Bouncing Universes in Loop Quantum Cosmology

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Testing Quantum Gravity

It is generally expected that quantum gravity effects will only become important when

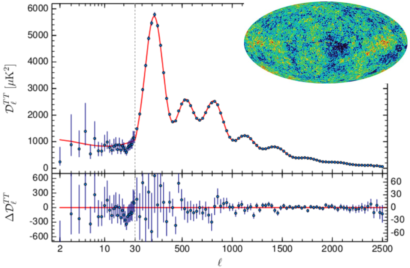
- the space-time curvature becomes very large,
- or at very small scales / very high energies.

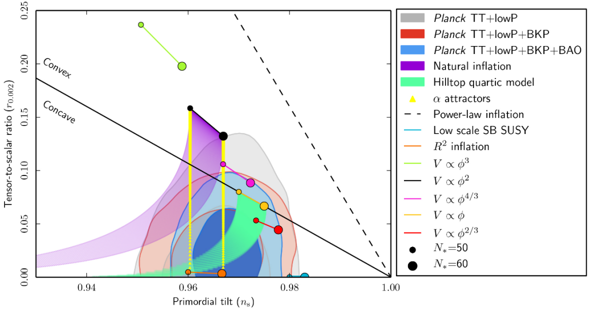
Since we cannot probe sufficiently small distances with accelerators, or even with cosmic rays, the best chance of testing any theory of quantum gravity appears to be observing regions with high space-time curvature.

The two obvious candidates are black holes and the early universe. However, since the strong gravitational field near the center of astrophysical black holes is hidden by a horizon, it seems that observations of the early universe are the best option.

The Cosmic Microwave Background

High precision observations of the cosmic microwave background (CMB) have taught us a lot about the early universe.





[Planck2015+BICEP2/Keck]

In fact, it has been possible to rule out a number of cosmological models, including some of the simplest models for inflation, as well as alternatives to inflation.

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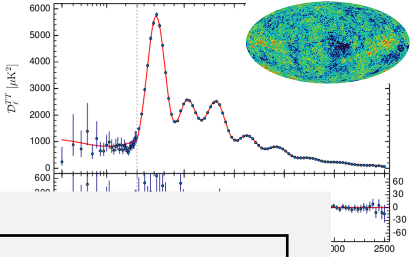
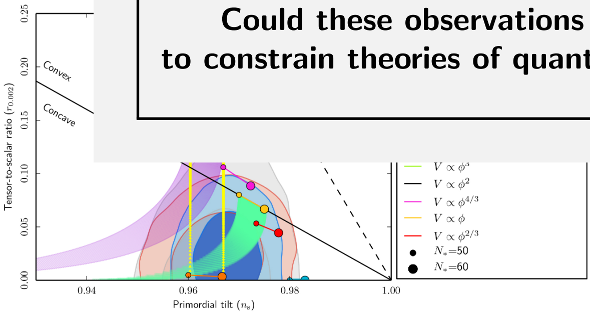


Could these observations be used to constrain theories of quantum gravity?



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Outline

Loop Quantum Cosmology

In loop quantum cosmology, we mimic the quantization techniques of loop quantum gravity (LQG) in order to study simple space-times that are of cosmological interest in the Planck regime [Bojowald; Ashtekar, Pawłowski, Singh; ...].

The key steps are the following:

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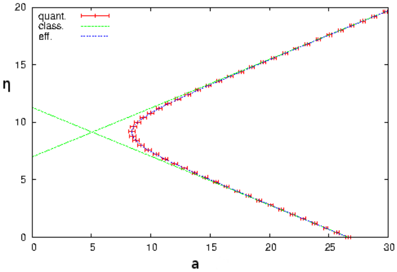
The key steps are the following:

1. Use the Hamiltonian framework and express all (symmetry-reduced) geometric quantities in terms of areas (a^2) and holonomies of the Ashtekar-Barbero $SU(2)$ connection A_a^i,
2. In particular, the field strength is written in terms of the holonomy of A_a^i around a loop of minimal area $\sim \ell_{\rm Pl}^2$,
3. Promote holonomies and areas to operators,
4. Solve (numerically) the resulting Hamiltonian constraint operator for physically interesting initial states.

LQC Dynamics

For example, the quantum dynamics of a sharply-peaked wave function representing a radiation-dominated space-time are shown here:



[Pawłowski, Pierini, WE]

An important result is that the big-bang and big-crunch singularities are generically resolved and replaced by a bounce [Ashtekar, Pawłowski, Singh; Ashtekar, Corichi, Singh, . . .].

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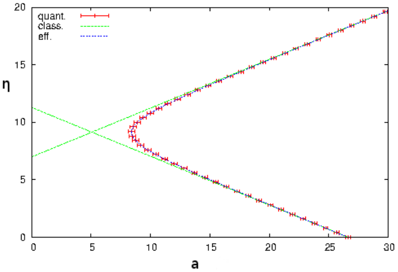

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For semi-classical states, the Friedmann equation becomes [Ashtekar, Pawłowski, Singh; Taveras, . . .]

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right).$$

The continuity equation is unchanged in LQC.

Quantum Gravity Effects in the Early Universe

Of course, the dynamics of LQC (just like general relativity) depend on the matter content. Therefore the predictions of LQC will strongly depend on what the dominant matter field (radiation, inflaton, . . .) is during the bounce.

If we assume the existence of an inflaton field with an appropriate potential, inflation naturally occurs [Bojowald, Vandersloot; Tsujikawa, Singh, Maartens; Ashtekar, Sloan; Corichi, Karami; Barrau, Linsefors, . . .] and after an inflationary epoch there might be quantum gravity effects that could be detected in the CMB [Bojowald, Calcagni, Tsujikawa; Barrau, Cailleteau, Grain, Mielczarek; Agulló, Ashtekar, Nelson; Gupt, Bonga, . . .].

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But inflation is not the only cosmological paradigm that fits together nicely with LQC. In fact, there are two alternatives to inflation that require a bounce, and that to me appear to be very nicely complementary to LQC: the **matter bounce** and **ekpyrotic** scenarios.

Alternatives to Inflation

In both the matter bounce and ekpyrotic scenarios, (nearly) scale-invariant perturbations are generated in a classical contracting Friedmann-Lemaître-Robertson-Walker (FLRW) space-time.

Then, a bounce is assumed to occur in order to link that contracting space-time with the current expanding branch of our universe, and it is hoped that the bounce doesn't ruin the scale-invariance.

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At this point, there remain many questions that must be addressed:

- Where does the bounce come from?
- Are we sure that scale-invariance is preserved across the bounce?
- And are there any effects from the bounce that could show up in the CMB today?

LQC can provide answers to these questions.

Cosmological Perturbation Theory and LQC

A convenient gauge-invariant variable to study scalar curvature perturbations is the co-moving curvature perturbation $\mathcal{R}$, whose dynamics are given by the Mukhanov-Sasaki equation

$$v_k = z\mathcal{R}_k, \qquad z = \frac{a\sqrt{\rho + P}}{c_s\, H}, \qquad v_k'' + c_s^2 k^2\, v_k - \frac{z''}{z} v_k = 0.$$

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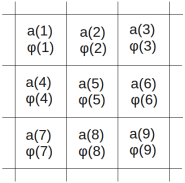
There are several approaches to cosmological perturbation theory in LQC:

- Effective equations from the anomaly-freedom approach [Bojowald, Hossain, Kagan, Shankaranarayanan; Cailleteau, Mielczarek, Barrau, Grain, Vidotto],
- Hybrid quantization [Fernández-Méndez, Mena Marugán, Olmedo; Agulló, Ashtekar, Nelson; Castello Gomar, Martín-Benito, Mena Marugán, ...],
- Separate universe approximation [WE].

In each case, the goal is to determine the LQC-corrected form of the Mukhanov-Sasaki equation.

Separate Universes in Loop Quantum Cosmology

In the separate universe framework, we make the approximation that long-wavelength perturbations can be modeled by separate Friedmann universes [Salopek, Bond; Wands, Malik, Lyth, Liddle]. This can be adapted to LQC:



Take a cubic lattice and assume that each cell is homogeneous and isotropic. (The small variations between the parameters in each cell correspond to the perturbations.) Then the usual LQC quantization of a flat FLRW space-time can be done in each cell [WE].

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The resulting LQC effective equations for scalar perturbations are expected to be valid for Fourier modes whose wavelength remains much larger than $\ell_{\rm Pl}$ [Rovelli, WE]. For these modes, the long-wavelength Mukhanov-Sasaki equation in LQC is

$$v_k'' - \frac{z''}{z} v_k = 0, \qquad z = \frac{a\sqrt{\rho + P}}{c_s H}.$$

The Matter Bounce Scenario

Vacuum fluctuations of the co-moving curvature perturbation $\mathcal{R}_k$ in a contracting matter-dominated ($P = 0$) FLRW space-time become scale-invariant. Then, if there is a bounce, matching arguments suggest the result is an expanding universe with scale-invariant perturbations [Wands; Finelli, Brandenberger].

One of the drawbacks of the matter bounce scenario is that, in the contracting branch, it typically produces scale-invariant tensor modes with a large amplitude that would violate the observational bound $r < 0.09$ (95% CL) [Planck2015+BICEP2/Keck].

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Loop quantum cosmology can provide the bounce, and it will be possible to check whether the scale-invariance is preserved across the bounce. But can quantum gravity effects during the bounce decrease the amplitude of the tensor modes?

Evolution through the Bounce

Using the LQC-corrected effective Mukhanov-Sasaki equation, it is possible to evolve $\mathcal{R}_k$ through the bounce. A non-trivial calculation shows that scale-invariance is preserved across the bounce.

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The evolution of the tensor modes across the bounce can also be calculated using the holonomy-corrected [Cailleteau, Barrau, Grain, Vidotto]

$$\mu_k'' - \frac{z_T''}{z_T}\mu_k = 0, \qquad z_T = \frac{a}{\sqrt{1 - 2\rho/\rho_c}}.$$

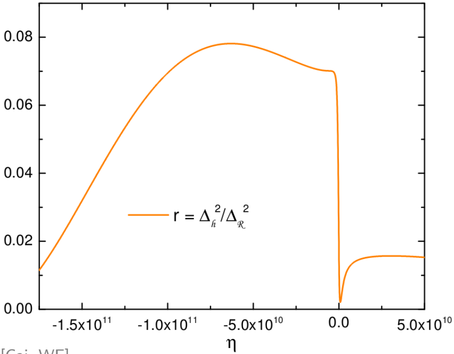
The result is that the tensor modes also remain scale-invariant, but their amplitude is significantly damped during the bounce. Thus, LQC effects can render the matter bounce scenario viable.

The exact factor the amplitude is damped by depends on the precise realization of the matter bounce.

Damping of the Tensor Modes

To be precise, the key physical ingredient is the equation of state $\omega = P/\rho$ of the matter field that dominates the background dynamics during the bounce [WE; Cai, WE].

- $\omega = 0$: the tensor modes are completely damped;
- $\omega = 1/3$: the amplitude of the tensor modes is damped by a factor of $1/4$;
- $\omega \gg 1$: the tensor modes are not significantly damped.



[Cai, WE]

This is a quantum gravity effect that, if the matter bounce is correct, can be tested!

The Matter Bounce vs Inflation

In order to test this prediction, we first have to be sure that the matter bounce scenario is correct, not inflation. One way to differentiate these two scenarios is by calculating the running of the scalar spectral index α_s,

$$\alpha_s = \frac{dn_s}{d(\ln k)}.$$

Typical realizations of the matter bounce scenario with more than one matter field predict a postive α_s [Lehners, WE] (although exceptions exist [de Haro, Cai]).

On the other hand, the currently preferred models of inflation predict a small negative running of n_s.

Thus, a measurement of α_s can allow us to differentiate between typical realizations of inflation and of the matter bounce.

The Ekpyrotic Universe

The ekpyrotic universe is another bouncing cosmology that is an alternative to inflation [Khoury, Ovrut, Steinhardt, Turok, ...]. Although it was originally motivated by string theory, it can easily be incorporated within the framework of LQC.

For ekpyrotic models where (nearly) scale-invariant $\mathcal{R}$ are generated from entropy perturbations, the perturbations post-bounce are also (nearly) scale-invariant. Note that single field ekpyrotic models, which have no entropy perturbations, are not viable.

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However, there are no imprints on the resulting spectrum coming from LQC. In particular, the damping of the tensor modes does not affect the predictions here since tensor modes are not generated in ekpyrotic models.

Conclusions

1. LQC automatically provides the bounce that matter bounce and ekpyrotic models need.
2. The LQC effective Mukhanov-Sasaki equation can be used to evolve the perturbations through the bounce and determine their post-bounce form.
3. In LQC, the amplitude of the tensor perturbations is generically damped during the bounce. This is an effect that can be used to test LQC in matter bounce scenarios.

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Observations of the CMB have already ruled out a large number of cosmological models; this process will continue. If we are lucky, the CMB may soon teach us something about quantum gravity.

Thank you for your attention!