Cosmic Inflation in Hybrid Metric-Palatini Gravity

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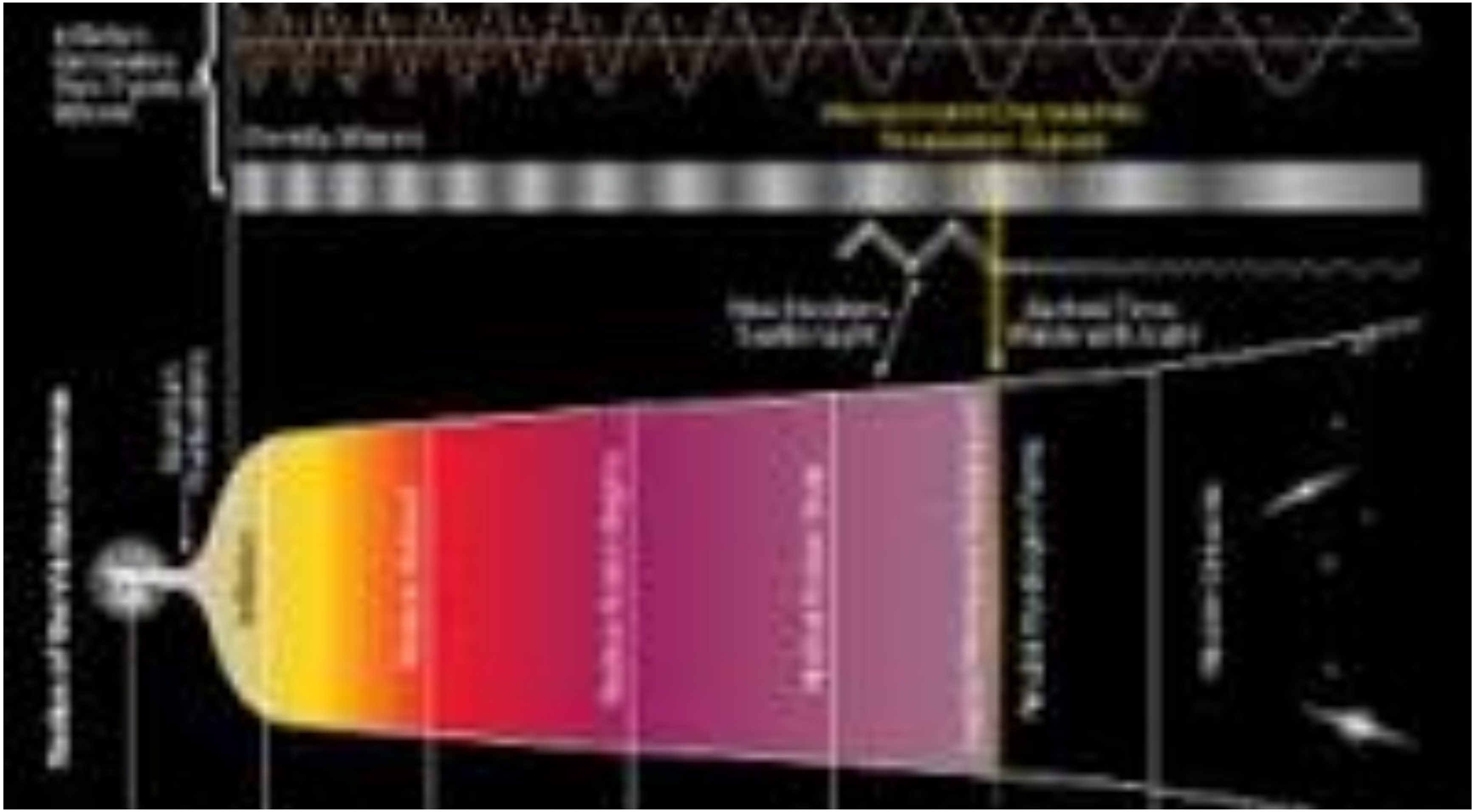
Abstract

Hybrid metric-Palatini gravity theory is a newly proposed theory, whose action depend linearly on the metric scalar curvature and non-linearly on the arbitrary function of Palatine scalar curvature. The fascinating property of this theory is to maintain all the positive results of GR at solar system and compact object scale as well as to add explanation to the recent cosmological observations. Looking at the recent popularity of this theory, we study the issue of cosmic inflation in this theory. For this purpose, we calculate the slow-roll parameter, scalar-to-tensor ratio and spectral index and then investigate their behavior graphically to check the growth of the universe. We also check our findings with the recent observational data.

Cosmic Inflation



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COSMIC INFLATION EXPLAINED

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Hybrid Metric Palatini Gravity

Field Equations

$f(X)$ theory is the hybrid combination of Palatini $f(R)$ gravity and Einstein-Hilbert action. The hybrid action is defined as

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,[R + f(\mathcal{R})]\,, \quad (1)$$

where R is the Ricci scalar and $\mathcal{R}$ is the Palatini scalar curvature by $f(R)$ theory. Variation of this action with respect to the metric yields the following

set of field equations

$$\begin{aligned}[1+f(\mathcal{R})]G_{\mu\nu} &= \kappa^2 T_{\mu\nu} + \nabla_\mu F(\mathcal{R})_{,\nu} - \Box F(\mathcal{R})g_{\mu\nu} - \frac{3}{2}\frac{F(\mathcal{R})_{,\mu}F(\mathcal{R})_{,\nu}}{F(\mathcal{R})} \\ &+ \frac{3}{4}\frac{F(\mathcal{R})_{,\sigma}F(\mathcal{R})^{,\sigma}g_{\mu\nu}}{F(\mathcal{R})} - \frac{1}{2}[F(\mathcal{R})\mathcal{R} - f(\mathcal{R})]g_{\mu\nu}. \quad (2)\end{aligned}$$

Trace of the above equation is obtained as

$$F(\mathcal{R})\mathcal{R} - 2f(\mathcal{R}) = \kappa^2 T + R \equiv X. \tag{3}$$

We may name this hybrid approach as $f(X)$ theory by writing $\mathcal{R}$ algebraically in terms of X if $f(\mathcal{R})$ supposed to have analytical solutions. In this way, we have direct way to measure deviation of $f(X)$ theory by GR. Thus taking $\mathcal{R} = \mathcal{R}(X)$, the field equations (2) can be written as

$$\begin{aligned} [1 + F(X)]G_{\mu\nu} &= \kappa^2 T_{\mu\nu} + \nabla_\mu F(X)_{,\nu} - \Box F(X) g_{\mu\nu} - \frac{3}{2}\frac{F(X)_{,\mu}F(X)_{,\nu}}{F(X)} \\ &+ \frac{3}{4}\frac{F(X)_{,\sigma}F(X)^{;\sigma}g_{\mu\nu}}{F(X)} - \Phi\frac{1}{2}[F(X)X - f(X)]g_{\mu\nu}. \end{aligned} \tag{4}$$

We may write, its scalar-tensor presentation by transforming as

$$\Phi = F(X), \quad V(\Phi) = F(X)X - f(X), \tag{5}$$

by which field equation will take the shape

$$\begin{aligned} [1 + \Phi]G_{\mu\nu} &= \kappa^2 T_{\mu\nu} + \nabla_\mu \Phi_{,\nu} - \Box\Phi g_{\mu\nu} - \frac{3}{2}\frac{\Phi_{,\mu}\Phi_{,\nu}}{\Phi} \\ &+ \frac{3}{4}\frac{\Phi_{,\sigma}\Phi^{;\sigma}g_{\mu\nu}}{\Phi} - \frac{1}{2}V(\Phi)g_{\mu\nu}. \end{aligned} \tag{6}$$

Hybrid FRW Equations

The field equations for the Friedmann-Robertson-Walker (FRW) cosmological model are obtained as

$$\kappa^2\rho = 3\kappa^2 H^2(1+\Phi) + \frac{3}{4\Phi}\dot{\Phi}^2 + 3H\dot{\Phi} + \kappa^2 V(\Phi), \quad (7)$$

$$\kappa^2 p = -\kappa^2(1+\Phi)\left(3H^2 + 2\dot{H}\right) + \frac{3}{4\Phi}\dot{\Phi}^2 + \kappa^2 V(\Phi) - \ddot{\Phi} - 2H\ddot{\Phi}. \quad (8)$$

Similarly, the equation of continuity for this universe model is obtained as

$$\ddot{\Phi} + 3H\dot{\Phi} - \frac{\dot{\Phi}^2}{2\Phi} + \Phi V(\Phi) - \frac{2V'(\Phi)\Phi^2}{3} + H\Phi\dot{\Phi} + \frac{\dot{\Phi}^2}{4} - \frac{2\kappa^2\Phi V'(\Phi)}{3} = 0\,. \quad (9)$$

Applying some necessary limits for getting static universe, i.e., $\rho_\Phi \approx V(\Phi)$, slow-roll limit, $V(\Phi) \gg \dot{\Phi}^2$, $3H\dot{\Phi} \gg \ddot{\Phi}$ and $\Phi \ll 1$ on Eq. (9), we get following important equations

$$\dot{\Phi} = \frac{\Phi}{3H}\left(\frac{2V'(\Phi)}{3} - V(\Phi)\right), \quad (10)$$

$$3H^2 = \frac{1}{1+\Phi}\left[\frac{2\Phi V'(\Phi)}{3} - (\Phi - \frac{1}{2})V(\Phi)\right], \quad (11)$$

$$\dot{H} = 3\dot{\Phi}^2\frac{[V(\Phi) + V'(\Phi)](\Phi + \frac{3}{2})}{6\Phi V(\Phi) - 4\Phi V'(\Phi)}. \quad (12)$$

Slow-Roll Parameters

For hybrid metric Platini theory, these slow-roll parameters will take the following form. Also, for convenience, we write, $V = V(\Phi)$, for further equations

$$\epsilon_1 = \frac{9\left[V'\left(\Phi+\frac{3}{2}\right)+V\right]\dot{\Phi}^2}{(4V'\Phi-6V\Phi)\left[\frac{2}{3}V'\Phi-V\left(\Phi-\frac{1}{2}\right)\right]}, \tag{13}$$

$$\epsilon_2 = \frac{(1+\Phi)}{\left[V\left(\Phi-\frac{1}{2}\right)-\frac{2V'\Phi}{3}\right]}\left[\frac{-4\Phi V'\left(V-\frac{2V'}{3}\right)}{3\left\{V'\left(\Phi+\frac{3}{2}\right)+V\right\}}-\left(\Phi-\frac{2}{3}\right)V'+V\right], \tag{14}$$

$$\begin{aligned}\epsilon_3 &= \left[\frac{6\left\{-V\left(-\frac{1}{2}+\Phi\right)+\frac{2\Phi V'}{3}\right\}+\frac{3\Phi}{2}\left\{-V+\left(\frac{7}{6}-\Phi\right)V'+\frac{2\Phi V''}{3}\right\}}{\Phi\left\{-V\left(-\frac{1}{2}+\Phi\right)+\frac{2\Phi V'}{3}\right\}}\right. \\ &- \left.\frac{\frac{3V}{2}+(1+2\Phi)V'-\frac{4V''}{3}}{(1+\Phi)\left(V-\frac{2V'}{3}\right)}\right]\frac{-\Phi(1+\Phi)\left(V-\frac{2V'}{3}\right)}{5\left[V\left(-\frac{1}{2}+\Phi\right)-\frac{2\Phi V'}{3}\right]}.\end{aligned} \tag{15}$$

Scalar to Tensor Ratio and Spectral Index

Based on the slow roll parameters, other two useful parameters for inflation are the scalar to tensor ratio r and the spectral index n_s, which are defined as [4],

$$r = 16(\epsilon_1 + \epsilon_3)\,, \qquad n_s = 1 - \frac{9\epsilon_1}{4} + \frac{3\epsilon_2}{2} - \frac{9\epsilon_3}{4}\,. \tag{16}$$

Perturbation

Since, the early universe has been originated from small seed perturbations, which over time grew to become all of the structure we observe. We investigate inhomogeneous perturbations in the direction of FRW and evaluate the scalar perturbations at the least possible background by the following relation [5, 6]

$$V(\Phi) = V_0 + V_1\Phi^2. \tag{17}$$

The other main parameter of the cosmic inflation, scalar-to-tensor ratio r, achieved as

$$\begin{aligned} r &= -2^{1/5} 32\Phi^2(1+\Phi)\Bigg[3^{7/10}\pi^{3/2}\left\{\frac{b\Phi^4(1+\Phi)^{3/2}\left(V_0-\frac{4\Phi V_1}{3}+\Phi^2V_1\right)^2}{c\left(\frac{4\Phi^2V_1}{3}-\left(-\frac{1}{2}+\Phi\right)\left(V_0+\Phi^2V_1\right)\right)^{3/2}}\right\}^{1/5} \\ &\times\ b\Phi^2\left\{\frac{c\left(\frac{4\Phi^2V_1}{3}-\left(-\frac{1}{2}+\Phi\right)\left(V_0+\Phi^2V_1\right)\right)^{3/2}}{b\Phi^4(1+\Phi)^{3/2}\left(V_0-\frac{4\Phi V_1}{3}+\Phi^2V_1\right)^2}\right\}^{1/10}\Bigg\{\left(V_0+\Phi^2V_1\right) \\ &-\ \Phi^2V_1+\left(-\frac{1}{2}+\Phi\right)\Bigg\}\left\{\frac{\frac{4}{3}\Phi^2V_1-\left(-\frac{1}{2}+\Phi\right)\left(V_0+\Phi^2V_1\right)}{1+\Phi}\right\}^{1/4}\Bigg]^{-1} \\ &\times\ \left(V_0-\frac{4\Phi V_1}{3}+\Phi^2V_1\right)^2. \end{aligned} \tag{18}$$

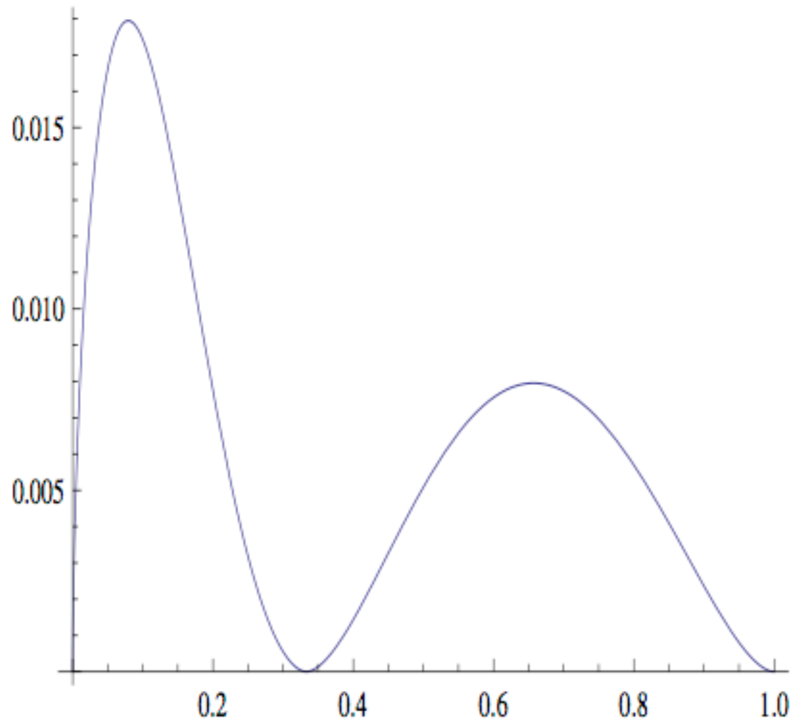

Figure 1: This is the plot of tensor to scalar ratio with Φ. The value of this parameter is lying between 0.2 to 0.36.

$$
\begin{aligned}
n_s &= \frac{\left[V_0 + \Phi^2 V_1 + 2\Phi\left(\frac{3}{2} + \Phi\right) V_1\right]\left[8\Phi^2 V_1 - 6\Phi\left(V_0 + \Phi^2 V_1\right)\right]^{-1}}{4\left[\frac{4\Phi^2 V_1}{3} - \left(-\frac{1}{2} + \Phi\right)\left(V_0 + \Phi^2 V_1\right)\right]\left[-\frac{4}{3}\Phi^2 V_1 + \left(-\frac{1}{2} + \Phi\right)\left(V_0 + \Phi^2 V_1\right)\right]} \\
&\times\ 27\Phi^2(1+\Phi)\left(V_0 - \frac{4\Phi V_1}{3} + \Phi^2 V_1\right)^2 - 4\Phi^2(1+\Phi)V_1 \\
&\times\ \frac{\left(V_0 - \frac{4\Phi V_1}{3} + \Phi^2 V_1\right)\left\{V_0 + \Phi^2 V_1 + 2\Phi\left(\frac{3}{2} + \Phi\right) V_1\right\}^{-1}}{\left\{-\frac{4}{3}\Phi^2 V_1 + \left(-\frac{1}{2} + \Phi\right)\left(V_0 + \Phi^2 V_1\right)\right\}} \\
&+\ \frac{3}{2}\frac{(1+\Phi)\left\{V_0 + 2\left(-\frac{2}{3} + \Phi\right)\Phi V_1 + \Phi^2 V_1\right\}}{\frac{4}{3}\Phi^2 V_1 - \left(-\frac{1}{2} + \Phi\right)\left(V_0 + \Phi^2 V_1\right)}. \qquad (19)
\end{aligned}
$$

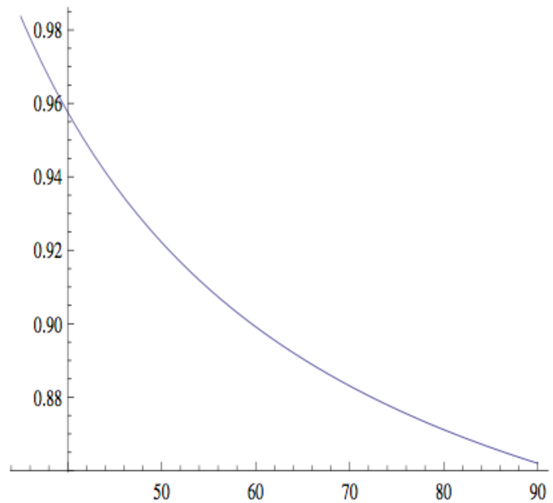

Figure 2: This is the plot of the spectral index with Φ. Its value is equal to the 0.960.0073. This graph shows that the value of the spectral index is compatible with the observational data.

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