local
- Computation is long and difficult, especially near extrema



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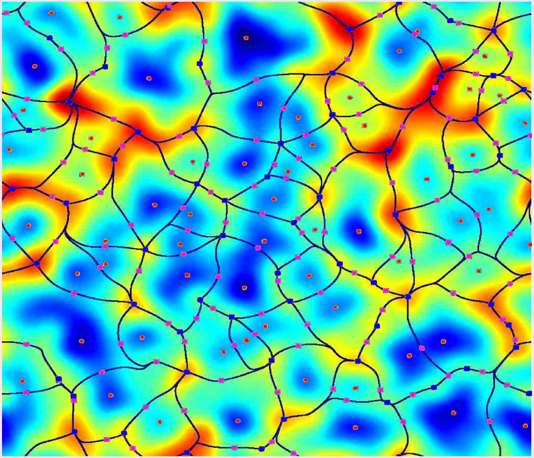

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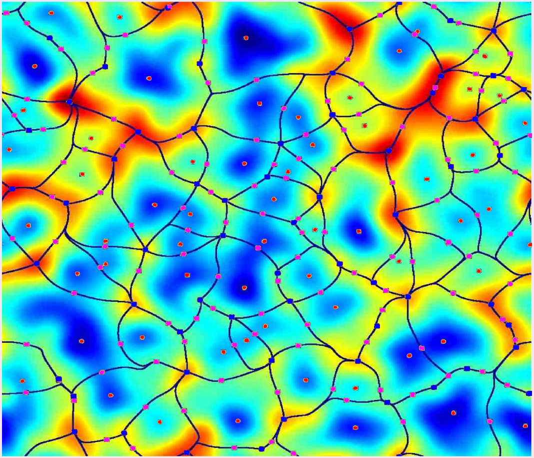

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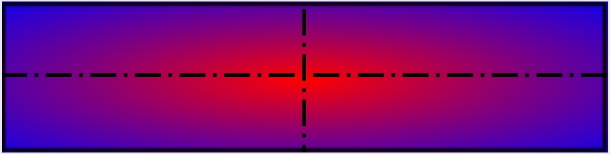

Local approximation

- Second order
- $\mathcal{H}_{ij}, \nabla_i(\rho), \rho$
- Two approaches

Using isocontours $r(x(t), y(t))$

- Gradiant is extremal along an isocontour
- $\frac{\partial}{\partial t}(|\nabla \rho|) = 0$
- Gradiant is minimal

Using field curvature

- Gradiant is an eigenvector of Hessian
- $\mathcal{H}\nabla\rho = \lambda\nabla\rho$

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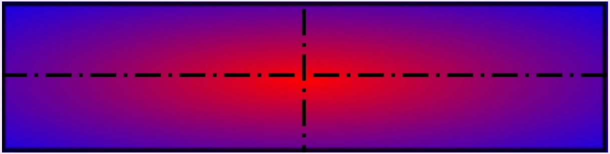

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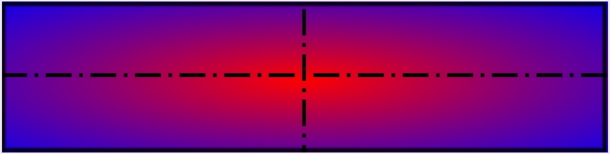

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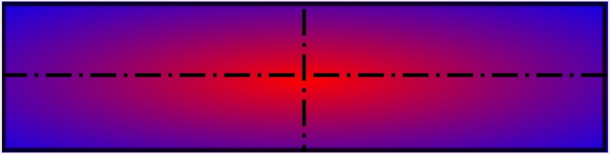

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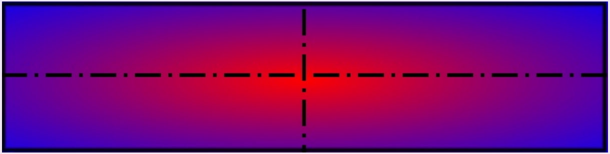

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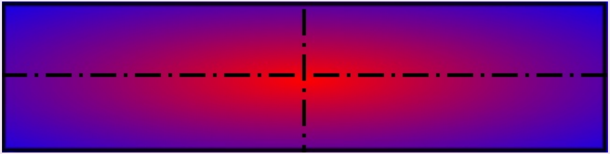

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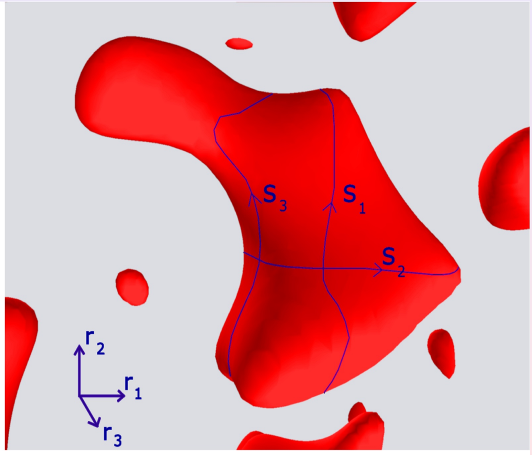
The 3D local skeleton



Isodensity surface

$$S(r_1(s_i, s_j), r_2(s_i, s_j), r_3(s_i, s_j))$$

$$i \neq j \in \{1, 2, 3\}$$

Definition

$$\begin{cases} \frac{d}{ds_i}(|\nabla\rho|^2) = 0 \\ \frac{d}{ds_j}(|\nabla\rho|^2) = 0 \end{cases}$$

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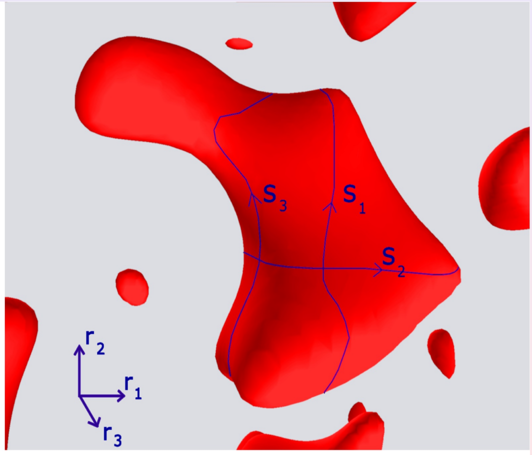

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Geometric approach

$$\mathcal{H}_{ij}\nabla_j\rho = \lambda_n\nabla_j\rho$$

Result

$$\begin{aligned} \mathcal{S}_i &= \frac{\partial^2\rho}{\partial r_j \partial r_k}\left(\frac{\partial\rho}{\partial r_j}^2 - \frac{\partial\rho}{\partial r_k}^2\right) \\ &+ \frac{\partial\rho}{\partial r_j}\frac{\partial\rho}{\partial r_k}\left(\frac{\partial^2\rho}{\partial r_k^2} - \frac{\partial^2\rho}{\partial r_j^2}\right) \\ &- \frac{\partial\rho}{\partial r_i}\left(\frac{\partial\rho}{\partial r_k}\frac{\partial^2\rho}{\partial r_i \partial r_j} - \frac{\partial\rho}{\partial r_j}\frac{\partial^2\rho}{\partial r_i \partial r_k}\right) = 0 \end{aligned}$$

with $i \neq j \neq k \in \{1..3\}$.

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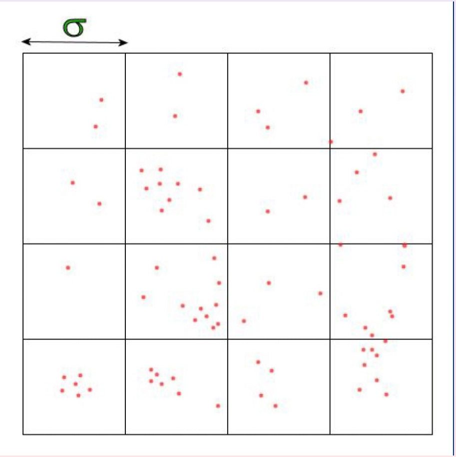

Smoothing

- CIC affectation
- Gaussian smoothing:

$$K(r) = \frac{1}{(2\pi L)^{3/2}} e^{-\frac{r^2}{2L}}$$

Two important parameters

- σ : Grid cell size
- L : Smoothing length

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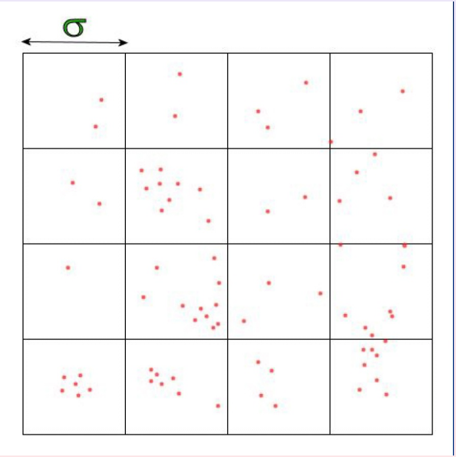

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Drawing isosurfaces, marching cubes algorithm

Field extrema

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[width=6cm]./pictures/mcubes.jpg

Demonstration

Densityfield
Gradiant
Skeleton

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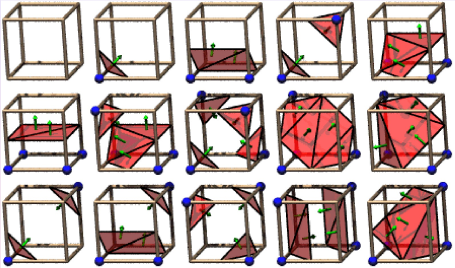

The 15 Cube Combinations

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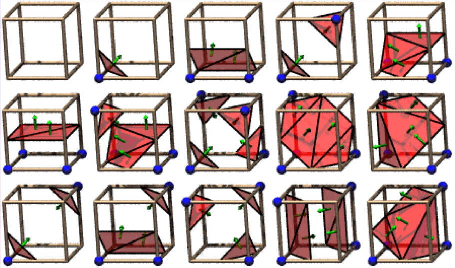
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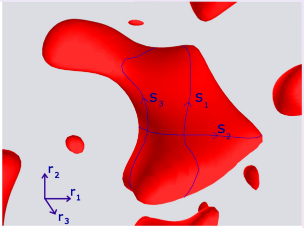
Final results



Selecting the *total* skeleton

Maximizing $d_i = \left|det(\mathbf{r_j}, \mathbf{r_k}, \nabla\rho)\right|$, $i \neq j \neq k$

Selecting only the filaments

$\lambda_1 > 0$, $\lambda_2 < 0$ and $\lambda_3 < 0$

Post treatement

- Starting from saddle points, reconnect properly aligned and neighbouring segements.
- Recover connectivity

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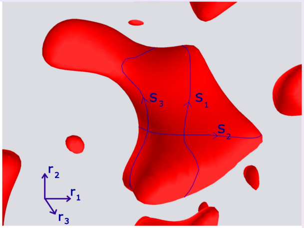

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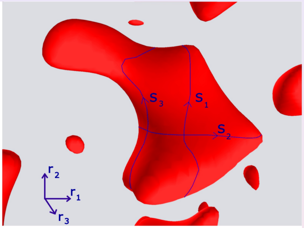

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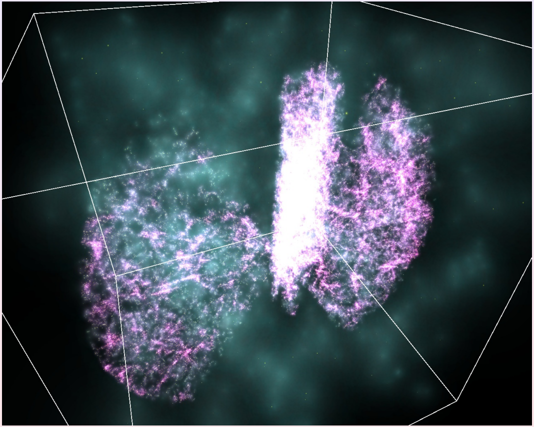

- Survey is embeded in a square box
- low density random field within voids

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