The role of internal clocks in early Universe cosmology

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1. Introduction

As we look back in time we see that to describe the evolution of the Universe it is not enough to follow the equations of General Relativity, as at some point quantum effects appear in the picture. One of the fundamental obstacles to overcome in merging relativistic and quantum theories is the problem of time [2-5]. In short, the time-reparametrization invariance in relativistic theories conflicts with the absolute status of Newtonian time we use in quantum theories.

The question is whether one can introduce a relativistic time-reparametrization to classical and quantum mechanics?

2. Internal clocks

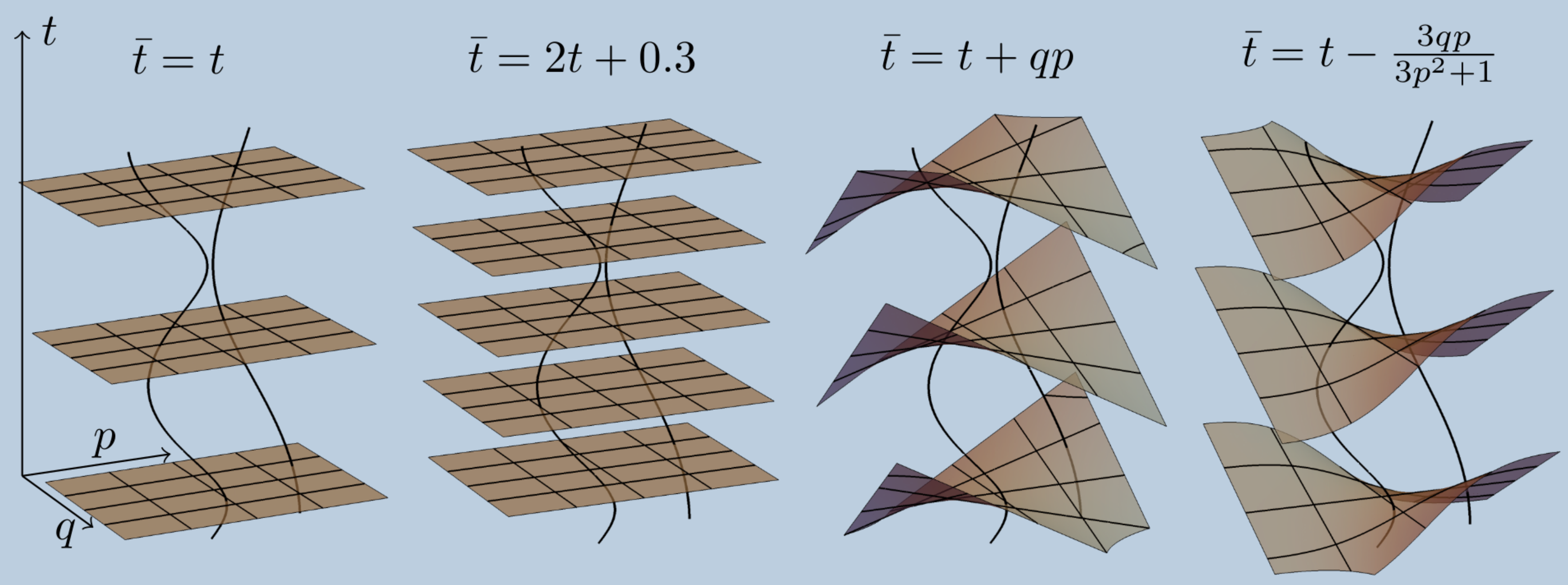

Figure 1: Different choice of time-slicing corresponds to different choice of internal clock.

The phase space formalism of general relativity involves a Hamiltonian constraint C which is constrained to vanish. The Hamiltonian constraint plays two roles in this formalism: (i) generating the dynamics and (ii) constraining the space of physically admissible states,

$$\frac{d}{d\tau}O(q_i,p^i) = \{O(q_i,p^i), C(q_i,p^i)\}, \qquad C(q_i,p^i) = 0. \tag{1}$$

The Hamiltonian constraint formalism can be brought to the ordinary canonical formalism upon identifying the constraint surface C= 0 with a contact manifold made of a lower-dimensional phase space and a time manifold. This procedure is called the reduced phase space approach. Due to time-reparametrization invariance, the choice of time parameter is arbitrary and corresponds to the time-slicing of the manifold. **The degree of freedom with respect to which we describe the evolution of the system is called an internal clock.**

3. Change of internal clocks in classical theories

Recalling the basic framework of classical mechanics, a phase space (q_i, p^i) is equipped with a symplectic form $\omega = dq_i \wedge dp^i$ and a Hamiltonian $H(q_i, p^i)$ which generates dynamics in t. The natural symmetry of a such system is the canonical symmetry allowing the change of phase-space variables

$$q_i \mapsto \tilde{q}_i(q_j, p^j)$$
$$p_i \mapsto \tilde{p}_i(q_j, p^j)$$

in such a way, that the symplectic form stays the same

$$\omega = dq_i \wedge dp^i = d\tilde{q}_i \wedge d\tilde{p}^i = \tilde{\omega}$$

In order to implement the time-reparametrization in classical mechanics one has to realize that the symplectic form description corresponds to the system after the choice of internal clock t (after phase space reduction). If we replace the symplectic form with the more general contact form $\omega_C = \omega - dt \wedge dH(q_j, p^j)$ [6], **the canonical symmetry is lifted to the symmetry ($\omega_C = \tilde{\omega}_C$) which includes choices of the internal clock**,

$$q_i \mapsto \tilde{q}_i(q_j, p^j, t),$$
$$p_i \mapsto \tilde{p}_i(q_j, p^j, t),$$
$$t \mapsto \tilde{t}(q_j, p^j, t),$$

4. Change of internal clocks in quantum mechanics

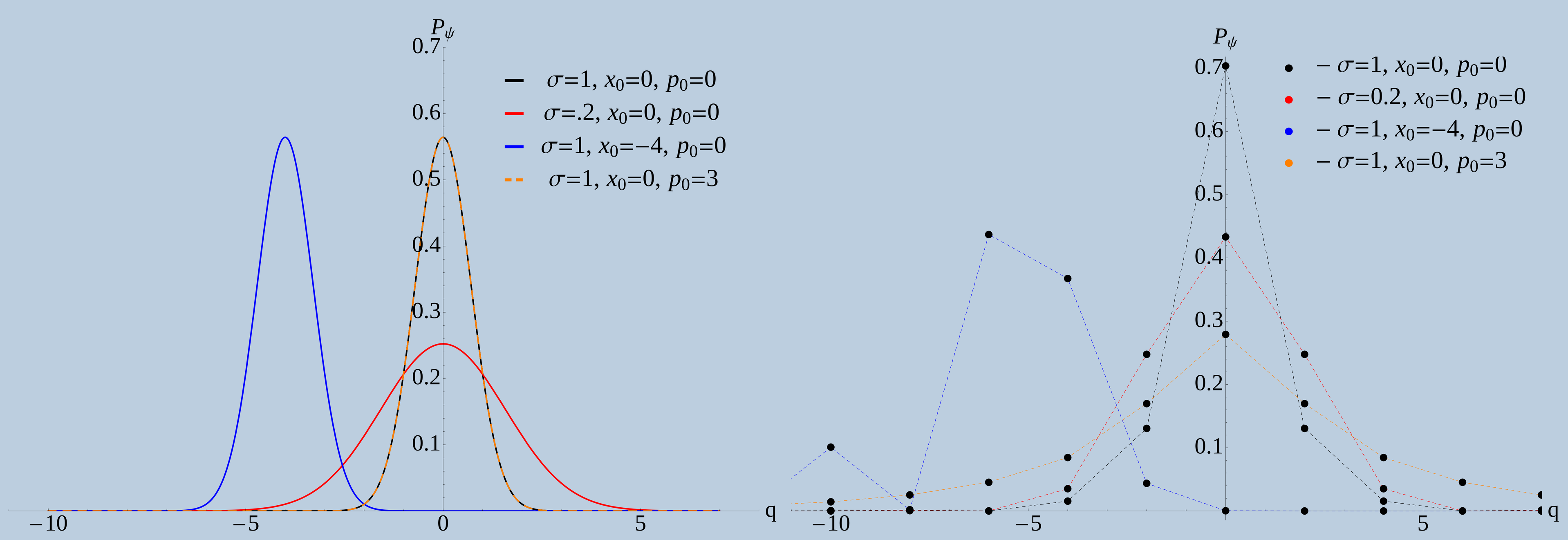

Figure 2: Probability distribution $P_\psi = |\langle q|\psi\rangle|^2$ of position eigenvalues for a gaussian state in the old clock t (on the left) and in the new clock $\tilde{t} = t + qp$ (on the right). On the right: the probability for eigenvalues is marked with dots. The spectrum is discrete.

First we set the internal clock transformations in such a way that the non-dynamical operators have the same form in any clock. Now we can see how the dynamical information about quantum states transforms when the internal clock is changed. Investigating specific examples (see Figure 2) suggests that not only the shape of the wavefunctions change in different quantum clocks, but also the spectrum of the dynamical operators (like position in non-trivially evolving systems) can change its character (from continuous to discrete). **The description of quantum mechanical systems in different quantum clocks are not equivalent.**

5. Conclusions

In this work, we have proposed a reformulation of classical and quantum mechanics in such a way as to remove the absolute time from its formalism and replace it with an arbitrarily chosen internal degree of freedom, the internal clock [1].

- In the case of classical mechanics we find an entirely consistent description of internal clock transformations.
- The choice of different internal clocks does not affect the Schrödinger equation.
- In purely quantum systems we find that the descriptions in different clocks are inequivalent. We call it the clock effect.
- In systems composed of classical and quantum degrees of freedom, the choice of different internal clock is equivalent.
- For the early quantum universe, there may be a fundamental obstacle (the inconsistency of different choices of internal clocks) which will prohibit us from inferring what happened near the initial singularity.
- When the universe is well approximated by classical picture all the discrepancies related to the clock effect converge to the unambiguous description.

References

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