Primordial black hole formation from cosmological fluctuations

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This talk is based on
Harada, Yoo, Nakama and Koga, arXiv:1503.03934
Harada, Yoo and Kohri, arxiv:1309.4201.

Primordial black holes and cosmology

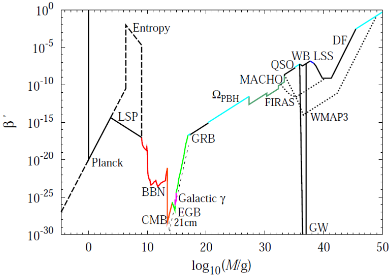
- Black holes may have formed in the early universe. (Zeldovich & Novikov 1967, Hawking 1971)
- PBHs the source of emission due to Hawking radiation and the source of gravitational field and gravitational waves
- Hawking radiation: nearly black-body radiation

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}, \quad \frac{dE}{dt} = -\frac{dM}{dt} = g_{\rm eff} 4\pi R_g^2 \sigma T_H^4, \quad R_g = \frac{2GM}{c^2}$$

- Mass accretion: important only immediately after the formation

Observational constraints on the PBH abundance

- PBHs of mass M is formed at the epoch when the mass contained within the Hubble length is M.
- Observational data can constrain the abundance of PBHs. Complementary to CMB observation. (Carr (1975), Carr et al. (2010))



Production rate of PBHs

- PBHs of mass M are formed from $\delta(M)$, the density perturbation of mass M.
- A Gaussian-like probability distribution for $\delta(M)$ (Carr (1975), cf. Kopp, Hofmann and Weller (2011))
- PBH production rate

$$\beta_0(M) \simeq \sqrt{\frac{2}{\pi}}\frac{\sigma(M)}{\delta_c(M)}\exp\left(-\frac{\delta_c^2}{2\sigma^2(M)}\right),$$

where $\delta_c = O(1)$ is the PBH threshold of δ and σ is the standard deviation of δ.

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Dynamics in PBH formation

1. Fluctuations in super-horizon scales are generated by inflation.
2. The fluctuations enter the Hubble horizon in the decelerated phase of the univese.
3. The Jeans instability sets in and the fluctuation collapses if its amplitude is nonlinearly large enough.
4. A black hole apparent horizon is formed.

3-zone model

Simplified model for analytic approach

- Background: a flat FRW

$$ds^2 = -c^2 dt^2 + a_b^2(t)(dr^2 + r^2 d\Omega^2)$$

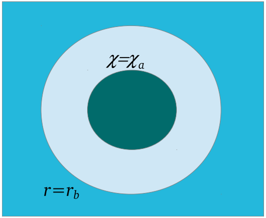
for $r > r_b$

- Overdense region: a closed FRW

$$ds^2 = -c^2 dt^2 + a^2(t)(d\chi^2 + \sin^2\chi d\Omega^2)$$

for $0 \le \chi < \chi_a$.

- Compensating region in between



Jeans instability argument

$t_{\rm ff}$ (free-fall time) versus $t_{\rm sc}$ (sound-crossing time)

Jeans criterion

- If and only if the overdensity reaches maximum expansion before a sound wave crosses over its radius from the big bang, it collapses to a black hole.
- Equivalently, if and only if the overdense region ends in singularity before a sound wave crosses its diameter from the big bang, it collapses to a black hole.

Analytic formula

- We assume an EOS $p = (\Gamma - 1)\rho$ for simplicity. We define $\tilde{\delta}$ as the density perturbation at the horizon entry in the comoving slicing. $\Gamma = 4/3$ for radiation.
- Carr's formula (1975): partially Newtonian estimate

$$\tilde{\delta}_c = \frac{3\Gamma}{3\Gamma + 2}(\Gamma - 1),$$

  or $\tilde{\delta}_{\mathrm{CMC},c} = \Gamma - 1$ in the constant-mean-curvature slicing.
- Harada, Yoo and Kohri (2013): fully GR

$$\tilde{\delta}_c = \frac{3\Gamma}{3\Gamma + 2}\sin^2\left(\frac{\pi\sqrt{\Gamma - 1}}{3\Gamma - 2}\right), \quad \tilde{\delta}_{max} = \frac{3\Gamma}{3\Gamma + 2}.$$

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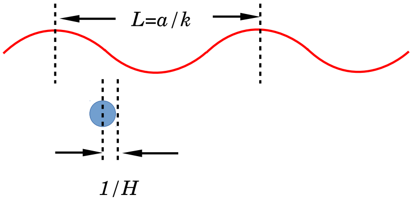
Cosmological long-wavelength solutions



- Generic initial conditions for numerical simulations
- The specetime is assumed smooth in the scales larger than $L = a/k$, which is much longer than the local Hubble length H^{-1}. By gradient expansion, the exact solution is expanded in powers of $\epsilon \sim k/(aH)$.

Construction of the CWLW solutions

- 3+1 and cosmological conformal decomposition

$$ds^2 = -\alpha^2 dt^2 + \psi^4 a^2(t)\tilde{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt),$$

where $\tilde{\gamma} = \eta$, $\tilde{\gamma} = \det(\tilde{\gamma}_{ij})$, $\eta = \det(\eta_{ij})$, and η_{ij} is the metric of the 3D flat space.
- We assume the spacetime approach the flat FRW in the limit $\epsilon \to 0$ (Lyth, Malik & Sasaki (2005)).
- Einstein eqs in $O(1)$ imply the Friedmann eq.
- We can deduce $\psi = \Psi(x^i) + O(\epsilon^2)$ for a perfect fluid with barotropic EOS. $\Psi(x^i)$ generates the solution.

Two approaches to spherically symmetric system

- Shibata and Sasaki (1999)
  - CMC slicing + conformally flat spatial coordinates (ϖ)
  - Initial conditions: The CLWL soln is generated by $\Psi(\varpi)$, where $\psi = \Psi(\varpi) + O(\epsilon^2)$.
- Polnarev and Musco (2007) (and many others)
  - Comoving slicing + comoving threading (r)
  - Initial conditions: The metric is assumed to approach

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - K(r)r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

in the limit $\epsilon \to 0$. The exact solution is expanded in powers of ϵ and generated by $K(r)$.

Equivalence of the two approaches

- The CLWL solutions and the Polnarev-Musco solutions are equivalent with each other through the relation

$$\begin{cases} r = \Psi^2(\varpi)\varpi, \\ K(r)r^2 = 1 - \left(1 + 2\dfrac{\varpi}{\Psi(\varpi)}\dfrac{d\Psi(\varpi)}{d\varpi}\right)^2. \end{cases}$$

- One of the correspondence relations is given by

$$\delta_{\rm C} = \frac{3\Gamma}{3\Gamma + 2}\delta_{\rm CMC} + O(\epsilon^4).$$

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Amplitude of the perturbation

- $\tilde{\delta}$: "the density perturbation at the horizon entry"

$$\tilde{\delta} := \lim_{\epsilon\to 0} \bar{\delta}_{\rm C}(t, r_0)\epsilon^{-2},$$

where $\bar{\delta}_{\rm C}(t, r_0)$ is the density in the comoving slicing averaged over r_0, the radius of the overdense region. $\tilde{\delta}$ is directly calculated from $\Psi(\varpi)$ or $K(r)$.

- ψ_0: the initial peak value of the curvature variable

$$\psi_0 := \Psi(0)$$

Note $\psi = \Psi(x^i) + O(\epsilon^2)$.

- The PBH threshold is determined by numerical simulations.

Carr's formula and numerical result

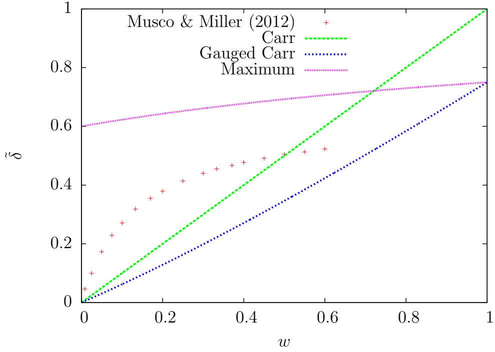


Figure: The gauged Carr's formula underestimates $\tilde{\delta}_c$ by a factor of 2 for $w(=\Gamma-1)=1/3$ and by a factor of 10 for the smaller values of $\Gamma-1$. The numerical result is taken from Musco and Miller (2013) for the Gaussian curvature profile.

HYK formula and numerical result

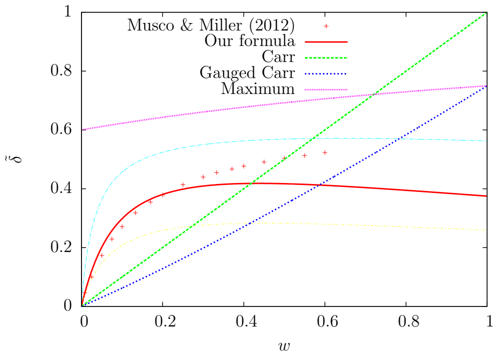
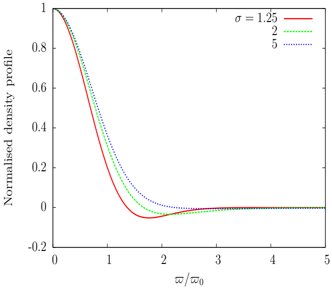


Figure: Harada-Yoo-Kohri formula agrees with the numerical result within $10 - 20\%$ accuracy for $0.01 \leq \Gamma - 1 \leq 0.6$.

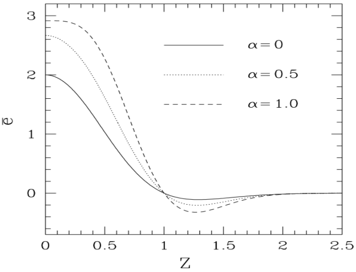
Initial density profiles

Similar Gaussian-type profiles with different parametrisations

a) SS99



b) PM07



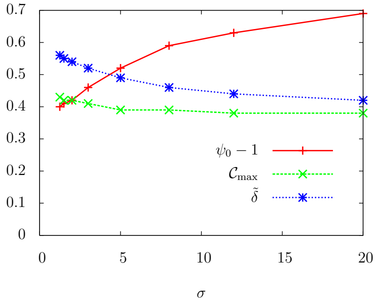
Gentler transition for larger $\sigma (> 1)$.

Gentler transition for smaller $\alpha (\geq 0)$.

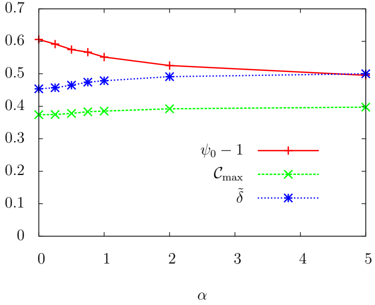
$\tilde{\delta}_c$ and $\psi_{0,c}$

- Our numerical results are consistent with SS99 and PM07.
- $\tilde{\delta}_c$ is smaller but $\psi_{0,c}$ is larger for the gentler transition.

a) SS99 initial data



b) PM07 initial data



Interpretation of $\tilde{\delta}_c$

- $\tilde{\delta}_c$ is larger if the transition is sharper. This is because the pressure gradient force impedes gravitational collapse.
- $\tilde{\delta}_{c,\mathrm{min}}$ is close to Harada-Yoo-Kohri formula. $\tilde{\delta}_{c,\mathrm{max}}$ is close to the possible maximum value in the 3-zone model.

$$\tilde{\delta}_{c,\mathrm{min}} < \tilde{\delta}_c < \tilde{\delta}_{c,\mathrm{max}},$$

where

$$\tilde{\delta}_{c,\mathrm{min}} \simeq \frac{3\Gamma}{3\Gamma+2}\sin^2\left(\frac{\pi\sqrt{\Gamma-1}}{3\Gamma-2}\right), \quad \tilde{\delta}_{c,\mathrm{max}} \simeq \frac{3\Gamma}{3\Gamma+2}.$$

Interpretation of $\psi_{0,c}$

- $\psi_{0,c}$ is smaller if the transition is sharper in contrast to $\tilde{\delta}_c$. This is because ψ is analogous to a Newtonian potential, which is affected by the perturbation in the far region.
- The PBH threshold should be determined by quasi-local dynamics within the local Hubble length. Since $\tilde{\delta}$ is a quasi-local quantity, $\tilde{\delta}_c$ is insensitive to the environment, while $\psi_{0,c}$ is sensitive to the environment.

Conclusion

- PBHs carry the information of the Early Universe.
- The Jeans criterion gives analytic threshold formulas.
- The CLWL solutions naturally give primordial fluctuations.
- $\tilde{\delta}_c$ is larger if the transition is sharper.

$$\tilde{\delta}_{c,\mathrm{min}} \simeq \frac{3\Gamma}{3\Gamma+2}\sin^2\left(\frac{\pi\sqrt{\Gamma-1}}{3\Gamma-2}\right), \quad \tilde{\delta}_{c,\mathrm{max}} \simeq \frac{3\Gamma}{3\Gamma+2}.$$

$\psi_{0,c}$ is subjected to environmental effect.