Dark energy and dark matter in the ghost-free bigravity theory

Bigravity theories

Cosmology in bigravity theories

- Cosmic No-Hair Conjecture and Dark Energy
- Twin Matter as Dark Matter

Conclusion and Remarks

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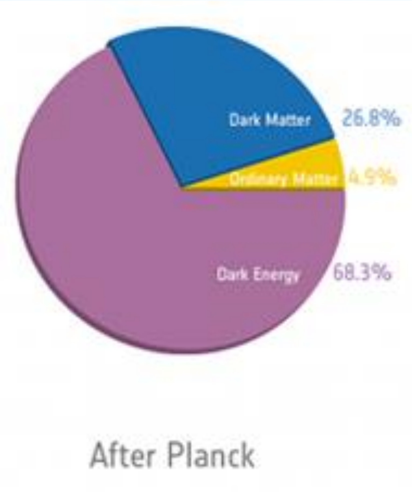
Ghost Free Bigravity Theory

Two big mysteries in cosmology

- Dark Energy → cosmological constant $\Lambda \leq 10^{-120} m_{PL}^2$
- Dark Matter → unknown particles



Bigravity theory with twin matter ?

General Relativity ⇒ graviton: massless spin 2

2 modes (x and +)

massive ?

$$m < 7 \times 10^{-23}\ \mathrm{eV}$$

(solid constraint
from a solar system test)

massive gravity theory

5 modes (massive spin 2)

Fierz-Pauli

M. Fierz and W. Pauli,Proc.Roy.Soc.Lond.A173, 211 (1939) ghost-free linear theory

de Rham-Gabadadze-Tolley

C. de Rham, G. Gabadadze, A.J. Tolley, PRL 106, 231101 (2011) non-linear extension

Introduction of four Stockelberg scalar fields $\phi^a \ (a = 0-3)$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b + H_{\mu\nu}$$

π : helicity-0 mode of the graviton $\quad \phi^a = x^a - \eta^{a\mu}\partial_\mu\pi$

$H_{\mu\nu}$:covariant description of metric perturbation

$$H_{\mu\nu} = h_{\mu\nu} + 2\Pi_{\mu\nu} - \eta^{\rho\sigma}\Pi_{\mu\rho}\Pi_{\nu\sigma} \qquad \Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \sqrt{\delta^\mu{}_\nu - H^\mu{}_\nu}$$

Fierz-Pauli mass term $\quad +\frac{m^2}{2}([\mathcal{K}]^2 - [\mathcal{K}^2])$

Non-linear ghost free term $\quad \mathcal{U} = \sum_{k=2}^{4} c_k\,\mathcal{U}_k(\mathcal{K}),$ $\quad \{c_k\}$ coupling constants

$$\mathcal{U}_2(\mathcal{K}) = -\frac{1}{2!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\rho\sigma}\mathcal{K}^\mu{}_\alpha\mathcal{K}^\nu{}_\beta = \frac{1}{2!}([\mathcal{K}]^2 - [\mathcal{K}^2]),$$

$$\mathcal{U}_3(\mathcal{K}) = -\frac{1}{3!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}\mathcal{K}^\mu{}_\alpha\mathcal{K}^\nu{}_\beta\mathcal{K}^\rho{}_\gamma = \frac{1}{3!}([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]),$$

$$\mathcal{U}_4(\mathcal{K}) = -\frac{1}{4!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\mathcal{K}^\mu{}_\alpha\gamma^\nu{}_\beta\mathcal{K}^\rho{}_\gamma\gamma^\sigma{}_\delta = \frac{1}{4!}([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4])$$

cosmology:

There exists no flat Friedmann universe

⇨ Fictitious metric η_{ab} : de Sitter (or FLRW metric)

There exists three types of Friedmann universe

Question:

Which spacetime we should adopt for the fictitious metric ?

⇨ bigravity

S.F. Hassan and R.A. Rosen, JHEP 1202, 126 (2012)

Dynamics for both metrics !

ghost-free bigravity theory

$$S[g,f,\text{matter}] = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}\,R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\,\mathcal{R}(f)$$

$$-\frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\,\mathscr{U}[g,f] + S_g^{[\mathrm{m}]}[g,\text{g-matter}] + S_f^{[\mathrm{m}]}[f,\text{f-matter}]$$

Twin Matter Fluids

Interaction term

$$\kappa^2 = \kappa_g^2 + \kappa_f^2$$

$$\mathscr{U} = \sum_{k=0}^{4} b_k\,\mathscr{U}_k(\gamma), \qquad \gamma^\mu{}_\nu = \sqrt{g^{\mu\alpha} f_{\alpha\nu}} \qquad \lambda_A : \text{eigenvalues of } \gamma^\mu{}_\nu$$

$$\mathscr{U}_0(\gamma) = -\frac{1}{4!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 1 \qquad \mathscr{U}_1(\gamma) = -\frac{1}{3!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\nu\rho\sigma}\gamma^\mu{}_\alpha = \sum_A \lambda_A$$

$$\mathscr{U}_2(\gamma) = -\frac{1}{2!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\rho\sigma}\gamma^\mu{}_\alpha\gamma^\nu{}_\beta = \sum_{A<B}\lambda_A\lambda_B$$

$$\mathscr{U}_3(\gamma) = -\frac{1}{3!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}\gamma^\mu{}_\alpha\gamma^\nu{}_\beta\gamma^\rho{}_\gamma = \sum_{A<B<C}\lambda_A\lambda_B\lambda_C$$

$$\mathscr{U}_4(\gamma) = -\frac{1}{4!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\gamma^\mu{}_\alpha\gamma^\nu{}_\beta\gamma^\rho{}_\gamma\gamma^\sigma{}_\delta = \lambda_0\lambda_1\lambda_2\lambda_3$$

m : graviton mass

a flat space is a solution $\Rightarrow$

$$b_0 = 4c_3 + c_4 - 6, \quad b_1 = 3 - 3c_3 - c_4,$$

$$b_2 = 2c_3 + c_4 - 1, \quad b_3 = -(c_3 + c_4), \quad b_4 = c_4.$$

Basic equations Two Einstein equations

$$G_{\mu\nu} = \kappa_g^2 \left[T^{[\gamma]}_{\mu\nu} + T^{[\mathrm{m}]}_{\mu\nu} \right] \qquad \mathcal{G}_{\mu\nu} = \kappa_f^2 \left[\mathcal{T}^{[\gamma]}_{\mu\nu} + \mathcal{T}^{[\mathrm{m}]}_{\mu\nu} \right]$$

$$m_g^2 = \frac{\kappa_g^2}{\kappa^2} m^2$$

$$m_f^2 = \frac{\kappa_f^2}{\kappa^2} m^2$$

Energy-momentum tensor from interactions

$$\kappa_g^2 T^{[\gamma]\mu}{}_\nu = m_g^2 (\tau^\mu{}_\nu - \mathscr{U} \, \delta^\mu_\nu) \qquad \kappa_f^2 \mathcal{T}^{[\gamma]\mu}{}_\nu = -m_f^2 \frac{\sqrt{-g}}{\sqrt{-f}} \tau^\mu{}_\nu \, ,$$

$$\tau^\mu{}_\nu = \{ b_1 \, \mathscr{U}_0 + b_2 \, \mathscr{U}_1 + b_3 \, \mathscr{U}_2 + b_4 \, \mathscr{U}_3 \} \gamma^\mu{}_\nu - \{ b_2 \, \mathscr{U}_0 + b_3 \, \mathscr{U}_1 + b_4 \, \mathscr{U}_2 \} (\gamma^2)^\mu{}_\nu$$
$$+ \{ b_3 \, \mathscr{U}_0 + b_4 \, \mathscr{U}_1 \} (\gamma^3)^\mu{}_\nu - b_4 \, \mathscr{U}_0 \, (\gamma^4)^\mu{}_\nu$$

$$\overset{(g)}{\nabla}_\mu \, T^{[\mathrm{m}]\mu}{}_\nu = 0, \qquad \overset{(f)}{\nabla}_\mu \, \mathcal{T}^{[\mathrm{m}]\mu}{}_\nu = 0$$

Conservation equations

Bianchi id. $\Rightarrow$

$$\overset{(g)}{\nabla}_\mu T^{[\gamma]\mu}{}_\nu = 0. \qquad \overset{(f)}{\nabla}_\mu \mathcal{T}^{[\gamma]\mu}{}_\nu = 0$$

Homothetic metrics:

$$f_{\mu\nu} = K^2 g_{\mu\nu} \Rightarrow \gamma^\mu{}_\nu = (\sqrt{g^{-1}f})^\mu{}_\nu = K\,\delta^\mu{}_\nu$$

$$\kappa_g^2 T^{[\gamma]\mu}{}_\nu = -\Lambda_g(K)\delta^\mu{}_\nu\,,\ \ \kappa_f^2 \mathcal{T}^{[\gamma]\mu}{}_\nu = -\Lambda_f(K)\delta^\mu{}_\nu$$

$$\Lambda_g(K) = m_g^2\left(b_0 + 3b_1\,K + 3b_2\,K^2 + b_3\,K^3\right)$$

$$\Lambda_f(K) = m_f^2\left(b_1/K^3 + 3b_2/K^2 + 3b_3/K + b_4\right)$$

$$\overset{(g)}{\nabla}_\mu T^{[\gamma]\mu}{}_\nu = 0. \qquad \overset{(f)}{\nabla}_\mu \mathcal{T}^{[\gamma]\mu}{}_\nu = 0 \Rightarrow$$

K: constant

$$\Rightarrow \begin{array}{c}\Lambda_g(b_i)\\ \Lambda_f(b_i)\end{array}$$

cosmological constants

“GR” with a cosmological constant

$$G_{\mu\nu} + \Lambda_g g_{\mu\nu} = \kappa_g^2 T^{[\mathrm{m}]}_{\mu\nu}$$

$\Lambda_g(K) = K^2\Lambda_f(K)$: quartic equation for *K*

$$\mathcal{T}^{[\mathrm{m}]}_{\mu\nu} = K^2 T^{[\mathrm{m}]}_{\mu\nu}$$

This does not give GR theory !

perturbations around a homothetic solution (vacuum)

$$g_{\mu\nu} = \overset{(0)}{g}_{\mu\nu} + \epsilon h_{\mu\nu}\,, \quad f_{\mu\nu} := K^2 \tilde{f}_{\mu\nu} = K^2 \left(\overset{(0)}{g}_{\mu\nu} + \epsilon k_{\mu\nu} \right)$$

$$\overset{(0)}{g}{}^{\mu\rho} \overset{(1)}{R}{}_{\rho\nu}(h) - \overset{(0)}{R}{}^{\rho(\mu} h_{\nu)\rho} = -\frac{m_g^2}{4}(b_1 K + 2b_2 K^2 + b_3 K^3)\left[2(h^\mu{}_\nu - k^\mu{}_\nu) + (h-k)\delta^\mu{}_\nu\right]$$

$$\overset{(0)}{g}{}^{\mu\rho} \overset{(1)}{R}{}_{\rho\nu}(k) - \overset{(0)}{R}{}^{\rho(\mu} k_{\nu)\rho} = +\frac{m_f^2}{4K^2}(b_1 K + 2b_2 K^2 + b_3 K^3)\left[2(h^\mu{}_\nu - k^\mu{}_\nu) + (h-k)\delta^\mu{}_\nu\right]$$

linear combinations

$$\psi_{\mu\nu} := m_f^2 h_{\mu\nu} + K^2 m_g^2 k_{\mu\nu}$$

$$\varphi_{\mu\nu} := h_{\mu\nu} - k_{\mu\nu}$$

$$\overset{(0)}{g}{}^{\mu\rho} \overset{(1)}{R}{}_{\rho\nu}(\psi) - \overset{(0)}{R}{}^{\rho(\mu} \psi_{\nu)\rho} = 0$$

massless mode

$$\overset{(0)}{g}{}^{\mu\rho} \overset{(1)}{R}{}_{\rho\nu}(\varphi) - \overset{(0)}{R}{}^{\rho(\mu} \varphi_{\nu)\rho} = -\frac{1}{4} m_{\rm eff}^2 \left[2\varphi^\mu{}_\nu + \varphi\delta^\mu{}_\nu\right]$$

massive mode

$$m_{\rm eff}^2 = \left(m_g^2 + \frac{m_f^2}{K^2} \right)(b_1 K + 2b_2 K^2 + b_3 K^3)$$

Minkowski (or de Sitter, AdS) background $\overset{(0)}{R}_{\mu\nu} = \Lambda_g \overset{(0)}{g}_{\mu\nu}$ $\overset{(0)}{C}_{\mu\nu}{}^{\alpha\beta} = 0$

massless mode

$$\bar{\psi}_{\mu\nu} := \psi_{\mu\nu} - \frac{1}{2}\psi g^{(0)}_{\mu\nu}$$

$\bar{\psi}^{(\mathrm{TT})}_{\mu\nu}$ transverse-traceless $\begin{cases} \nabla^\mu \bar{\psi}^{(\mathrm{TT})}_{\mu\nu} = 0 \\ g^{\mu\nu}\bar{\psi}^{(\mathrm{TT})}_{\mu\nu} = 0 \end{cases}$

$$\overset{(0)}{\Box}\bar{\psi}^{(\mathrm{TT})}_{\mu\nu} - \frac{2}{3}\Lambda_g \bar{\psi}^{(\mathrm{TT})}_{\mu\nu} = 0$$

2 degrees of freedom

usual graviton

massive mode $\varphi_{\mu\nu}$

perturbed Bianchi id $\overset{(0)}{\nabla}_\mu \overset{(1)}{G}{}^\mu_\nu(\varphi) = 0$ $\Rightarrow$ $\overset{(0)}{\nabla}{}^\mu \varphi_{\mu\nu} - \overset{(0)}{\nabla}_\nu \varphi = 0$

4 constraints

trace $\Rightarrow$ $0 = 2\overset{(0)}{\nabla}{}^\nu \left(\overset{(0)}{\nabla}{}^\mu \varphi_{\mu\nu} - \overset{(0)}{\nabla}_\nu \varphi \right) = -3\left(m_{\text{eff}}^2 - 2H_g^2\right)\varphi$ $\quad H_g^2 = \Lambda_g/3$

$\Rightarrow$ $\left\{ \begin{array}{l} \varphi = 0 \\ \overset{(0)}{\Box}\varphi_{\mu\nu} - \left(\frac{2}{3}\Lambda_g + m_{\text{eff}}^2\right)\varphi_{\mu\nu} = 0 \end{array} \right.$

1 constraint (no BD ghost)

degrees of freedom

10–4–1=5

massive spin 2

or

Higuchi bound $m_{\text{eff}}^2 = 2H_g^2$

$\Lambda_g(K) = K^2 \Lambda_f(K)$ trivial

new gauge symmetry $\Rightarrow$ degrees of freedom 5–1=4

partially massless

Cosmology in Bigravity Theory

Homothetic metrics: $f_{\mu\nu} = K^2 g_{\mu\nu}$

GR with a cosmological constant

$$G_{\mu\nu} + \Lambda_g g_{\mu\nu} = \kappa_g^2 T^{[\mathrm{m}]}_{\mu\nu}$$

$$\Lambda_g(K) = m_g^2 \left(b_0 + 3b_1\, K + 3b_2\, K^2 + b_3\, K^3\right)$$

$$\Lambda_f(K) = m_f^2 \left(b_1/K^3 + 3b_2/K^2 + 3b_3/K + b_4\right)$$

$\Lambda_g(K) = K^2 \Lambda_f(K)$: quartic equation for *K*

$$\mathcal{T}^{[\mathrm{m}]}_{\mu\nu} = K^2 T^{[\mathrm{m}]}_{\mu\nu}$$

dS spacetime is a solution

It could be the present acceleration of the universe if $m \sim 10^{-33}\ \mathrm{eV}$

Question:

- de Sitter : attractor ? ⇨ **cosmic no hair conjecture**

In GR, if a cosmological constant exists, it was proved by Wald for Bianchi models

- Homothetic solution (de Sitter) is an attractor
  - **FLRW universe** Aoki, KM ('14)
  - **Bianchi universe** KM, Volkov ('13)
    - The shear density drops as a^{-3} (In GR a^{-6})
    - Chaotic behaviour in Type IX

FLRW universe

$$ds_g^2 = -N_g^2(t)dt^2 + a_g^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$$ds_f^2 = -N_f^2(t)dt^2 + a_f^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$$A = N_f/N_g\,, \quad B = a_f/a_g$$

$$N_g = 1\,(\text{gauge choice})$$

Friedmann equations

$$H_g^2 + \frac{k}{a_g^2} = \frac{\kappa_g^2}{3}\left[\rho_g^{[\gamma]} + \rho_g\right] \qquad \rho_g^{[\gamma]} = \frac{m^2}{\kappa^2}(b_0 + 3b_1\tilde{B} + 3b_2\tilde{B}^2 + b_3\tilde{B}^3)$$

$$H_f^2 + \frac{k}{a_f^2} = \frac{\kappa_f^2}{3}\left[\rho_f^{[\gamma]} + \rho_f\right] \qquad \rho_f^{[\gamma]} = \frac{m^2}{\kappa^2}\left(b_4 + \frac{3b_3}{\tilde{B}} + \frac{3b_2}{\tilde{B}^2} + \frac{b_1}{\tilde{B}^3}\right)$$

$$H_g = \frac{\dot{a}_g}{a_g}\,, \quad H_f = \frac{\dot{a}_f}{N_f a_f}$$

conservation equations

$$\left(\frac{\dot{a}_f}{\dot{a}_g} - A\right)(b_1 + 2b_2\tilde{B} + b_3\tilde{B}^2) = 0 \;\Rightarrow\; \frac{\dot{a}_f}{\dot{a}_g} = A \qquad H_g = BH_f$$

$$\kappa_g^2 \left[\rho_g^{[\gamma]}(\tilde{B}) + \rho_g(a_g)\right] - {\kappa_f}^2 \tilde{B}^2 \left[\rho_f^{[\gamma]}(\tilde{B}) + \rho_f(a_f)\right] = 0$$

$$a_g = a_g(\tilde{B}) \qquad a_f = B a_g$$

$$\left(\frac{d\tilde{B}}{dt}\right)^2 + V_g(\tilde{B}) = 0$$

dust radiation

$$V_g(\tilde{B}) = \frac{a_g^2}{a_g'^2}\left[\frac{k}{a_g^2(\tilde{B})} - \frac{1}{3}\left(\kappa_g^2 \rho_g^{[\gamma]}(\tilde{B}) + \frac{c_{g,\mathrm{m}}}{a_g^3(\tilde{B})} + \frac{c_{g,\mathrm{r}}}{a_g^4(\tilde{B})}\right)\right]$$

$$a_g' = -\frac{\left(C_\Lambda + \tilde{B} C_\Lambda'\right) a_g^4 + (2c_{g,\mathrm{m}}\tilde{B} - \epsilon c_{f,\mathrm{m}}) a_g + 2c_{g,\mathrm{r}}\tilde{B}}{\tilde{B}(4C_\Lambda a_g^3 + C_\mathrm{m})}$$

$$C_\Lambda(\tilde{B}) = \tilde{B}\left[\kappa_g^2 \rho_g^{[\gamma]}(\tilde{B}) - {\kappa_f}^2 \tilde{B}^2 \rho_f^{[\gamma]}(\tilde{B})\right]$$

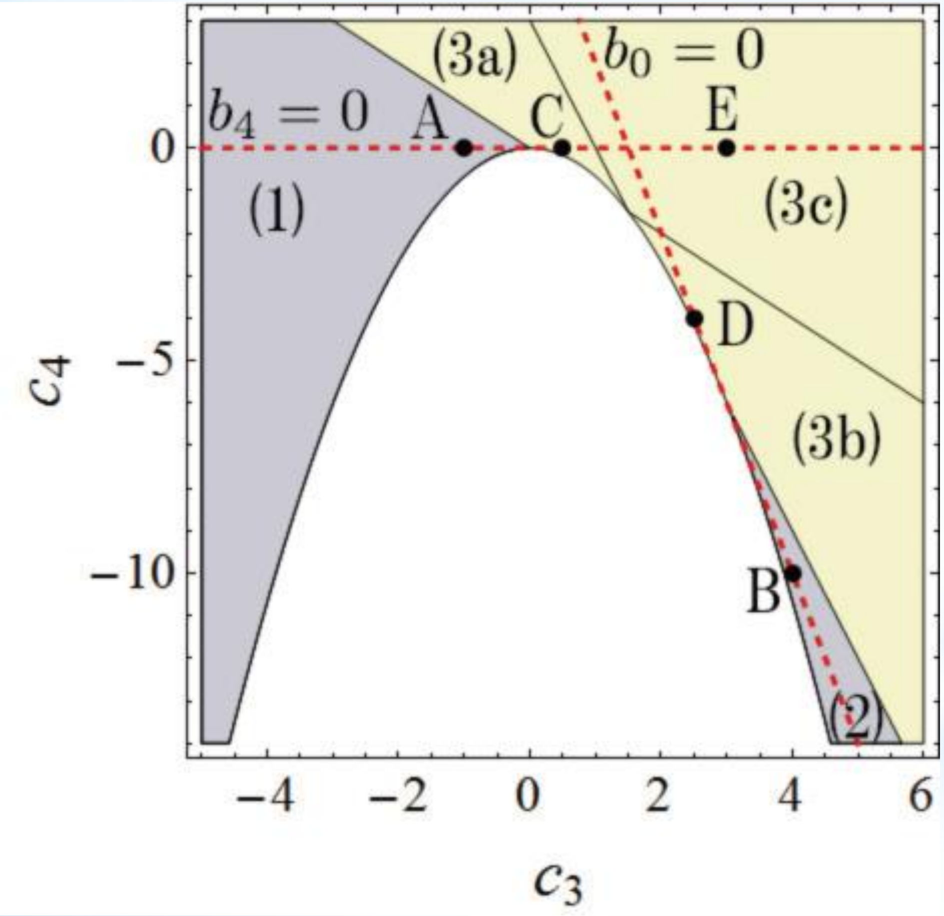
$$C_\mathrm{m}(\tilde{B}) = c_{g,\mathrm{m}}\tilde{B} - \epsilon c_{f,\mathrm{m}}$$

vacuum solution $C_\Lambda(\tilde{B}) = 0$ ⇒ **Homothetic solution (*B*=*A*=*K*)**

quadratic eq.

dS, M, AdS

Parameter region for dS



$B = 1$ $\Lambda_g(1) = 0$

M (stable)

$B \neq 1$

1 dS (stable)

$\Lambda_g(B_{dS}) > 0$

2 AdS

$\Lambda_g(B_{AdS}) < 0$

The evolution of the universe

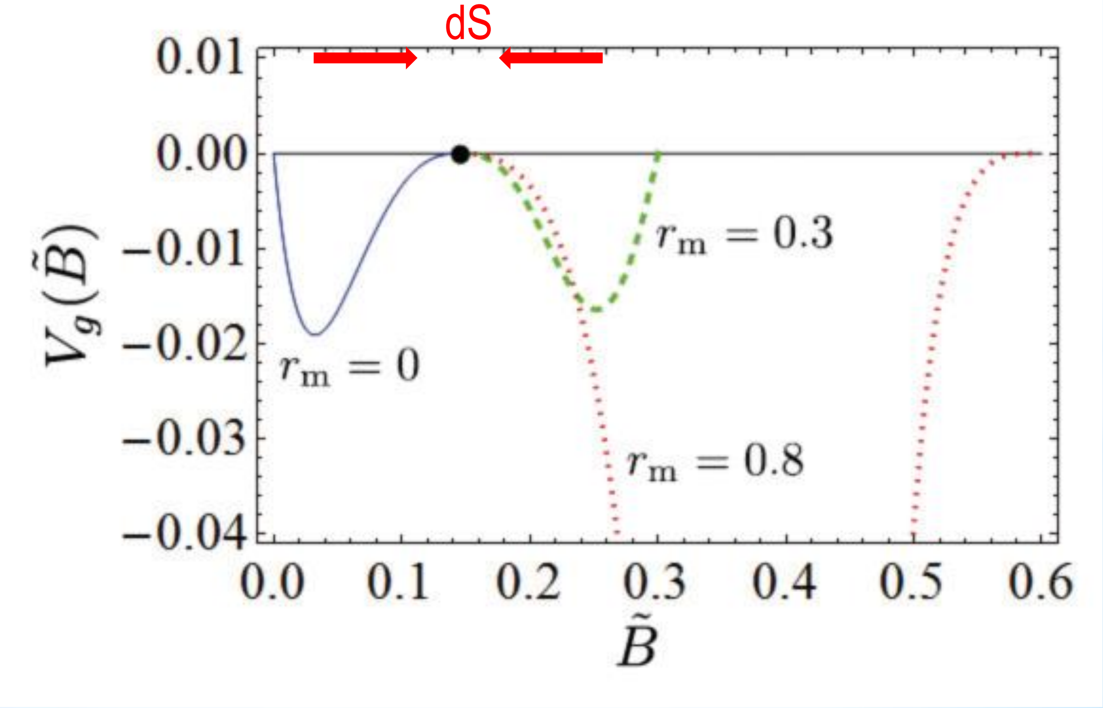
Twin matter fluids: **dust fluid**

$$\rho_g = \frac{c_{g,\mathrm{m}}}{a_g^3} \qquad \rho_f = \frac{c_{f,\mathrm{m}}}{a_f^3}$$

$$r_\mathrm{m} \equiv \frac{c_{f,\mathrm{m}}}{c_{g,\mathrm{m}}} \propto \left.\frac{\rho_f}{\rho_g}\right|_0$$

$$\left(\frac{d\tilde{B}}{dt}\right)^2 + V_g(\tilde{B}) = 0$$

potential for $B = a_f/a_g$

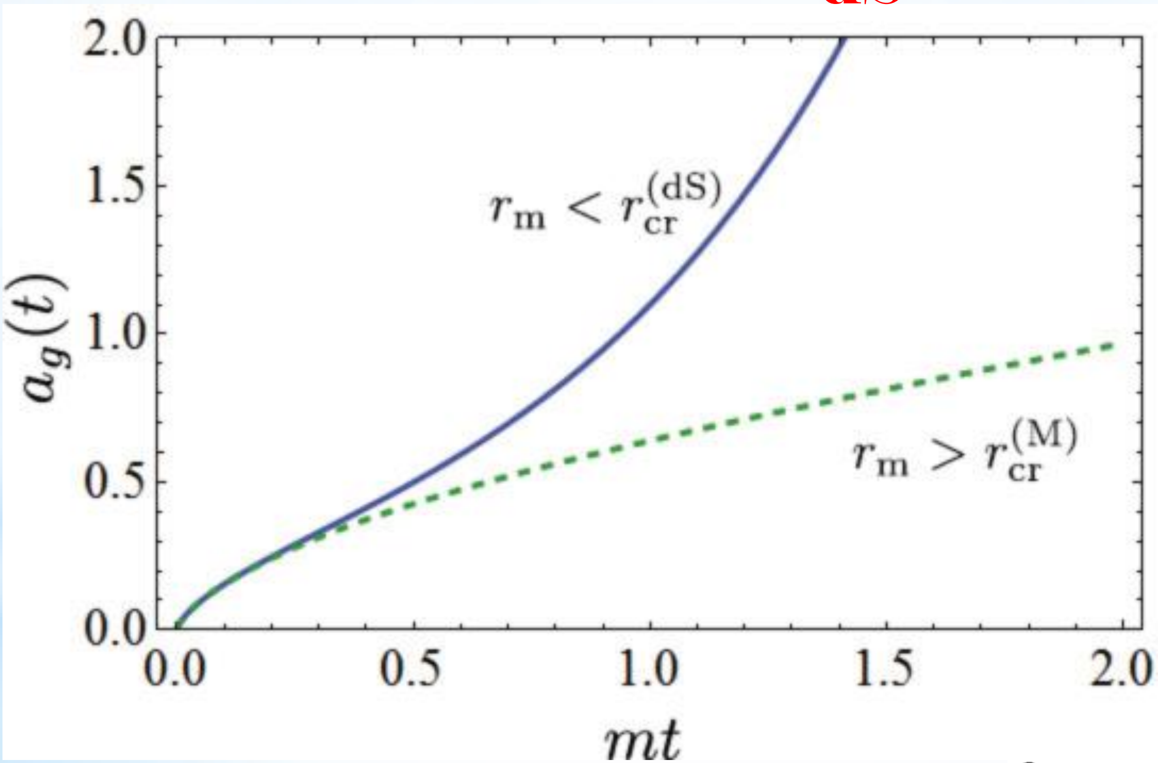


dS: de Sitter
M: matter dominant universe

There are two attractor solutions

The fate depends on the ratio of two matter fluids $r_{\mathrm{m}} \propto \left.\frac{\rho_f}{\rho_g}\right|_0$

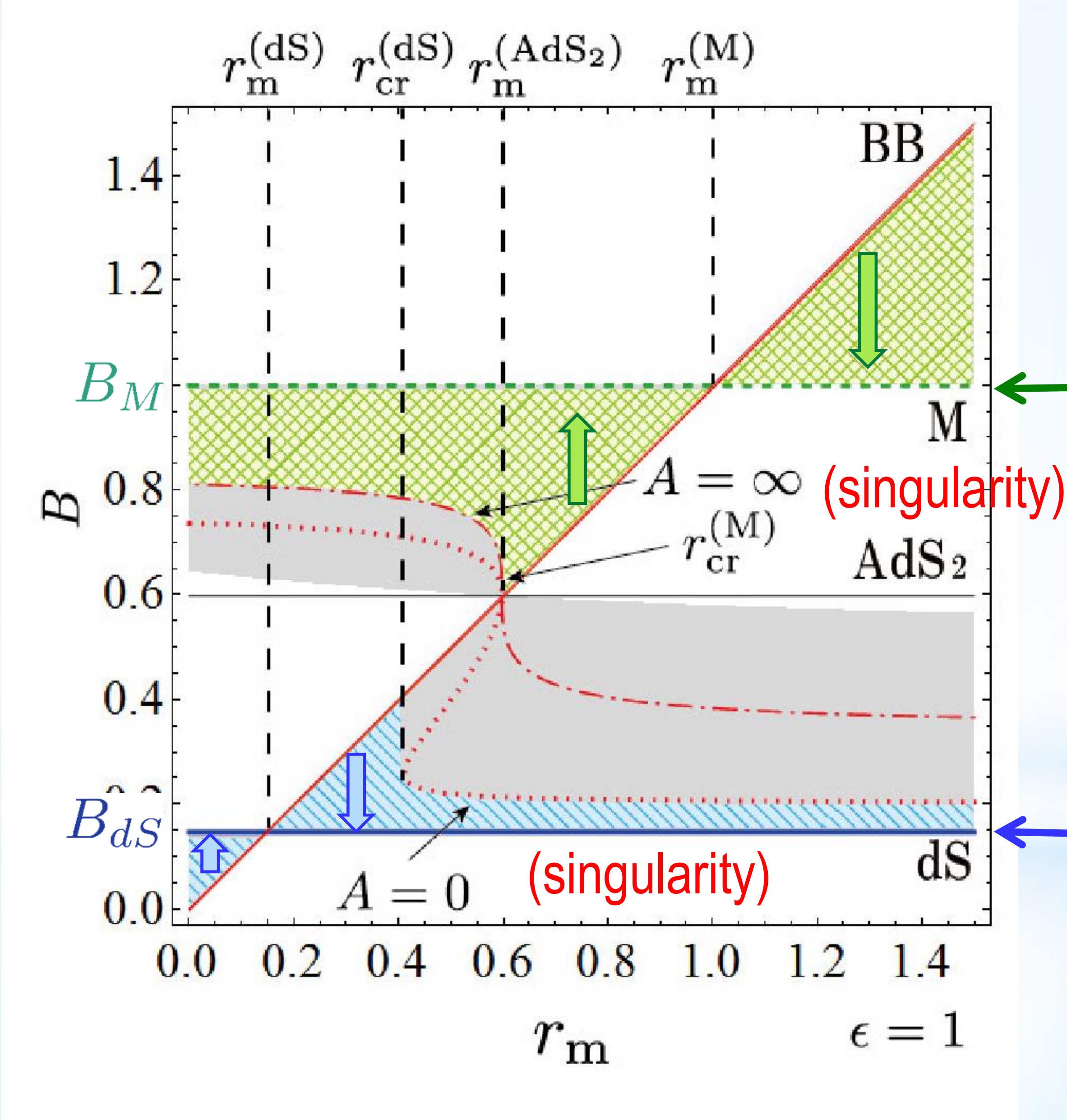
dS



M

$c_3 = 4\,, \quad c_4 = -10$

Model B $c_3 = 4\,,\quad c_4 = -10$



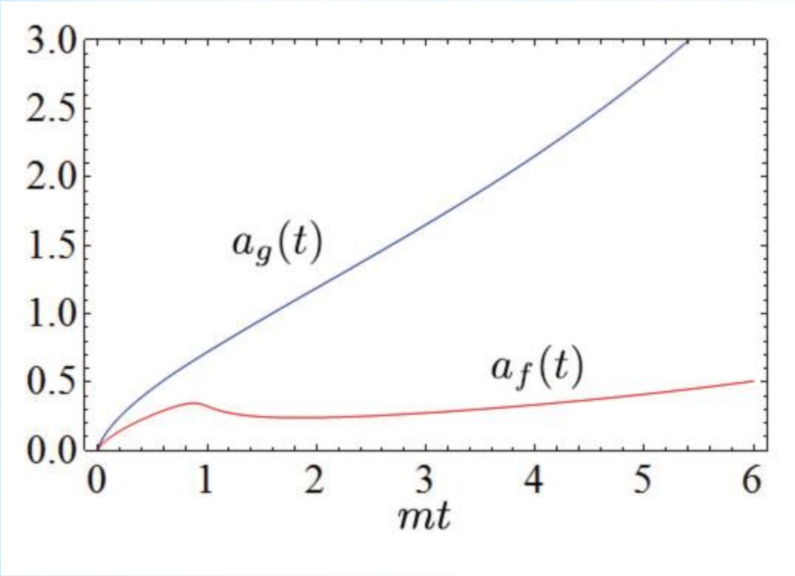
matter dominant
(homothetic sol)

de Sitter
(homothetic sol)

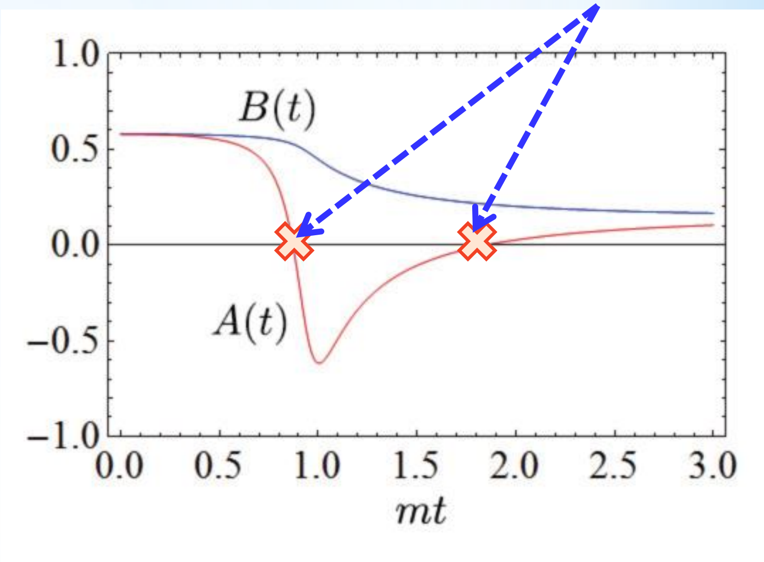
The universe evolves into a singularity

Model B $c_3 = 4\,, \quad c_4 = -10$

$r_{\mathrm{m}} = 0.58$



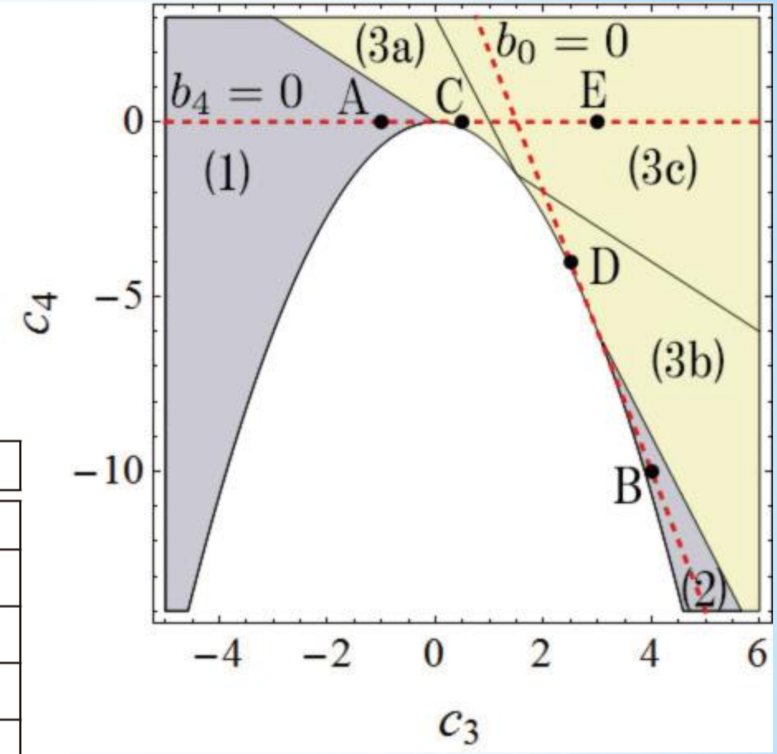
singularity



$A = N_f / N_g\,, \quad B = a_f / a_g$

The conditions for
de Sitter accelerating universe
or
the matter dominant universe

region	ϵ	$r_{\rm m}$	B
de Sitter accelerating universe			
(1)	1	$r_{\rm m} > r_{\rm cr}^{(\rm dS)}$	$B > r_{\rm m}^{(\rm AdS_2)}$
(2)	1	$r_{\rm m} < r_{\rm cr}^{(\rm dS)}$	$B < r_{\rm m}^{(\rm AdS_2)}$
(3a)	-1	$r_{\rm m} > r_{\rm cr}^{(\rm dS)}$	$B > r_{\rm m}^{(\rm AdS_1)}$
(3b)	-1	$r_{\rm m} < r_{\rm cr}^{(\rm dS)}$	$B < r_{\rm m}^{(\rm AdS_1)}$
(3c)	-1	no condition	no condition
matter dominant universe			
(1)	1	$r_{\rm m} < r_{\rm cr}^{(\rm M)}$	$B < r_{\rm m}^{(\rm AdS_2)}$
(2)	1	$r_{\rm m} > r_{\rm cr}^{(\rm M)}$	$B > r_{\rm m}^{(\rm AdS_2)}$
(3a)	1	$r_{\rm m} < r_{\rm cr}^{(\rm M)}$	$B < r_{\rm m}^{(\rm AdS_2)}$
(3b)	1	$r_{\rm m} > r_{\rm cr}^{(\rm M)}$	$B > r_{\rm m}^{(\rm AdS_2)}$
(3c)	1	$r_{\rm cr}^{(\rm AdS_1)} < r_{\rm m} < r_{\rm cr}^{(\rm AdS_2)}$	$r_{\rm cr}^{(\rm AdS_1)} < B < r_{\rm cr}^{(\rm AdS_2)}$



dS is an attractor and reached from reasonable initial conditions

Bianchi universe

Bianchi Spacetimes

$$[\xi_a, \xi_b] = C^c{}_{ab}\xi_c \qquad \xi_a : \text{Killing vectors}$$

$$C^c{}_{ab} = n^{cd}\epsilon_{dab} + a(\delta^1_a\delta^c_b - \delta^1_b\delta^c_a) \qquad n^{ab} = \mathrm{diag}[n^{(1)}, n^{(2)}, n^{(3)}]$$

$a = 0$ ⇒ Class A

	I	II	VI_0	VII_0	VIII	IX
$n^{(1)}$	0	1	1	1	1	1
$n^{(2)}$	0	0	-1	1	1	1
$n^{(3)}$	0	0	0	0	-1	1

$$ds_g^2 = -\alpha^2 dt^2 + e^{2\Omega}e^{2\beta_{ij}}\omega_i\omega_j \qquad ds_f^2 = -\mathcal{A}^2 dt^2 + e^{2\mathcal{W}}e^{2\mathcal{B}_{ij}}\omega_i\omega_j$$

$$(\beta_{ij}) = \begin{pmatrix} \beta_+ + \sqrt{3}\beta_- & 0 & 0 \\ 0 & \beta_+ - \sqrt{3}\beta_- & 0 \\ 0 & 0 & -2\beta_+ \end{pmatrix}$$

$$(\mathcal{B}_{ij}) = \begin{pmatrix} \mathcal{B}_+ + \sqrt{3}\mathcal{B}_- & 0 & 0 \\ 0 & \mathcal{B}_+ - \sqrt{3}\mathcal{B}_- & 0 \\ 0 & 0 & -2\mathcal{B}_+ \end{pmatrix}.$$

Bianchi I

$$\frac{1}{2}\dot{\Omega}^2 = \frac{1}{2}\left(\dot{\beta}_+^2 + \dot{\beta}_-^2\right) + \frac{m_g^2}{6}\alpha^2 e^{-3\Omega} V_g + \frac{\alpha^2\kappa_g^2}{6}\rho_g^{(\mathrm{m})}$$

$$\ddot{\Omega} - \frac{\dot{\alpha}}{\alpha}\dot{\Omega} + 3\dot{\Omega}^2 = \frac{m_g^2}{6}\alpha e^{-3\Omega}\left[\alpha\left(3V_g + \frac{\partial V_g}{\partial \Omega}\right) + \mathcal{A}\frac{\partial \mathcal{V}_f}{\partial \Omega}\right] + \frac{\alpha^2\kappa_g^2}{2}\left(\rho_g^{(\mathrm{m})} - P_g^{(\mathrm{m})}\right)$$

$$\ddot{\beta}_\pm - \frac{\dot{\alpha}}{\alpha}\dot{\beta}_\pm + 3\dot{\Omega}\dot{\beta}_\pm = -\frac{m_g^2}{6}\alpha e^{-3\Omega}\left(\alpha\frac{\partial V_g}{\partial \beta_\pm} + \mathcal{A}\frac{\partial \mathcal{V}_f}{\partial \beta_\pm}\right)$$

$$\frac{1}{2}\dot{\mathcal{W}}^2 = \frac{1}{2}\left(\dot{\mathcal{B}}_+^2 + \dot{\mathcal{B}}_-^2\right) + \frac{m_f^2}{6}\mathcal{A}^2 e^{-3\mathcal{W}}\mathcal{V}_f + \frac{\mathcal{A}^2\kappa_f^2}{6}\rho_f^{(\mathrm{m})}$$

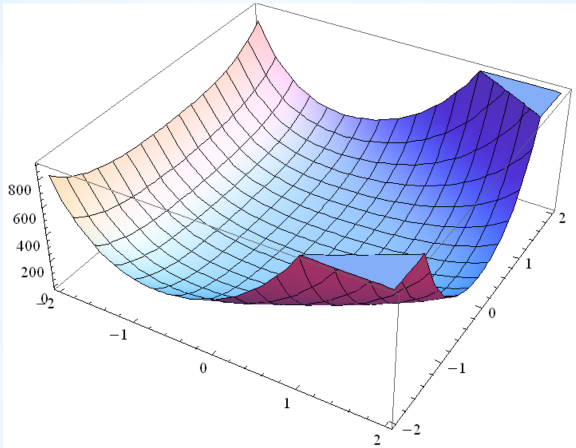
$$\ddot{\mathcal{W}} - \frac{\dot{\mathcal{A}}}{\mathcal{A}}\dot{\mathcal{W}} + 3\dot{\mathcal{W}}^2 = \frac{m_f^2}{6}\mathcal{A}e^{-3\mathcal{W}}\left[\alpha\frac{\partial V_g}{\partial \mathcal{W}} + \mathcal{A}\left(3\mathcal{V}_f + \frac{\partial \mathcal{V}_f}{\partial \mathcal{W}}\right)\right] + \frac{\mathcal{A}^2\kappa_f^2}{2}\left(\rho_f^{(\mathrm{m})} - P_f^{(\mathrm{m})}\right)$$

$$\ddot{\mathcal{B}}_\pm - \frac{\dot{\mathcal{A}}}{\mathcal{A}}\dot{\mathcal{B}}_\pm + 3\dot{\mathcal{W}}\dot{\mathcal{B}}_\pm = -\frac{m_f^2}{6}\mathcal{A}e^{-3\mathcal{W}}\left(\alpha\frac{\partial V_g}{\partial \mathcal{B}_\pm} + \mathcal{A}\frac{\partial \mathcal{V}_f}{\partial \mathcal{B}_\pm}\right)$$

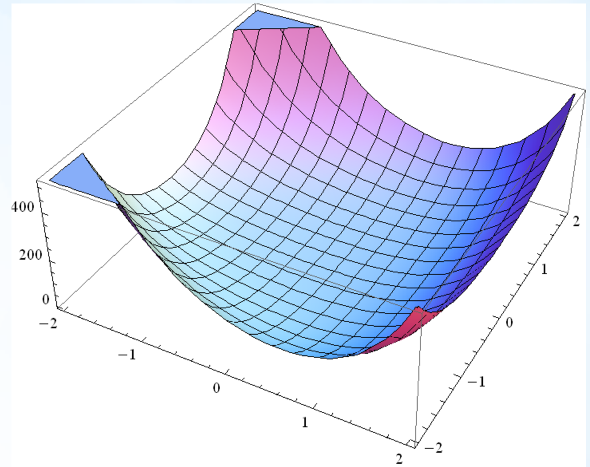
$$m_g^2 = \frac{m^2\kappa_g^2}{\kappa^2}, \quad m_f^2 = \frac{m^2\kappa_f^2}{\kappa^2}$$

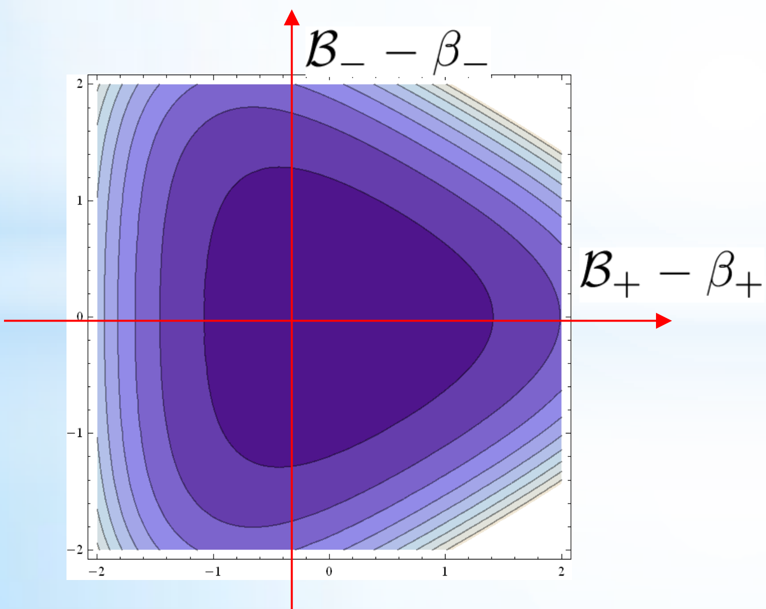
$b_1, b_2, b_3 < 0$

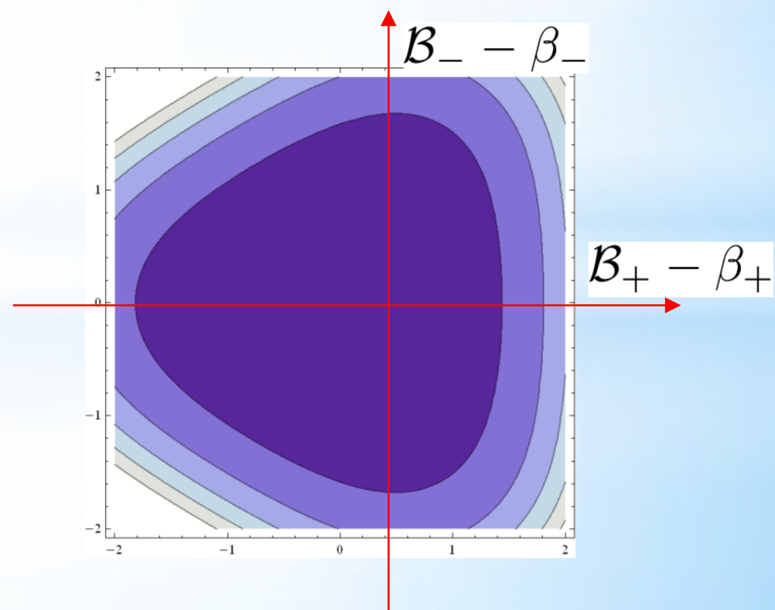
V_g



$\mathcal{V}_f$







homothetic solution=vacuum Bianchi I with a cosmological constant Λ in GR

analytic solution

$\Lambda > 0$

$$e^{\Omega} = \frac{1}{2^{1/3}} e^{\pm H_0(t-t_0)} \left(1 - \frac{\sigma_0^2}{H_0^2} e^{\mp 6H_0(t-t_0)}\right)^{1/3}$$

$$H_0 = \sqrt{\Lambda/3}$$

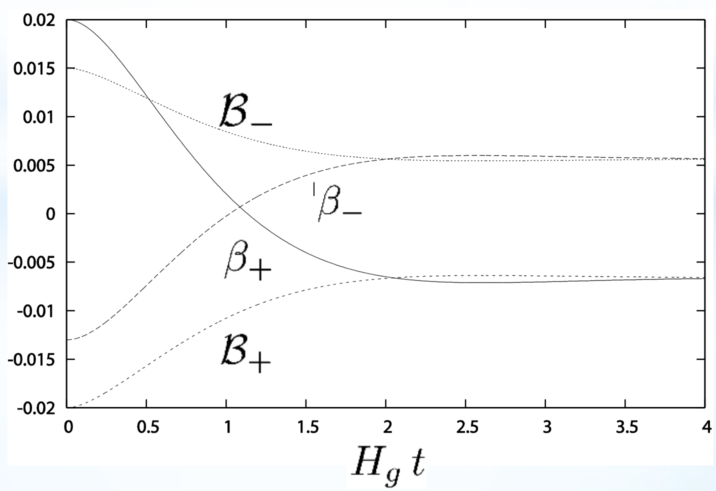
$$e^{\beta_\pm} = e^{\beta_{\pm(0)}} \left(\frac{1 - \frac{\sigma_0}{H_0} e^{\mp 3H_0(t-t_0)}}{1 + \frac{\sigma_0}{H_0} e^{\mp 3H_0(t-t_0)}}\right)^{\pm \frac{\sigma_\pm^{(0)}}{3\sigma_0}}$$

$$\sigma_0^2 = \sigma_{+(0)}^2 + \sigma_{-(0)}^2$$

$$\Sigma^2 = \frac{\sigma^2}{H^2} \propto e^{-6\Omega}$$

Homothetic solution is an attractor in Bianchi I

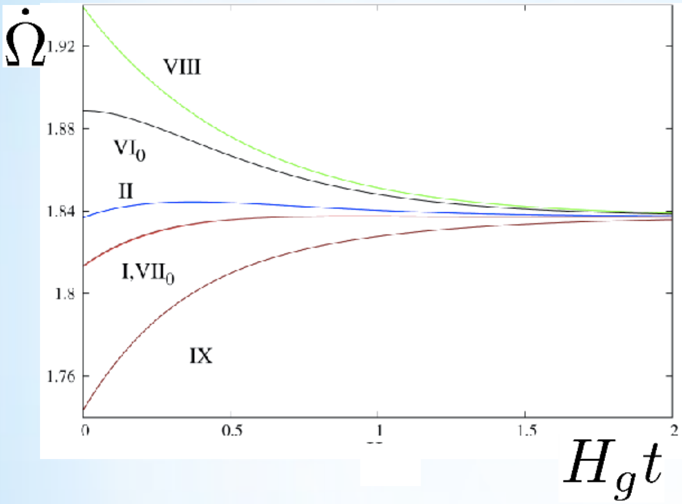
$$\mathcal{B}_\pm - \beta_\pm \to 0$$

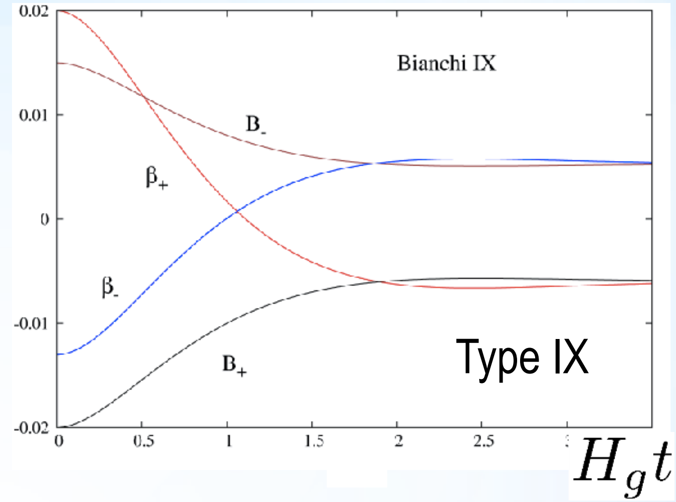


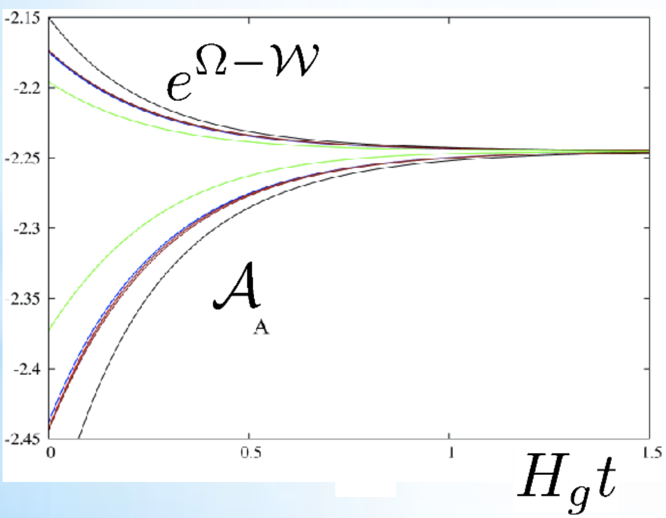
$$H_g = \sqrt{\Lambda_g/3}$$

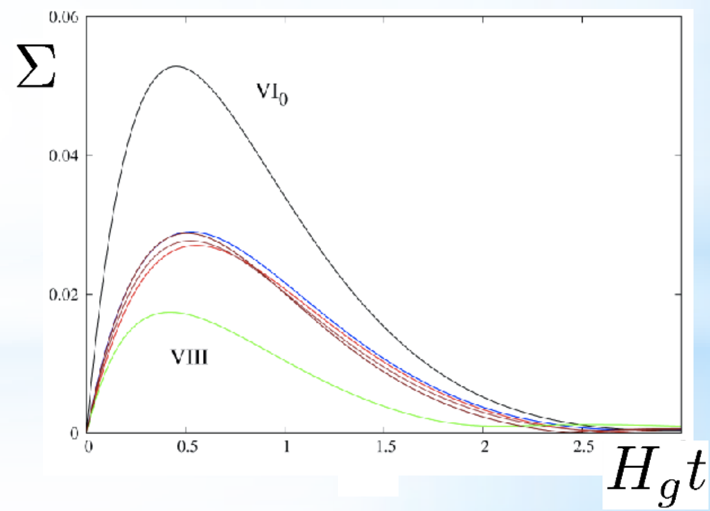
More General Bianchi Types

Numerical Results $c_3 = 1\,,\quad c_4 = 0.3\,,\quad m_g = 0.54m\,,\quad m_f = 0.84m$









Approach to homothetic metrics

homothetic solution $\Rightarrow$ GR with a cosmological constant

Shear drops fast as $$\sigma^2 \sim \dot\beta_+^2 + \dot\beta_-^2 \sim e^{-6\Omega}$$

However, it does not drop so fast:

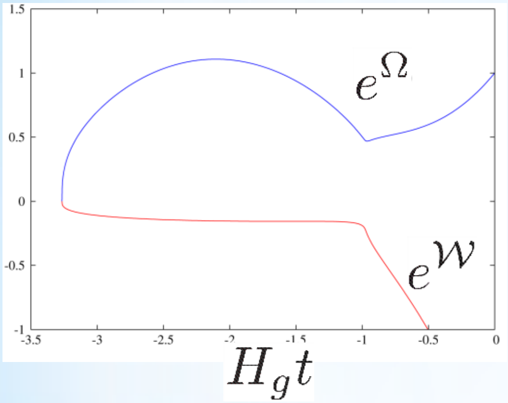
$$\sigma^2 \sim \dot\beta_+^2 + \dot\beta_-^2 \sim e^{-3\Omega}$$

This is the same as matter fluid

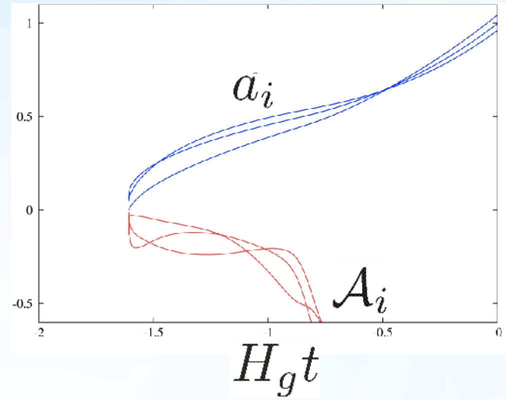
Any Observational Effect ?

Initial Stage (near singularity)

Bianchi I

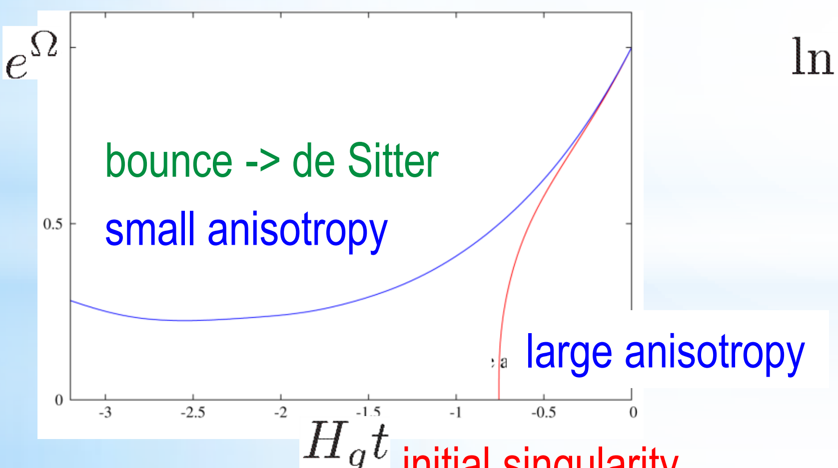


Bianchi IX

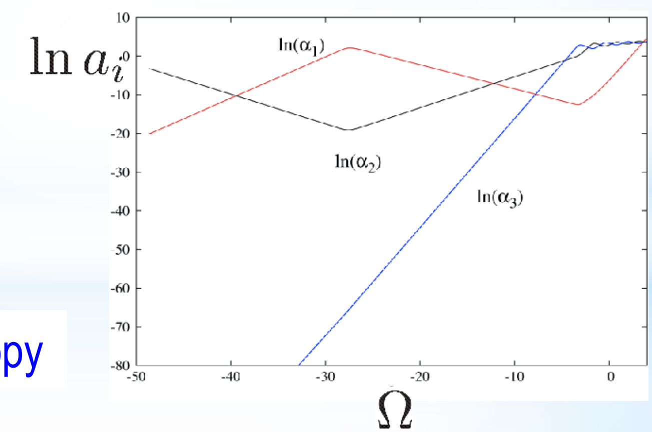


Three spatial components of metrics

vacuum Bianchi IX



initial singularity



chaotic behaviour near singularity

Twin Matter as Dark Matter

Effective Friedmann Equation

$$B = \frac{a_f}{a_g}$$

Near an attractor point (B_{dS}),

$$\kappa_g^2 \left[\rho_g^{[\gamma]}(\tilde{B}) + \rho_g(a_g)\right] - {\kappa_f}^2 \tilde{B}^2 \left[\rho_f^{[\gamma]}(\tilde{B}) + \rho_f(a_f)\right] = 0$$

$$\Rightarrow \quad \tilde{B} - \tilde{B}_{dS} \propto \frac{\alpha}{a_g^3} + \frac{\beta}{a_f^3} + O\left(\frac{1}{a_g^6}\right)$$

$$\Downarrow$$

$$H_g^2 + \frac{k}{a_g^2} = \frac{\Lambda_g}{3} + \frac{\kappa_{\rm eff}^2}{3}\left[\rho_g + \rho_{\rm D}\right]$$

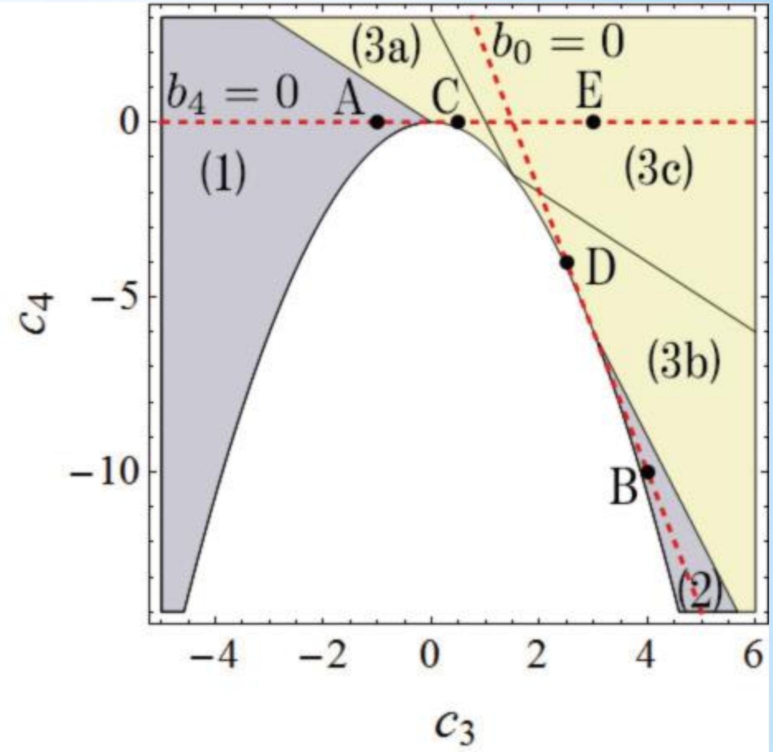
$$\kappa_{\rm eff}^2 = \kappa_g^2 \left[1 - \frac{1}{\left(1 - \frac{2\Lambda_g}{3m_{\rm eff}^2}\right)\left(1 + \frac{\kappa_f^2}{\tilde{B}_{dS}^2 \kappa_g^2}\right)}\right]$$

$$\rho_{\rm D} = \frac{\kappa_f^2 \tilde{B}_\ell^2}{\kappa_g^2 \left[\left(1 - \frac{2\Lambda_g}{3m_{\rm eff}^2}\right)\left(1 + \frac{\kappa_f^2}{\tilde{B}_{dS}^2 \kappa_g^2}\right) - 1\right]} \boxed{\rho_f}$$

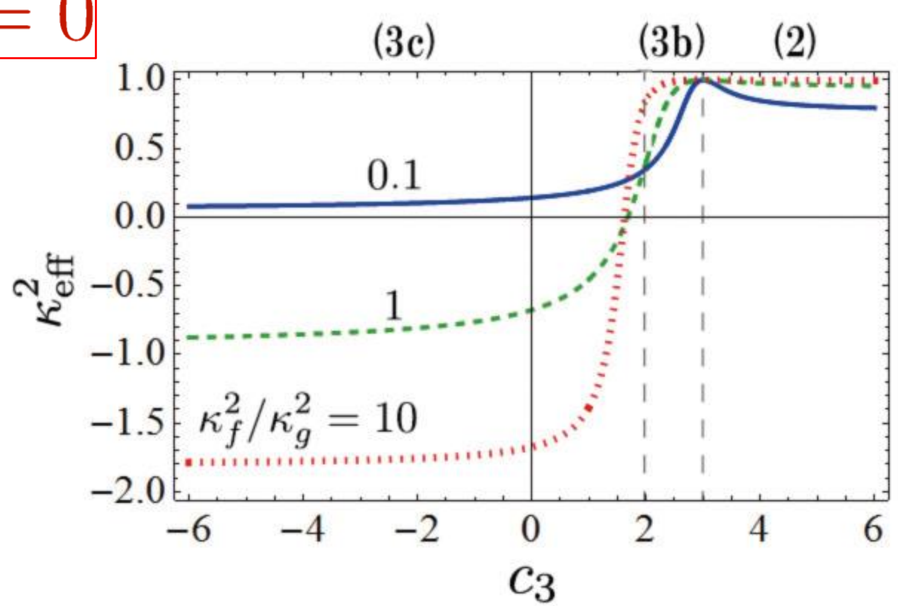
twin matter

Effective Gravitational Constant

Model	$\kappa_{\rm eff}^2/\kappa_g^2$	$m_{\rm eff}/m$	
A	-0.0880972	6.43151	X
B	0.976813	0.938869	○
C	-0.108741	3.30578	X
D	0.920396	0.885782	○
E	0.0283764	3.99107	△



$\boxed{b_0 = 0}$



Dark Matter

twin matter is dark matter ? $\rho_D/\rho_g \sim 5$

Model	$\rho_{\rm D}/\rho_g$
B	$0.16261\, r_{\rm m}$
D	$0.32278\, r_{\rm m}$
E	$44.9613\, r_{\rm m}$

B, D: x

E: ○

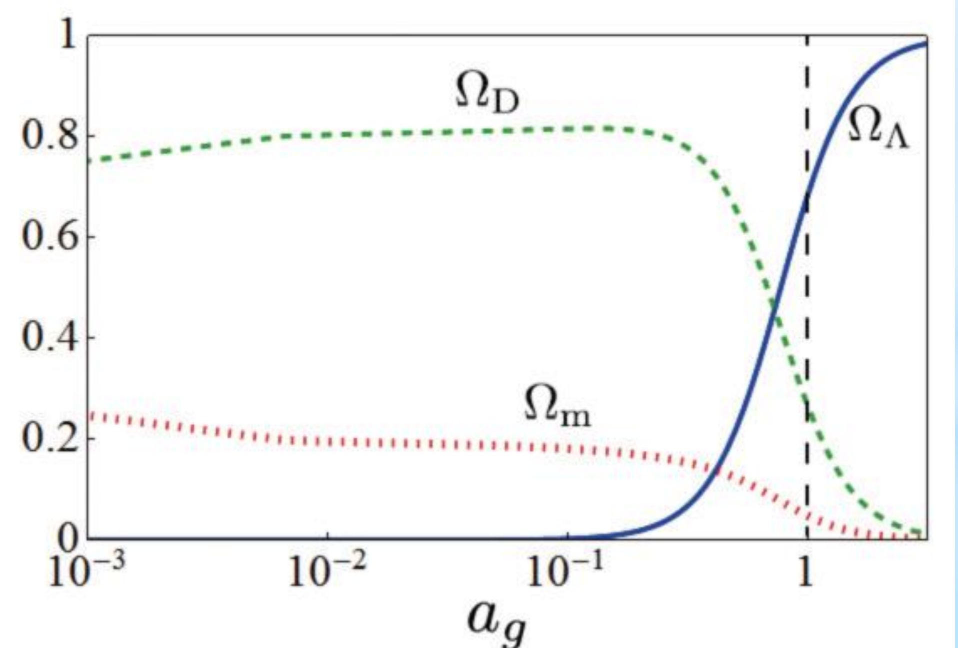
attractor condition

$$r_{\rm m} < r_{\rm m}^{\rm (cr)}$$

Model H

$$c_3 = -4\,, \quad c_4 = -10$$

$$\frac{\kappa_f^2}{\kappa_g^2} = 1000\,, \quad r_{\rm m} = 3000$$



Twin matter can be “dark matter” in Friedmann eq.

Twin matter can really be dark matter ?

Dark matter is required in three situations:

- “Dark matter” in Friedmann equation ○

- Dark matter at a galaxy scale

  rotation curve

- Dark matter in structure formation

Dark matter at a galaxy scale $\triangle$

$$\Delta\Phi_+ = 4\pi G \frac{m_f^2}{m_{\rm eff}^2}\rho_g + 4\pi\mathcal{G}K^2\frac{m_g^2}{m_{\rm eff}^2}\rho_f$$

massless mode

$$(\Delta - m_{\rm eff}^2)\Phi_- = \frac{4}{3}(4\pi G\rho_g - 4\pi\mathcal{G}K^2\rho_f)$$

massive mode

Newton potential

$$\Phi_g = \Phi_+ + \frac{m_g^2}{m_{\rm eff}^2}\Phi_-$$

$$= -\frac{GM_g}{r}\left(\frac{m_f^2}{m_{\rm eff}^2} + \frac{4m_g^2}{3m_{\rm eff}^2}e^{-m_{\rm eff}r}\right) - \frac{m_g^2}{m_{\rm eff}^2}\frac{K^2\mathcal{G}\mathcal{M}_f}{r}\left(1 - \frac{4}{3}e^{-m_{\rm eff}r}\right)$$

positive definite

~~positive definite~~

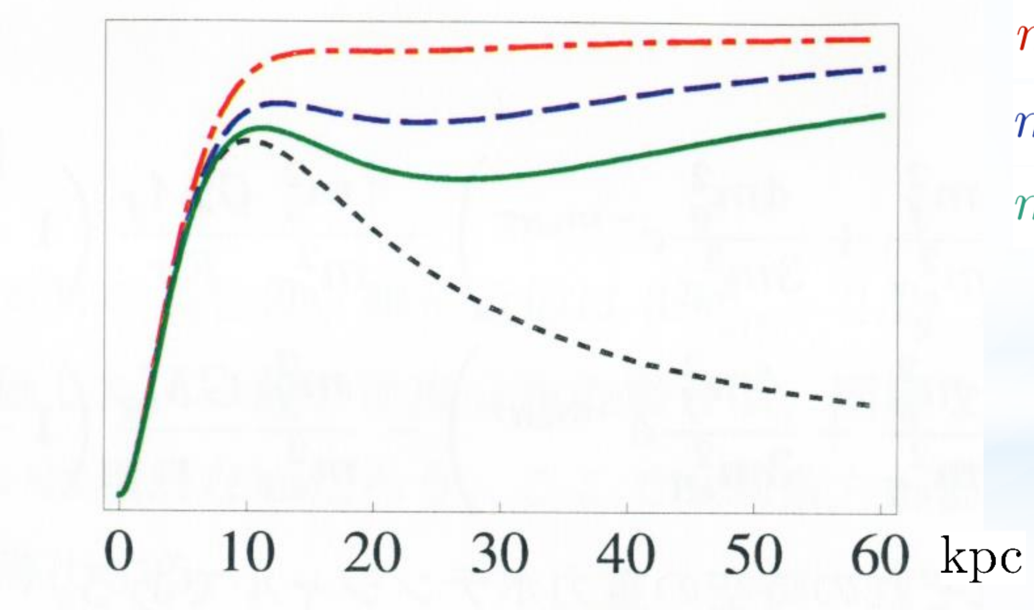
scale > $m_{\rm eff}^{-1}$

$$\Phi_g \approx -\frac{G_{\rm eff}}{r}\left(M_g + K M_f\right)$$

$$G_{\rm eff} = \frac{m_f^2}{m_{\rm eff}^2} G$$

Twin matter can be dark matter

rotation curve



$m_{eff}^{-1} = 5\mathrm{kpc}$

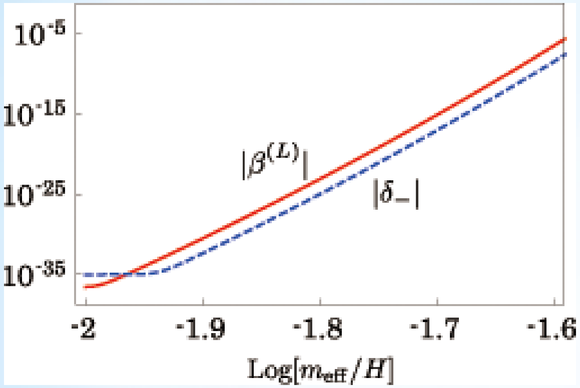
$m_{eff}^{-1} = 10\mathrm{kpc}$

$m_{eff}^{-1} = 15\mathrm{kpc}$

fine tuning

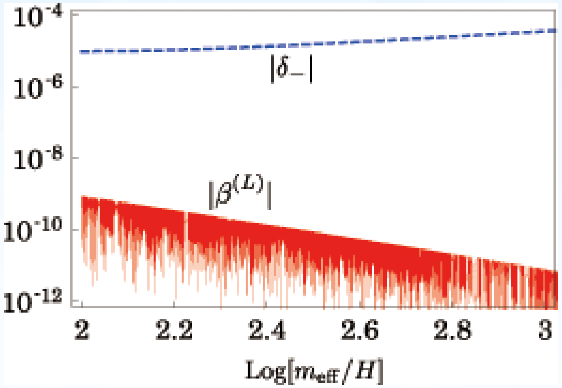
◈ Dark matter for structure formation

Numerical simulation for linear perturbations



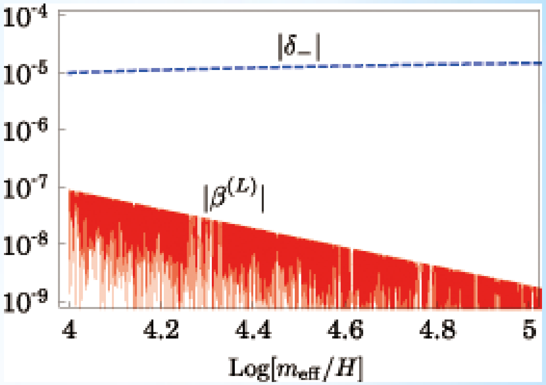
$$a/k \ll H^{-1} \ll m_{\mathrm{eff}}^{-1}$$

(a)



$$a/k \ll m_{\mathrm{eff}}^{-1} \ll H^{-1}$$

(b)



$$m_{\mathrm{eff}}^{-1} \ll a/k \ll H^{-1}$$

(c)

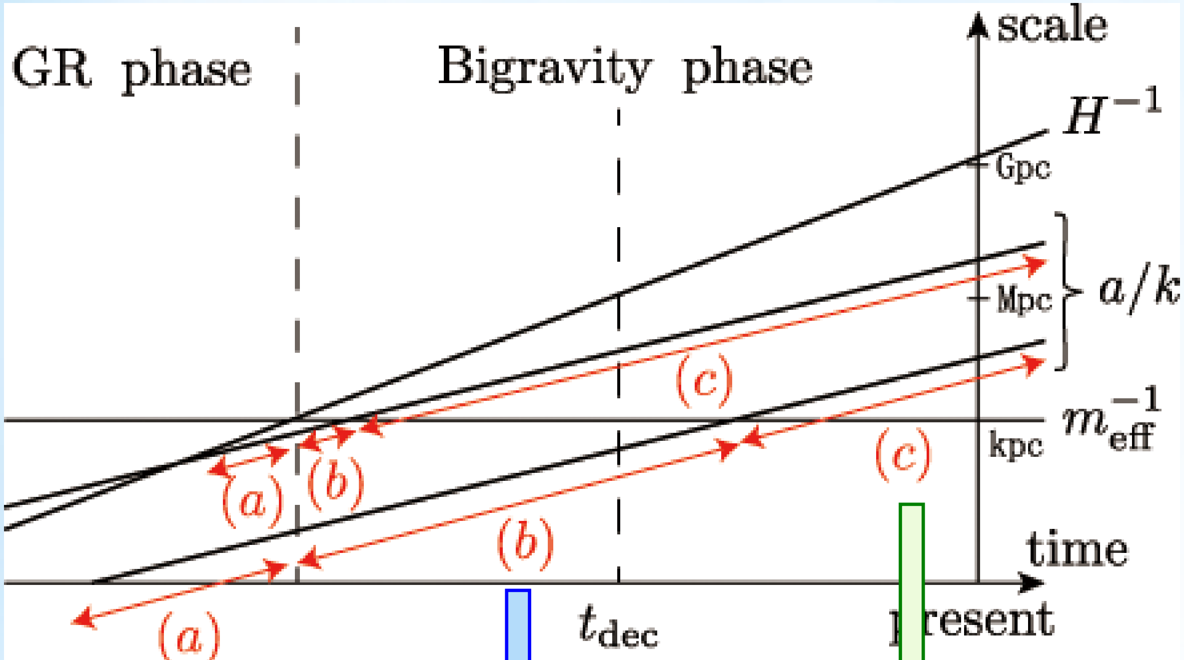
unstable

Vainshtein mechanism

oscillations: damping

adiabatic potential approx

evolution of density perturbations



(a)

$a/k \ll H^{-1} \ll m_{\rm eff}^{-1}$

(b)

$a/k \ll m_{\rm eff}^{-1} \ll H^{-1}$

(c)

$m_{\rm eff}^{-1} \ll a/k \ll H^{-1}$

Vainshtein mechanism

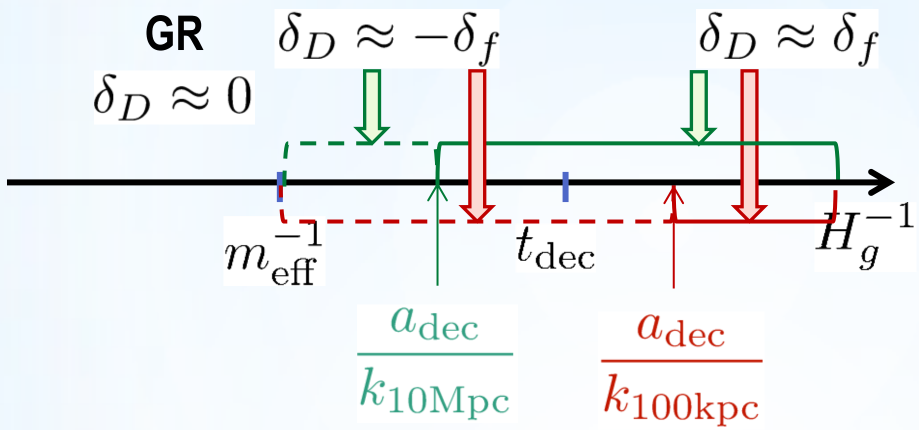
repulsive

attractive

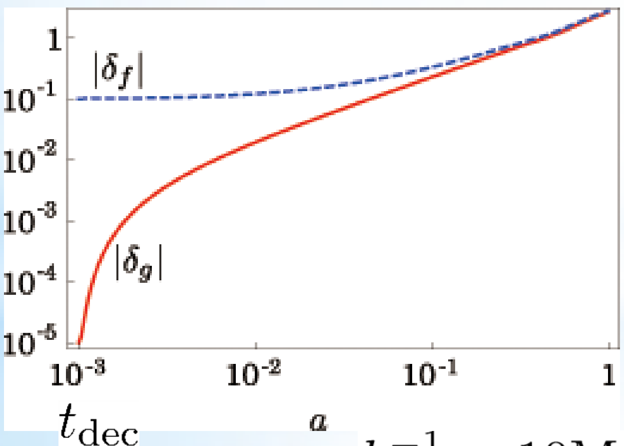
evolution equation for density perturbations

adiabatic potential approx

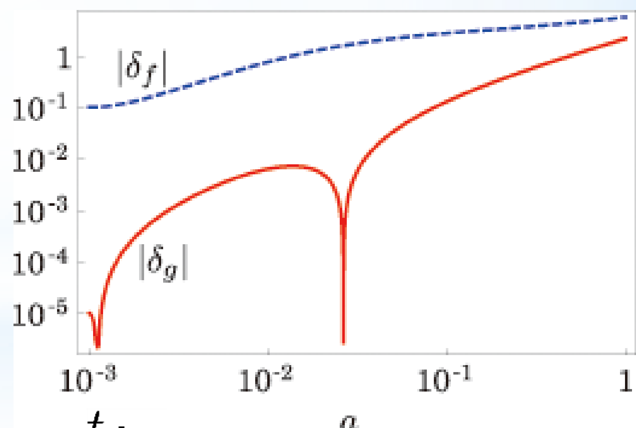
$$\ddot{\delta}_g + 2H\dot{\delta}_g - 4G_{\rm eff}\left(\rho_g\delta_g + \rho_D\delta_D\right) = 0$$



$m_{\rm eff}^{-1} = 1{\rm kpc}$



$k^{-1} = 10{\rm Mpc}$



$k^{-1} = 100{\rm kpc}$

Density fluctuations can evolve into non-linear stage.

Conclusion and Remarks

Conclusion

- We discuss cosmology in ghost-free bigravity theory

  - The homothetic solution (de Sitter spacetime) is an attractor

    ⟹ “cosmic no hair”

  - “Dark matter” could be explained by twin matter

    But fine-tuning is indispensable

Large graviton mass and small cosmological constant ?

κ_g^2/κ_f^2	$2c_3^2+3c_4$	K_{dS}	$\Lambda_g/m_{\mathrm{eff}}^2$	m_g^2/m_f^2
1	1	5.08	0.0815	25.8
10^{-12}	1	8.85	5.11×10^{-11}	7.84×10^{-11}
1	10^{-12}	4.00	9.34×10^{-14}	16.0
10^{-6}	10^{-6}	4.00	8.10×10^{-11}	1.60×10^{-5}

Either small ratio of κ_g^2/κ_f^2 or fine-tuning of $2c_3^2+3c_4$

Remarks

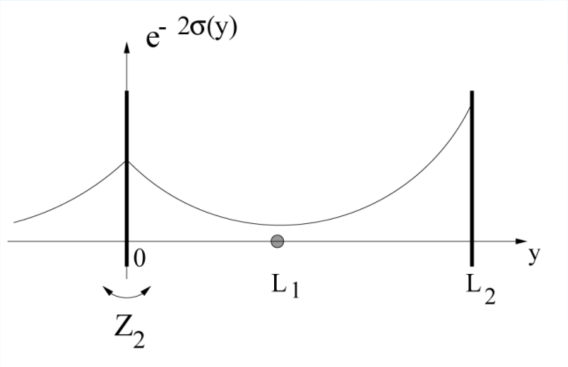
T. Damour and I.I. Kogan, PRD 66, 104024 (2002)

T. Damour, I.I. Koganans A. Papazoglou,

PRD 66,104025 (2002); PRD 67, 064009 (2003)

- From more fundamental theory ?

◆ brane world



complicated coupling

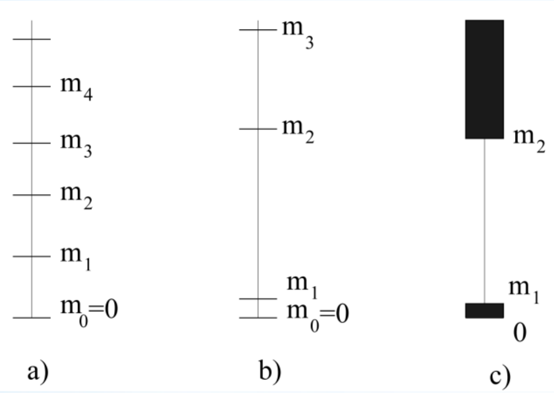
Yamashita, Tanaka (2014)

DGP two-brane model with a bulk scalar field

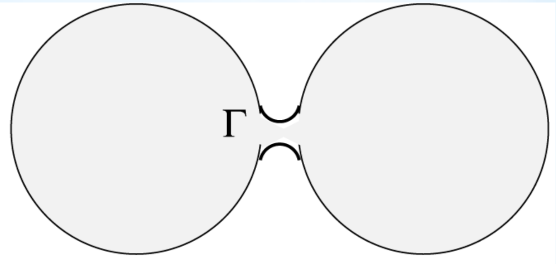
⇨ dRGT bigravity ?

Higher-dimensional theory

- Kaluza-Klein theories



KK modes



- 6D Einstein-Maxwell-Λ

Kan, Maki, Shiraishi ('14)

$M_4 \times S^2$ compactification

Interaction of two gauge fields

$$S_{int} = -\frac{\alpha}{96}\int d^6x\, \epsilon^{MNRSTL}\epsilon_{ABCDEF}$$

$$\times e_g{}^A_M e_f{}^B_N e_g{}^C_R e_f{}^D_S \underline{F_{(g)\,TL} F_{(f)}}_{\,JK} e_g{}^{EJ} e_f{}^{FL}$$

$\Rightarrow$ $\mathcal{U}_2(\gamma)$

Effective Riemann geometry

- Non-comutative type theories

Connes' non-comutative geometry

$\Downarrow$ macroscopic

Riemann geometry

$$X = Y \times Z_2$$

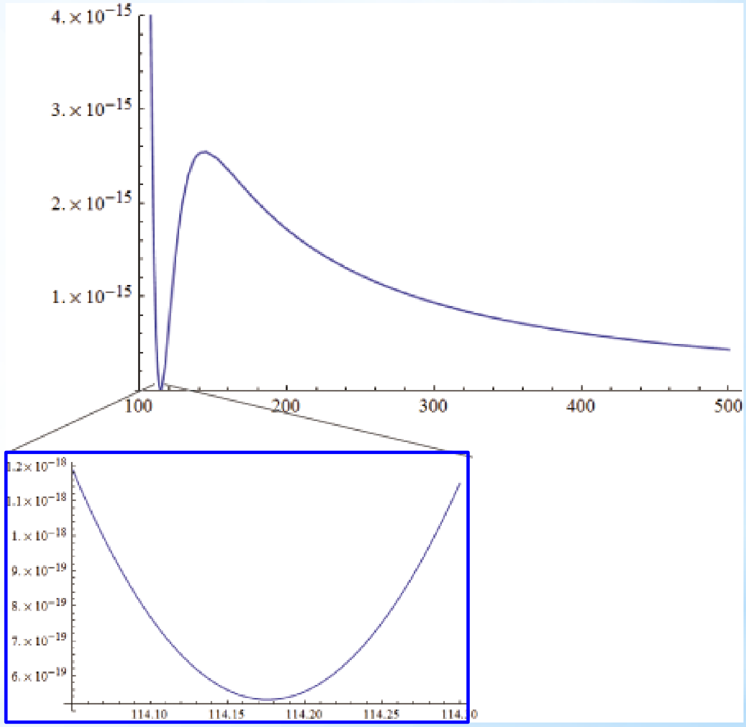
Y : continuous space $\Rightarrow$ bigravity theory

Two types of strings

g-string & f-string ⟹ bigravity theory in 10-dim

- similar interactions
- KKLT compactification

Not need to introduce anti-branes



Thank you for your attention