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Workshop Fundamental Issues of the Standard Cosmological Model, Cargese

Nontopological solitons: explicit solutions and stability issues

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Localized field configuration (soliton) $\rightarrow$ DM candidate

motivation:

- to avoid restrictions on WIMP,
- possible objects in the new field content,
- baryogenesis, cogenesis,
- new mechanism of production?

main issues:

- stability
- energy spectrum
- production in Early Universe

Stability $\boxed{Charge}$ or Topology

Static solutions $\to$ problem with Derrick theorem

(nonlinear kinetic term, gauge fields)

For pure scalar field theory scaling arguments restrict number of space-time dimensions $D < 3$

Stationary solution for $U(1)$-invariant scalar field theory:

$\phi = e^{i\omega t} f(r)$ in ordinary $(3+1)$ space-time

(we also turn off gravity) $\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$

only r dependence $\to$ Q-ball

Charge (not electric!) $\rightarrow$ global $U(1)$ symmetry,

G. Rosen, J. Math. Phys. **9** (1968) 996

or Q-balls, S.R. Coleman, Nucl. Phys. B **262** (1985) 263 [Erratum **269** (1986) 744]

overshoot-undershoot method, where r corresponds to time in Eq. of motion

Eq. is nonlinear, how to check (numerical) result?

$$\boxed{\partial E/\partial Q = \omega}$$

here

$$E = \int d^3x(\partial_0\phi^*\partial_0\phi + \partial_i\phi^*\partial_i\phi + V),$$

$$\text{and } Q = \mathrm{i}\int d^3x(\partial_0\phi^*\phi - \phi^*\partial_0\phi)$$

with simple generalisation for additional mediator scalar field $\longrightarrow$ NONTOPOLOGICAL SOLITONS

R.Friedberg, T.D. Lee, A.Sirlin, PRD **13**(1976) 2739

Time-dependent background: how to investigate stability?

$$V = m^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

D.L.T. Anderson, G.H. Derrick, J. Math. Phys. **11** (1970) 1336

all solutions are unstable,

$$\delta\phi = e^{i\omega t + |\gamma| t}(\psi_1(r) + \psi_2^*(r))$$

$$\begin{pmatrix} \hat{O}_{11} & \hat{O}_{12} \\ \hat{O}_{21} & \hat{O}_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} (\omega - \gamma)^2\psi_1 \\ (\omega + \gamma)^2\psi_2 \end{pmatrix}$$

Are the stable solutions in any other theory?

- Yes

Q-criterion of stability: $\frac{dQ}{d\omega} < 0$

N.G. Vakhitov, A.A. Kolokolov, Radiophys. Quantum Electron. **16** (1973) 783 -NSE

R.Friedberg, T.D. Lee, A.Sirlin, PRD **13**(1976) 2739

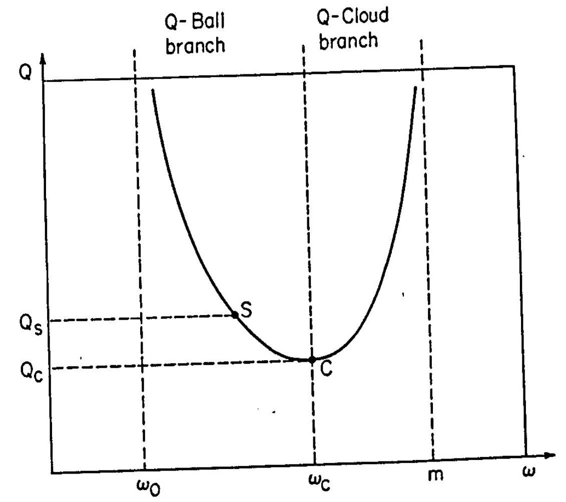


Fig. 1: Expected form of $Q(\omega)$, showing Q-ball branch and Q-cloud branch, and the critical point C where they join.

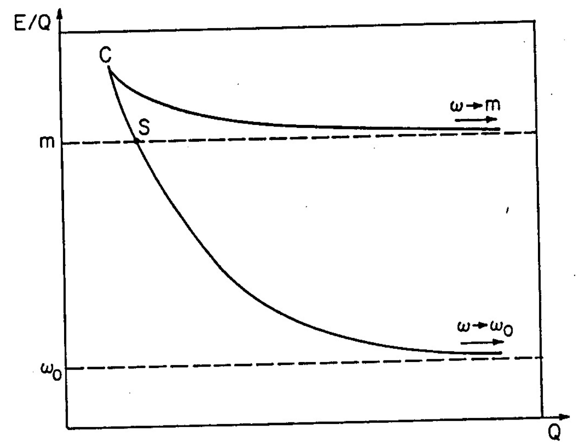
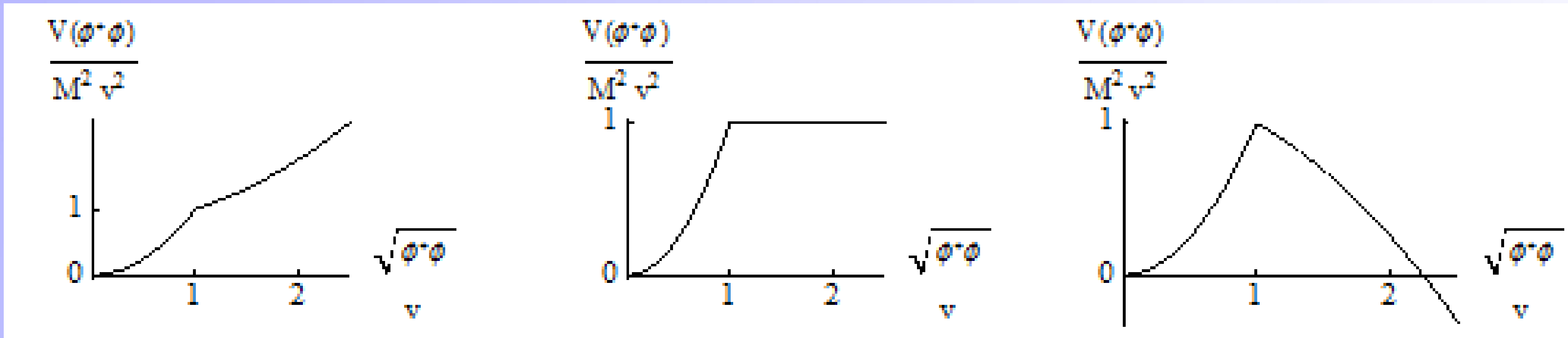


Fig. 2: Expected form of E/Q as a function of Q.

R.Friedberg, T.D. Lee, A.Sirlin, PRD **13**(1976) 2739; M.G. Alford, Nucl. Phys. B **298** (1988) 323

Explicit solution: Parabolic-piecewise potential (it was in original Rosen's paper!)

$$V(\phi^*\phi) = M^2\phi^*\phi\,\theta\left(1-\frac{\phi^*\phi}{v^2}\right) + (m^2\phi^*\phi + \Lambda)\theta\left(\frac{\phi^*\phi}{v^2}-1\right),$$



In this approach $\hat{O}_{12}, \hat{O}_{21}$ are proportional to $\delta(\phi^*\phi - v^2) \to \delta(r-R)$, where $R : f(R) = v$ and problem for linear excitations can be solved, although full operator is not hermitian! Q-criterion was confirmed.

I. Gulamov, E.N., M. Smolyakov, PRD **87** (2013) 085043

For flat potential $E(Q)$ is similar to previous works. Spectrum of classically stable solutions starts from cusp point and crosses condensate line at some Q_S. As a result, these solution has no main particle-like feature:

How to make $\delta Q \ll Q$

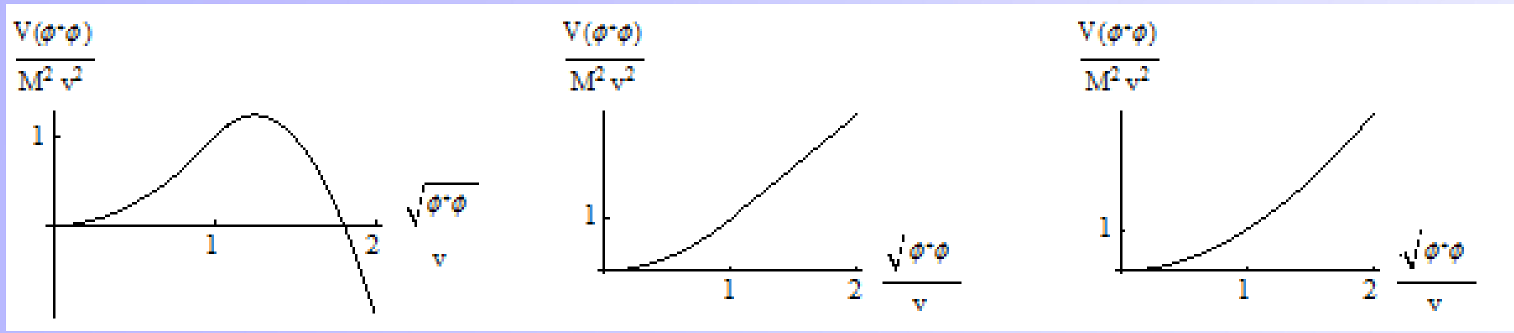
Indeed, for monopole we have precisely one value of the mass! Let us consider gauge case! Coulomb potential enlarge energy for large Q.

K.-M. Lee, J.A. Stein-Schabes, R. Watkins and L.M. Widrow, PRD 39 (1989) 1665

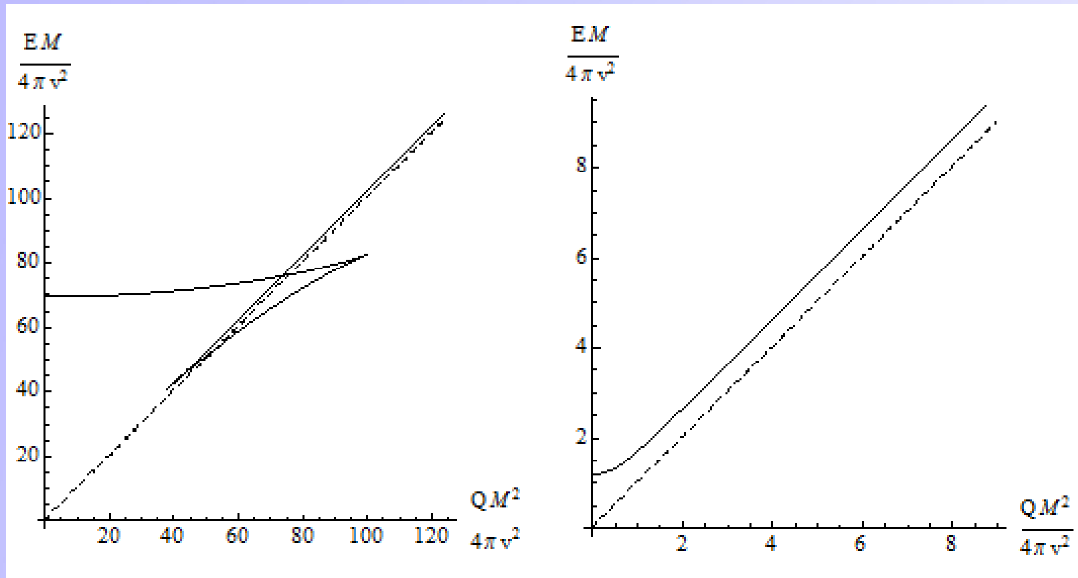
Perturbation theory works well till $Q \ll e^4$. Check $\partial E/\partial Q = \omega < M$ also holds and this is a obstacle for the second cross of condensate line from below.

I. Gulamov, E.N., M. Smolyakov, PRD **89** (2014) 085006

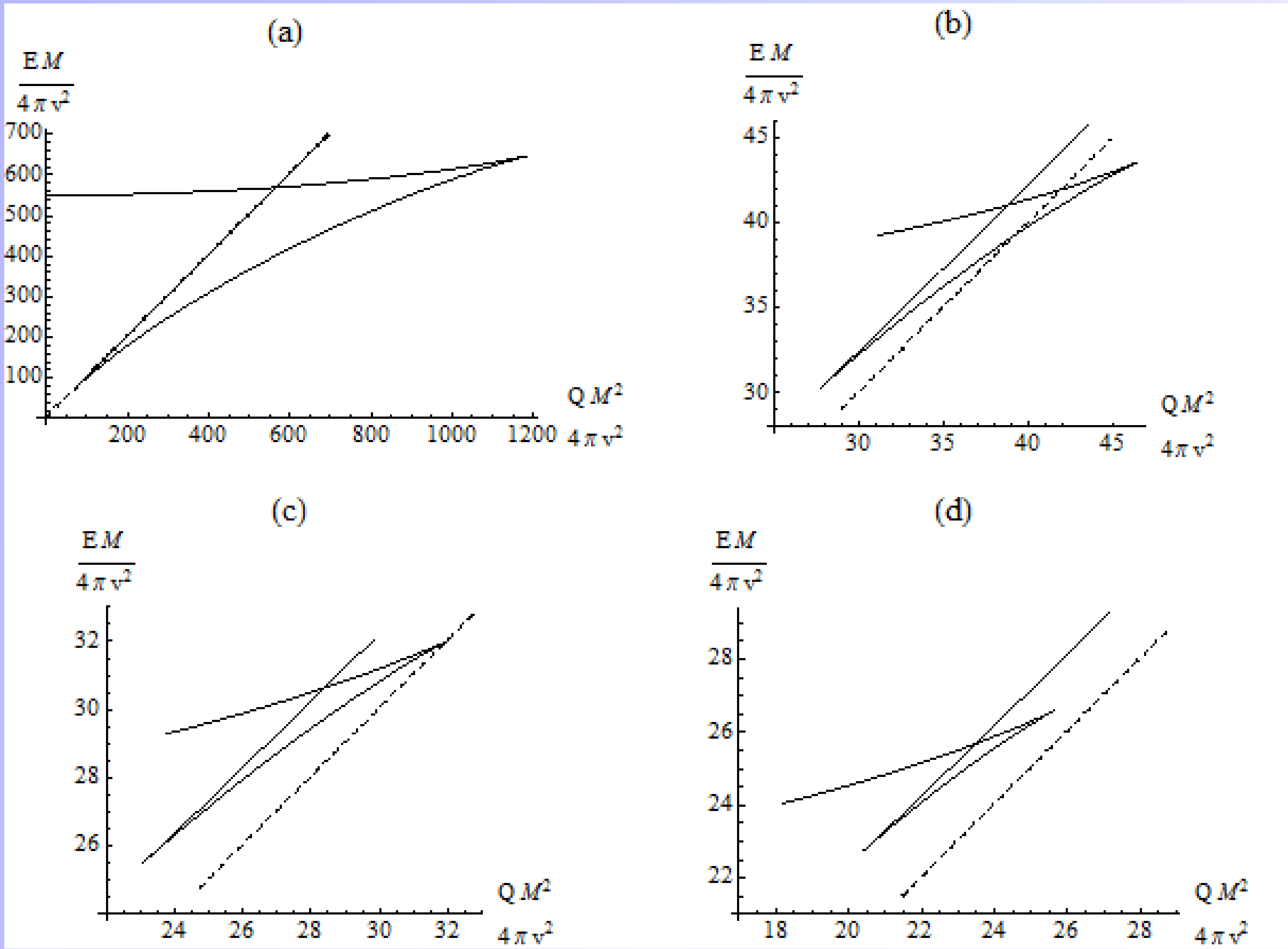
In weak coupling regime $e \ll 1$ difference between Q_S and Q_{Max} still large.



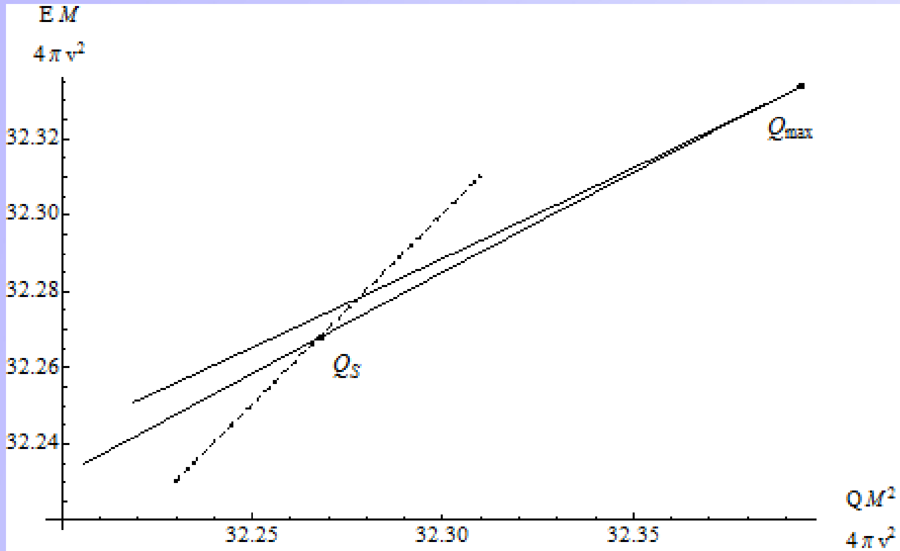
$\frac{V(\phi^*\phi)}{M^2 v^2}$
1
1
2
$\frac{\sqrt{\phi^*\phi}}{v}$
$\frac{V(\phi^*\phi)}{M^2 v^2}$
1
1
2
$\frac{\sqrt{\phi^*\phi}}{v}$
$\frac{V(\phi^*\phi)}{M^2 v^2}$
1
1
2
$\frac{\sqrt{\phi^*\phi}}{v}$



$\frac{EM}{4\pi v^2}$
120
100
80
60
40
20
20
40
60
80
100
120
$\frac{QM^2}{4\pi v^2}$
$\frac{EM}{4\pi v^2}$
8
6
4
2
2
4
6
8
$\frac{QM^2}{4\pi v^2}$



(a)
E M / 4π v²
700
600
500
400
300
200
100
200
400
600
800
1000
1200
Q M² / 4π v²
(b)
E M / 4π v²
45
40
35
30
30
35
40
45
Q M² / 4π v²
(c)
E M / 4π v²
32
30
28
26
24
26
28
30
32
Q M² / 4π v²
(d)
E M / 4π v²
28
26
24
22
18
20
22
24
26
28
Q M² / 4π v²


$\frac{E M}{4\pi v^2}$
32.32
32.30
32.28
32.26
32.24
32.25
32.30
32.35
$\frac{Q M^2}{4\pi v^2}$
Q_{max}
Q_S

Thus, particle-like spectrum for Q-ball can be obtained in potential with turnover (or false vacuum at $\phi = 0$). E.N., M. Smolyakov, JHEP 1407 (2014) 009

At large values one can cure potential by growing terms. This procedure does not disturb Q-ball, which has relatively small field value in the core.

What we can learn from the analogy with monopole?

Are string-like objects are possible?

- Yes, M. Volkov, E Wohnert. Spinning Q balls. PRD66:085003,2002.

$$\phi = e^{i\omega t} e^{in\theta} f(\rho)$$

here ρ is two-dimensional radius.

more interesting analogy in classical mechanics

What about stability:

- in $(2+1)$-dimensional theory
- in our space-time

We have tools to investigate classical stability in $(2+1)$ dimensions – just more matching points and Bessel functions. The same check $\frac{dE}{d\omega} = \omega \frac{dQ}{d\omega}$ holds for values per unit length.

Deviation from Q-criterion was found $n > 0$. E.N., A. Shkerin, PRD 90 (2014) 016002
– Only $n = 0$ solution is stable in $(2+1)$-dimensions.

This is similar to the Abrikosov vortices in superconductor.

$(3+1)$-dimensional case. No additional windings $\rightarrow$ Coleman analysis. The chain of Q-balls is energetically more preferable.

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Conclusions

- Parabolic-pieswise potential allows us to find analitic solution
- Meanwhile, general check $\frac{dE}{d\omega} = \omega \frac{dQ}{d\omega}$ is useful
- Moreover, stability issue can be reformulated as algebraic problem
- There is a possibility to tune E and Q of solutions in parametrically small region
- In such a case the spectra of stable Q-balls look similar to the one of free particles

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THANK YOU!