

*Several astronomical problems
solved by calibrating
the distribution in projected phase space
with cosmological simulations*



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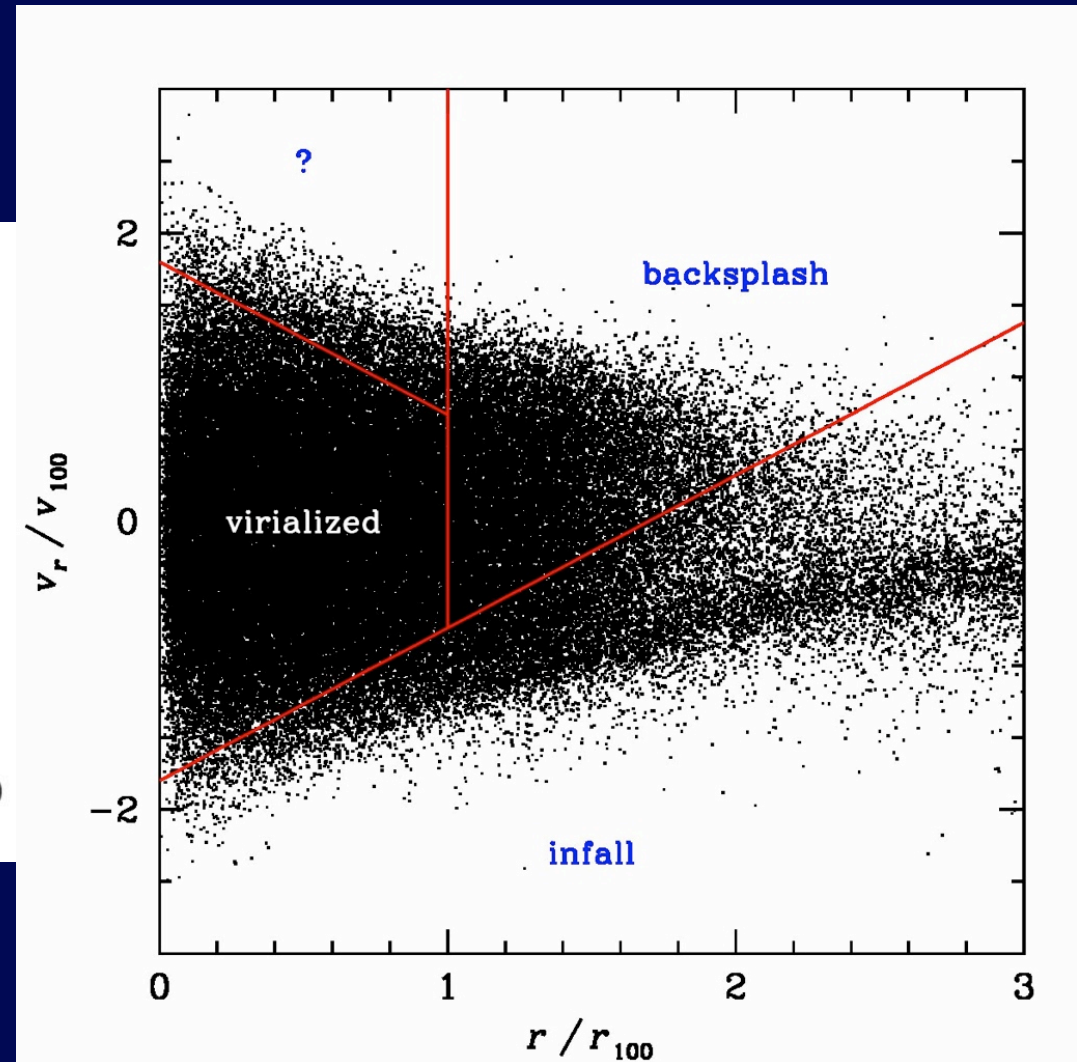
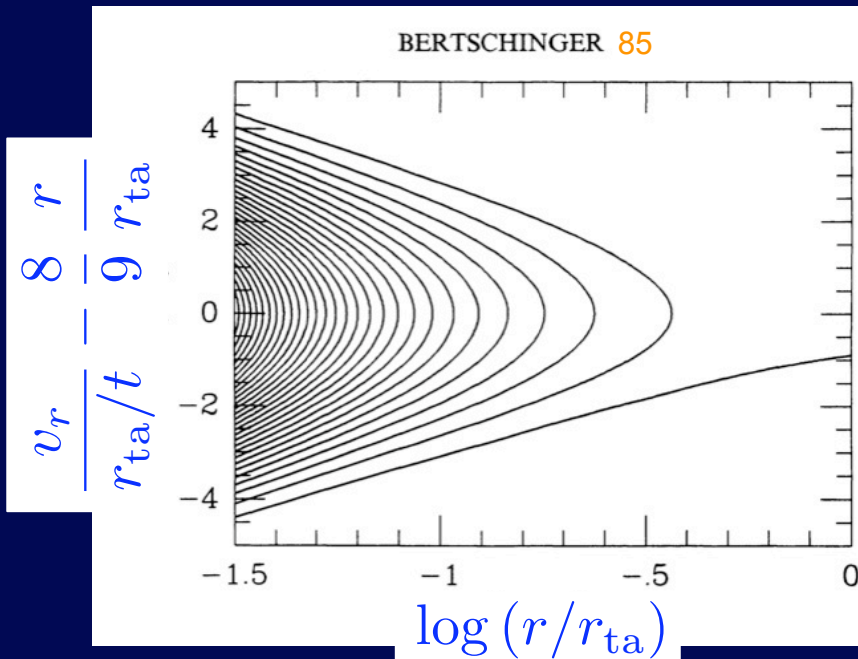


Somak RAYCHAUDHURY
(Birmingham, UK)

Outline

- Phase space & observers projected phase space
- Universal distribution of halo interlopers
- New mass modelling techniques see also R. Wojtak
- Applications:
 - ➔ Dark matter & orbits in quasi-spherical systems
from galaxy clusters see also R. Wojtak *to dwarf spheroidals*
 - ➔ Neutral gas deficient spiral galaxies “in” Virgo cluster
 - ➔ Star formation history of galaxies falling into clusters

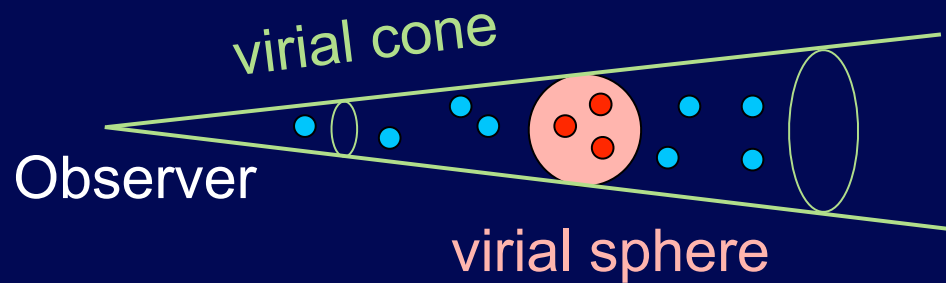
Radial phase space



stack of 93 regular halos of dark matter particles
within hydrodynamical cosmological simulations by **Borgani et al. 04**

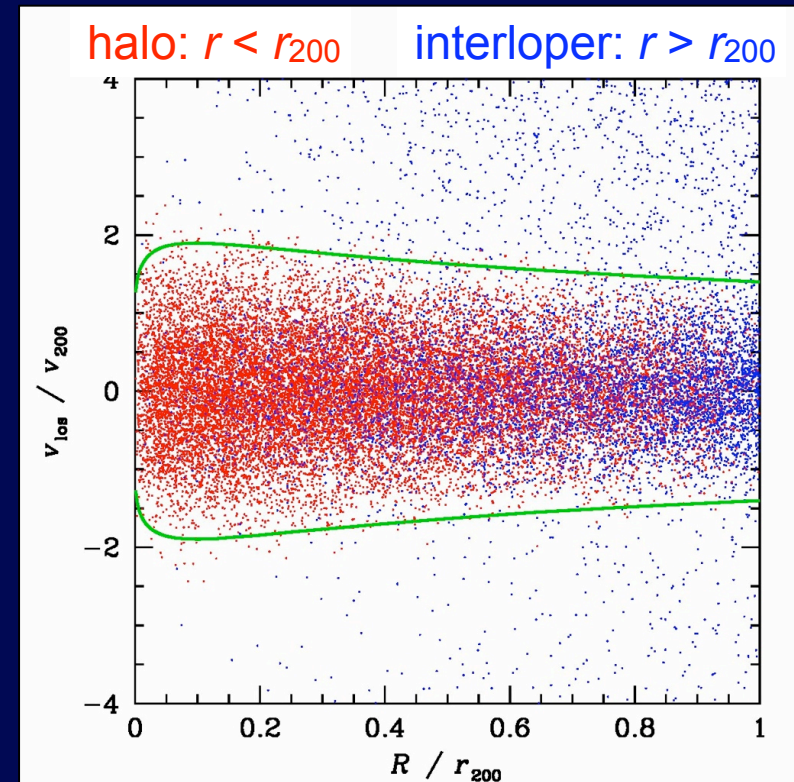
Projected phase space

= observer's viewpoint



- halo particle
- interloper

line-of-sight-velocity

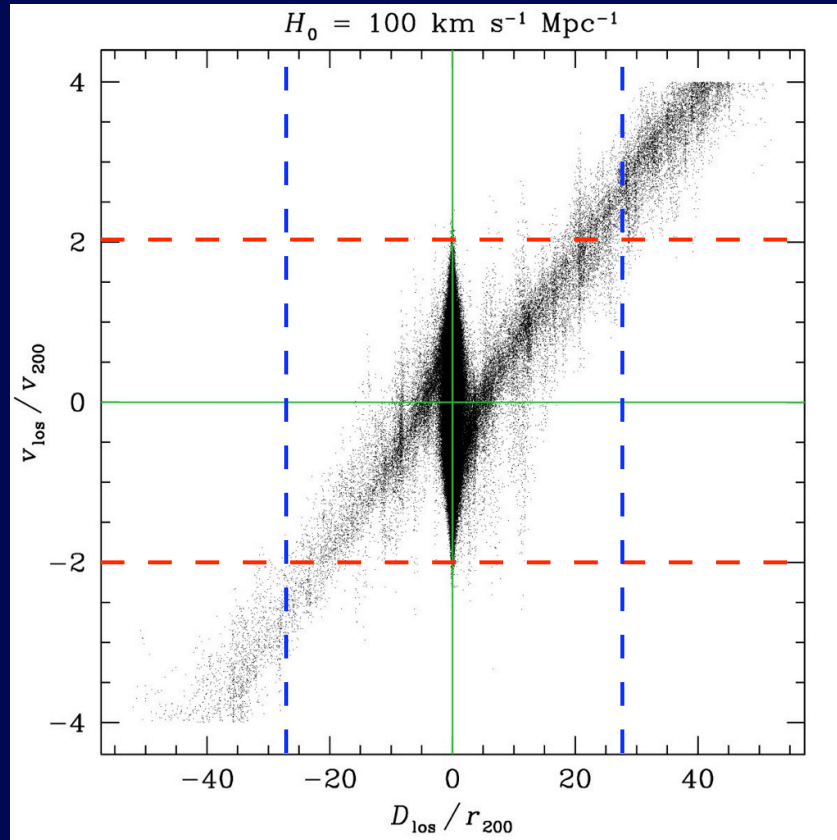


projected radius

How does one learn about full phase space when limited to projected phase space view?

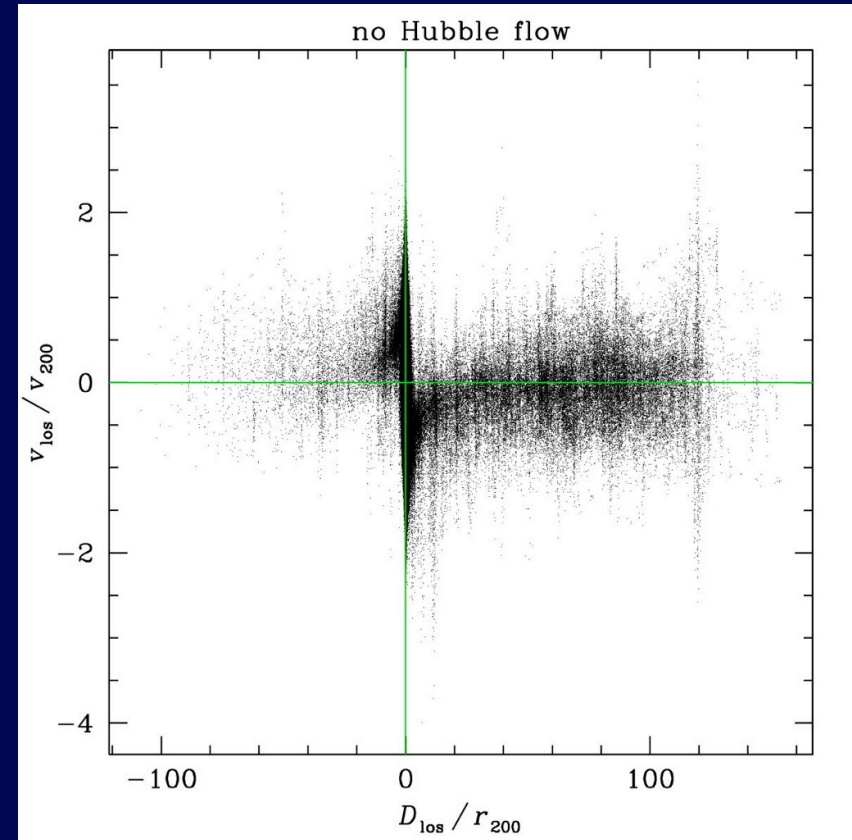
Hubble flow distortion

line-of-sight velocity



line-of-sight distance

line-of-sight velocity



line-of-sight distance

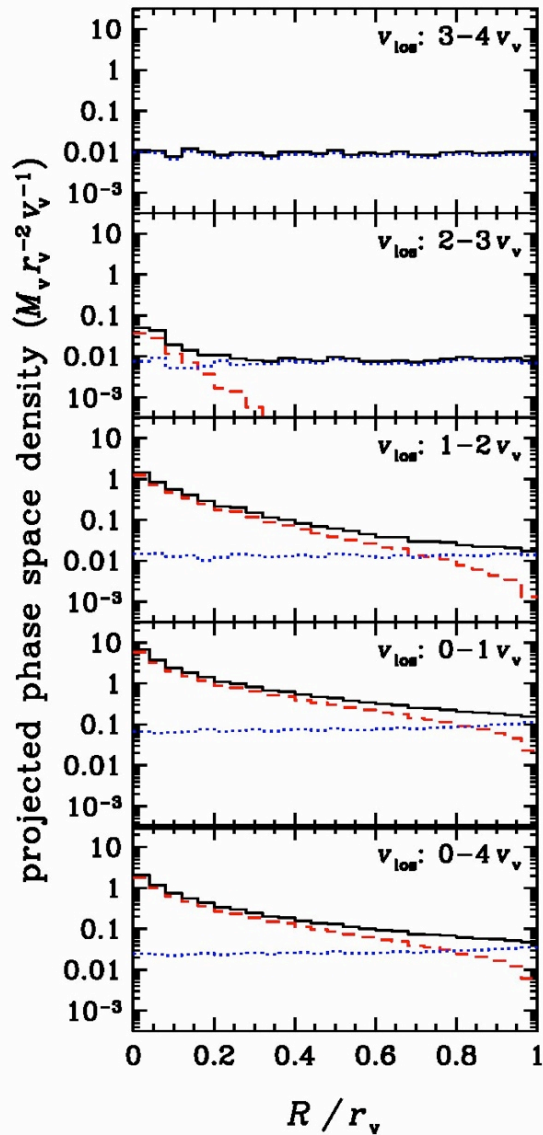
Hubble flow:

- $\pm 3\sigma_v$ cuts ~all particles beyond $\sim 27 r_{200}$
- what effect on projected phase space?

*Universal distribution
of halo & interloper particles
in projected phase space*

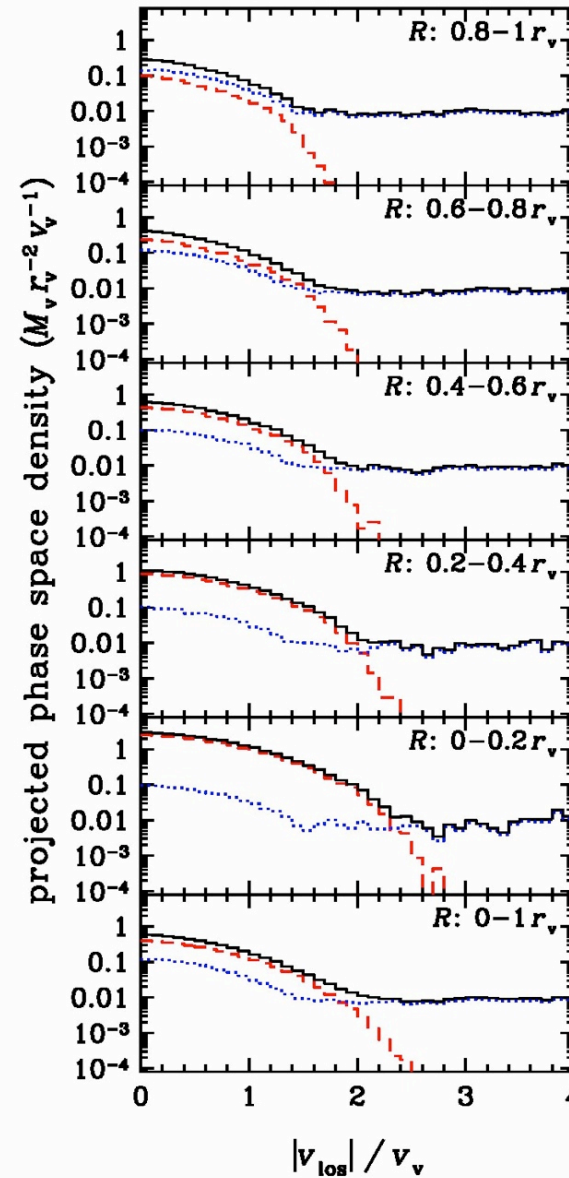
Cuts through projected phase space

Mamon, Biviano & Murante in prep.



nearly constant surface density

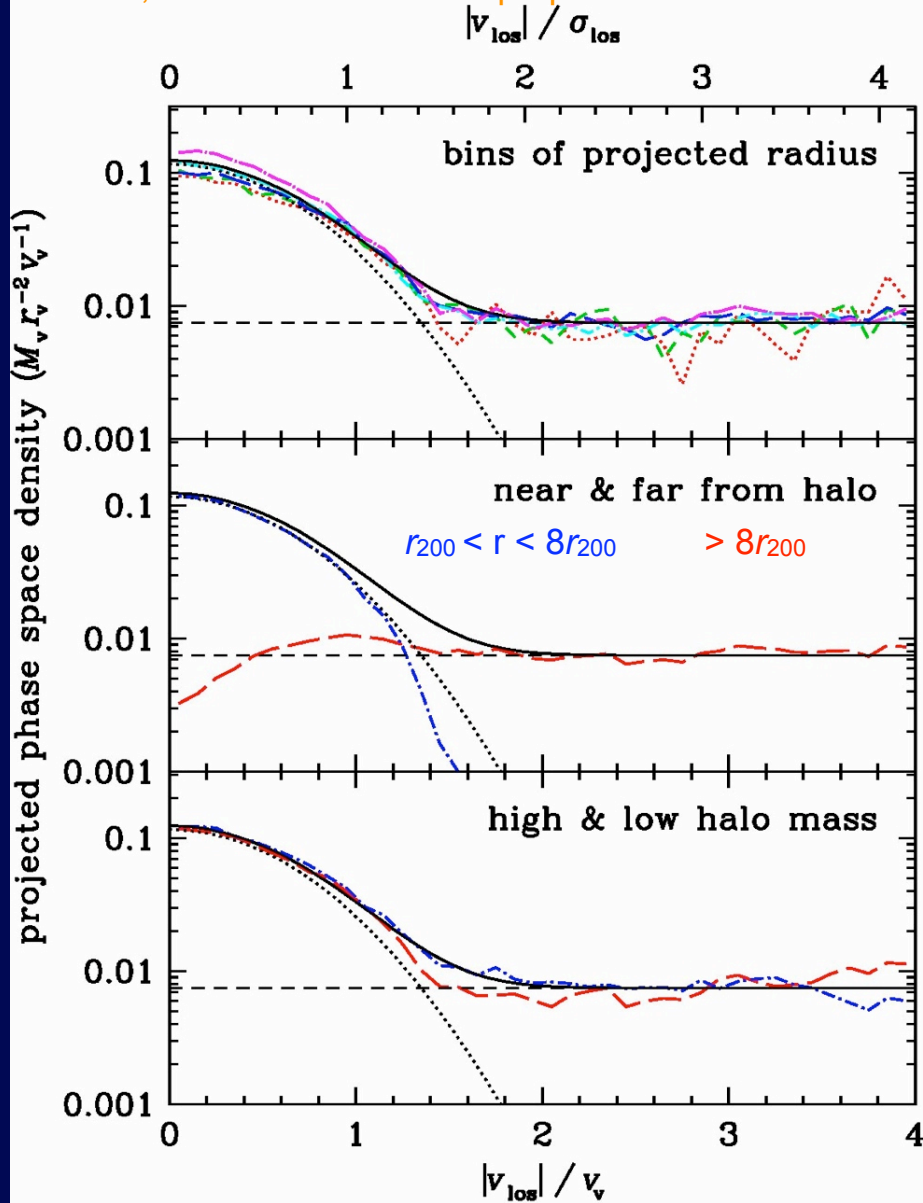
Mamon, Biviano & Murante in prep.



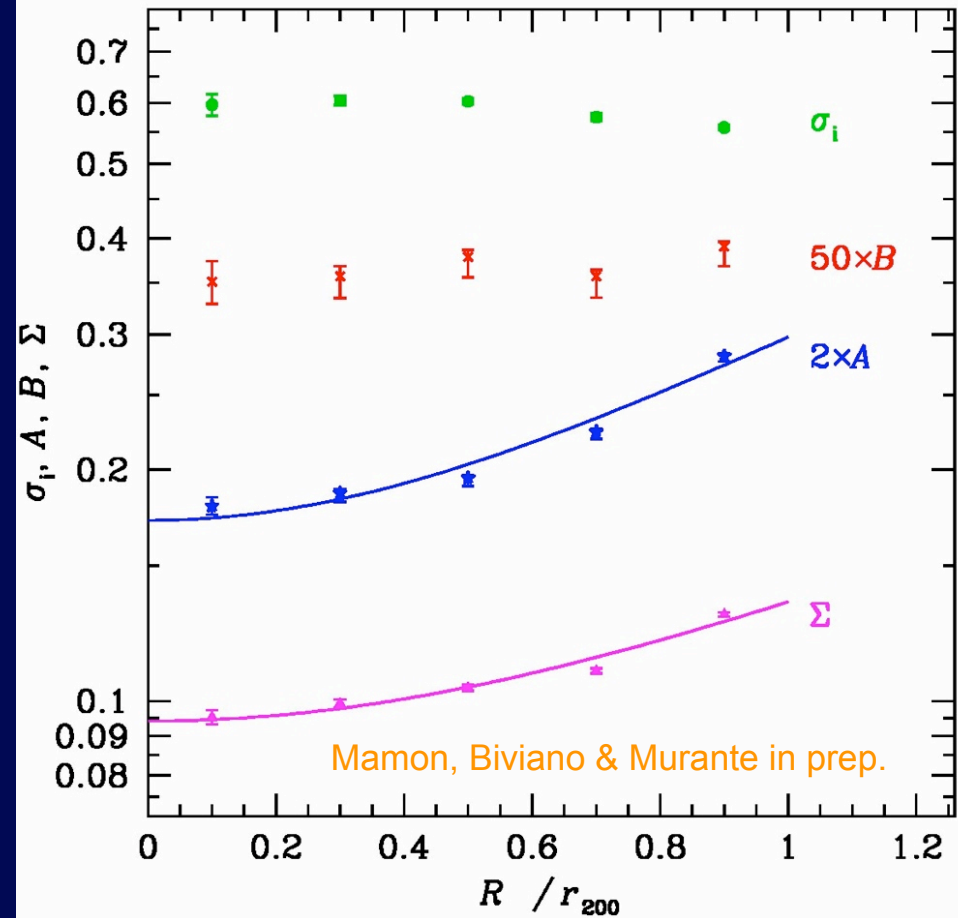
gaussian + constant velocity distribution

Universality

Mamon, Biviano & Murante in prep.



$$g(R, |v_{\text{los}}|) = A \exp \left[-\frac{1}{2} \left(\frac{|v_{\text{los}}|}{\sigma_i} \right)^2 \right] + B$$



*Old & New
Mass Modeling
Techniques
using internal motions*

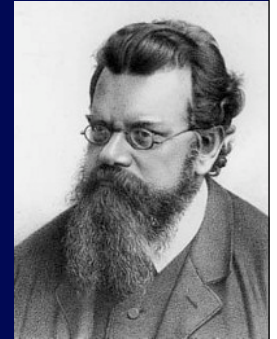
From phase space to local space

$f = f(r, v) \equiv$ distribution function = 6D phase space density

Collisionless Boltzmann Equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla \Phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

test particles in potential
= incompressible 6D fluid

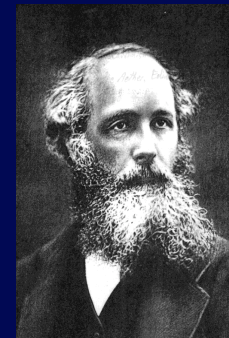


$$\int v_j \text{CBE } d^3 \mathbf{v}$$

$$\nabla \cdot \mathbf{P} = -\nu \nabla \Phi$$

Jeans Equation

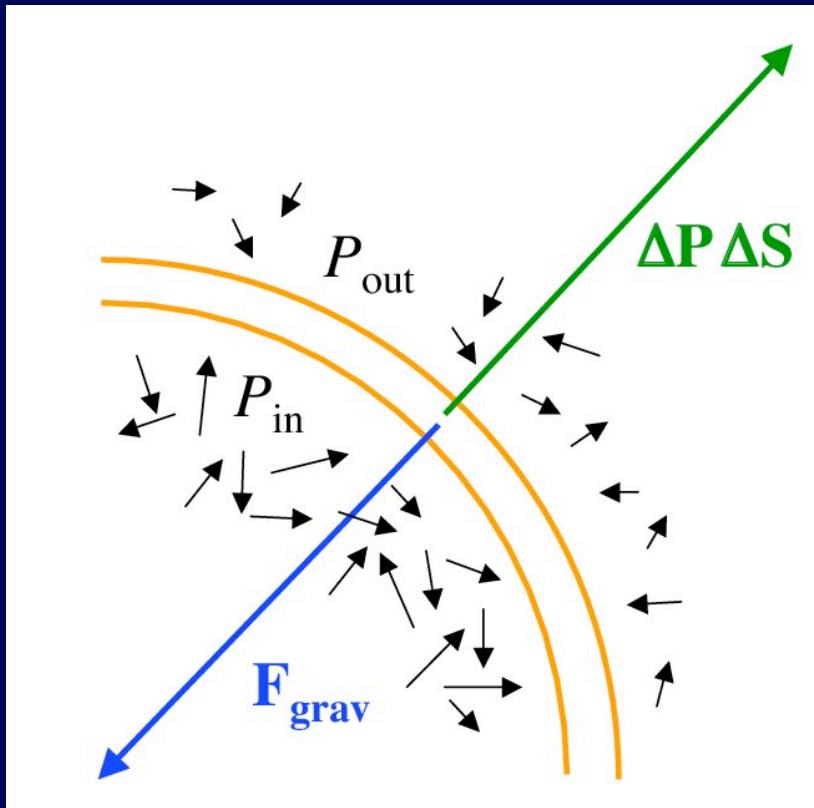
tracer density



Maxwell



Jeans



2 classes of mass & anisotropy modelling

- Jeans analysis
- Fitting the projected phase space distribution

Spherical stationary Jeans equation

tracer density

anisotropic dynamical pressure

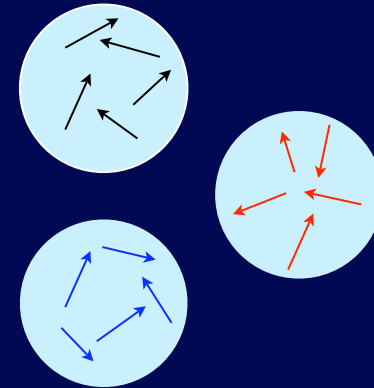
$$\frac{d(\nu\sigma_r^2)}{dr} + 2\frac{\beta(r)}{r}\nu\sigma_r^2 = -\nu\frac{GM(r)}{r^2}$$

$$\beta(r) = 1 - \frac{\sigma_\theta^2(r)}{\sigma_r^2(r)} = \text{velocity anisotropy}$$

isotropic orbits: $\beta = 0$

radial orbits: $\beta = 1$

circular orbits: $\beta \rightarrow -\infty$



mass / anisotropy degeneracy

MAD

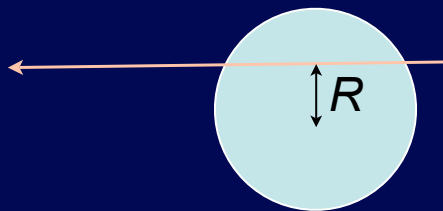
Jeans 1) assume both $M(r)$ & $\beta(r)$ & fit the projected velocity dispersions

for $\beta = 0$

line-of-sight velocity dispersion

Tremaine et al. 94; Prugniel & Simien 97

surface density



$$\Sigma(R) \sigma_{\text{los}}^2(R) = 2G \int_R^{\infty} \frac{\sqrt{r^2 - R^2}}{r^2} v(r) M(r) dr$$

kernels for other simple $\beta(r)$

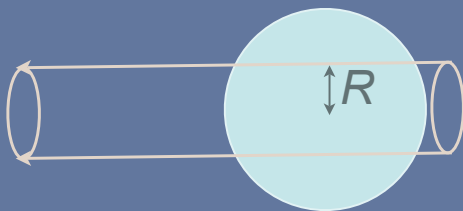
Mamon & Łokas 05b

axisymmetric models

Cappellari 08

aperture velocity dispersion

Mamon & Łokas 05a



$$\frac{3}{4\pi G} M_2(R) \sigma_{\text{ap}}^2(R) = \int_0^{\infty} r v M dr - \int_R^{\infty} \frac{(r^2 - R^2)^{3/2}}{r^2} v M dr$$

Jeans 2) assume either $M(r)$ or $\beta(r)$

given projected observations:

surface density $\Sigma(R)$

line of sight velocity dispersion $\sigma_{\text{los}}(R)$

2a) Anisotropy inversion

assume $M(r) \rightarrow \beta(r)$

Binney & Mamon 82

Tonry 83; Bicknell et al. 89

Solanes & Salvador-Solé 90

Dejonghe & Merritt 92

Jeans 2b) Mass inversion

Mamon & Boué 09, MNRAS in press
Wolf et al. 09, MNRAS submitted

$p = \rho \sigma_r^2 =$ dynamical pressure

$P = \Sigma \sigma_{\text{los}}^2 =$ projected pressure

anisotropic kinematic projection

$$P(R) = 2 \int_R^{\infty} \left(1 - \beta \frac{R^2}{r^2}\right) p \frac{r dr}{\sqrt{r^2 - R^2}}$$

anisotropic kinematic deprojection

$$(1 - \beta) p(r) = \int_r^{\infty} K_{\beta}(R, r) \frac{dP}{dR} dR$$

$$\rho v_c^2 = -p' - 2 \frac{\beta}{r} p$$

insert dynamical pressure into **Jeans equation** → mass profile

Jeans 3) combine dispersion & kurtosis assuming cst β

4th moment Jeans equations

$$\frac{d(\nu \overline{v_r^4})}{dr} + 2 \frac{\beta}{r} (\nu \overline{v_r^4}) = -3 \nu \sigma_r^2 \frac{GM(r)}{r^2}$$

Lokas 02; Lokas & Mamon 03

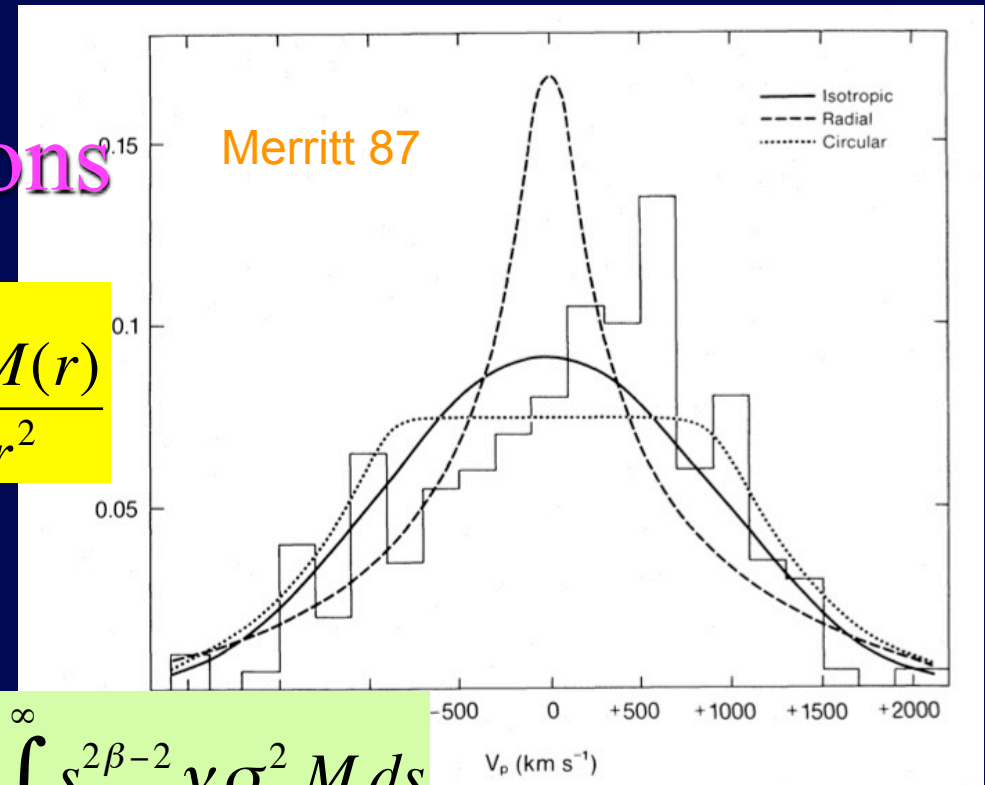
if $\beta = \text{cst}$



$$\overline{v_r^4}(r) = \frac{3Gr^{-2\beta}}{\nu(r)} \int_r^\infty s^{2\beta-2} \nu \sigma_r^2 M ds$$

line of sight kurtosis

$$\kappa_{\text{los}}(R) = \frac{\overline{v_{\text{los}}^4}(R)}{\sigma_{\text{los}}^4(R)} - 3$$



fit to distribution in projected phase space

- Clean method:
projected phase space from distribution function and potential

$$g(R, v_z) = 2 \int_R^\infty \frac{r dr}{\sqrt{r^2 - R^2}} \int_{-\infty}^\infty dv_R \int_{-\infty}^\infty dv_\theta f [1/2v^2 + \Phi(r), J]$$

Dejonghe & Merritt 92

which $f(E, J)$?

- fit from combination of orbits Schwazschild 79; Syer & Tremaine 94
De Lorenzi, Gerhard et al. 07
- fit from combination of elementary $f_i(E, J)$ Merritt & Saha 93; Gerhard et al. 98
- fit from non-parametric $f(E, J)$ Wu & Tremaine 06
- simple $f(E, J)$ from Λ CDM halos Wojtak, Lokas, Mamon, Gottlöber & Klypin 08

fit to distribution in projected phase space

- Clean method:
projected phase space from distribution function and potential

$$g(R, v_z) = 2 \int_R^\infty \frac{r dr}{\sqrt{r^2 - R^2}} \int_{-\infty}^\infty dv_R \int_{-\infty}^\infty dv_\theta f [1/2v^2 + \Phi(r), J]$$

Dejonghe & Merritt 92

which $f(E, J)$?

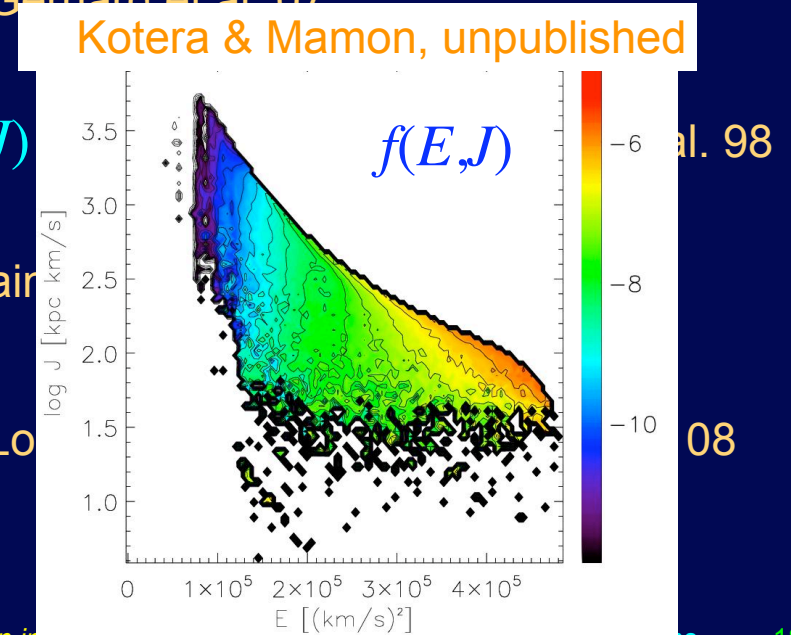
- fit from combination of orbits Schwazschild 79; Syer & Tremaine 94
De Lorenzi, Gerhard et al. 07

- fit from combination of elementary $f_i(E, J)$

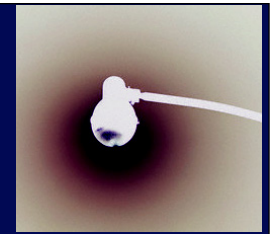
- fit from non-parametric $f(E, J)$ Wu & Tremaine

- simple $f(E, J)$ from Λ CDM halos Wojtak, Lo

- dissipative mergers?



fit to distribution in projected phase space (2)



- Quick method: **MAMPOSSt**
projected phase space from velocity distribution, anisotropy & potential
Mamon, Biviano & Boué, to be submitted soon

assume radial profiles: $M_{\text{tot}}(r)$, $\beta(r)$, $\{v\}(r)$

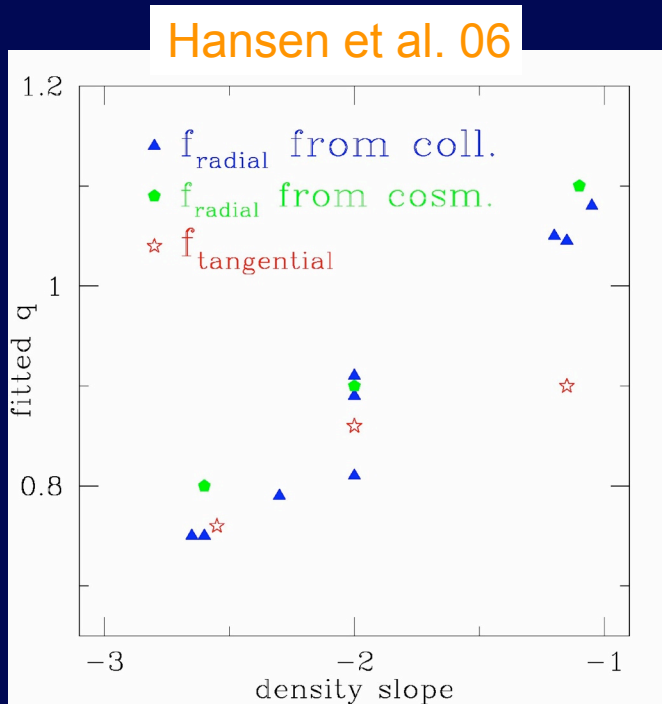
gaussian $\{v\}(r)$

$$p(v_z|R) = \frac{\sqrt{2/\pi}}{\Sigma(R)} \int_R^\infty \frac{r \nu}{\sqrt{r^2 - R^2}} \frac{(1 - \beta R^2/r^2)^{-1/2}}{\sigma_r} \exp\left[-\frac{v_z^2}{2(1 - \beta R^2/r^2) \sigma_r^2}\right] dr$$
$$\sigma_z^2(R, r) = \left[1 - \beta \left(\frac{R}{r}\right)^2\right] \sigma_r^2(r)$$

for academic case of $\beta=\text{cst}$, gaussian-MAMPOSSt predicts:
correct I.o.s. velocity dispersion profile
but incorrect I.o.s. velocity kurtosis profile

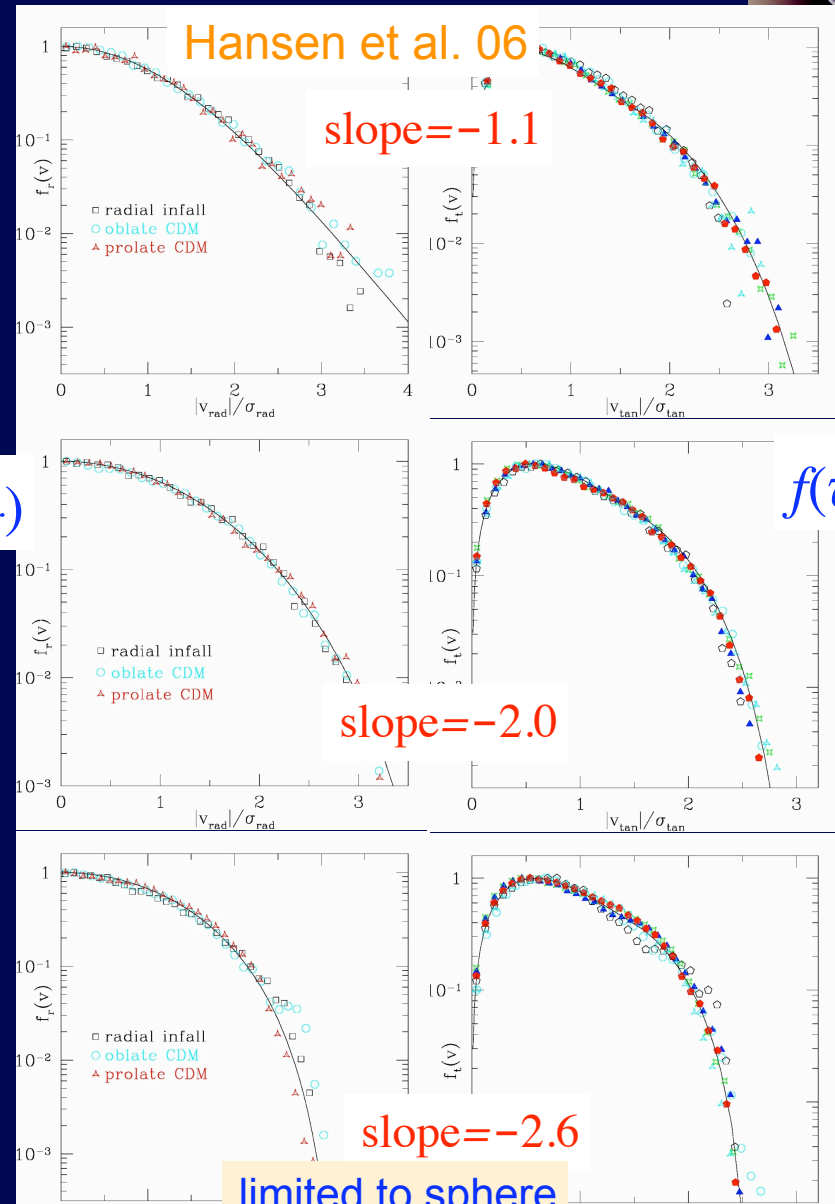
Possible improvements to MAMPOSSt

- More realistic velocity profiles: **variable-index Tsallis**



$$q = a + b \text{ slope}$$

- Universal distribution of interlopers in projected phase space



$$p(v | R) = \frac{g(R, v)}{\Sigma(R)} = \frac{g_{\text{halo}}(R, v) + g_{\text{interlopers}}(R, v)}{\Sigma(R)}$$

kinematical effects of:

- * *non-sphericity*
- * *projected infalling filaments*
- * *substructure*
- * *streaming motions (infall, rebound)*



test with halos from cosmological N -body simulations:
measure in 3D & reestimate in 2D
mass follows number, $\beta = \text{cst}$

dispersion-kurtosis

Sanchis, Łokas & Mamon 04

10 halos \times 3 projections (400 pts / halo)

$$\Delta \log M_{100} = -0.07 \pm 0.10$$

$$\Delta \log c = 0.08 \pm 0.24$$

$$\Delta \log \left(\frac{\sigma_r}{\sigma_\theta} \right) = -0.04 \pm 0.11$$

MAMPOSSt

Mamon, Biviano & Boué in prep.



11 halos \times 3 projections (500 pts / halo)

$$\Delta \log M_{200} = 0.01 \pm 0.09$$

$$\Delta \log c = 0.07 \pm 0.20$$

$$\Delta \log \left(\frac{\sigma_r}{\sigma_\theta} \right) = -0.03 \pm 0.08$$

dispersion-kurtosis: very uncertain concentration

MAMPOSSt: somewhat better!

Comparison with orbit modeling

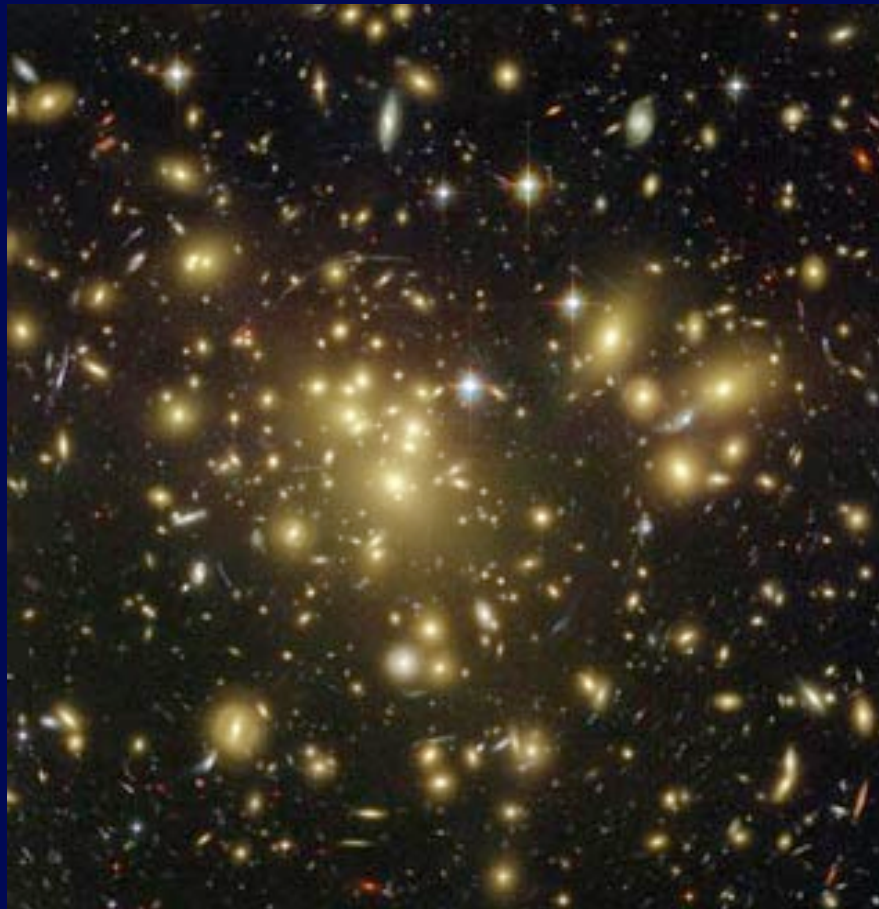
- requires systems not far from spherical symmetry
- rotation, substructure not incorporated

BUT

- dispersion-kurtosis, *MAMPOSSt* & Wojtak method tested on cosmological halos
- mass & anisotropy inversions: non-parametric
- less a black box and more intuition
- fast enough to probe set of solutions (MCMC)

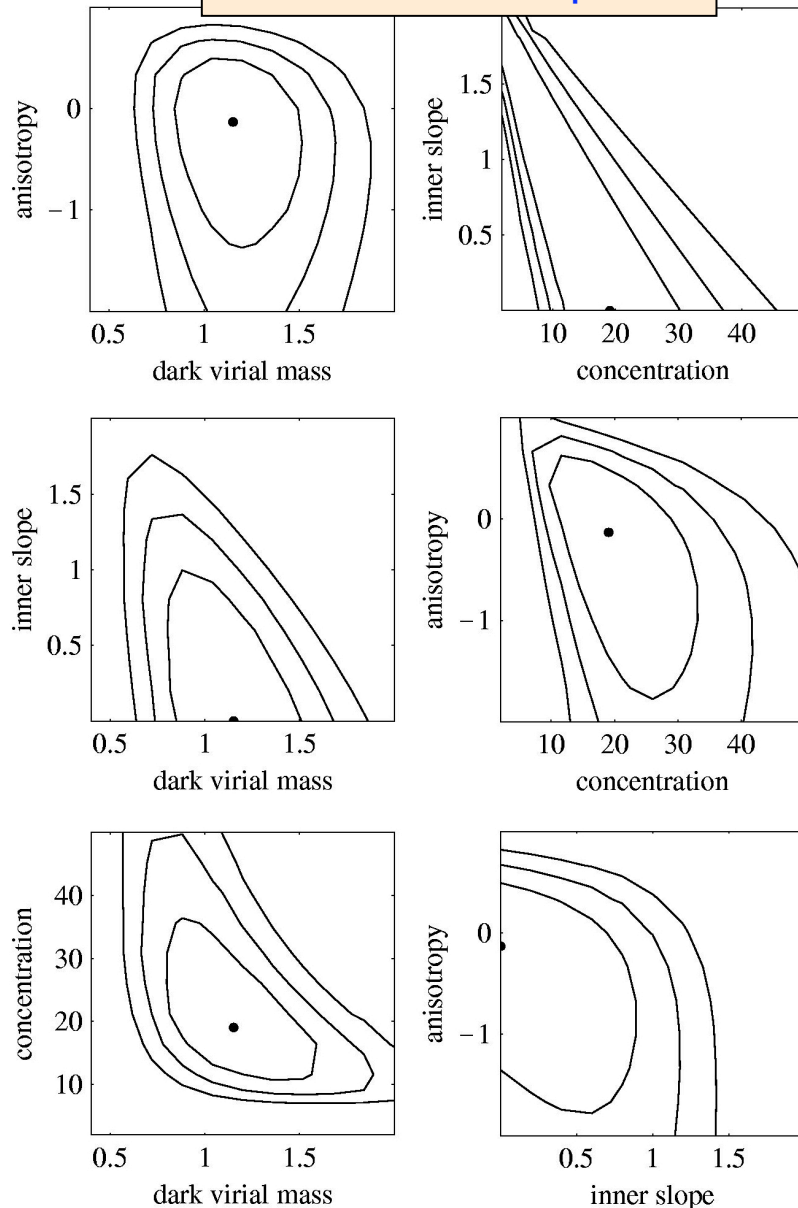
Applications of new mass & anisotropy modelling techniques

Mass & anisotropy modeling of Clusters of galaxies



Partially lifting the anisotropy / mass degeneracy: joint fits to velocity dispersion & kurtosis profiles

Coma cluster ellipticals



isotropic fits best!

cusplike and core
both agree with data

NFW:

$$c = \frac{r_{100}}{r_s} \approx 9.4$$

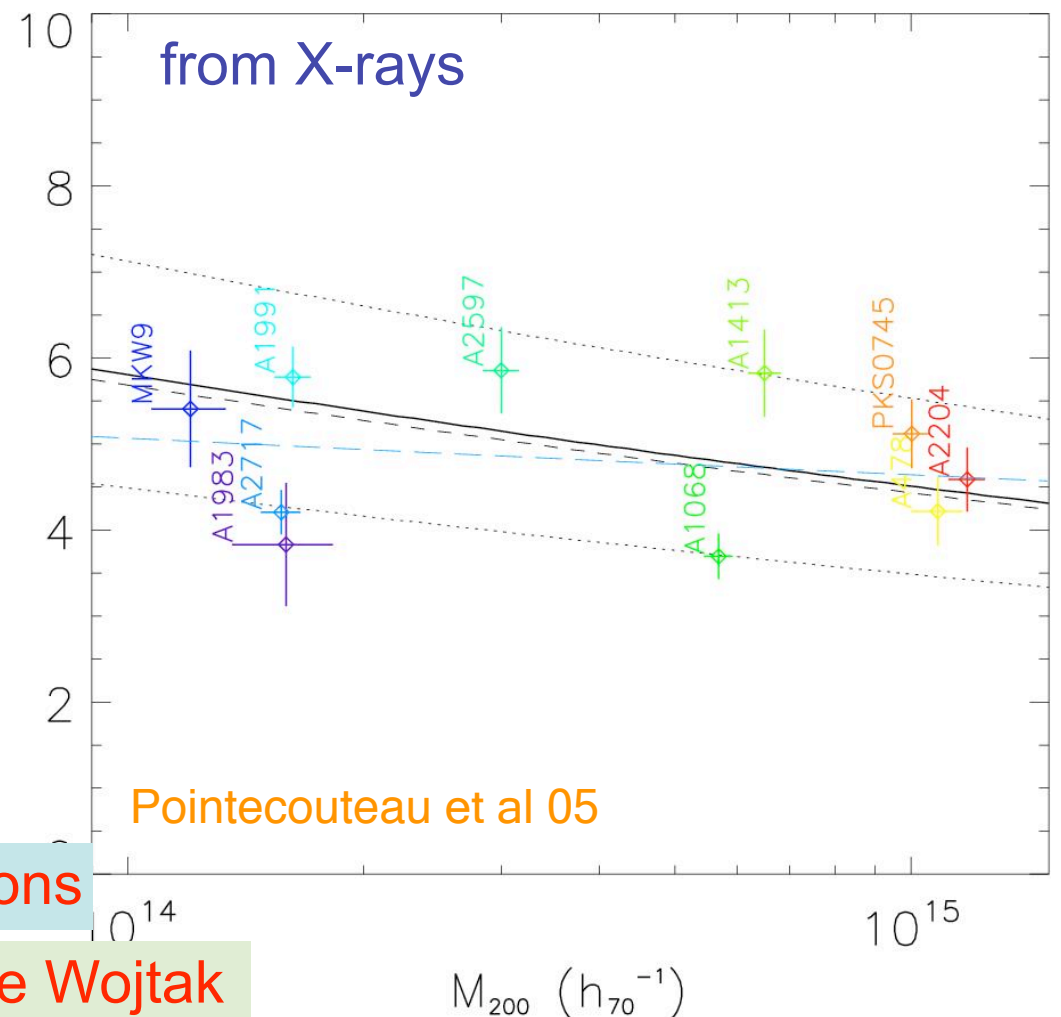
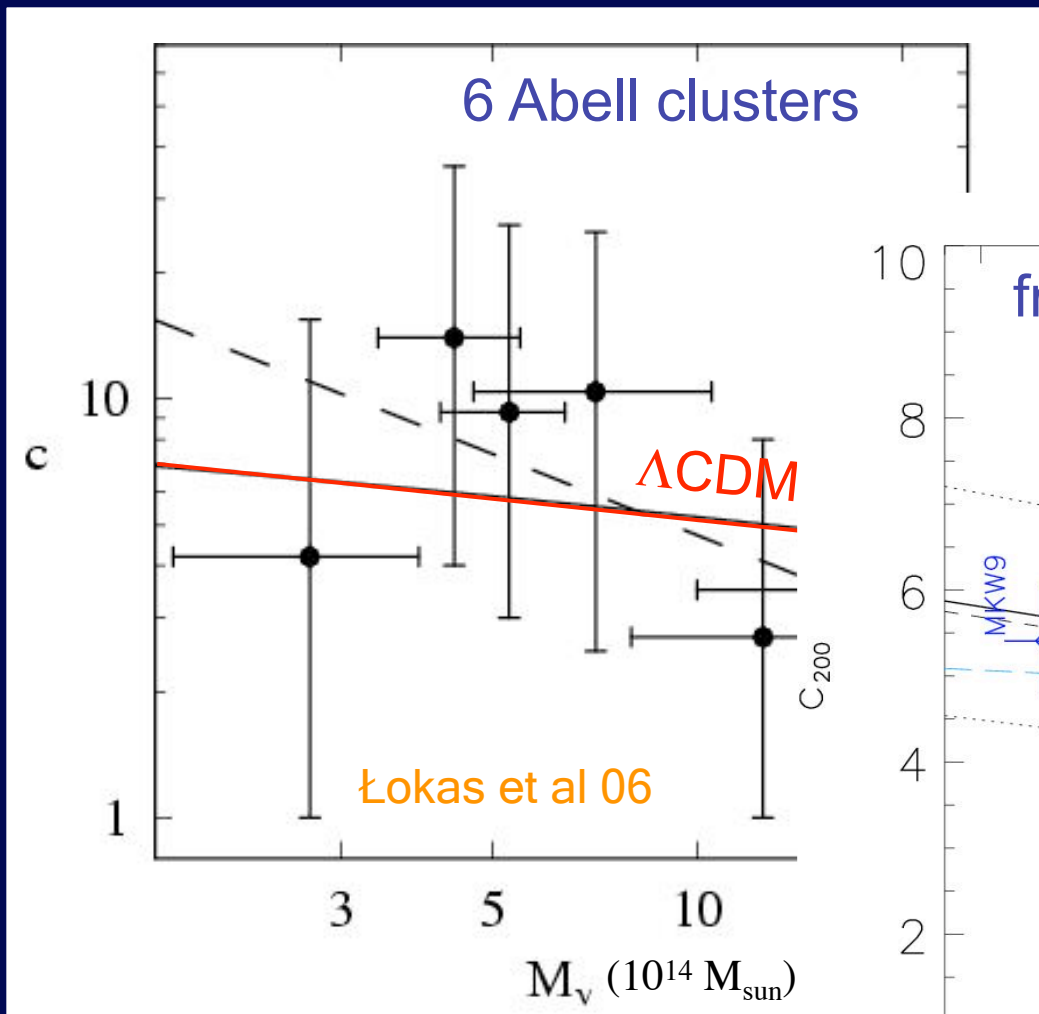
vs.

- $c = 6$ (cosmo sims) Bullock et al. 01
- $c = 5.5$ (stacked 2dFGRS)
- $c = 4$ (stacked ENACS) Biviano & Girardi 03

Katgert, Biviano & Mazure 04

Cluster concentration vs. mass

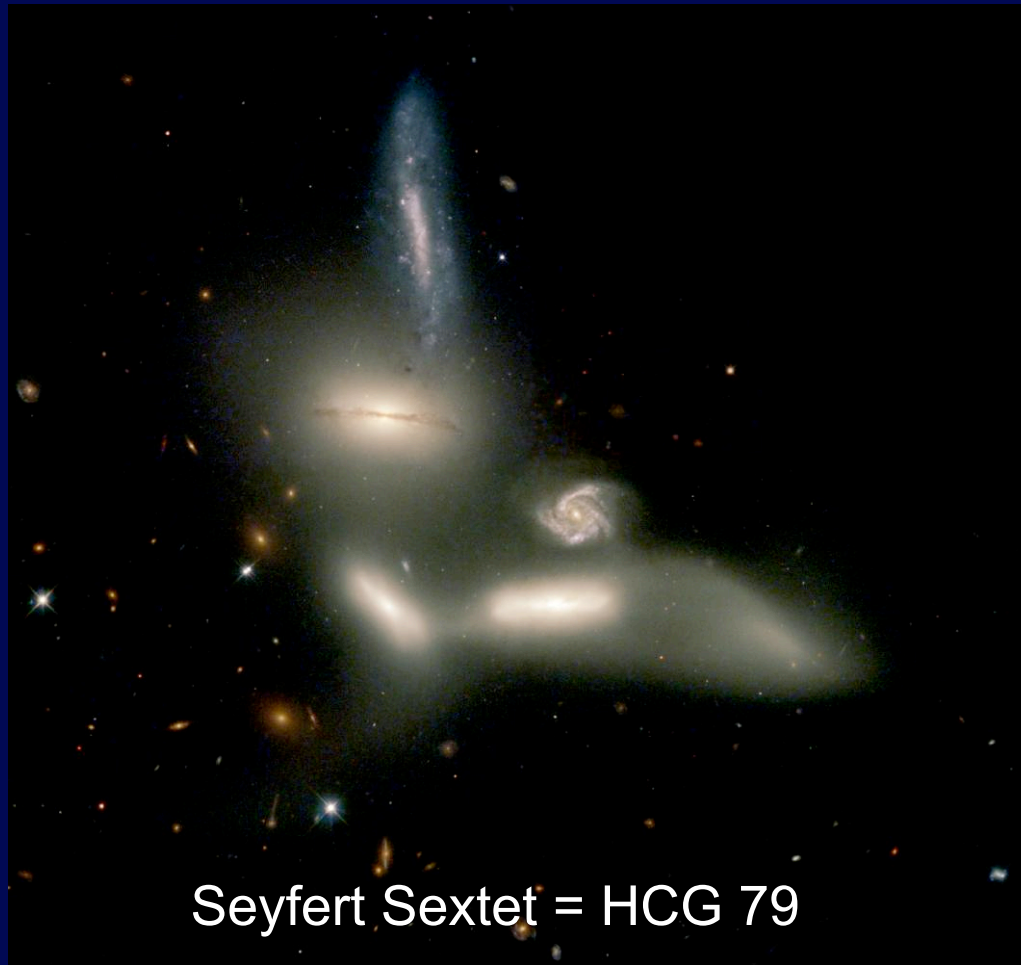
$$= r_{200}/r_{-2}$$



consistent with ΛCDM predictions

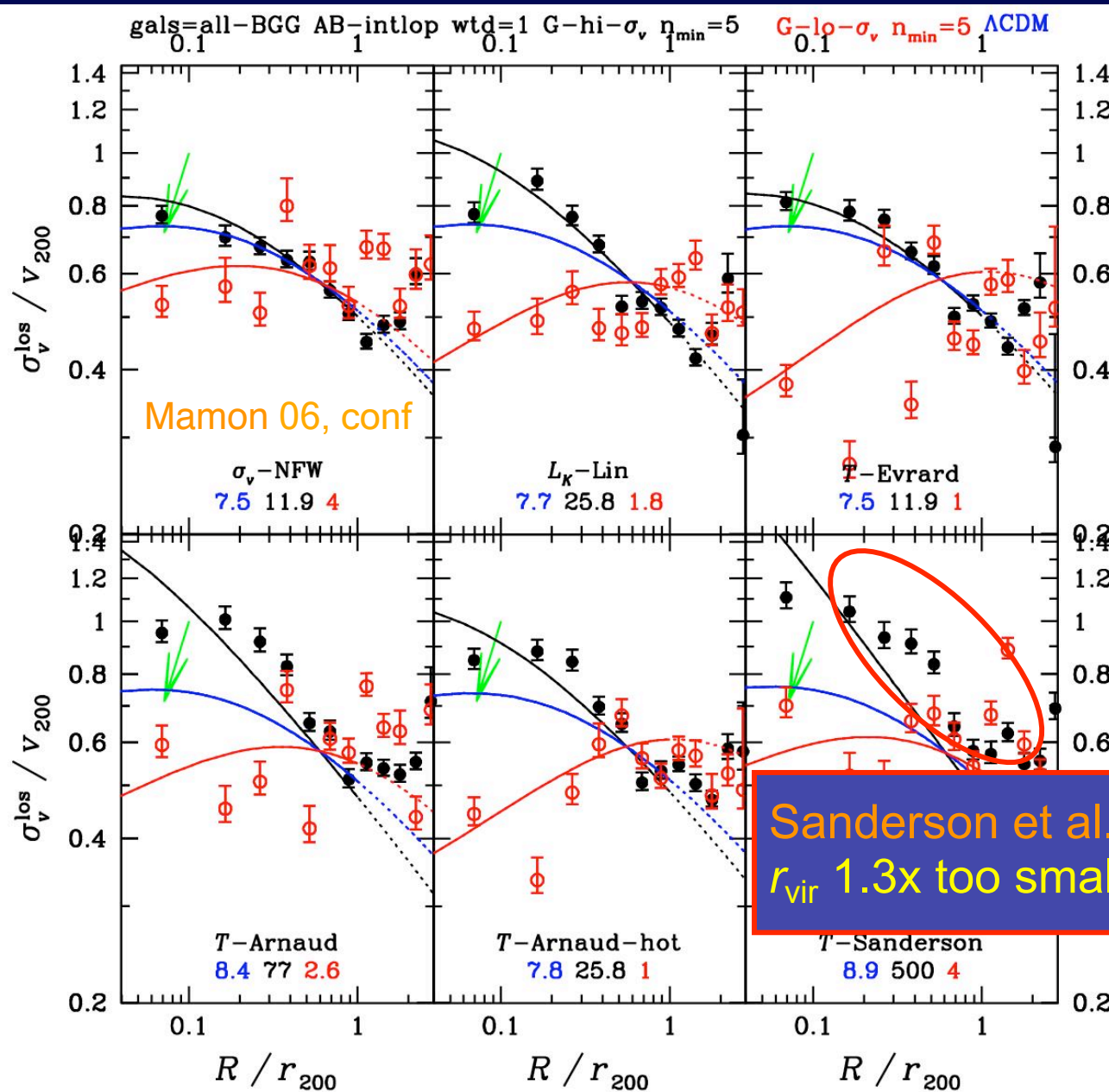
X-rays: less scatter, but see Wojtak

Mass & anisotropy modelling of Groups of Galaxies



Seyfert Sextet = HCG 79

Line-of-sight velocity dispersion profiles of groups: effect of global velocity dispersion



low σ_v (< 300 km/s) groups:
shallower $\rho(r)$
OR
tangential orbits

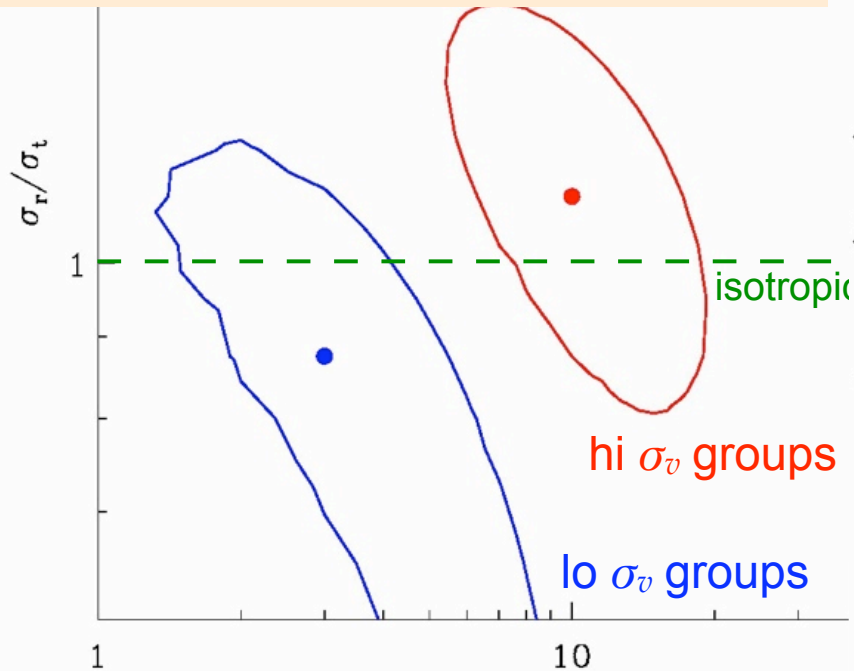
Sanderson et al.:
 r_{vir} 1.3x too small $\Rightarrow M_{\text{vir}}$ 2x too small

Concentration vs. Mass

High- vs Low- σ_v groups - Ah scaling

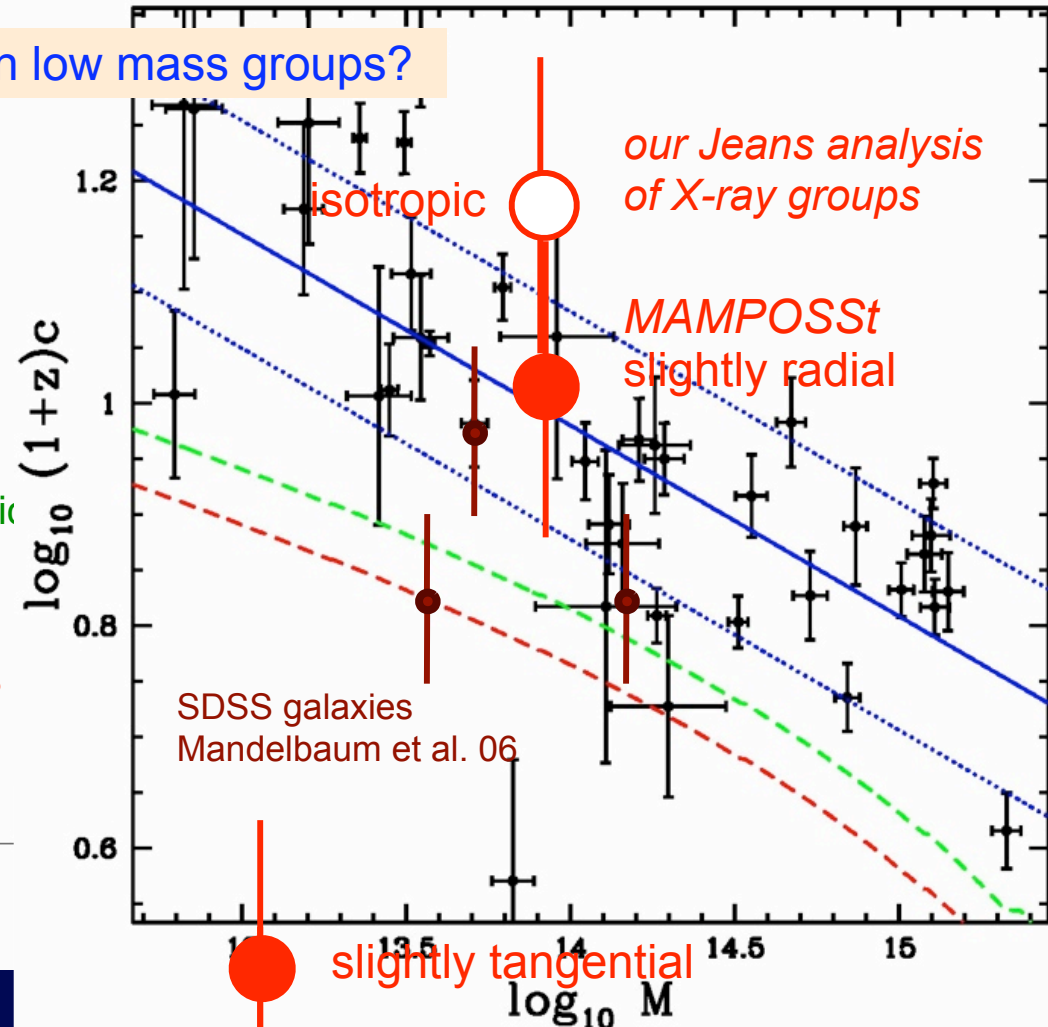
Energy dissipation by dynamical friction in low mass groups?

Irregular potential of low mass groups?

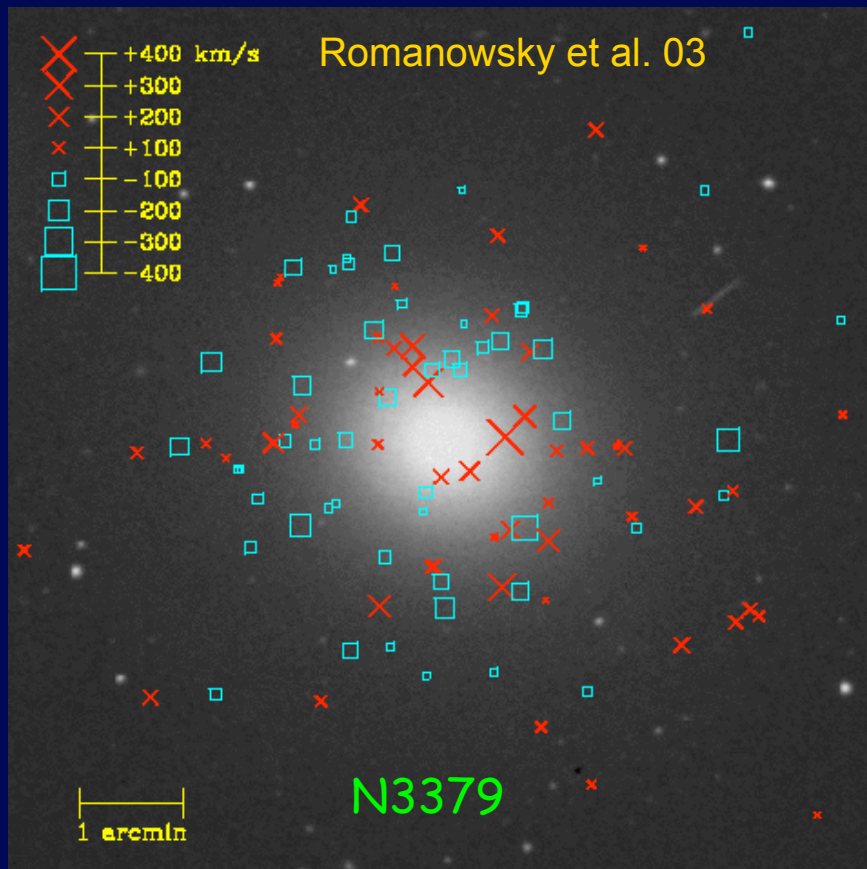


Biviano, Mamon & Ponman in prep

X-ray mass analysis Buote et al. 07

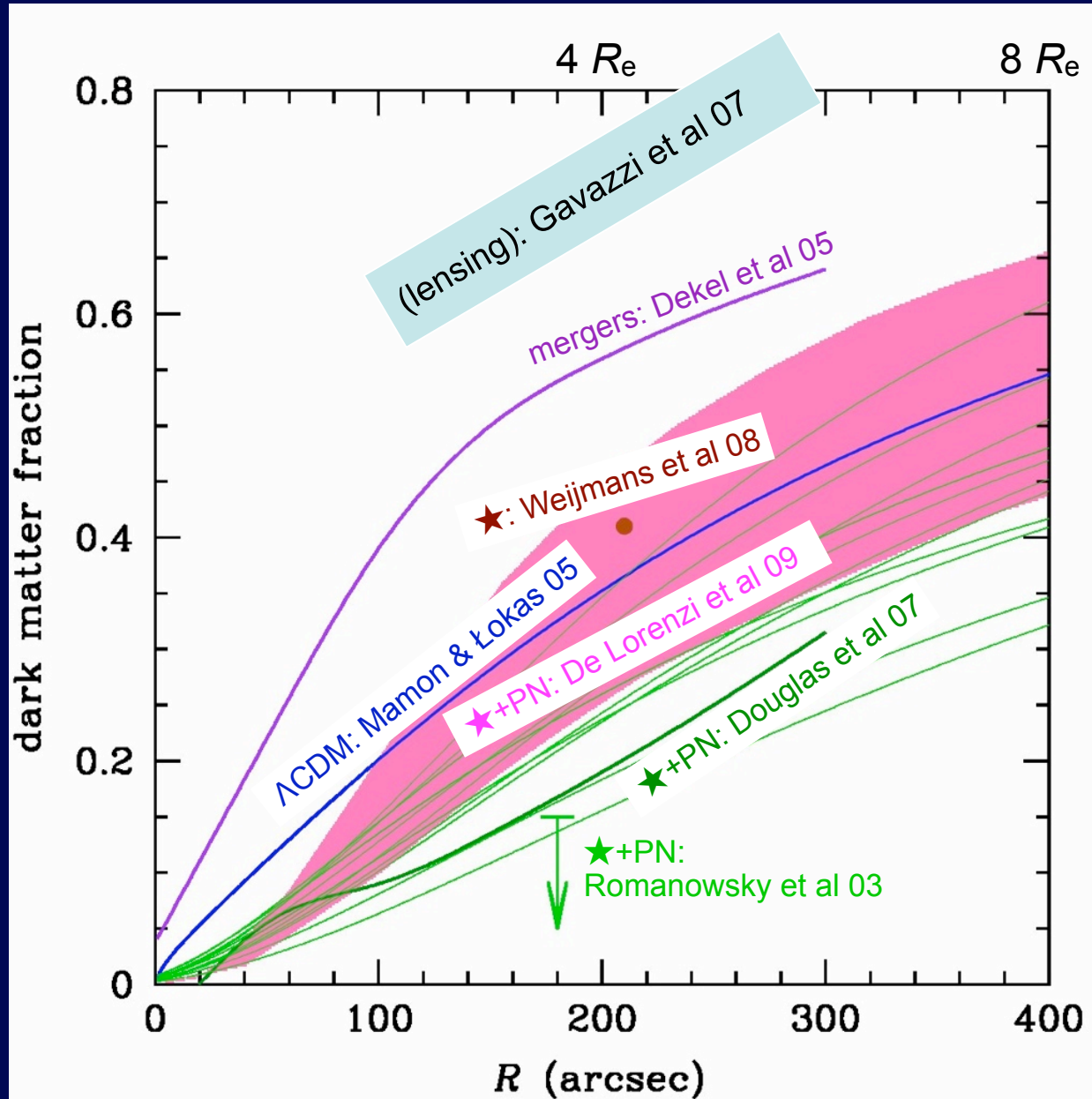


Mass & anisotropy modeling of Elliptical galaxies



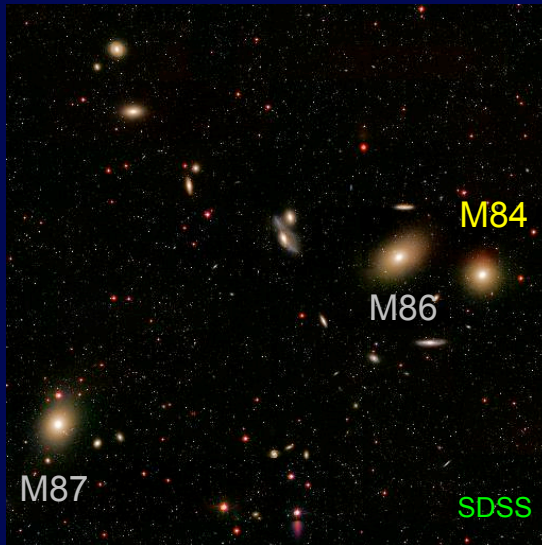
A dearth of dark matter
in ordinary elliptical galaxies
Romanowsky et al. 03, Science

Recent analyses of NGC 3379

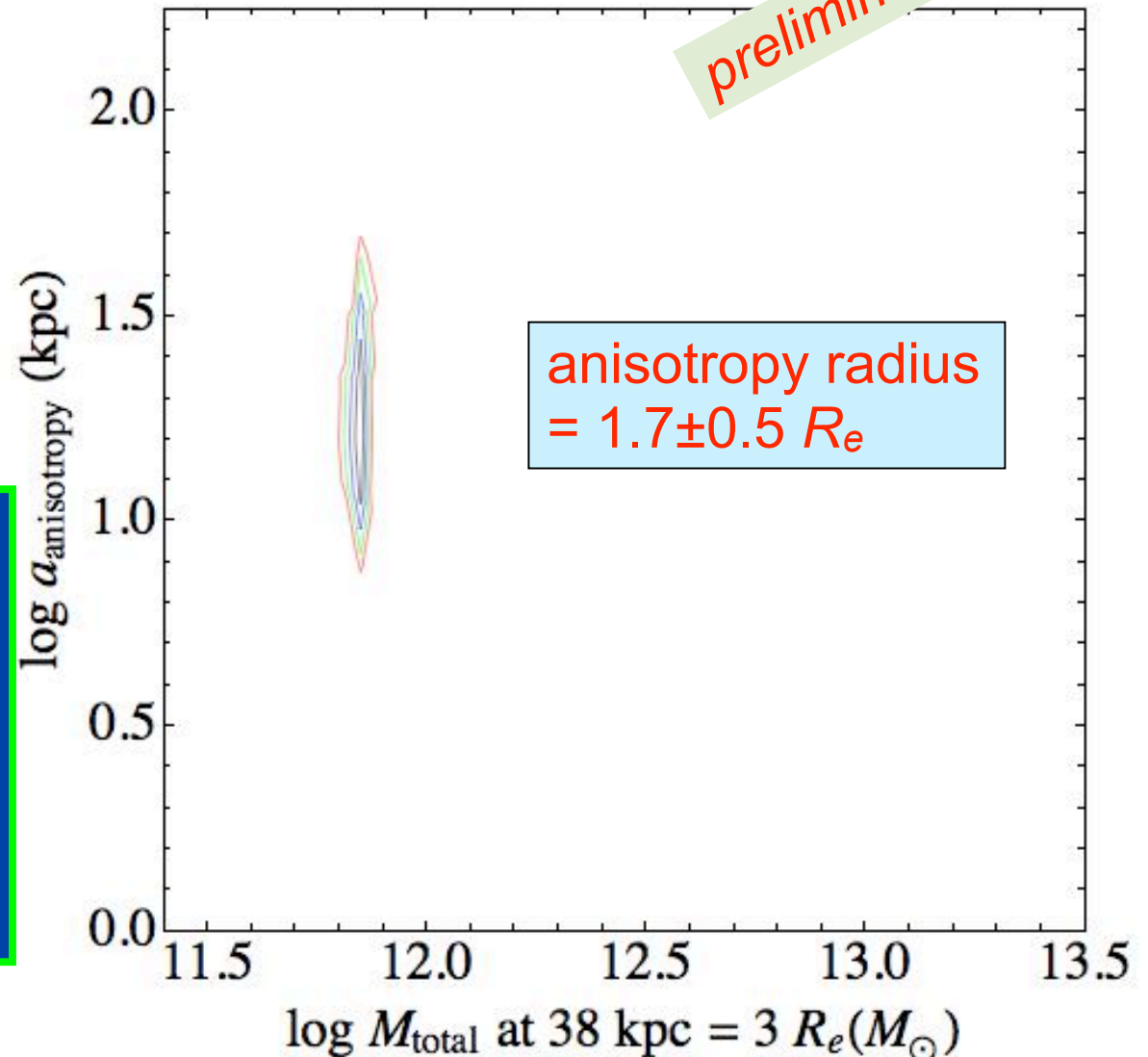


dark matter content @ 4 R_e still very uncertain!

anisotropy vs. mass in M84



M84 with MAMPOSSt
452 PN velocities (PN.S)
gaussian 3D velocities
 $\beta = r^2/(r^2+a^2)$ Osipkov-Merritt
cst M_{stars}/L
Einasto dark matter
 $C = C_{\Lambda\text{CDM}}(M_{\text{vir}})$

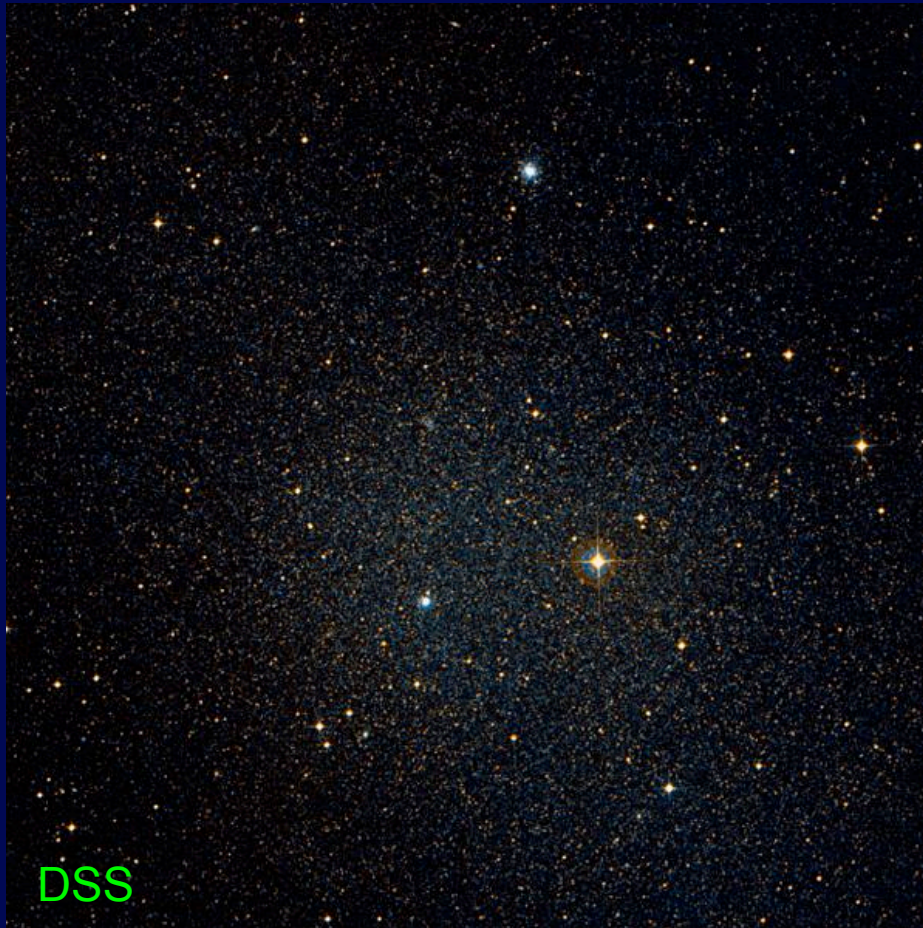


Mass & anisotropy modelling of the Fornax dwarf spheroidal

with

Chris GORDON (*Oxford*)

Andrea BIVIANO (*Trieste*)



Fornax data

2633 velocities

2278 Fornax members

$$L_V = 1.9 \times 10^7 L_{\text{sun}}$$

Irwin & Hatzimiditriou 95

$$L_V = 0.9 \times 10^7 L_{\text{sun}}$$

Walcher et al. 03

$m = 0.7$ Sersic distribution

Walcher et al. 03;
Battaglia et al. 06

ellipticity: 0.21 \rightarrow 0.36

Battaglia et al. 06

main starburst: age = 5.4 Gyr

Saviane et al. 00

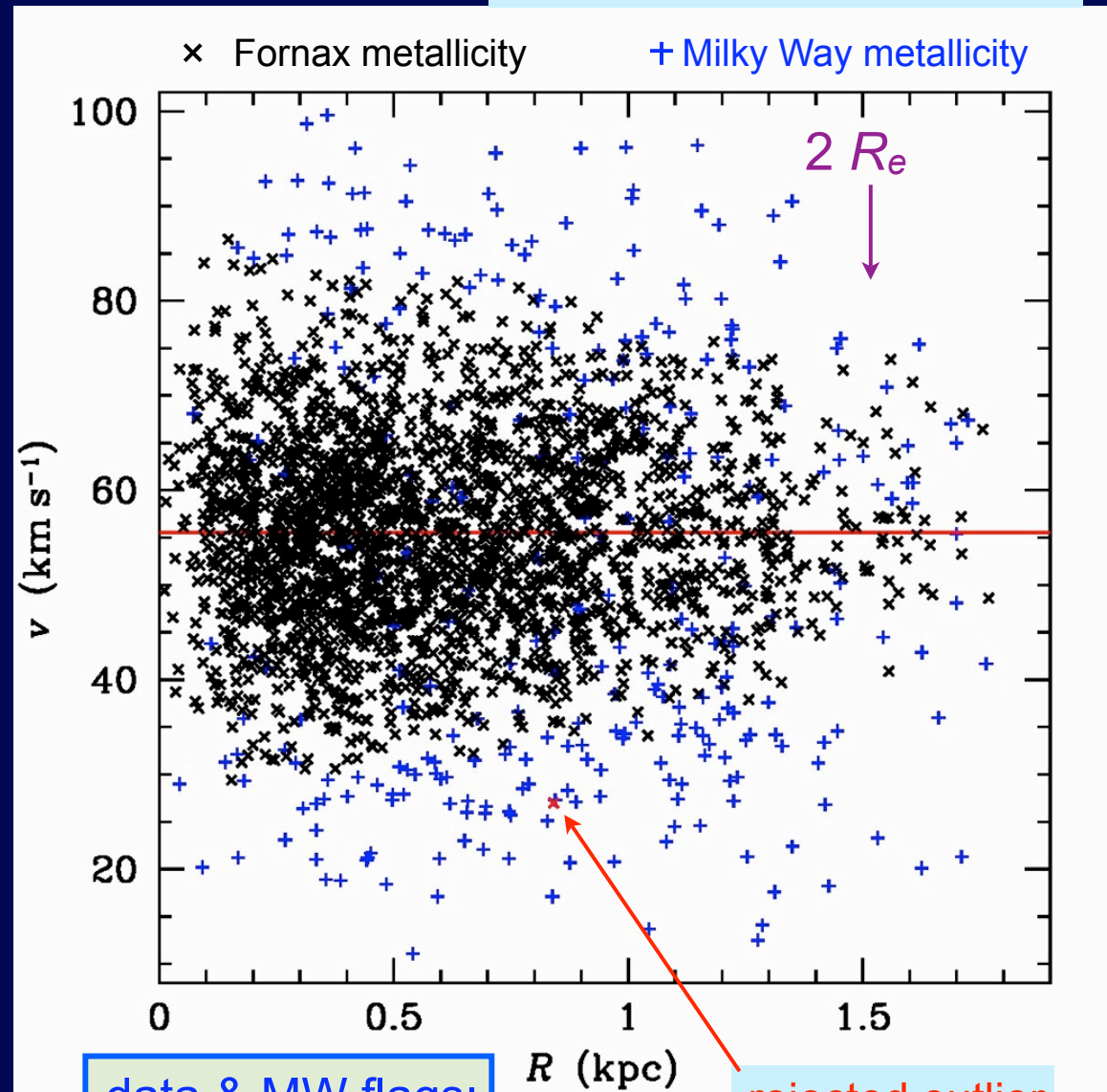


$$M_{\text{stars}}/L_V = 4.8$$

Walcher et al. 03

(uncertain) center:

Battaglia et al. 06

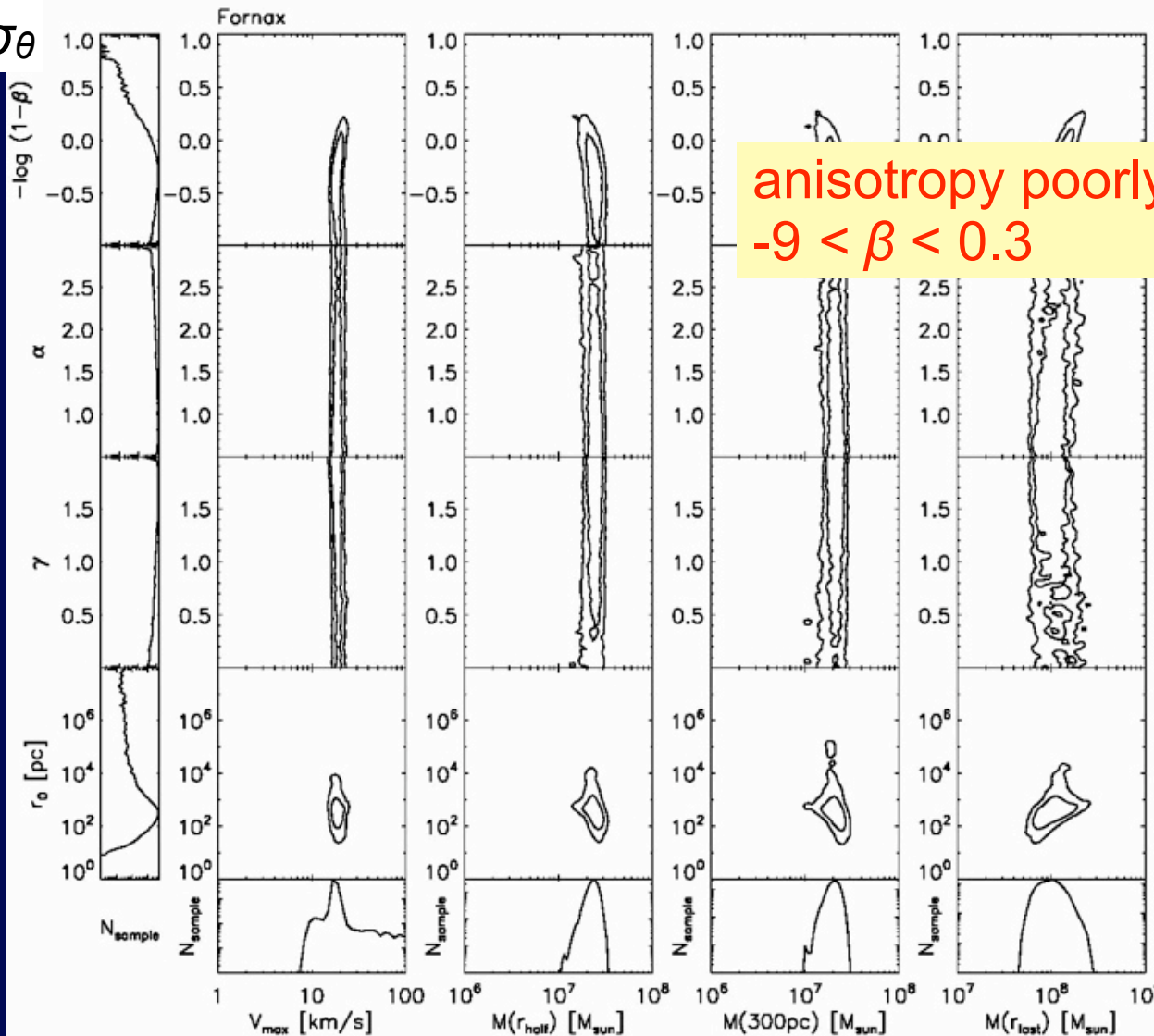


data & MW flags:
Walker et al. 09
Magellan/MMFS

Fitting velocity dispersion only

Walker et al. 09

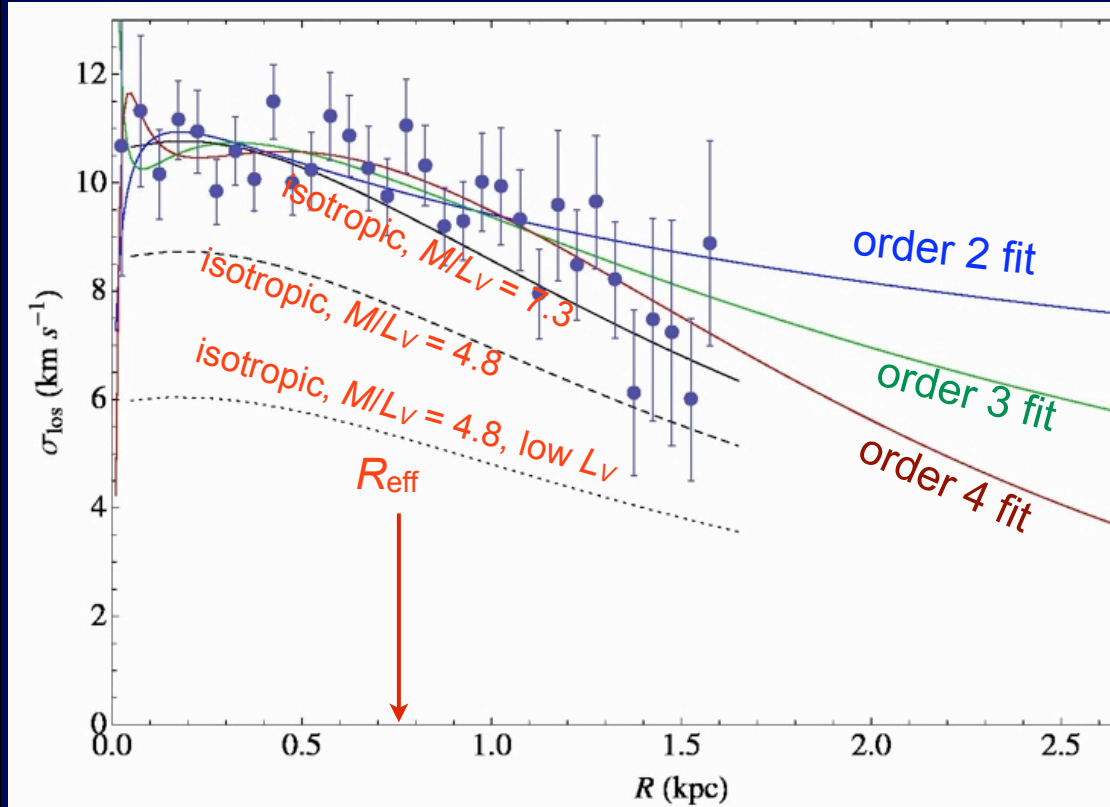
$\log \sigma_r / \sigma_\theta$



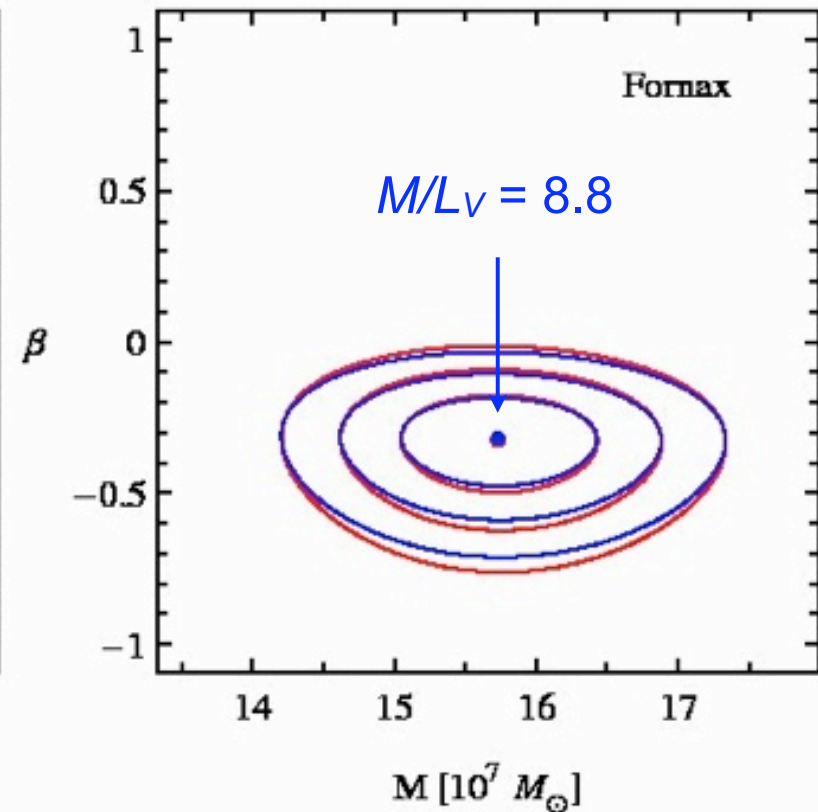
anisotropy poorly constrained:
 $-9 < \beta < 0.3$

Fornax: velocity dispersion profile

out to $2 R_{\text{eff}}$



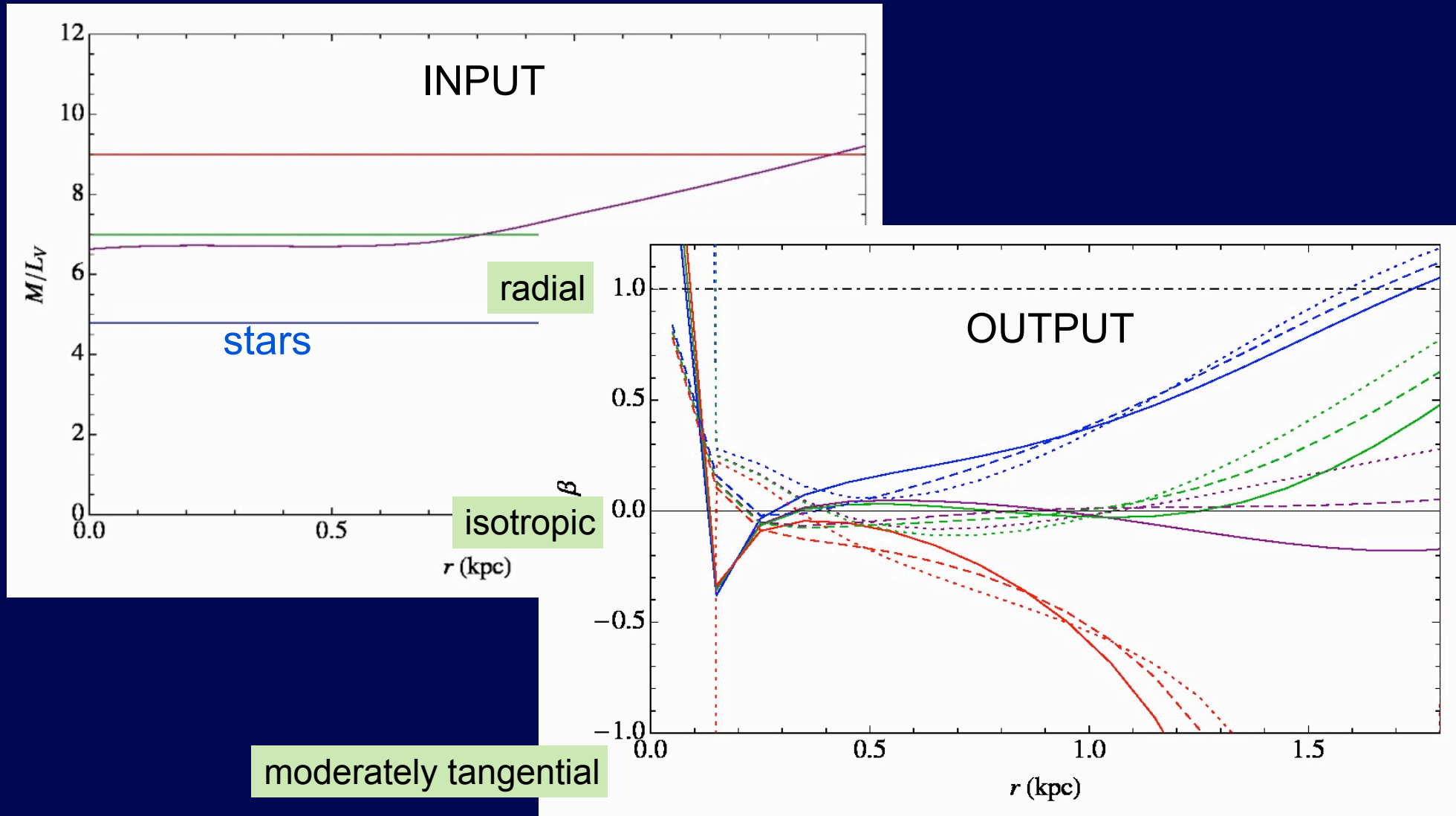
Lokas 09



1/3 fraction of dark matter in inner regions? OR L_V underestimated by 40%?

M/L increases outwards? OR tangential outer orbits?

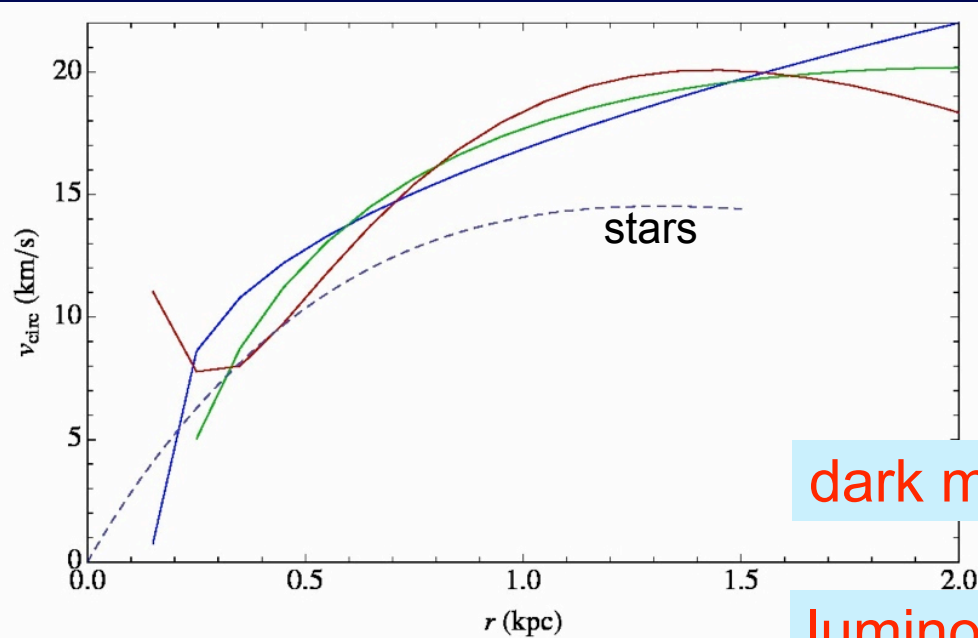
Fornax: anisotropy inversion



moderately tangential

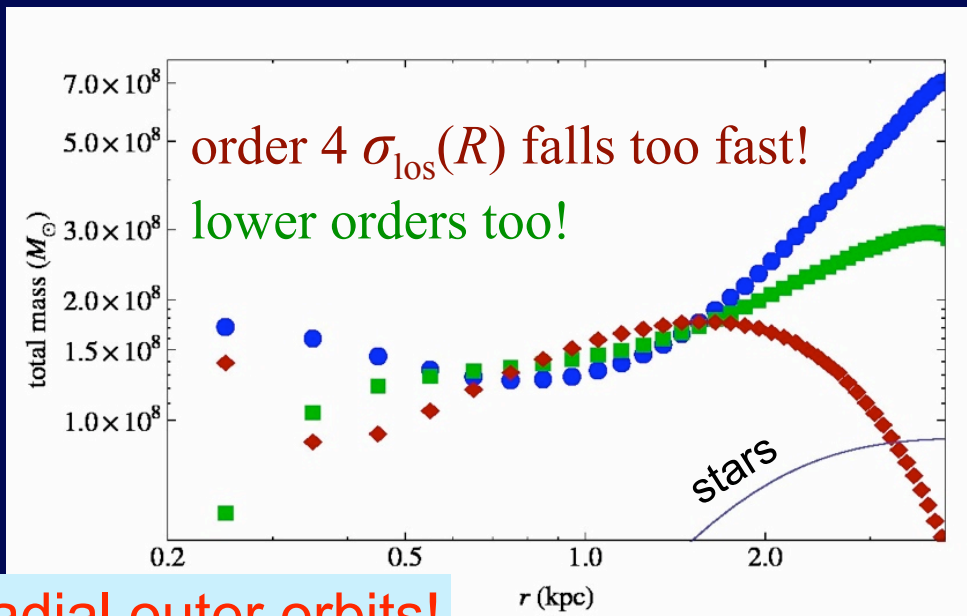
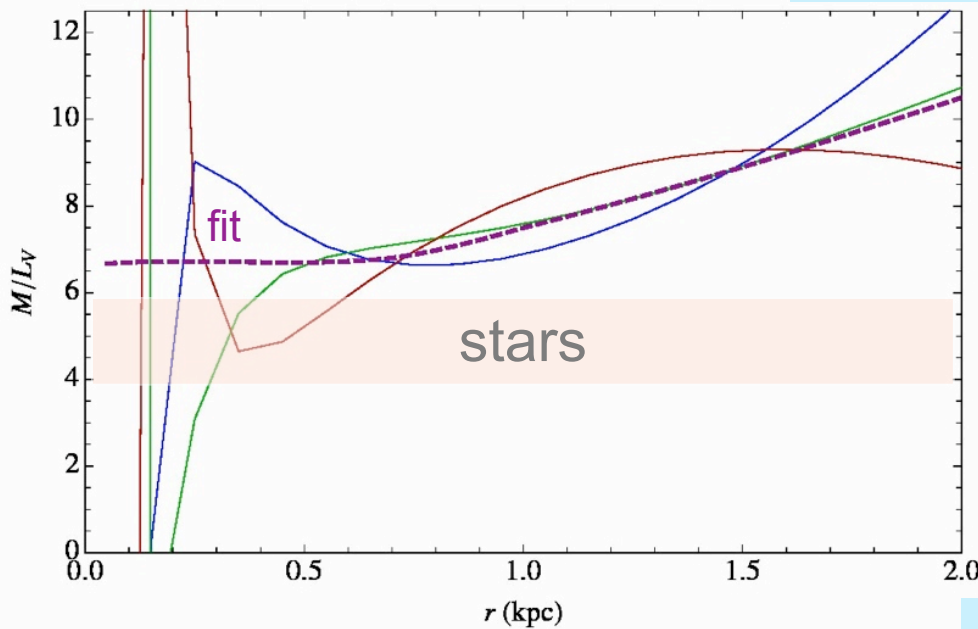
cst $M/L \rightarrow$ radial (low M/L) or tangential (high M/L) orbits
no dark matter solution fails at large radius

Fornax: isotropic mass inversion



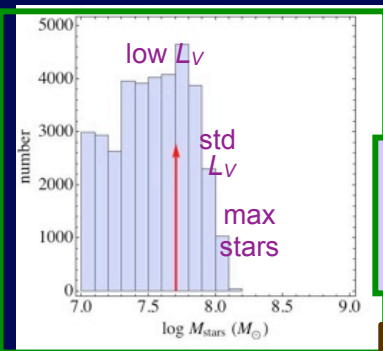
dark matter, even in inner regions?

luminosity and/or M_{stars}/L too low?



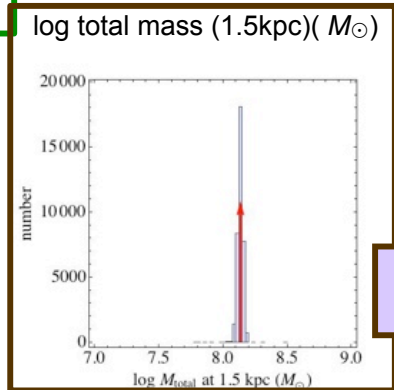
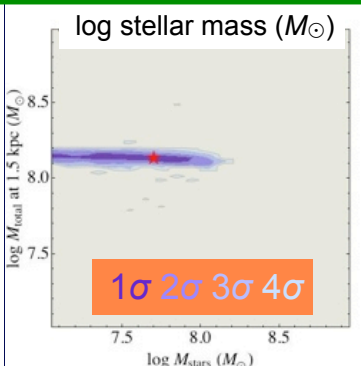
radial outer orbits!

Fornax with MAMPOSSt
 gaussian 3D velocities
 cst β
 cst M/L
 Kazantzidis DM: $\rho \sim \exp(-r/a) / r$
 MCMC: 7 chains of 5000



low L_V
 ($P=0.95$)

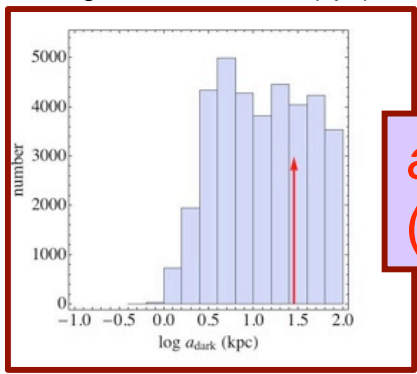
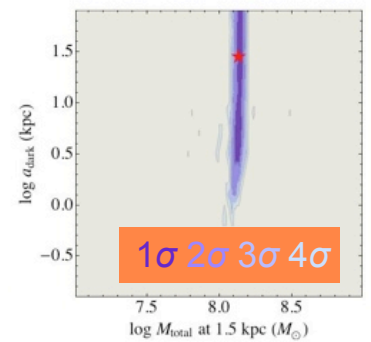
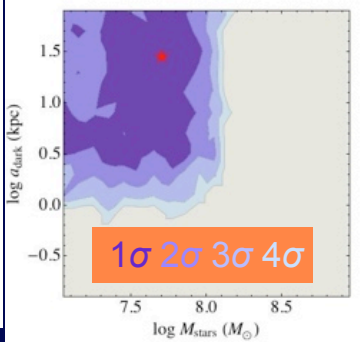
log total mass (1.5kpc) (M_\odot)



$M(2R_e)$ well constrained

see Wolf et al. 09

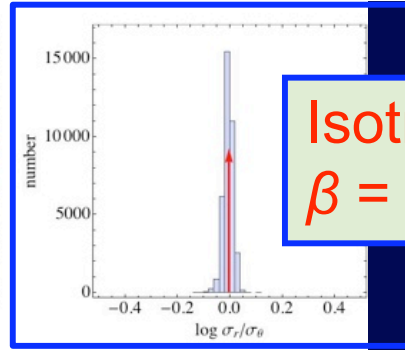
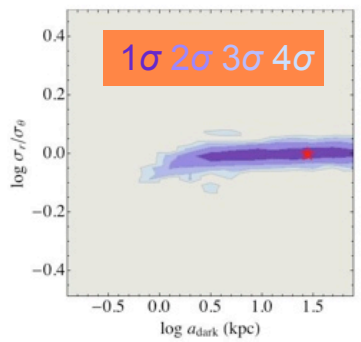
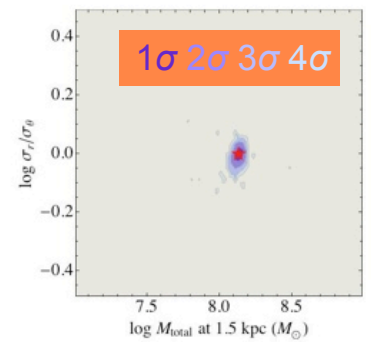
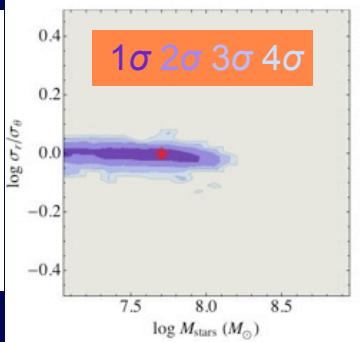
log DM scale radius (kpc)



$a > 2$ kpc
 (95% conf.)

anisotropy: $(1/2)\log(1-\beta)$

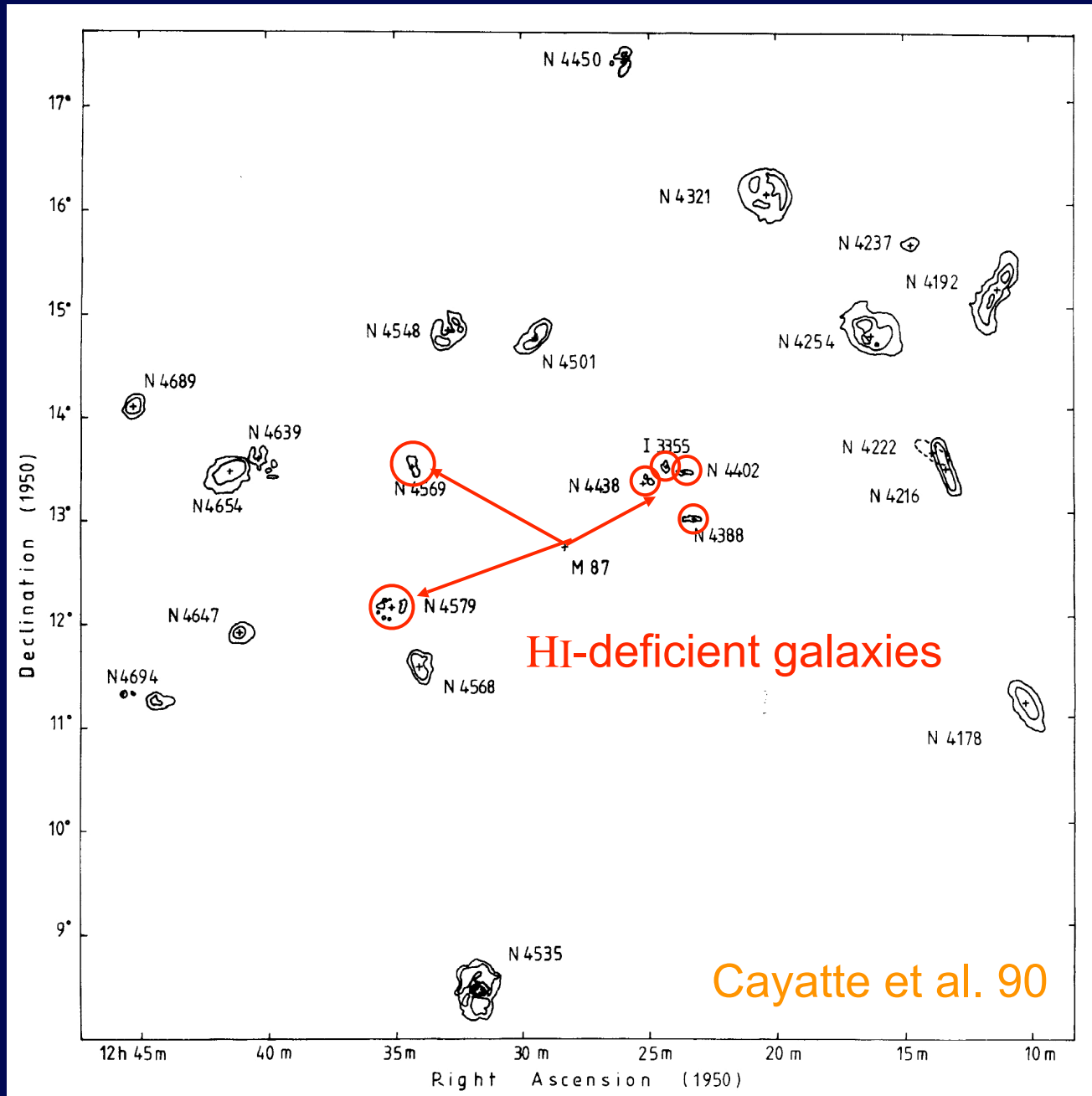
anisotropy: $\log(1-2\beta)$



Isotropic orbits!
 $\beta = -0.03 \pm 0.09$

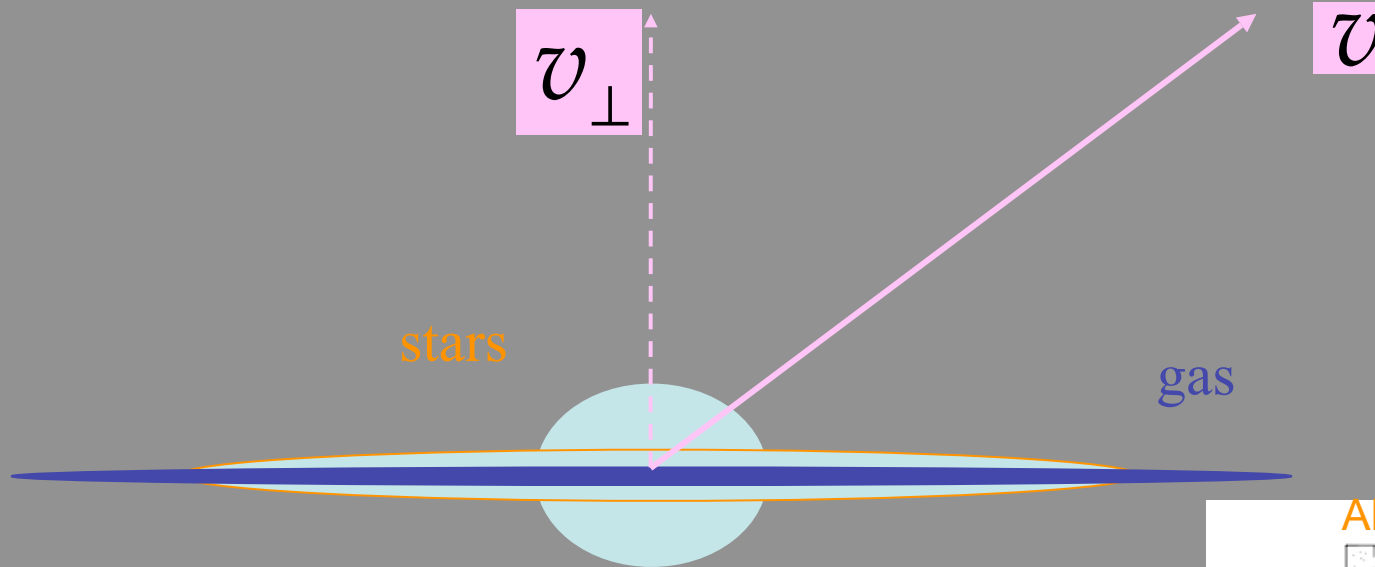
Neutral gas deficient spiral galaxies towards the Virgo cluster

Neutral gas (HI) maps in Virgo cluster



Ram pressure stripping

Gunn & Gott 72

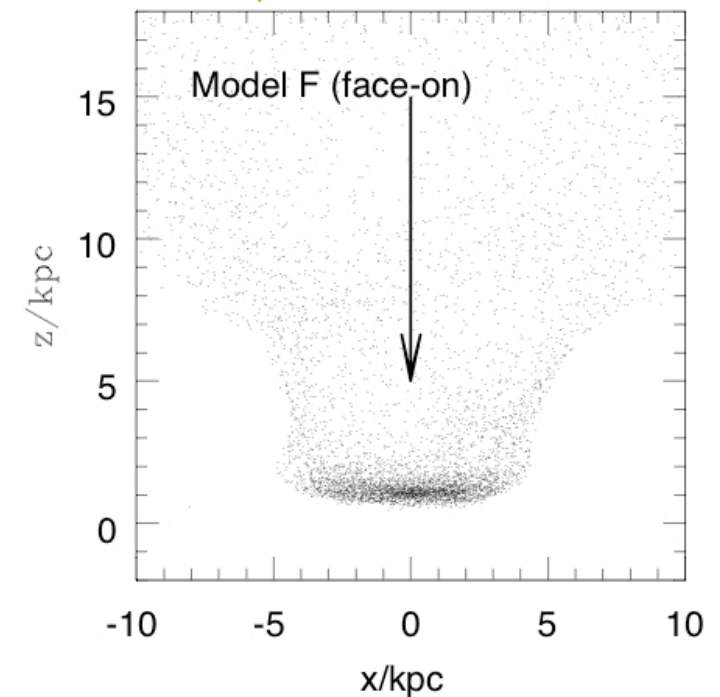


$$\rho_{\text{gas}}^{\text{ICM}} v^2 \cos \theta > 2\pi G \Sigma_* \Sigma_{\text{gas}}$$

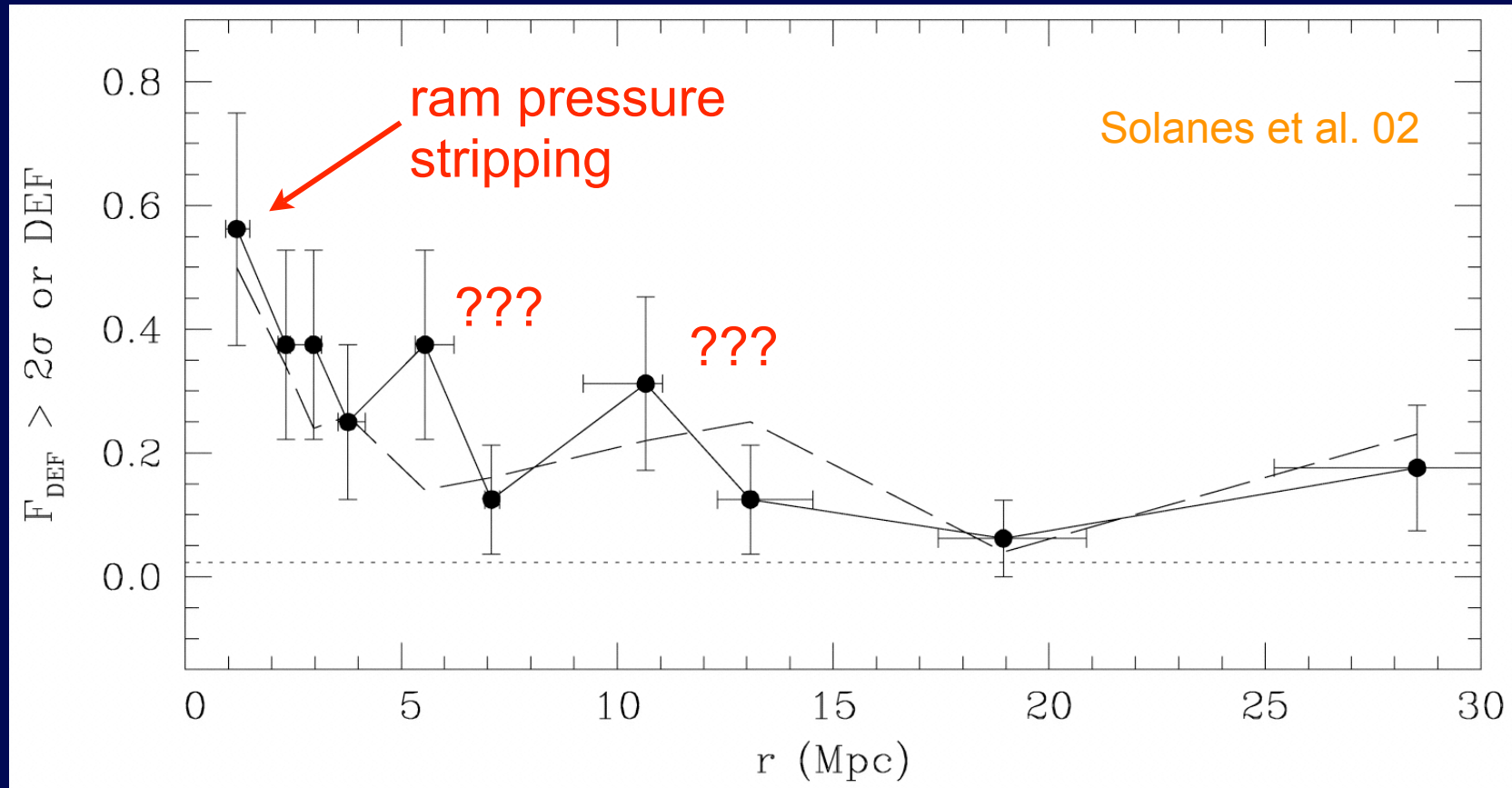
explains pattern of HI-deficiency in Virgo

Chamaraux, Balkowski & Gérard 80

Abadi, Moore & Bower 99

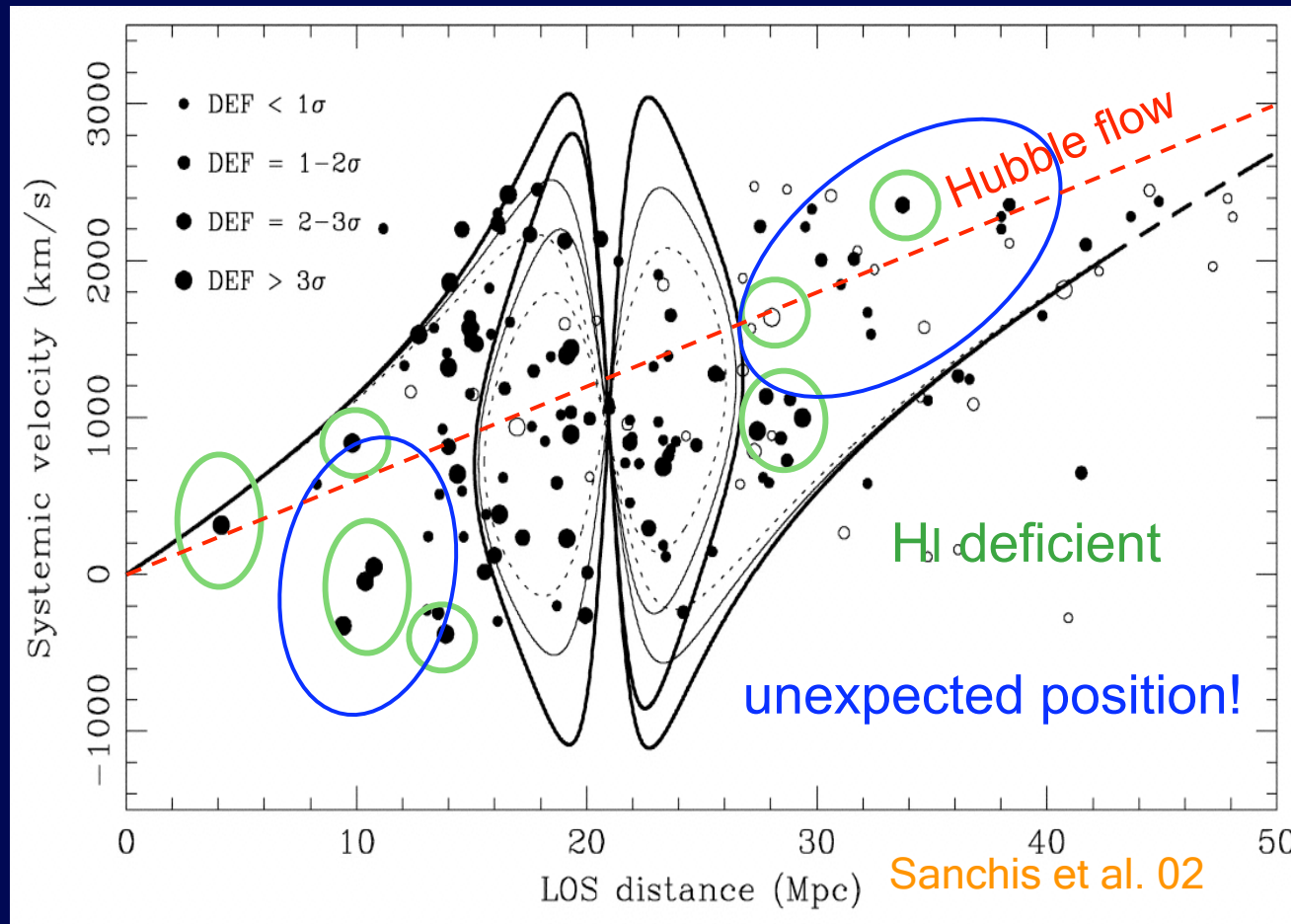


HI deficiency vs. line-of-sight distance



What causes HI-deficiency in distant galaxies?







HI-deficient galaxies on the outskirts of the Virgo cluster!



Could distant HI-deficient spirals have passed through cluster & been stripped?

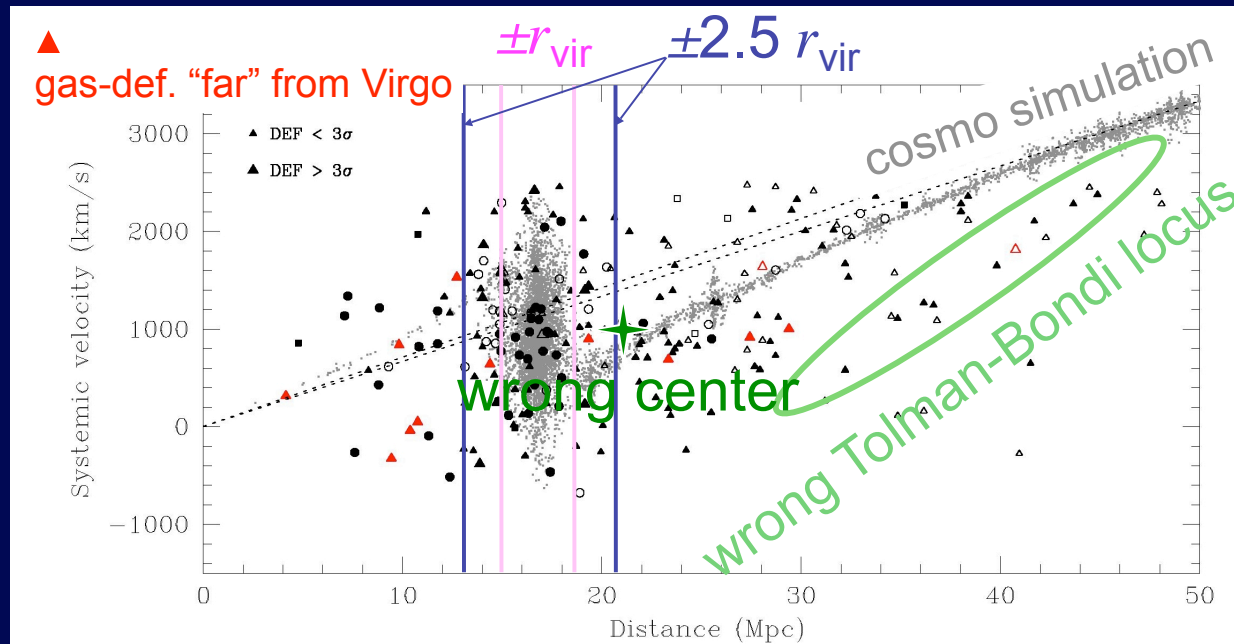
How far can galaxies bounce out of clusters?

Mamon et al. 04

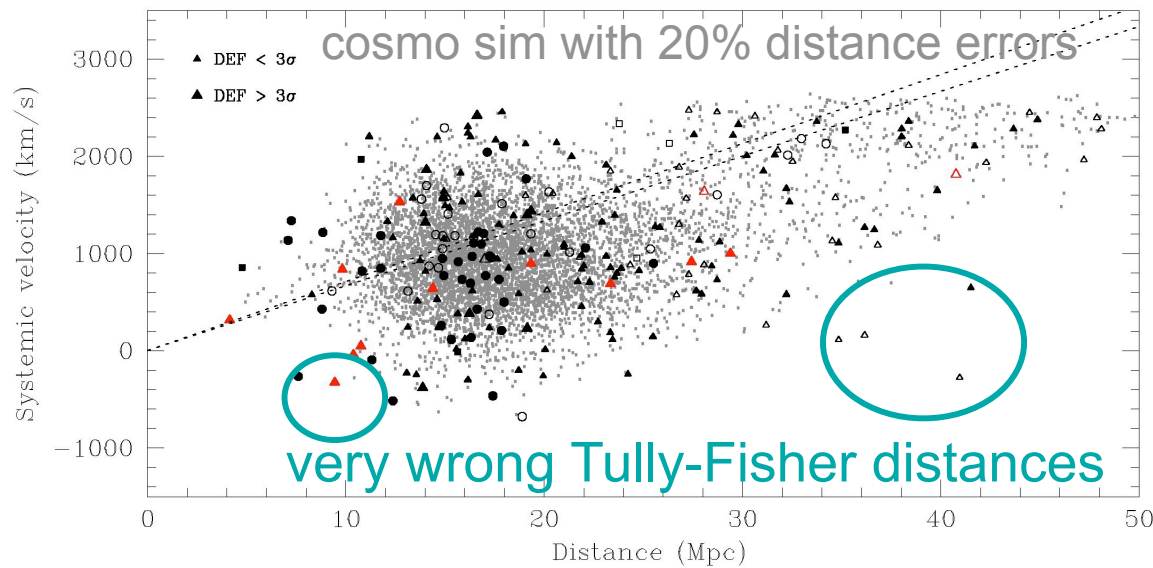
- hand-waving  $r_{\text{rebound}} = r_{\text{vir}}$
- phase-space at $z = 0$  $r_{\text{rebound}} = 1.7 r_{\text{vir}}$
- particle turnaround & rebound times & radii
 - toy models  $r_{\text{rebound}} = 1 \text{ to } 2.5 r_{\text{vir}}$
 - in simulations  $r_{\text{rebound}} = 0.5 \text{ to } 1.3 r_{\text{vir}}$
- **Summary**  $r_{\text{rebound}} = 1 \text{ to } 2.5 r_{\text{vir}}$
- direct following of orbits in simulations  $r_{\text{rebound}} = 2.5 r_{\text{vir}}$
Gill et al. 05

Comparison with cosmological simulations

including ellipticals



Sanchis, Mamon, Salvador-Solé & Solanes 04, A&A



13 HI-deficient outliers

neighboring particles in $\{D_{\text{los}}, R, v_{\text{los}}\}$ space (w errors):
 what fraction in Virgo before including the distance errors?

Galaxy		RA	Dec	T	B_T^c	D_{25}	θ	v	D	N	P_1	$P_{2.5}$	
VCC	NGC	(J2000)						(km s^{-1})	(Mpc)				
—	4064	12 ^h 04 ^m 11 ^s .8	18°26'33"	1	11.73	4'0	8°8	837	9.8	942	0.00	0.81	Merger
522	4305	12 ^h 22 ^m 03 ^s .5	12°44'27"	1	12.87	2'0	2°2	1814	40.7	200	0.00	0.00	Companion
524	4307	12 ^h 22 ^m 06 ^s .3	09°02'27"	3	11.84	3'5	4°0	913	27.4	1846	0.41	0.48	Companion
559	4312	12 ^h 22 ^m 32 ^s .0	15°32'20"	2	11.76	4'7	3°7	47	10.8	754	0.95	1.00	in Virgo
713	4356	12 ^h 24 ^m 15 ^s .9	08°32'10"	6	13.02	2'6	4°2	998	29.4	1377	0.19	0.23	Companion
979	4424	12 ^h 27 ^m 13 ^s .3	09°25'13"	1	11.99	3'4	3°1	314	4.1	106	0.00	0.00	Merger
1043	4438	12 ^h 27 ^m 46 ^s .3	13°00'30"	1	10.55	8'7	1°0	-45	10.4	755	1.00	1.00	in Virgo
1330	4492	12 ^h 30 ^m 58 ^s .9	08°04'41"	1	13.04	1'9	4°3	1638	28.1	1261	0.08	0.11	Companion
1569	—	12 ^h 34 ^m 31 ^s .4	13°30'23"	5	14.62	0'8	1°4	687	23.3	8967	0.72	0.94	in Virgo
1690	4569 ^a	12 ^h 36 ^m 50 ^s .5	13°09'54"	2	9.63	10'4	1°7	-328	9.4	143	1.00	1.00	in Virgo
1730	4580	12 ^h 37 ^m 49 ^s .5	05°22'09"	2	12.39	2'0	7°2	893	19.3	8724	0.00	0.84	Companion
1760	4586	12 ^h 38 ^m 28 ^s .1	04°19'09"	1	12.06	3'9	8°3	639	14.4	4657	0.00	0.98	Companion
1859	4606	12 ^h 40 ^m 57 ^s .7	11°54'46"	1	12.28	2'9	2°5	1528	12.7	7648	0.95	1.00	in Virgo

at least 5 out of 13 must be in Virgo: suffered ram pressure stripping

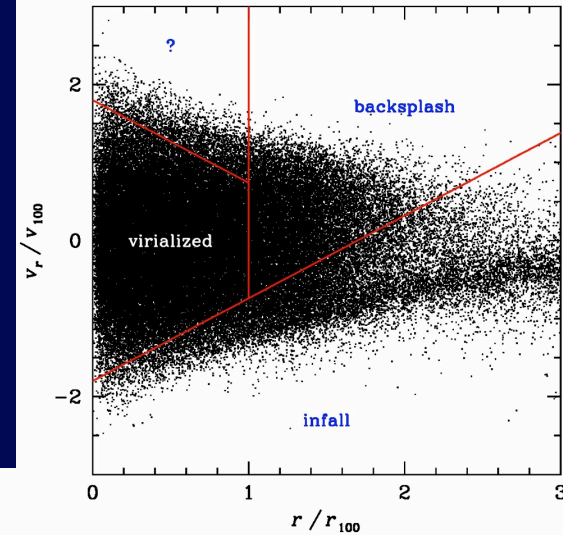
6/13 have companions: induce *tidal* stripping of outer gas

2/13: minor mergers heat up & ionize neutral Hydrogen

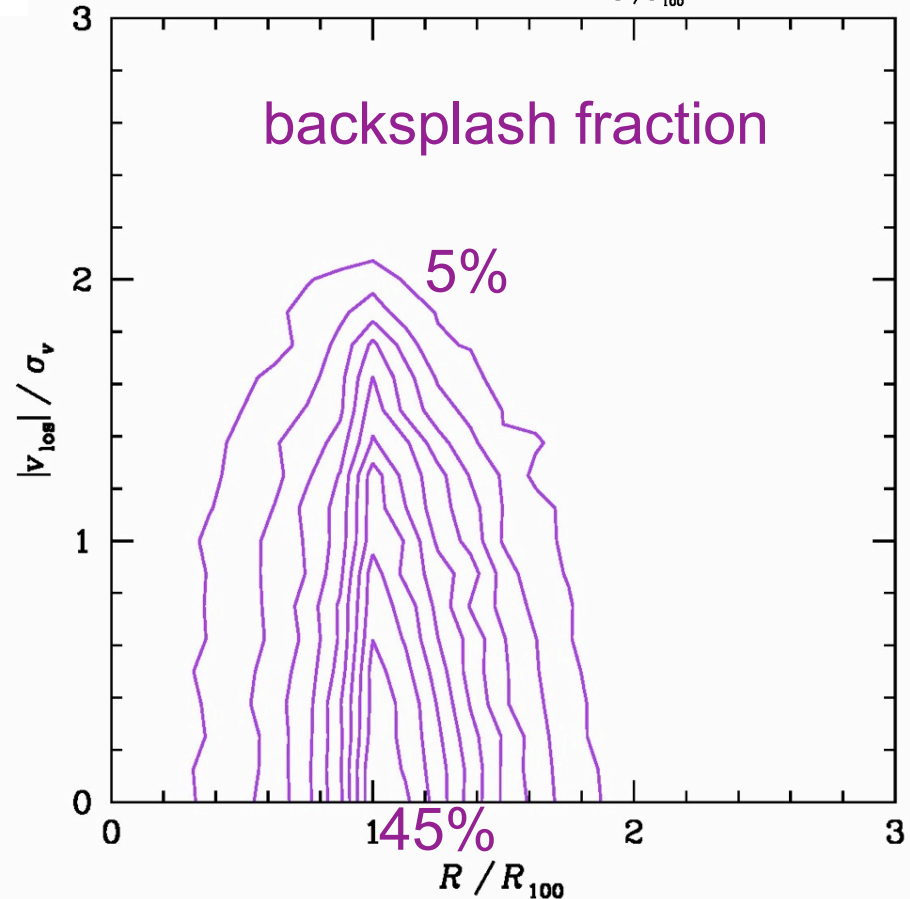
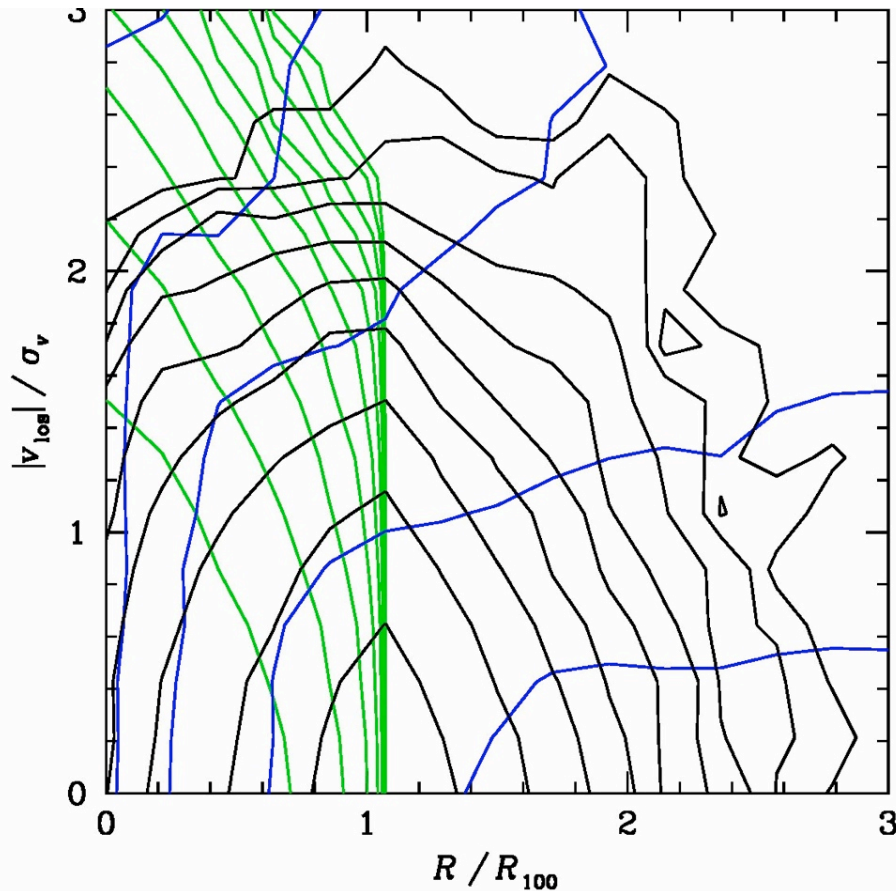
Star formation history of galaxies falling in clusters

with Smriti MAHAJAN & Somak RAYCHAUDHURY

Virialized, infalling & backsplash galaxies



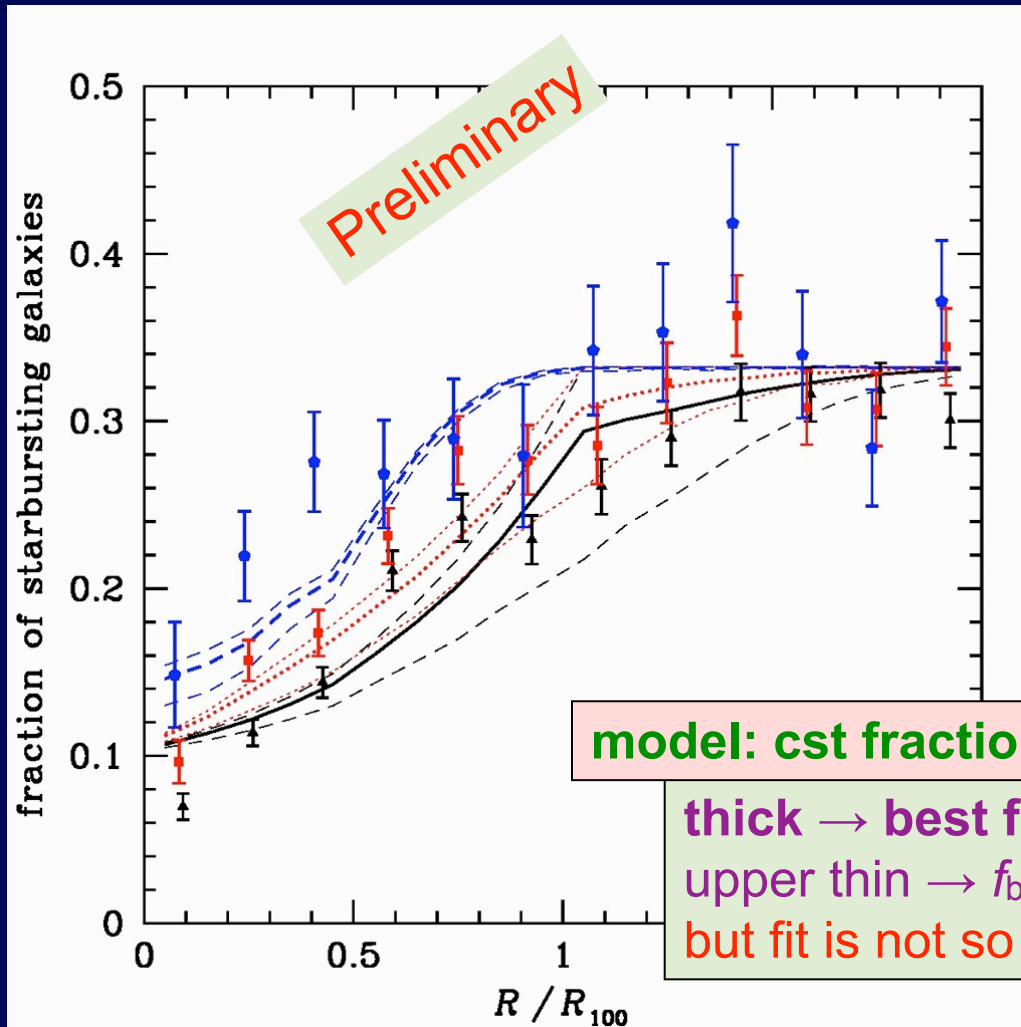
densities of virialized, infall & backsplash



Fraction of starburst galaxies vs. projected radius & line-of-sight velocity

SDSS data: galaxies more luminous than $M_r = -20.5$
around $M > 10^{14} M_{\odot}$ clusters Yang et al. 07

efficient recent
star formation:
 $EW(H\delta) > 2\text{\AA}$
& $D_n4000 < 1.5$



$$2 < |v| / \sigma_{\text{cluster}} < 3$$

$$1 < |v| / \sigma_{\text{cluster}} < 2$$

$$0 < |v| / \sigma_{\text{cluster}} < 1$$

model: cst fraction of star forming galaxies per class

thick \rightarrow best fit: $f_{\text{vir}}=10\pm 1\%$, $f_{\text{inf}}=33\pm 1\%$, $f_{\text{bsp}}=26\pm 4\%$
upper thin $\rightarrow f_{\text{bsp}}=f_{\text{inf}}$ lower thin $\rightarrow f_{\text{bsp}}=f_{\text{vir}}$
but fit is not so good...

Conclusions

Mass modelling = *exploratory data analysis*:

try \neq methods: * mass & β inversions

* dispersion-kurtosis

* *MAMPOSSt* or Wojtak+09

Coma cluster

- E/S0s on isotropic orbits

X-ray groups

- very low concentration of low mass ones:
⇒ irregular potential or energy dissipation

M84

- Strong radial anisotropy beyond $1.5 R_e$
- Dark matter dominates at $4 R_e$

Fornax

- Isotropic inner orbits, \approx radial outer orbits
- Dark matter present:
 - likely $> 50\%$ at all radii

seen in simulations: Klimentowski+07

Conclusions (2)

- **Universal** distribution of interlopers in projected phase space
→ **incorporate in mass & anisotropy modelling**
- Half of neutral gas deficient spirals behind Virgo are in Virgo!
- Single passage through cluster decreases fraction of recent starbursts by only $< 20\%$