Numerical modeling of binary neutron star mergers

Kenta Kiuchi (YITP)

Masaru Shibata (YITP), Yuichiro Sekiguchi (Toho Univ.), Koutarou Kyutoku (KEK), Kyohei Kawaguchi (AEI)
Dawn of the GW astronomy

- **O₂ run of advance LIGO.**
- **Worldwide GW detector network in 2018-2019**
- **NS-NS merger**: \(8^{+10}_{-5}\) events/yr (Kim et al. 15)
- **BH-NS merger**: 0.2-300 event/yr (Abadie et al. 10)
Role of simulation in GW physics
Figuring out a realistic picture of BH-BH, NS-NS, BH-NS mergers

Numerical relativity simulations on super-computer with a code implementing all the fundamental interactions

- Einstein eq.
- MHD
- Neutrino radiation transfer
- Nuclear EOS

The NR simulations of the BH-BH merger played an essential role for the first detection.
Science target of GWs from compact binary

Exploring the theory of gravity
▶ GW150914 is consistent with GR prediction (Abott et al. 16)

Exploring the equation of state of neutron star matter
▶ Determination of NS radius (NS tidal deformability) (Flanagan & Hinderer 08 etc.)

Revealing the central engine of SGRBs
▶ Merger hypothesis (Narayan, Paczynski, and Piran 92)

Origin of the heavy elements
▶ R-process nucleosynthesis site (Lattimer & Schramm 76)
▶ Electromagnetic counter part (Li & Paczynski 98)
Exploring a realistic picture of NS-NS mergers

(Bartos et al. 13)

Evolution path depends on the total mass and maximum mass of NSs

Science target: Measuring a tidal deformability of NS
From inspiral to late inspiral phase

Tidal deformation: NS just before the merger could be deformed by a tidal force of its companion.

Tidal deformability depends on NS constituent, i.e., EOS.

Tidal deformation

<table>
<thead>
<tr>
<th>Stiff EOS (larger R)</th>
<th>Soft EOS (small R)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="NS" /></td>
<td><img src="image2" alt="NS" /></td>
</tr>
<tr>
<td><img src="image3" alt="NS" /></td>
<td><img src="image4" alt="NS" /></td>
</tr>
</tbody>
</table>

Easily tidally deformed | Hard to be tidally deformed
How is tidal deformability imprinted in GWs?

\[ h = A(t)e^{i\Phi(t)} \]

Amplitude Phase

Tidal deformation accelerates the phase evolution

Post Newton (cf. EOB); Low cost, but inaccurate @ merger

NR; Robust, but high cost

Template bank based on NR simulations should be built
Large tidal deformability $\Rightarrow$ Rapid phase evolution
Numerical diffusion $\Rightarrow$ Rapid phase evolution

For the calibration of EOS waveforms

Requirement: $\Delta \Phi_{\text{error}} < \Delta \Phi_{\text{tidal}}$
Convergence study $\Rightarrow$ Continuum limit
Current status tidal deformability of NSs

Hotokezaka et al. 13, 15, 16, see also Dietrich et al. 17, Beruzzi et al. 15

GW phase and phase shift

Extrapolated data vs EOB

- $\Delta \Phi_{\text{error}} \approx 3 - 4$ radian

Still not sufficient for the template
⇒ Need higher res. simulation
A step towards accurate late inspiral waveform

Super computers accelerate NR waveform production.

32 TFlops month/model for “best” resolution (2.2 times higher resolution than in Hotokezaka et al.) ⇒ Systematic study is possible

Waveform production: over 100 waveforms/yr

Key ingredients

▷ Resolution study (4-5 res.)
▷ Low eccentricity initial data (e ~ 10^{-3})
▷ Long term evolution (15-16 orbits before the merger)
Phase shift of GWs $1.35M_\odot - 1.35M_\odot$

- Merger time = Time at maximum amplitude of GWs
- Phase shift is $\sim 0.4$ radian over $200$ radian
- Merger before $\sim 0.5$ ms may not be described by the analytic modeling (c.f., EOB)
Current status of NR simulations

- $\Delta x = 78$-$104$ m for the model similar to that in Hotokezaka et al. 15, 16, c.f. $\Delta x = 140$-$183$ m
- $\Delta \Phi_{\text{error}} \approx 0.3 - 0.5$ rad.
- Higher res. ($\Delta x = 64$-$86$ m) run will finish within 1 month
  $\Rightarrow \Delta \Phi_{\text{error}} \approx 0.1$ rad.?
Unequal-mass case \(1.21M_\odot - 1.51M_\odot\)

\[ R_{1.35M_\odot} = 13.69\text{km} \]

- \(\Delta \Phi_{\text{error}} \approx 0.2\text{ rad.}\)
- Other models are on going

To do list
- Take continuum limit
- Calibration EOB and construct a template bank
Exploring a realistic picture of NS-NS mergers

(Bartos et al. 13)

- MHD instability-driven viscosity drives the angular momentum transport of remnant massive NSs.
- Neutrino radiation determines the chemical composition as well as the thermodynamical properties of the ejecta.

B-field and neutrino play an essential role.

▶ Time axis

- MHD instability-driven viscosity drives the angular momentum transport of remnant massive NSs.
- Neutrino radiation determines the chemical composition as well as the thermodynamical properties of the ejecta.
B-field amplification @ the merger

Kelvin Helmholtz instability (Rasio and Shapiro 99, Price & Rosswog 05)

Minimum wave number of the unstable mode :

\[ k_{\text{min}} \propto g \frac{(\rho_1 - \rho_2)}{(v_1 - v_2)^2} \]

⇒ If \( g = 0 \), all the mode are unstable. \( \sigma \propto k \)
Magnetization of the remnant massive NS

Kelvin-Helmholtz instability (KK et al. 14, 15)

Finer resolution ($\Delta x=17.5\text{m, } N=1,024^3/2$)

$\Delta x = 17.5\text{m}$, $N = 1,024^3/2$

$\log_{10}[\rho \text{ (g/cm}^3\text{)]}$

$t - t_{\text{mrg}} = -1.05 \text{ ms}$

Small scale vortices develop rapidly $\Rightarrow$ Efficient amplification of the B-field
Magnetization of the remnant massive NS

Kelvin-Helmholtz instability (KK et al. 14, 15)

Low resolution (Δx=150m)

Small scale vortices develop rapidly ⇒ Efficient amplification of the B-field
Magnetic field amplification

Maximum field is almost virial value $\sim 10^{17} \text{G}$. The magnetic field energy is amplified by a factor of $10^6$ times at least; The averaged value of the B-fields is amplified by a factor of $10^3$ times. Fitting $E_B(t) \propto \exp(\sigma t)$ for $0 \leq t - t_{\text{mrg}} \lesssim 1[\text{ms}]$. The growth rate shows the divergence. c.f. $\sigma \propto$ wave-number for KH instability.
Saturation of magnetic-field energy

Saturation $\gtrsim 4 \times 10^{50}$ erg ($B_{\text{RMS}}=10^{16}$G)

- The back reaction turns on at 1 (2) ms for B15 (B14) run.
- The saturation energy is likely to be $\sim 10^{50}$ erg = 0.1% of the bulk kinetic energy.
- RMS value of the magnetic field strength of the HMNS is $\sim 10^{16}$G.
Long term evolution of remnant massive NS

Our strategy

▶ High res. GRMHD simulation ⇒ Evaluation of alpha viscosity

▶ Relativistic viscous simulation ⇒ Given a viscosity parameter, systematic study is doable.
Importance of MHD turbulence

EOM: \( \partial_t (\rho R^2 \Omega) + \partial_R (\rho R^2 \Omega v_A - \eta R^2 \partial_R \Omega) = 0 \)

\( \rho = \) density, \( \Omega = \) angular velocity, \( \eta = \) viscosity

- Angular momentum transfer by the viscous term.
- Energy dissipation due to the viscosity

Q. What is the “viscosity” in this system?
A. Magnetohydrodynamical turbulence;

\( q = q_{ave} + \delta q \) s.t. \( <q> = q_{ave} \) and \( <\delta q> = 0 \) where \( < \cdot > \) denotes the time average.

EOM: \( \partial_t <\rho R^2 \Omega> + \partial_R ( <\rho R^2 \Omega v_R> + R W_{R\phi} ) = 0 \)

\( W_{R\phi} = <\rho \delta v_R \delta v_\phi - B_R B_\phi / 4\pi> \) : Reynolds+Maxwell stress
High res. GRMHD simulation of remnant NS (KK et al. in prep.)

To do list: Read $\alpha$-viscosity parameter from MHD simulation data

$$\alpha = \left\langle \frac{W_{R\phi}}{P} \right\rangle$$

$W_{R\phi}$: Reynolds + Maxwell stress

Caution: neutrino viscosity and dragging effect on MRI (Guilet et al. 16); Growth rate could be suppressed if $B_{ini} \lesssim 10^{13}$G

Caveat: Resolution study is essential again because numerical diffusion kills the "turbulence".
Structure of the remnant massive NS

Space-time diagram on the orbital plane

MRI

Stable

Unstable

Core

Envelope
Magnetic field amplification
Power spectrum (merger time= 13.7ms, Δx=12.5m, N=1,400 × 1,400 × 700 & 12 levels)

Early phase: KH instability amplifies the small scale magnetic field efficiently

Late phase: Magneto Rotational Instability amplifies the B-field
\( \alpha \) - viscosity parameter

\[ 13 \leq \log_{10}[\rho \ (g \ cm^{-3})] < 14 \]

\[ \langle \alpha_{13} \rangle \geq 4 \times 10^{-3} \] for the core

\[ t_{\text{vis}} \lesssim 120 \text{ ms} \ (\langle \alpha \rangle/4 \times 10^{-3})^{-1} \]
\[ \times (\langle j \rangle/1.7 \times 10^{16} \text{ cm}^2\text{s}^{-1}) (\langle c_s \rangle/0.2c)^{-2} \]
\( \alpha \)-viscosity parameter

\[ 12 \leq \log_{10}[\rho \ (g \ cm^{-3})] < 13 \]

\[ \langle \alpha_{12} \rangle \approx 1 \times 10^{-2} \text{ for the envelope} \]
Relativistic viscous hydro. simulation (Shibata & KK 17a, b. see also Radice 17)

- Israel-Stewart formulation ⇒ Causality preserving formulation

- Systematic study is possible because of low computational cost.

Set up.

Hydro simulation of BNS merger without viscosity up to \( \sim 5 \text{ms} \) after the merger.

⇒ Switch on the viscosity \( \nu = \alpha \frac{c_s^2}{\Omega} \)
Non-axisymmetric structure of the HMNS remains.
Relativistic viscous hydro. simulation (Shibata & KK 17a, b)

\[ \alpha = 0.02 \]

Angular momentum transfer due to the viscosity
⇒ Nearly axi-symmetric configuration
Angular velocity evolution

\[ \alpha = 0.00 \quad \alpha = 0.02 \]

- Inner part quickly relaxes into an uniform rotation
  \[ t_{\text{vis}} \approx 4.4 \text{ ms} (\alpha/0.01)^{-1}(c_s/0.5 \text{ c})^{-2}(R/10 \text{ km})^2(\Omega/10^4 \text{ rad/s}) \]
- The density structure relaxes into an axi-symmetric structure.
Impact of viscosity on GWs from HMNS

Ideal hydro. case

- HMNS emits quasi periodic GWs.
- Peak frequency around 2-4 kHz depends of the EOS.

Shibata 05, Shibata & Tanguchi 09, Hotokezaka et al. 13, Bawswein et al. 12, 13, 15, Takami et al. 14, 15, 16
Impact of viscosity on GWs from HMNS

Waveforms

Amplitude

- Axisymmetric structure of the HMNS due to the angular momentum transport $\Rightarrow$ Damp of the GW amplitude
- Damping timescale is consistent with the viscous timescale
Viscous hydro. simulation of BNS merger

GW spectrum

Remnant massive NS could not be a strong GW emitter?

Caveat
No physical modeling of remnant massive NSs because of the lack of many ingredients
Summary

- Deriving a realistic picture of compact binary mergers is an urgent issue

**BNS(BH-NS) merger**

- High-precision GW forms in inspiral and late inspiral phase ⇒ Template bank

- Evolution in post merger phase (B-field, Neutrino) Remnant massive NS is strongly magnetized ⇒ Angular momentum transport due to MRI. Neutrino radiation is important for the dynamical ejecta and disk wind from the HMNS.
Exploring a realistic picture of BH-NS merger

(Bartos et al. 13)

- **Inspiral and early merger waveforms**
  - Tidal deformability of NSs

- **Post merger evolution:**
  - Mass ejection driven by neutrino, viscous, MHD
  - Modeling of the central engine of SGRBs

B-field and neutrino are irrelevant

B-field and neutrino play an essential role
$t = 0.0000 \text{ ms}$
BH-NS merger as a central engine of SGRBs

Funnel wall formation by the torus wind

Torus wind $\Rightarrow$ Coherent poloidal B-field $\Rightarrow$ Formation of the magnetosphere

The BH rotational energy is efficiently extracted as the outgoing Poynting flux $; \approx 2 \times 10^{49}$ erg/s (Blandford-Znajek 77)
R-process nucleosynthesis in BH-NS mergers
(Kyutoku et al. in prep.)

- Dynamical ejecta ⇒ Low $Y_e$
- Torus wind ⇒ High $Y_e$
Tidal deformability of NSs

Lackey et al. 12, 14

\[ P = 3.5966 \times 10^{13} \rho^{1.3569} (\rho \leq \rho_0) \]
\[ = \kappa_1 \rho^{\Gamma} (\rho \geq \rho_0) \]

\[ p_1 = \kappa_1 \rho_1^{\Gamma} (\rho_1 = 10^{14.7} \text{ gcm}^{-3}) \]

\[ \Lambda = \frac{2}{3} k_2 \left( \frac{GM_{NS}}{Rc^2} \right)^{-5} \]

NR simulation data

1 σ error circle

Error contour for Advanced LIGO with D=100Mpc, M_{BH}/M_{NS} = 2, and M_{NS}=1.35M_{☉}
Tidal deformability of NSs

Lackey et al. 12, 14

\[ P = 3.5966 \times 10^{13} \rho^{1.3569} \quad (\rho \leq \rho_0) \]
\[ = \kappa_1 \rho^\Gamma \quad (\rho \geq \rho_0) \]
\[ p_1 = \kappa_1 \rho_1^\Gamma \quad (\rho_1 = 10^{14.7} \text{ gcm}^{-3}) \]
\[ \Lambda = \frac{2}{3} k_2 \left( \frac{GM_{\text{NS}}}{R_c^2} \right)^{-5} \]

NR simulation data

1σ error circle

- Error circle of ET with D=100Mpc, \( \frac{M_{\text{BH}}}{M_{\text{NS}}} = 2 \), \( M_{\text{NS}} = 1.35M_\odot \)
- Need high-precision GW waveforms and large parameter study (\( \frac{M_{\text{BH}}}{M_{\text{NS}}} \), \( M_{\text{NS}} \), EOS, BH spin (dir., mag))