Evolution of spherical self-gravitating collisionless systems in phase-space

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Through a phase-space analysis based both on a spherical Vlasov solver, a shell code and a N-body code, we revisit the evolution of collisionless self-gravitating spherical systems with initial power-law density profiles and Gaussian velocity dispersion. We are able, for the first time, to show the clear separation between two or three well known dynamical phases: (i) the establishment of a spherical quasi-steady state through a violent relaxation phase during which the phase-space density displays a smooth spiral structure presenting a morphology consistent with predictions from self-similar dynamics, (ii) a quasi-steady state phase during which radial instabilities can take place at small scales and destroy the spiral structure but do not change quantitatively the properties of the phase-space distribution at the coarse grained level and (iii) relaxation to non spherical state due to radial orbit instabilities for cold cases with steep initial density profiles.

Simulations

Initial phase-space density: $f(r,v) = \frac{\rho_0(r)}{(2\pi\sigma_r^2)^{3/2}} \exp\left(-\frac{1}{2}\frac{v^2}{\sigma_r^2}\right), r \leq R_0$

$\rho_0(r) \propto r^n$ with n=0,-0.5,-1,-1.5, initial virial ratio $\eta = \frac{2T}{|W|}$ of 0.5 ("warm" case) and 0.1 ("cool" case).

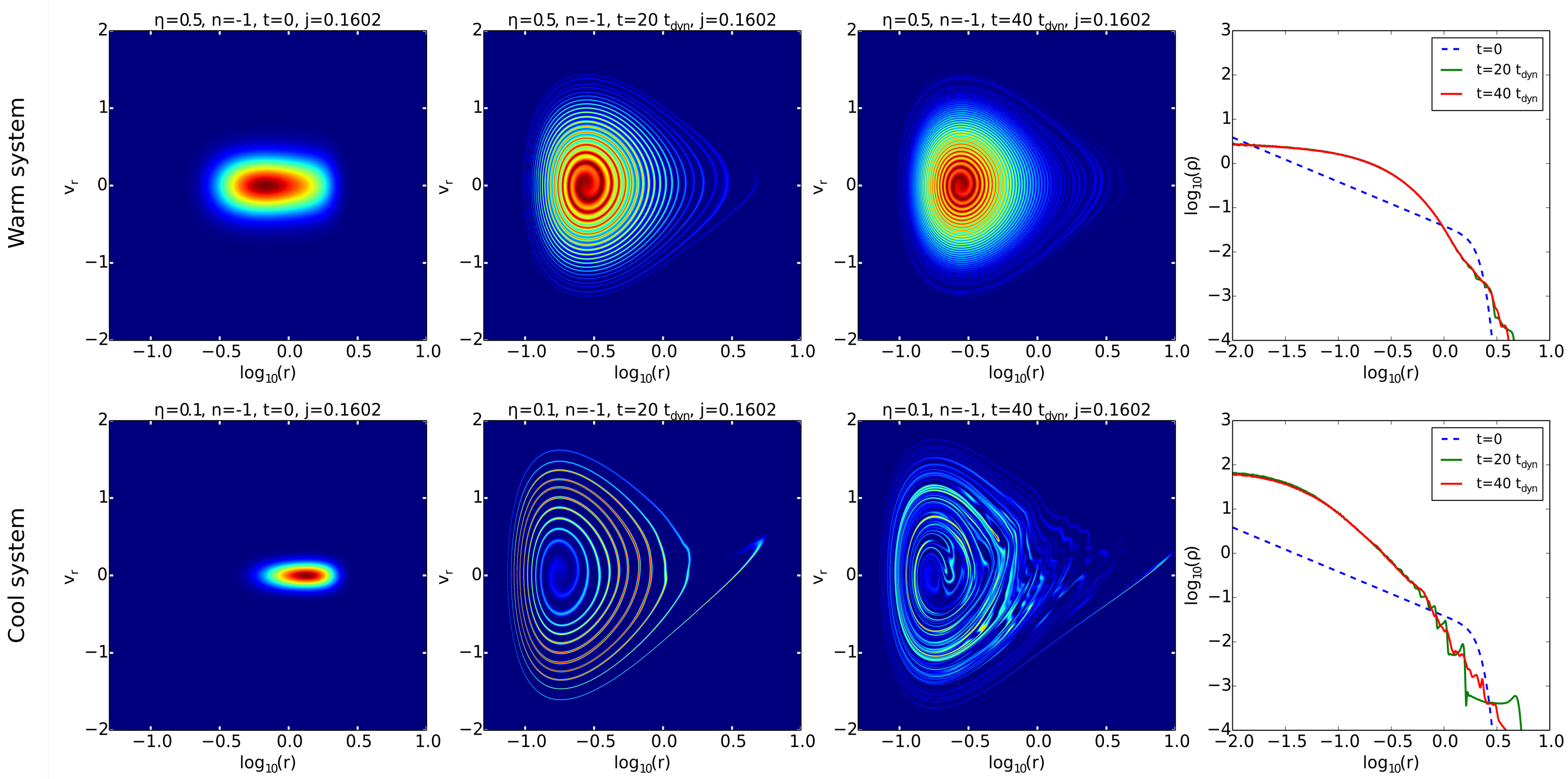
Simulations on a 2048x2048x128 grid in $\ln(r), v_r, \sqrt{j}$, using the semi-lagrangian Vlasove code by T. Sousbie (Colombi et al 2015).

Vlasov-Poisson in spherical symmetry

$$\begin{cases} \frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \left(\frac{j^2}{r^3} - \frac{\partial \phi}{\partial r}\right)\frac{\partial f}{\partial v_r} = 0 \\ \triangle\, \phi = 4\pi G \rho \end{cases}$$

j : angular momentum (conserved), gravitational force $-\frac{\partial \phi}{\partial r} = -\frac{GM(<r)}{r^2}$

Fine phase-space structure and coarse-grained properties



The systems first experience a quiescent mixing phase during which the phase-space density in a slice in angular momentum is a smooth spiral.

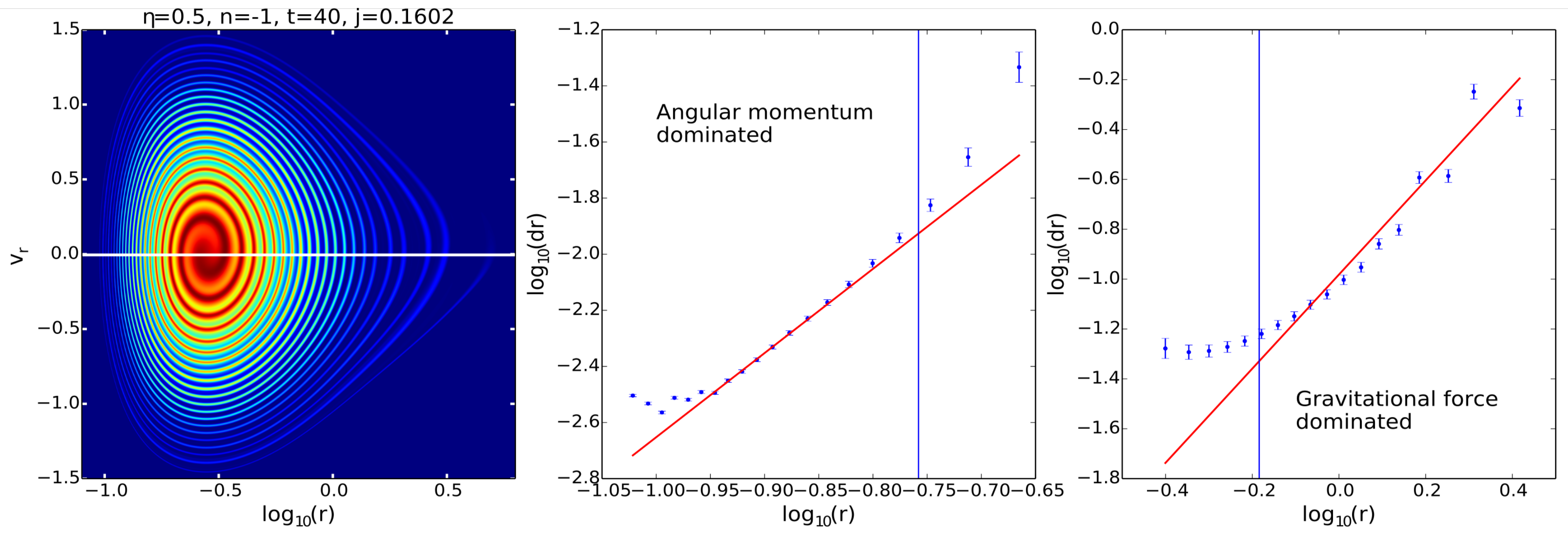
The simulations then all display some level of instability, particularly in the "cool" cases, where some resonant modes destroy the spiral structure *without affecting the coarse-grained density*.

Self-similarity properties of the spiral

Analytic prediction (Alard 2013) for interfold distance as a function of fold position, using self-similar solutions of the Vlasov-Poisson system:

$\mathrm{d}r \propto r_0^{\frac{3-\gamma}{2}}$ for a power-law force $F = \left(\frac{j^2}{r^3} - \frac{\partial\phi}{\partial r}\right) \propto r^\gamma$

At fixed angular momentum, the repulsive force dominates at low radii: $\gamma = -3 \Rightarrow \mathrm{d}r \propto r_0^3$, while the gravitational force dominates at large radii.



Comparison with N-body codes

For n=-1 and n=-1.5, we find the "cool" systems develop a radial orbit instability in simulations with Gadget-2 (Springel et al 2005), which makes the central parts of the density profiles deviate from the spherical codes (Vlasolve, or a N-body spherical shells code following the algorithm of Hénon et al 1964).