Modified Newtonian gravity and field theory

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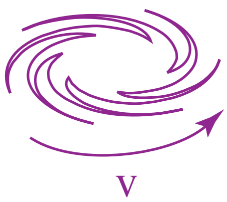
Based on **Phys. Rev. D 76 (2007) 124012** in collaboration with J.-P. Bruneton,
on my study of scalar-tensor theories & binary-pulsar tests with T. Damour since 1991,
on a work in progress with C. Deffayet & R. Woodard about nonlocal models,
and discussions with many colleagues, notably L. Blanchet, B. Fort,
G. Mamon, Y. Mellier, M. Milgrom, J. Moffat, R. Sanders, J.-P. Uzan, *etc.*

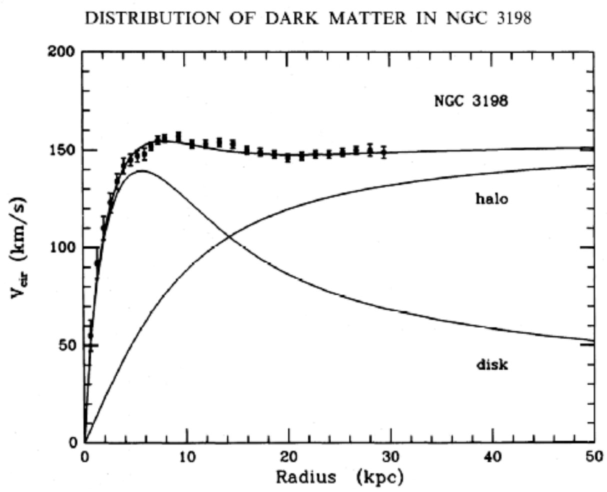
This PDF file is a summary of my lectures at

IESC, Cargèse, November 2008

Dark matter and galaxy rotation curves

- $\Omega_\Lambda \approx 0.7$ (SNIa) and $\Omega_\Lambda + \Omega_m \approx 1$ (CMB) $\Rightarrow$ $\Omega_m \approx 0.3$, at least $10\times$ greater than estimates of baryonic matter.
- Rotation curves of galaxies and clusters: almost rigid bodies





- $\exists$ many theoretical candidates for dark matter (e.g. from SUSY)
- Numerical simulations of structure formation are successful while incorporating (noninteracting, pressureless) dark matter

Milgrom's MOND proposal [1983]

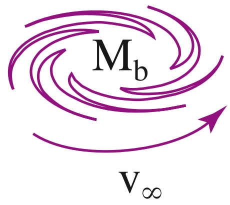
MOdified Newtonian Dynamics
for small accelerations (i.e., at large distances)

$$a = a_N = \frac{GM}{r^2} \quad \text{if } a > a_0 \approx 1.2 \times 10^{-10}\,\text{m.s}^{-2}$$

$$a = \sqrt{a_0 a_N} = \frac{\sqrt{GMa_0}}{r} \quad \text{if } a < a_0$$

- Automatically recovers the Tully-Fisher law [1977]

  $$v_\infty^4 \propto M_{\text{baryonic}}$$



- Superbly accounts for galaxy rotation curves
  (but clusters still require some dark matter)

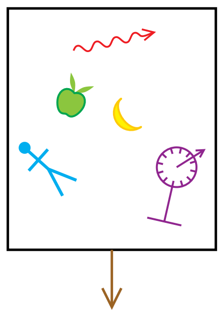
  [Sanders & McGaugh, Ann. Rev. Astron. Astrophys. **40** (2002) 263]

Modified gravity or modified inertia?

General Relativity

$$S = \underbrace{\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\,R}_{\text{Einstein-Hilbert}} + \underbrace{S_{\text{matter}}[\text{matter}, g_{\mu\nu}]}_{\text{Metric coupling}}$$

freely falling elevator



Einstein-Hilbert ⇒ pure spin 2
Metric coupling ⇒ equivalence principle
Example: $S_{\text{point particle}} = -\int mc\sqrt{-g_{\mu\nu}(x)\, v^\mu v^\nu}\, dt$

- Modified inertia [Milgrom 1994, 1999]:
  Keep $S_{\text{Einstein-Hilbert}}[g_{\mu\nu}]$, but look for $S_{\text{point particle}}(\mathbf{x}, \mathbf{v}, \mathbf{a}, \dot{\mathbf{a}}, \dots)$.

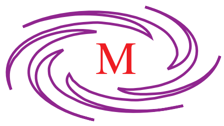
  Galileo invariance ⇒ nonlocal! (⇒ causality?)

- Modified gravity:
  Keep metric coupling, but $S_{\text{gravity}} \neq$ Einstein-Hilbert (⇒ extra fields)

- Other possibilities: Both modifications, or none?

Mass-dependent models?

- *A priori* easy to predict a force $\propto 1/r$:
  If $V(\varphi) = -2a^2 e^{-b\varphi}$, unbounded by below
  then $\Delta\varphi = V'(\varphi) \Rightarrow \varphi = (2/b)\ln(abr)$.
  Constant coefficient $2/b$ instead of $\sqrt{M}$.



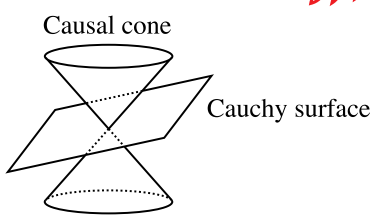
  Some papers write actions which depend on the galaxy mass M
  $\Rightarrow$ They are actually using a different theory for each galaxy!

- Moffat [2004] proposes a consistent field theory (nonsymmetric $g_{\mu\nu}$)
  but predicts $a = kM^2/r$ instead of $\sqrt{M}/r$,
  and assumes then $k = M^{-3/2}$!
- In 2005, he introduced a potential in his model
  to derive $k = M^{-3/2}$, but the potential depends on M

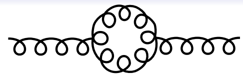
Consistent field theories

- Field theories
  All predictions deriving from a single action
  $\exists$ proposed models in which 2 field equations are inconsistent with each other
  $\Rightarrow$ violation of conservation laws
- Stability
  Full Hamiltonian should be bounded by below:
  no tachyon ($m^2 \geq 0$), no ghost ($E_{\text{kinetic}} \geq 0$)

- Well-posed Cauchy problem
  Hyperbolic field equations



Quadratic gravity

- 't Hooft & Veltman [1974]: Divergence of  needs

Counterterm

$$\Delta\mathcal{L} = \frac{\sqrt{-g}}{8\pi^2(d-4)}\left[\frac{53}{90}R^2_{\mu\nu\rho\sigma} - \frac{361}{180}R^2_{\mu\nu} + \frac{43}{72}R^2\right]$$

$$= \frac{\sqrt{-g}}{8\pi^2(d-4)}\left[\frac{7}{40}C^2_{\mu\nu\rho\sigma} + \frac{1}{8}R^2 + \frac{149}{360}\mathrm{GB}\right]$$

$C_{\mu\nu\rho\sigma}$: Weyl tensor (fully traceless)
$\mathrm{GB} \equiv R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$: Gauss-Bonnet topological invariant

- Stelle's thesis [1977]: If $\alpha \neq 0$ and $\beta \neq 0$,

Quadratic gravity is renormalizable

$$S_{\text{gravity}} = \int d^4x\sqrt{-g}\left[R + \alpha C^2_{\mu\nu\rho\sigma} + \beta R^2 + \gamma\mathrm{GB}\right]$$

f(curvature) and ghosts

But quadratic gravity is unstable!

$$S_{\text{gravity}} = \int d^4x\sqrt{-g}\left[R + \alpha C^2_{\mu\nu\rho\sigma} + \beta R^2 + \gamma\mathrm{GB}\right]$$

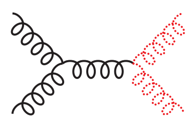
- Intuitive argument:

Propagator $\dfrac{1}{p^2 + \alpha\, p^4} = \dfrac{1}{p^2} \underset{\text{ghost!}}{-} \dfrac{1}{p^2 + \frac{1}{\alpha}}$

N.B.: $\frac{1}{\alpha} = m^2$ of extra d° of freedom $\Rightarrow$ negative α gives a tachyon, but anyway a ghost

- Full calculation

[Stelle 1977; Hindawi, Ovrut, Waldram 1996; Tomboulis 1996]:



  - $R + f(R_{\mu\nu}, R_{\mu\nu\rho\sigma}) \Rightarrow$ extra massive spin-2 ghost $\Rightarrow$ unstable vacuum
  - $R + f(R) \Rightarrow$ extra massive spin-0 scalar with $E_{\text{kin}} > 0$

N.B.: Strings predict $C^2_{\mu\nu\rho\sigma}$, but also any higher derivative (nonlocal)

$f(R)$ as scalar-tensor theories

- Introduction of a Lagrange multiplier Φ [Teyssandier & Tourrenc 1983]

$$
\begin{aligned}
S_{\text{gravity}} &= \int d^4x\sqrt{-g}\,f(R) \\
&\Leftrightarrow \int d^4x\sqrt{-g}\,\{f(\Phi) + (R-\Phi)f'(\Phi)\} \\
&= \int d^4x\sqrt{-g}\Big\{f'(\Phi)R - \underbrace{0\,(\partial_\mu\Phi)^2}_{\omega_{\text{BD}}=0} - \underbrace{[\Phi f'(\Phi) - f(\Phi)]}_{\text{potential}}\Big\}
\end{aligned}
$$

N.B.: If potential $= 0$, solar system $\Rightarrow \omega_{\text{BD}} > 40000$

- Similarly $f(R, \Box R, \ldots, \Box^n R) \Rightarrow$ Einstein plus $n+1$ scalar fields
  [Gottlöber, Schmidt, Starobinsky 1990; Wands 1994]

- Such scalar fields give generically Yukawa potentials $\propto \dfrac{e^{-mr}}{r}$
  $\Rightarrow$ not MOND (potential $\propto \ln r$)

$f(R)$ as scalar-tensor theories (continued)

We saw that $f(R)$ theories are equivalent to

$$
\begin{aligned}
S = &\int d^4x\sqrt{-g}\Big\{f'(\Phi)R - 0\,(\partial_\mu\Phi)^2 - [\Phi f'(\Phi) - f(\Phi)]\Big\} \\
&+ S_{\text{matter}}[\text{matter}, g_{\mu\nu}]
\end{aligned}
$$

Let $g^*_{\mu\nu} \equiv f'(\Phi)g_{\mu\nu}$, $\varphi \equiv \sqrt{3}\,\ln f'(\Phi)$, $V(\varphi) \equiv \dfrac{\Phi f'(\Phi) - f(\Phi)}{f'^2(\Phi)}$

$\Rightarrow$ **Standard scalar-tensor theory**

$$
\begin{aligned}
S = &\int d^4x\sqrt{-g^*}\Big\{R^* - \frac{1}{2}\,g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\Big\} \\
&+ S_{\text{matter}}[\text{matter}, g_{\mu\nu} = e^{\varphi/\sqrt{3}}g^*_{\mu\nu}]
\end{aligned}
$$

graviton

scalar

Relativistic aquadratic Lagrangians

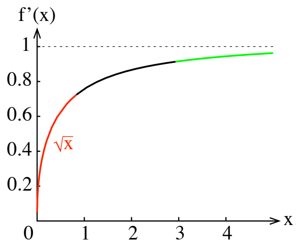
“RAQUAL” models

$$S = \int d^4x\sqrt{-g^*}\Big\{R^* - \underbrace{f\left(g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right)}_{\text{“k-essence”}} - V(\varphi)\Big\}$$
$$+ S_{\text{matter}}[\text{matter}, g_{\mu\nu} \equiv A^2(\varphi)g^*_{\mu\nu}]$$

The nonlinearity of $f(\partial_\mu\varphi\partial^\mu\varphi)$ now allows us to reproduce a MOND-like potential $\sim \sqrt{GMa_0}\ln r$ [Bekenstein & Sanders]:

$$\frac{1}{r^2}\partial_r\left(r^2 f'\left[(\partial_r\varphi)^2\right]\partial_r\varphi\right) \propto T \text{ (matter source)}$$

- $f'(x) \longrightarrow$ constant for large x : Newtonian limit
- $f'(x) \propto \sqrt{x}$ for small x : MOND regime

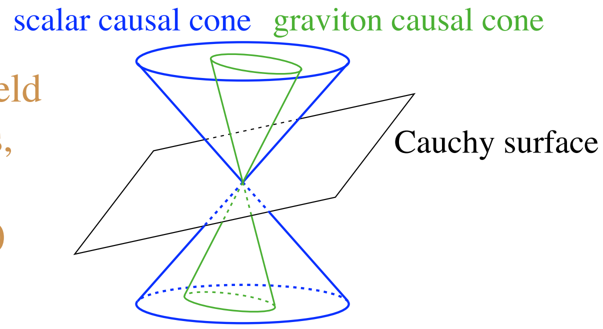


Consistency conditions on $f(\partial_\mu\varphi\partial^\mu\varphi)$

Hyperbolicity of the field equations + Hamiltonian bounded by below

- $\forall x, \quad f'(x) > 0$
- $\forall x, \quad 2\,x f''(x) + f'(x) > 0$

N.B.: If $f''(x) > 0$, the scalar field propagates faster than gravitons, but still causally
$\Rightarrow$ no need to impose $f''(x) \leq 0$



These conditions become much more complicated *within matter*

Action and reaction principle

- Replace gravitational field $g_{\mu\nu}$ by its value imposed by material sources $\Rightarrow$ Fokker Lagrangian

$$\mathcal{L} = -\sum_A m_A c^2 \sqrt{1 - \frac{\mathbf{v}_A^2}{c^2}} + \frac{1}{2}\sum_{A\neq B} \underbrace{V(m_A, m_B, r_{AB}, \mathbf{v}_A, \mathbf{v}_B)}_{\frac{Gm_A m_B}{r_{AB}} - 2m_A\sqrt{Gm_B a_0}\,\ln r_{AB}} + \frac{1}{2}\sum_{A\neq B\neq C} \dots$$

$$\Rightarrow m_A a_A = \frac{Gm_A m_B}{r_{AB}^2} + \frac{m_A\sqrt{Gm_B a_0}}{r_{AB}} + \frac{m_B\sqrt{Gm_A a_0}}{r_{AB}}$$

$$\Rightarrow \quad a_A = \frac{Gm_B}{r_{AB}^2} + \frac{\sqrt{Gm_B a_0}}{r_{AB}} + \frac{m_B\sqrt{Ga_0/m_A}}{r_{AB}}$$

Divergent acceleration for test masses $m_A \to 0$!

- But if self-field effects are treated consistently [Milgrom 1994–97], RAQUAL models actually predict in the MOND regime ($a < a_0$)

$$m_A a_A = \frac{2}{3}\frac{\sqrt{Ga_0}}{r_{AB}}\left[(m_A + m_B)^{3/2} - m_A^{3/2} - m_B^{3/2}\right]$$

$$\Rightarrow \quad a_A = \frac{\sqrt{Gm_B a_0}}{r_{AB}} + \mathcal{O}(\sqrt{m_A})$$

Light deflection

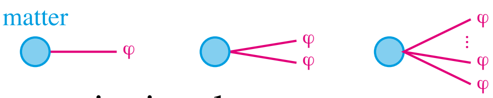
RAQUAL models

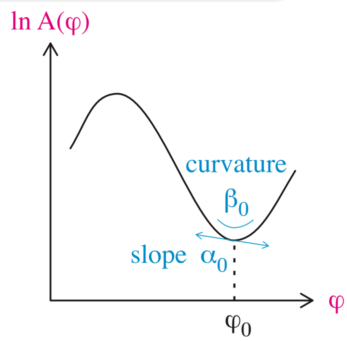
$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g^*}\left\{R^* - f\left(g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) - V(\varphi)\right\}$$

$$+ S_{\text{matter}}[\text{matter}, \underbrace{g_{\mu\nu}}_{\text{physical metric}} \equiv \underbrace{A^2(\varphi)}_{\text{matter-scalar coupling}} \times \underbrace{g^*_{\mu\nu}}_{\text{spin-2 metric}}]$$

- Matter-scalar coupling function

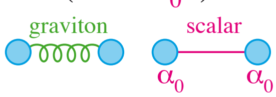
$\ln A(\varphi) = \alpha_0\,(\varphi-\varphi_0) + \frac{1}{2}\beta_0\,(\varphi-\varphi_0)^2 + \dots$





- Effective gravitational constant

$G_{\text{eff}} = G\,(1 + \alpha_0^2)$



Scalar field $\Rightarrow$ extra attractive force

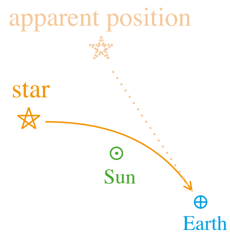
Light deflection (continued)

- Conformally related metrics $g_{\mu\nu} \equiv A^2(\varphi) g^*_{\mu\nu}$
  $\Rightarrow$ Light rays do not feel the scalar field:
  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = 0 \Leftrightarrow g^*_{\mu\nu} dx^\mu dx^\nu = 0$

- Light deflection angle

$$\begin{aligned} \Delta\theta &= \frac{4GM}{bc^2} \quad \text{same as G.R.} \\ &= \underbrace{\frac{4G_{\text{eff}}M}{bc^2(1+\alpha_0^2)} < \frac{4G_{\text{eff}}M}{bc^2}} \end{aligned}$$

interpreted as smaller than G.R. because $G_{\text{eff}} > G_{\text{bare}}$



[N.B.: $\exists$ an erroneous theorem (overstatement) by Bekenstein & Sanders about this]

"Disformal" coupling

Stratified theories
[Ni, Sanders, Bekenstein (TeVeS)]

$$\begin{aligned} S &= \frac{c^3}{16\pi G} \int d^4x \sqrt{-g^*} \left\{ R^* - 2 f(g_*^{\mu\nu} \partial_\mu\varphi \partial_\nu\varphi) \right\} \\ &\quad + S_{\text{matter}} \left[\text{matter}\ ;\ g_{\mu\nu} \equiv A^2(\varphi, U) g^*_{\mu\nu} + B(\varphi, U) U_\mu U_\nu \right] \end{aligned}$$

- U_μ is either a new vector field, or $\partial_\mu\varphi$ itself
- $A^2 > 0$ and $A^2 + B g_*^{\mu\nu} U_\mu U_\mu > 0$ necessary for hyperbolicity (in addition to the previous conditions on f).

Stratified theories

Trick to increase light deflection (as dark matter does)

- Since Schwarzschild is such that $-g_{00} = g_{rr}^{-1} = \left(1 - \frac{2GM}{rc^2}\right)$,
  let us couple φ inversely to g^*_{00} and g^*_{ij}, say

$$g_{00} \equiv e^{2\varphi} g^*_{00} \quad \text{and} \quad g_{ij} \equiv e^{-2\varphi} g^*_{ij}$$

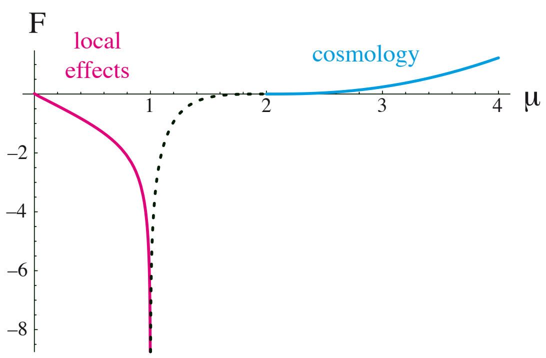
- Covariant rewriting:
  Let a vector $U_\mu = (1, 0, 0, 0)$ in this preferred frame.
  Then $-U_\mu U_\nu = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ behaves as g^*_{00}, and $\left(g^*_{\mu\nu} + U_\mu U_\nu\right)$ as g^*_{ij}
  $\Rightarrow$ Define the physical metric (minimally coupled to matter) as

$$\begin{aligned} g_{\mu\nu} &= e^{-2\varphi}\left(g^*_{\mu\nu} + U_\mu U_\nu\right) - e^{2\varphi} U_\mu U_\nu \\ &= e^{-2\varphi} g^*_{\mu\nu} - 2\, U_\mu U_\nu \sinh(2\varphi) \end{aligned}$$

- N.B.1: Other factors depending on φ would give a different light deflection
  $\Rightarrow$ this is *ad hoc*!
- N.B.2: Preferred-frame effects strongly constrained in solar system

Bekenstein's TeVeS model [2004–05]

- Many years of work $\Rightarrow$ complicated
  $S_{\text{scalar}} = -\frac{1}{2}\int d^4x\sqrt{-g^*}\left[\sigma^2(g_*^{\alpha\beta} - U^\alpha U^\beta)\partial_\alpha\varphi\partial_\beta\varphi + \frac{1}{2}G\ell^{-2}\sigma^4 F(kG\sigma^2)\right]$
- Potential F discontinuous
  $F(\mu) = \frac{3}{8}\,\frac{\mu(4 + 2\mu - 4\mu^2 + \mu^3) + 2\ln[(1-\mu)^2]}{\mu^2}$

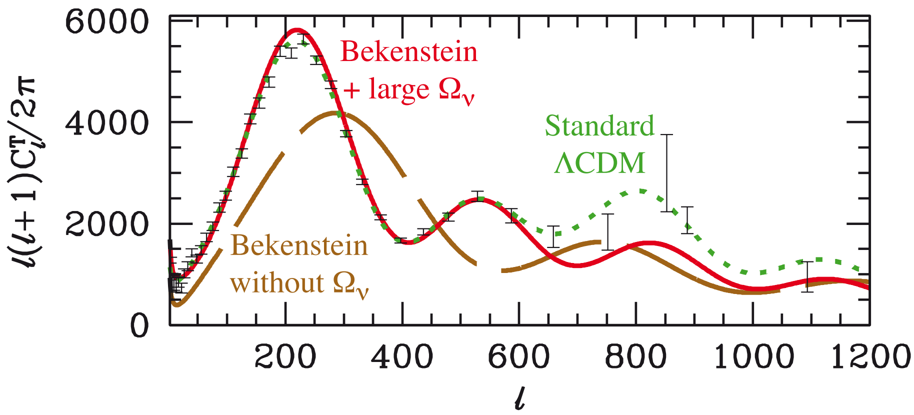


- Sanders defines smoother functions, and promotes σ to a 2nd dynamical scalar field, but this is a tachyon

MOND cosmology?

- No MOND regime in dense Universe at high redshifts
  $\Rightarrow$ Too small $\Omega_{\rm baryon}$ to account for structure formation
  [Lue & Starkman 2004]

- Skordis *et al.* [2006] need $\Omega_\nu = 0.17$ for CMB
  (not far from $\Omega_{\rm DM} = 0.24$) $\Rightarrow \exists$ dark matter!

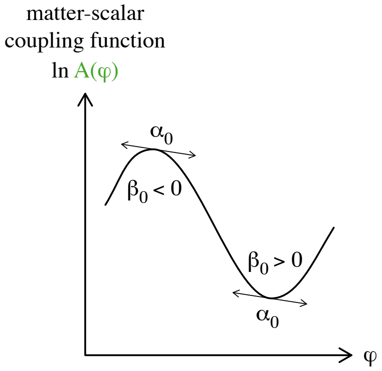


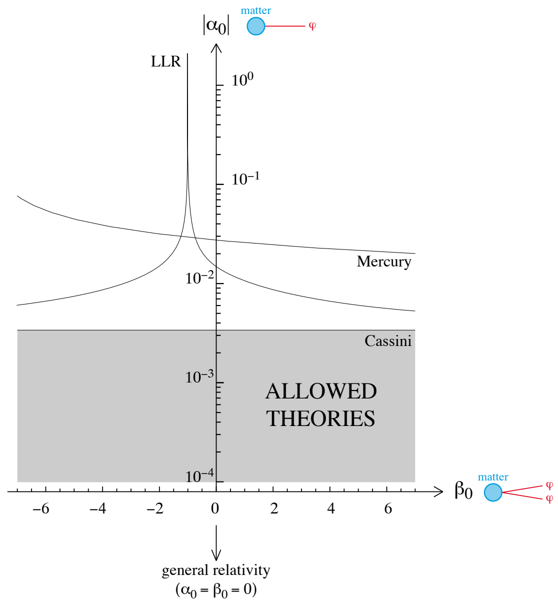
Difficulties of RAQUAL models

- Action/reaction, light deflection & CMB: $\exists$ solutions
- Complicated Lagrangians (unnatural)
- Fine tuning ($\approx$ fit rather than predictive models):
  Possible to predict different lensing and rotation curves
- Discontinuities: can be cured
- In TeVeS [Bekenstein], gravitons & scalar are slower than photons
  $\Rightarrow$ gravi-Cerenkov radiation suppresses high-energy cosmic rays
  [Moore *et al.* 2001–05]
  Solution: Accept slower photons than gravitons
- $\exists$ preferred frame (ether) where vector $U_\mu = (1, 0, 0, 0)$
  Maybe not too problematic if U_μ is dynamical
- Vector contribution to Hamiltonian unbounded by below
  [Clayton 2001] $\Rightarrow$ unstable model
- Post-Newtonian tests very constraining

Post-Newtonian constraints

- Solar-system tests $\Rightarrow$ matter *a priori* weakly coupled to φ
- TeVeS *tuned* to pass them even for strong matter-scalar coupling
- Binary-pulsar tests $\Rightarrow$ matter must be weakly coupled to φ





$$\tilde{g}_{\mu\nu} \equiv A^2(\varphi) g_{\mu\nu}$$

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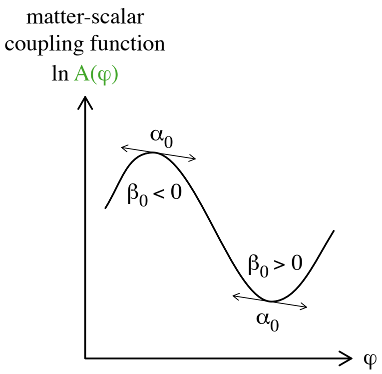

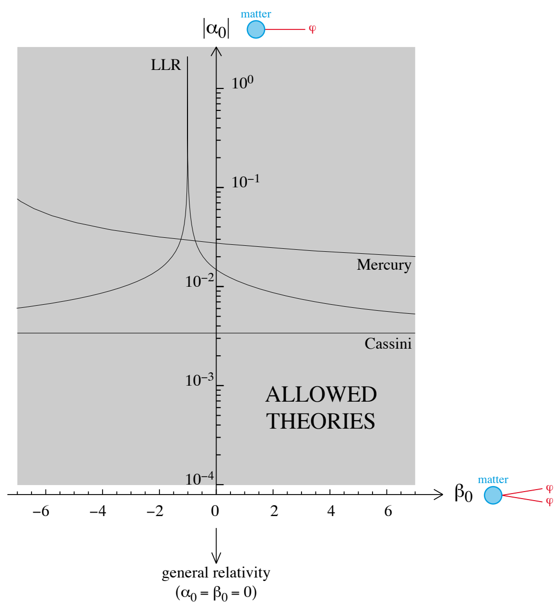

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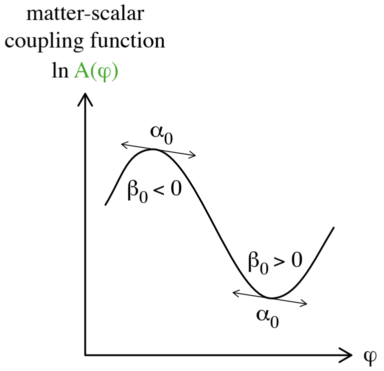

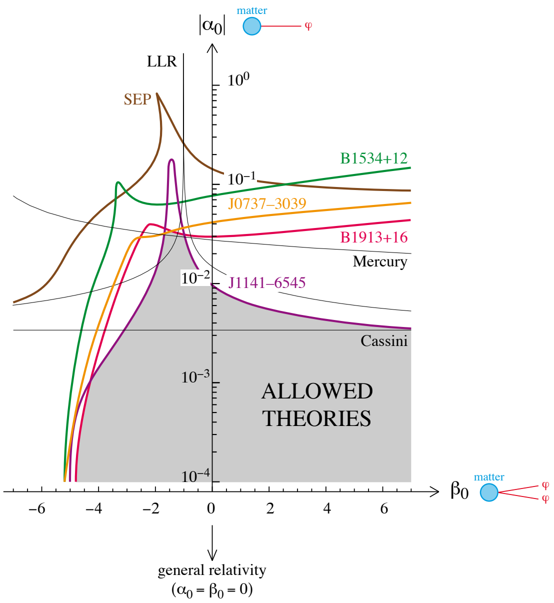

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Introduction Model building Kinetic terms **Difficulties** New routes Conclusions

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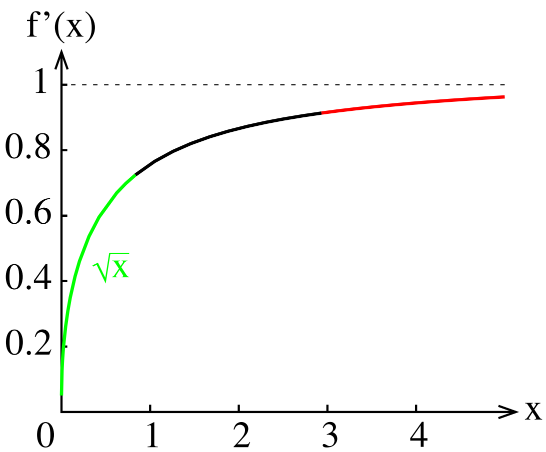

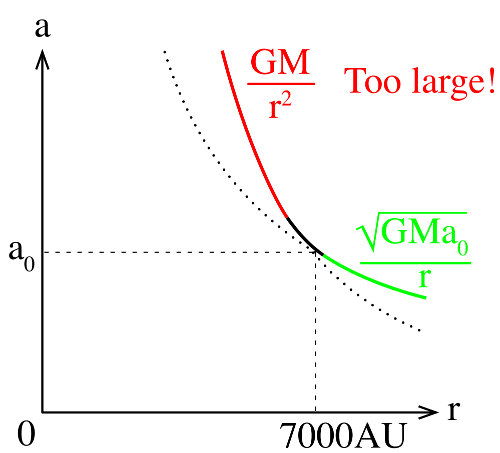

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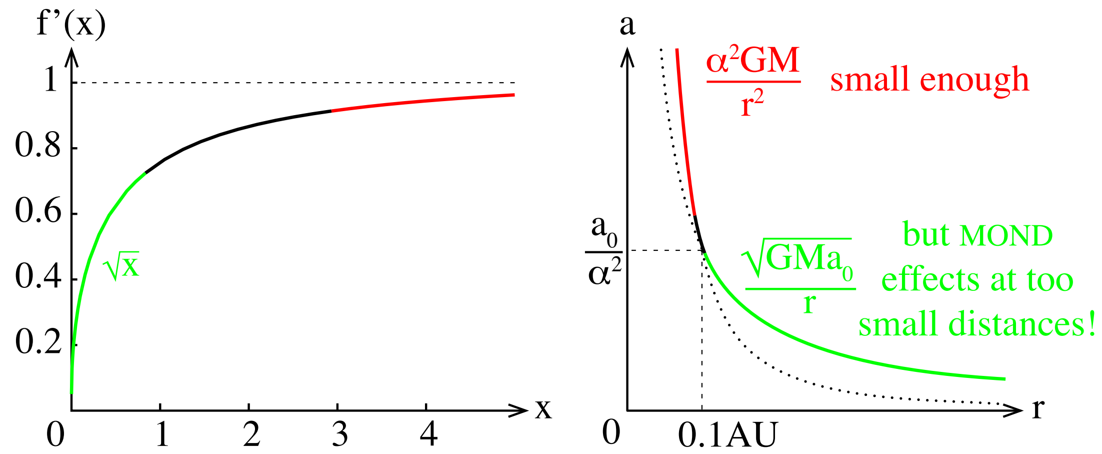

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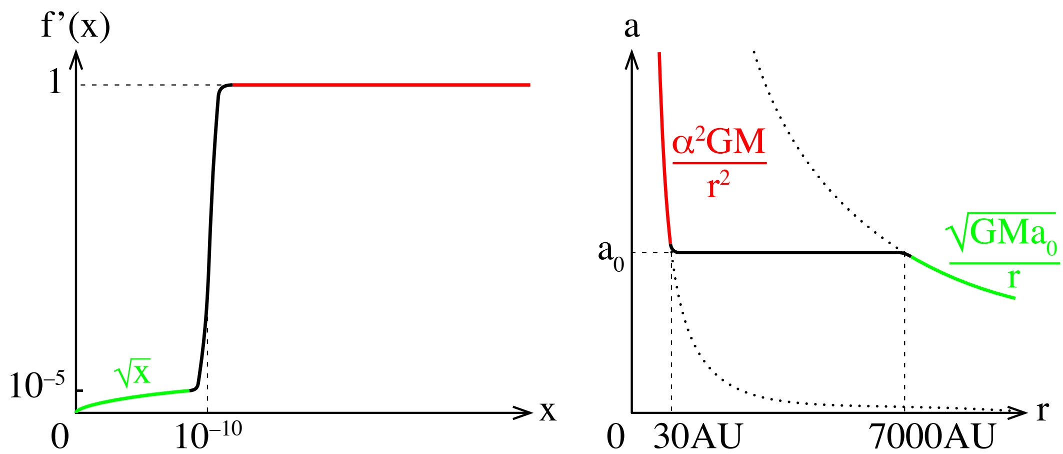


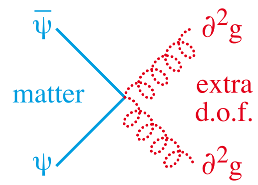
Quite unnatural! (and not far from being experimentally ruled out)

Non-minimal metric coupling

Neither "modified gravity" nor "modified inertia"

$$S = \underbrace{\frac{c^3}{16\pi G}\int d^4x\sqrt{-g^*}\,R^*}_{\text{Einstein-Hilbert}} + \underbrace{S_{\text{matter}}[\text{matter}, g_{\mu\nu} \equiv g^*_{\mu\nu} + f(R^*_{\mu\nu\rho\sigma}, \dots)]}_{\text{Metric coupling}}$$

- Strictly same spectrum as G.R. in vacuum $\Rightarrow$ standard Schwarzschild solution (no extra field, no tachyon nor ghost)
- Equivalence principle satisfied
- $\exists$ vertices coupling matter fields to curvature
  $\Rightarrow$ extra degrees of freedom confined within matter (no free propagator)



- Same structure as finite-size effects in G.R.

Non-minimal metric coupling: problems

But exhibits all generic problems
$\Rightarrow$ quite useful toy model to locate hidden assumptions in the literature!

- Near a spherical body, $R^*_{\mu\nu\rho\sigma}$ and its covariant derivatives give access to M and r independently
  $\Rightarrow$ One can reproduce the MOND phenomenology,
  but also any other potential and any light deflection: not predictive!
- MOST IMPORTANTLY, although this model does not involve any tachyon nor ghost, it is anyway unstable: Hamiltonian unbounded by below

Generic instability of higher-derivative theories

- Consider a "non-degenerate" $\mathcal{L}(q, \dot{q}, \ddot{q})$:

$$p_2 \equiv \frac{\partial \mathcal{L}}{\partial \ddot{q}} \text{ invertible} \Rightarrow \ddot{q} = f(q, \dot{q}, p_2)$$

- Ostrogradski [1850] defines

$$q_1 \equiv q \qquad p_1 \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \ddot{q}}\right)$$

$$q_2 \equiv \dot{q} \qquad p_2 \equiv \frac{\partial \mathcal{L}}{\partial \ddot{q}}$$

- Then $\mathcal{H} = p_1 \dot{q}_1 + p_2 \dot{q}_2 - \mathcal{L}(q, \dot{q}, \ddot{q})$
  $= p_1 q_2 + p_2 f(q_1, q_2, p_2) - \mathcal{L}\left(q_1, q_2, f(q_1, q_2, p_2)\right)$

  is such that $\dot{q}_i = \partial\mathcal{H}/\partial p_i$ and $\dot{p}_i = -\partial\mathcal{H}/\partial q_i$ reproduce the Euler-Lagrange equations derived from $\mathcal{L}$.

Nonminimal scalar-tensor model

Nonminimal metric coupling (unstable within matter)

$$S = \frac{c^3}{16\pi G}\int d^4x \sqrt{-g^*}\, R^* \qquad \text{pure G.R. in vacuum}$$

$$+ S_{\text{matter}}\left[\text{matter}\,;\; g_{\mu\nu} \equiv f(g^*_{\mu\nu}, R^\lambda_{*\mu\nu\rho}, \nabla^*_\sigma R^\lambda_{*\mu\nu\rho}, \ldots)\right]$$

Nonminimal scalar-tensor model

$$S = \frac{c^3}{16\pi G}\int d^4x \sqrt{-g^*}\left\{R^* - 2\, g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right\} \qquad \text{Brans-Dicke in vacuum}$$

$$+ S_{\text{matter}}\left[\text{matter}\,;\; g_{\mu\nu} \equiv A^2 g^*_{\mu\nu} + B\,\partial_\mu\varphi\partial_\nu\varphi\right]$$

Avoids Ostrogradskian instability

- because $g_{\mu\nu}$ depends only on φ and $\partial\varphi$
- and because S_{matter} only involves ∂g linearly

Nonminimal scalar-tensor model (continued)

Nonminimal scalar-tensor model

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g^*}\left\{R^* - 2\,s\right\} \quad \text{Brans-Dicke in vacuum}$$
$$+ S_{\text{matter}}\left[\text{matter}\,;\; g_{\mu\nu} \equiv A^2 g^*_{\mu\nu} + B\,\partial_\mu\varphi\partial_\nu\varphi\right]$$

$$s \equiv g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \qquad A(\varphi,\partial\varphi) \equiv e^{\alpha\varphi} - \frac{\varphi X}{\alpha}\ln X$$

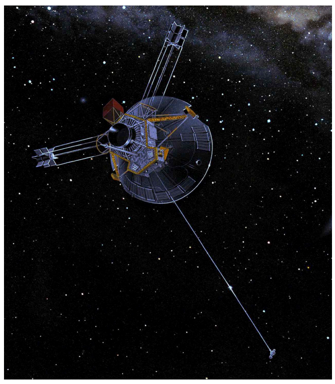
$$X \equiv \frac{\sqrt{\alpha a_0}}{c}\,s^{-1/4} \qquad B(\varphi,\partial\varphi) \equiv -4\,\frac{\varphi X}{\alpha}\,\frac{1}{s}$$

Reproduces MOND while avoiding Ostrogradskian instability

but field equations not always hyperbolic within outer dilute gas!

Pioneer 10 & 11 anomaly

- Extra acceleration $\sim 8.5 \times 10^{-10}$ m.s^{-2} towards the Sun between 30 and 70 AU
- Simpler problem than galaxy rotation curves ($M_{\text{dark}} \propto \sqrt{M_{\text{baryon}}}$), because we do not know how this acceleration is related to $M_\odot$
- $\Rightarrow$ *several* stable & well-posed solutions



Nonminimal scalar-tensor model

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g^*}\left\{R^* - 2\,g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right\} \quad \text{Brans-Dicke in vacuum}$$
$$+ S_{\text{matter}}\left[\text{matter}\,;\; g_{\mu\nu} \equiv e^{2\alpha\varphi} g^*_{\mu\nu} - \lambda\,\frac{\partial_\mu\varphi\partial_\nu\varphi}{\varphi^5}\right]$$

- $\alpha^2 < 10^{-5}$ to pass solar-system & binary-pulsar tests
- $\lambda \approx \alpha^3 (10^{-4}\text{m})^2$ to *fit* Pioneer anomaly

Other recent ideas

- Einstein-æther gravity [T. Jacobson, arXiv:0801.1547 (gr-qc)]

$$S = \int d^4x\sqrt{-g}\Big\{R + c_1(\nabla_\mu U_\nu)^2 + c_2(\nabla_\mu U^\mu)^2 + c_3\nabla_\mu U_\nu \nabla^\nu U^\mu + c_4(U^\mu\nabla_\mu U_\nu)^2 + \lambda(U_\mu U^\mu + 1)\Big\} + S_{\text{matter}}\big[\text{matter}\,;\; g_{\mu\nu}\big]$$

  Nonstandard kinetic term for vector U_μ but constant norm.
  Metric coupling of matter, but $\exists\, T_{\mu\nu}(U) \neq 0$.
  H.S. Zhao generalizes this framework to nonconstant coefficients $c_i(x)$ to reproduce MOND, but predictions & stability still quite unclear.

- Special kind of dark matter reproducing MOND predictions?
  For instance, fluid of gravitational dipoles [L. Blanchet 2006–08].
  $\exists$ nice relativistic version, although a few difficulties remain.

- Nonlocal models? [C. Deffayet, GEF, R. Woodard 2008]
  Actions involving $\frac{1}{\Box}R$, or more generally $f_1(R^\lambda_{\mu\nu\rho})\Box^{\text{power}} f_2(R^\lambda_{\mu\nu\rho})$.
  $\exists$ too naive reasoning showing that this should involve a ghost, but this is not always the case.

Conclusions

A consistent field theory should satisfy different kinds of constraints:

- Mathematical: stability, well-posedness of the Cauchy problem, no discontinuous nor adynamical field
- Experimental: solar-system & binary-pulsar tests, galaxy rotation curves, gravitational lensing by "dark matter" haloes, CMB
- Esthetical: natural model, rather than fine-tuned *fit* of data

Best present candidate: TeVeS [Bekenstein–Sanders], but it has still some mathematical *and* experimental difficulties

$\exists$ simpler models, useful to exhibit the generic difficulties of all MOND-like field theories

By-product of our study: a consistent class of models for the Pioneer anomaly (but *not* natural!)

$\exists$ new ideas (æther, nonlocal, ...), but stability still needs to be proven