Holography without SUSY and GR

Miok Park

Korea Institute of Advanced Study(KIAS), Seoul, S. Korea

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Contents

Properties of GR
- Properties of Black Hole
- Black Hole thermodynamics
- Padmanabhan's observation on Einstein Equation
- RN-AdS black hole vs Van der Waals system

Hologram

AdS/CFT : provides a well-defined dictionary for holography

Applied AdS/CFT
- Hairy Black Holes $\Longleftrightarrow$ Superconductor
- Negative Energy of AdS Soliton $\Longleftrightarrow$ Casimir Energy
- Lifshitz Spacetime $\Longleftrightarrow$ QCP and Lifshitz Theory

Summary

I am working on AdS/CMT, which is an application of AdS/CFT.


AdS/CMT


AdS
/CMT

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In order to do so, we need to take a look back of the time line for holography.

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- 2008, Sean A. Hartnoll, Christopher P. Herzog, and Gary T. Horowitz, *"Building an AdS/CFT superconductor"*

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"Let's go back to the start"

Properties of Black Hole [1]

- Black Hole Energy : global charge of the spacetime
  e.g. ADM formalism(1959), Komar formalism(1963), ADT formalism(1982, 2002), BY boundary stress tensor formalism(1993), Noether formalism(1993), etc.
- Black Hole Entropy
  Bekenstin Entropy(1972,1973), Wald formalism(*"Black Hole Entropy is Noether Charge"*, 1993)
- Black Hole Temperature
  e.g. Hawking's method, Periodic (thermal) Greens functions, Euclidean black hole metric (impose a period 2π for Euclidean time direction to avoid a conical singularity)

[1] S. Carlip, "Black Hole Thermodynamics," Int. J. Mod. Phys. D **23**, 1430023 (2014) [arXiv:1410.1486 [gr-qc]].

Four laws of black hole mechanics [2]

For a stationary asymptotically flat black hole in four dimensions, uniquely characterized by a mass M, an angular momentum J, and a charge Q

0. The surface gravity κ is constant over the event horizon
1. For two stationary black holes differing only by small variations in the parameters M, J, and Q,

$$\delta M = \frac{\kappa}{8\pi G}\delta A_{\mathrm{hor}} + \Omega_{\mathrm{H}}\delta J + \Phi_{\mathrm{H}}\delta Q$$

   where Ω_{H} is the angular vel. and Φ_{H} is the electric potential at the horizon
2. The area of the event horizon of a black hole never decreases $A_{\mathrm{hor}} \geq 0$
3. It is impossible by any procedure to reduce the surface gravity κ to zero in a finite number of steps.

These can be extended to more dimensions, more charges and angular momenta and to other "black" objects such as black strings, rings, and branes.

[2] J. M. Bardeen, B. Carter and S. W. Hawking, "The Four laws of black hole mechanics," Commun. Math. Phys. **31**, 161 (1973).

Gravitational field eqns as a thermodynamic identity [3]

Let us consider a static, spherically symmetric spacetime with a horizon, described by a metric

$$ds^2 = -f(r)c^2dt^2 + f^{-1}(r)dr^2 + r^2d\Omega^2 \tag{1}$$

rr component of the Einsteins equation becomes

$$(1-f) - rf'(r) = -(8\pi G/c^4)Pr^2, \qquad \text{where} \quad P = T^r_r \tag{2}$$

$$\frac{c^4}{G}\left[\frac{1}{2}f'(a)a - \frac{1}{2}\right] = 4\pi Pa^2, \tag{3}$$

$$\frac{c^4}{2G}f'(a)ada - \frac{c^4}{2G}da = P(4\pi a^2 da) \tag{4}$$

$$\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{k_B T}\ \underbrace{\frac{c^3}{G\hbar}d\left(\frac{1}{4}4\pi a^2\right)}_{dS}\ \underbrace{-\ \frac{1}{2}\frac{c^4 da}{G}}_{-dE} = \underbrace{Pd\left(\frac{4\pi}{3}a^3\right)}_{P\,dV}$$

[3] T. Padmanabhan,"Emergent perspective of Gravity and Dark Energy," Res. Astron. Astrophys. **12**, 891 (2012) [arXiv:1207.0505 [astro-ph.CO]]

RN-AdS black hole vs Van der Waals system [4]

- Van der Waals system

[4] D. Kubiznak and R. B. Mann, "P-V criticality of charged AdS black holes," JHEP **1207**, 033 (2012) [arXiv:1205.0559 [hep-th]]

RN-AdS black hole vs Van der Waals system [4]

- Van der Waals system

$$(P + \frac{a}{v^2})(v - b) = kT, \text{ where } v = \frac{V}{N},$$

$$kT_c = \frac{8a}{27b}, \; v_c = 3b, \; P_c = \frac{a}{27b^2} \quad \Rightarrow \quad \frac{P_c v_c}{kT_c} = \frac{3}{8}$$

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- RN-AdS black hole for $k=1$

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- RN-AdS black hole for $k=1$

$$T=\frac{1}{4\pi r_+}\left(1+\frac{3r_+^2}{l^2}-\frac{Q^2}{r_+^2}\right)$$
$$P=\frac{T}{v}-\frac{1}{2\pi v^2}+\frac{2Q^2}{\pi v^4},\quad\left(P=-\frac{\Lambda}{8\pi}=\frac{3}{8\pi l^2},\quad r_+=\left(\frac{3V}{4\pi}\right)^{1/3}\ \text{if}\ v=2l_p^2 r_+\right)$$
$$T_c=\frac{\sqrt{6}}{18\pi Q},\ v_c=2\sqrt{6}Q,\ P_c=\frac{1}{96\pi Q^2}\quad\Rightarrow\quad\frac{P_c v_c}{T_c}=\frac{3}{8}$$

[4] D. Kubiznak and R. B. Mann, "P-V criticality of charged AdS black holes,"

Hologram

Black hole's characteristic temperatures and entropies are

$$kT_H = \frac{\hbar\kappa}{2\pi}, \qquad S_{\rm BH} = \frac{A_{\rm hor}}{4\hbar G}$$

where κ is the surface gravity and $A_{\rm hor}$ is the area of the horizon.

- **G. 't Hooft (1993), Leonard Susskind (1995)**
  $\Rightarrow$ **"the combination of quantum mechanics and gravity requires the three-dimensional world to be an image of data that can be stored on a two-dimensional projection much like a holographic image."**

AdS/CFT Conjecture

Type IIB supergravity on $AdS_5 \times S^5$, which is low energy limit of string theory, is dual to $N = 4$ $d = 3 + 1$ $U(N)$ super-Yang-Mills.

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- Gravity/Gauge duality
  Isometry group for Type IIB supergravity on $AdS_5 \times S^5$ is equivalent to $N = 4$ $d = 3 + 1$ $U(N)$ super-Yang-Mills.

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- Gravity/Gauge duality
  Isometry group for Type IIB supergravity on $AdS_5 \times S^5$ is equivalent to $N = 4$ $d = 3 + 1$ $U(N)$ super-Yang-Mills.
- Weak/Strong coupling duality
  Identification of the parameters reads

$$g_s = g_{YM}^2, \qquad (L/l_s)^4 = 4\pi g_{YM}^2 N = 4\pi\lambda$$

When the effective coupling $g_s N$ becomes large we cannot trust perturbations in the Yang-Mills theory but we can trust calculations in supergravity on $AdS_5 \times S^5$.

Holographic Dictionary : Hologram + AdS/CFT

- Let us consider now any bulk field $\phi(z,x)$ fluctuating in AdS. Let $\phi_0(x)$ be the boundary value of ϕ

$$\phi_0(x) = \phi(z=0,x) = \phi|_{\partial AdS}(x)$$

The field $\phi_0(x)$ is related to a source for some dual operator O in the QFT.

- The AdS/CFT prescription for the generating functional is

$$Z_{CFT}[\phi_0] = \left\langle \exp[\int \phi_0 O] \right\rangle_{CFT} = Z_{gravity}[\phi \to \phi_0]$$

where $Z_{gravity}[\phi \to \phi_0]$ is the partition function in the gravity theory

- One point function is

$$\langle O(x) \rangle_\phi = \frac{\delta S^{ren}_{gravity}[\phi]}{\delta \phi(x)}$$

1. RN-AdS/Hairy Black Holes

⟺ Normal/Superconducting phase [5] [6] [7]

In collaboration with
Keun-Young Kim and Kyung Kiu Kim (GIST)

[5] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, "Holographic Superconductors," JHEP **0812**, 015 (2008) [arXiv:0810.1563].

[6] K. Y. Kim, K. K. Kim and M. Park, "A Simple Holographic Superconductor with Momentum Relaxation," JHEP **1504**, 152 (2015)[arXiv:1501.00446].

[7] 1508.XXXXX

Holographic Conjecture

- $U(1)$ symmetry breaking
- It is a local $U(1)$ for a gravity, but it is a global $U(1)$ for a gauge.

Gravitational Setup

- Original model by HHH

$$S = \int d^4x\sqrt{-g}\Big(R - 2\Lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\Big),$$

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2), \quad \text{where} \quad g(r) = r^2 - \frac{M}{r} + \frac{Q^2}{4r^2}$$

- Momentum Relaxation Model

Gravitational Setup

- Original model by HHH

$$S = \int d^4x \sqrt{-g}\left(R - 2\Lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |D\Phi|^2 - m^2\Phi^2\right),$$

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2), \quad \text{where} \quad g(r) = \text{numeric sol.}$$

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- Momentum Relaxation Model

$$S=\int_M d^{d+1}x\sqrt{-g}\left[R+\frac{d(d-1)}{L^2}-\frac{1}{4}F^2-\frac{1}{2}\sum_{I=1}^{d-1}(\partial\psi_I)^2\right],$$

$$ds^2=-g(r)e^{-\chi(r)}dt^2+\frac{dr^2}{g(r)}+\frac{r^2}{L^2}(dx^2+dy^2)\,,\quad \psi_I=\beta_{Ii}x^i=\frac{\beta}{l^2}\delta_{Ii}x^i$$

$$\text{where}\quad g(r)=\frac{1}{l^2}\left(r^2-\frac{\beta^2}{2}-\frac{m_0}{r}+\frac{\mu^2}{4}\frac{r_h^2}{r^2}\right)$$

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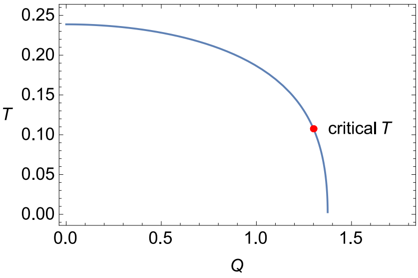
where $g(r) =$ numeric sol.

RN-AdS BH Instability : Second Order Phase Transition

: Basic Mechanism for forming Hairy Black Hole from RN-AdS black hole suggested by Gary Horowitz

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2+dy^2), \quad \text{where} \quad f(r) = r^2 - \frac{M}{r} + \frac{Q^2}{4r^2}$$



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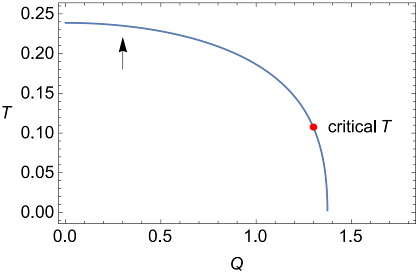

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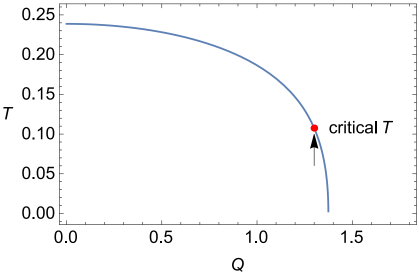

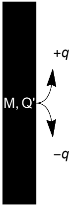

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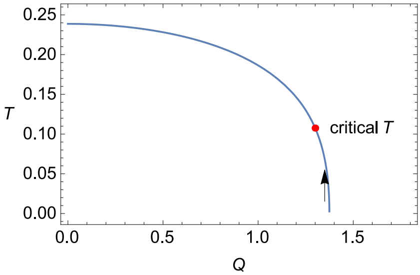

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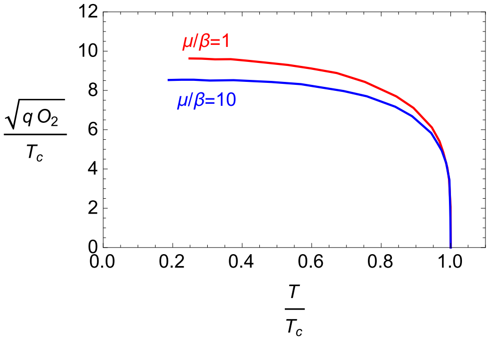

Figure: condensation for $\mu/\beta = 1$ for Red and $\mu/\beta = 10$ for Blue with $\Delta = 2$ and $q = 3$

Perturbation on Hairy BH : Momentum Dissipation

Linearized perturbed equations of motion without an Anxion field are

$$a_x'' + \left(\frac{g'}{g} - \frac{\chi'}{2}\right)a_x' + \left(\frac{\omega^2}{g^2}e^{\chi} - \frac{2q^2\Phi^2}{g}\right)a_x + \frac{r^2 e^{\chi} A_t'}{g}h_{tx}' = 0\,, \tag{5}$$

$$h_{tx}' + \frac{A_t'}{r^2}a_x = 0\,, \tag{6}$$

(7)

Combining the first and second becomes

$$a_x'' + \left(\frac{g'}{g} - \frac{\chi'}{2}\right)a_x' + \left(\frac{\omega^2}{g^2}e^{\chi} - \frac{2q^2\Phi^2}{g} - \frac{e^{\chi}A_t'^2}{g}\right)a_x = 0 \tag{8}$$

Perturbation on Hairy BH : Momentum Dissipation

Linearized perturbed equations of motion with an Axion field are

$$a_x'' + \left(\frac{g'}{g} - \frac{\chi'}{2}\right) a_x' + \left(\frac{\omega^2}{g^2} e^{\chi} - \frac{2q^2\Phi^2}{g}\right) a_x + \frac{r^2 e^{\chi} A_t'}{g} h_{tx}' = 0\,, \tag{5}$$

$$h_{tx}' + \frac{A_t'}{r^2} a_x + \frac{i\beta g e^{-\chi}}{r^2\omega} \xi' = 0\,, \tag{6}$$

$$\xi'' + \left(\frac{g'}{g} - \frac{\chi'}{2} + \frac{2}{r}\right)\xi' - \frac{i\beta\omega e^{\chi}}{g^2} h_{tx} + \frac{\omega^2 e^{\chi}}{g^2}\xi = 0 \tag{7}$$

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Perturbation on Hairy BH : Linear Response Theory

Quadratic term for the renormalized action becomes

$$S^{(2)}_{\text{ren}} = \frac{V_2}{2}\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\left(-\rho a_x^{(0)} h_{tx}^{(0)} + 2g_1 h_{tx}^{(0)} h_{tx}^{(0)} + a_x^{(0)} a_x^{(1)} - 3h_{tx}^{(0)} h_{tx}^{(3)} + 3\xi^{(0)}\xi^{(3)}\right),$$

where V_2 is the two dimensional spatial volume $\int \mathrm{d}x\mathrm{d}y$.

- From the linear response theory (or quantum field theory),

$$\delta O^i(k) = \tilde{G}^{ij}_R(k)\tilde{\phi}_j(k) + O(\phi^2), \tag{9}$$

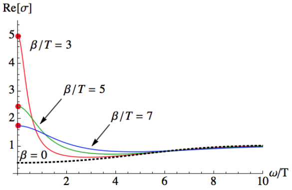
$$\begin{pmatrix} \langle J_x \rangle \\ \langle Q_x \rangle \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E_x \\ -(\nabla_x T)/T \end{pmatrix}, \tag{10}$$

- We relate these Green's functions to the electric (σ), thermal ($\bar{\kappa}$), thermoelectric ($\alpha, \bar{\alpha}$) conductivities defined as

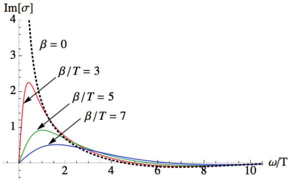
$$S^{(2)}_{\text{ren}} \equiv \frac{1}{2}\int_{\omega\geq 0} \frac{\mathrm{d}^d k}{(2\pi)^d}\left[J^a_{-k} G^R_{ab} J^b_k\right] \tag{11}$$

$$\begin{pmatrix} \langle J_x \rangle \\ \langle T_{tx} \rangle \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} a_x^{(0)} \\ h_{tx}^{(0)} \end{pmatrix}. \tag{12}$$

Normal Phase : Conductivity [8]



(a) Re σ. A delta function at $\omega = 0$ for $\beta = 0$ is not drawn. The red dots at $\omega = 0$ are the analytic DC values (4.16).



(b) Im σ. There is a $1/\omega$ pole for $\beta = 0$ corresponding to a delta function in (a)

[8] K.Y.Kim, K.K.Kim, Y.Seo and S.J.Sin, JHEP **1412**, 170 (2014)

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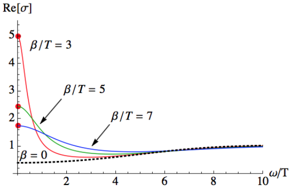

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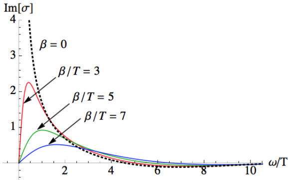

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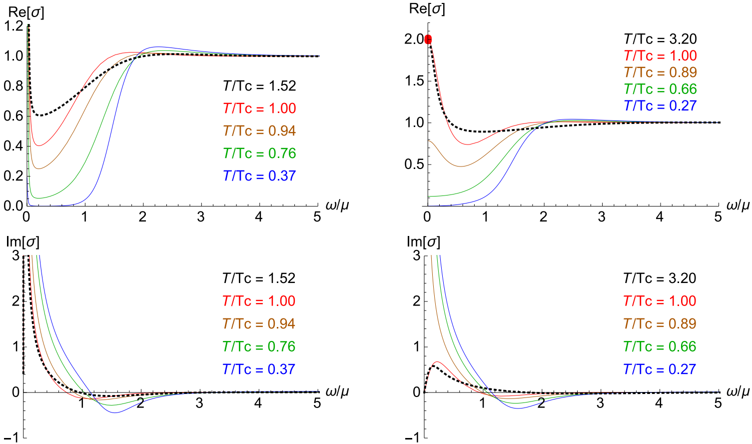
- **Kramers-Kronig relations**

$$\mathrm{Im}[\sigma(\omega)] = -\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{\mathrm{Re}[\sigma(\omega')]d\omega'}{\omega'-\omega}$$

From this formula we can see that the real part of the conductivity contains a delta function $\mathrm{Re}[\sigma(\omega)] = \pi\delta(\omega)$, if and only if the imaginary part has a pole, $\mathrm{Im}[\sigma(\omega)] = 1/\omega$

[8] K.Y.Kim, K.K.Kim, Y.Seo and S.J.Sin, JHEP **1412**, 170 (2014)

Superconducting Phase : Conductivity for $\frac{\beta}{\mu} = 0.1, 1$ [9]



[9] K. Y. Kim, K. K. Kim and M. Park, "A Simple Holographic Superconductor with Momentum Relaxation," JHEP **1504**, 152 (2015) [arXiv:1501.00446].

Momentum dissipation : Linear Response Theory

$$\int d^3x \left(\frac{\delta W}{\delta \bar{h}_{\mu\nu}(x)} (L_\zeta \bar{h})_{\mu\nu} + \frac{\delta W}{\delta \bar{A}_\mu(x)} (L_\zeta A)_\mu + \frac{\delta W}{\delta \bar{\psi}_I(x)} (L_\zeta \bar{\psi}_I) + \frac{\delta W}{\delta \bar{\Phi}(x)} (L_\zeta \bar{\Phi}) \right) = 0$$

$$D_\mu \langle T^{\mu\nu} \rangle + \bar{F}_\lambda{}^\nu \left\langle J^\lambda \right\rangle + \left\langle O^I \right\rangle \bar{h}^{\nu\lambda} \partial_\lambda \bar{\psi}_I + \left\langle O^\Phi \right\rangle \bar{h}^{\nu\lambda} \partial_\lambda \bar{\Phi} = 0 \tag{13}$$

$$\langle QJ \rangle = \omega i \alpha_{xx} T \ \ , \langle JQ \rangle = \omega i \bar{\alpha}_{xx} T \ \ , \ \ \langle QQ \rangle = \omega i \tilde{\kappa}_{xx} T \ \ , \ \ \langle JJ \rangle = \omega i \sigma_{xx} \tag{14}$$

$$\alpha_{xx} + \frac{\mu}{T}\sigma_{xx} - i\frac{n}{\omega T} + \beta \frac{\langle JS \rangle}{\omega^2 T} = 0 \tag{15}$$

$$\frac{\tilde{\kappa}_{xx}}{T} + \frac{\mu\alpha_{xx} + \mu\bar{\alpha}_{xx}}{T} + \frac{\mu^2\sigma_{xx}}{T^2} - i\frac{\epsilon}{\omega T^2} + \beta\frac{\langle QS \rangle}{\omega^2 T^2} + \beta\frac{\mu \langle JS \rangle}{\omega^2 T^2} = 0 \tag{16}$$

$$\langle ST \rangle + i\beta \frac{\langle SS \rangle}{\omega} = 0 \ \ , \tag{17}$$

2. Negative Energy of AdS Soliton

⟺ Casimir Energy [10]

[10] G. T. Horowitz and R. C. Myers, "The AdS / CFT correspondence and a new positive energy conjecture for general relativity," Phys. Rev. D **59**, 026005 (1998) [hep-th/9808079]

Holographic Conjecture

- Gravity
  - Gravitational soliton solution
  - The topology of geometry is $R^{p-1} \times S^1$
- Gauge
  - Casimir Energy
  - The gauge theory on $R^2 \times S^1$

AdS Soliton/Casimir Energy

- AdS Soliton

  Analytically continue this metric with both $t \to i\tau$ and $x^p \to it$

$$ds^2 = \frac{r^2}{l^2}\left[\left(1-\frac{r_0^{p+1}}{r^{p+1}}\right)d\tau^2 + (dx^i)^2 - dt^2\right] + \left(1-\frac{r_0^{p+1}}{r^{p+1}}\right)^{-1}\frac{l^2}{r^2}dr^2 \quad (18)$$

  where $i = 1, \cdots, p-1$. Energy is calculated by

$$E = -\frac{1}{8\pi G}\int N(K-K_0) \quad \Rightarrow \quad \rho_{\text{sugra}} = \frac{E}{V_2\beta} = -\frac{\pi^2}{8}\frac{N^2}{\beta^4} \quad (19)$$

- Casimir Energy

  The ground state energy of the gauge theory on $S^1 \times R^2$ where the length of the S^1 is β. This can only be calculated directly at weak gauge coupling, where to leading order, it reduces to the problem of determining the Casimir Energy of the free field theory.

$$\rho_{\text{gauge}} = -\frac{\pi^2}{6}\frac{N^2}{\beta^4} \quad (20)$$

Holographic Results

- The factor of 3/4 discrepancy between the two calculations does not conflict the AdS/CFT correspondence.
- Rather the supergravity result corresponds to the energy density of the gauge theory in a regime of strong coupling.
- To extrapolate the AdS results to weak coupling, one must include all of the higher order (in the string scale) corrections to the geometry induced by the Type IIB string theory.

3. Lifshitz Spacetime

⟺ QCP and Lifshitz Theory [11]

[11] S. Kachru, X. Liu and M. Mulligan, "Gravity duals of Lifshitz-like fixed points," Phys. Rev. D **78**, 106005 (2008) [arXiv:0808.1725 [hep-th]].

Lifshitz Theory

$$S = \int dt d^2x \left((\partial_t \phi)^2 - \kappa (\nabla^2 \phi)^2 \right)$$

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  - If ρ is positive then the **theory flows to a Lorentz invariant theory in the IR**.
  - If ρ is negative then one obtains ‘a tiled phase’ that spontaneously breaks spatial isotropy. The Lifshitz theory is then understood as the quantum critical theory separating tilted and untilted phases at zero temperature.

Lifshitz Spacetime

The anisotropic symmetry can be geometrically configured by

$$ds^2 = l^2\Big(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx_1^2 + dx_2^2}{r^2} \Big)$$

where z is the dynamical critical exponent and $z \neq 1$.

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where z is the dynamical critical exponent and $z \neq 1$. When $z = 1$ it restores the AdS spacetime.

Holographic Conjecture

Spatial translations P_i, time translations H, rotations M_{ij}, and Galilean boosts K_i. A scale invariant theory will also have the dilatation generator D.

$$[M^{ij},M^{kl}] = i(\delta^{ik}M^{jl} + \delta^{jl}M^{ik} - \delta^{il}M^{jk} - \delta^{jk}M^{il}), \tag{21}$$

$$[M^{ij},P^{k}] = i(\delta^{ik}P^{j} - \delta^{jk}P^{i}), \;\; [M^{ij},K^{k}] = i(\delta^{ik}K^{j} - \delta^{jk}K^{i}), \tag{22}$$

$$[K^{i},P^{j}] = i\delta^{ij}N, \quad [H,K^{i}] = -iP^{i}, \tag{23}$$

$$[D,P^{i}] = -iP^{i}, \; [D,K^{i}] = (z-1)iK^{i}, \;\; [D,H] = -ziH, [D,N] = i(z-2)N \tag{24}$$

This algebra is referred to as the Lifshitz algebra.

Gravity duals of Lifshitz-like fixed point III

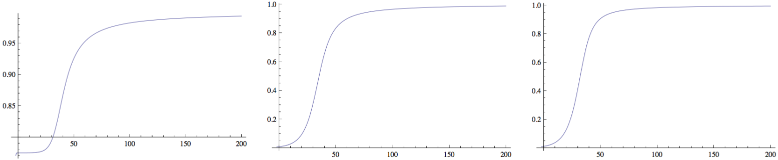

Fig. 1. Plotted are respectively $g(r)$, $h(r)$ and $j(r)$ as functions of $log(r)$. Their initial values are set at $r = 1$ to be respectively $\sqrt{\frac{3}{5}}$, 0.00728 and 0.01, i.e. small deviations away from the AdS_4 fixed point along the unique irrelevant direction. As the figure shows, all three functions asymptote to 1 at large r. Numerical integration of the system (4.5) suggests that the marginal direction at the Lifshitz fixed point becomes relevant at the nonlinear level.

Summary

GR and QG

- Einstein equation gives spacetime geometry curved by matter and energy.
- Then why GR shows thermodynamic or statistical properties?
- this means that spacetime can be understood by thermodynamics or statistics?
- Now holography works focus on strong/weak coupling duality. Still $S \propto A$ is a unsolved problem. What is the interpretation of this then?
- We don't care quantum gravity in holography now. Hologram interpretation for quantum gravity is not right?

AdS/CMT

- holographic conjecture becomes weaker and weaker in this area unlike AdS/CFT.
- Then how holographic result is reliable?
- Or holographic conjecture is necessary?
- Anyway, AdS/CMT could be a good testing stage for learning more theoretical structures of GR.