Black Holes and Conductivity

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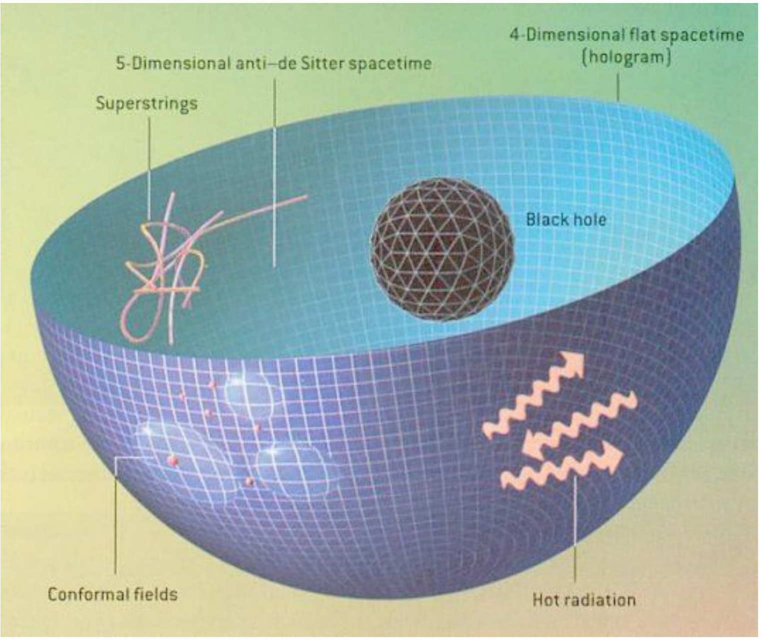
Hot Topics in General Relativity and Gravitation
Quy Nhon, Vietnam, August 2015

The Context: Holography

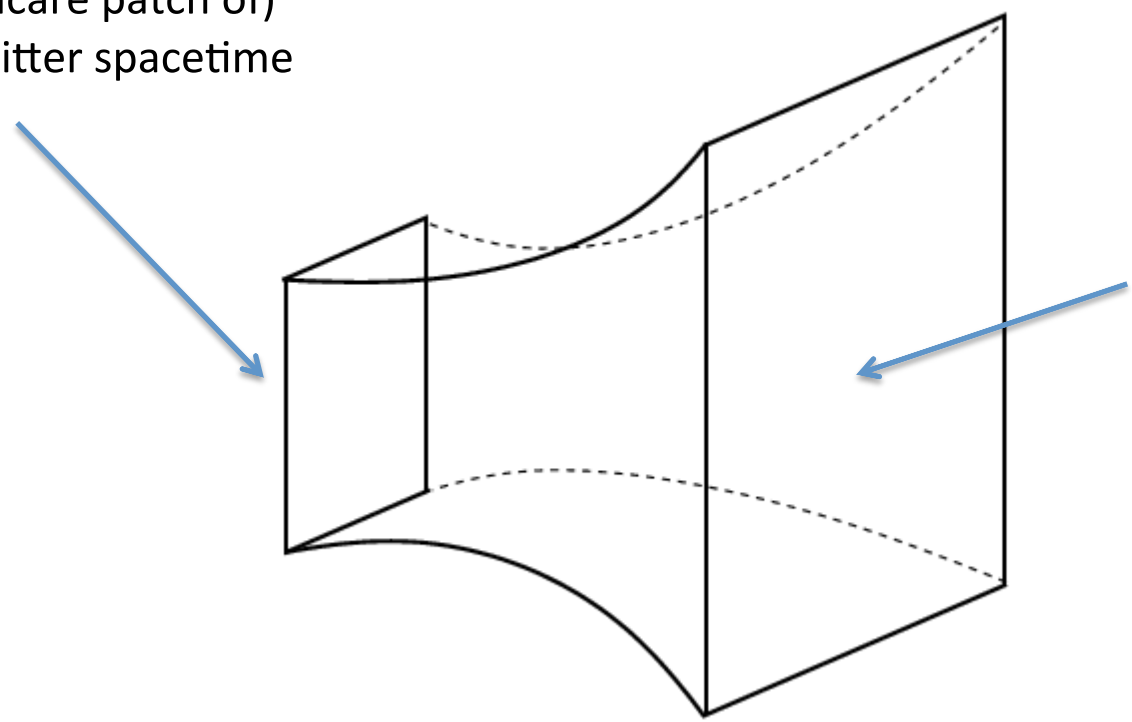
also known as: gauge gravity duality, or AdS/CFT correspondence

Strongly interacting quantum field theory ⟷ Gravity in (at least) one dimension higher



The Vacuum State

My attempt at drawing
(the Poincare patch of)
anti de-Sitter spacetime

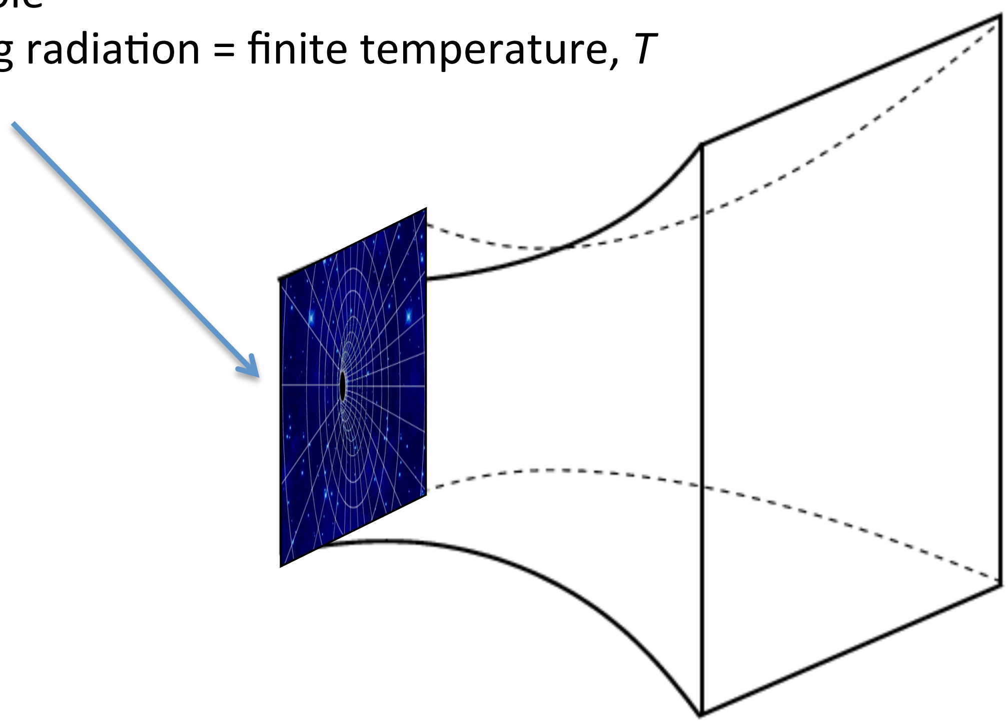


Boundary field theory

$$ds^2 = \frac{L^2}{r^2}\left(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

Heating up the Boundary Theory

- Black hole
- Hawking radiation = finite temperature, *T*

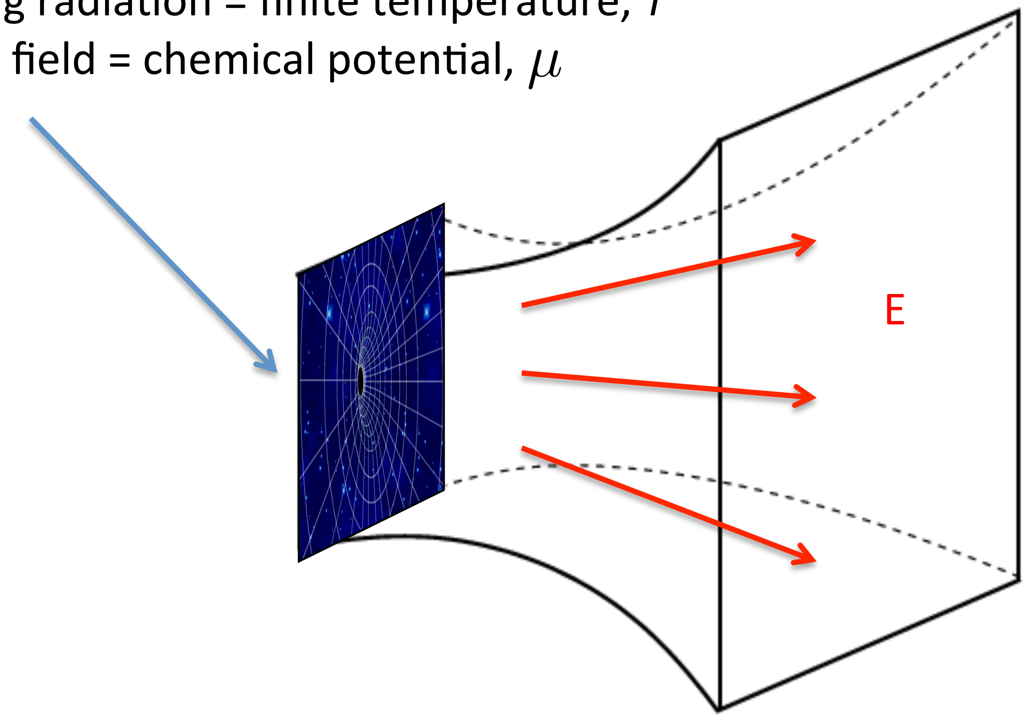


Boundary field theory

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + \eta_{\mu\nu}dx^\mu dx^\nu\right) \qquad f(r) = 1 - \left(\frac{r}{r_h}\right)^3$$

Finite Density Matter

- Reissner-Nordstrom black hole
- Hawking radiation = finite temperature, T
- Electric field = chemical potential, μ

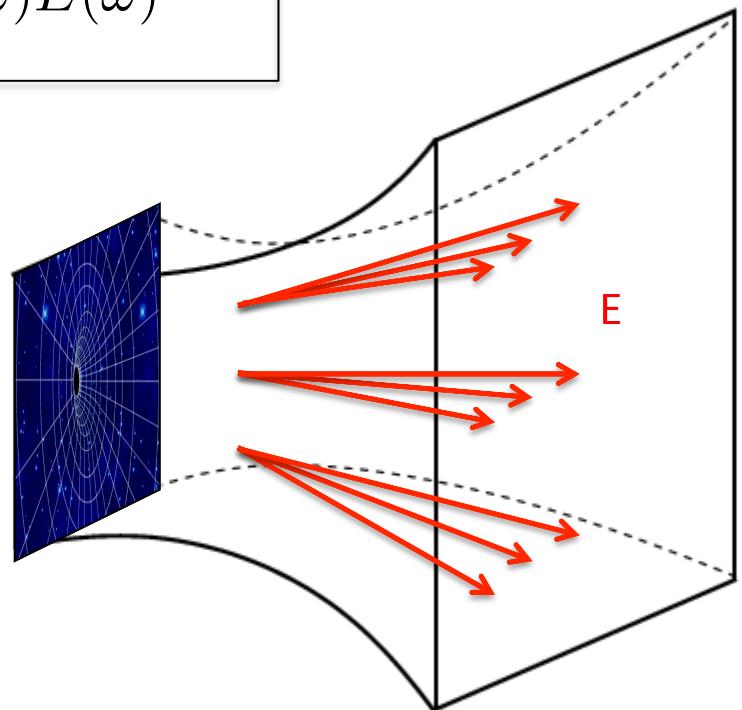


Boundary field theory

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + \eta_{\mu\nu}dx^\mu dx^\nu\right) \qquad f(r) = 1 - \left(1 + \frac{r_h^2\mu^2}{\gamma^2}\right)\left(\frac{r}{r_h}\right)^3 + \frac{r_h^2\mu^2}{\gamma^2}\left(\frac{r}{r_h}\right)^4$$

Ohm's Law

$$J(\omega) = \sigma(\omega)E(\omega)$$



Boundary field theory

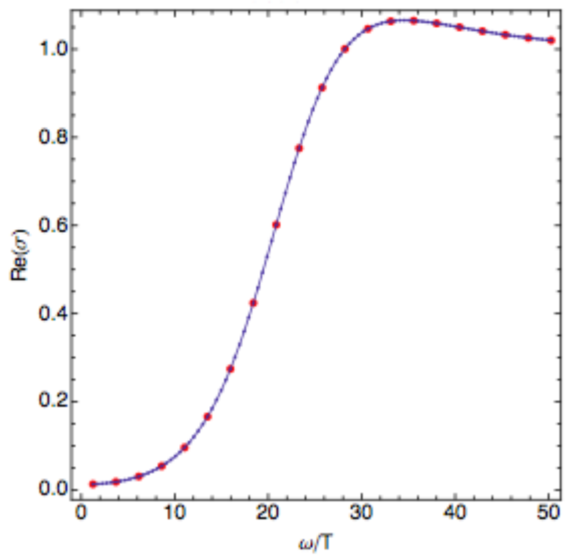
Perturb the background…

$$A_x = \frac{E}{i\omega}e^{i\omega t} + \langle J_x \rangle r + \ldots$$

Fix this

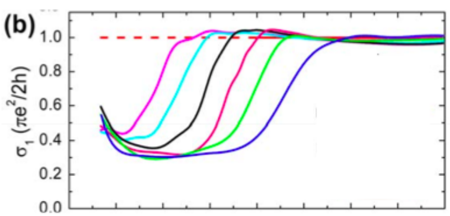
Solve for this

Optical Conductivity in d=2



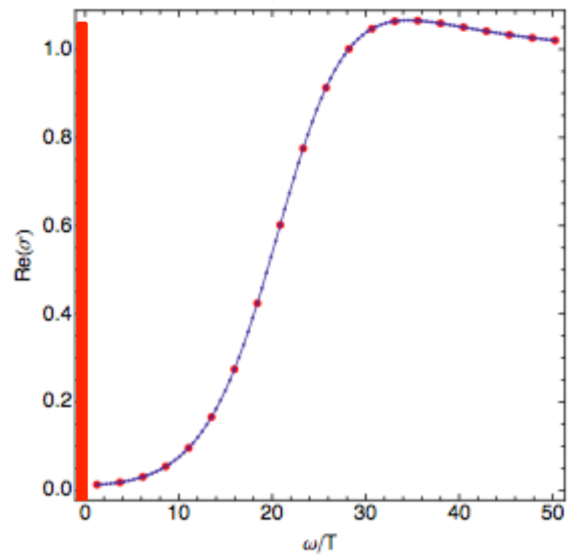
Herzog, Kovtun, Sachdev, Son
Hartnoll, 2007

$$J(\omega) = \sigma(\omega) E(\omega)$$



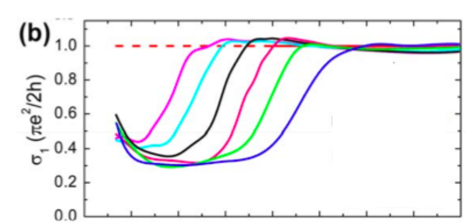
c.f. graphene

Optical Conductivity in d=2



Herzog, Kovtun, Sachdev, Son
Hartnoll, 2007

$$\mathrm{Re}\,\sigma(\omega) \sim K\,\delta(\omega)$$

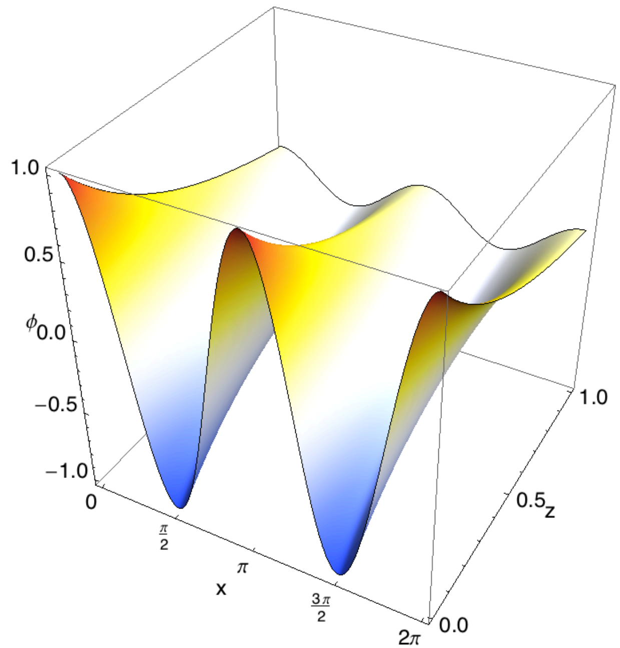


Finite charge density + momentum conservation

c.f. graphene

Breaking Translational Invariance

$\mu = \mu(x, y)$: spatially varying chemical potential



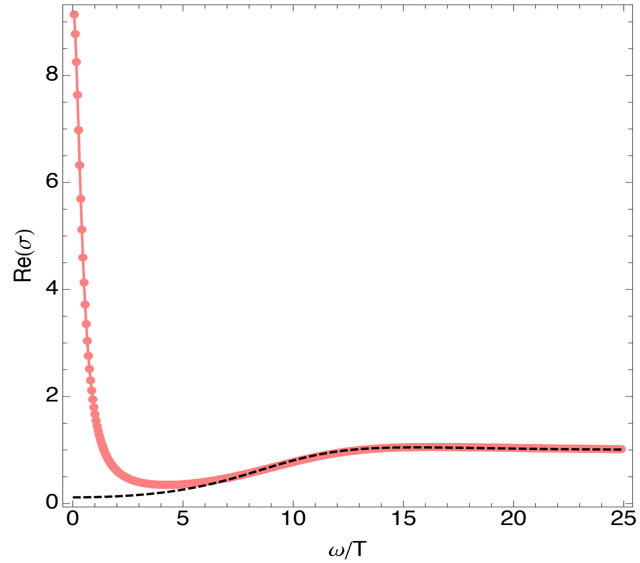
Horowitz, Santos, Tong, 2012

A rippled black hole

$$ds^2 = \frac{L^2}{z^2}\Big[- g_{tt}(z,x)dt^2 + g_{zz}(z,x)dz^2 + g_{xx}(z,x)(dx + a(z,x)dz)^2 + g_{yy}(z,x)dy^2 \Big]$$

Optical Conductivity

$$J(\omega) = \sigma(\omega)E(\omega)$$



Horowitz, Santos, Tong, 2012

Delta function spreads out. The low-frequency curve is a perfect fit to the Drude model!

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

DC Conductivity

Subsequently we managed to compute DC conductivity analytically.

Related to massive gravity

Vegh
Blake and Tong
Blake, Tong and Vegh, 2013

DC Conductivity

$$\sigma_{\rm DC} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

Blake, Tong
Blake, Tong and Vegh
Donos and Gauntlett, 2013
+ many more now

Analytic DC Conductivity

charge density

scattering time*

$$\sigma_{\rm DC} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

energy density

pressure

*Related to the mass of the graviton at the horizon

Blake, Tong
Blake, Tong and Vegh
Donos and Gauntlett, 2013
+ many more now

Analytic DC Conductivity

Due to scattering of charged stuff

Due to "pair creation"

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E}+P}\tau + \sigma_0$$

	holes	particles
momentum	←	→
electric current	→	→

Damle and Sachdev, 1997

Some Surprises

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

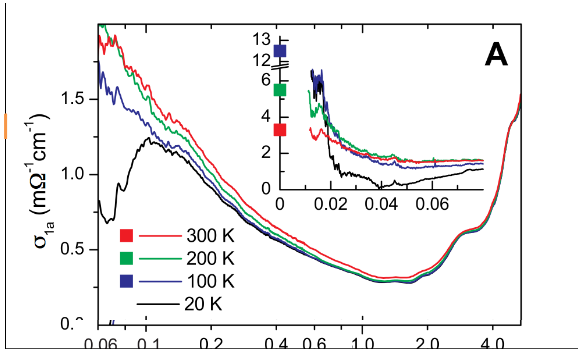
This formula is surprising:

- First term is power law in temperature
- Second term is power law in temperature
- Inverse Matthiessen rule

Some Uses

LSCO

Takenaka et al



Data suggests that second term is responsible for DC conductivity in cuprates

Hartnoll (2014)

Possible explanation for linear resistivity?

Hartnoll 2014 (see also Bruin et al; Sachdev, Zaanen)

But, in the presence of magnetic field, first term dominates

Blake and Donos, 2014

"Over broad regions of doping, the two kinds of relaxation rates, the one for the conductivity and the one for the Hall rotation, seem to add as inverses: conductivity is proportional to $1/T + 1/T^2$. That is, it obeys an anti-Matthiessen law"

P.W. Anderson

Much much more…

Black holes offer a framework to answer the simple question:

“What can strongly coupled matter do?”

They are providing new ways to think about old problems in condensed matter physics and fluid dynamics.

Thank you for your attention

Additional Material

DC Conductivity: Surprise 1

The first term varies as a power-law in temperature.

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

$$\tau(T) \sim T^{-2\Delta(k_L)}$$

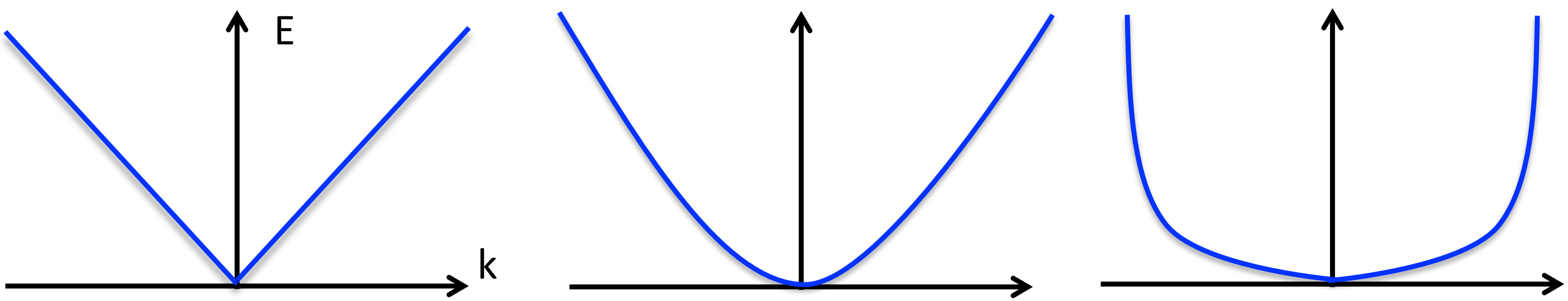
There must be low-energy degrees of freedom at finite momentum *k*

In a metal, these come from the Fermi surface. But not in a black hole…

Low-Energy Excitations in a black hole

Finite momentum excitations arise in a more exotic way. Consider dispersion relations

$E \sim k$ $\qquad$ $E \sim k^2$ $\qquad$ $E \sim k^z \quad z > 2$



relativistic $\qquad$ non-relativistic $\qquad$ unusual!

Excitations around the black hole have:

$$E \sim k^z \qquad z \to \infty$$

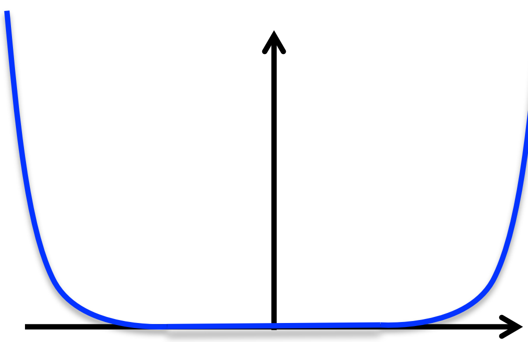
Faulkner, Liu, McGreevy, Vegh, 2009

This is known as *local criticality*.

Si et al. 2001

Low-Energy Excitations in a black hole

$E \sim k^z \qquad z \to \infty$



The DC conductivity for such a system is

$$\sigma_{\rm DC} \sim T^{-2\Delta(k_L)}$$

ugly function depending
on lattice spacing

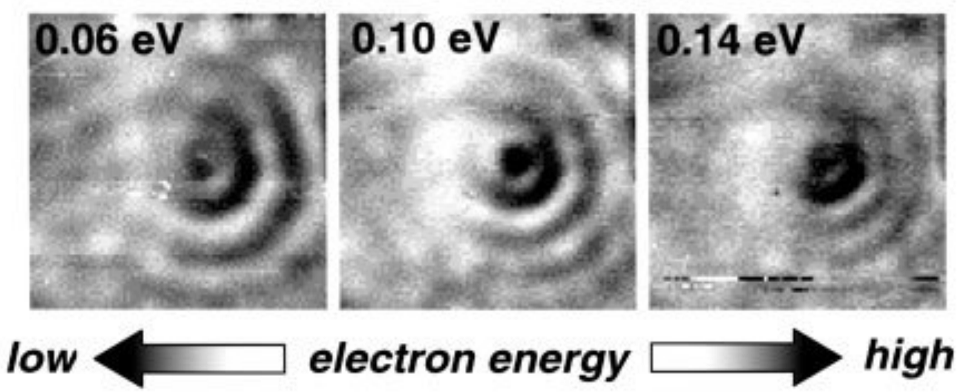
Hartnoll and Hofman, 2012

This agrees with the first term of the black hole calculation

$$\sigma_{\rm DC} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

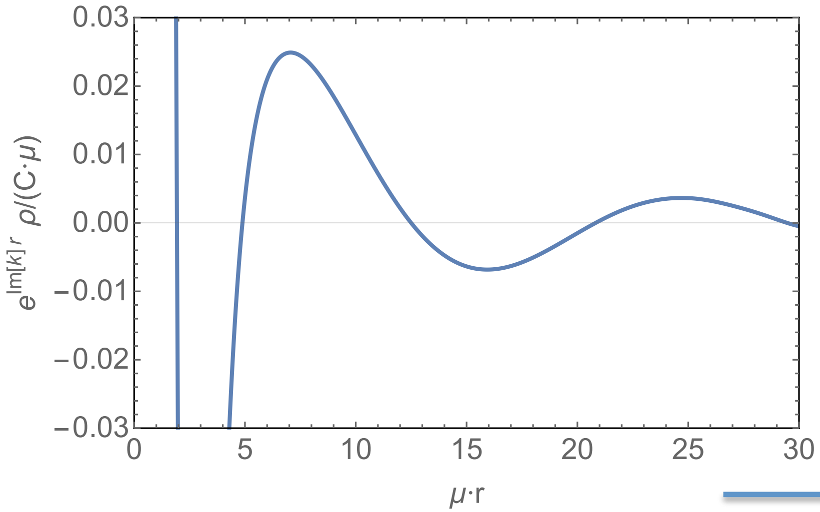
Something Fun About Black Holes

In metals, a charged impurity gives *Friedel Oscillations*



Friedel Oscillations for Black Holes

charge density
on horizon



Blake, Donos, Tong, 2014

See also Horowitz, Iqbal, Santos, Way 2014

distance from
impurity

DC Conductivity: Surprise 2

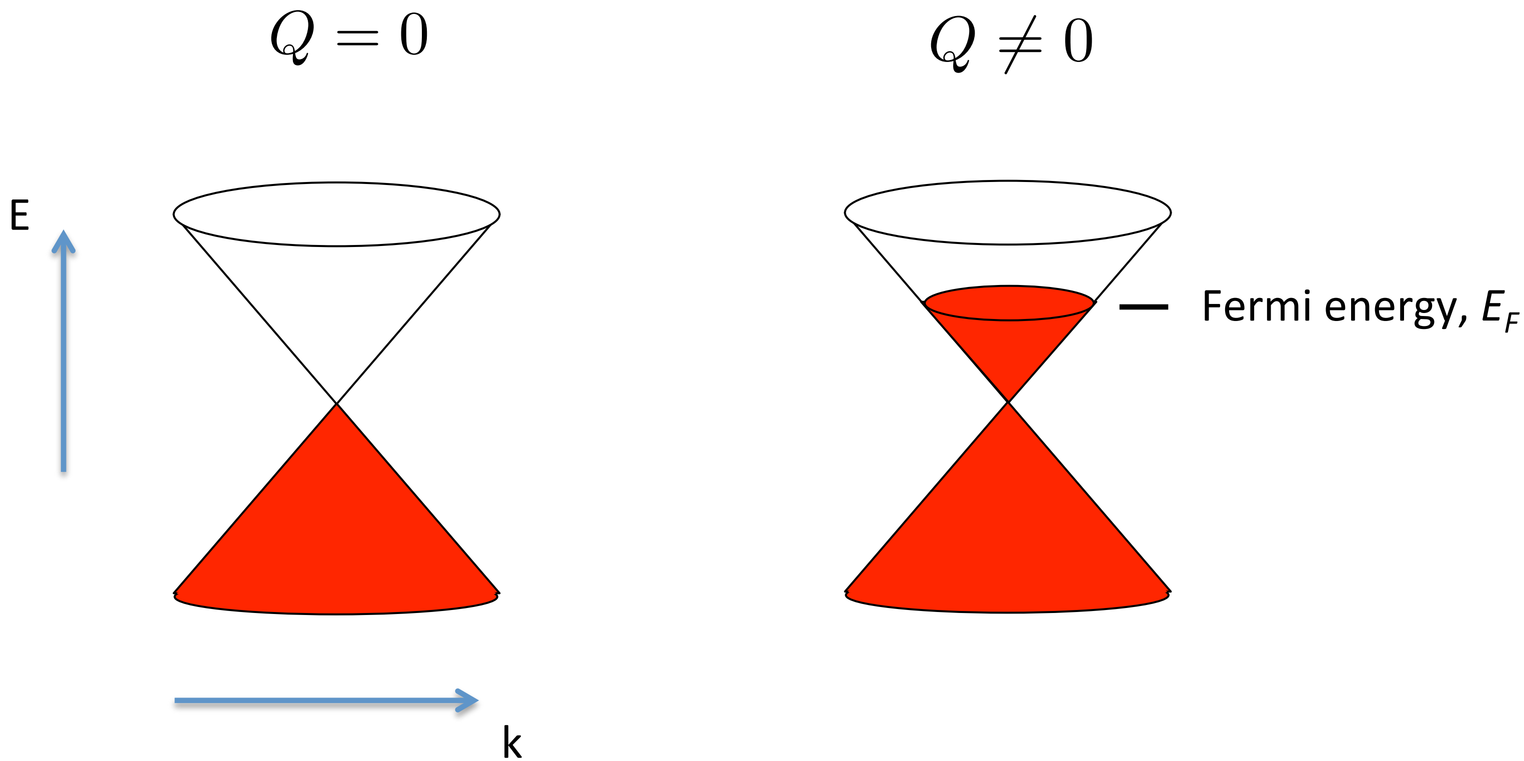
The second term also varies as a power-law in temperature

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

$$\sigma_0(T) \sim T^{\#}$$

Expected at Q=0 but surprising at finite charge density

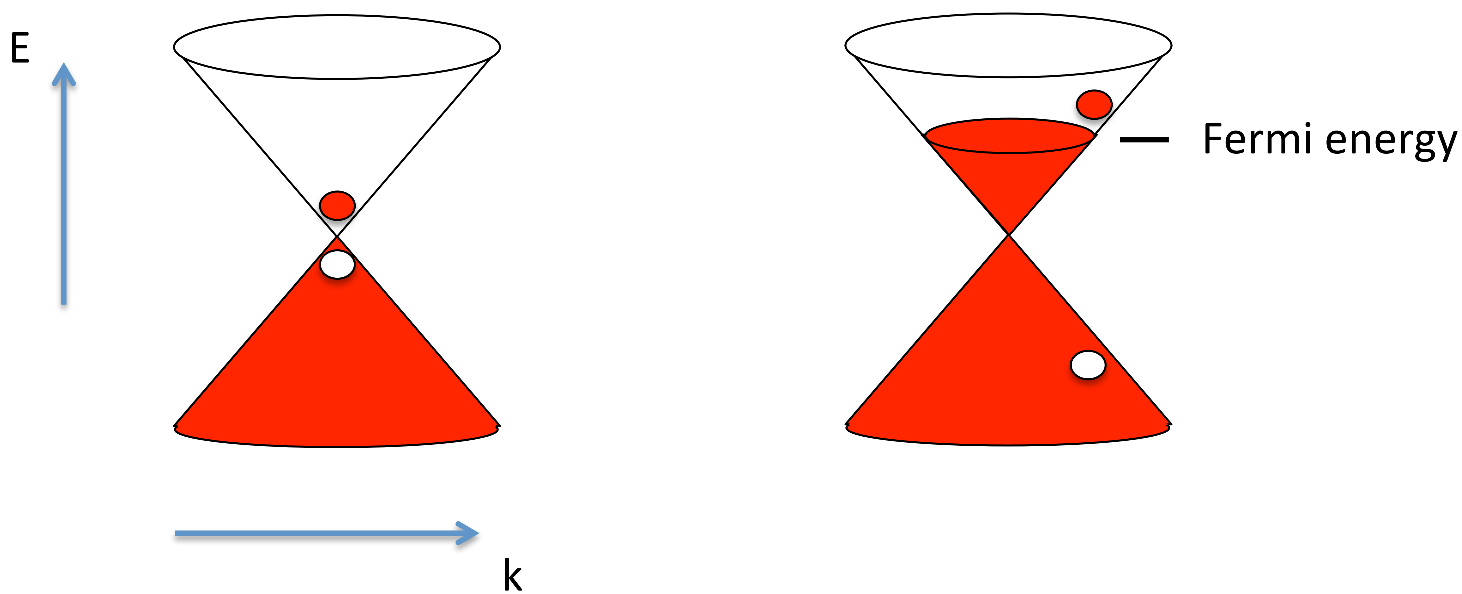
Pair Creation at Weak Coupling



c.f. graphene

Pair Creation at Weak Coupling

$Q = 0$ $\qquad\qquad$ $Q \neq 0$



$\sigma_{DC} \sim T^{d-2}$ $\qquad\qquad$ $\sigma_{DC} \sim e^{-2E_F/T}$

c.f. graphene

“Pair Creation” at Strong Coupling

$$Q \neq 0 \quad \Longrightarrow \quad \sigma_0(T) \sim T^{\#}$$

Intuition behind this remains unclear.

Is there also a lesson here for strongly coupled electron systems?

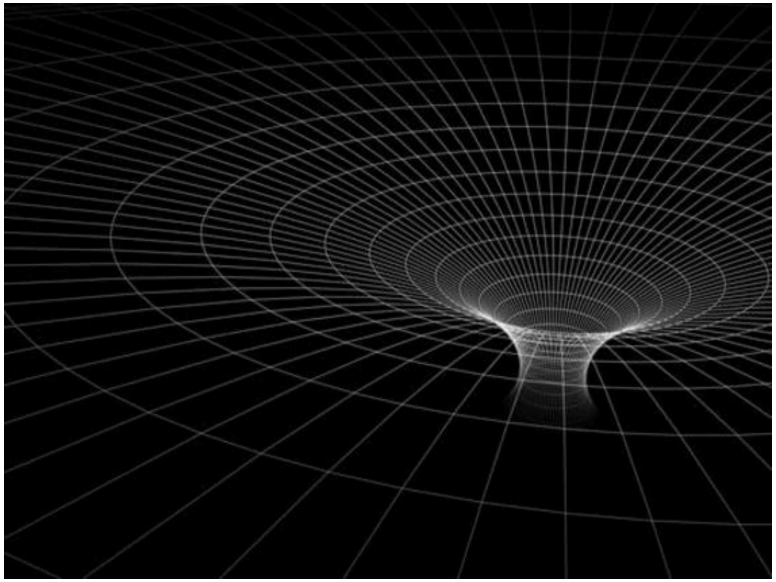
Summary of Black Hole Conductivity

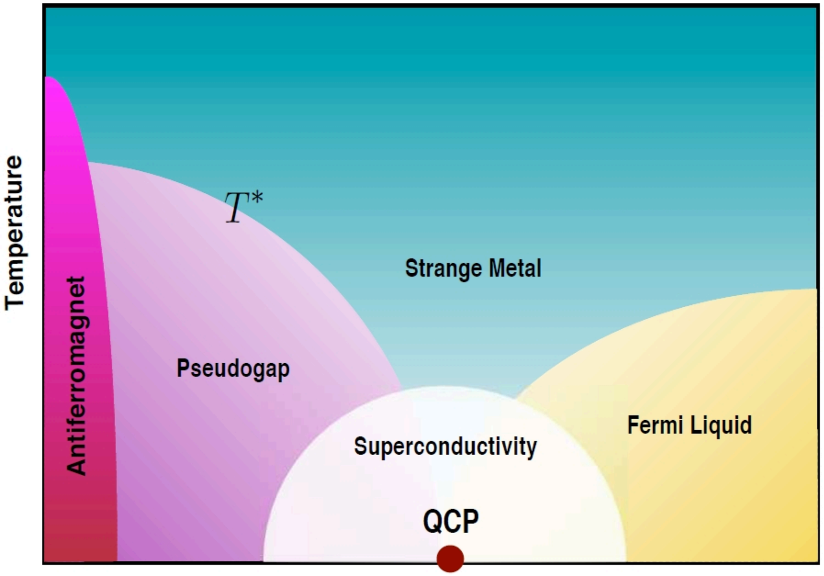
Two Processes

- Low energy modes at finite momentum
  - But not a Fermi surface

- Low energy pair creation even at finite Q

Are there any similarities?

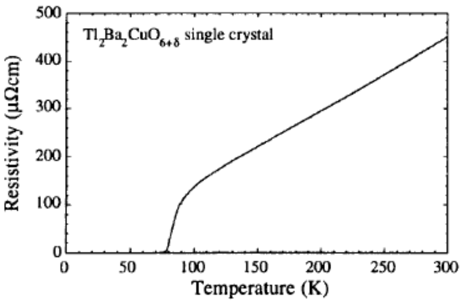


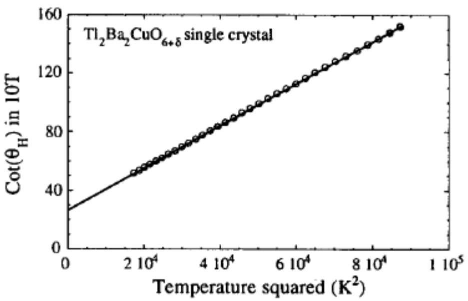


Strange Properties of Strange Metals

DC Conductivity

$$\sigma_{DC} \sim \frac{1}{T}$$





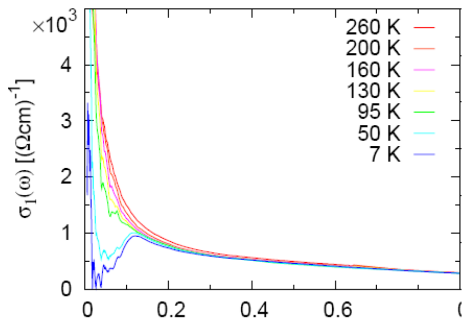
Mackenzie et al 1997

Hall Conductivity

$$\frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$

AC Conductivity

$$\sigma(\omega) \sim 1/(i\omega)^{2/3}$$



Van der Marel et al 2001

Lesson 1: Hall Angle

Drude model
(or Fermi liquid theory)

$$\sigma_{DC} \sim \frac{\sigma_{xy}}{\sigma_{xx}} \sim \tau$$

Experimental data
on strange metals

$$\sigma_{DC} \sim \frac{1}{T} \qquad \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$

Suggests two time scales at play?

Anderson, 1991
Coleman, Schofield, Tsvelik, 1996

"Over broad regions of doping, the two kinds of relaxation rates, the one for the conductivity and the one for the Hall rotation, seem to add as inverses: conductivity is proportional to $1/T + 1/T^2$. That is, it obeys an anti-Matthiessen law"

P.W. Anderson

Lesson 1: Hall Angle

Drude model
(or Fermi liquid theory)

$$\sigma_{DC} \sim \frac{\sigma_{xy}}{\sigma_{xx}} \sim \tau$$

Experimental data
on strange metals

$$\sigma_{DC} \sim \frac{1}{T} \qquad \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$

Black Holes

$$\sigma_{DC} = \sigma_0 + \frac{Q^2}{\mathcal{E} + P}\tau \qquad \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{BQ}{\mathcal{E} + P}\tau$$

Blake and Donos, 2014

If this term dominates DC transport,
we get two time scales

Lesson 2: (In)coherent Transport

There is another interpretation of these two terms*

Hartnoll, 2014

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

<u>Coherent Transport</u>

due to (almost) conserved momentum

$$\tau^{-1} \gg T$$

<u>Incoherent Transport</u>

due to charge diffusion

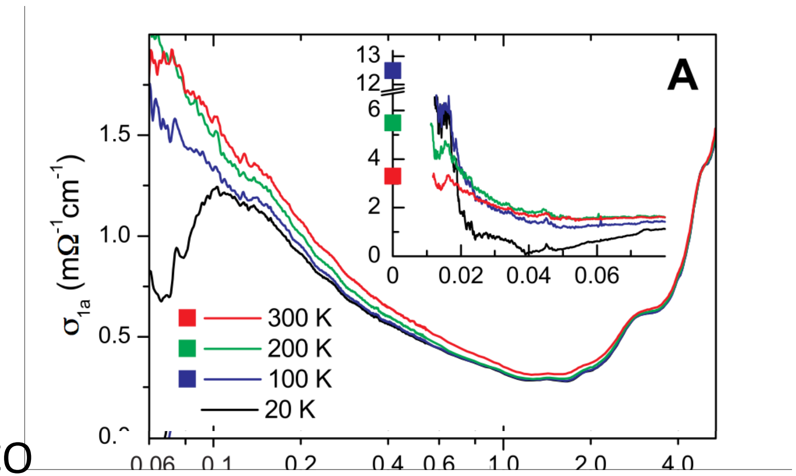
$$\tau_{\mathrm{eff}}^{-1} \sim T$$

which of these processes describes actual materials?

*actually it's slightly more complicated

Davison and Gouteraux (last week)
Blake (today)

Lesson 2: Incoherent Transport



LSCO

Takenaka et al

$$\tau^{-1} \sim \frac{k_B T}{\hbar}$$

Suggests incoherent transport

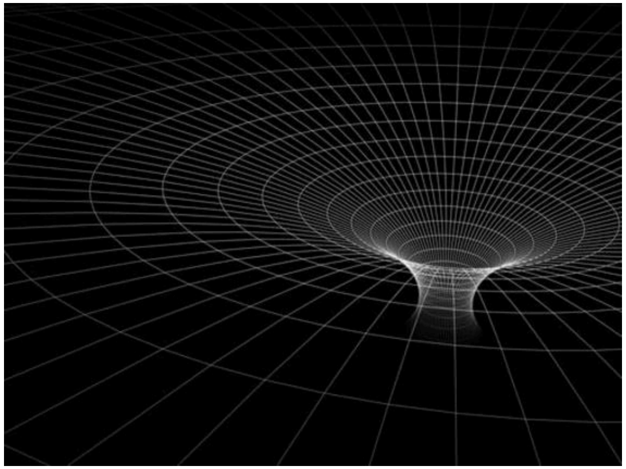
Conjecture: there is a quantum bound for diffusion $\sigma_0 \gtrsim \frac{\hbar}{k_B T}$

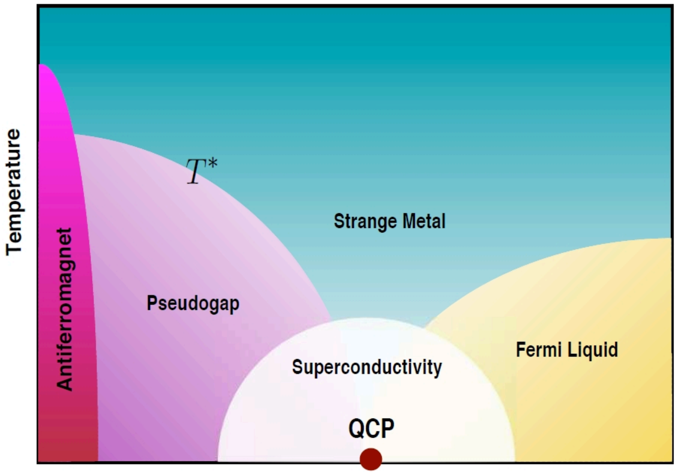
Hartnoll 2014 (see also Bruin et al; Sachdev, Zaanen)

Does this explain linear relativity? Evidence far from conclusive

Summary

- We're understanding better the conductivity properties of black holes
- Are there lessons here for strongly interacting electrons?





The End (for real now)