Relativistic Stars
in dRGT Massive Gravity (dRGT MG)

Nagoya University @ Japan

Masashi Yamazaki

In collaborate with
T. Katsuragawa, S. Nojiri, S. D. Odintsov

Reference:

T. Katsuragawa, S. Nojiri, S. D. Odintsov, MY,
PRD 93 124013 (2016)
Point of view for relativistic stars in dRGT massive gravity

- The Vainshtein mechanism and k-mouflage
- The dRGT massive gravity and its decoupling limit
- UV behaviors in dRGT massive gravity
- Modified TOV eqs. (including hydrostatic eq.) in the theory
- Future works and summery
A theory for massive spin-2 particle

Naively, we expect ...

The Compton length

MG behaviors
- Weaken gravitational force
- Graviton condensation etc.

Radial coordinate

\[ V \sim \frac{1}{r} e^{-mr} \]
Gravity theory in astrophysics (inside the Compton length)
The Neutron Star’s Inner Structures

Neutron Stars’ Maximum Mass

A observation result by a binary pulsar

[Demorest et al., Nature 467 (2010) 1081]
Neutron Stars in F(R) Gravity

\[ S = \int d^4x \sqrt{-\det(g)} R^{1+\epsilon} \]

\[ M/M_R (\text{km}) \]

\[ \text{SLy Nucleons model} \]

\[ \text{GR} = -0.001 \]
\[ = -0.002 \]
\[ = -0.005 \]
\[ = -0.008 \]

The maximum mass is drastically changed.

[S. Capozziello et al., (2016)]
The Purpose of This Research

Previous researches

By using modified gravity \((f(R)\) gravity), heavier neutron stars are made for passing observational constraints.

This research

In massive gravity, neutron stars’ maximum mass should be the same with GR for restoring GR behaviors.
Contents

- Point of view for relativistic stars in dRGT massive gravity
- **The Vainshtein mechanism** and k-mouflage
- The dRGT massive gravity and its decoupling limit
- **UV behaviors** in dRGT massive gravity
- Modified TOV eqs. (including *hydrostatic eq.*) in the theory
- Future work and summery
The degrees of freedom (DOF)

Gravitational Wave (GW) polarizations in MG

- (a) plus mode
- (b) cross mode
- (c) breathing mode
- (d) longitudinal mode
- (e) vector-x mode
- (f) vector-y mode

Up to 6 modes
[Chamberlin et. al. (2012)]

The mode is a ghost DOF.
In ghost-free theory, it’s absent.

The extra DOF makes a fifth force.
It should be hidden in short distances.

The vector modes don’t couple with matters.
The Screening Mechanisms

Making the scalar DOF (the breathing mode) **not effective in short distance**.

There are several ideas to achieve it.

In short distance, the environment ...

- makes the graviton **heavier**.
- suppress **the graviton-matter coupling**.
- changes **the scalar field’s effective metric** drastically.

The massive gravity’s case
The Vainshtein Mechanism

Gravity

Pressure

The tensor modes as in GR

The scalar mode (The fifth force)

Non-linear derivative couplings become relevant in short-range. The effects “screen” the scalar DOF.
The lowest energy that a new interaction with additional DOF in MG is the Planck energy. Energies that new interactions emerge in the decoupling limit (DL). Focusing the lowest interaction only. The theory is a scalar-tensor theory.
Kinetic screening in scalar-tensor theories

**Brans-Dicke term**

\[ S_{\text{k-mouflage}} = M_P^2 \int d^4x \sqrt{-g} \left[ R + \phi R + m^2 K_{NL}(\phi, \partial\phi, \partial^2\phi, \ldots) \right] + S_m[g] \]

**Non-linear derivative couplings**

\[ \sim M_P^2 \int (h \partial^2 h + \phi \partial^2 h + m^2 K_{KL} + O(h^3)) + hT \]

**Equation of Motion (EOM)**

in abstract

\[ \partial^2 (h + \phi) = \frac{T}{M_P^2}, \quad \partial^2 h + m^2 \frac{\delta K_{NL}}{\delta \phi} = 0 \]

\[ \equiv \mathcal{E}_\phi \]

\[ \partial^2 \phi + m^2 \mathcal{E}_\phi = \frac{T}{M_P^2} \]
The Effect of Non-linear Kinetic Terms

\[ \partial^2 \phi + m^2 \mathcal{E}_\phi = \frac{T}{M_P^2}, \quad \partial^2 (h + \phi) = \frac{T}{M_P^2} \]

- \( m^2 \mathcal{E}_\phi \gg \partial^2 \phi \sim 0 \Rightarrow \partial^2 h \sim \frac{T}{M_P^2} \)  
  **GR Restoring**

- \( m^2 \mathcal{E}_\phi \ll \partial^2 \phi \sim \frac{T}{M_P^2} \Rightarrow \partial^2 h \sim 0 \)
  The scalar DOF couples with matters strongly

Non-linear derivative couplings dominate

\[ \downarrow \]
Non-linear derivative couplings screen the scalar DOF
- Point of view for relativistic stars in dRGT massive gravity
- The Vainshtein mechanism and k-mouflage
- The dRGT massive gravity and its decoupling limit
- UV behaviors in dRGT massive gravity
- Modified TOV eqs. (including hydrostatic eq.) in the theory
- Future work and summery
- It is written by a symmetric Lorentz tensor field.
- Ghosts by higher derivative (the Ostrogradsky ghosts) are absent.
- In Minkowski spacetime

The linearized Einstein-Hilbert action

\[ \mathcal{L} = -\frac{M_\text{Pl}^2}{4} h^{\mu\nu} \hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} \]  
(The normalization is chosen as usual.)

\[ \hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} = -\frac{1}{2} \left( \Box h_{\mu\nu} - 2 \partial_{(\mu} \partial_{\alpha} h_{\nu)}^{\alpha} + \partial_{\mu} \partial_{\nu} h - \eta_{\mu\nu} (\Box h - \partial_{\alpha} \partial_{\beta} h^{\alpha\beta}) \right) \]
The nonlinear completion that the massless spin-2 field couples matters is the Einstein-Hilbert action.

\[ \mathcal{L} = -\frac{M_P^2}{4} h^{\mu\nu} \hat{\mathcal{E}}_{\mu\nu}^\alpha{}^\beta h_{\alpha\beta} \]

\[ \mathcal{L} = \frac{M_P^2}{2} \sqrt{-g} R[g] \]

- Self-interaction consistency POV [Deser 1970]
- The diffeomorphism’s nonlinear completion POV [Wald 1986]
From the condition that \textit{ghosts by higher derivatives are absent},

\[ \mathcal{L}_{\text{mass}} = -\frac{1}{8} m^2 M_P^2 (h_{\mu\nu}^2 - h^2) \]

Nonlinear completion by $g_{\mu\nu}$ is ...
(non-derivative way)

\[ g_{\mu\nu} g^{\mu\nu} = D, \quad \det(g) \ ? \]

The trivial quantity or the cosmological constant term

\textbf{Another rank-2 symmetric tensor is needed!}
The Theory Space [Arkani-Hamed et al., 2003]

GC → General Coordinate Invariance Symmetry

$$(\mathcal{M}_4, g_{\mu\nu})$$  \hspace{1cm} $$(\mathcal{M}'_4, f_{AB})$$

These are called as the “link fields” or the Stuckelberg fields. These are needed for the pullback.

\[
f_{\mu\nu}(x) = \partial_\mu X^A(x) \partial_\nu X^B(x) f_{AB}(X(x))
\]
There are some candidates for NLFP.

\[ H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu} \]

\[ S_{\text{int}} = -\frac{1}{8} m^2 M^2_P \int d^4x \sqrt{-f} H_{\mu\nu} H_{\sigma\tau} (f^{\mu\sigma} f^{\nu\tau} - f_{\mu\nu} f_{\sigma\tau}) \]

These are just inverse matrixes of these metric.

\[ S_{\text{int}} = -\frac{1}{8} m^2 M^2_P \int d^4x \sqrt{-g} H_{\mu\nu} H_{\sigma\tau} (g^{\mu\sigma} g^{\nu\tau} - g_{\mu\nu} g_{\sigma\tau}) \]

Taking decoupling limit

\[ \text{The Gallileon Theories} \]
The Goldstone Expansion

\[ X^A(x) = X_0^A(x) + \pi^A(x) \]

The identity map

\[ X_0^A(x) \equiv \delta^A_\mu x^\mu \]

The scalar-vector decomposition

\[ \pi^A(x) = \delta^A_\mu (A^\mu(x) + \eta^{\mu\nu} \partial_\nu \phi) \]

\[ H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu} \]

\[ = h_{\mu\nu} - (\partial_\mu A_\nu + \partial_\nu A_\mu) - 2\partial_\mu \partial_\nu \phi - \partial_\mu A_\sigma \partial_\nu A^\sigma \]

\[ - \partial_\mu \partial_\sigma \phi \partial_\nu \partial^\sigma \phi - (\partial_\nu A^\sigma \partial_\mu \partial_\sigma \phi + \partial_\mu A^\sigma \partial_\nu \partial_\sigma \phi) \]
The canonical normalizations for fields
\[ \hat{h}_{\mu\nu} = M_P h_{\mu\nu}, \quad \tilde{A}^\mu = M_P m A^\mu, \quad \tilde{\phi} = M_P m^2 \phi \]

In the limit of \( M_P \to \infty, m \to 0 \) (\( m \ll M_P \))

The strongest interaction
or
The lowest energy scale interaction

\[ \mathcal{L}_{\text{int}} \supset m^2 M_P^2 \left( \frac{\partial_\mu \partial_\nu \tilde{\phi}}{m^2 M_P} \right) \left( \frac{\partial_\mu \partial_\sigma \tilde{\phi}}{m^2 M_P} \frac{\partial_\nu \partial_\sigma \tilde{\phi}}{m^2 M_P} \right) \]

\[ \sim \frac{1}{(m^4 M_P)^{1/5 \times 5}} \left( \partial^2 \tilde{\phi} \right)^3 \]

\[ \equiv \Lambda_5 \]

This should be constant for DL to remain this coupling.

It leads the Ostrogradsky ghost. (especially BD ghost)
The dRGT Massive Gravity eliminates the Ostrogradsky ghost that emerge in nonlinear.

\[ S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( R - m^2 \sum_{k=0}^{k=4} \beta_k e_k \left( \sqrt{g^{-1}f} \right) \right) \]

\[ e_k(X) = \frac{1}{k!} X^{I_1} [I_1 \cdots X^{I_k} I_k] \]

cf. \( e_0(X) = 1 \), \( e_1(X) = \text{Tr}(X) \), \( e_2(X) = \frac{1}{2} \left( \text{Tr}(X)^2 - \text{Tr}(X^2) \right) \), \cdots \]

Although, it remains not ghost-like scalar DOF (the fifth force).
The Condition for Flat Sol. In Vacuum

\[ G_{\mu\nu} + m_0^2 I_{\mu\nu} (\sqrt{g^{-1}f}, \beta_n) = \kappa^2 T_{\mu\nu} \]

\[ g_{\mu\nu} = f_{\mu\nu} = \eta_{\mu\nu} \Rightarrow \sqrt{g^{-1}f} = 1 \]

\[ \Rightarrow I_{\mu\nu} \left( \sqrt{g^{-1}f} \right) = (\beta_0 + 3\beta_1 + 3\beta_2 + \beta_3) \eta_{\mu\nu}. \]

\[ G_{\mu\nu} = T_{\mu\nu} = 0 \Rightarrow \beta_0 + 3\beta_1 + 3\beta_2 + \beta_3 = 0. \]

There is a relationship between free parameters.
The special choices of interaction terms rise the lowest strong coupling scale.

\[
m^2 M_P^2 \frac{\hat{h}}{M_P} \left( \frac{\partial^2 \tilde{\phi}}{m^2 M_P} \right)^2 = \frac{1}{(m^2 M_P)^{1/3} \times 3} \hat{h} \left( \partial^2 \tilde{\phi} \right)^2
\]

This is fixed for DL.

The multiplication of the power of

\[
\frac{\partial^2 \tilde{\phi}}{m^2 M_P} = \frac{1}{(m^2 M_P)^{1/3} \times 3} \partial^2 \tilde{\phi} \equiv \Lambda_3
\]

for the above term is also remained in the DL.
If we choose \( \tilde{\alpha} = \tilde{\beta} = 0 \) \( \Rightarrow \) \( \beta_2 = \beta_3 = 0 \) \( \Rightarrow \) \( \beta_0 + 3\beta_1 = 0 \) the model becomes \text{trivial theory in DL.}

This is called as \text{the minimal model.}
The scalar and tensor interactions in minimal model and static and spherical symmetric (SSS) configurations are absent not only in DL but also until the Planck energy. [S. Renaux-Petel (2014)]

There is possibility that the system does not have the Vainshtein mechanism because of the absence of higher derivative couplings.

The minimal model in non SSS configurations can have the scalar and tensor interactions. [S. Renaux-Petel (2014)]

The Vainshtein mechanism in the minimal model should be checked by making solutions
Contents

- Point of view for relativistic stars in dRGT massive gravity
- The Vainshtein mechanism and k-mouflage
- The dRGT massive gravity and its decoupling limit
- UV behaviors in dRGT massive gravity
- Modified TOV eqs. (including hydrostatic eq.) in the theory
- Future work and summery
The MG models in short distances may be influenced by quantum corrections.

- **The Gallileon non-renormalization thm.** protects the graviton mass at 1-loop order. *(Technically natural graviton mass)*

- **The destabilization of the potential is suppressed** at 1-loop order.

  - Matter loop structures are the same with GR for 1-loop order.

  - Graviton loops leads new ops., but it’s suppressed.

[C. de Rham, L. Heisenberga, and R. H. Ribeiroa (2013)]

The effective field theory (EFT) description in DL can also be accepted in beyond DL.
\[ \mathcal{L}(\phi) = \sum_{i=1}^{5} c_i \mathcal{L}_i, \quad \text{The Gallileon Lagrangian} \]

\[ \mathcal{L}_1 = \phi, \quad \mathcal{L}_2 = (\partial \phi)^2, \quad \mathcal{L}_3 = \partial^2 \phi (\partial \phi)^2, \]
\[ \mathcal{L}_4 = (\partial \phi)^2 \left[ (\partial^2 \phi)^2 - (\partial_\mu \partial_\nu \phi)^2 \right], \]
\[ \mathcal{L}_5 = (\partial \phi)^2 \left[ (\partial^2 \phi)^3 + 2(\partial_\mu \partial_\nu \phi)^3 - \partial^2 \phi (\partial_\mu \partial_\nu \phi)^2 \right] \]

The coefficients for linear combination are protected w.r.t. quantum corrections.

The Gallileon theories can be treated by tree-level only.
The interactions $h^2(\partial^2 \pi)^n$ in full theory are suppressed by the Planck mass for DL interactions.

\[ \delta m^2 \lesssim m^2 \left( \frac{m}{M_{Pl}} \right)^2 \]
Contents

- Point of view for relativistic stars in dRGT massive gravity
- **The Vainshtein mechanism** and k-mouflage
- The dRGT massive gravity and its decoupling limit
- **UV behaviors** in dRGT massive gravity
- Modified TOV eqs. (including *hydrostatic eq.*) in the theory
- Future work and summery
The Static and Spherical Symmetric (SSS) configuration

\[
\begin{align*}
    g_{\mu\nu} \, dx^\mu \, dx^\nu &= -A(\chi) \, dt^2 + B(\chi) \, d\chi^2 + D(\chi)^2 \, d\Omega^2 \\
    f_{\mu\nu} \, dx^\mu \, dx^\nu &= -dt^2 + d\chi^2 + \chi^2 \, d\Omega^2
\end{align*}
\]

The reference metric is fixed as flat.

\[
D(\chi)^2 \equiv r^2 \Rightarrow \chi \equiv \chi(r)
\]

\[
\begin{align*}
    A(\chi) &\equiv e^{2\nu(r)} \\
    B(\chi) \chi'(r)^2 &\equiv e^{2\lambda(r)} = \left(1 - \frac{2GM(r)}{r}\right)^{-1}
\end{align*}
\]

\[
\begin{align*}
    g_{\mu\nu} \, dx^\mu \, dx^\nu &= -e^{2\nu(r)} \, dt^2 + \left(1 - \frac{2GM(r)}{r}\right)^{-1} \, dr^2 + r^2 \, d\Omega^2 \\
    f_{\mu\nu} \, dx^\mu \, dx^\nu &= -dt^2 + (\chi'(r))^2 \, dr^2 + \chi(r)^2 \, d\Omega^2
\end{align*}
\]

This function is a gauge function.
The additional terms come from mass and interaction terms

\[ G_{\mu\nu} + m_0^2 I_{\mu\nu}(\sqrt{g^{-1}} f, \beta_n) = \kappa^2 T_{\mu\nu} \]

\[ \nabla_\mu G^{\mu\nu} = \nabla_\mu T^{\nu\mu} = 0 \]

\[ \nabla_\mu I^{\mu\nu} = 0 \quad (m_0 \neq 0) \]

The equations in SSS becomes a constraint equation that determine the gauge function
The EOM

Assuming perfect fluid as matter

\[ T^\mu_\nu = (-\rho(r), p(r), p(r), p(r)), \]

\[ \nabla_\mu T^\mu_\nu = 0 \Rightarrow \nu'(r) = -\frac{p'(r)}{p(r) + \rho(r)}. \]

Substitute this to EOM

\[ GM' = 4\pi G\rho r^2 + \frac{m_0^2}{2} r^2 \int^t t \]

Modified mass conservation

\[ p' = -\frac{(4\pi G\rho r^3 + GM - m_0^2 r^3 I^r_r)(p + \rho)}{r(r - 2GM)} \]

Modified hydrostatic equation
\[ \nabla_{\mu} I^{\mu} = -\frac{\lambda'}{r^2} e^{-\lambda-\nu} \left\{ (\beta_0 + 3(\beta_1 + \beta_2)) \chi (2 - 2e^\lambda + \chi \nu' e^\nu) \right\} \]

These are set to 0 in the minimal model.

\[ -\beta_1 \left[ \left( \frac{2}{r} + \nu' \right) e^{-\lambda} - \frac{2}{r} \right] r^2 e^{\lambda+\nu} \]

\[ -2\beta_2 \left[ r(1 - e^\lambda) + \chi (1 - e^\lambda + r\nu') e^\nu \right] \}

= 0

Especially, this determines the mass parameter algebraically in the minimal model.

\[ -\frac{p'}{p + \rho} = \nu' = \frac{G}{r^2} \frac{4\pi pr^3 + M - \tilde{m}_0^2 r^3 \tilde{I}^r_r}{1 - \frac{2GM}{r}} \]

2\text{nd order for } \chi
The Equations and DOF in General Case

\[(\nu, M, \chi; \rho, p)\]

\[\begin{align*}
G_{\mu\nu} + m_0^2 I_{\mu\nu} &= \kappa^2 T_{\mu\nu} \\
\nabla_\mu T^{\mu\nu} &= 0 = \nabla_\mu I^{\mu\nu}
\end{align*}\]

1 nontrivial eq. \(\rightarrow\) Determine \(\chi\)

EOS

\[p = p(\rho(r))\]

\[\begin{align*}
G_{\mu\nu} + m_0^2 I_{\mu\nu} &= \kappa^2 T_{\mu\nu} \\
\nabla_\mu I^{\mu\nu} &= 0
\end{align*}\]

2 independent eqs. \(\rightarrow\) Determine \(M\) and \(p\)
The EOM in minimal model

The mass parameter $m(r)$ can be eliminated from EOM by the new constraint.

$$q(r) \equiv \frac{p'(r)}{p(r) + \tilde{\rho}(r)}, \quad m(r) = \frac{1}{2} r - \frac{1}{2} r \left(1 - \frac{1}{2} q(r) r\right)^{-2},$$

$$8\pi pq + 8\pi p' - 16\pi \frac{p''}{q} + 16\pi \frac{p'q'}{q^2} = \frac{q}{r^2} + \frac{2}{r^3} + 3\alpha^2 (r_g M_\odot)^2 q$$

Dimensionless graviton mass

$$\alpha \equiv \frac{m}{M_\odot} = \frac{\sqrt{\Lambda}}{M_\odot} \sim 10^{-99}$$

Solving 3rd-order ODE for $\rho(r)$

(EOS is used.)
The Boundary Condition

The GR case

The same radius

for the same central density

The minimal model in dRGT MG (not asymptotic flat)

Considering differences for the gravitational effects inside the stars
Solving 3\textsuperscript{rd}-order ODE for $\rho(r)$

- Solving from the center to outer

- Obtaining the radius
  (The pressure in there is equal to 0.)

- Improving $\rho''(r = 0)$

- Is the radius the same with GR?

The center of the star
(Initial conditions)
\[
\begin{align*}
\rho(r = 0) &= \rho_c \\
P'(r = 0) &= 0
\end{align*}
\]

The solution is consistent with BC.

NO!

NO!

YES!
Results 1: A Quark Star Case

by using MIT bag model

Decreasing the maximum mass

The minimal model of dRGT MG

[PRD 93 (2016) 124013]
Results 2: A Traditional Star Case

by using SLy model

Decreasing the maximum mass

The minimal model of dRGT MG

[PRD 93 (2016) 124013]
The minimal model in the dRGT MG has **smaller neutron stars’ maximum mass** than these in GR.

It is caused by **the algebraic equation** for mass parameter $m(r)$ that is proper to minimal model.

The minimal model in the dRGT MG **does not have screening mechanisms** in these case.

In the point of view of observation, **the GR solutions is preferred** to the minimal model’s solutions. Because the large neutron star’s mass is observed.
Point of view for relativistic stars in dRGT massive gravity

The Vainshtein mechanism and k-mouflage

The dRGT massive gravity and its decoupling limit

UV behaviors in dRGT massive gravity

Modified TOV eqs. (including hydrostatic eq.) in the theory

Future work and summery
Future Directions

- Considering **models around the minimal model**
  - How is the parameter choices crucial?

- Adding perturbations for the breaking SSS configuration
  - Is the Vainshtein mechanism restored by this?
  - **How strong** are perturbations needed?
    - It might be **uncommon strength in the solar-system**

- (Considering non-minimal models)
Summery

- **Relativistic stars’ maximum mass** is a good indicator for verifying the Vainshtein mechanism.

- **The Vainshtein mechanism** is caused by non-linear kinetic terms. (cf. k-mouflage)

- The decoupling limit of dRGT MG has **the Gallileon-type interactions** (non-linear kinetic terms).

- The minimal model is the special model that **has not the non-linear derivative coupling** below the Planck scale.

- The dRGT MG isn’t influenced by 1-loop quantum corrections. It can be treated as classical theory effectively.

- Modified TOV eqs. contains **the new constrains for fixing the gauge function**.

- The minimal model’s maximum mass is smaller than GR’s maximum mass.