Quantum Correlation of Unruh detector

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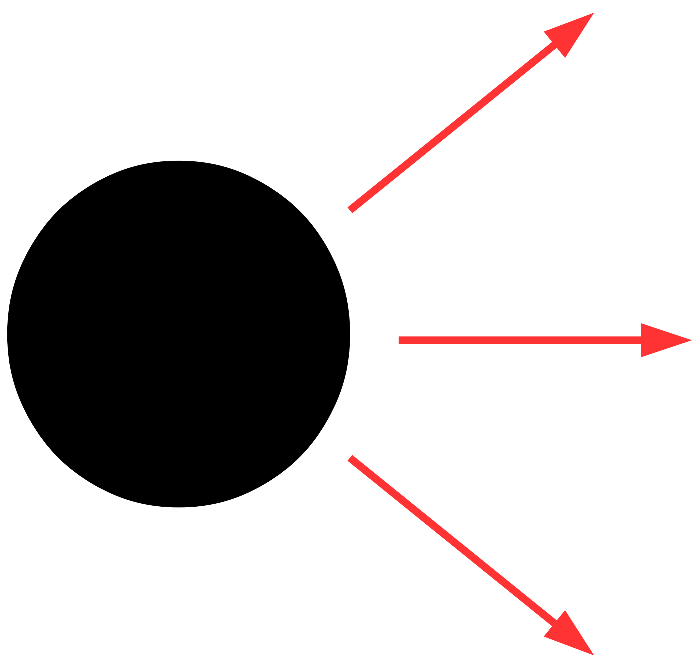
INTRODUCTION

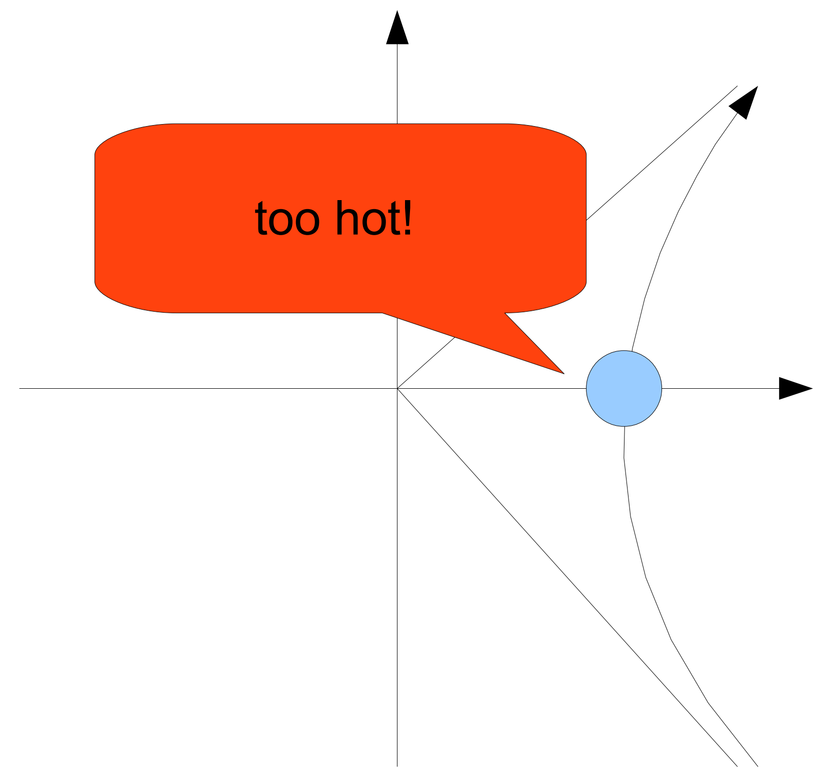
Introduction

Quantum fields on a curved space time have non trivial properties

Hawking radiation



Unruh effect



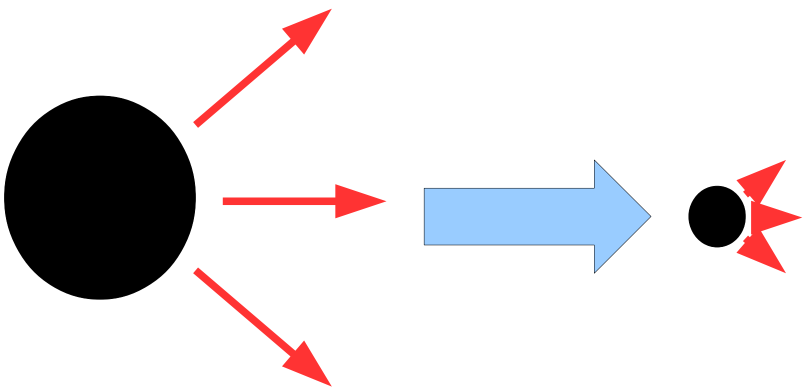
Thermal radiation

Thermal bath like behaviour

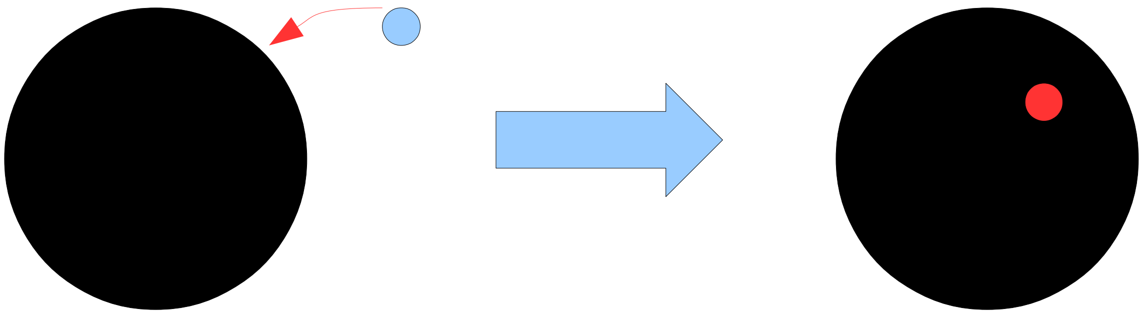
Introduction

the entanglement of quantum fields has impotant role.

Infomration loss paradox



Fire wall

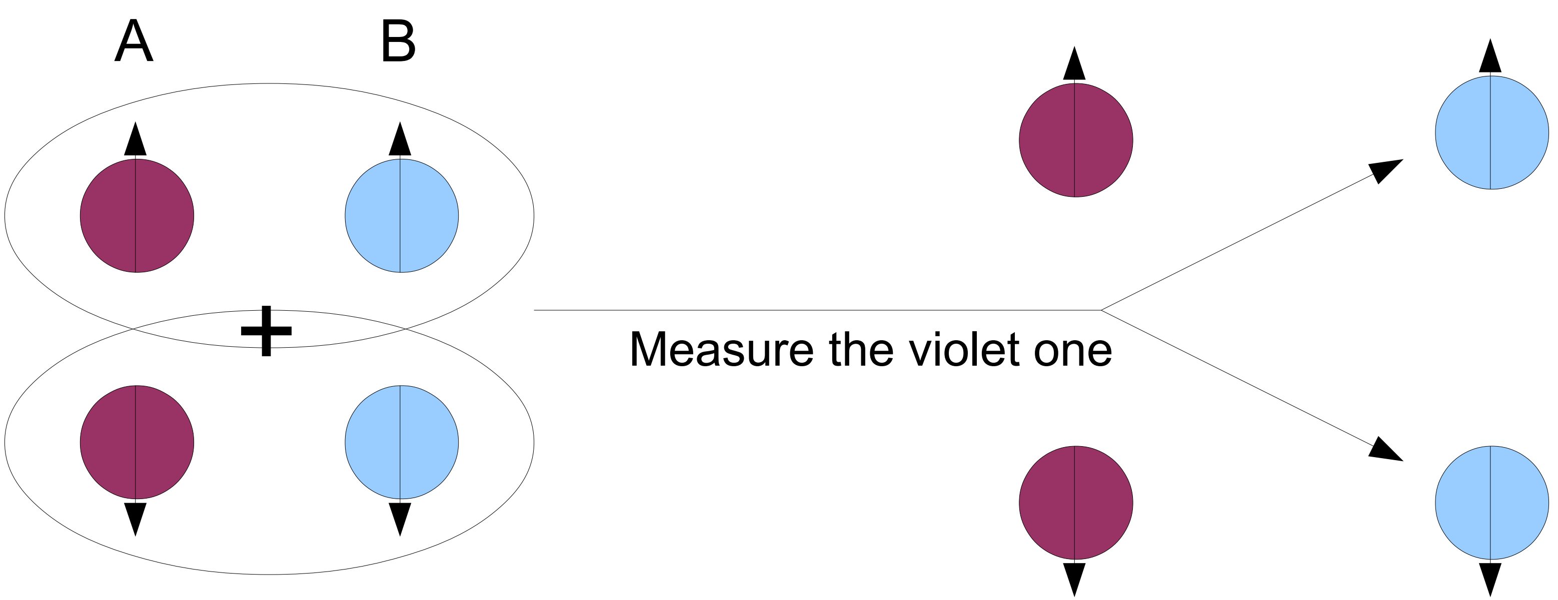


Introduction

Brief review of entanglement

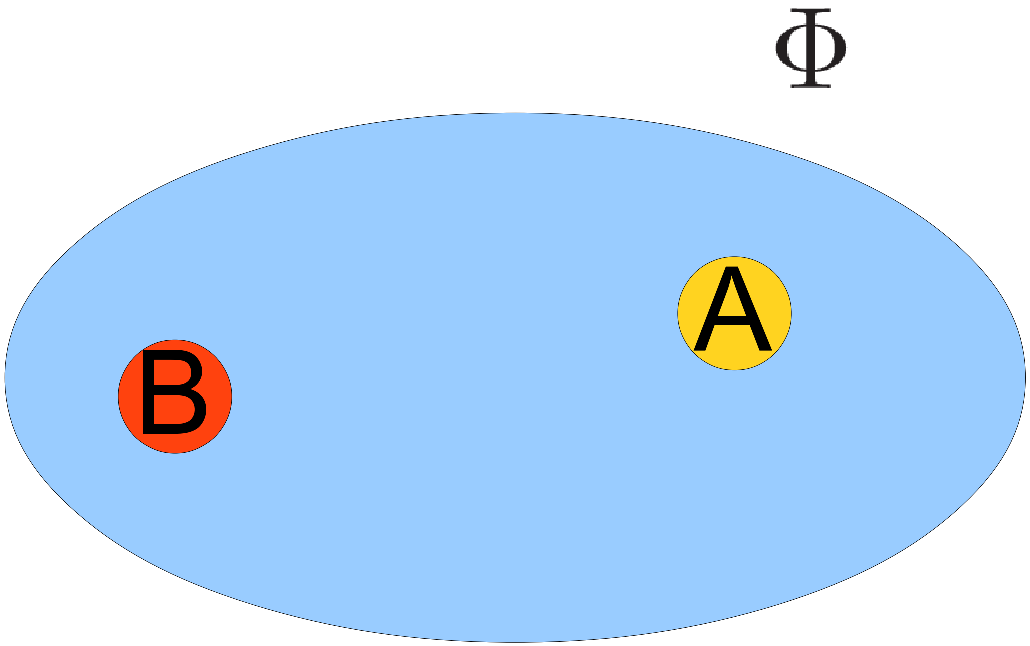
Entanglement is one kind of quantum corrlations

Example: Bell state $$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle)$$



Introduction

Entanglement for quantum fields



How can we see or use the entanglement of quantum fields?

Unruh de Witt detector model

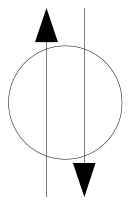
Unruh de Witt detector

Experimentally, we get the information of quantum fields by detectors (just like thermometer)

detector model (Unruh De Witt detector)

This model represents a “detector”

Easiest one is qubit, which has two internal levels,



$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

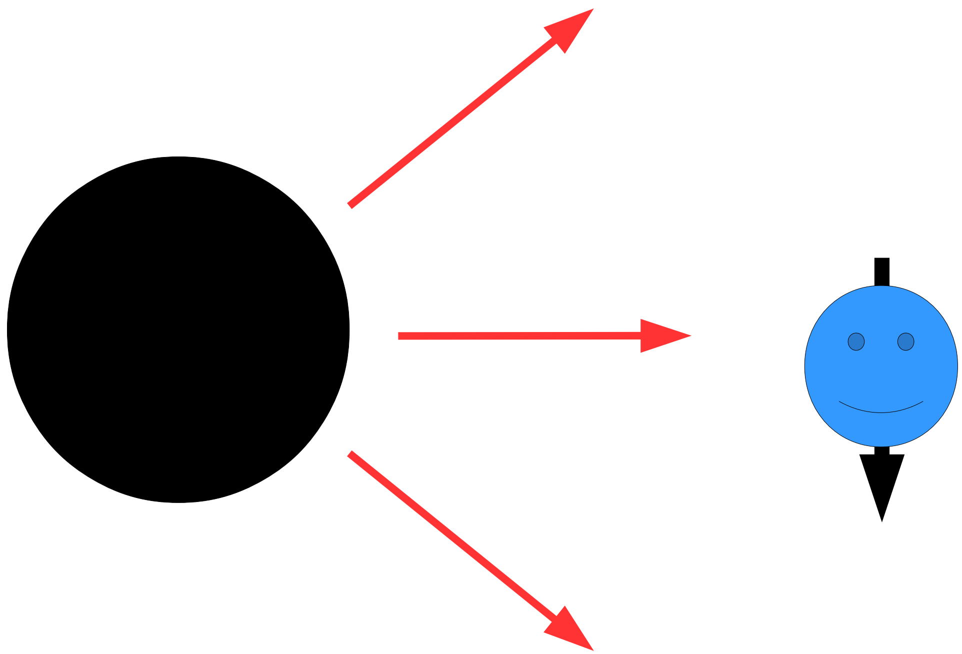
energy gap

and coupled with a quantum field.

Unruh de Witt detector

Hawking radiation

Thermal radiation



Unruh de Witt detector

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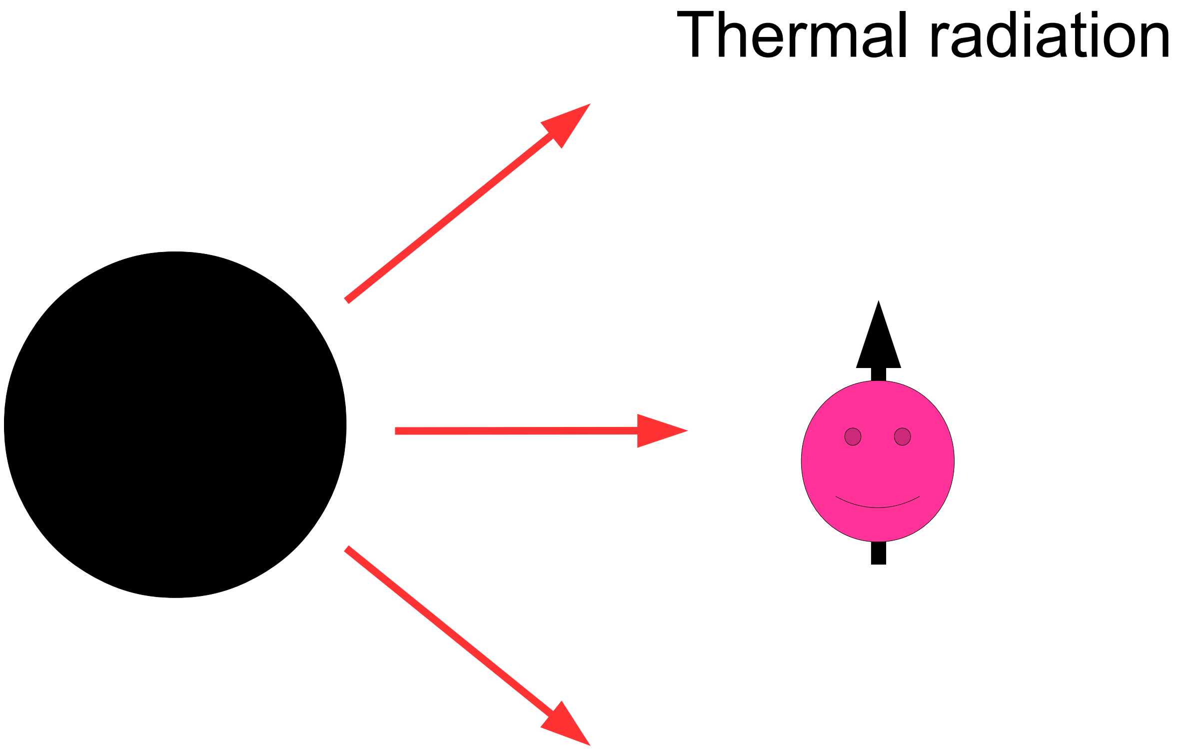
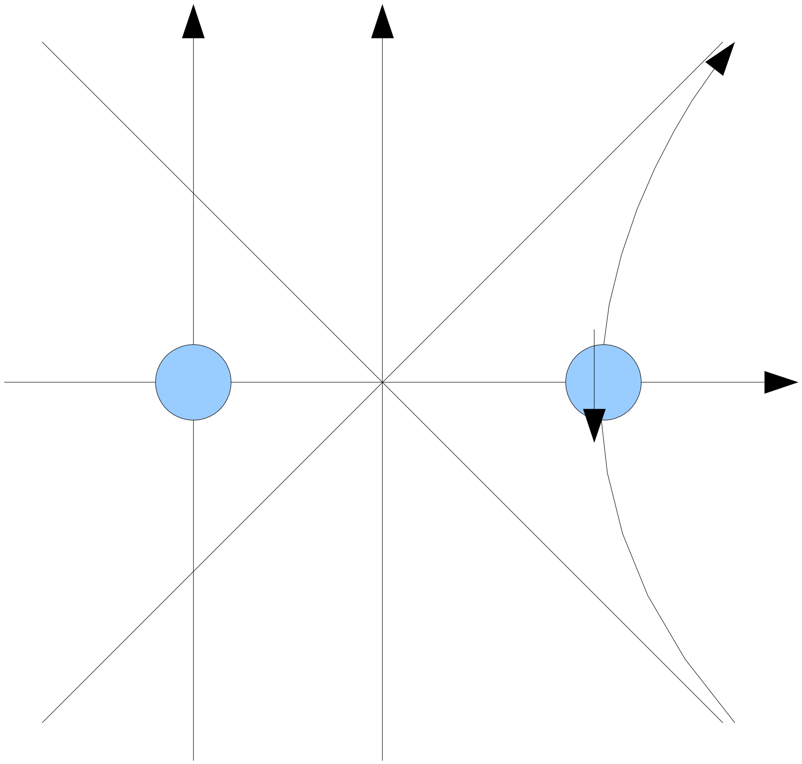


Unruh de Witt detector

Unruh effect

An uniformly accelerated observer detects the Minkowski vacuum as a thermal state



Unruh temperature

$$T = \frac{a}{2\pi}$$

Unruh de Witt detector

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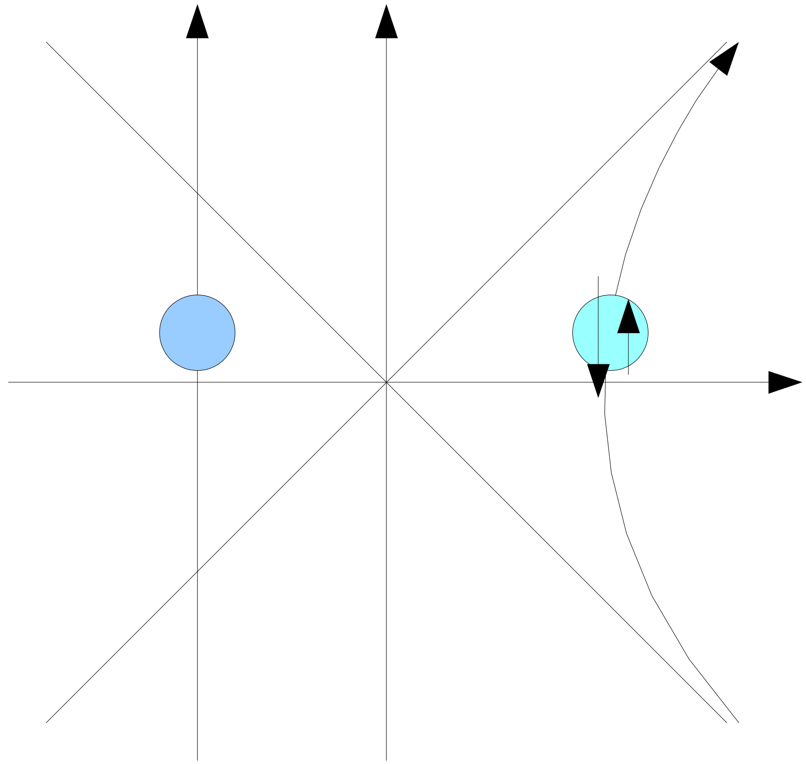

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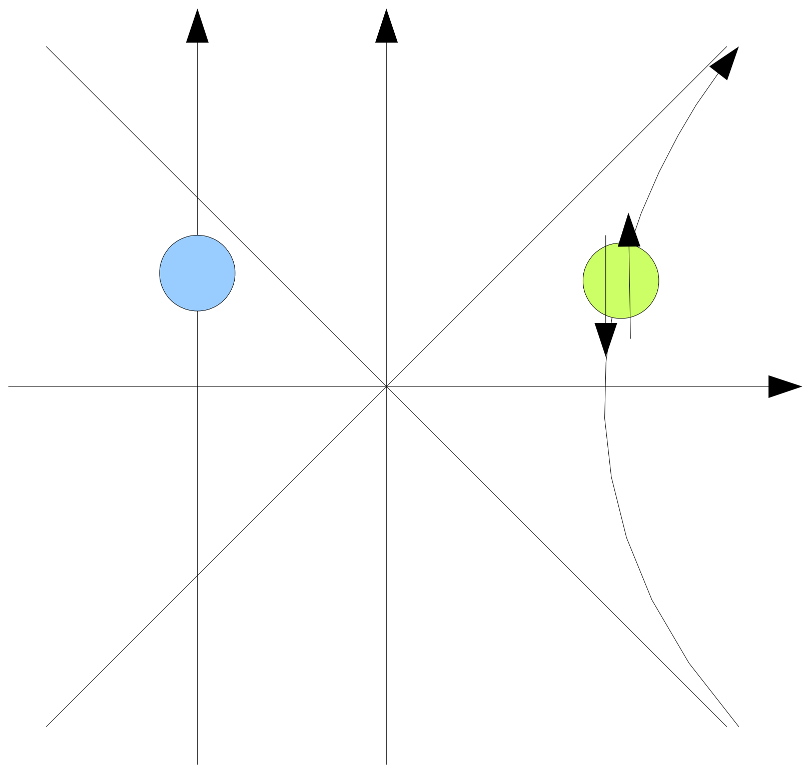

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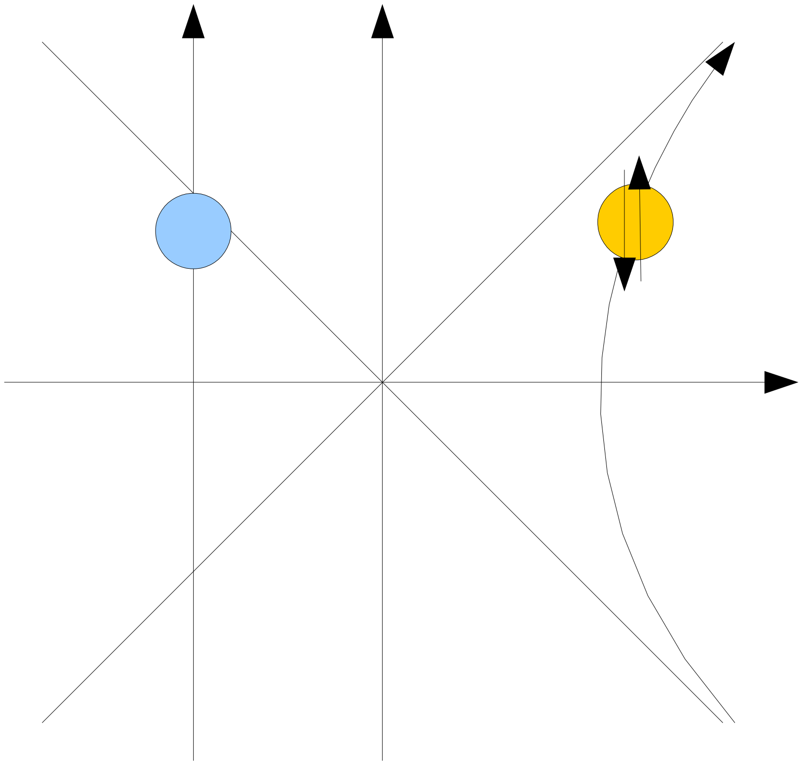

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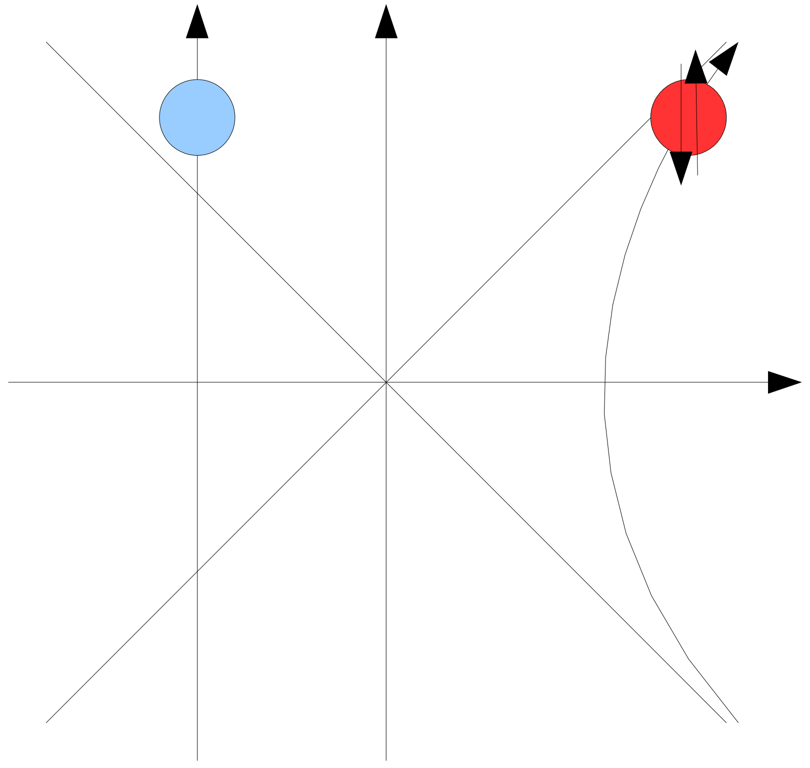
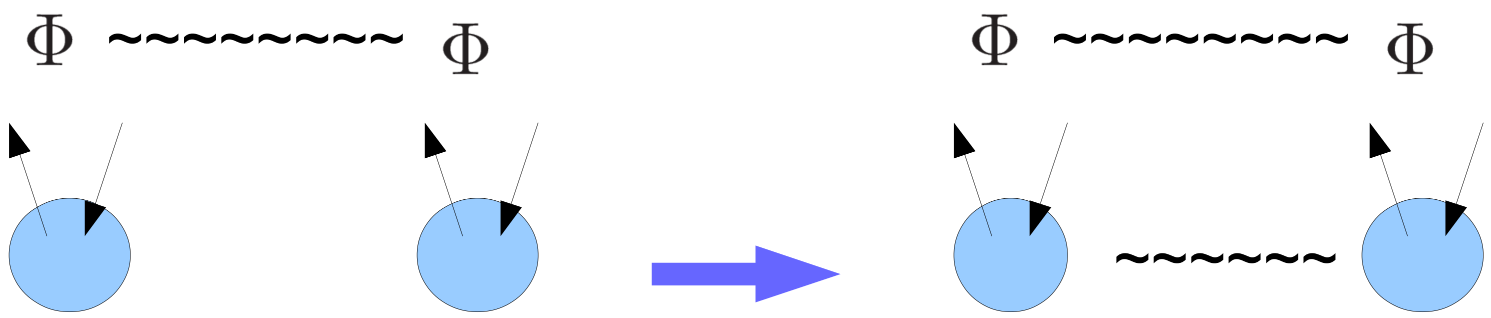


Unruh temperature

$$T = \frac{a}{2\pi}$$

Unruh de Witt detector

detector model also tells us the property of the entanglement of quantum fields



entanglement of the fields　　　entanglement between detectors

(entanglement swapping)

The entanglement of two qubits is more accessible than that of quantum fields

No one to one correspondence

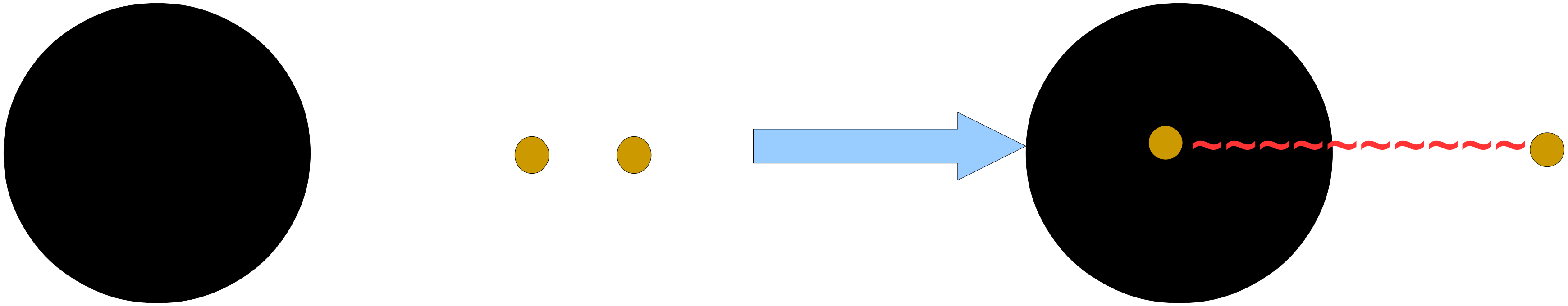
Our Goal

By an intutive way,such as Unruh detector, What kind of information of quantum fields can we get?

DREAM: Understand and reformulate ploblems, such as Information loss, Firewall, in a more intutive manner

Causally disconnected detector can be entangled and sustain the entangled state?

Our Goal



We can access the information inside black hole indirectly???

Method

a theoretical model discribing the dynamics of the detector model

Method

A quantum mechanical system is described by Liouville von Neumann equation

total system = detector system + field system

$$H_{tot} = \underbrace{H_s + H_\Phi}_{H_0} + \lambda H_{int} \qquad \rho_{tot} \in \mathcal{C}(\mathcal{H}_s) \otimes \mathcal{C}(\mathcal{H}_\Phi)$$

The time evoltion of the total system is described by

$$\frac{\partial \rho_{tot}(t)}{\partial t} = -iL_{H_{tot}}[\rho_{tot}(t)] := -i[H_{tot}, \rho_{tot}(t)]$$

Method

2 particle detector model

Benatti(2004)

Hamiltonian

$$H_{tot} = H_s + H_\phi + H_{int}$$

$$H_s = H_s^{(1)} + H_s^{(2)} \qquad H_s^{(\alpha)} = \frac{\omega}{2}\sigma_3^{(\alpha)}$$

$$\sigma_i^{(1)} = \sigma_i \otimes \sigma_0 \qquad \sigma_i^{(2)} = \sigma_0 \otimes \sigma_i$$

the field is a massless free scalar fields

$$H_{int} = \sigma_1^{(1)}\Phi(\vec{x_1}(t), t) + \sigma_1^{(2)}\Phi(\vec{x_2}(t), t)$$

Position of detector1 Position of detector2

Generally, the system is hard to solve exactly

Method

Liouville von Neumann equation in interaction picture

$$\frac{\partial \tilde{\rho}_{tot}(t)}{\partial t} = -i[\tilde{H}_{int}(t), \tilde{\rho}_{tot}(t)]$$

The solution:

$$\tilde{\rho}_{tot}(t) = \sum_n (\frac{\lambda}{i\hbar})^n \int_0^t dt_1 \cdots \int_0^{t_{n-1}} dt_n [\tilde{H}_{int}(t_1), \cdots, [\tilde{H}_{int}(t_n), \tilde{\rho}_{tot}(t_0)] \cdots]$$

We can calculate the dynamics perturbatively with $\lambda \ll 1$

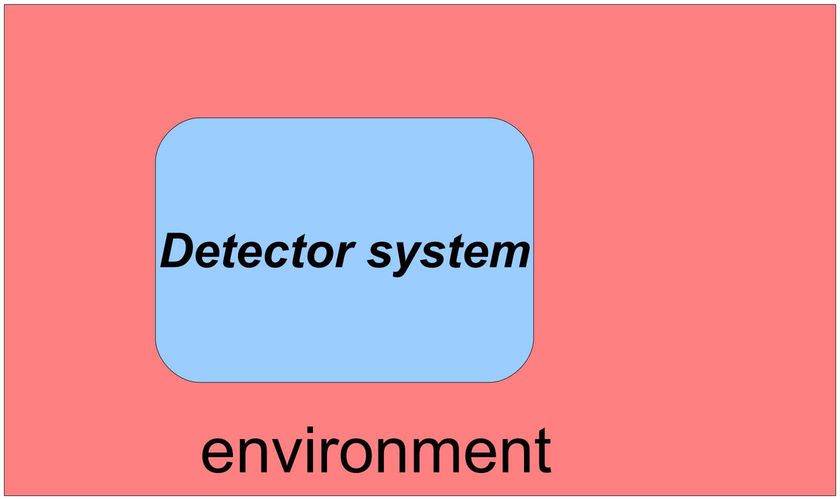
$$\tilde{\rho}(t) = \tilde{\rho}(0) + (i\lambda)^2 \int_0^t ds_1 \int_0^{s_1} ds_2 \mathrm{Tr}_B [\tilde{H}_{int}(s_1), [\tilde{H}_{int}(s_2), \tilde{\rho}(0) \otimes \rho_B]]$$

can not evaluate the long time evolution due to existence of ***secular terms***

$\lambda^2 t$

Method

Open quantum system



We can treat the background field as a environment.

Benatti et al 2004

Assuming that we can observe only the detector

$$\rho(t) := Tr_{\Phi}\rho_{tot}(t)$$ ➡ Quantum master equation

Method

quantum master equation(No secular term)

: a popular way to describe open quantum systems

(= this method is closely related with elimination of secular terms)

Simplification often used

1)Markovian

~~2)Rotating Wave Approximation(RWA)~~

RWA neglect transittion by quantum fluctuation

Quantum fluctuation plays an impotant role in entanglement generation.

We use a generalization of RWA which is called coarse graining approximation.

Method:Coarse graining approximation

$$\dot{\tilde{\rho}}(t) = -\frac{1}{\hbar^2}\frac{\lambda^2}{\Delta t}\int_t^{t+\Delta t} dt_1 \int_t^{t_1} dt_2 \mathrm{Tr}_\phi\left([\tilde{H}_{int}(t_1), [\tilde{H}_{int}(t_2), \tilde{\rho}(t)\otimes\tilde{\rho}_B(t)]]\right)$$

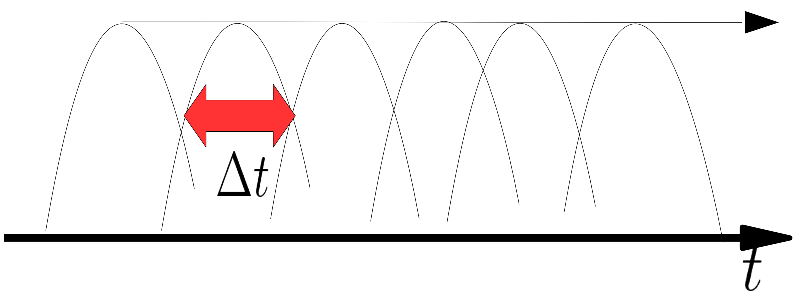
Quantum master equation(CGA , Markov) Schaller(2008)

This equation has the free parameter Δt

$\Delta t \to \infty$ corresponds to the rotating wave approximation.

Δt means the coarse graining time scale.

ΔtΔE~h



Method:Coarse graining approximation

$$\dot{\tilde{\rho}}(t) = -i[H_{eff}, \rho(t)] + \mathcal{L}[\rho(t)]$$

$$H_{eff} = H_s + i \sum_{\alpha,\beta=1}^{2} \sum_{i,j=1}^{3} \Gamma_{ij}^{(\alpha\beta)} \sigma_i^{(\alpha)} \sigma_j^{(\beta)}$$

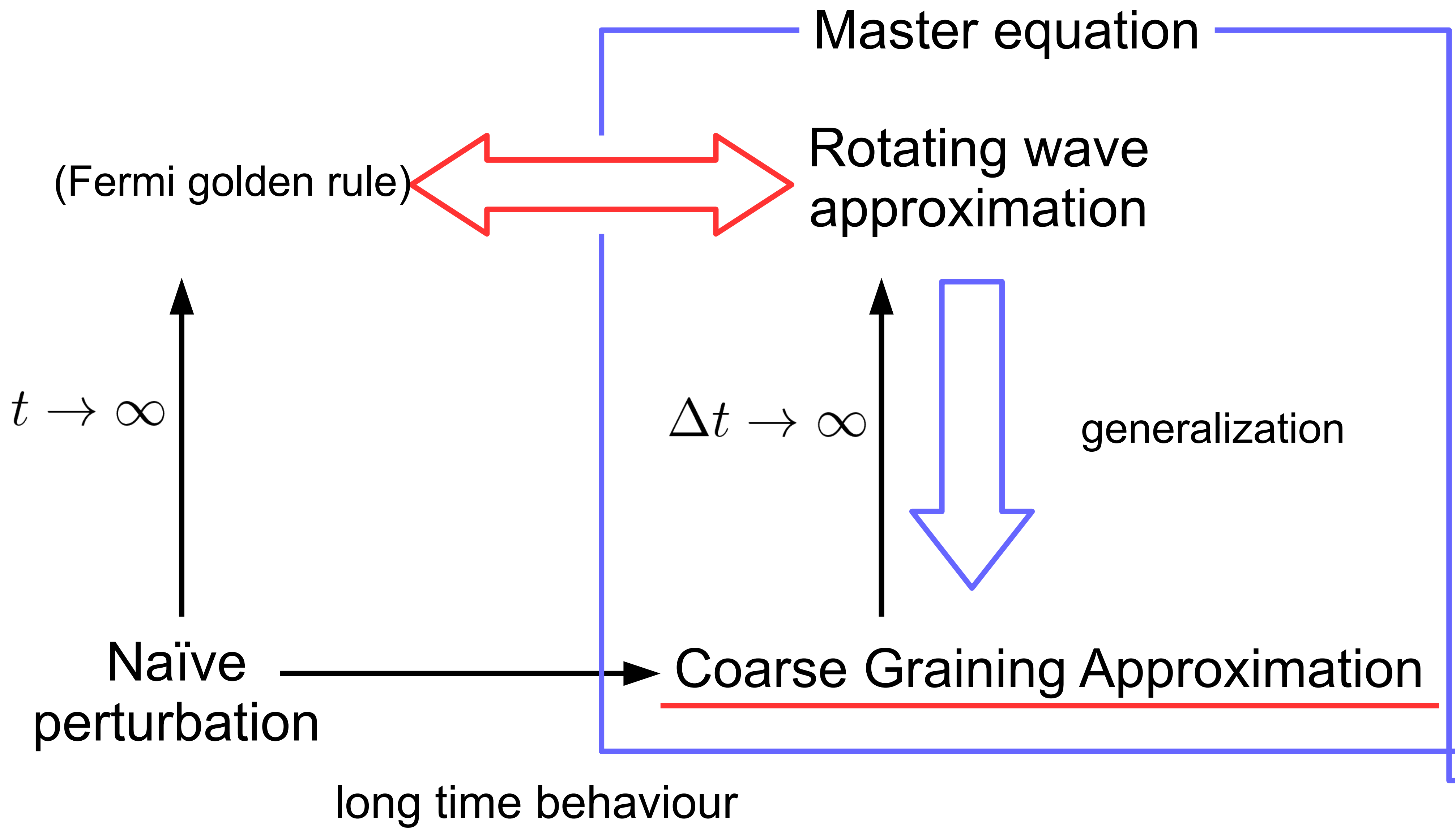
$$\mathcal{L}[\rho] = \frac{1}{2} \sum_{\alpha,\beta=1}^{2} \sum_{i,j=\pm} C_{ij}^{(\alpha\beta)} [2\sigma_j^{(\beta)} \rho \sigma_i^{(\alpha)} - \sigma_i^{(\alpha)} \sigma_j^{(\beta)} \rho - \rho \sigma_i^{(\alpha)} \sigma_j^{(\beta)}]$$

This type equation called Lindblad form

$$C_{\xi_1\xi_2}^{(\alpha\beta)} = \frac{\lambda^2}{\Delta t} \int_0^{\Delta t} dt_1 \int_0^{\Delta t} dt_2 e^{i\xi_1\omega t_1} e^{i\xi_2\omega t_2} \langle \phi(x_\alpha(t_1), t_1) \phi(x_\beta(t_2), t_2) \rangle$$

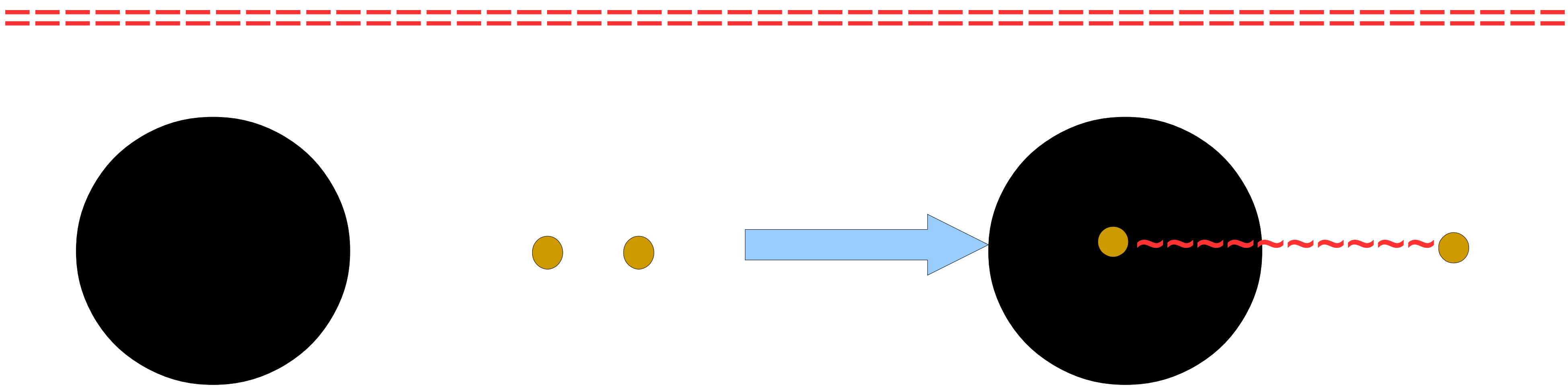
$$\Gamma_{\xi_1\xi_2}^{(\alpha\beta)} = \frac{\lambda^2}{\Delta t} \int_0^{\Delta t} dt_1 \int_0^{\Delta t} dt_2 e^{i\xi_1\omega t_1} e^{i\xi_2\omega t_2} \theta(t_1 - t_2) \langle \phi(x_\alpha(t_1), t_1) \phi(x_\beta(t_2), t_2) \rangle$$

Relation


Master equation
(Fermi golden rule)
Rotating wave approximation
$t \to \infty$
$\Delta t \to \infty$
generalization
Naïve perturbation
Coarse Graining Approximation
long time behaviour

Our results

Entanglement of causally disconnected detectors (simple case)



We can access the information inside black hole indirectly???

Entanglement between causaly disconnected detector

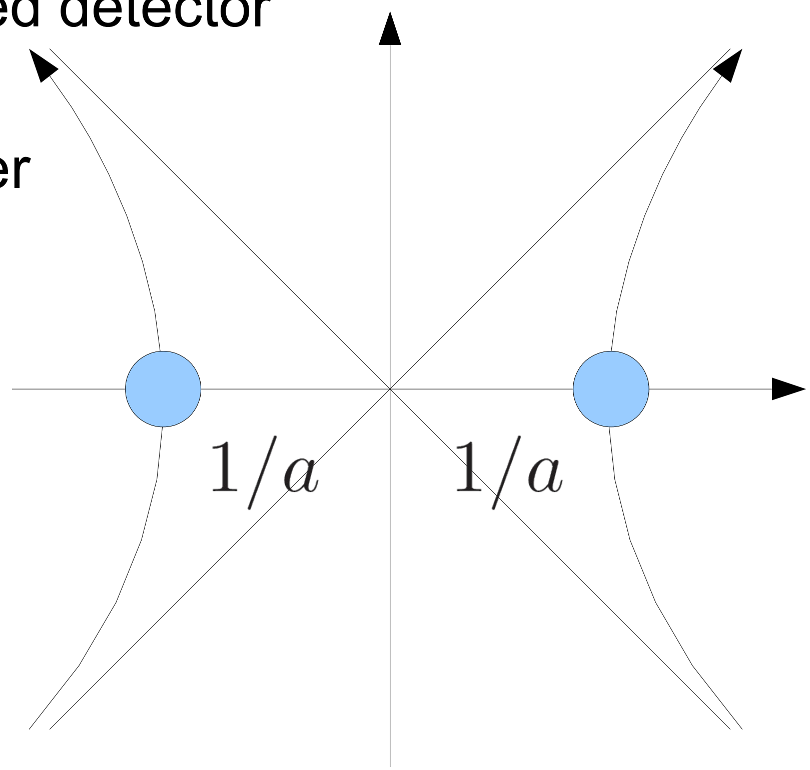
Considering two uniformly accelerated detector

The detectors can not see each other
due to the rindler horizon.

trajectory

$$x_1 = \frac{1}{a}\cosh(a\tau)$$

$$x_2 = -\frac{1}{a}\cosh(a\tau)$$



According to a previous study(Reznik,2002), The detector(initially ground state) can be entangled. (can not evaluate long time evolution)

initial setup

Initial state: ground state(not entangled)

$$\rho(0) = |\downarrow\rangle|\downarrow\rangle\langle\downarrow|\langle\downarrow|$$

time evolution

$$\begin{array}{c} \quad\quad\quad\;\; \uparrow\uparrow \quad \downarrow\uparrow \quad \uparrow\downarrow \quad \downarrow\downarrow \\ \rho(t) = \begin{pmatrix} \rho_1 & 0 & 0 & \rho_{14} \\ 0 & \rho_2 & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_3 & 0 \\ \rho_{14}^* & 0 & 0 & \rho_4 \end{pmatrix} \begin{matrix} \uparrow\uparrow \\ \downarrow\uparrow \\ \uparrow\downarrow \\ \downarrow\downarrow \end{matrix} \end{array}$$

Entanglement measure

We can not use entanglement entropy to evaluate the entanglement of the detectors

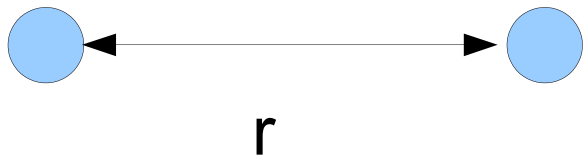
entanglement negativity $\mathcal{N}$ N=max(λ_1,λ_2)

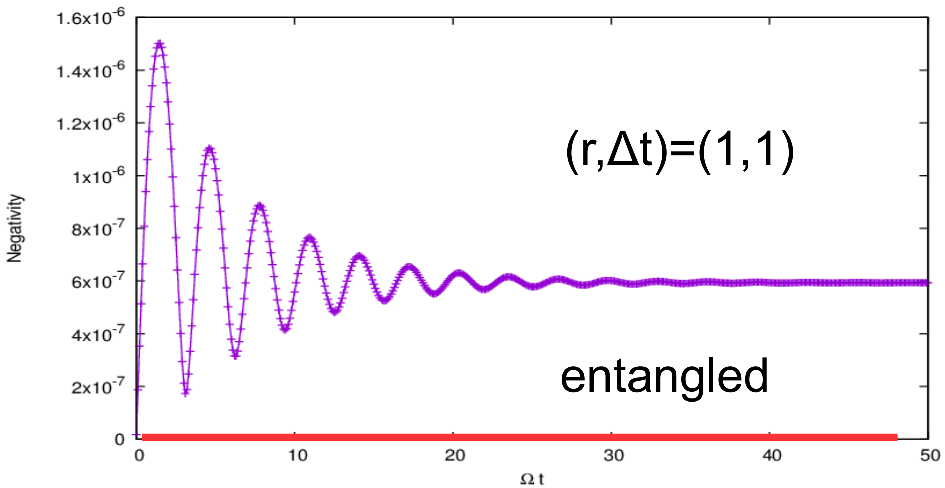
$$\lambda_1 = \frac{\sqrt{(\rho_1 - \rho_4)^2 + 4|\rho_{23}|^2} - (\rho_1 + \rho_4)}{2}$$

$$\lambda_2 = \frac{\sqrt{(\rho_2 - \rho_3)^2 + 4|\rho_{14}|^2} - (\rho_2 + \rho_3)}{2}$$

$\mathcal{N} > 0$ Entangled

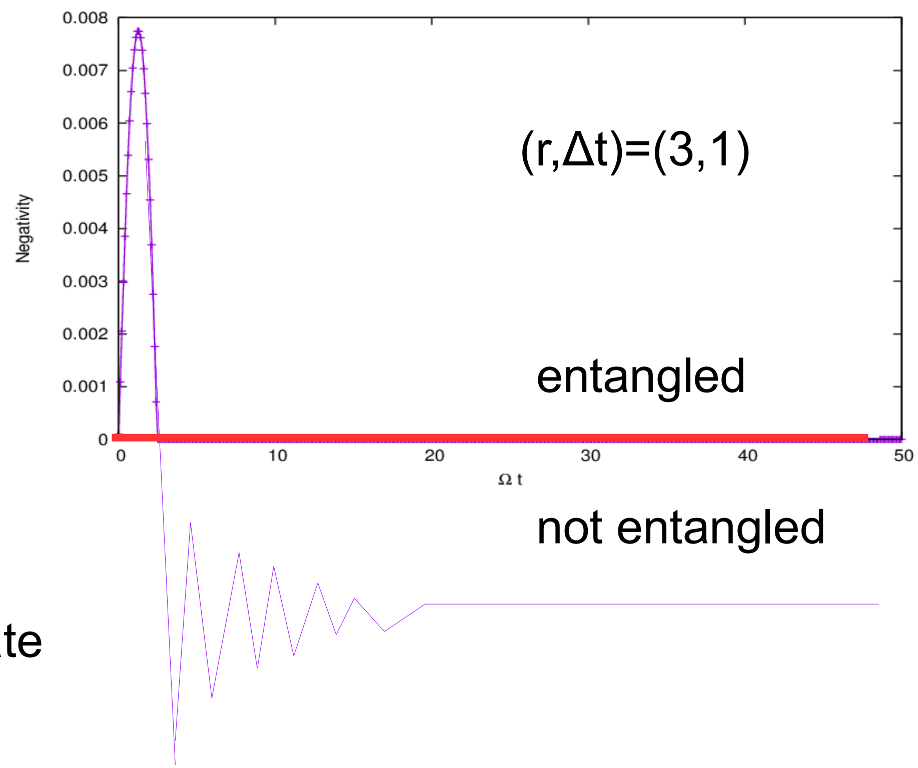
Minkowski vacuum case





not entangled

ground state →entangled state → entangled state

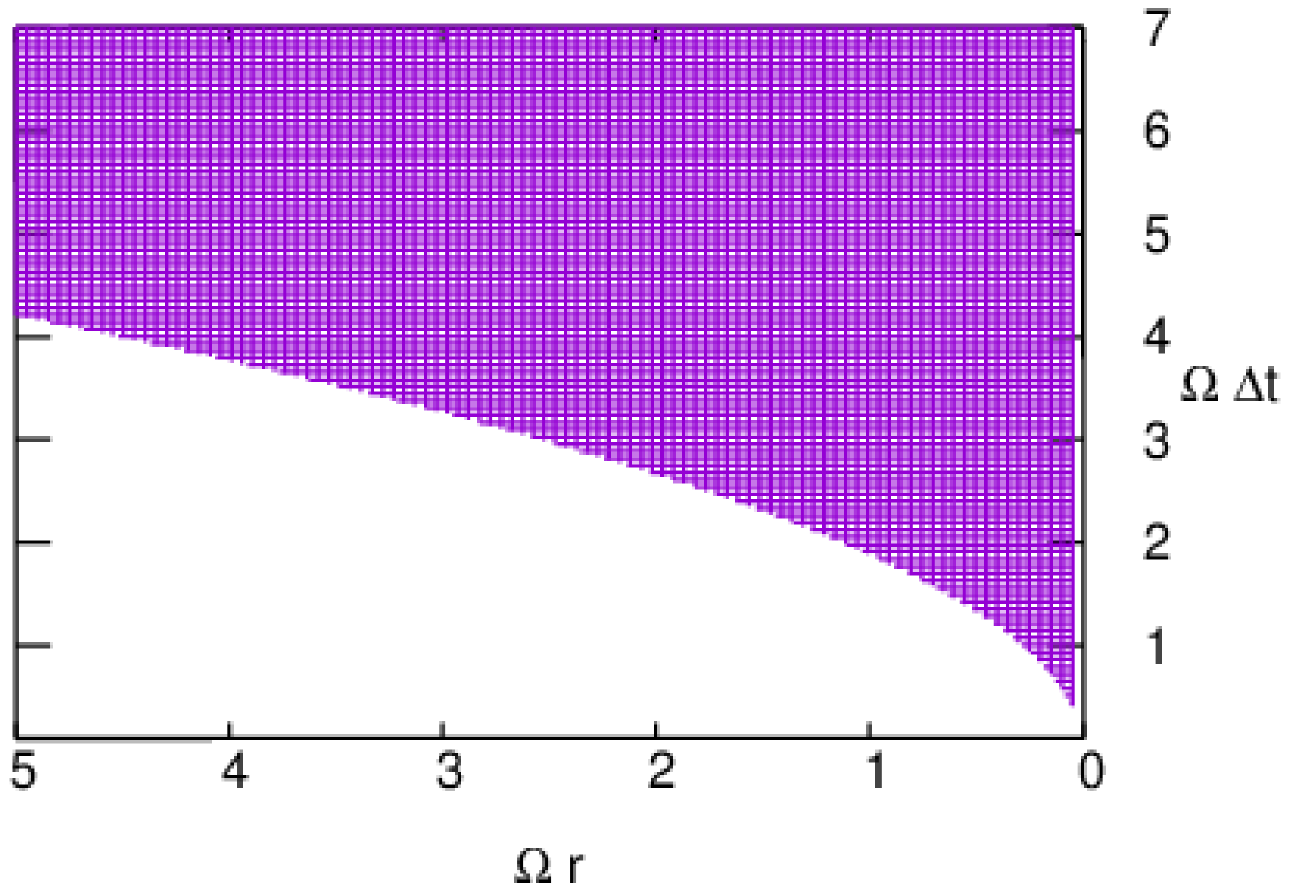


not entangled

ground state →entangled state → separable state

Minkowski vacuum case

Stationary state($t \to \infty$)



Independent of initial states

Entanglement between causaly disconnected detector

Considering two uniformly accelerated detector

The detectors can not see each other due to the rindler horizon.

trajectory

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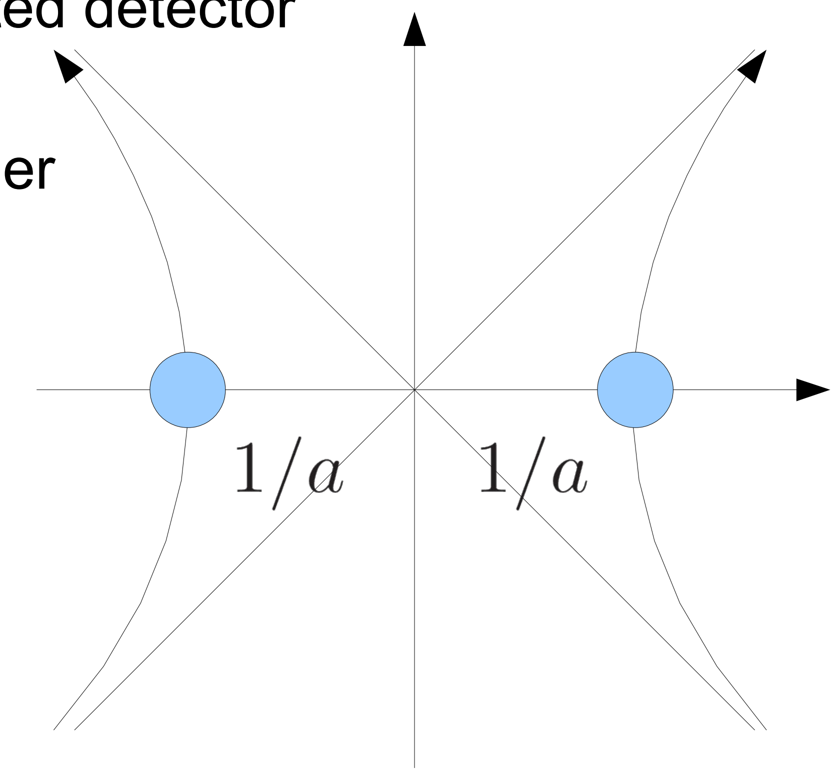

Whether the detectors become entangled state or not?

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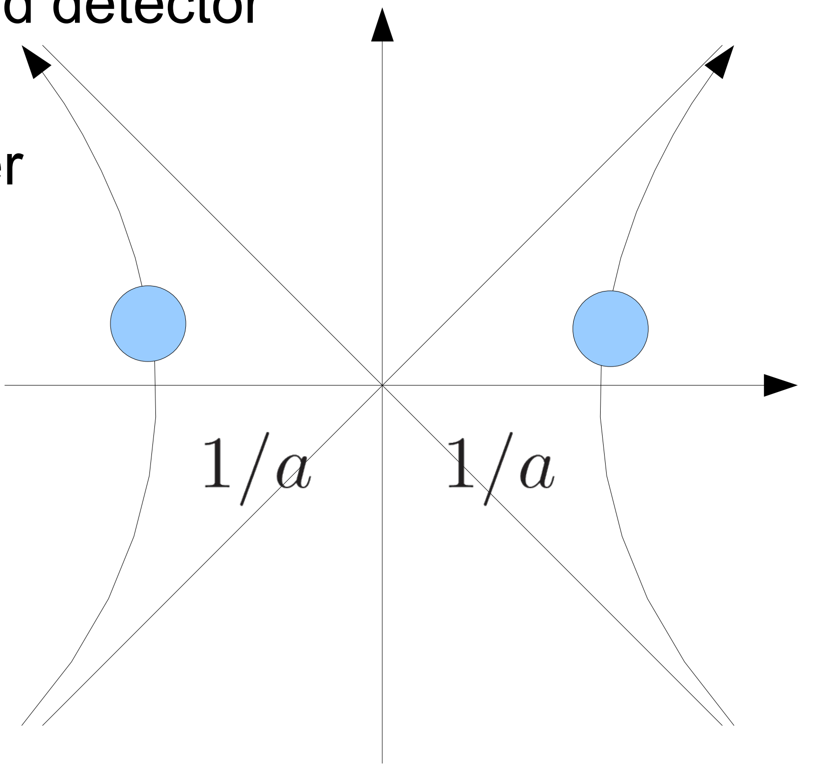

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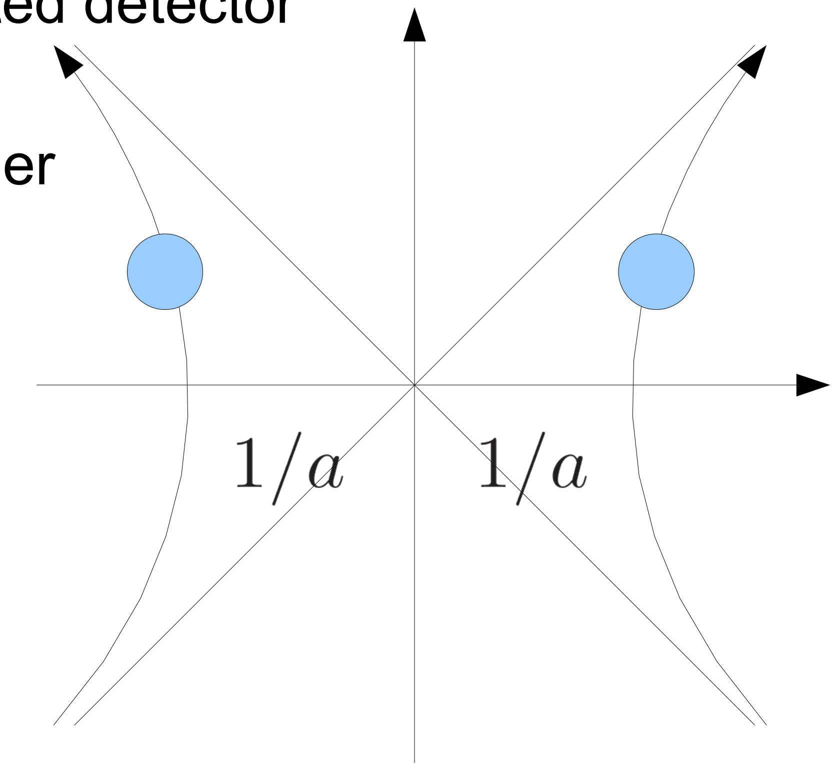

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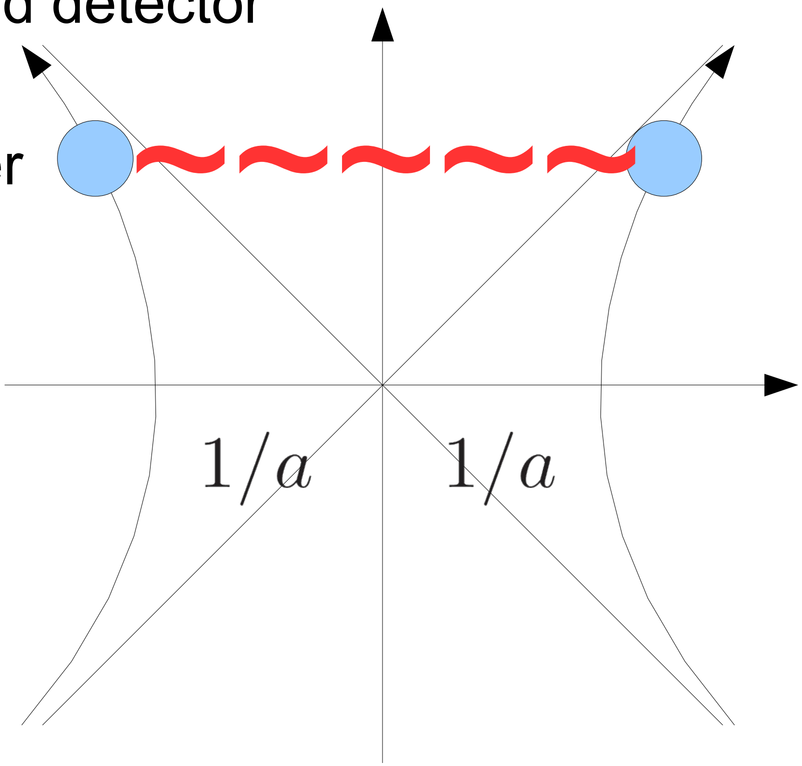

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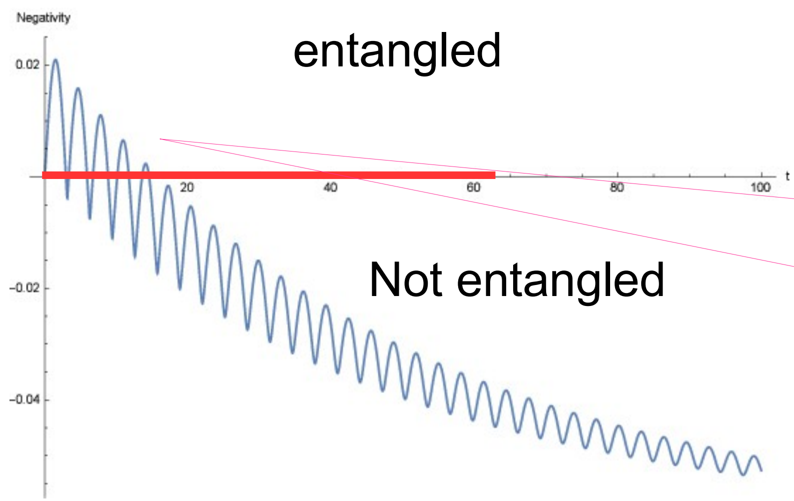
Whether the detectors become entangled state or not?

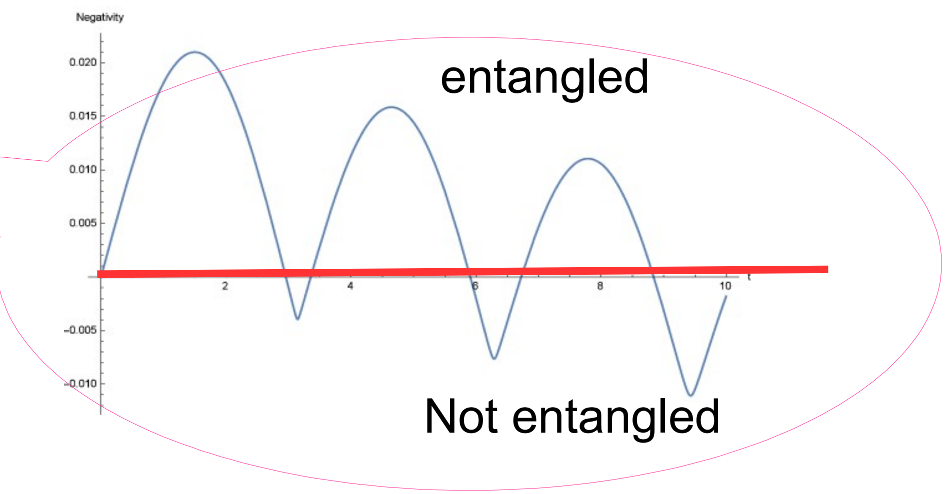
Result: dynamics

Entanglement negativity

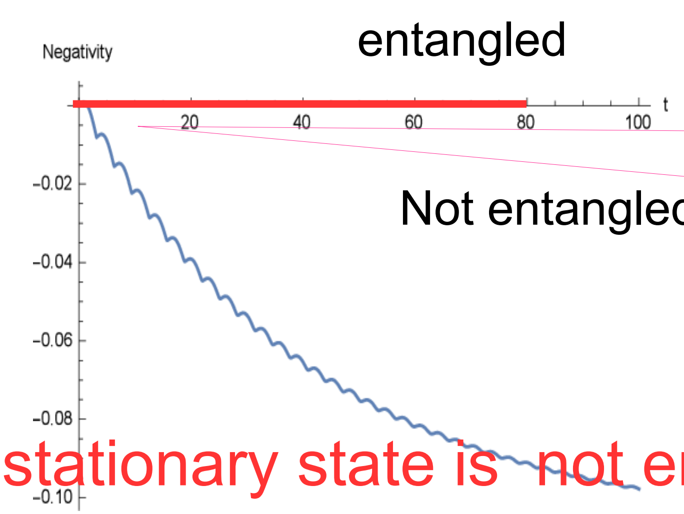
$$T := a/2\pi$$

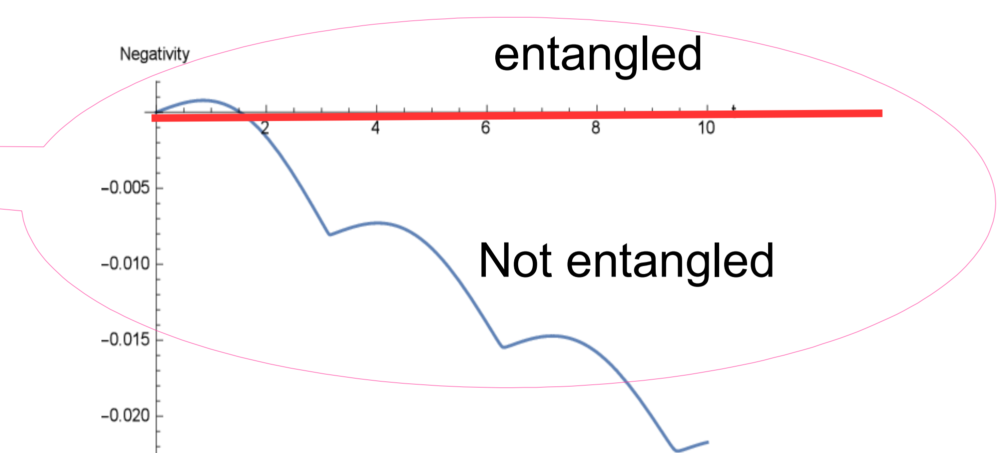
$$\Delta t = 1 \qquad T/\omega = 0.1$$





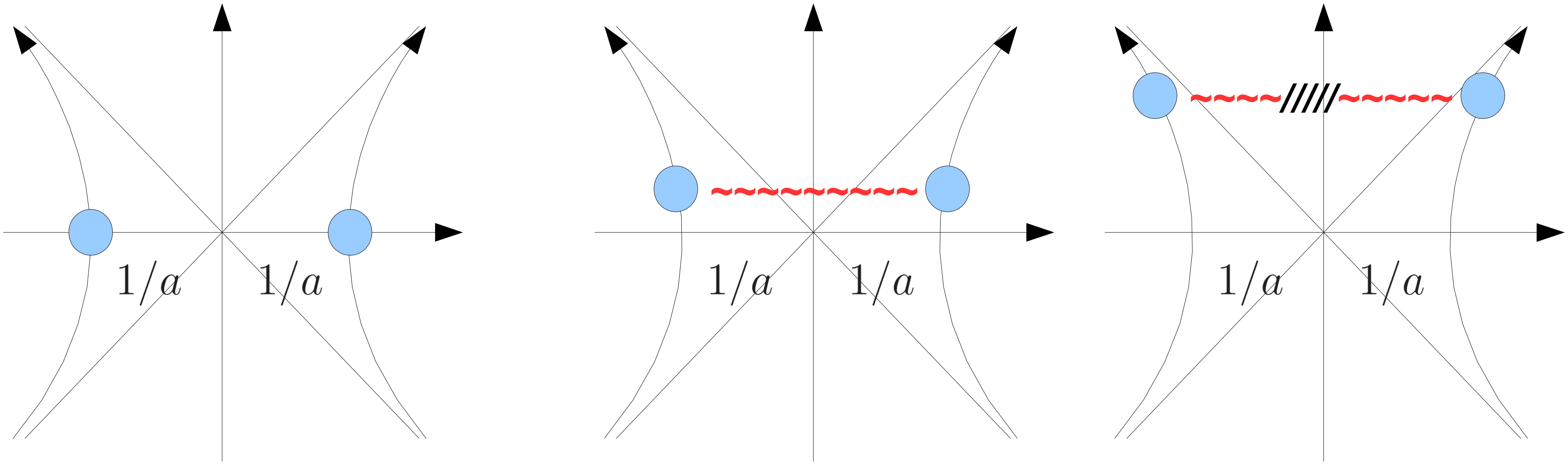
$$\Delta t = 3 \qquad T/\omega = 0.5$$





The stationary state is not entangled for any (Δt,T) and initial states

Our results



Summary

The detector model gives us a intuitive way to consider the quantum field theory on curved space time.

The CGA master equation allows us evaluate long time behaviour of the detector system.

Which parameter is physically realized?

Two causally disconnected detector can be entangled even so initially ground state. However, the stationary state is separable for any parameters.