Global and simple paradigm for gravitational structures formation

Pr. J. Perez
Ecole Nationale Supérieure de Techniques Avancées

Applied Mathematics Laboratory

jerome.perez@ensta.fr

Phase Space at CIRM - Nov 2009

Outline

- Step 1 : Some facts from dynamical stellar systems
- Step 2 : Some instabilities in dynamical stellar systems
- Step 3 : Some key N Body experiences

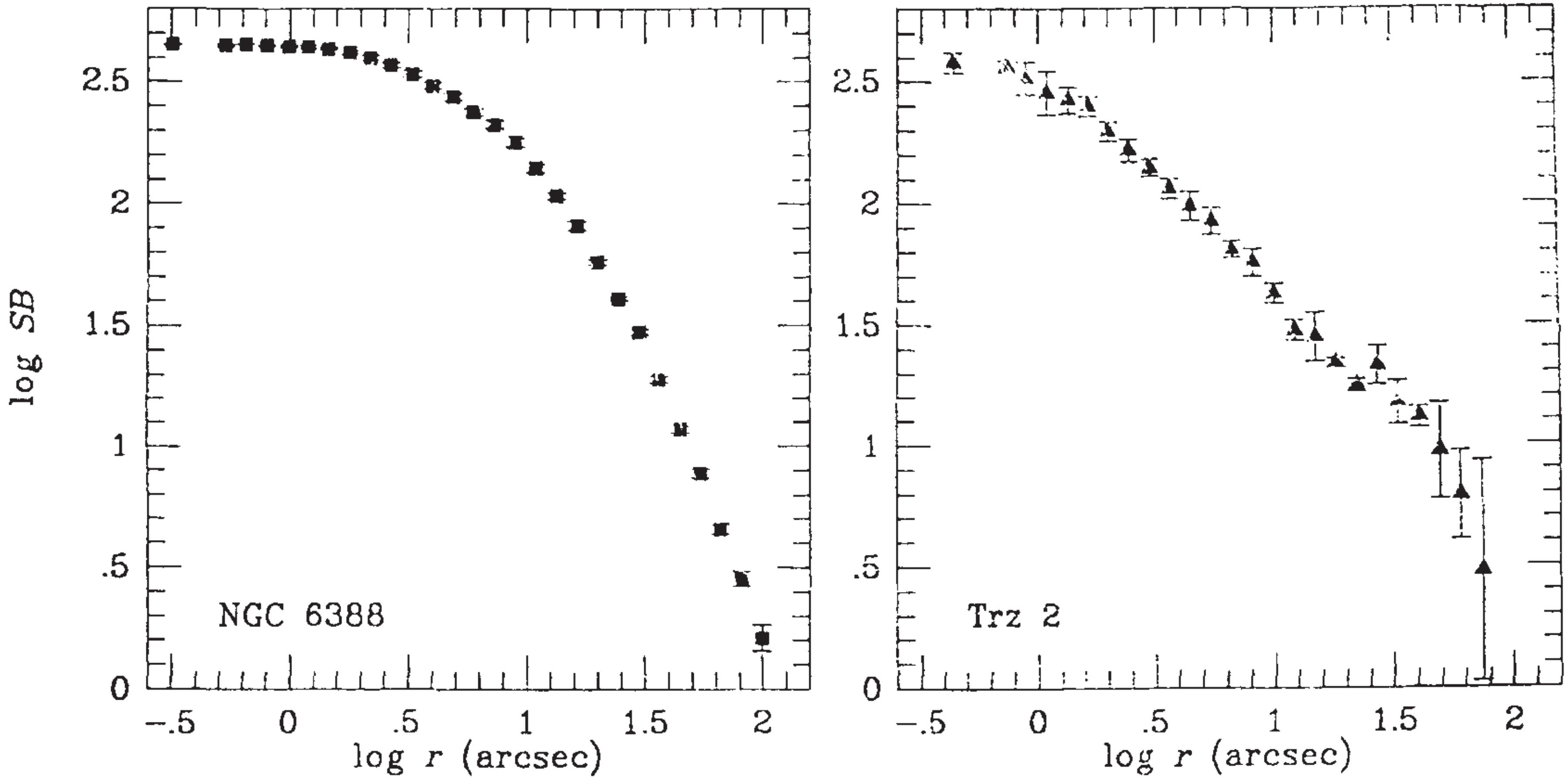
Step 1

Some facts ...

PhaseSpace at CIRM

Globular clusters

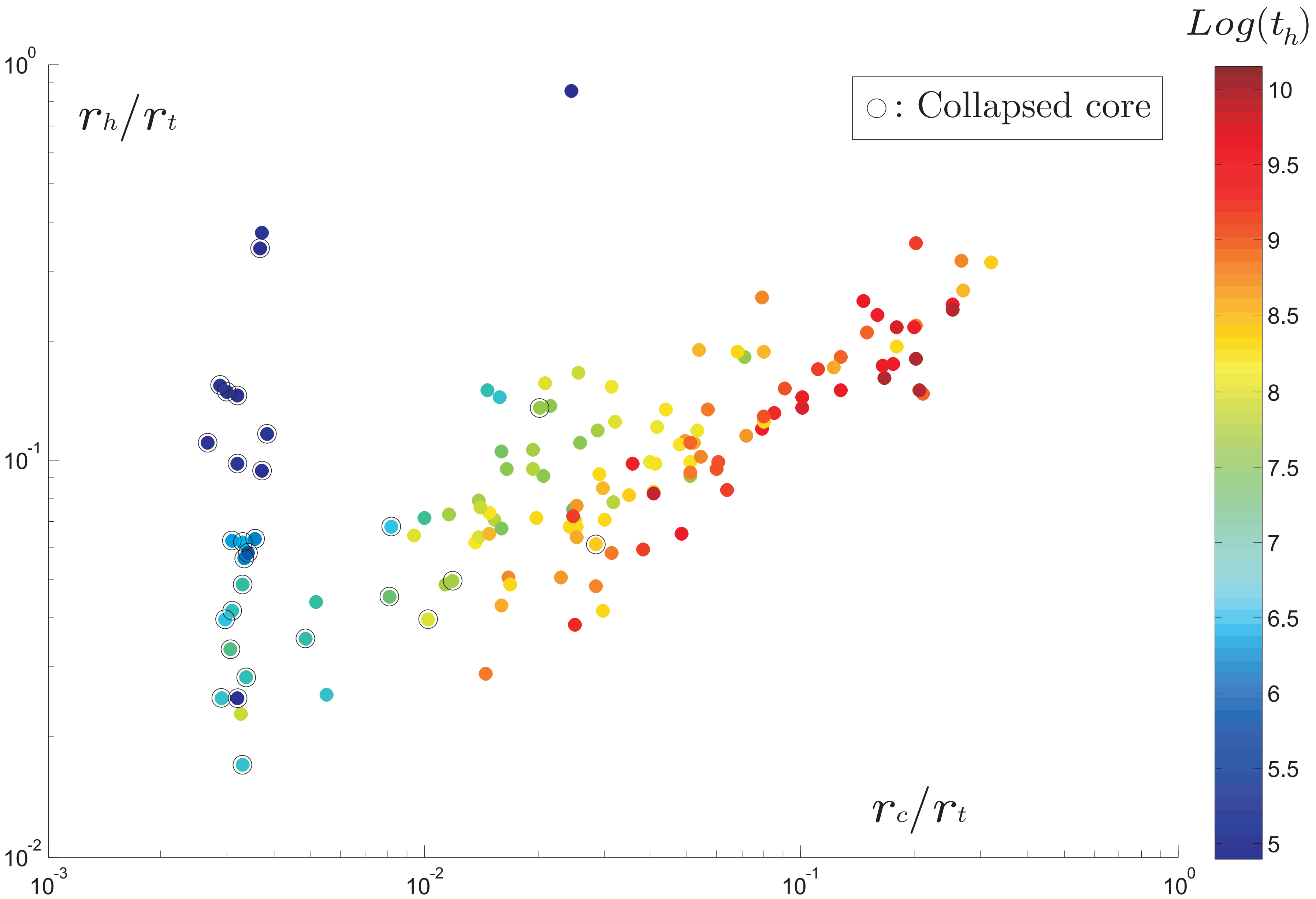
80% : Core-halo , 20% Collapsed Core, Generically no intermediate mass BH



From Djorgowski et al. ’86

PhaseSpace at CIRM

This effect is due to the evolution of GC through the galaxy.

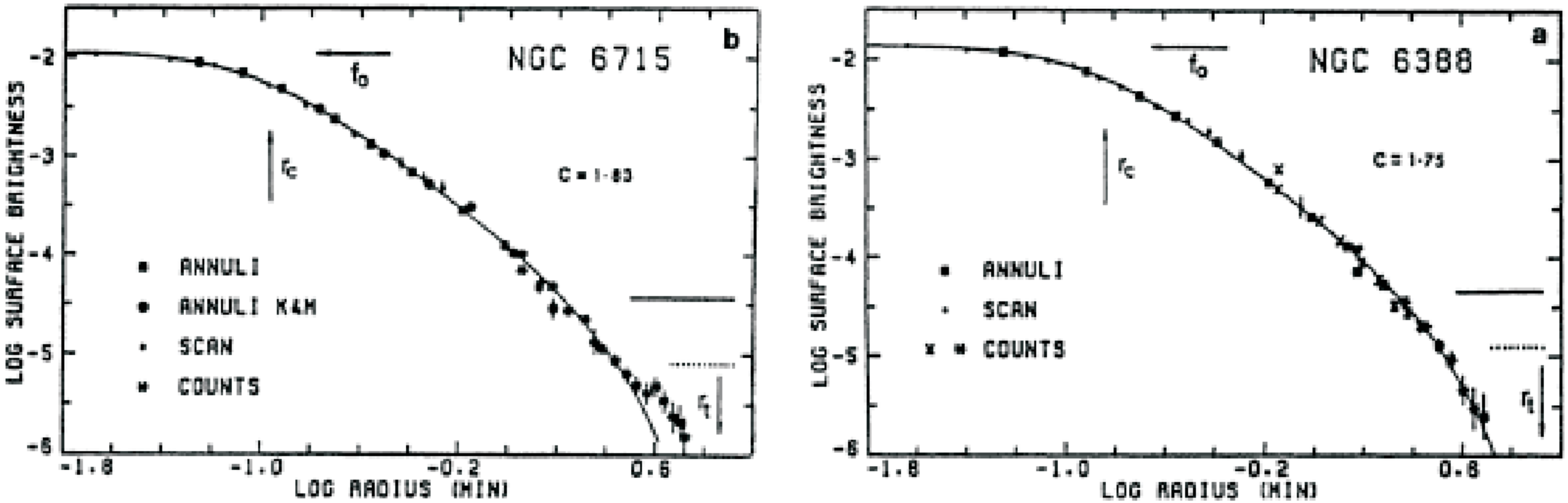


Shape of Globular Clusters in the Harris '91 catalogue

Color means 2 body relaxation time at r_h

Globular clusters

Core-Hallo structures are King models (from Elson et al. '87)



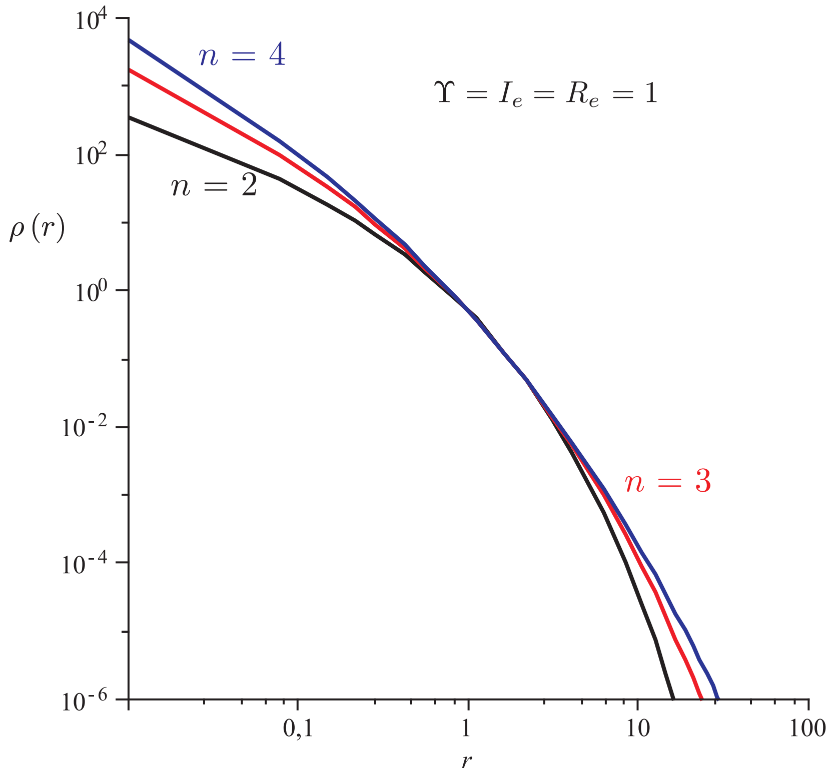
Collapsed core structures are like Singular Isothermal Spheres

Galaxies

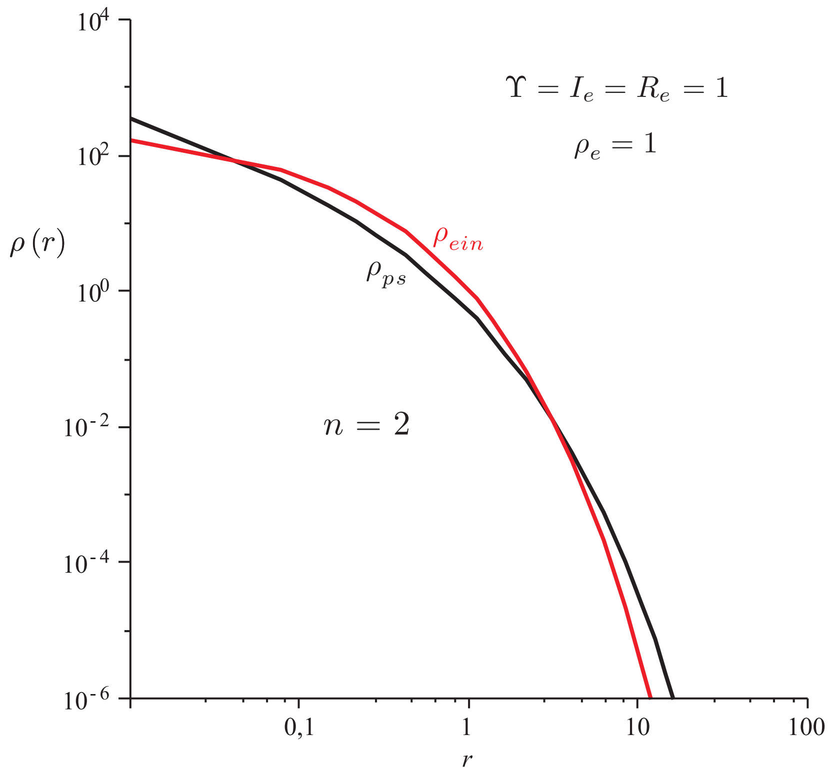
Observations : Luminosity profile : $R^{1/4}$ Law, Supermassive BH, No Core.
Simulations : The best model is Prugniel-Simien (see Merritt et al. '06)

$$\rho_o \left(\frac{r}{R_e}\right)^{-p_n} \exp\left[-b_n\left\{\left(\frac{r}{R_e}\right)^{1/n} - 1\right\}\right]$$ Triple power law like

Deprojection of Sersic $R^{1/n}$ Law,... Einasto-Sersic



Prugniel-Simien Model with various n



Comparison Prugniel-Simien / Einasto

Step 2

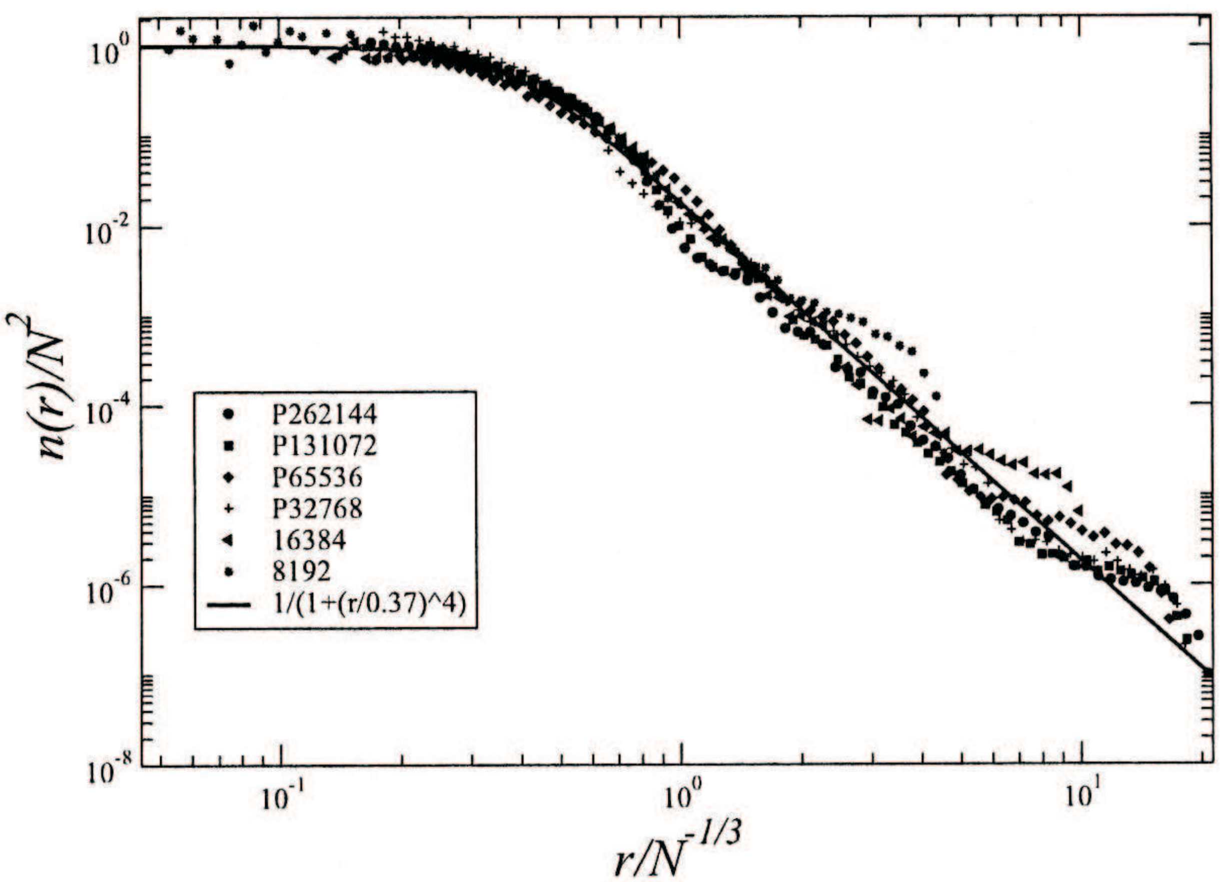
Some instabilities ...

Jeans Instability

See Kiessling '03 for modern improvement (?)

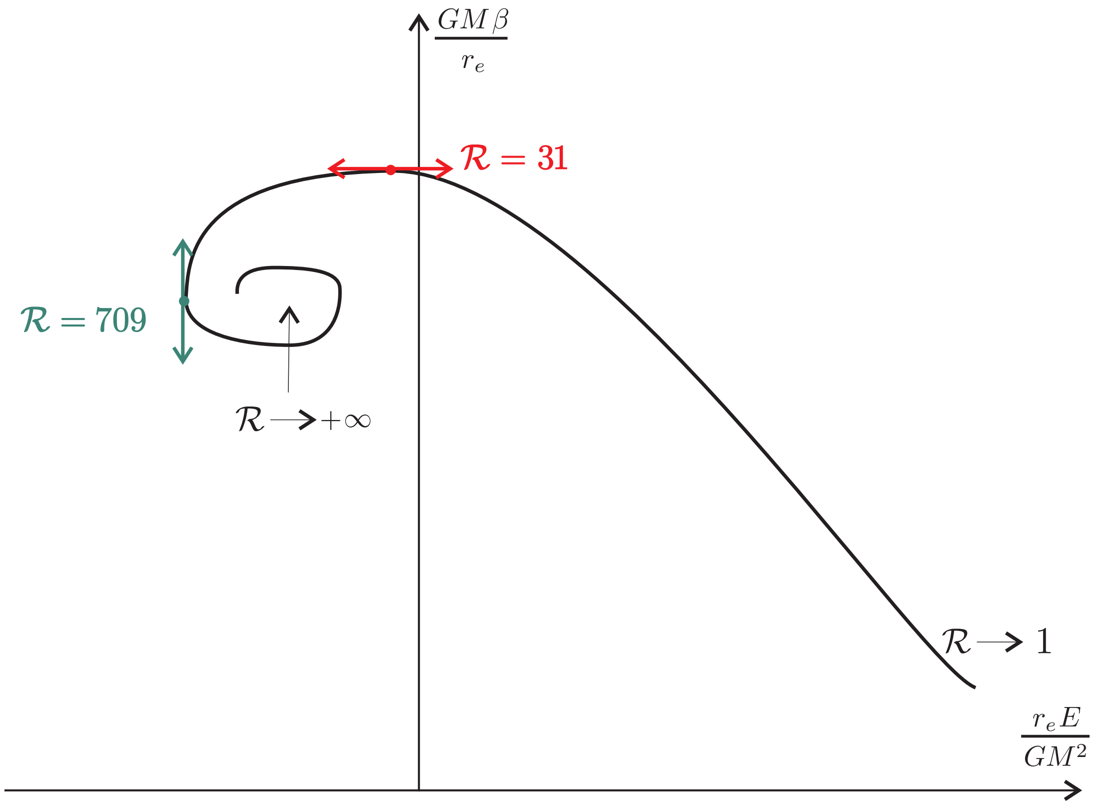
$$R > \left(\frac{\sigma^2}{4\pi G \rho} \right)^{1/2}$$

Sufficiently "Large" or "Cold" or "Dense" *homogeneous* system (Top Hat) collapses and forms a core-hallo 4 structure (..., Roy & P. '04, Joyce et al. '09)



Antonov Instability

1. Isothermal sphere in a box is unstable if the density contrast $\mathcal{R} > 709$
2. Isothermal sphere in a thermal bath is unstable if $\mathcal{R} > 32$

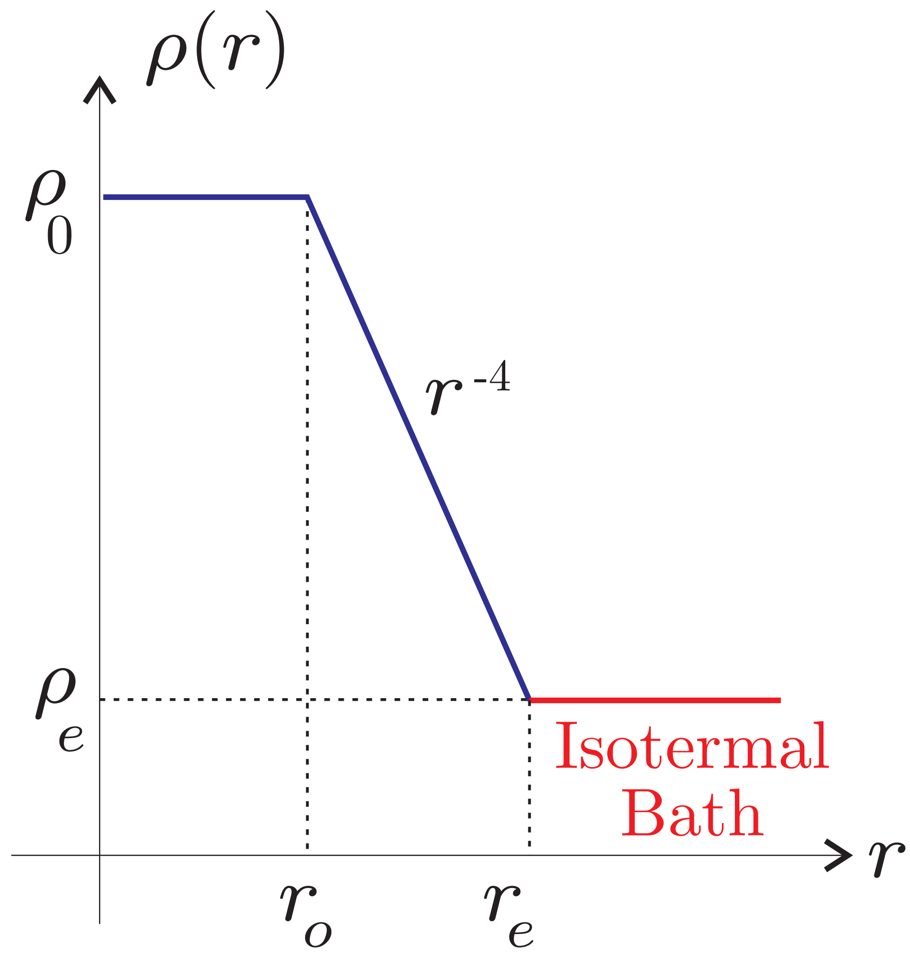


Look for horizontal or vertical tangents in $(E\,,\beta)$ plane (see Katz '78).

PhaseSpace at CIRM

Fundamental Example

Stability of core-hallo 4 structure of mass M in an isothermal bath



Fundamental Example

Fundamental Example

Mass density

$$\rho\left(\mathbf{r}\right)=\begin{cases}\rho_o=\dfrac{3M}{4\pi r_e^3}\dfrac{x^4}{(4x-3)} & \text{if } r<r_o\\[2ex] \dfrac{3M}{4\pi r_e^3}\dfrac{1}{(4x-3)\,\xi^4} & \text{if } r_o<r<r_e\end{cases}$$

with $x=\dfrac{r_e}{r_o}\geq 1$ and $r=|\mathbf{r}|$
and $\xi=\dfrac{r}{r_e}$

Fundamental Example

Mass density

$$\rho\left(\mathbf{r}\right)=\begin{cases}\rho_o=\dfrac{3M}{4\pi r_e^3}\dfrac{x^4}{(4x-3)} & \text{if } r<r_o\\[2ex] \dfrac{3M}{4\pi r_e^3}\dfrac{1}{(4x-3)\,\xi^4} & \text{if } r_o<r<r_e\end{cases}$$

with $x=\dfrac{r_e}{r_o}\geq 1$ and $r=|\mathbf{r}|$
and $\xi=\dfrac{r}{r_e}$

Gravitationnal potential

$$\psi\left(r\right)=\begin{cases}\psi_o+\dfrac{GM}{2r_e}\dfrac{x^4\xi^2}{(4x-3)} & \text{if } r<r_o\\[2ex] \psi_o+\dfrac{GM}{2r_e}\dfrac{\left[6x^2\xi^2-8x\xi+3\right]}{(4x-3)\,\xi^2} & \text{if } r_o<r<r_e\end{cases}$$

Fundamental example

PhaseSpace at CIRM

Fundamental example

Potential Energy : $U_{jk} = -\int \rho(\mathbf{r}) \; (\mathbf{r}.\mathbf{e}_k)(\nabla\psi \;.\mathbf{e}_j) \; d\mathbf{r} \qquad W = \mathrm{Tr}(U_{jk})$.
in our spherical bounded case one have

$$W = -4\pi G \int_0^{r_e} s^3 \rho(s) \frac{d\psi}{ds} ds = -\frac{3GM^2}{5r_e} \frac{x^3(5x-4)}{(4x-3)^2}$$

PhaseSpace at CIRM

Fundamental example

Potential Energy : $U_{jk} = -\int \rho(\mathbf{r})\ (\mathbf{r}.\mathbf{e}_k)(\nabla\psi\ .\mathbf{e}_j)\ d\mathbf{r}$ $\qquad W = \mathrm{Tr}(U_{jk})$.
in our spherical bounded case one have

$$W = -4\pi G \int_0^{r_e} s^3 \rho(s) \frac{d\psi}{ds} ds = -\frac{3GM^2}{5r_e} \frac{x^3(5x-4)}{(4x-3)^2}$$

Virial theorem for a system confined in a thermal bath : $2T + W = 4\pi r_e^3 P_e$

Fundamental example

Potential Energy : $U_{jk} = -\int \rho(\mathbf{r}) \ (\mathbf{r}.\mathbf{e}_k)(\nabla\psi \ .\mathbf{e}_j) \ d\mathbf{r} \qquad W = \mathrm{Tr}(U_{jk})$.
in our spherical bounded case one have

$$W = -4\pi G \int_0^{r_e} s^3 \rho(s) \frac{d\psi}{ds} ds = -\frac{3GM^2}{5r_e} \frac{x^3(5x-4)}{(4x-3)^2}$$

Virial theorem for a system confined in a thermal bath : $2T + W = 4\pi r_e^3 P_e$
Isothermal bath $\beta = 1/k\theta$

$$\rho = \beta P \quad \Rightarrow \quad P_e = \frac{\rho(r_e)}{\beta} = \frac{1}{4\pi r_e^3} \frac{3M}{\beta} \frac{1}{(4x-3)}$$

Fundamental example

Potential Energy : $U_{jk} = -\int \rho\left(\mathbf{r}\right)\ \left(\mathbf{r}.\mathbf{e}_{k}\right)\left(\nabla\psi\ .\mathbf{e}_{j}\right)\ d\mathbf{r} \qquad W = \mathrm{Tr}\left(U_{jk}\right)$.
in our spherical bounded case one have

$$W = -4\pi G \int_{0}^{r_e} s^3 \rho\left(s\right) \frac{d\psi}{ds} ds = -\frac{3GM^2}{5r_e} \frac{x^3\left(5x-4\right)}{\left(4x-3\right)^2}$$

Virial theorem for a system confined in a thermal bath : $2T + W = 4\pi r_e^3 P_e$
Isothermal bath $\beta = 1/k\theta$

$$\rho = \beta P \quad \Rightarrow \quad P_e = \frac{\rho\left(r_e\right)}{\beta} = \frac{1}{4\pi r_e^3} \frac{3M}{\beta} \frac{1}{\left(4x-3\right)}$$

Kinetic energy : $T = \dfrac{3}{2}\dfrac{M}{\beta}$

Fundamental example

Potential Energy : $U_{jk} = -\int \rho(\mathbf{r}) \; (\mathbf{r}.\mathbf{e}_k)(\nabla\psi \; .\mathbf{e}_j) \; d\mathbf{r} \qquad W = \mathrm{Tr}(U_{jk})$.
in our spherical bounded case one have

$$W = -4\pi G \int_0^{r_e} s^3 \rho(s) \frac{d\psi}{ds} ds = -\frac{3GM^2}{5r_e} \frac{x^3(5x-4)}{(4x-3)^2}$$

Virial theorem for a system confined in a thermal bath : $2T + W = 4\pi r_e^3 P_e$
Isothermal bath $\beta = 1/k\theta$

$$\rho = \beta P \quad \Rightarrow \quad P_e = \frac{\rho(r_e)}{\beta} = \frac{1}{4\pi r_e^3} \frac{3M}{\beta} \frac{1}{(4x-3)}$$

Kinetic energy : $T = \dfrac{3}{2}\dfrac{M}{\beta}$

Virial relation then writes

$$\beta = \frac{r_e}{GM} \frac{20(4x-3)(x-1)}{x^3(5x-4)}$$

PhaseSpace at CIRM

Fundamental example

Potential Energy : $U_{jk} = -\int \rho(\mathbf{r})\ (\mathbf{r}.\mathbf{e}_k)(\nabla\psi\ .\mathbf{e}_j)\ d\mathbf{r}$ $\qquad W = \mathrm{Tr}(U_{jk})$.
in our spherical bounded case one have

$$W = -4\pi G \int_0^{r_e} s^3 \rho(s) \frac{d\psi}{ds} ds = -\frac{3GM^2}{5r_e} \frac{x^3(5x-4)}{(4x-3)^2}$$

Virial theorem for a system confined in a thermal bath : $2T + W = 4\pi r_e^3 P_e$
Isothermal bath $\beta = 1/k\theta$

$$\rho = \beta P \quad \Rightarrow \quad P_e = \frac{\rho(r_e)}{\beta} = \frac{1}{4\pi r_e^3} \frac{3M}{\beta} \frac{1}{(4x-3)}$$

Kinetic energy : $T = \frac{3}{2}\frac{M}{\beta}$
Virial relation then writes

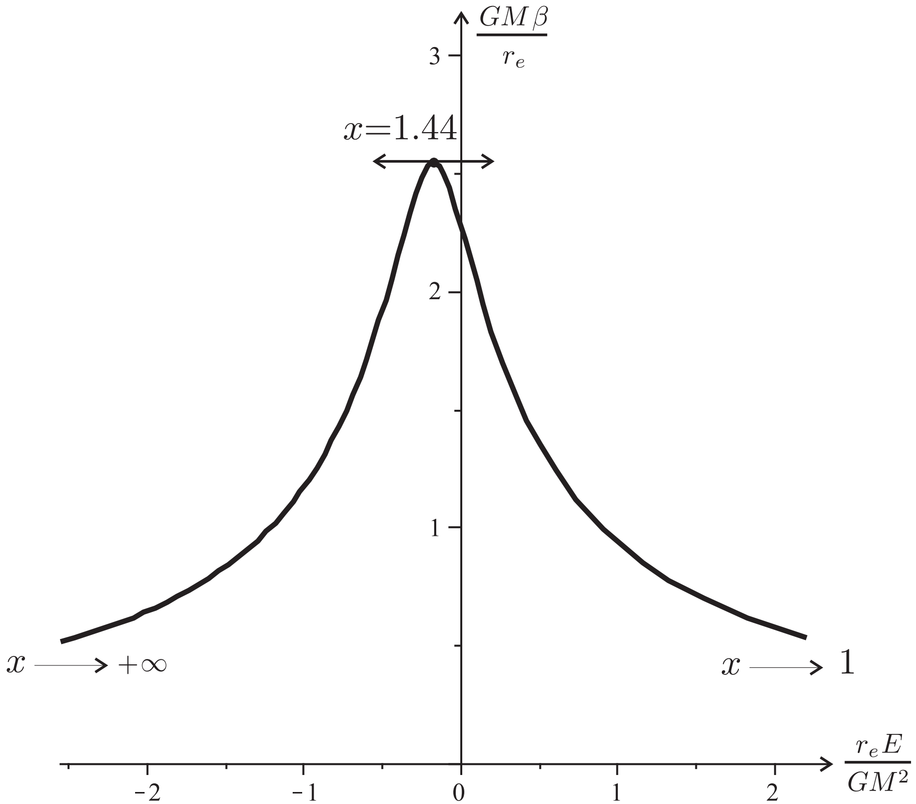
$$\beta = \frac{r_e}{GM} \frac{20(4x-3)(x-1)}{x^3(5x-4)}$$

Total energy is $E = T + W$, one then have

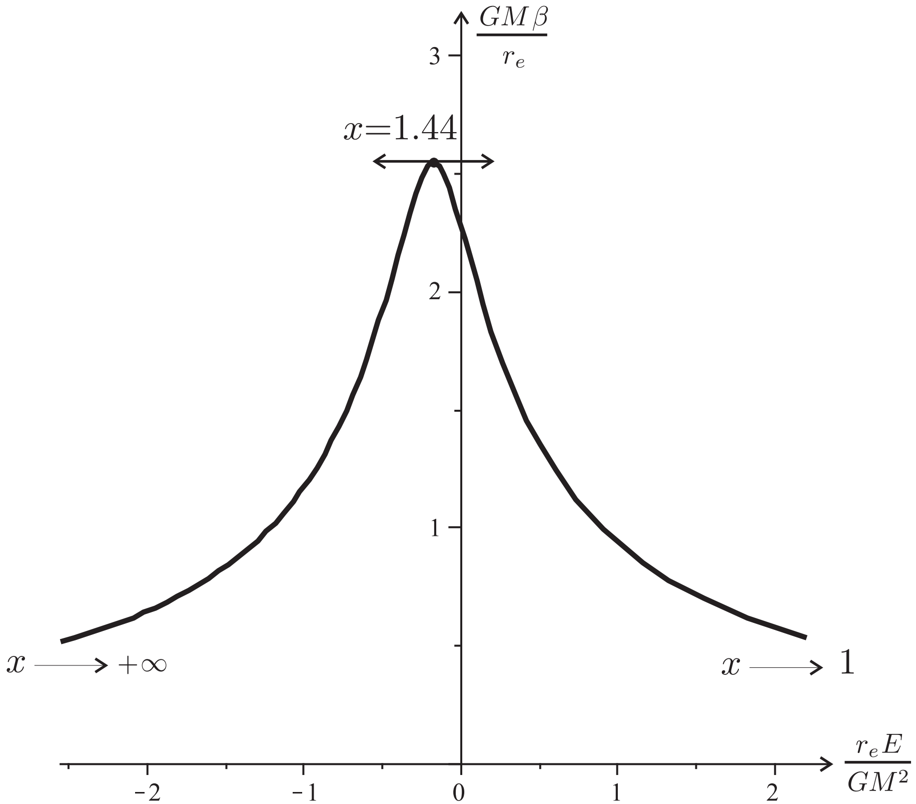
$$E = \frac{3GM^2}{40r_e} \frac{x^3(5x-4)(5-4x)}{(4x-3)^2(x-1)}$$

let define $\lambda = \frac{r_e E}{GM^2}$ and $\mu = \frac{GM\beta}{r_e}$ and plot $(\lambda(x), \mu(x))$

let define $\lambda = \dfrac{r_e E}{GM^2}$ and $\mu = \dfrac{GM\beta}{r_e}$ and plot $(\lambda(x), \mu(x))$



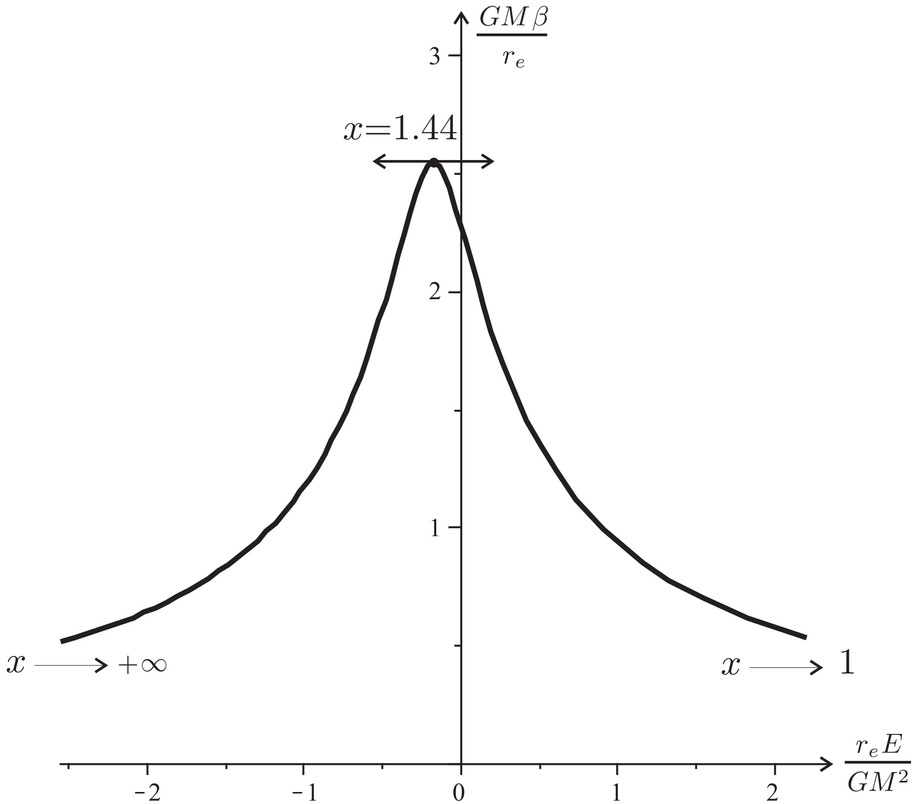
let define $\lambda = \frac{r_e E}{GM^2}$ and $\mu = \frac{GM\beta}{r_e}$ and plot $(\lambda(x), \mu(x))$



If $x = \frac{r_e}{r_o} > x_c = 1.44$ the system becomes unstable, it corresponds to

$$\mathcal{R} = \left. \frac{\rho(r_e)}{\rho(0)} \right|_{x=x_c} = x_c^4 \approx 4.3$$

let define $\lambda = \dfrac{r_e E}{GM^2}$ and $\mu = \dfrac{GM\beta}{r_e}$ and plot $(\lambda(x), \mu(x))$



If $x = \dfrac{r_e}{r_o} > x_c = 1.44$ the system becomes unstable, it corresponds to

$$\mathcal{R} = \left.\frac{\rho(r_e)}{\rho(0)}\right|_{x=x_c} = x_c^4 \approx 4.3$$

A core-halo 4 structure in a isothermal bath is unstable if $\mathcal{R} \gtrsim 4$

Radial orbit instability (ROI)

Radial orbit instability (ROI)

see Maréchal & P. ’09a for a large review ...

An anisotropic spherical equilibrium system becomes triaxial if it is too radial in velocity space

Radial orbit instability (ROI)

see Maréchal & P. ’09a for a large review ...

An anisotropic spherical equilibrium system becomes triaxial if it is too radial in velocity space

Proof (see Maréchal & P. ’09b for details)

Radial orbit instability (ROI)

see Maréchal & P. ’09a for a large review ...

An anisotropic spherical equilibrium system becomes triaxial if it is too radial in velocity space

Proof (see Maréchal & P. ’09b for details)

1 The second order energy variation

$$H^{(2)}[f_0] = -\int \{g, E\}\{g, f_0\} \mathrm{d}\mathbf{\Gamma} - Gm^2 \iint \frac{\{g, f_0\}\{g', f_0'\}}{|\mathbf{q} - \mathbf{q}'|} \mathrm{d}\mathbf{\Gamma}\, \mathrm{d}\mathbf{\Gamma}'$$

associated to a perturbation generated by g of an *equilibrium* system with

$$f_o\left(E, L^2\right) = f_0^a\left(E, L^2\right) = \varphi\left(E\right) \delta_a\left(L^2\right) \qquad \text{with} \qquad \delta_a(L^2) = \frac{1}{\pi a^2} \exp\left(-\frac{L^2}{a^2}\right)$$

becomes negative when $a \to 0$ (radial orbits system) $\Rightarrow$ Existence of negative energy modes.

Radial orbit instability (ROI)

see Maréchal & P. '09a for a large review ...

An anisotropic spherical equilibrium system becomes triaxial if it is too radial in velocity space

Proof (see Maréchal & P. '09b for details)

1. The second order energy variation

$$H^{(2)}[f_0] = -\int \{g,E\}\{g,f_0\}\mathrm{d}\mathbf{\Gamma} - Gm^2 \iint \frac{\{g,f_0\}\{g',f_0'\}}{|\mathbf{q}-\mathbf{q}'|}\mathrm{d}\mathbf{\Gamma}\,\mathrm{d}\mathbf{\Gamma}'$$

associated to a perturbation generated by g of an *equilibrium* system with

$$f_o\left(E,L^2\right) = f_0^a\left(E,L^2\right) = \varphi\left(E\right)\delta_a\left(L^2\right) \qquad \text{with} \qquad \delta_a(L^2) = \frac{1}{\pi a^2}\exp\left(-\frac{L^2}{a^2}\right)$$

becomes negative when $a \to 0$ (radial orbits system) $\Rightarrow$ Existence of negative energy modes.

2. Vlasov-Poisson system *is* hamiltonian

Radial orbit instability (ROI)

see Maréchal & P. ’09a for a large review ...

An anisotropic spherical equilibrium system becomes triaxial if it is too radial in velocity space

Proof (see Maréchal & P. ’09b for details)

1 The second order energy variation

$$H^{(2)}[f_0] = -\int \{g, E\}\{g, f_0\}\mathrm{d}\mathbf{\Gamma} - Gm^2 \int\int \frac{\{g, f_0\}\{g', f_0'\}}{|\mathbf{q}-\mathbf{q}'|}\mathrm{d}\mathbf{\Gamma}\,\mathrm{d}\mathbf{\Gamma}'$$

associated to a perturbation generated by g of an *equilibrium* system with

$$f_o\left(E, L^2\right) = f_0^a\left(E, L^2\right) = \varphi\left(E\right)\delta_a\left(L^2\right) \qquad \text{with} \qquad \delta_a(L^2) = \frac{1}{\pi a^2}\exp\left(-\frac{L^2}{a^2}\right)$$

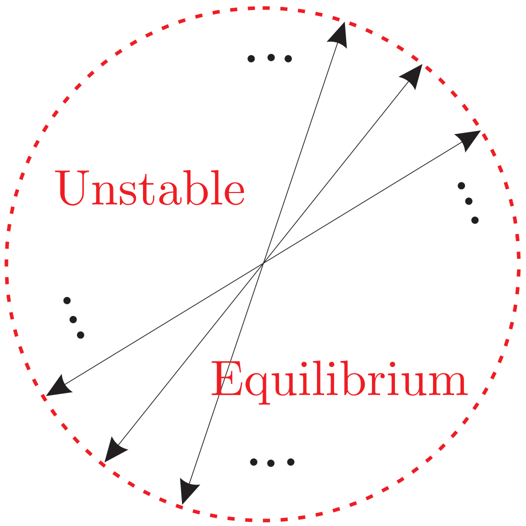
becomes negative when $a \to 0$ (radial orbits system) $\Rightarrow$ Existence of negative energy modes.

2 Vlasov-Poisson system *is* hamiltonian

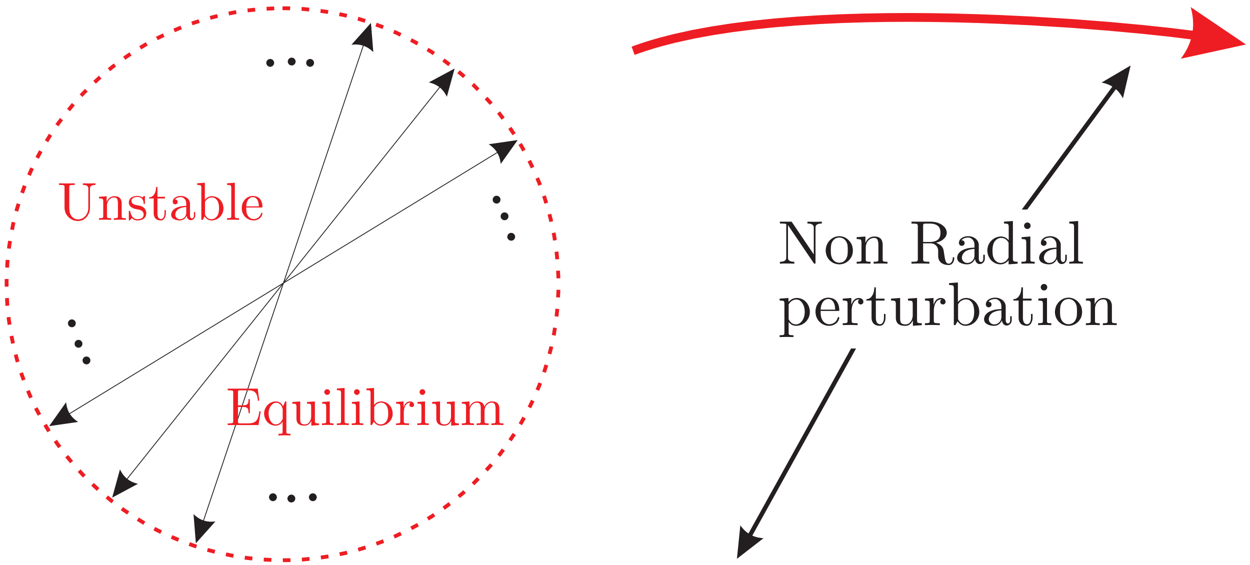
Conclusion : The system is Dissipation-Induced Unstable (spectral instability) and that cause it to lose its spherical symmetry.

Mechanism of ROI

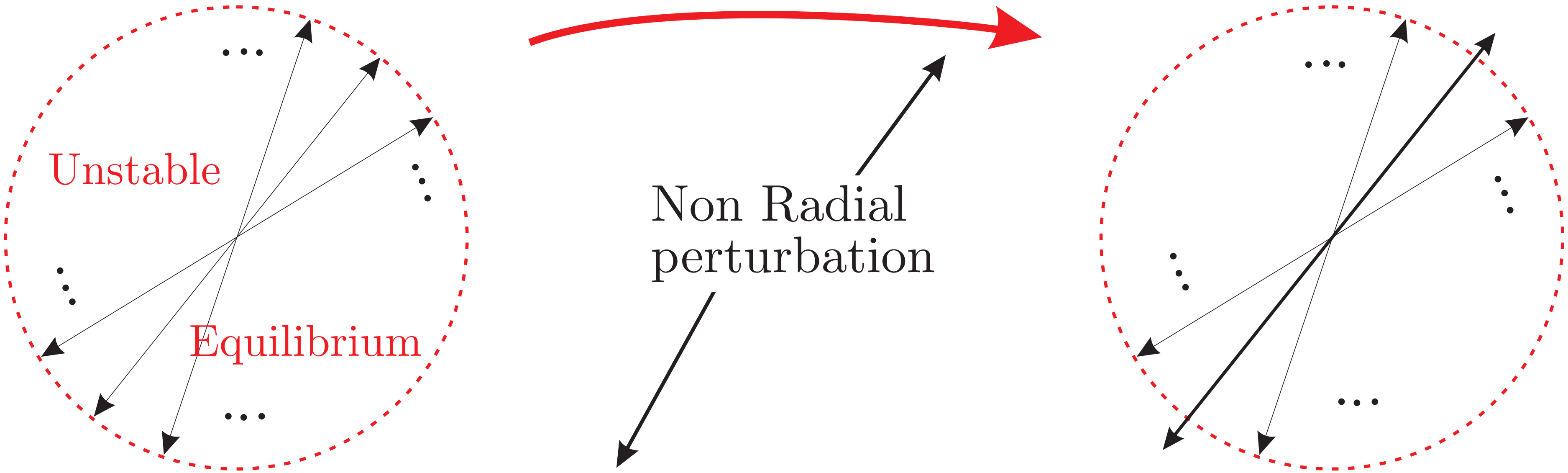
Mechanism of ROI



Mechanism of ROI

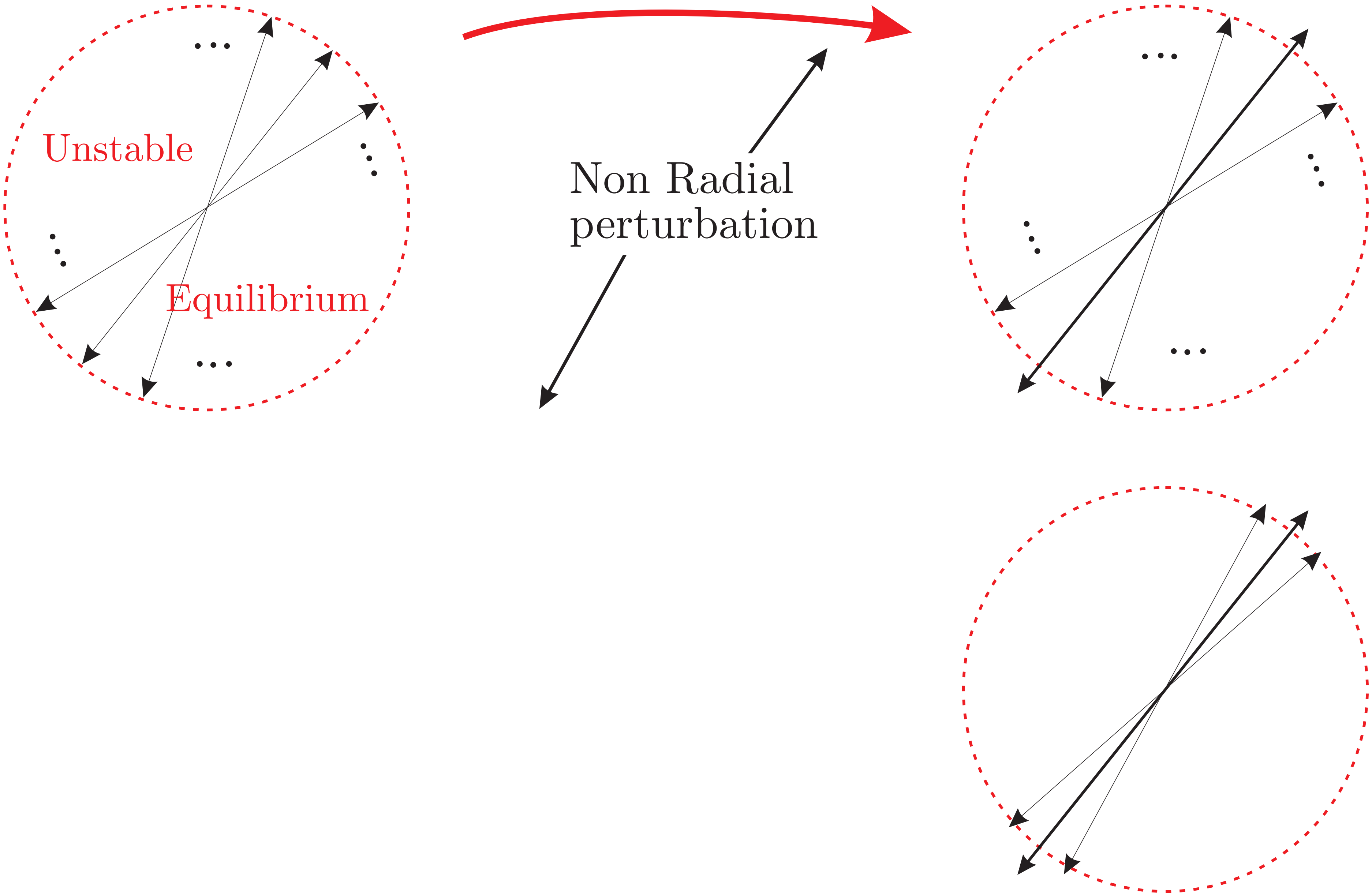


Mechanism of ROI

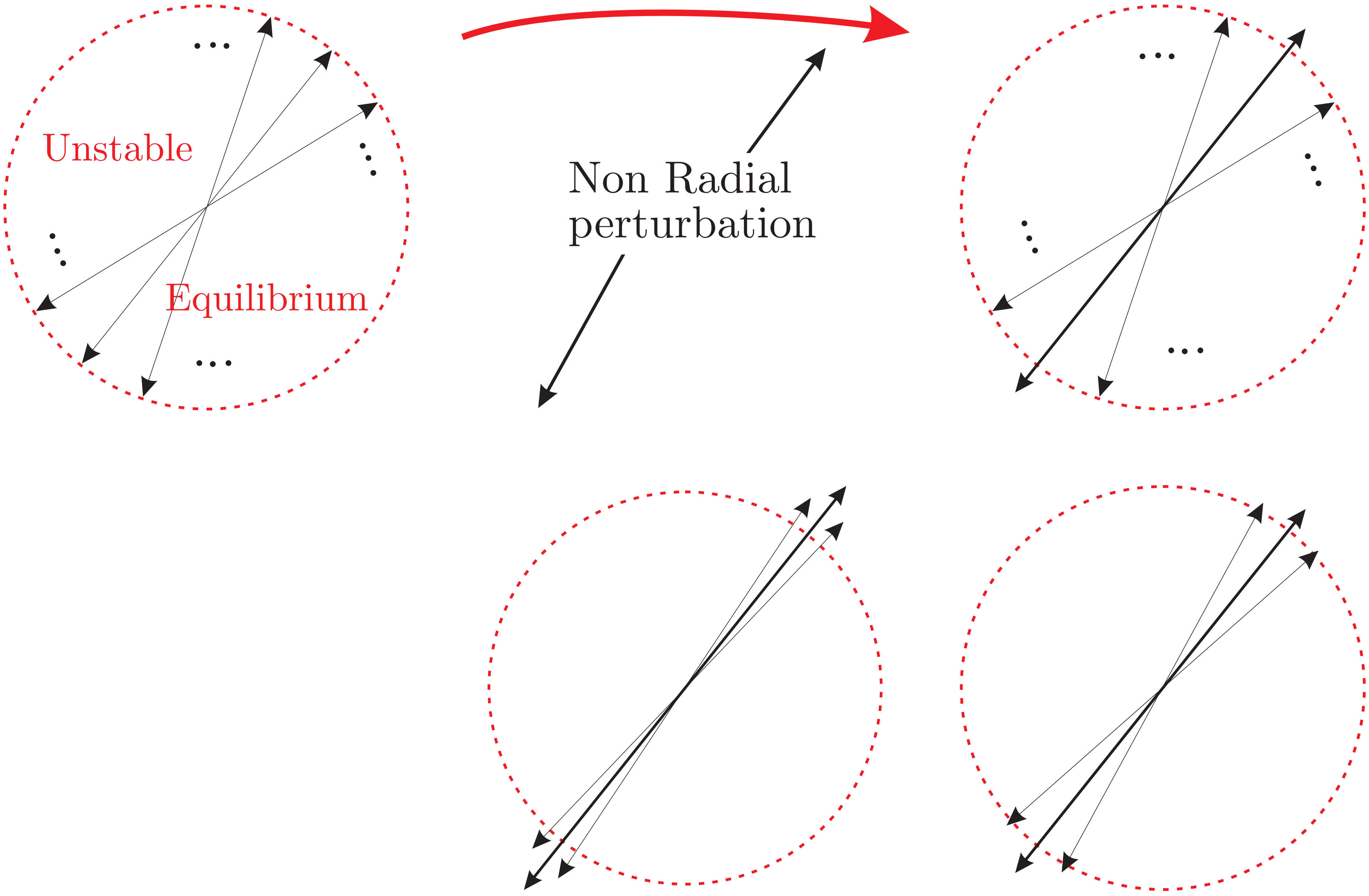


PhaseSpace at CIRM

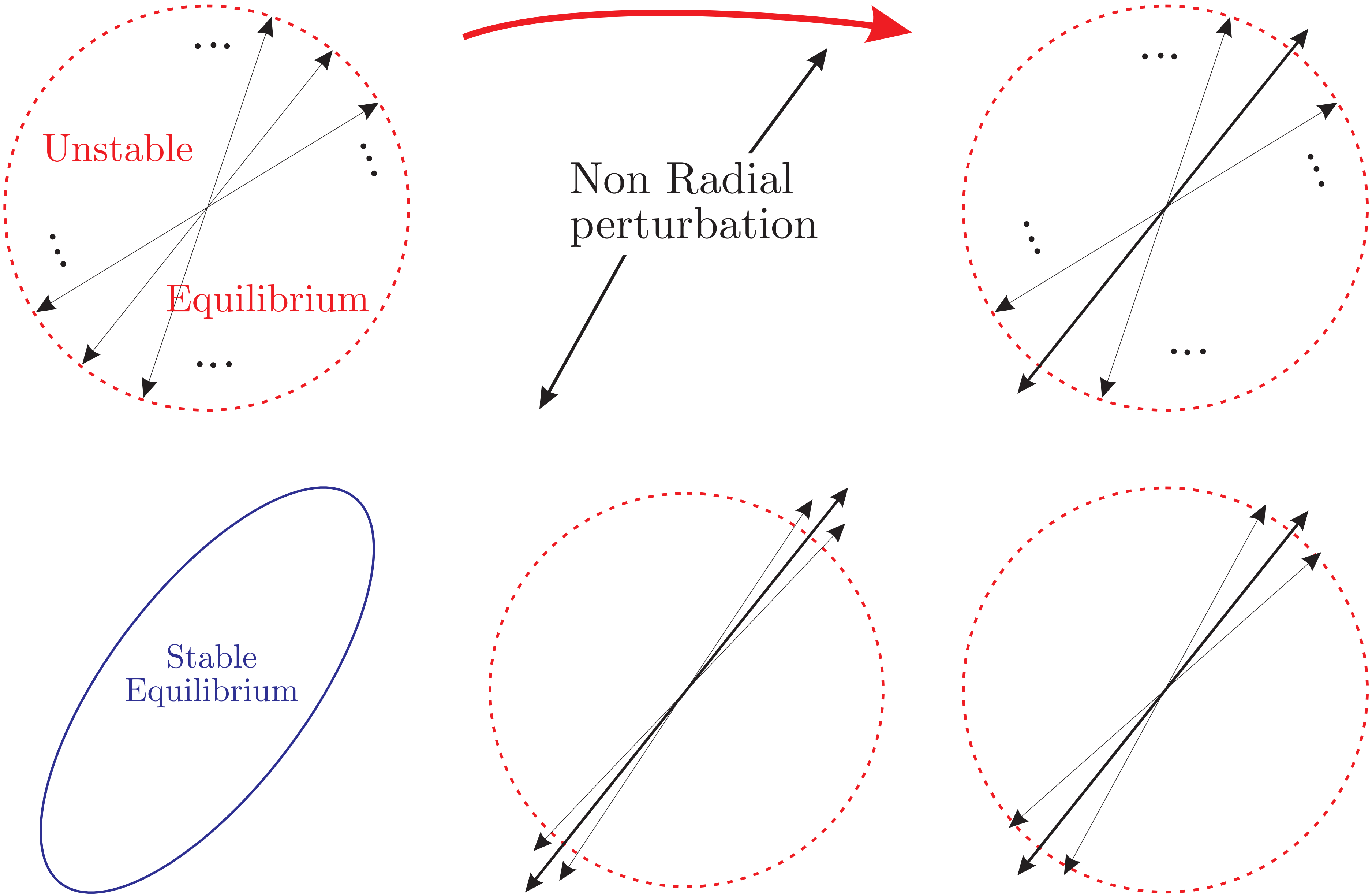
Mechanism of ROI



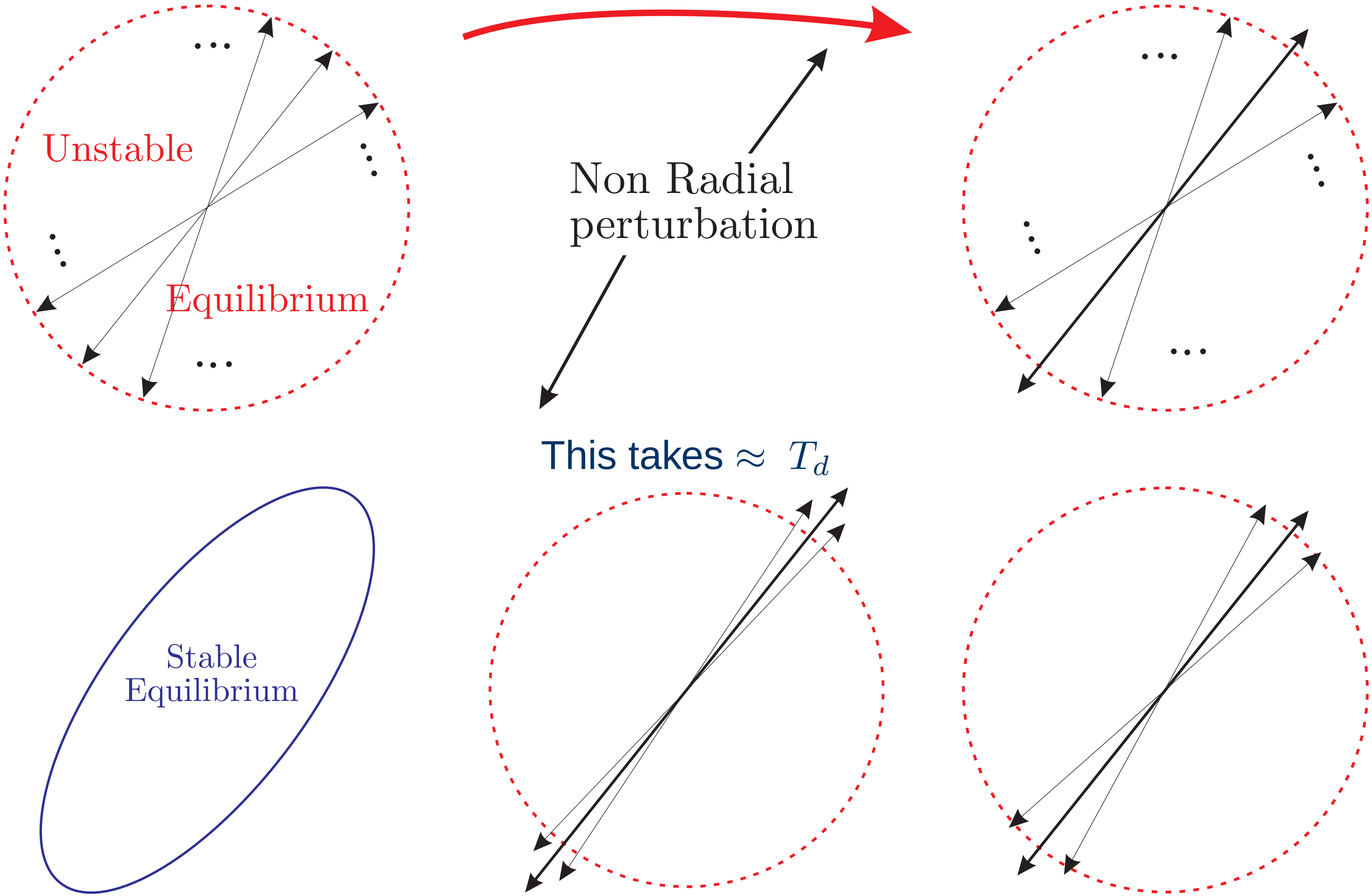
Mechanism of ROI



Mechanism of ROI



Mechanism of ROI



Facts about ROI

Facts about ROI

1. Huss et al. ’91, Mac Millan et al. ’06 : The effects of ROI are attenuated by the merging process (eg Katz ’91), but this instability is needed to obtain the "good" density profile of structures in simulations and not a pure power law.

Facts about ROI

1. Huss et al. ’91, Mac Millan et al. ’06 : The effects of ROI are attenuated by the merging process (eg Katz ’91), but this instability is needed to obtain the "good" density profile of structures in simulations and not a pure power law.

2. Roy & P. ’04,... ,Barnes et al. ’09 : Generic Top Hat Collapse don’t suffer ROI. Most popular (?) explanation by Barnes et al. ’09 : "*Roi is suppressed because sufficient mass occupying radial anisotropic orbits is not retained during the collapse*" (because they leave the system)

Facts about ROI

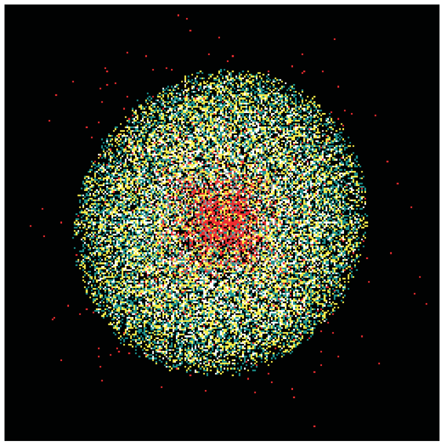
1. Huss et al. '91, Mac Millan et al. '06 : The effects of ROI are attenuated by the merging process (eg Katz '91), but this instability is needed to obtain the "good" density profile of structures in simulations and not a pure power law.

2. Roy & P. '04,... ,Barnes et al. '09 : Generic Top Hat Collapse don't suffer ROI. Most popular (?) explanation by Barnes et al. '09 : "*Roi is suppressed because sufficient mass occupying radial anisotropic orbits is not retained during the collapse*" (because they leave the system)
My explanation : This instability needs an equilibrium seed to grow : only actual inhomogeneous collapse (with several timescales) is affected by ROI.

Facts about ROI

1 Huss et al. '91, Mac Millan et al. '06 : The effects of ROI are attenuated by the merging process (eg Katz '91), but this instability is needed to obtain the "good" density profile of structures in simulations and not a pure power law.

2 Roy & P. '04,... ,Barnes et al. '09 : Generic Top Hat Collapse don't suffer ROI. Most popular (?) explanation by Barnes et al. '09 : "*Roi is suppressed because sufficient mass occupying radial anisotropic orbits is not retained during the collapse*" (because they leave the system)
My explanation : This instability needs an equilibrium seed to grow : only actual inhomogeneous collapse (with several timescales) is affected by ROI.
Example : A Plummer sphere could be triaxialized by ROI :



At $t = 0$,
a red Plummer is surrounded by
an hollow cold and homogeneous bowl

Step 3

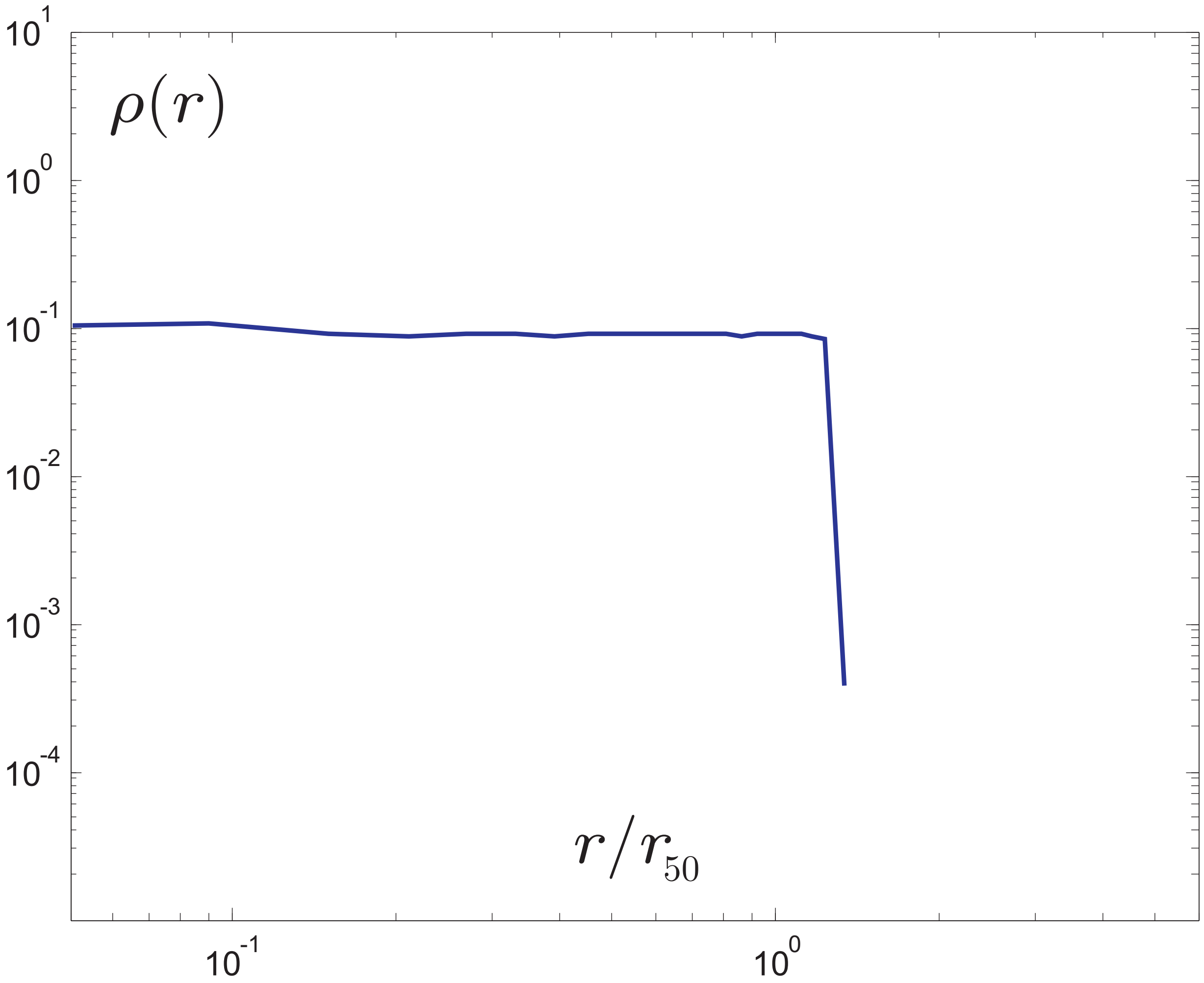
Some fundamental experiences ...

N bodies collapses

Set 1 : One Top Hat of density ρ_0 with $N = 3 \times 10^4$ particles a virial $\vartheta = -0.5$.

N bodies collapses

Set 1 : One Top Hat of density ρ_0 with $N = 3 \times 10^4$ particles a virial $\vartheta = -0.5$.

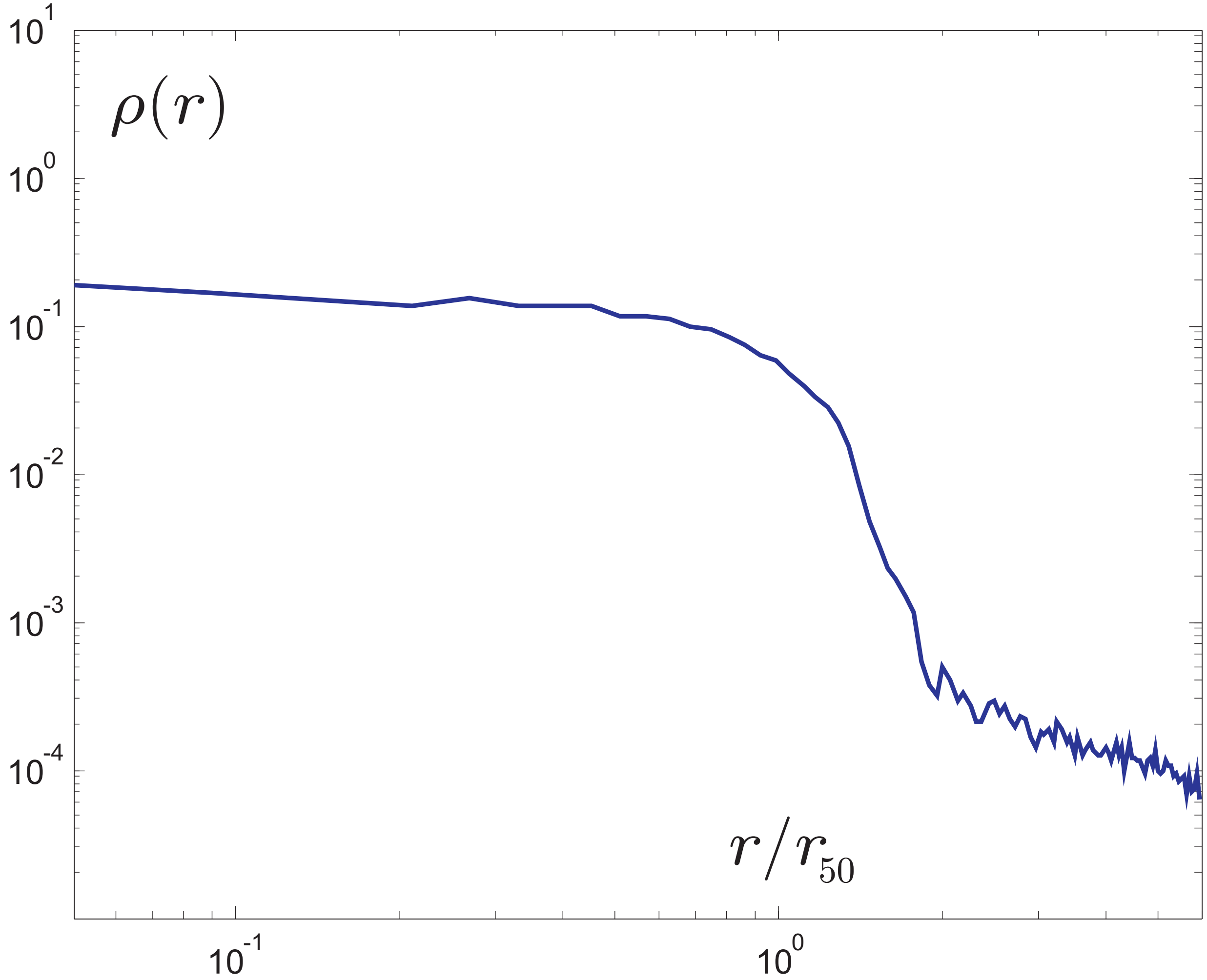


$t = 0$

PhaseSpace at CIRM

N bodies collapses

Set 1 : One Top Hat of density ρ_0 with $N = 3 \times 10^4$ particles a virial $\vartheta = -0.5$.



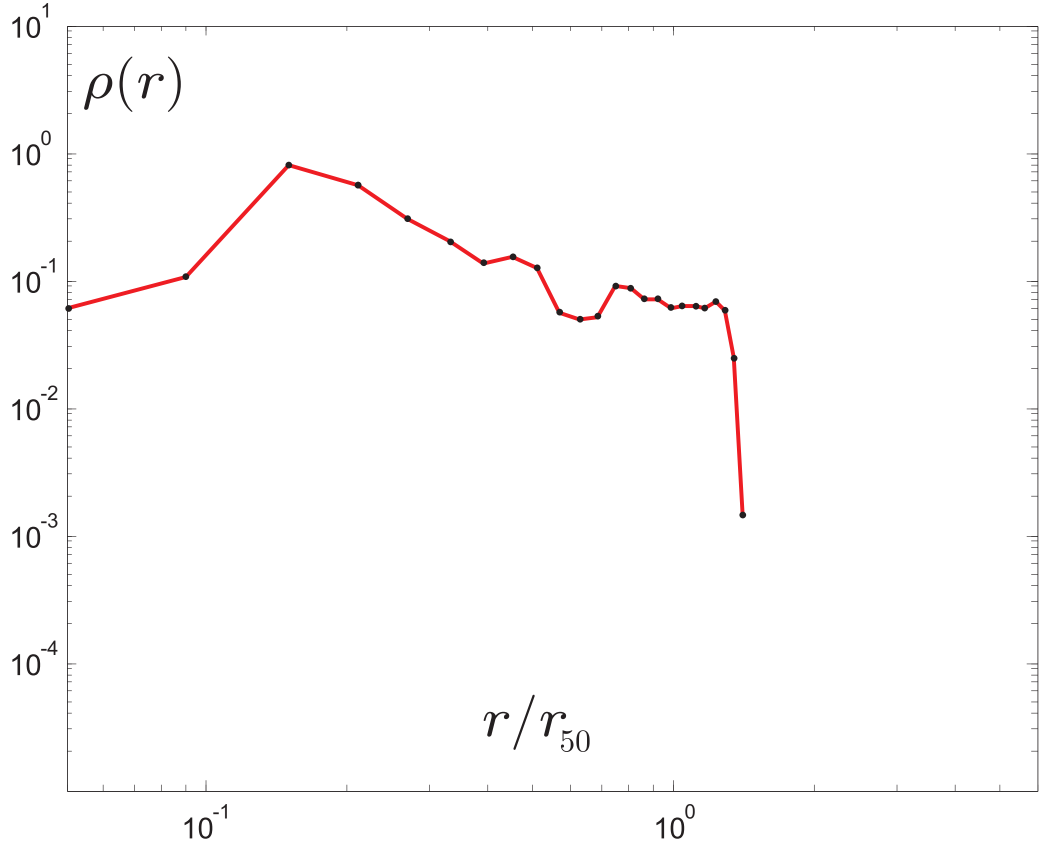
$t \gtrsim T_0 = (G\rho_0)^{-1/2}$

N bodies collapses

Set 2 : Set 1 + 20 small top hats of density $\rho_1 > \rho_0$ the whole at virial $\vartheta = -0.5$.

N bodies collapses

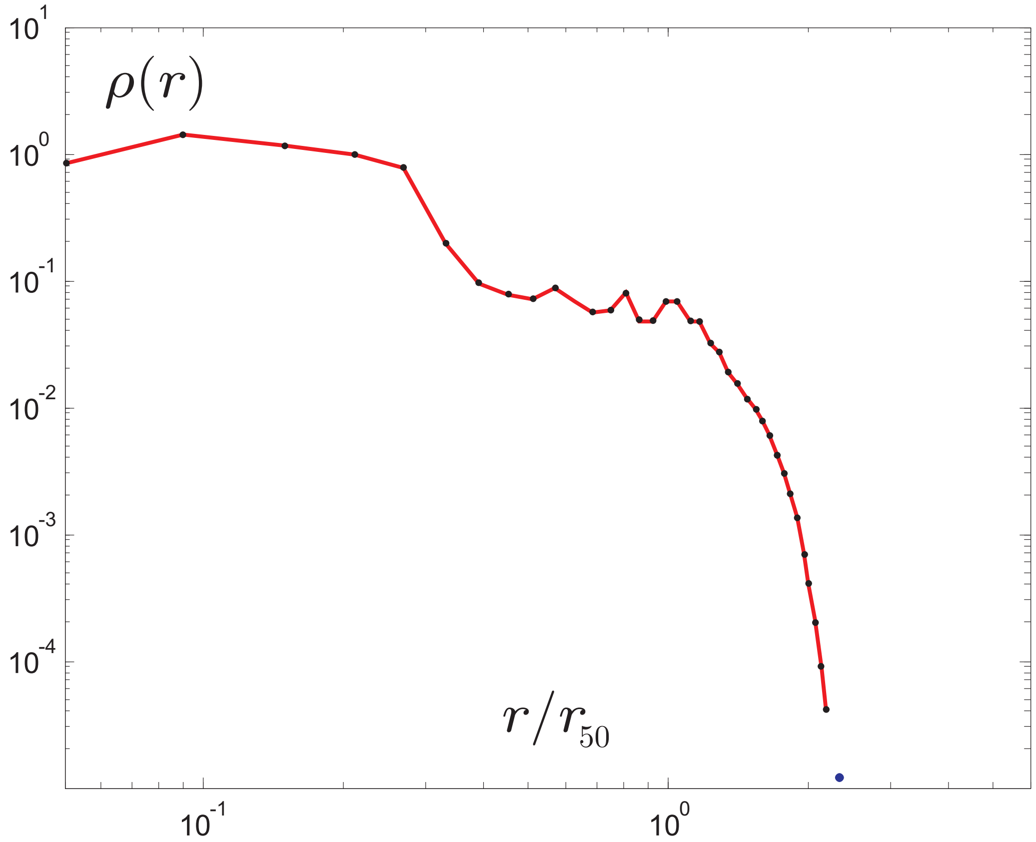
Set 2 : Set 1 + 20 small top hats of density $\rho_1 > \rho_0$ the whole at virial $\vartheta = -0.5$.



$t = 0$

N bodies collapses

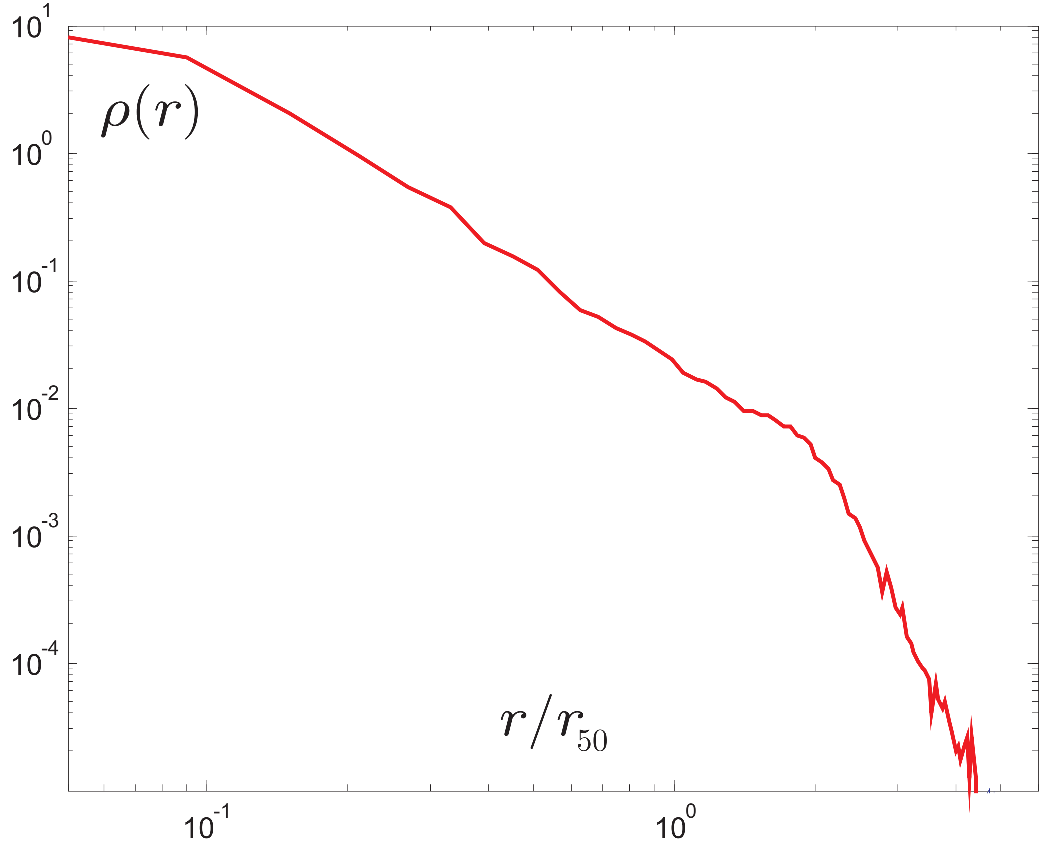
Set 2 : Set 1 + 20 small top hats of density $\rho_1 > \rho_0$ the whole at virial $\vartheta = -0.5$.



$t \gtrsim T_1 = (G\rho_1)^{-1/2}$ and $t < T_0$

N bodies collapses

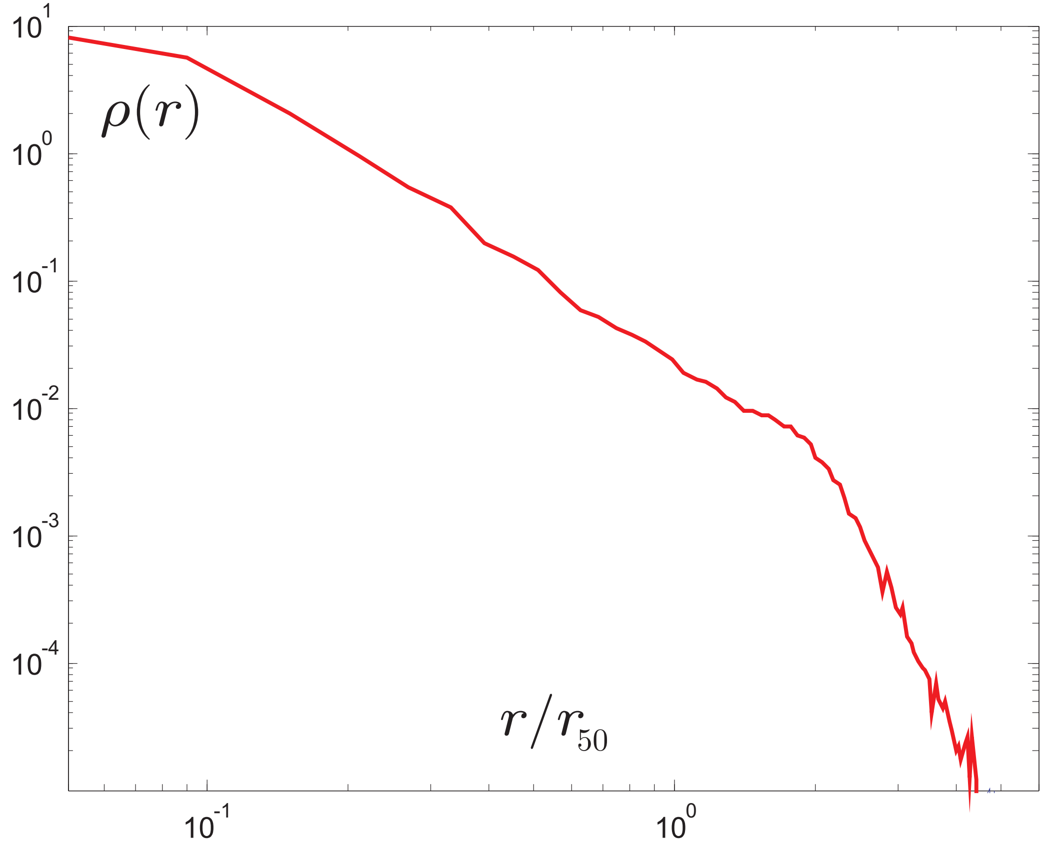
Set 2 : Set 1 + 20 small top hats of density $\rho_1 > \rho_0$ the whole at virial $\vartheta = -0.5$.



$$t \gtrsim T_0$$

N bodies collapses

Set 2 : Set 1 + 20 small top hats of density $\rho_1 > \rho_0$ the whole at virial $\vartheta = -0.5$.



$t \gtrsim T_0$

Additional remarks

- With less than 10 small top hats, the results are similar to the one of set 1 ;
- Allowing more evaporation produces the collapse of Set 1 Top Hat's core ;
- More cold collapses of set 2 $(\vartheta \lesssim 0.2)$ become triaxial ; only for set 2 !

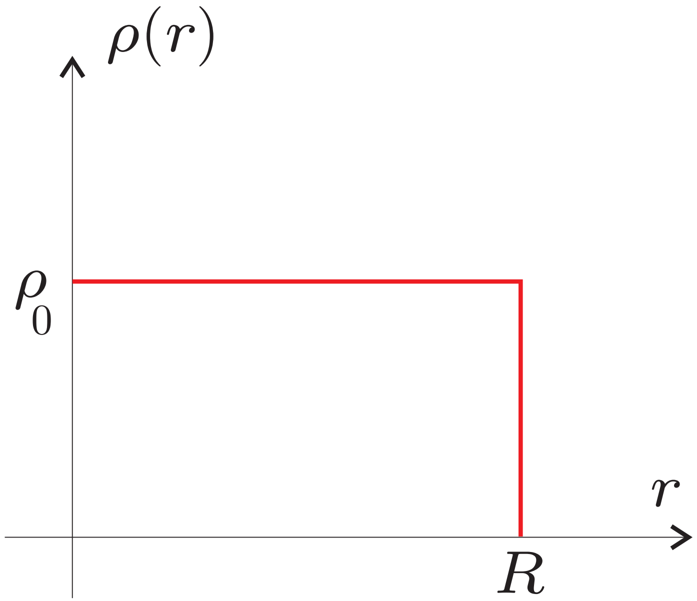
Analysis : what's happened

1. Homogeneous collapse

PhaseSpace at CIRM

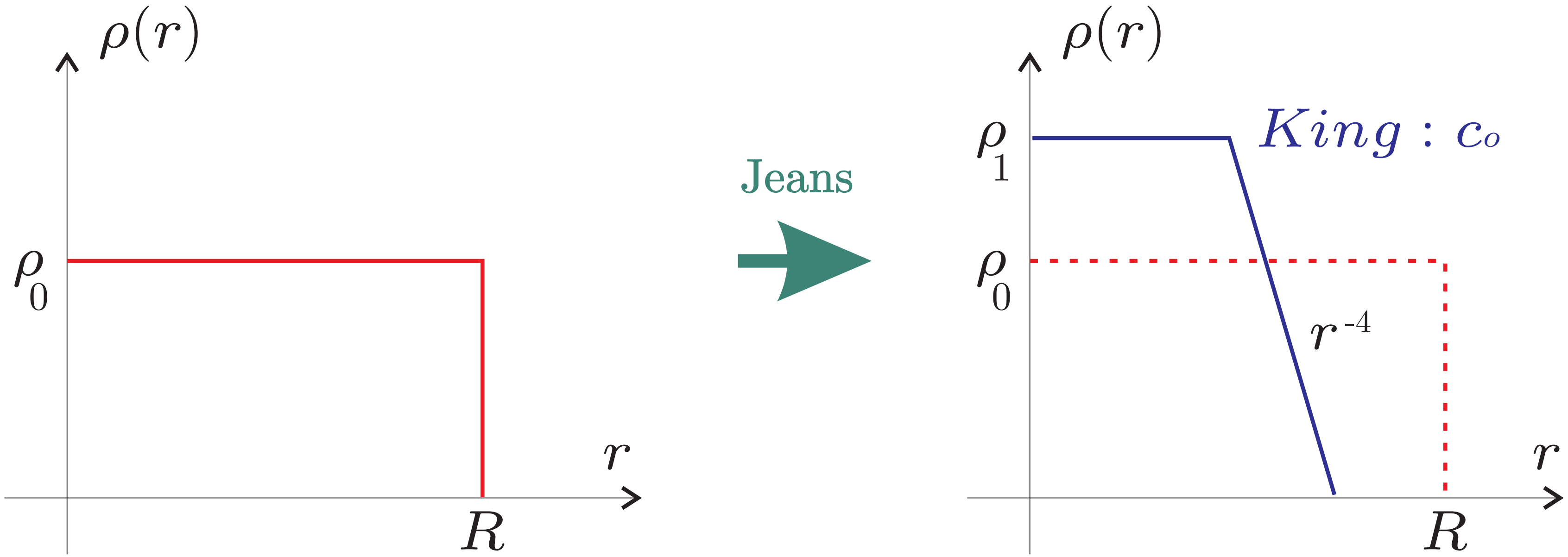
Analysis : what's happened

1. Homogeneous collapse



Analysis : what's happened

1 Homogeneous collapse

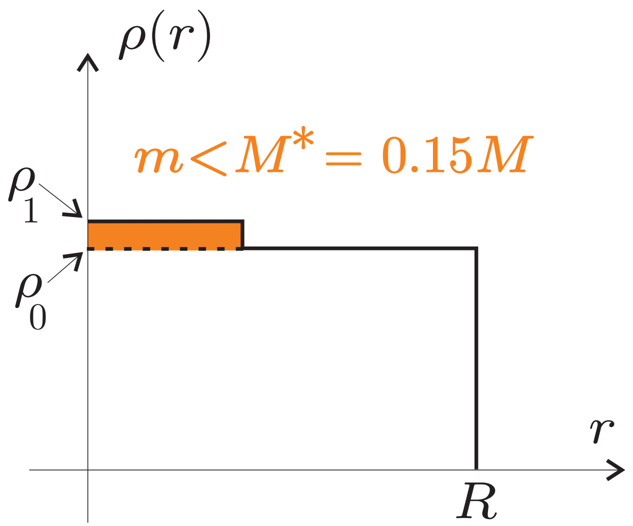


Analysis : What's happened

2. Not enough mass in the small top hats

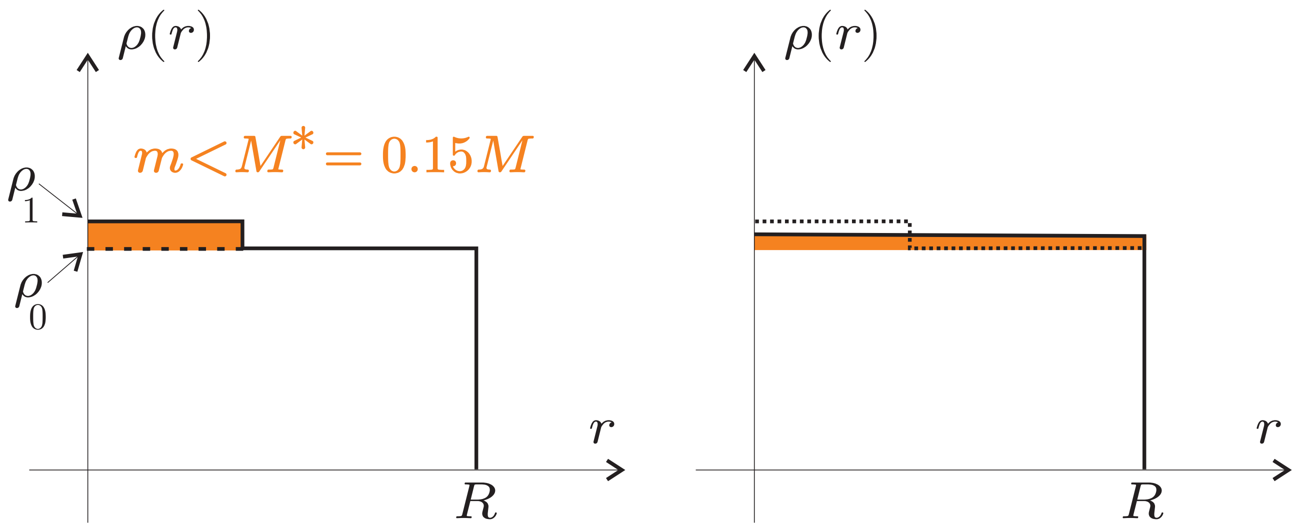
Analysis : What's happened

2 Not enough mass in the small top hats



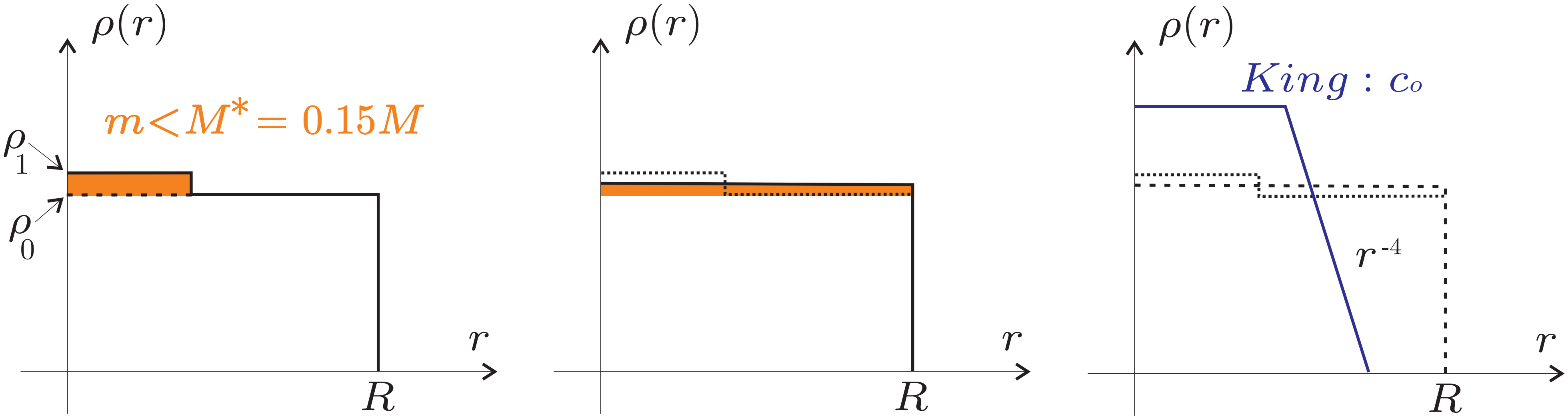
Analysis : What's happened

2. Not enough mass in the small top hats



Analysis : What's happened

2 Not enough mass in the small top hats

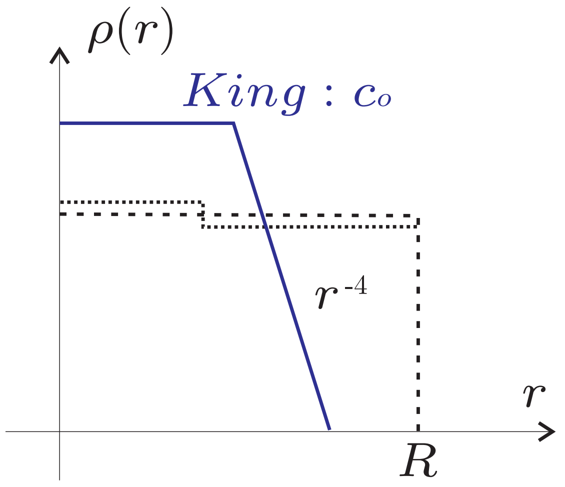


Analysis : What's happened

① and ② on more longer time (2 body relaxation)

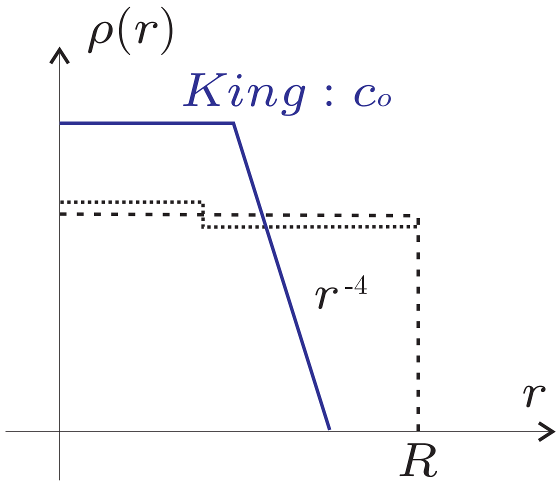
Analysis : What's happened

1 and 2 on more longer time (2 body relaxation)



Analysis : What's happened

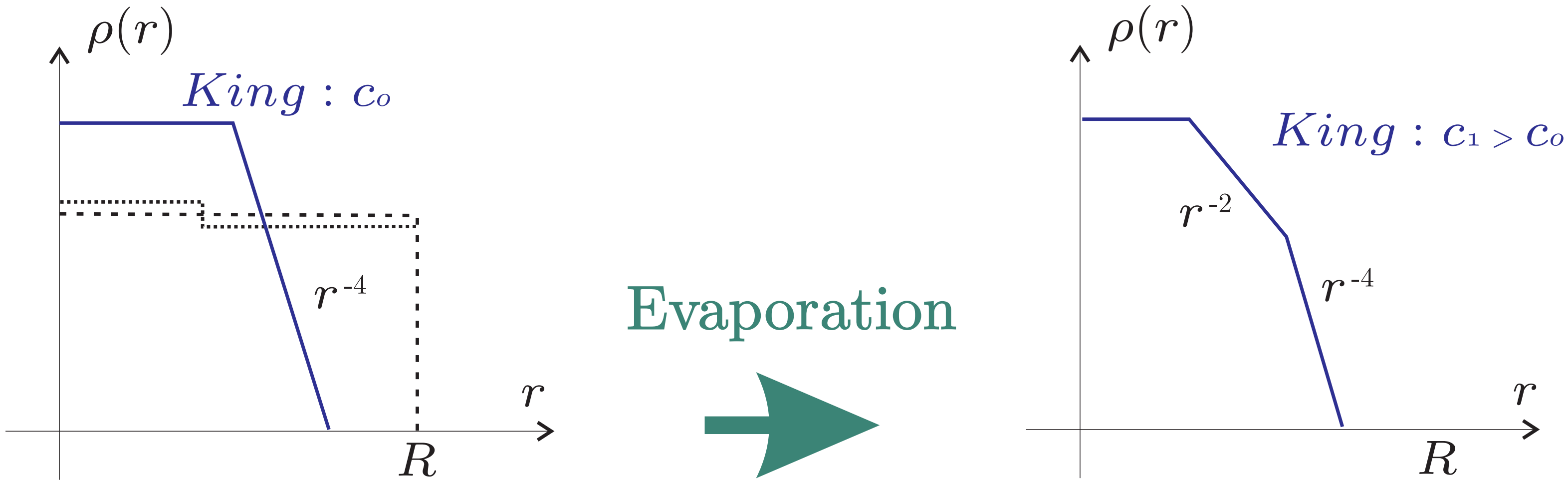
1 and 2 on more longer time (2 body relaxation)



Evaporation

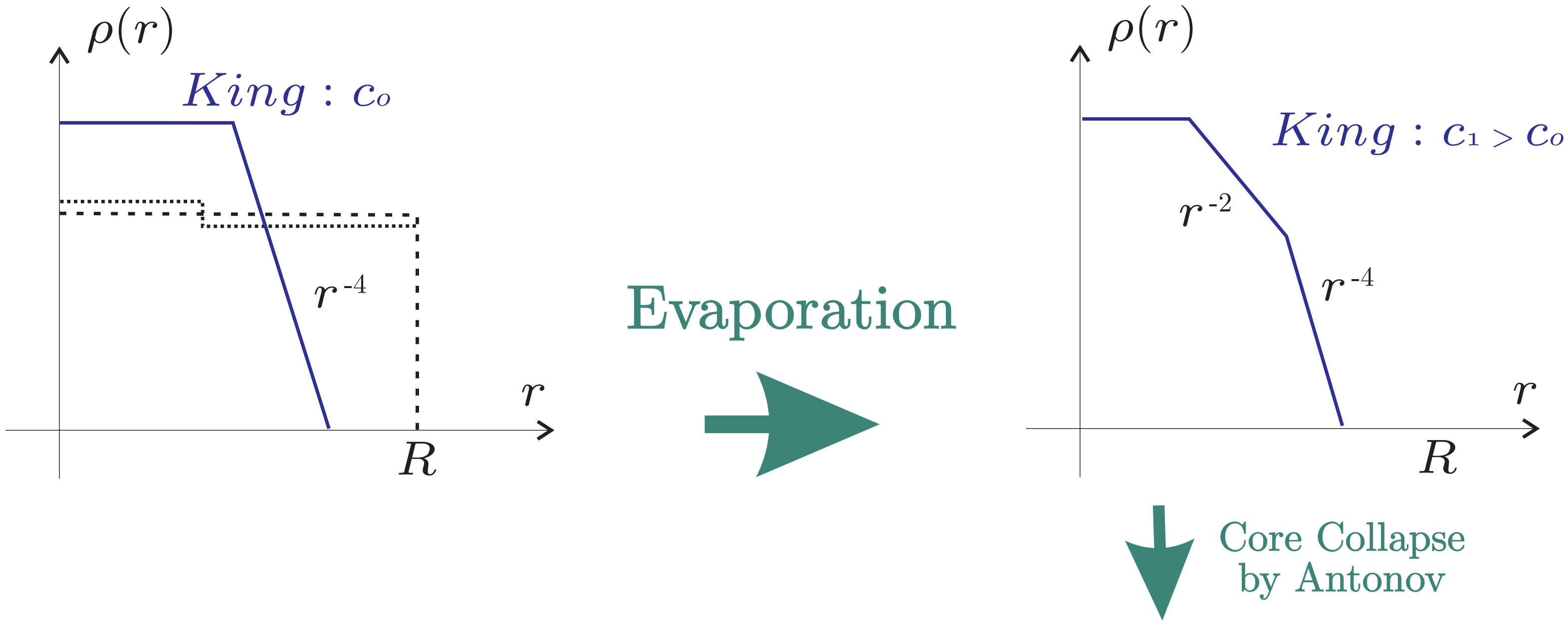
Analysis : What's happened

1 and 2 on more longer time (2 body relaxation)



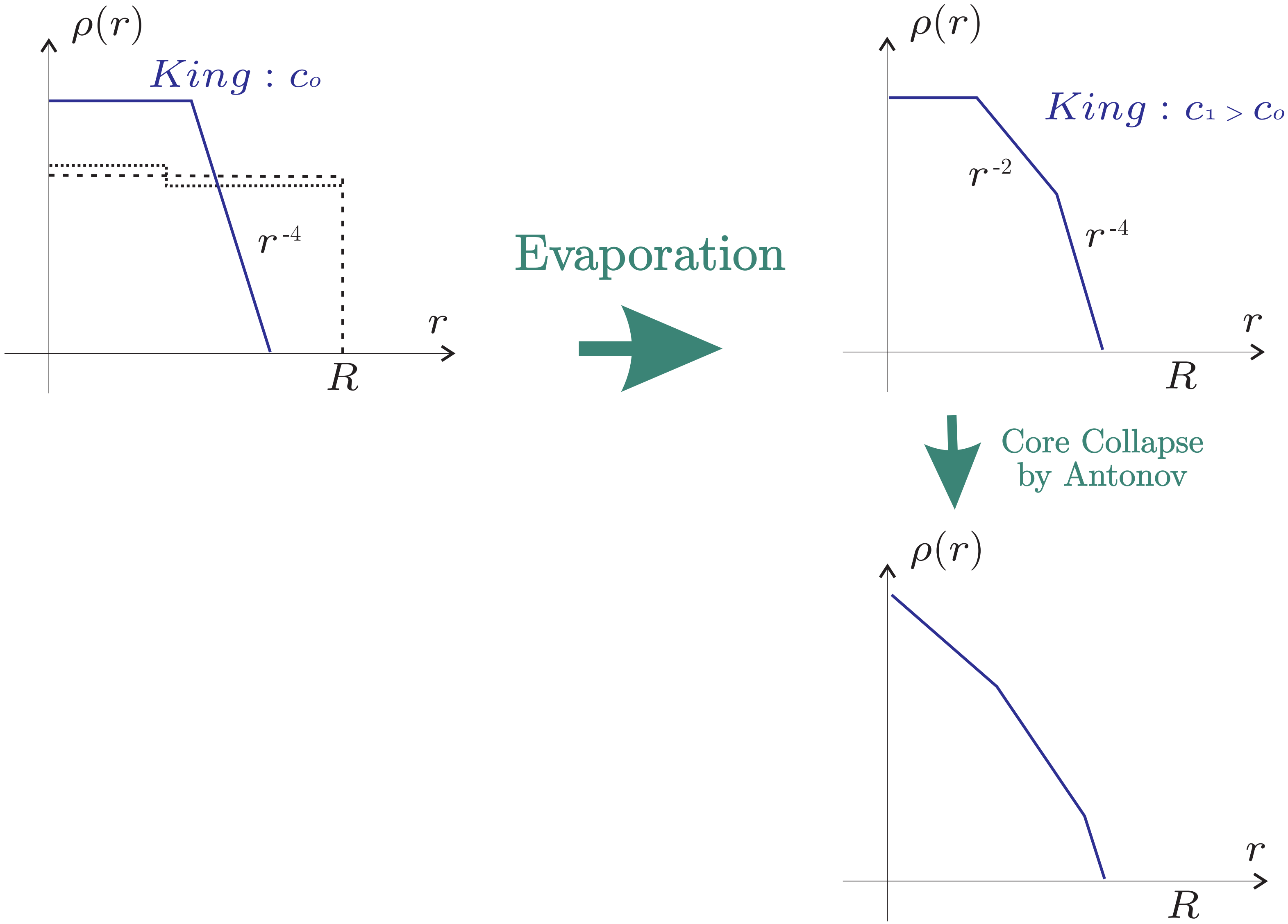
Analysis : What's happened

(1) and (2) on more longer time (2 body relaxation)



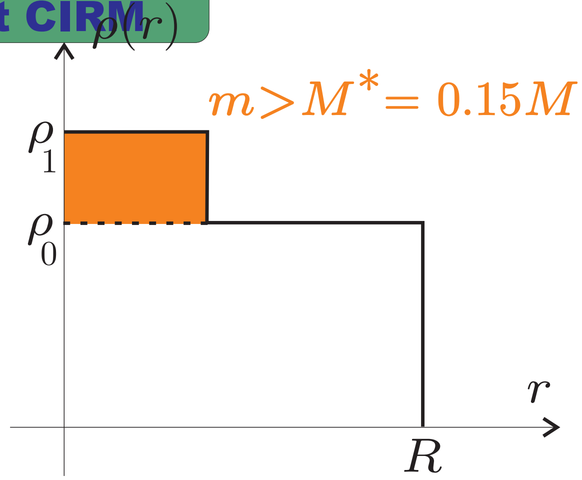
Analysis : What's happened

1 and 2 on more longer time (2 body relaxation)

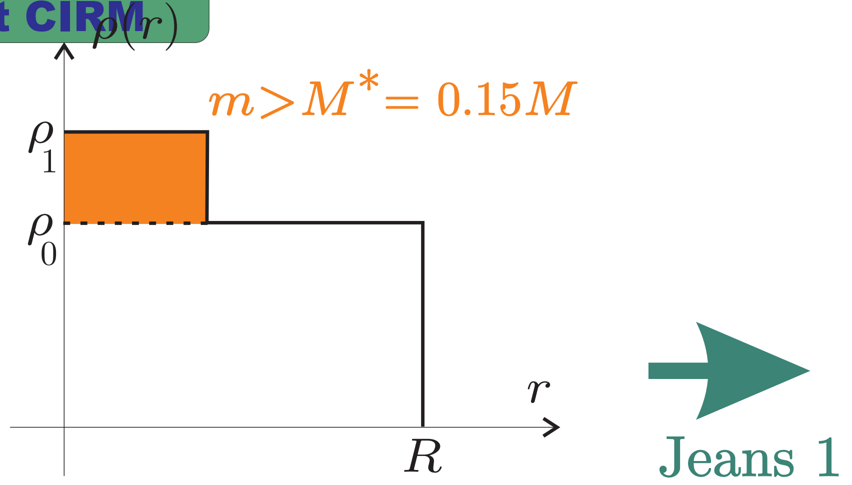


3 Enough mass in the small top hats

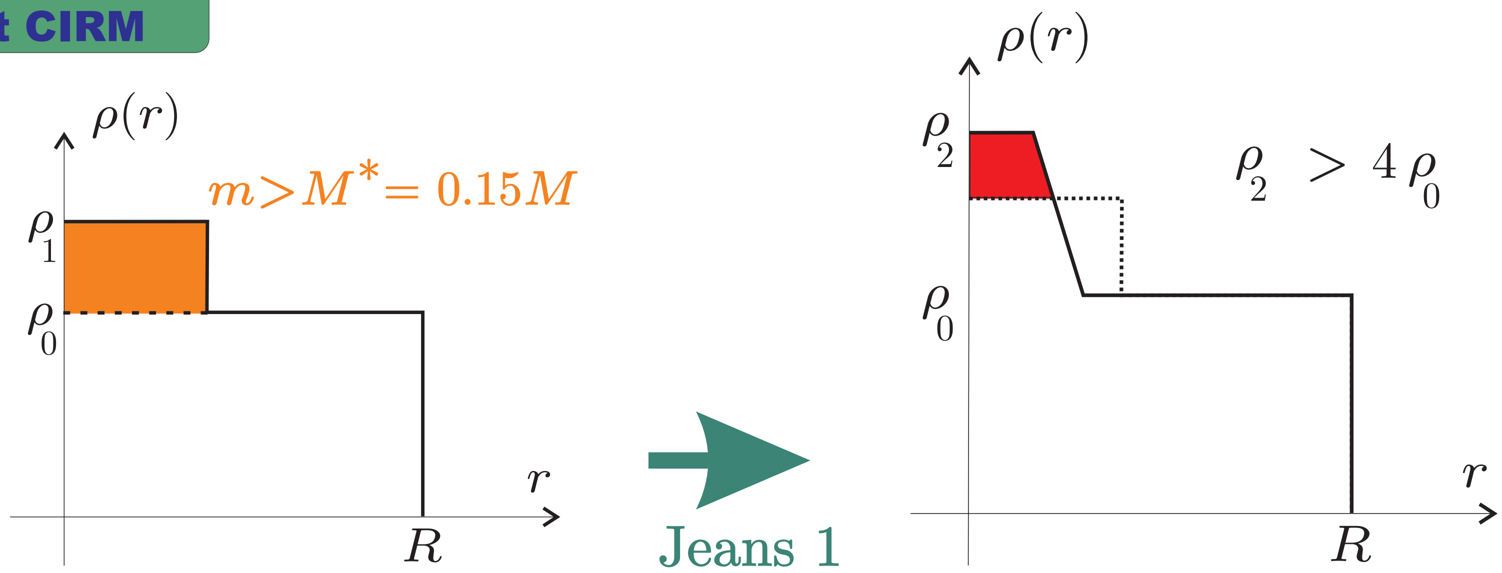
③ Enough mass in the small top hats



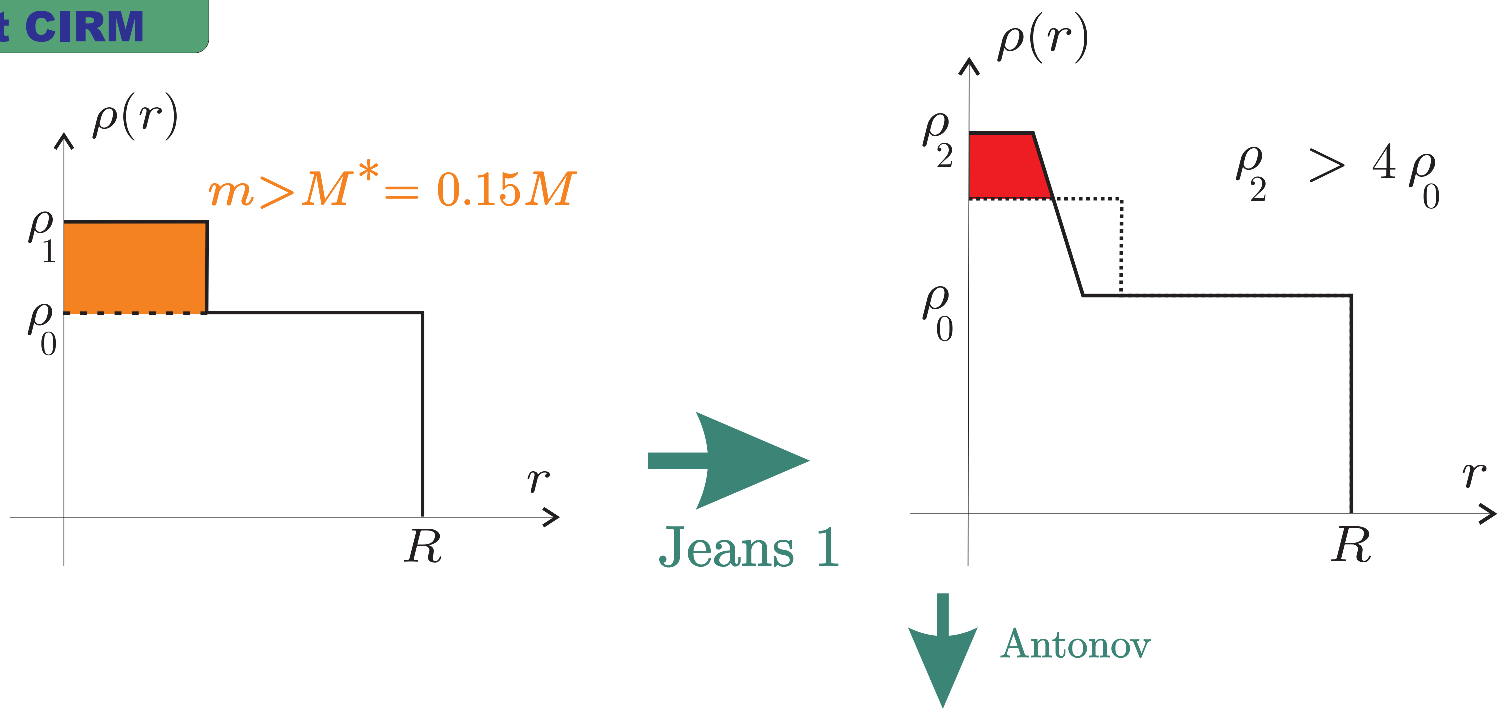
3 Enough mass in the small top hats



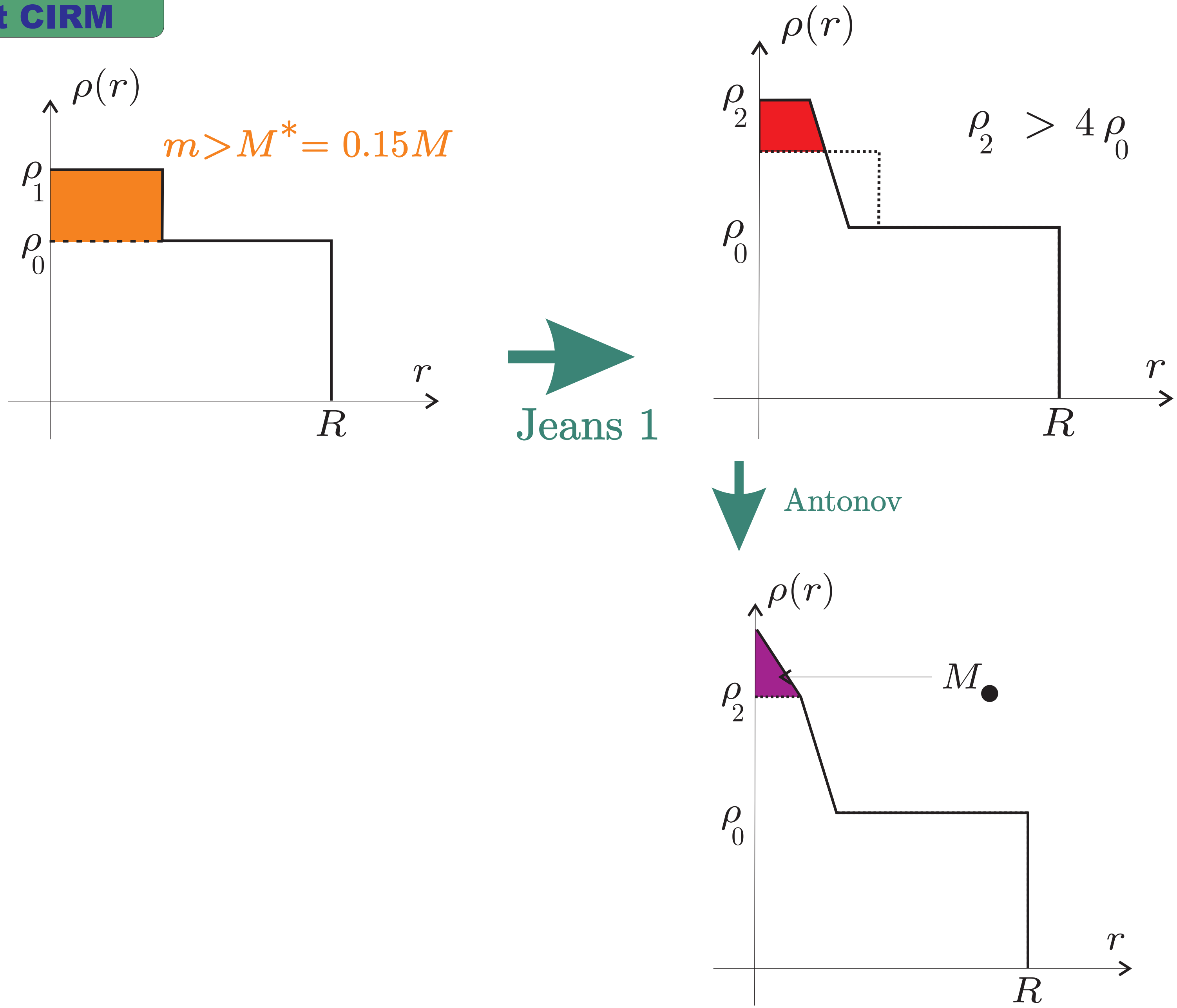
3 Enough mass in the small top hats



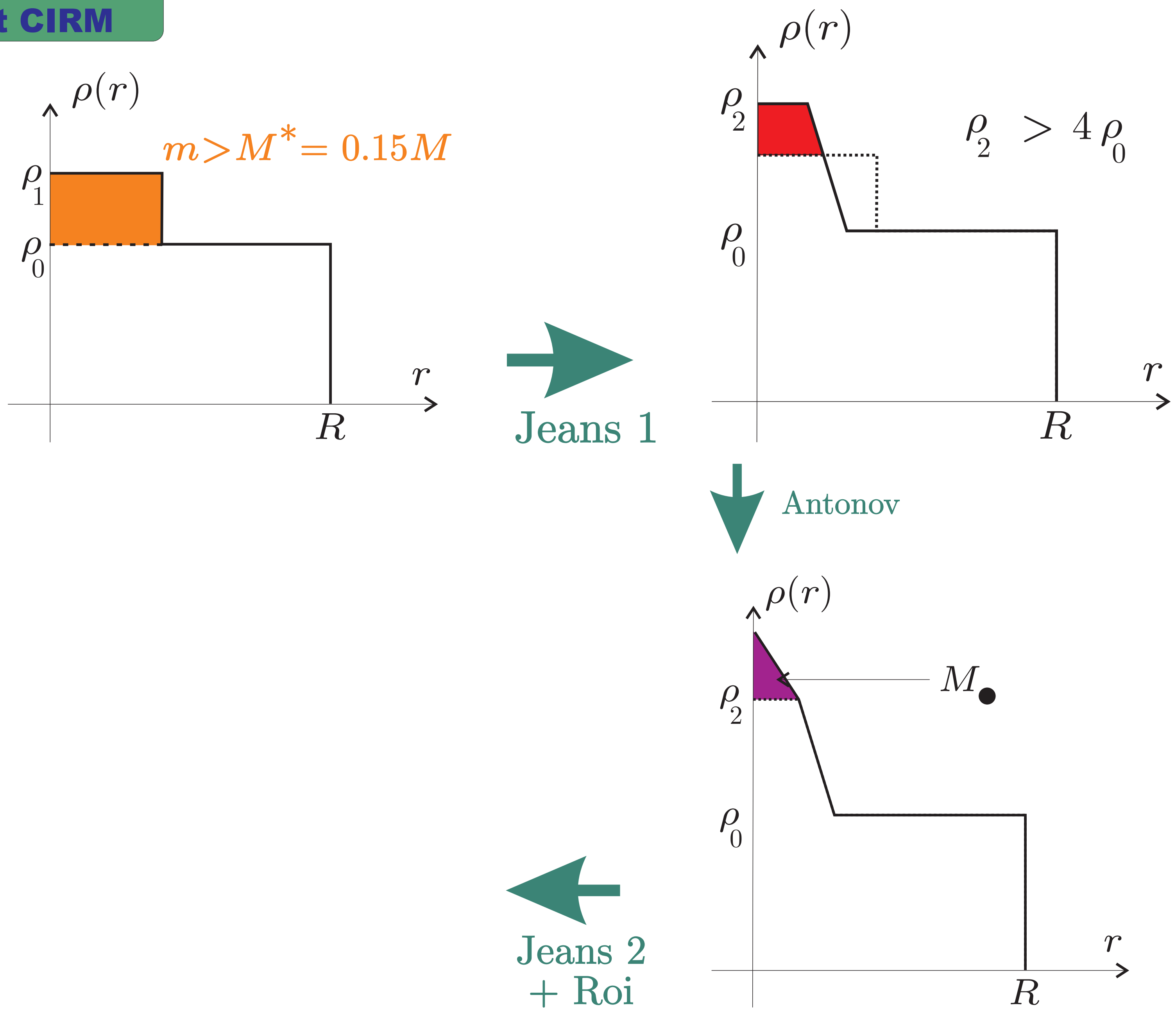
3 Enough mass in the small top hats



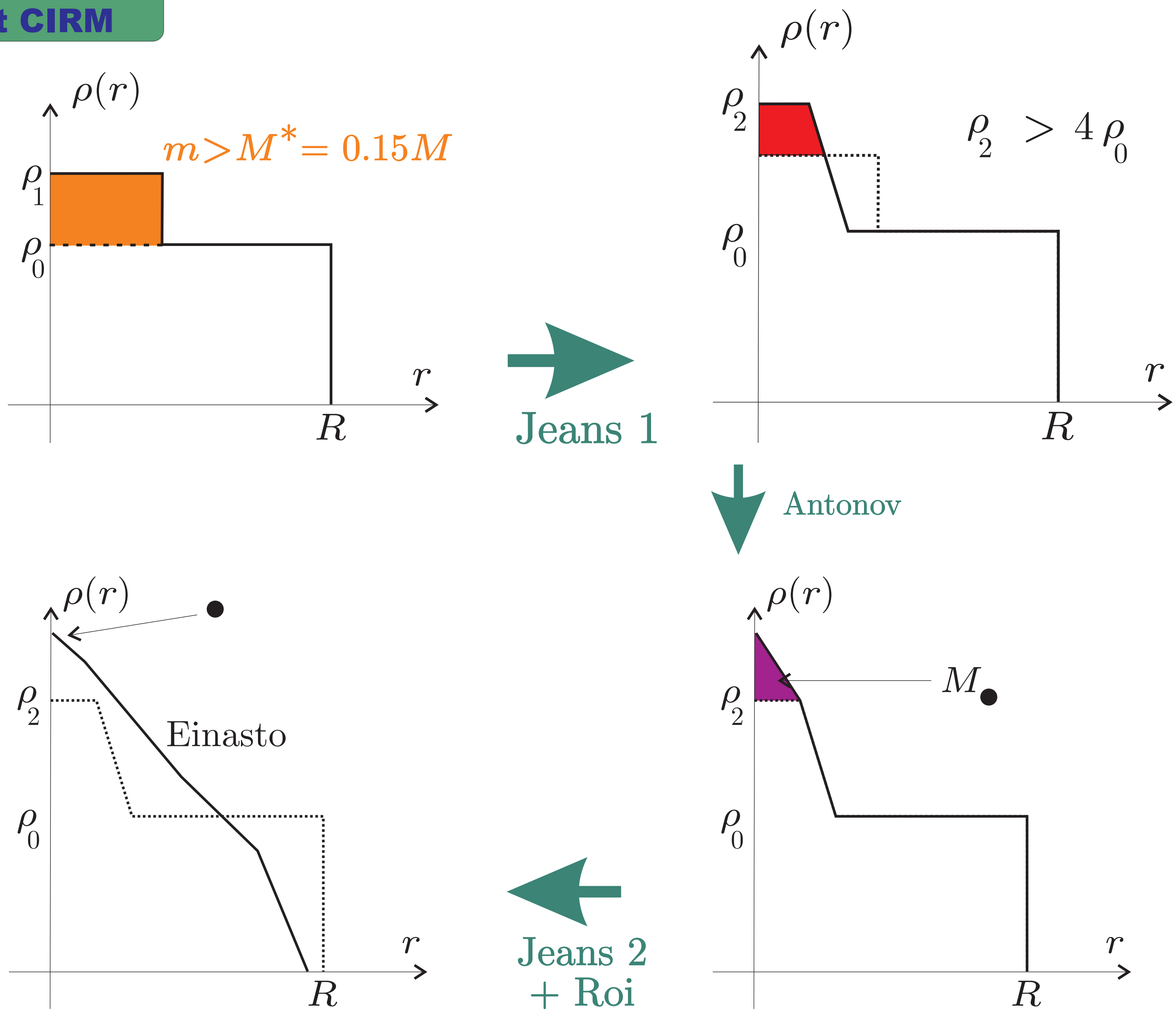
3 Enough mass in the small top hats



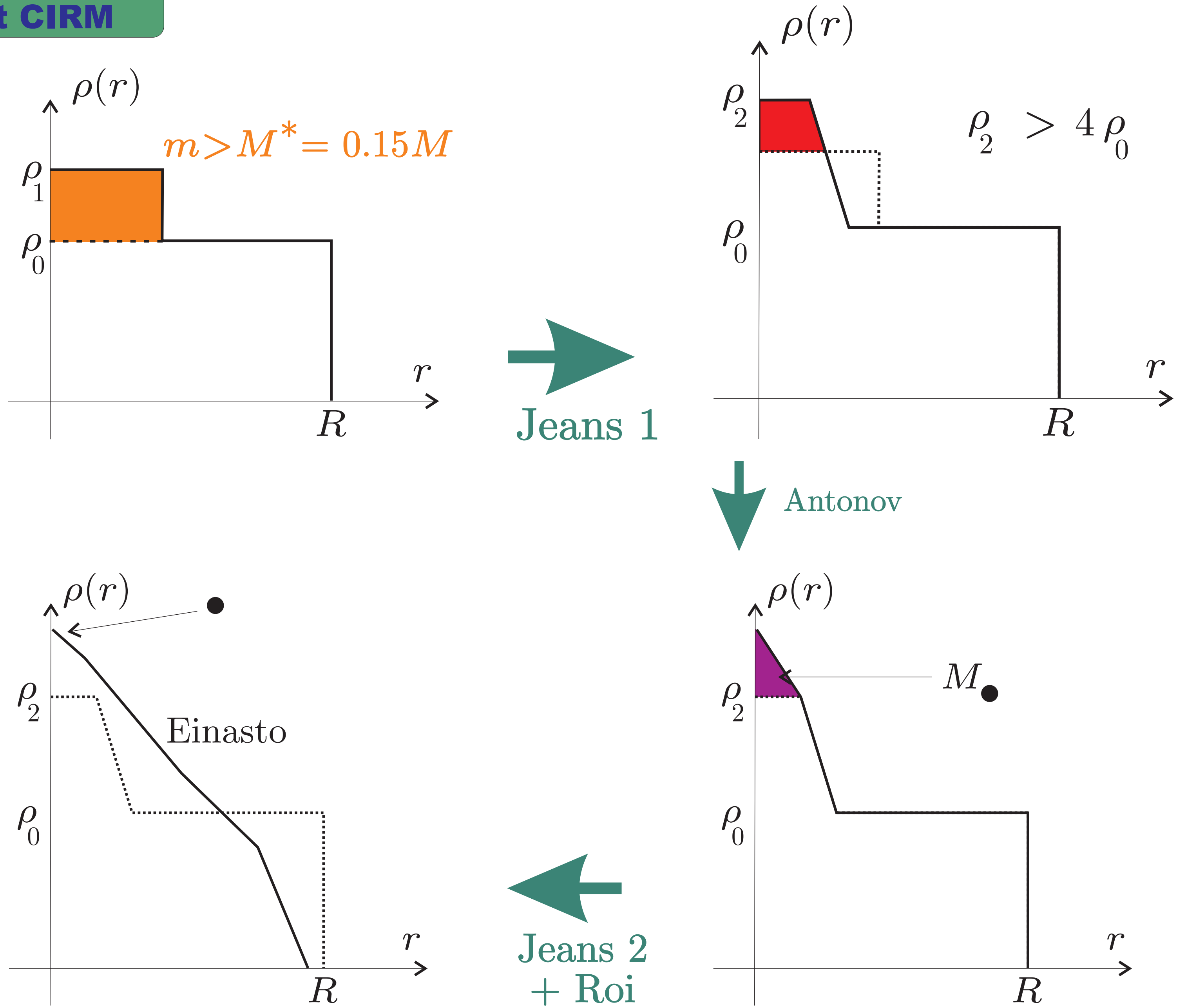
3 Enough mass in the small top hats



3 Enough mass in the small top hats



3 Enough mass in the small top hats



A rough calculus gives $M_\bullet \approx 10^{-5} \times M$ ...!

Conclusion

Conclusion

- Homogeneous isolated collapse (small astrophysical scales)
  - forms King profile ;
  - cannot suffer Roi ;
  - long time evolution can produce core collapse ;
  - generically *no contain* a high mass concentrated object.

Conclusion

- Homogeneous isolated collapse (small astrophysical scales)
  - forms King profile ;
  - cannot suffer Roi ;
  - long time evolution can produce core collapse ;
  - generically *no contain* a high mass concentrated object.

  This could be an idealization of the Globular Clusters formation and evolution.

Conclusion

- Homogeneous isolated collapse (small astrophysical scales)
  - forms King profile ;
  - cannot suffer Roi ;
  - long time evolution can produce core collapse ;
  - generically *no contain* a high mass concentrated object.

  This could be an idealization of the Globular Clusters formation and evolution.

- "'Hierarchic"' collapse
  - forms Einasto profile (or multiple power law) ;
  - can suffer ROI $\Rightarrow$ Ellipticity and good profile;
  - generically *contain* a high mass concentrated object.

Conclusion

- Homogeneous isolated collapse (small astrophysical scales)
  - forms King profile ;
  - cannot suffer Roi ;
  - long time evolution can produce core collapse ;
  - generically *no contain* a high mass concentrated object.

  This could be an idealization of the Globular Clusters formation and evolution.

- "'Hierarchic"' collapse
  - forms Einasto profile (or multiple power law) ;
  - can suffer ROI $\Rightarrow$ Ellipticity and good profile;
  - generically *contain* a high mass concentrated object.

  This could be an idealization of the Galaxies formation and evolution.

PhaseSpace at CIRM

Elements of bibliography

- J. Katz, *On the number of unstable modes of an equilibrium*, MNRAS, **183**, p.765, *1978*
- S. Djorgovski, I.R. King, C. Vuosalo, A. Oren & H. Penner, *The Structure of Globular Clusters*, in "Instrumentation and Research Programmes for Small Telescopes", IAU Symposium 118, eds. J.B. Hearnshaw & P.L. Cottrell, p. 281,*1986*
- R. Elson, P.Hut and S. Inagaki, *Dynamical evolution of globular clusters*, ARAA, **25**, pp. 565-601, *1987*
- N. Katz, *Dissipationless collapse in an expanding universe*, ApJ, **368**, 325, *1991*
- W.E. Harris, AJ, 112, p. 1487, *1996*
- A. Huss ,B. Jain & M. Steinmetz, *How universal are the density profile of dark halos?*, ApJ, **517**, p. 64, *1999*
- M. K.-H. Kiessling, *The Jeans Swindle. A True Story: Mathematically Speaking*, Adv. Applied Math., **31**, pp. 132-149, *2003*
- F. Roy & J. Perez, *Dissipationless collapse of a set of N massive particles*, MNRAS, **348**, p. 62, *2004*

...

- D. Merritt, A.W. Graham, B. Moore, J. Diemand, B. Terzic, *Empirical Models for Dark Matter Halos. I. Nonparametric Construction of Density Profiles and Comparison with Parametric Models*, AJ, 132, pp 2685-2700, *2006*
- J. MacMillan, L.M. Widrow, R. N. Henriksen, *On universal halos and the radial orbit instability*, ApJ, **653**, p. 43, *2006*
- M. Joyce, B. Marcos & F. Sylos Labini, *Energy ejection in the collapse of a cold spherical self-gravitating cloud*, MNRAS, **397**, p. 775, *2009*
- E. I. Barnes, P. A. Lanzel, L. L. R. Williams, *The Radial Orbit Instability in Collisionless N-Body Simulations*, ApJ, **704**, p. 372-384, *2009*
- L. Maréchal & J. Perez, *Radial orbit instability : Review and perspectives*, Submited to Transport Theory and Statistical Physics, arXiv:0910.5177, *2009*
- L. Maréchal & J. Perez, *Radial orbit instability as a dissipation-induced phenomenon*, Submited to MNRAS, *2009*