Cargese Lectures

EFFECTIVE FIELD THEORY IN COSMOLOGY

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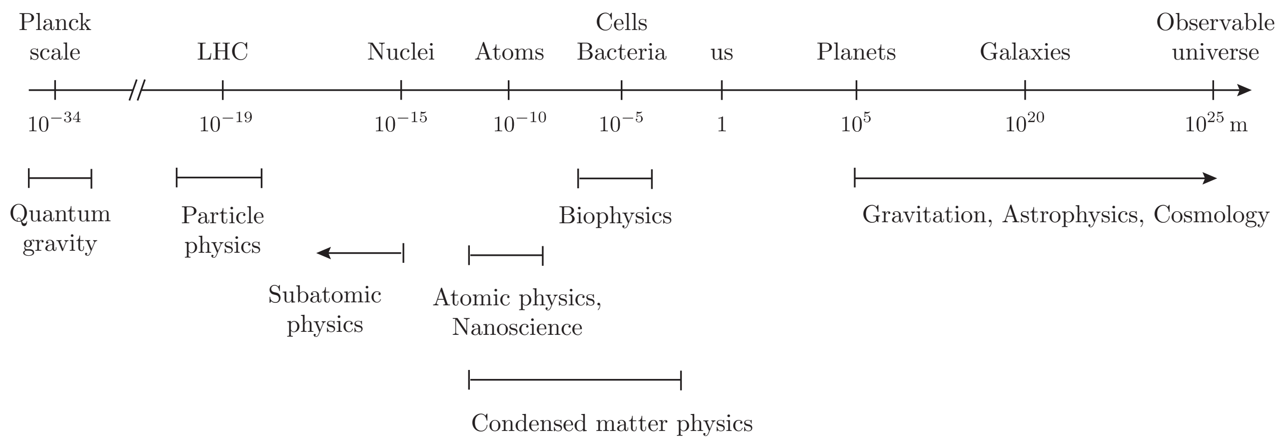
Lecture 1.
INTRODUCTION TO EFFECTIVE FIELD THEORY

1. MOTIVATION

Nature comes with many scales:



Science progresses because we can treat one scale at a time.

Coarse-graining over short scales (high energies) leads to an **effective field theories (EFTs)** at long distances (low energies).

Even if we don't know the full microscopic theory, we can parameterize our ignorance as an EFT.

Natural units: [see Nima Arkani-Hamed PIRSA/09080035]

Setting $\begin{aligned} c &= 3 \times 10^8\,\text{m/s} \equiv 1 \\ \hbar &= 10^{-34}\,\text{J}\cdot\text{s} \quad \equiv 1 \end{aligned}$ we have $[L] = [T] = [E^{-1}]$.

A useful conversion is $m_p \sim 1\,\text{GeV} \sim (10^{-16}\,\text{m})^{-1} \sim (10^{-24}\,\text{s})^{-1}$.

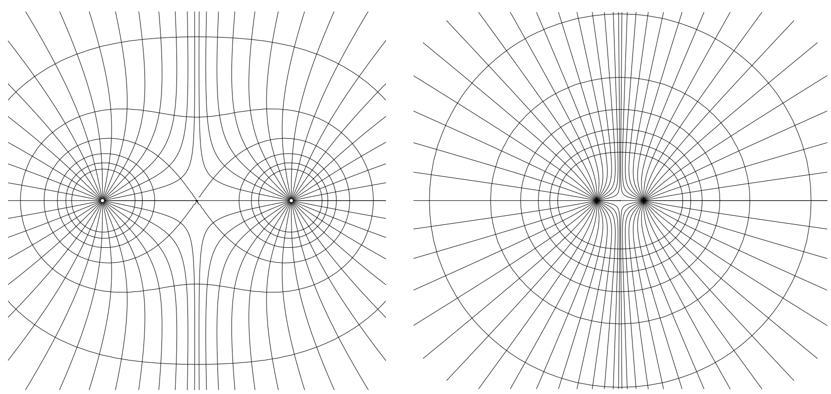
- **Examples of EFTs**:

  - *Hydrogen atom*

$$\text{Recall that} \quad H = \frac{p^2}{2m_e} - \frac{\alpha}{r} \quad \Rightarrow \quad E_n = -\frac{m_e \alpha^2}{n^2} \; + \; \text{Corrections}$$

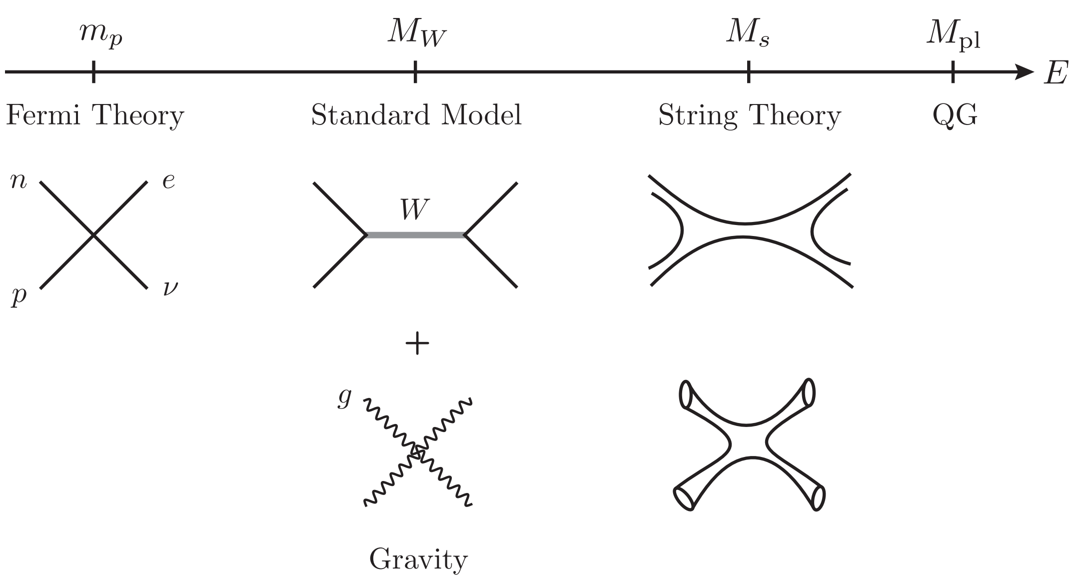
- Proton recoil: $O(m_e/m_p)$
- Fine structure: $O(\alpha^2)$
- Weak interactions: $O(m_p/M_W)$

  - *Multipole expansion*



$$V(\mathbf{r}) = \frac{1}{r} \sum_{l,m} c_{lm} \left(\frac{a}{r}\right)^l Y_{lm}(\Omega)$$

  - *High-energy physics*



- *Hydrodynamics*

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x^i}(\rho v^i)$$
$$\frac{\partial}{\partial t}(\rho v^i) = -\frac{\partial}{\partial x^j}\,\Pi^{ij}$$
$$\uparrow$$
$$\Pi^{ij} = P\delta_{ij} + \rho v_i v_j + \eta\left(\partial_{(i}v_{j)} - \frac{2}{3}\delta_{ij}(\partial_k v_k)\right) + O(\partial^2)$$

2. PRINCIPLES OF EFT

I will illustrate the basic principles of EFTs with the following toy model:

$$\mathcal{L}[\phi,\Psi] = \begin{array}{l} -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 \\ -\frac{1}{2}(\partial\Psi)^2 - \frac{1}{2}M^2\Psi^2\,, \end{array} \quad - \frac{1}{4}g\phi^2\Psi^2$$

where $m \ll M$ and $g \ll 1$.

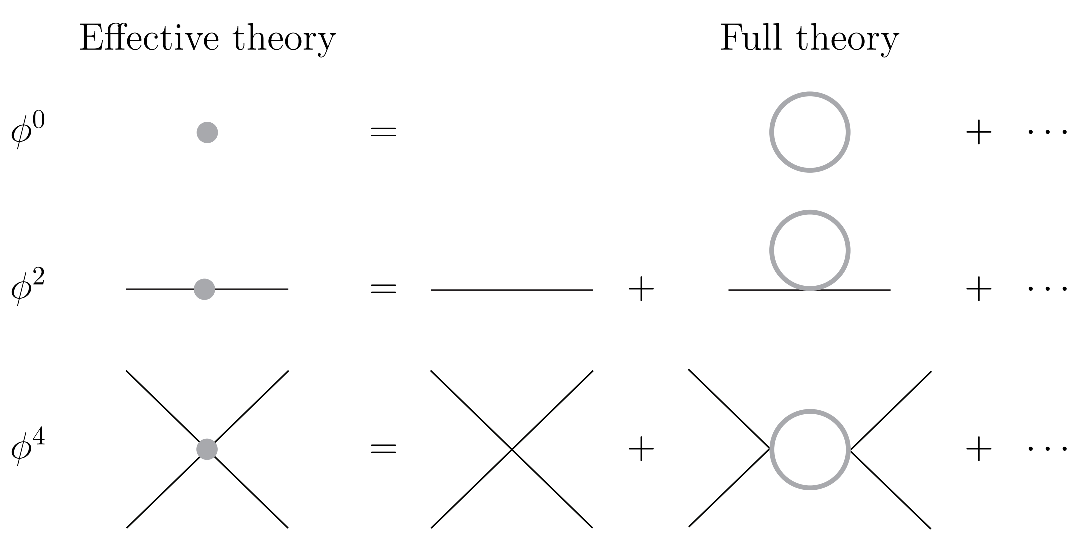
• Integrating out

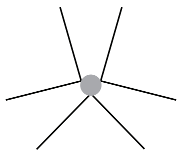
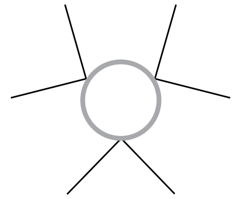
We can *integrate out* the heavy fields to get an EFT for the light fields:

$$e^{iS_{\rm eff}[\phi]} = \int \mathcal{D}\Psi\, e^{iS[\phi,\Psi]}\,.$$

• Matching

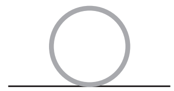
In practice, the effective action is usually found by *matching*:

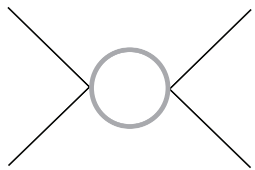


ϕ^6  $=$  $+ \cdots$

• Renormalization

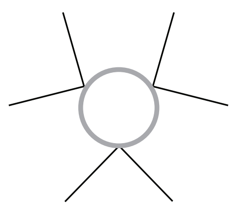
Heavy fields *renormalize* the IR couplings

$\Delta m^2 =$  $= +\frac{g}{32\pi^2}\left(\Lambda^2 - M^2 \log\left(\frac{\Lambda^2}{\mu^2}\right)\right)$

$\Delta\lambda =$  $= -\frac{3g^2}{32\pi^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)$

• Non-renormalizable interactions

Heavy fields also add new *non-renormalizable interactions*:

 $\sim \; g^3 \frac{\phi^6}{M^2}$

• Decoupling

These new higher-dimensional interactions *decouple* for $M \to \infty$.

• Power counting

EFTs are expansions in powers of $\delta \equiv E/M \ll 1$.
Only a finite number of terms are relevant for observations with finite precision.

• Effective actions

"Everything that is allowed is compulsory."

For example, in the toy model we generate all terms that are consistent with the $\phi \to -\phi$ symmetry of the full theory:

$$\mathcal{L}_{\text{eff}}[\phi] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_{\text{R}}^2 - \frac{1}{4!}\lambda_{\text{R}}\phi^4 - \sum_{i=1}^{\infty}\left(\frac{c_i}{M^{2i}}\phi^4 + 2i + \frac{d_i}{M^{2i}}(\partial\phi)^2\phi^{2i} + \cdots\right).$$

Even if the full theory is not known, we can still parameterize the EFT:

$$\mathcal{L}_{\text{eff}}[\phi] = \mathcal{L}_0[\phi] + \sum_i c_i \frac{O_i[\phi]}{\Lambda^{\delta_i - 4}} \,.$$

Wilson coefficient
cutoff
operator
dimension

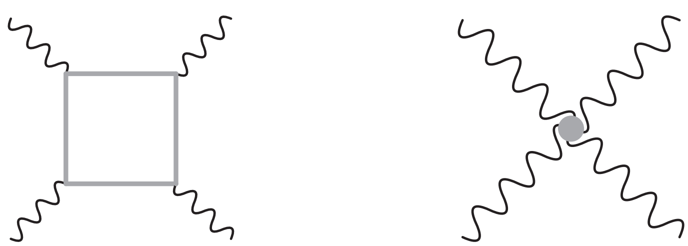
- **EFT approach**
  - Identify the relevant degrees of freedom.
  - Determine the relevant symmetries.
  - Write all operators compatible with the symmetries.
  - Compute observables.
  - Measure parameters.

3. EXAMPLES OF EFTS

- **Photon-photon scattering**

Consider $\gamma\gamma$ scattering at energies $E \ll m_e$.

The only dynamical degrees of freedom in the EFT are photons.
Photons can interact via electron loops:



Full theory　　　Effective theory

The EFT Lagrangian is

$$\mathcal{L}_{\text{eff}}[A_\mu] = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha^2}{m_e^4} c_1 (F_{\mu\nu} F^{\mu\nu})^2 + \cdots \,,$$

where $c_1 = 1/90$.

- **Rayleigh scattering**

Consider the scattering of photons off atoms at low energies.

Let $\psi(x)$ denote a field operator that creates an atom at the point x.
The effective Lagrangian for the atom is

$$\mathcal{L}_{\rm eff}[\psi] = \psi^\dagger \left(i\partial_t - \frac{\partial^2}{2M} \right) \psi + \mathcal{L}_{\rm int} \, .$$

At low energies, the dominant interaction with photons is

$$\mathcal{L}_{\rm int} = \underset{\substack{\uparrow \\ \text{size of the atom}}}{a_0^3} \, \psi^\dagger \psi F_{\mu\nu} F^{\mu\nu}$$

The corresponding cross section is

$$\sigma \propto a_0^6 \, \omega^4 \, .$$

That is why the sky is blue!

- **Gravity**

Like Fermi's theory, Einstein's gravity requires a UV completion.

However, at low energies, $E \ll M_{\rm pl}$, gravity is described by an EFT:

$$\mathcal{L}_{\rm eff}[g_{\mu\nu}] = \sqrt{-g} \left[\frac{M_{\rm pl}^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{1}{\Lambda^2} \left(d_1 R^3 + \cdots \right) + \cdots \right] ,$$

where $\Lambda \lesssim M_{\rm pl}$.

- **Particle physics**

The most conservative way to described BSM physics is as an EFT:

$$\mathcal{L}_{\rm eff}[\psi, A_\mu, H] = \mathcal{L}_{\rm SM} + \sum_i c_i \frac{O_i}{\Lambda^{\delta_i - 4}} \, .$$

- Dim-0: CC problem.
- Dim-2: Hierarchy problem.
- Dim-5: Neutrino masses.

$$\Delta\mathcal{L} \sim \frac{1}{\Lambda}(LH)(LH) \xrightarrow{H=v} m_\nu = \frac{v^2}{\Lambda} \sim 10^{-2}\,{\rm eV} \, , \ \text{for} \ \ \Lambda \sim 10^{15}\,{\rm GeV}.$$

- Dim-6: Proton decay.

$$\Delta\mathcal{L} \sim \frac{1}{\Lambda^2} QQQL \xrightarrow{\tau_p > 10^{33}\,{\rm yrs}} \Lambda > 10^{15}\,{\rm GeV} \, .$$

• Inflation

The most conservative way to described the physics of inflation is as an EFT:

$$\mathcal{L}_{\rm eff}[\phi, \Psi, g] = \sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V_0(\phi) + \sum_i c_i \frac{O_i[\phi, \Psi]}{\Lambda^{\delta_i - 4}}\right].$$

- Dim-6: Eta problem.

$$\Delta V = V_0 \frac{\phi^2}{\Lambda^2} \xrightarrow{\Lambda < M_{\rm pl}} \Delta\eta \equiv M_{\rm pl}^2 \frac{V''}{V} \approx \frac{M_{\rm pl}^2}{\Lambda^2} > 1.$$

- Dim-8: Non-Gaussianity.

$$\Delta\mathcal{L} = \frac{(\partial\phi)^4}{\Lambda^4} \xrightarrow{\Lambda^2 < \dot\phi} f_{\rm NL} \sim \frac{\dot\phi^2}{\Lambda^4} < 1 \ .$$

- Dim-∞: Lyth bound.

$$\Delta\phi \sim \left(\frac{r}{0.01}\right)^{1/2} M_{\rm pl} \xrightarrow{r > 0.01} \Delta\phi > M_{\rm pl}\,.$$

4. OUTLOOK

In the rest of the lectures, I will describe two important EFTs in more detail:

1. EFT of Inflation
2. EFT of Large-Scale Structure

References

A. Manohar, *Introduction to Effective Field Theories*, [arXiv:1804.05863]

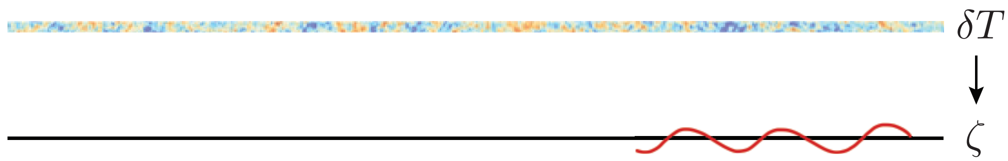
D. Baumann and L. McAllister, *Inflation and String Theory*, [arXiv:1404.2601]

Lecture 2.

EFFECTIVE FIELD THEORY OF INFLATION

1. MOTIVATION

The origin of structure in the universe is one of the biggest open questions in cosmology:



Although there is growing evidence that the primordial fluctuations originated from quantum fluctuations during inflation, the physics of inflation remains a mystery.

In this lecture, I will describe inflation as a symmetry breaking phenomenon and derive an effective action for the inflationary perturbations.

2. SPONTANEOUS SYMMETRY BREAKING

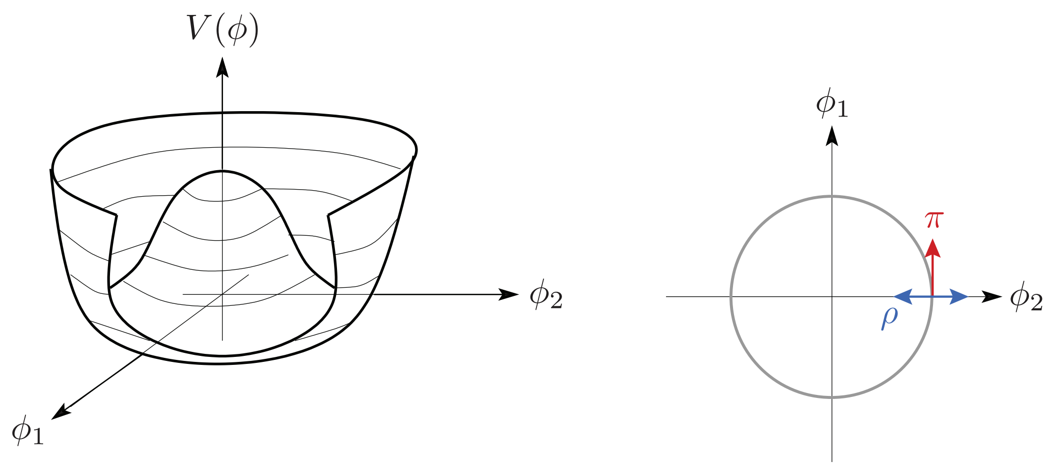
• Broken global symmetries

Consider

$$\mathcal{L} = -\partial_\mu \phi^\dagger \partial^\mu \phi + \left(\mu^2 \phi^\dagger \phi - \frac{1}{4}\lambda(\phi^\dagger\phi)^2 \right) ,$$

which is invariant under the $U(1)$ symmetry $\phi \to e^{i\beta}\phi$.

For $\mu^2 > 0$, the symmetry is spontaneously broken:



Substituting

$$\phi = \frac{1}{\sqrt{2}}(v + \rho(x))\, e^{i\pi(x)}\,, \quad \text{with} \quad v \equiv \frac{\mu}{\sqrt{\lambda}}\,,$$

we find

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \rho)^2 - \mu^2\rho^2 - \sqrt{\lambda}\mu\rho^3 - \lambda\rho^4 - \frac{1}{2}(v+\rho)^2 \underset{\substack{\uparrow \\ \text{massless} \\ \text{Goldstone boson}}}{(\partial_\mu \pi)^2}\,.$$

Integrating out the massive field ρ, we get an effective Lagrangian for π

$$\mathcal{L}_\pi = -\frac{1}{2}(\partial_\mu \pi_c)^2 + c_1 \frac{(\partial_\mu \pi_c)^4}{v^4} + \cdots\,,$$

where $\pi_c \equiv v\pi$ and $c_1 = v^2/\mu^2$.

From the bottom up, we can write the effective action of π as a derivative expansion of $U(x) \equiv e^{i\pi(x)}$:

$$\mathcal{L}_\pi = -\frac{f_\pi^2}{2}\partial_\mu U^\dagger \partial^\mu U + c_1(\partial_\mu U^\dagger \partial^\mu U)^2 + c_2(\partial_\mu U^\dagger \partial_\nu U)(\partial^\mu U^\dagger \partial^\nu U) + \cdots\,,$$

where f_π is the symmetry breaking scale.

If a symmetry G is broken to a subgroup H, we obtain one massless Goldstone boson π_a for each broken generator T^a. The effective Lagrangian of the Goldstone bosons is

$$\mathcal{L}_\pi = -\frac{f_\pi^2}{2}\mathrm{Tr}[\partial_\mu U^\dagger \partial^\mu U] + c_1 \mathrm{Tr}[(\partial_\mu U^\dagger \partial^\mu U)^2] + \cdots\,,$$

where $U(x) \equiv e^{i\pi_a(x)T^a}$.

• Broken gauge symmetries

Consider scalar electrodynamics

$$\mathcal{L} = -D_\mu \phi^\dagger D^\mu \phi - V(\phi) - \frac{1}{4}F_{\mu\nu}^2\,,$$

with $D_\mu = \partial_\mu + igA_\mu$. Let

$$\phi = \frac{1}{\sqrt{2}}(v + \rho(x))\, e^{i\pi(x)}\,.$$

and use the gauge symmetry to set $\pi \equiv 0$ (*unitary gauge*). After the SSB, the gauge field has become massive

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2 A_\mu^2 + \cdots ,$$

where $m^2 = g^2 v^2$. The Goldstone boson has become the longitudinal mode of the massive vector field (= Higgs mechanism).

• Stückelberg trick

To understand the behavior of the theory at high energies, it is useful to reintroduce the Goldstone boson. This is achieved by imposing the following transformation on the vector field (i.e. by 'undoing' the gauge fixing)

$$A_\mu \to A_\mu + \frac{\partial_\mu \pi}{g} \equiv \frac{i}{g} U D_\mu U^\dagger ,$$

where $U(x) \equiv e^{i\pi(x)}$. The Lagrangian then becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{f_\pi^2}{2} D_\mu U^\dagger D^\mu U ,$$

where $f_\pi \equiv m/g$.

At quadratic order, this can be written as

$$\mathcal{L}_2 = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial_\mu \pi_c)^2 - \frac{1}{2}m^2 A_\mu^2 + m\, \partial_\mu \pi_c A^\mu ,$$

where $\pi_c \equiv f_\pi \pi$.

• Decoupling limit

Because the mixing term $\partial_\mu \pi_c A^\mu$ has one fewer derivative than $(\partial_\mu \pi_c)^2$, we expect it to become irrelevant at high energies.

To see this, we take the so-called *decoupling limit*

$$g \to 0\, , \quad m \to 0\, , \quad \text{for} \quad f_\pi \equiv m/g = const.$$

In this limit, there is *no mixing* between π and A_μ.

For $E > E_{\rm mix} = m$, the scattering[1] of the longitudinal modes of the gauge fields is therefore described by the scattering of the Goldstone bosons, up to corrections of order m/E and g^2 (= *Goldstone boson equivalence theorem*).

[1]For non-Abelian gauge bosons, interactions of the form $f_\pi^2 \pi^2 (\partial_\mu \pi)^2 = \pi_c^2 (\partial_\mu \pi_c)^2 / f_\pi^2$ arise from expanding the universal kinetic term $f_\pi^2 {\rm Tr}[D_\mu U^\dagger D^\mu U]$, while for Abelian gauge bosons they only arise from the non-universal higher-derivative terms.

3. EFT OF INFLATION

• Broken time translations

Time-dependent matter fields, $\psi_m(t)$, break time translation invariance, i.e. the action isn't invariant under

$$t \to t + \pi(x) \equiv U(x)\,.$$

The field π is the *Goldstone boson* of the broken symmetry.

• Adiabatic perturbations

The field π parameterizes *adiabatic perturbations*

$$\delta\psi_m(t,\mathbf{x}) = \bar{\psi}_m(t + \pi(t,\mathbf{x})) - \bar{\psi}_m(t)\,.$$

In spatially flat gauge, $g_{ij} = a^2(t)\delta_{ij}$, all metric perturbations are related to $\pi(x)$ by the Einstein equations.

• Unitary gauge

For purely adiabatic fluctuations, we can perform a local time shift, $t \to t - \pi(x)$, to remove all matter fluctuations, $\delta\psi_m \to \delta\psi_m \equiv 0$ (*unitary gauge*).

This induces the following metric perturbation

$$\delta g_{ij} = a^2(t) e^{2\zeta(t,\mathbf{x})}\delta_{ij}\,,$$

where $\zeta = -H\pi$ is the *comoving curvature perturbation.*

• Effective action

The effective action after gauge fixing includes all terms that are invariant under spatial diffeomorphisms, e.g. g^{00}, K_{ij}, R_{ij}.

At leading order in derivatives, we have

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + \sum_{n=0}^{\infty}\frac{M_n^4(t)}{n!}(\delta g^{00})^n\right],$$

where $\delta g^{00} = g^{00} + 1$. During inflation, we have $M_n(t) \approx const.$

This is an expansion around the correct FRW background iff

$$M_0^4 = -M_{\rm pl}^2(3H^2 + 2\dot{H})\,,$$
$$M_1^4 = M_{\rm pl}^2\dot{H}\,.$$

• Slow-roll inflation

The universal part of the action is

$$S_0 = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - M_{\rm pl}^2(3H^2+\dot{H}) + M_{\rm pl}^2\dot{H}g^{00}\right].$$

For $\dot{H} \ll H^2$, this is just single-field slow-roll inflation in disguise.
To see this, consider

$$\mathcal{L}_0 = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \quad \xrightarrow{\phi=\bar{\phi}(t)} \quad -\frac{1}{2}\dot{\bar{\phi}}^2 g^{00} - V(\bar{\phi})$$
$$= M_{\rm pl}^2\dot{H}g^{00} - M_{\rm pl}^2(3H^2+\dot{H})\,.$$

• Stückelberg trick

We introduce the Goldstone boson by performing the transformation

$$t \to t + \pi \equiv U\,,$$
$$g^{00} \to g^{\mu\nu}\partial_\mu U\partial_\nu U\,,$$

so that the effective action becomes

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + \sum_{n=0}^{\infty}\frac{M_n^4(U)}{n!}(g^{\mu\nu}\partial_\mu U\partial_\nu U + 1)^n\right],$$

or, more explicitly,

$$S = \int \mathrm{d}^4x\sqrt{-g}\Bigg[\frac{M_{\rm pl}^2}{2}R - M_{\rm pl}^2\left(3H^2(t+\pi) + \dot{H}(t+\pi)\right)$$
$$+ M_{\rm pl}^2\dot{H}\left(g^{00} + 2\partial_\mu\pi g^{0\mu} + \partial_\mu\pi\partial_\nu\pi g^{\mu\nu}\right)$$
$$+ \sum_{n=2}^{\infty}\frac{M_n^4}{n!}\left(1 + g^{00} + 2\partial_\mu\pi g^{0\mu} + \partial_\mu\pi\partial_\nu\pi g^{\mu\nu}\right)^n\Bigg].$$

• Decoupling limit

The mixing with metric perturbations vanishes in the decoupling limit:

$$M_{\rm pl} \to \infty\,, \quad \dot{H} \to 0\,, \quad \text{for} \quad M_{\rm pl}^2\dot{H} = const.$$

For $\omega^2 > \omega_{\rm mix}^2 = |\dot{H}|$, we can therefore evaluate the action in the *unperturbed* spacetime. The Goldstone action then becomes

$$S = \int {\rm d}^4x \sqrt{-g}\left[\underset{\substack{\uparrow \\ \text{slow-roll inflation}}}{M_{\rm pl}^2 \dot{H} (\partial_\mu \pi)^2} + \sum_{n=2}^{\infty} \frac{M_n^4}{n!} \underset{\substack{\uparrow \\ \text{DBI, } P(X)\text{, etc.}}}{\left(-2\dot{\pi} + (\partial_\mu \pi)^2\right)^n}\right].$$

• Speed of Sound

At quadratic order, the Goldstone Lagrangian is

$$\mathcal{L}_\pi^{(2)} = M_{\rm pl}^2 \dot{H} (\partial_\mu \pi)^2 + \ 2M_2^4 \, \dot{\pi}^2 \ = \ \frac{M_{\rm pl}^2 \dot{H}}{c_s^2}\left(\dot{\pi}^2 - \frac{c_s^2}{a^2}(\partial_i \pi)^2\right),$$

where we have introduced a non-trivial *speed of sound*

$$c_s^2 \equiv \frac{M_{\rm pl}^2 \dot{H}}{M_{\rm pl}^2 \dot{H} - 2M_2^4} \leq 1\,.$$

The rescaling $x^i \to \tilde{x}^i \equiv c_s^{-1} x_i$ allows us to write

$$\tilde{\mathcal{L}}_\pi^{(2)} \equiv c_s^3 \mathcal{L}_\pi^{(2)} = \frac{f_\pi^4}{2}\left(\dot{\pi}^2 - \frac{(\tilde{\partial}_i \pi)^2}{a^2}\right) = -\frac{1}{2}(\tilde{\partial}_\mu \pi_c)^2\,,$$

where $f_\pi^4 \equiv 2M_{\rm pl}^2 |\dot{H}| c_s$ is the *symmetry breaking scale* and $\pi_c \equiv f_\pi^2 \pi$ is the canonically normalized field.

• Power spectrum

The dimensionless power spectrum of curvature perturbations, $\zeta = -H\pi$, is

$$\Delta_\zeta^2 = \frac{1}{4\pi^2}\left(\frac{H}{f_\pi}\right)^4 .$$

The observed value, $\Delta_\zeta^2 = 2 \times 10^{-9}$, implies $f_\pi \approx 58\, H$.

• Non-Gaussianity

Small c_s (or large M_2) implies large interactions

$$\tilde{\mathcal{L}}_\pi^{(3)} = -\frac{1}{2\Lambda^2} \frac{\dot{\pi}_c (\tilde{\partial}_i \pi_c)^2}{a^2} + \cdots\,,$$

where $\Lambda^2 \equiv f_\pi^2\, c_s^2/(1 - c_s^2)$ is the *strong coupling scale*.

At the single-derivative level, the complete cubic action is

$$\tilde{\mathcal{L}}_\pi^{(3)} = -\frac{1}{2\Lambda^2}\left[\frac{\dot{\pi}_c(\tilde{\partial}_i\pi_c)^2}{a^2} + A\,\dot{\pi}_c^3\right],$$

where

$$A \equiv \left(-1 + \frac{2}{3}\frac{M_3^4}{M_2^4}\right) c_s^2 \,.$$

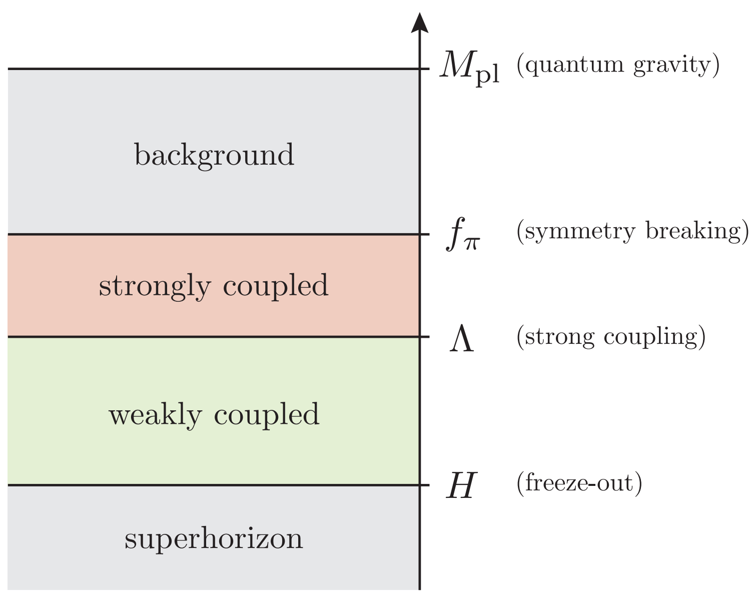
Naturalness demands $A = \mathcal{O}(1)$.

The typical size of the non-Gaussianity is

$$f_{\rm NL} \sim \left(\frac{f_\pi}{\Lambda}\right)^2 \lesssim 50\,.$$

• Energy scales

A nice way to summarize all single-field inflation models is in terms of the relevant energy scales:



The hierarchy of scales is determined by observations.

4. COSMOLOGICAL COLLIDER PHYSICS

In particle physics, the masses and spins of new particles are determined by measuring the positions and angular dependences of *resonances*.

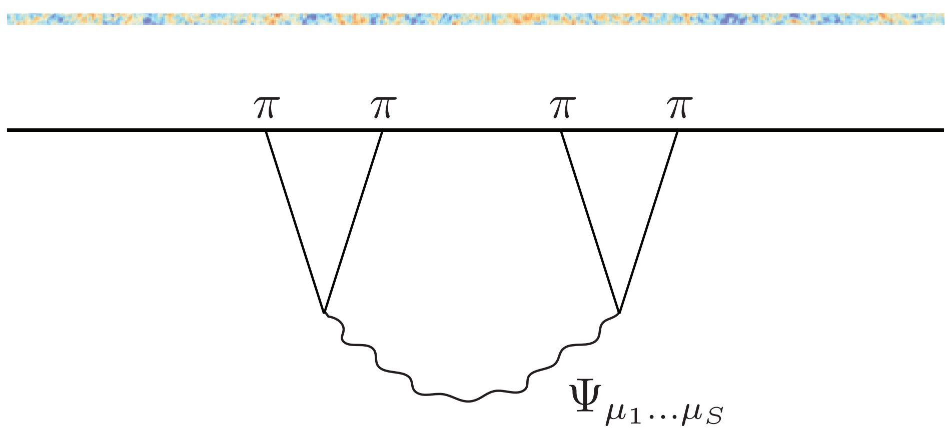
In cosmology, we can do something similar.

Extra massive particles during inflation can have two types of effects:

- $(H/M)^n$:
  They can lead to new self-interactions in the Goldstone effective action. These effects are captured by the EFT of inflation.

- $e^{-M/H}$:
  They can spontaneously be created by the expansion of the spacetime. This *particle production* is not captured by the EFT.

  The late-time decay of these massive particles leads to distinct imprints in the Goldstone correlation functions:



  The masses and spins of the new particles are encoded in the momentum dependence of the correlators (like in particle physics).

These signals will be hard to observe. Having said that, their detection would be a *direct* probe of the UV completion of inflation (SUSY, strings, ...).

5. OUTLOOK

• The EFT approach is the most conservative way to describe inflation.

• It is directly related to cosmological observables:

$$\pi \;\rightarrow\; \zeta \;\rightarrow\; \delta T \;\rightarrow\; \delta_g \,.$$

• The UV completion of inflation is encoded in subtle correlations inherited by the effective action of the Goldstone boson of broken time translations

$$\Psi \;\rightarrow\; \pi \,.$$

• We hope to measure these effects in future large-scale structure observations.

References

C. Cheung et al., *The Effective Field Theory of Inflation*, [arXiv:0709.0293]

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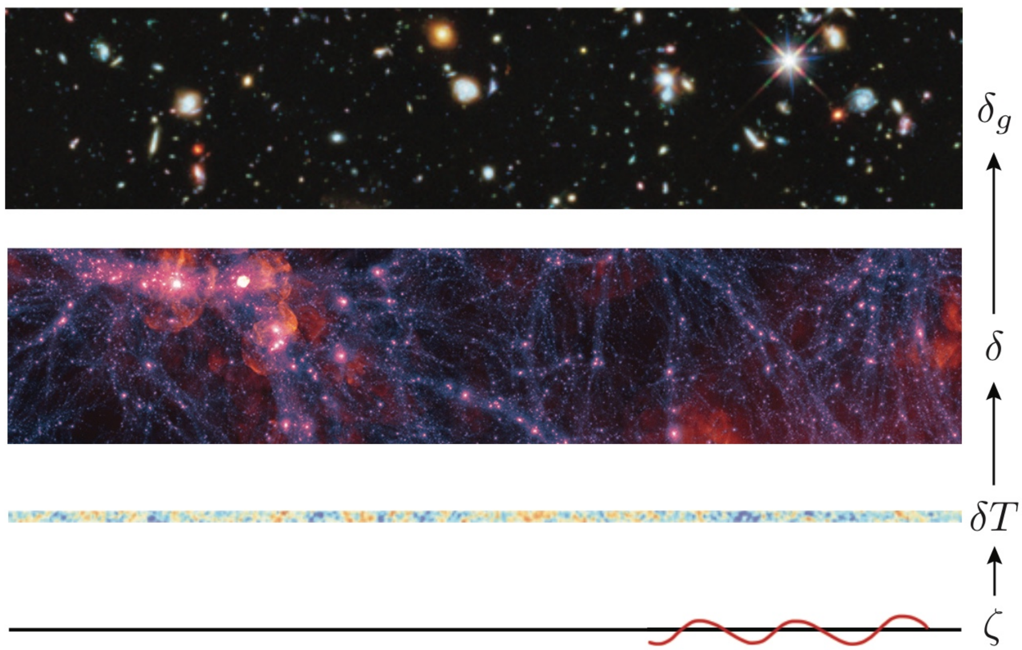
Lecture 3.
EFT OF LSS

1. MOTIVATION

Late-time observables are related to the primordial fluctuations through *non-linear evolution*:



At short distances, $k > k_{\rm max}$, standard perturbation theory breaks down.

It is essential to understand how nonlinearities on short scales feed into the evolution of long-wavelength modes. This is what an EFT does for a living.

The EFT of LSS extends perturbative control to larger $k_{\rm max}$, allowing more modes to be used in the analysis, $N \sim V k_{\rm max}^3$. This greatly enhances our ability to probe fundamental physics with LSS observations.

2. STANDARD PERTURBATION THEORY

The evolution of dark matter particles is described by the *collisionless Boltzmann equation.* The first two moments are

$$\begin{aligned} \text{continuity} \qquad & \dot\delta + \boldsymbol{\nabla}\cdot[\mathbf{v}(1+\delta)] = 0\,, \\ \text{Euler} \qquad & \dot v_i + \mathcal{H} v_i + \mathbf{v}\cdot\boldsymbol{\nabla} v_i = -\nabla_i \Phi - \frac{1}{\rho}\partial^j \tau_{ij}\,. \end{aligned}$$

In SPT, we set $\mathbf{w} \equiv \boldsymbol{\nabla}\times\mathbf{v} \equiv 0$ and $\tau_{ij} \equiv 0$.

Going to Fourier space, and introducing $\theta \equiv \partial_i v^i$, we get

$$\begin{aligned} \dot\delta + \theta \;&=\; -\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\alpha(\mathbf{k},\mathbf{p})\,\theta_{\mathbf{p}}\,\delta_{\mathbf{k}-\mathbf{p}} \;\equiv\; [\theta\star\delta]_{\mathbf{k}}\,, \\ \dot\theta + \mathcal{H}\theta + \frac{3}{2}\Omega_m\mathcal{H}^2\,\delta \;&=\; -\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\beta(\mathbf{k},\mathbf{p})\,\theta_{\mathbf{p}}\,\theta_{\mathbf{k}-\mathbf{p}} \;\equiv\; [\theta\star\theta]_{\mathbf{k}}\,, \end{aligned}$$

where the kernel functions are

$$\begin{aligned} \alpha(\mathbf{k},\mathbf{p}) &\equiv \frac{\mathbf{k}\cdot(\mathbf{k}+\mathbf{p})}{k^2}\,, \\ \beta(\mathbf{k},\mathbf{p}) &\equiv \frac{1}{2}(\mathbf{k}+\mathbf{p})^2\,\frac{\mathbf{k}\cdot\mathbf{p}}{k^2p^2}\,. \end{aligned}$$

Schematically, we can write

$$\mathcal{D}\phi = \phi\star\phi\,, \quad \text{where} \quad \phi \equiv \begin{pmatrix}\delta \\ \theta\end{pmatrix}.$$

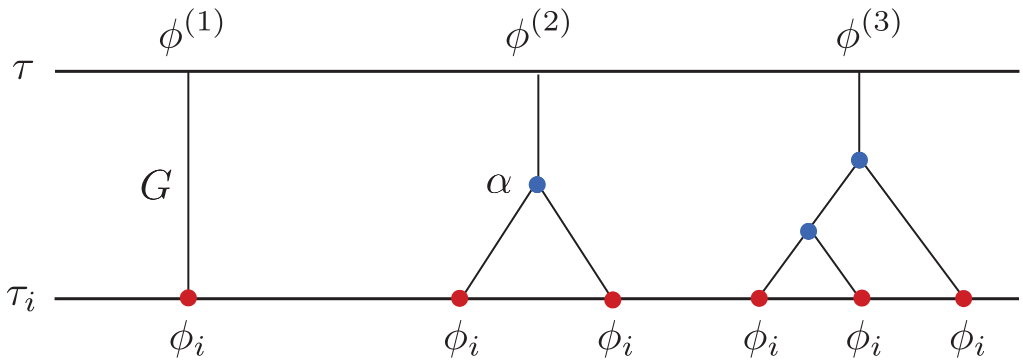
We solve this iteratively

$$\phi = \phi^{(1)} + \phi^{(2)} + \phi^{(3)} + \cdots$$

where

$$\begin{aligned} \phi^{(1)}(\tau) &= G(\tau,\tau_i)\phi_i\,, \\ \phi^{(2)}(\tau) &= \int_{\tau_i}^{\tau} \mathrm{d}\tau'\,G(\tau,\tau')\,\phi^{(1)}(\tau')\star\phi^{(1)}(\tau')\,, \\ \phi^{(3)}(\tau) &= \int_{\tau_i}^{\tau} \mathrm{d}\tau'\,G(\tau,\tau')\,\phi^{(1)}(\tau')\star\phi^{(2)}(\tau')\,. \end{aligned}$$

Diagrammatically, this can be represented as



• DM in EdS

For $\Omega_m = 1$, the Green's function simplifies and we can perform the time integrals:

$$\delta(\mathbf{k},\tau) = \sum_{n=1}^{\infty} \delta^{(n)}(\mathbf{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\, \delta_n(\mathbf{k})\,,$$

with

$$\delta_n(\mathbf{k}) \equiv \int_{\mathbf{p}_1} \dots \int_{\mathbf{p}_n} \delta_D(\mathbf{p}_1 + \dots + \mathbf{p}_n - \mathbf{k})\, F_n(\mathbf{p}_1, \dots, \mathbf{p}_n)\, \delta_{\rm in}(\mathbf{p}_1) \cdots \delta_{\rm in}(\mathbf{p}_n)\,.$$

The kernel functions satisfy

$$\lim_{q\to\infty} F_n(\mathbf{p_1}, \dots, \mathbf{p}_{n-2}, \mathbf{q}, -\mathbf{q}) \propto \frac{p^2}{q^2}\,,$$

where $\mathbf{p} \equiv \mathbf{p_1} + \dots + \mathbf{p}_{n-2}$.

• One-loop power spectrum

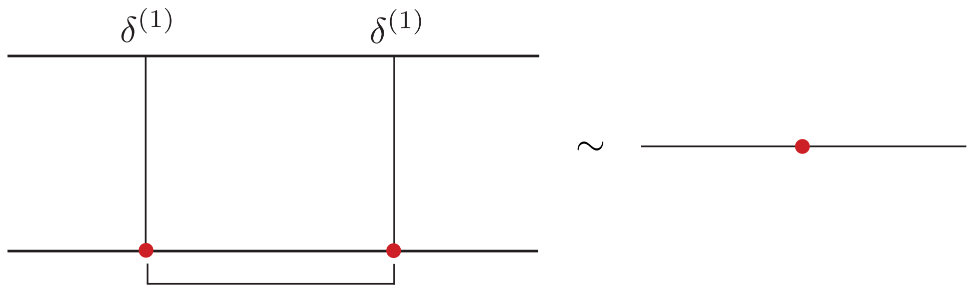
The power spectrum has the following perturbative expansion

tree level one-loop

$$\begin{aligned} \langle \delta\delta \rangle &= \langle \delta^{(1)}\delta^{(1)} \rangle + \langle \delta^{(2)}\delta^{(2)} \rangle + 2\,\langle \delta^{(1)}\delta^{(3)} \rangle + \cdots \\ &\equiv \quad P_{11}(k) \;+\; P_{22}(k) \;+\; 2\,P_{13}(k) \;+ \cdots \end{aligned}$$

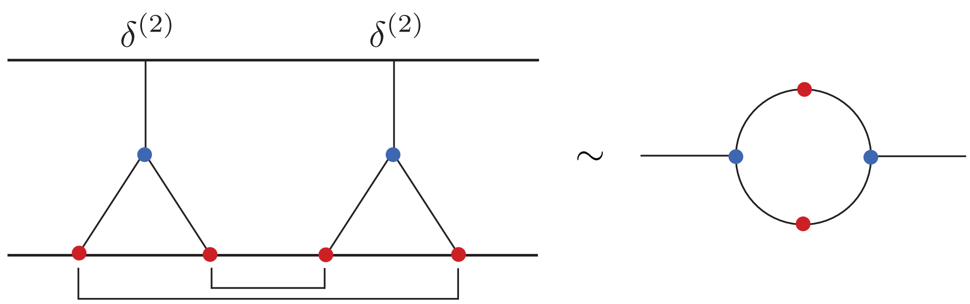
where

$$P_{11}(k) \;=$$



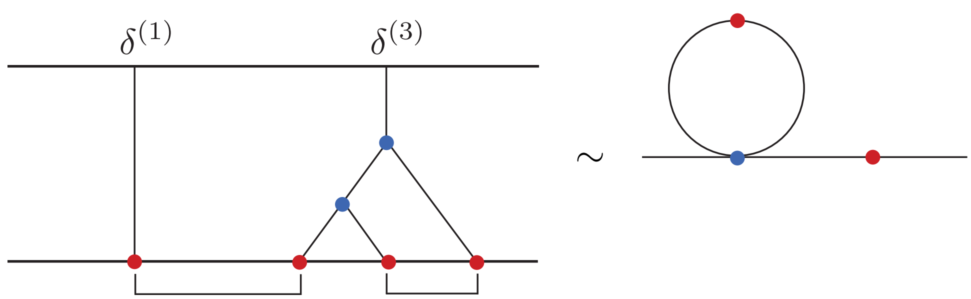
$$\equiv \; P(k)$$

$$P_{22}(k) \;=$$



$$= \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\, P(q)P(|\mathbf{k}-\mathbf{q}|)\, |F_2(\mathbf{q},\mathbf{k}-\mathbf{q})|^2$$

$$P_{13}(k) \;=$$



$$= \; 3P(k)\int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\, P(q)\, F_3(\mathbf{k},\mathbf{q},-\mathbf{q})$$

• Scaling universe

For purposes of illustration, let us use scale-invariant initial conditions

$$P(k) \propto k^n \,.$$

Using the $q \to \infty$ limit of the kernel functions, we find

$$\begin{aligned} &\qquad\qquad\qquad\quad \text{momentum cutoff} \\ &\qquad\qquad\qquad\qquad \downarrow \\ P_{13}(k) &\propto k^2 P(k) \Lambda^{n+1} \\ P_{22}(k) &\propto k^4 \Lambda^{2n-1} \\ &\quad\ \uparrow \\ &\ \text{contact term} \end{aligned}$$

which are both divergent for $n > -1$.

In the real universe, the result is finite, but cutoff dependent.

The theory needs to be *renormalized*.

• Renormalized dark matter

For $k < \Lambda$, we define a new perturbative expansion

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} - c_s^2(\Lambda) k^2 \delta^{(1)} + \delta_J \,.$$

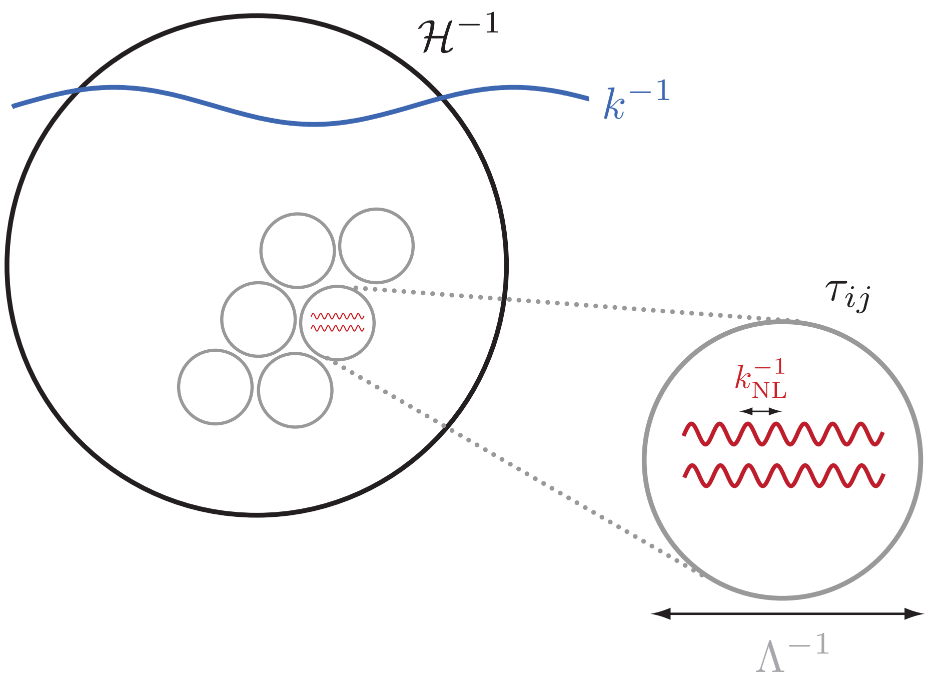
The renormalized power spectrum is

$$\begin{aligned} P_{13}(k) &= \langle \delta^{(1)} \delta^{(3)} \rangle - c_s^2(\Lambda) k^2 \langle \delta^{(1)} \delta^{(1)} \rangle \\ P_{22}(k) &= \langle \delta^{(2)} \delta^{(2)} \rangle + \langle \delta_J \delta_J \rangle \end{aligned}$$

Where do these counterterms come from?

3. EFFECTIVE FIELD THEORY APPROACH

Coarse-graining the Boltzmann equation for the dark matter gives an effective stress tensor τ_{ij} which includes the necessary counterterms:



To construct the effective theory, we split all fields into long and short modes

$$X = X_L + X_S\,,$$

with

$$X_L \equiv [X]_\Lambda = \int \mathrm{d}^3x'\, W_\Lambda(|\mathbf{x}-\mathbf{x}'|)\, X(\mathbf{x}')\,.$$

The effective stress tensor is made out of products of short modes:

$$\tau_{ij} \;=\; \bar\rho v_i^S v_j^S - \frac{1}{8\pi G}\Big[\Phi^S_{,k}\Phi^S_{,k}\delta_{ij} - 2\Phi^S_{,i}\Phi^S_{,j}\Big]\,.$$

The short modes are in the nonlinear regime and therefore not computable in perturbation theory.

We “integrate out” the short modes by writing

$$[\tau_{ij}]_\Lambda \;=\; \underset{\substack{\uparrow\\ \text{expectation value}}}{\langle[\tau_{ij}]_\Lambda\rangle} \;+\; \underset{\substack{\uparrow\\ \text{stochastic term}}}{\Delta\tau_{ij}}$$

The expectation value depends on the presence of the long-wavelength fields and can be written as

$$\partial^i \partial^j \langle [\tau_{ij}]_\Lambda \rangle = \bar{\rho} \left[c_s^2(\Lambda)\, \partial^2 \delta_L - \eta(\Lambda)\, \frac{\partial^2 \theta_L}{\mathcal{H}} + \cdots \right].$$

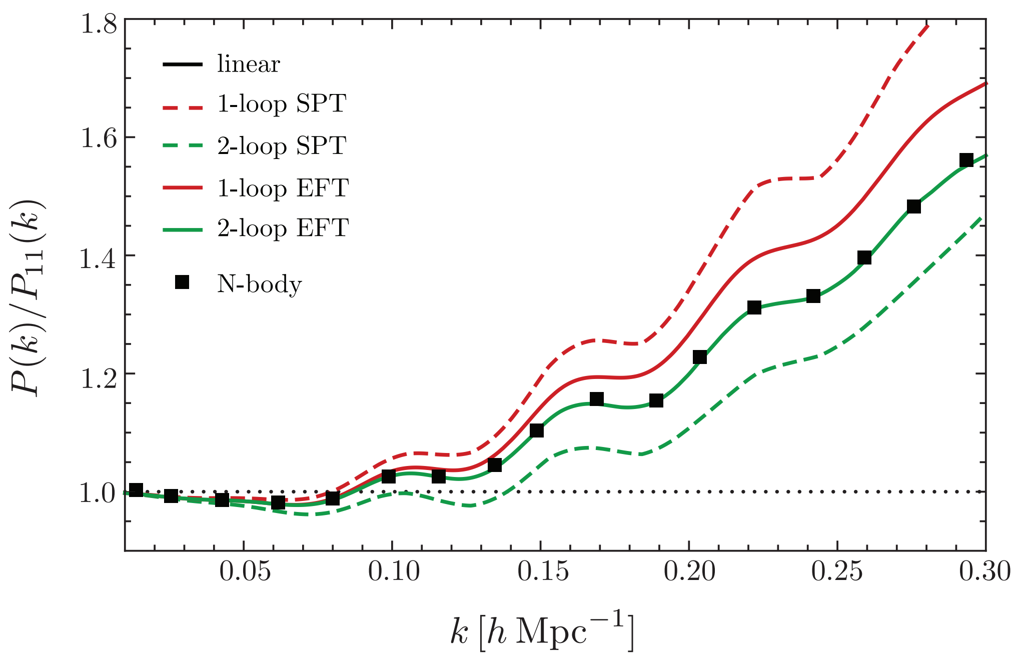
sound speed viscosity

The effective theory for the long-wavelength fields includes precisely the terms required for renormalization:

$$\begin{aligned} \dot{\theta}_L + \mathcal{H}\theta_L + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_L &= \theta_L \star \theta_L - \frac{1}{\rho}\partial^i \partial^j \tau_{ij} \\ &= \theta_L \star \theta_L - c_s^2\, \partial^2 \delta_L - \Delta J + \cdots \end{aligned}$$

The finite parts of the coefficients of the EFT have to measured (from simulations or data).

After renormalization, the theory is better-behaved in the UV:



4. OUTLOOK

The EFT of LSS is the right way to describe the effects on short-scale nonlinearities on quasi-linear modes.

It has been extended to include

- Biasing
- Redshift space distortions
- IR resummation
- Primordial non-Gaussianity
- Neutrinos
- Modified gravity

Its application to data analysis remains to be explored.

References

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