Non-Abelian SU(3) gauge field as a dark matter candidate

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One can give such definition for a dark matter: "…dark matter is matter of unknown composition that does not emit or reflect enough electromagnetic radiation to be observed directly, but whose presence can be inferred from gravitational effects on visible matter." The nature of the dark matter of the Universe is one of the most challenging problems facing modern physics.

Direct observational evidence for dark matter is found from a variety of sources:

- The observed flatness of the rotation curves of spiral galaxies is a clear indicator for dark matter.
- The orbital velocities of galaxies within galactic.
- The direct evidence of dark matter has been obtained through the study of gravitational lenses.

One of the strongest pieces of evidence for the existence of dark matter is following. Let us consider a rotational velocity $v(r)$ of stars in a galaxy. According to Newton law

$$v^2(r) \propto G_N \frac{M(r)}{r}$$

Schematically a typical rotation curves of spiral galaxies is shown in figure

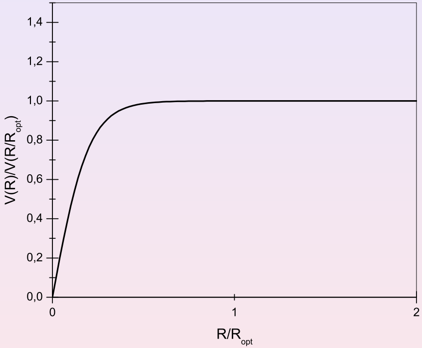


Рис.: Schematical rotation curve of spiral galaxies.

SU(3) Non-Abelian matter in the galaxy is in two phases: quantum one (nucleus, protons, neutrons and so on) and classical one (dark matter). We will:

- ansatz;
- equations;
- numerical solutions;
- energy density;
- asymptotical behaviour;
- v^2;
- cutting off;
- invisibility.

The corresponding Yang – Mills field equations are

$$\partial_\nu F^{A\mu\nu} = 0.$$

where $F^B_{\mu\nu} = \partial_\mu \mathcal{A}^B_\nu - \partial_\nu \mathcal{A}^B_\mu + g f^{BCD} \mathcal{A}^C_\mu \mathcal{A}^D_\nu$ is the field strength; A^B_μ is the non-Abelian gauge potential

Ansätze

$$
\begin{aligned}
A^2_0 &= -2\frac{z}{gr^2}\chi(r), \\
A^5_0 &= 2\frac{y}{gr^2}\chi(r), \\
A^7_0 &= -2\frac{x}{gr^2}\chi(r), \\
A^2_i &= 2\frac{\epsilon_{3ij}x^j}{gr^2}\left[h(r)+1\right], \\
A^5_i &= -2\frac{\epsilon_{2ij}x^j}{gr^2}\left[h(r)+1\right], \\
A^7_i &= 2\frac{\epsilon_{1ij}x^j}{gr^2}\left[h(r)+1\right]
\end{aligned}
$$

here $A^{2,5,7}_\mu \in SU(2) \subset SU(3)$.

The remaining components are belong to the coset $SU(3)/SU(2)$

$$(A_0)_{\alpha,\beta} = 2\left(\frac{x^\alpha x^\beta}{r^2} - \frac{1}{3}\delta^{\alpha\beta}\right)\frac{w(r)}{gr},$$

$$(A_i)_{\alpha\beta} = 2\left(\epsilon_{is\alpha}x^\beta + \epsilon_{is\beta}x^\alpha\right)\frac{x^s}{gr^3}v(r),$$

The corresponding Yang - Mills equations are

$$
\begin{aligned}
x^2 w'' &= 6wv^2, \\
x^2 v'' &= v^3 - v - vw^2, \\
\chi(r) &= h(r) = 0
\end{aligned}
$$

Numerical solution

The typical behavior of functions $v(x)$ and $w(x)$:

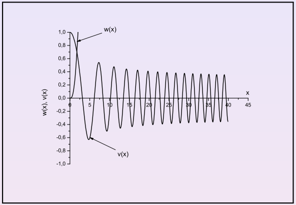


Рис.: The profiles of functions $w(x), v(x)$, $v_2' = -0.2$, $w_3' = 1$.

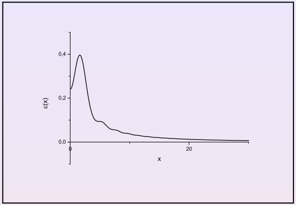


Рис.: The profile of the dimensionless energy density $\varepsilon(x)$.

The energy density $\epsilon(x)$ is

$$\epsilon(r) = -F^a_{0i}F^{a0i} + \frac{1}{4}F^a_{ij}F^{aij} = \frac{1}{g^2 r_0^4}\varepsilon(x).$$

The key point

The asymptotic behavior of the energy density is

$$\varepsilon_\infty(x) \approx \alpha \frac{r_0^4}{r^4} \left(\frac{r}{r_0} \right)^{2\alpha} .$$

The idea presented in this talk is that in a galaxy there exists an ordinary visible matter (quantum SU(3) non-Abelian gauge field inside of protons, neutrons, nucleus and so on) and an invisible matter (classic gauge field which does not interact with electromagnetic waves). The visible matter is immersed into a cloud of the classical gauge field.

A Universal Rotation Curve of spiral galaxies describes any rotation curve at any radius with a very small cosmic variance

$$\left[\frac{V_{URC}\left(\frac{r}{R_{opt}}\right)}{V(R_{opt})}\right]^2 =$$

$$\left(0.72 + 0.44 \log \frac{L}{L_*}\right) \frac{1.97 X^{1.22}}{\left(X^2 + 0.78^2\right)^{1.43}} +$$

$$1.6\, e^{-0.4(L/L_*)} \frac{X^2}{X^2 + 1.5^2 \left(\frac{L}{L_*}\right)^{0.4}} =$$

$$V_{LM}^2 + V_{DM}^2, \quad \mathrm{Km}^2/\mathrm{s}^2$$

Our goal is to compare the rotation curve for the color fields with the Universal Rotational Curve

$$
\begin{aligned}
V^2(r) &= G_N \frac{\mathcal{M}(r)}{r} \approx \\
& C(r_0, V_0, \alpha, L/L_*) \left(\frac{r}{r_0}\right)^{2\alpha-2} - V_0^2. \\
\mathcal{M} &= 4\pi \int\limits_0^\infty \epsilon(r) r^2 dr
\end{aligned}
$$

where $\mathcal{M}(r)$ is the mass of the color fields A^B_μ inside the sphere of radius r.

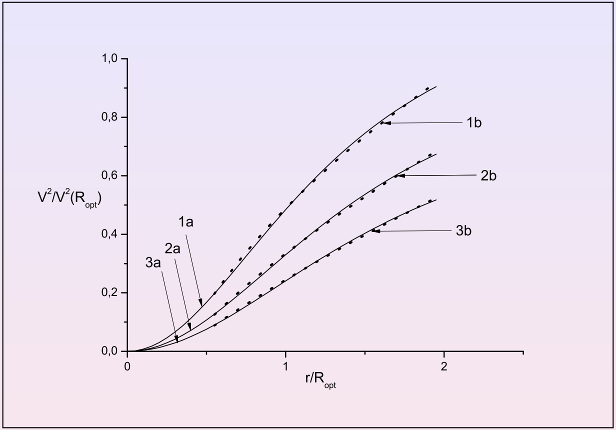


Рис.: Fitting parameters are $r_0, V_0, \alpha, LŁ_0$.

The gauge field distribution considered here has an infinite energy in the consequence of the asymptotic behavior of the energy density.

$$\varepsilon_\infty(x) \approx \alpha \frac{r_0^4}{r^4} \left(\frac{r}{r_0} \right)^{2\alpha} .$$

Consequently it is necessary to have a mechanism to cut-off the distribution of the classical gauge fields in the space.

The physical reason why such transition takes place is following. The gauge potentials are oscillating functions with increasing frequency.

Far away from the origin the frequency is so big that it is necessary quantum fluctuations take into account. In this way the transition from the classical state to quantum one takes place.

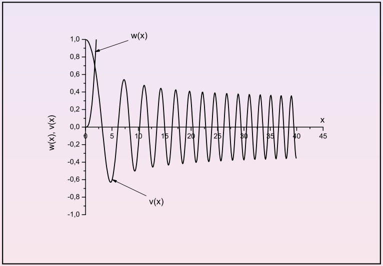


Рис.: The profiles of functions $w(x), v(x)$, $v_2' = -0.2$, $w_3' = 1$.

Cut-off the region filled with classical gauge field

To estimate a transition radius we follow to the Heisenberg uncertainty principle

$$\frac{1}{c}\,\Delta F^a_{ti}\,\Delta A^{ai}\,\Delta V \approx \hbar$$

To an accuracy of a numerical factor the fluctuations of the SU(3) color electric field are

$$\Delta \tilde{F}^2_{t\theta} \approx \frac{1}{g}\frac{1}{r^2}\left(\Delta v\; w + v\;\Delta w\right).$$

Cut-off the region filled with classical gauge field

Now we assume that the quantum fluctuations $\Delta \tilde{A}^2_\theta$ of the component $\tilde{A}^2_\theta$ have the same order as the quantum fluctuations of the components $\tilde{A}^1_\theta$

$$\Delta \tilde{A}^2_\theta \approx \Delta \tilde{A}^1_\theta \approx \frac{1}{g} \frac{\Delta v}{r}.$$

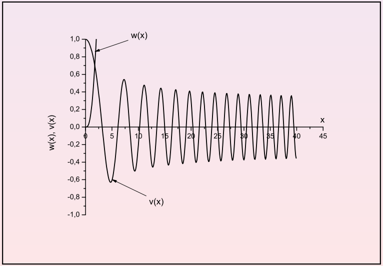
The volume ΔV where supposed quantum fluctuations take place is

$$\Delta V = 4\pi r^2 \Delta r.$$

Cut-off the region filled with classical gauge field

We suppose that the place where the SU(3) classical color field becomes quantum one is defined as the place where the quantum fluctuations in the volume $\Delta V = 4\pi r^2 \Delta r$ with

$$\frac{\Delta r}{r_0} \approx \lambda \approx \frac{1}{\alpha} \frac{2\pi}{x^{\alpha-1}}$$



of the corresponding field becomes comparable with magnitude of these fields

$$\Delta v \approx v, \quad \Delta w \approx w$$

After all calculations we obtain

$$\left(\frac{g'}{A}\right)^2 \approx 2\pi$$

where $\frac{1}{g'^2} = \frac{4\pi}{g^2\hbar c}$ is the dimensionless coupling constant. If we choose $1/g' \approx 1$ and from Fig. 6 we take $A \approx 0.4$ we see that

$$\left(\frac{g'}{A}\right)^2 \approx 6.25$$

that is comparable with $2\pi \approx 6.28$.

The main question in any dark matter model is its invisibility. For the model presented here the answer is very simple: the SU(3) color matter is invisible because color gauge fields interact with color charged particles only. But at the moment in the nature we do not know any particles with SU(3) color charge. In principle such particles can be SU(3) monopoles but up to now the monopoles are not experimentally registered.

Conlusions

In this work we have suggested the idea that the dark matter model is SU(3) gauge field.
(1) We have shown that in SU(3) Yang –Mills theory there exists a spherical symmetric distribution of the gauge potential with slow decreasing matter density.
(2) The asymptotic behavior of the density allow us to describe the rotational curve for the stars in elliptic galaxies.
(3) The fitting of the typical rotational curve gives us parameters which have a good agreement with the parameters of above mentioned spherical solution of Yang – Mills equations.