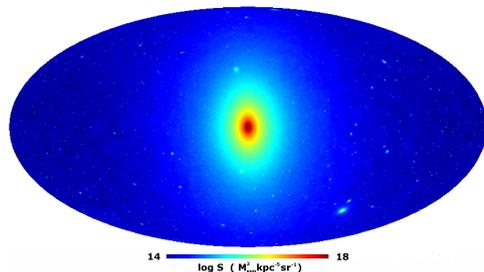
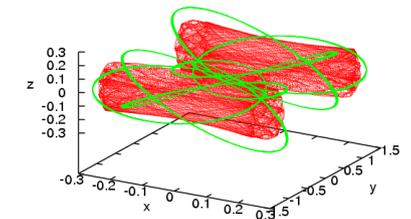
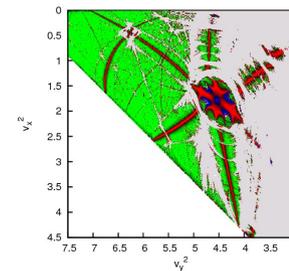
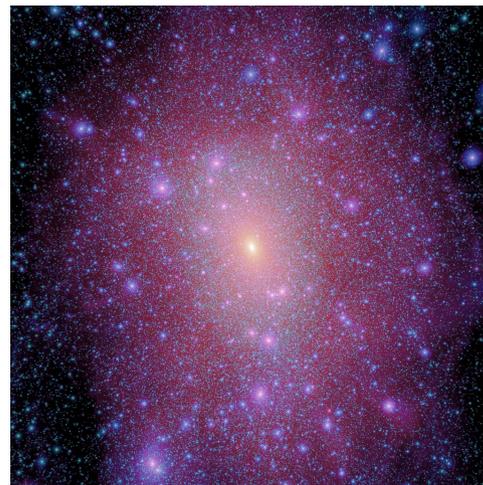
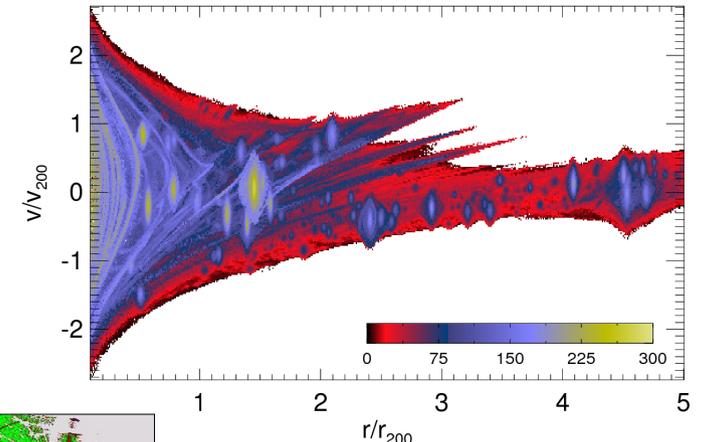
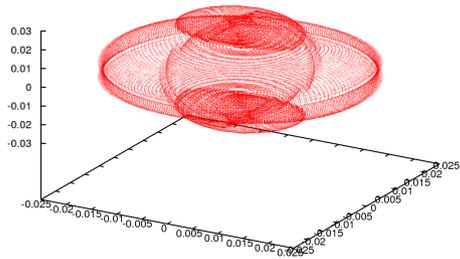
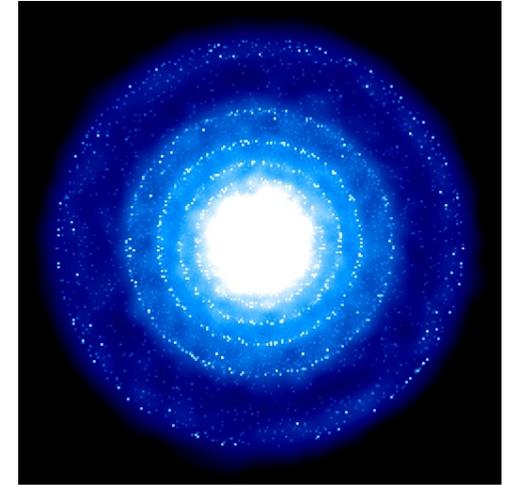
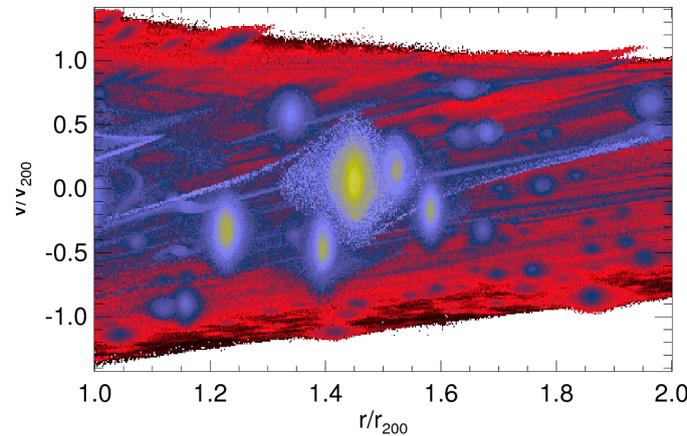
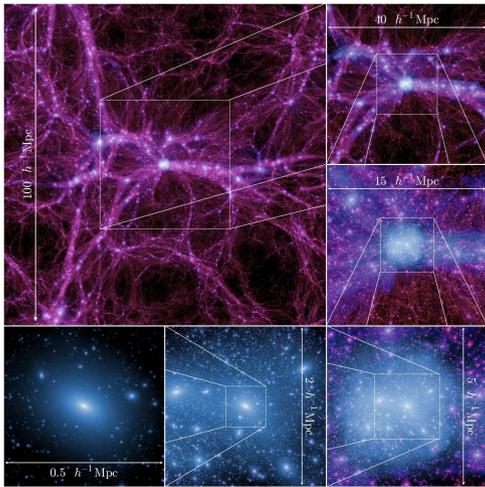


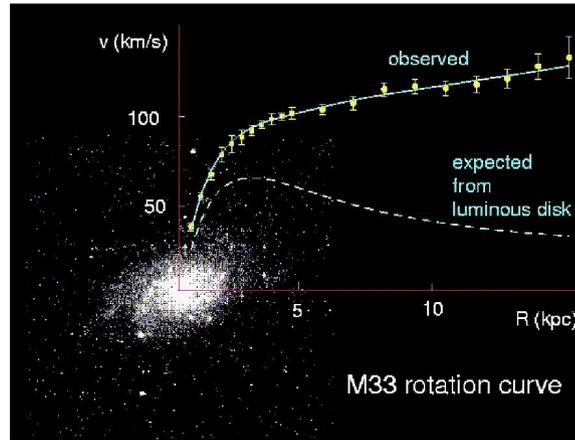
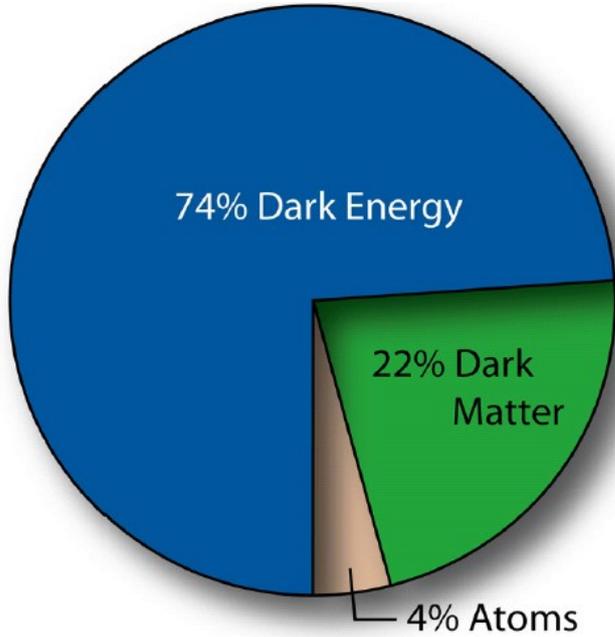
The (fine-grained) phase-space structure of Cold Dark Matter Halos

- and its influence on dark matter searches -

Mak Vogelsberger, Harvard/CfA



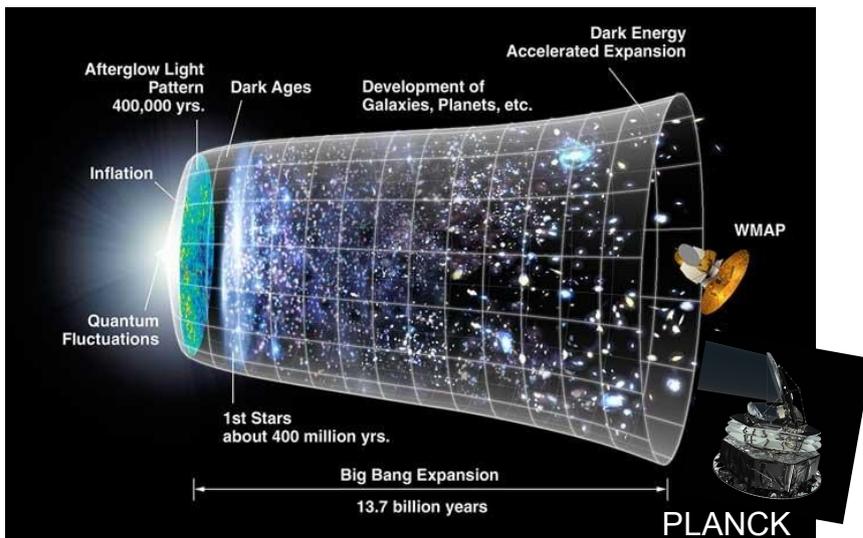
The content of the Universe



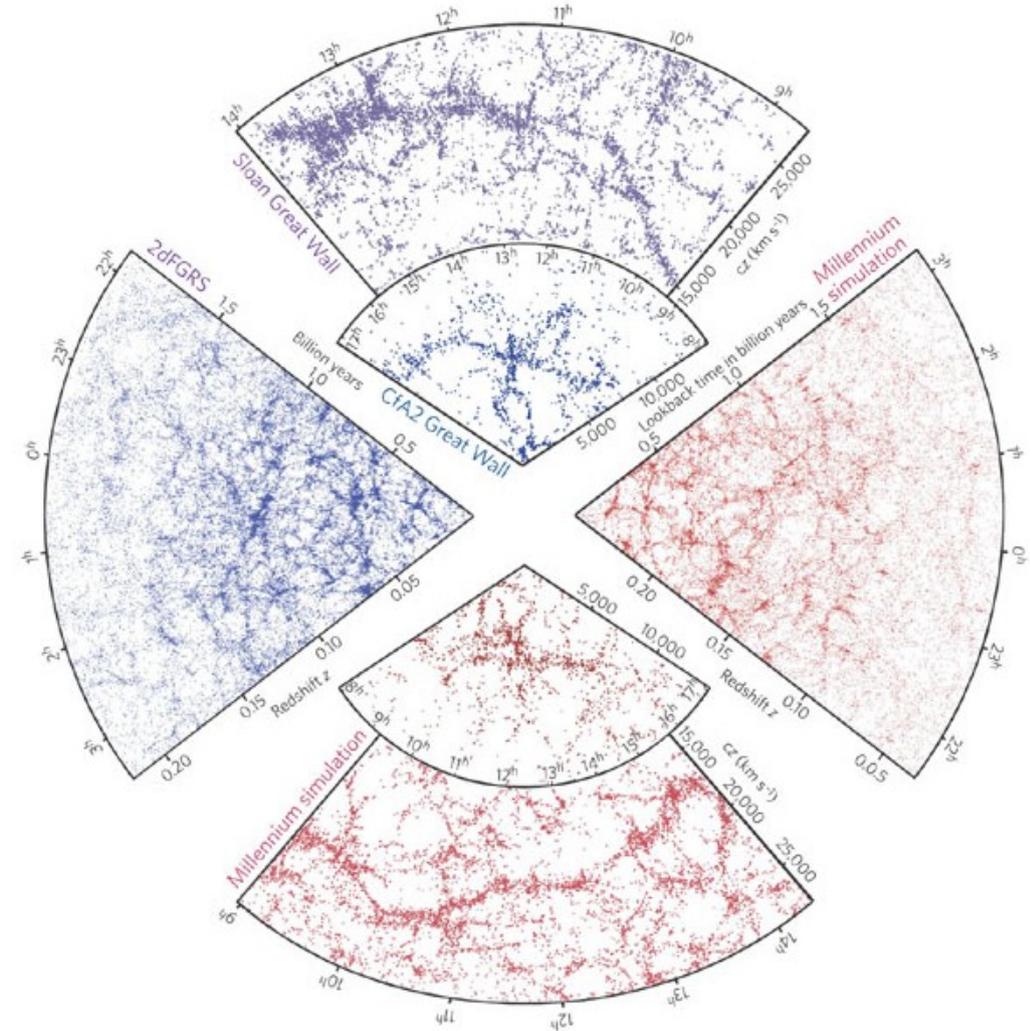
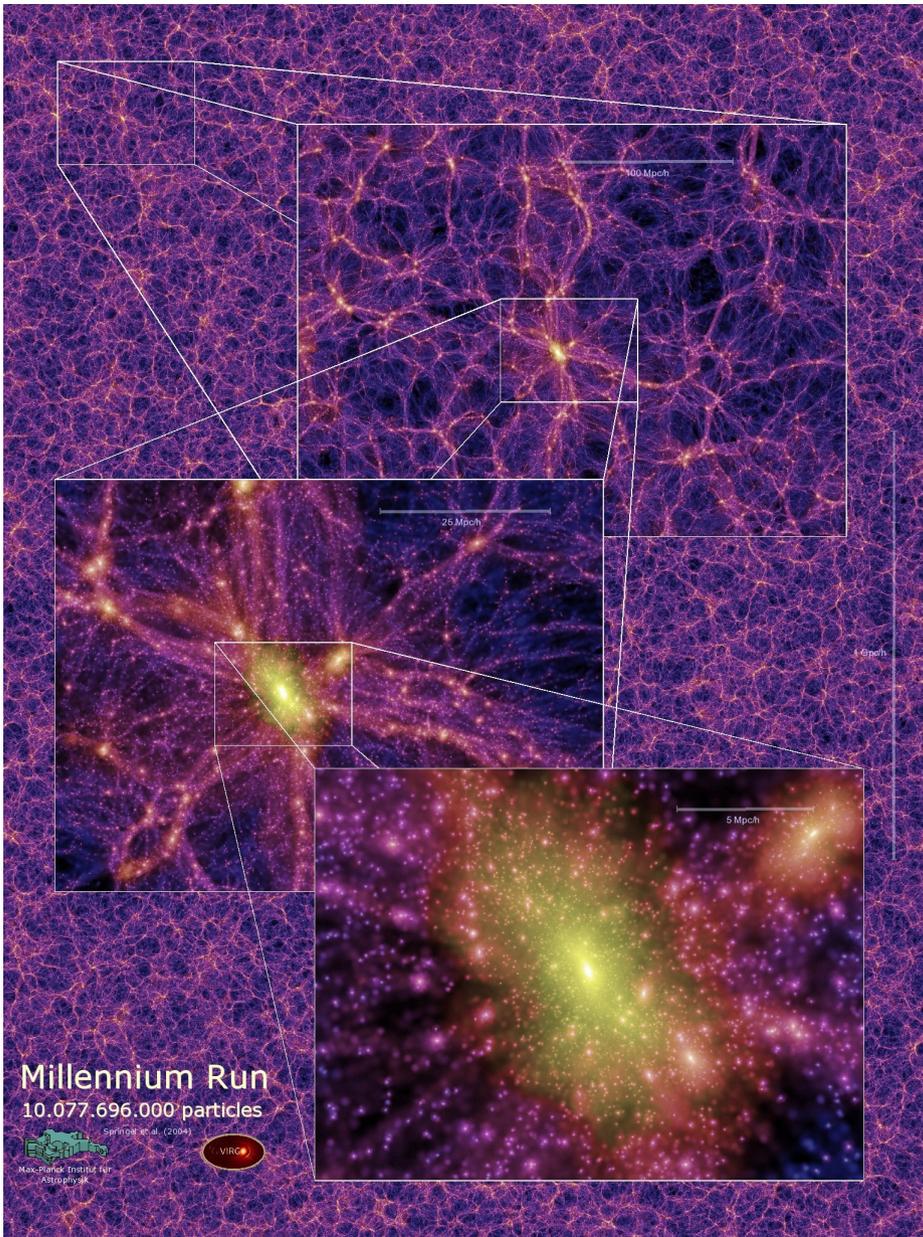
**Evidence \leftrightarrow Existence:
WIMP, Axion, ... ?**



N-body simulations:
→ calculate DM gravitational interaction
→ structure formation

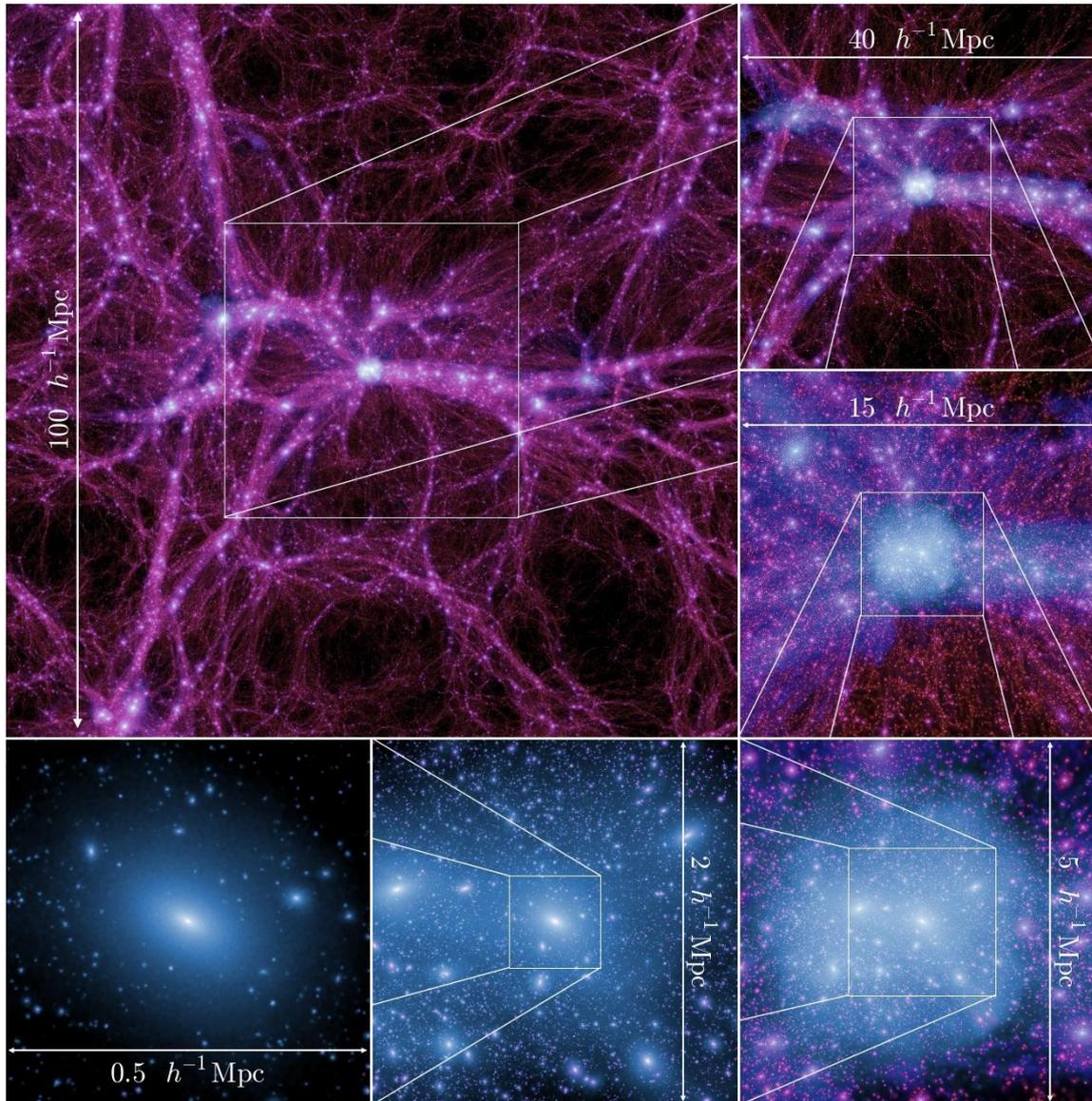


Large-scale structure



Springel et al (2005)

Large-scale structure



Boylan-Kolchin et al (2009)

a playground for
galaxy formation
(semi-analytic modeling)

~6.000 Milky Way-mass
halos with ~100.000 particles



use a large calculator

Millennium II

Do we need more?!

Do we simulate reality?

Is there really dark matter?

How to be sure?

Detect it!

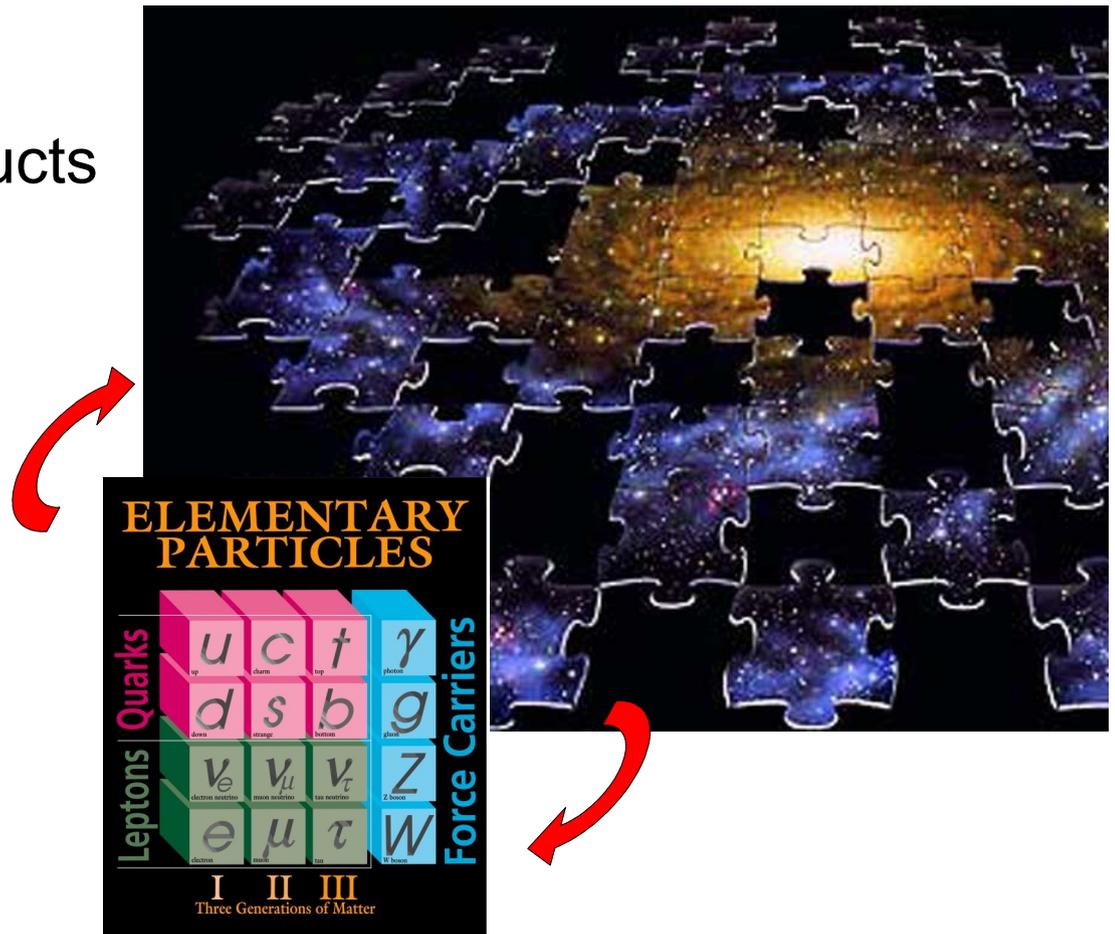
- indirectly via annihilation products
- directly via scattering in underground detectors

And ideally:

- produce a suitable particle at LHC



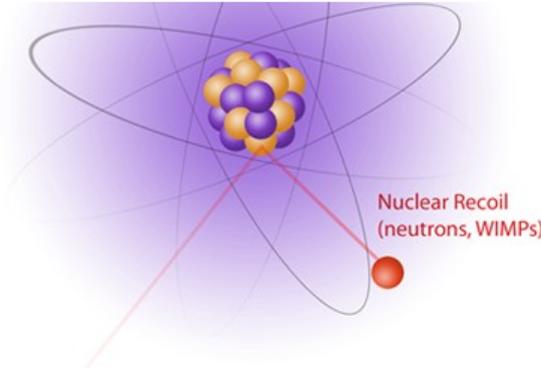
Complete the puzzle



The Hunt for Dark Matter

Direct searches: nuclear recoil events

CRESST, XENON, ZEPLIN,
EDELWEISS, CDMS, DAMA, ...



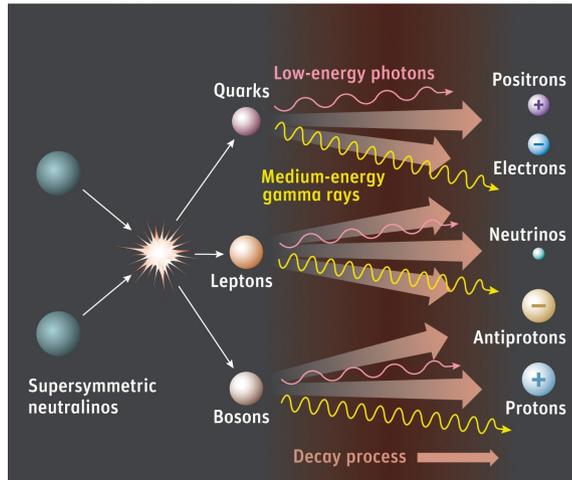
Accelerator searches: producing DM

LHC



Indirect searches: annihilation products

FERMI, PAMELA, ...



Usually assumed astrophysical input:

Density: $\sim 0.3 \text{ GeV} / \text{c}^2 / \text{cm}^3$

Velocity: Maxwellian

Standard Halo Model (SHM):

- Smooth mass distribution
- Smooth velocity distribution
- 'Featureless' phase-space

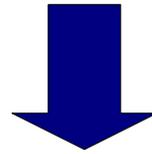


'Non-standard' Halo models

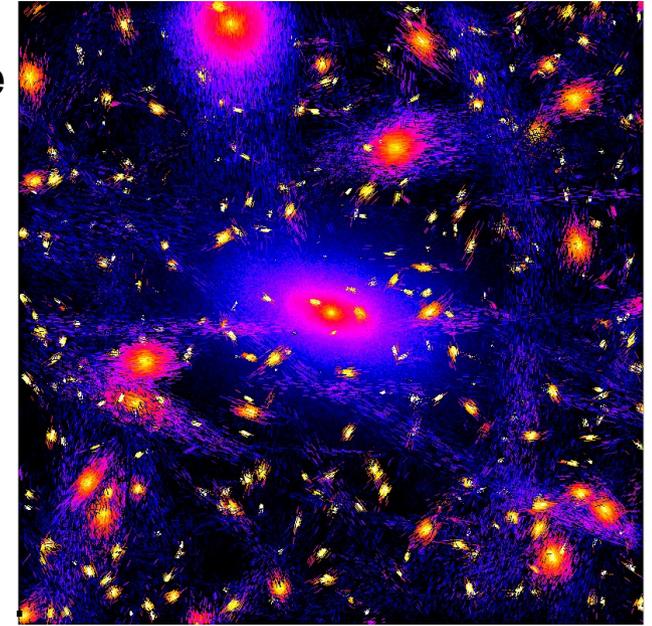


N-body simulations:
lots of phase-space substructure

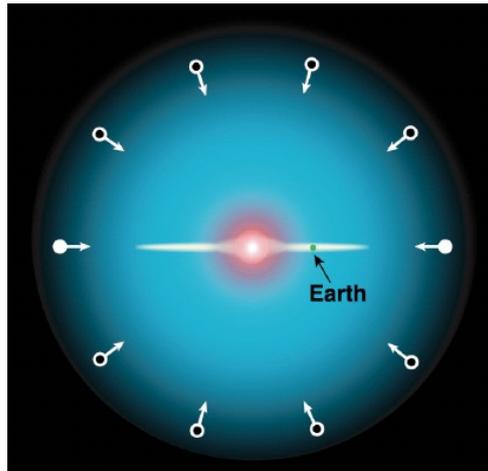
Standard halo model
assumption wrong?!



Dark matter
parameter-space
limits wrong?!

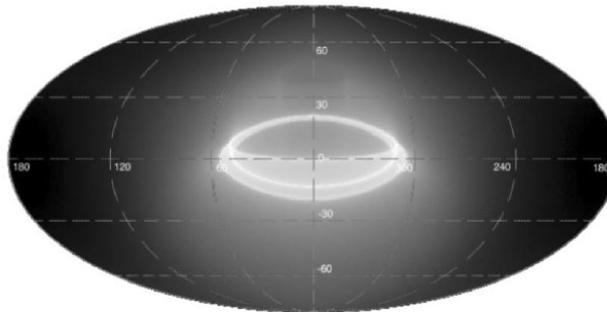


Diemand et al (2008)

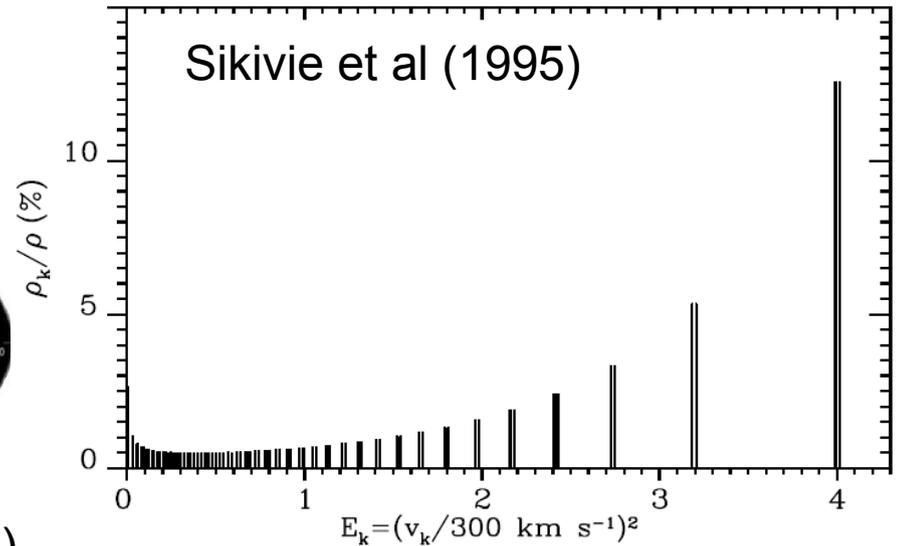


Van Bibber (2008)

Analytic models:
→ massive streams
→ caustic ring model



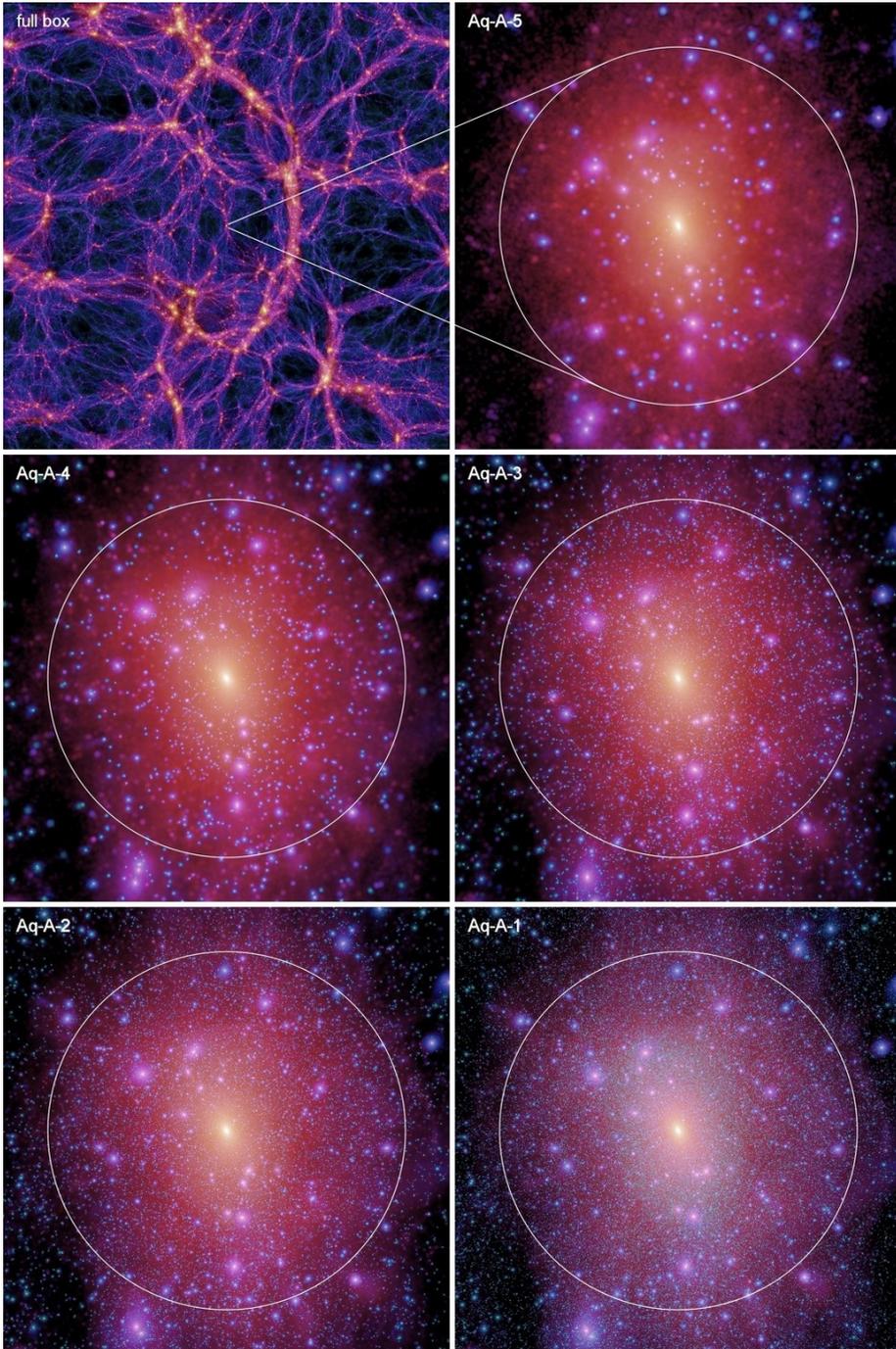
Natarajan & Sikivie (2008)



Outline

- 1) The coarse-grained structure of LCDM halos**
- 2) Towards the fine-grained structure of LCDM halos**
- 3) The fine-grained structure of LCDM halos**
- 4) A note: Dynamics with the Geodesic Deviation Equation**
- 5) Conclusions**

1) The coarse-grained structure of LCDM halos

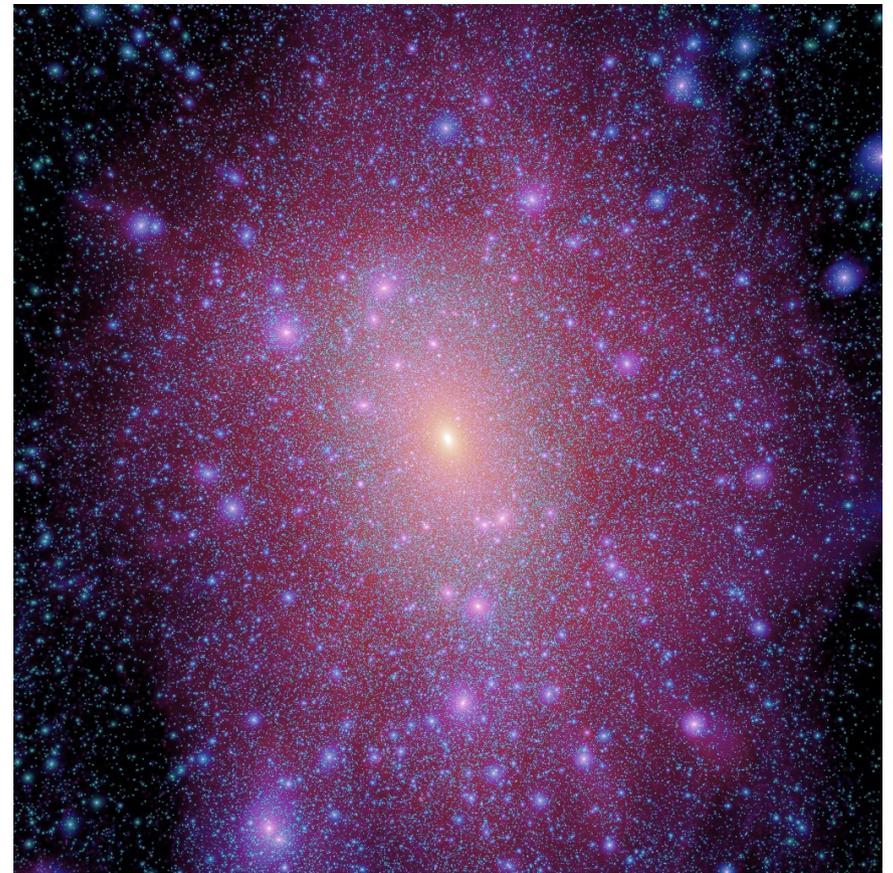


Springel et al (2008)

Aquarius Project

-six Milky Way-like Halos-

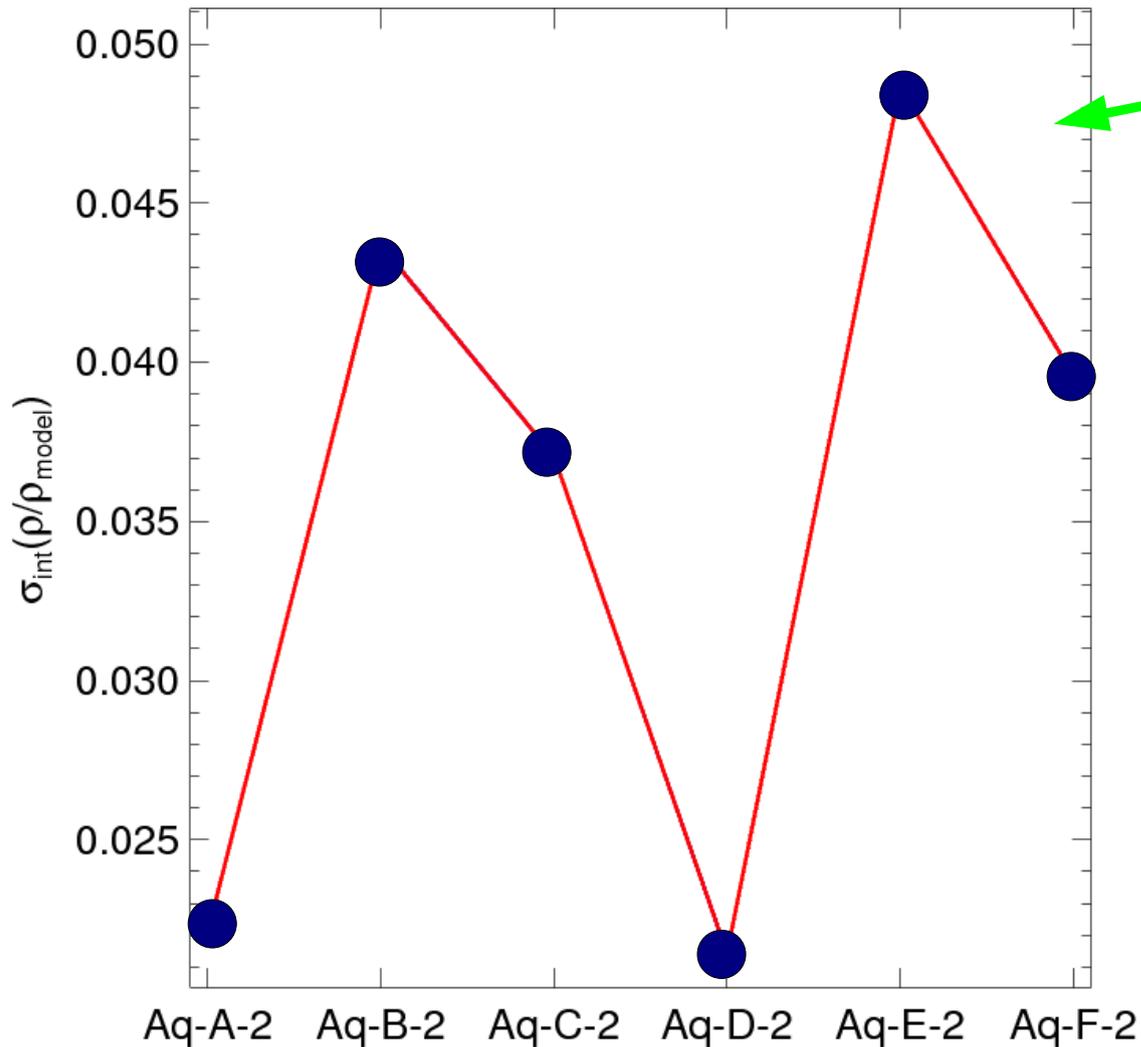
m_p [M_\odot]	ϵ [pc]	N_{hr}
1.712×10^3	20.5	4,252,607,000



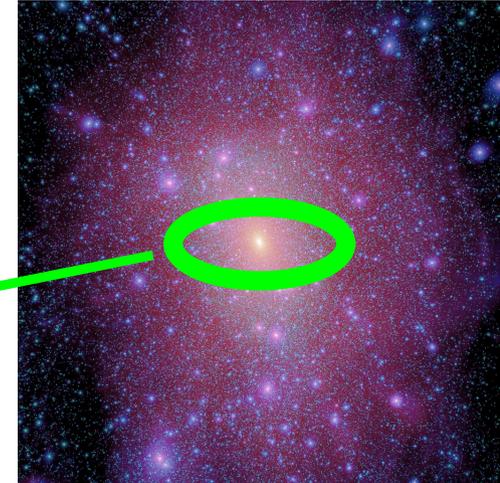
Probing DM near the Sun!

DM smoothness near the Sun

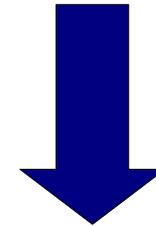
Scatter standard deviation $< 5\%$
(variation due to halo shape taken out)



6-12kpc



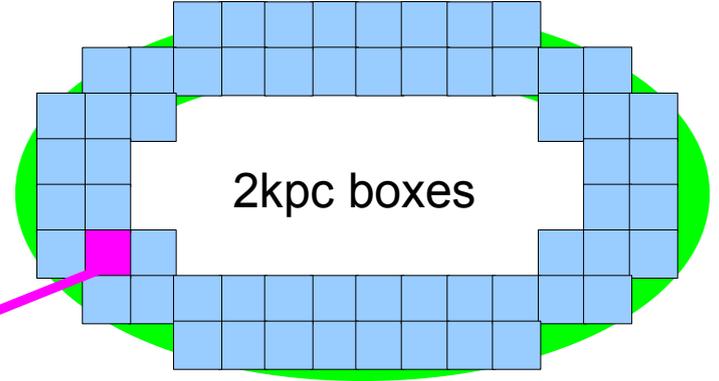
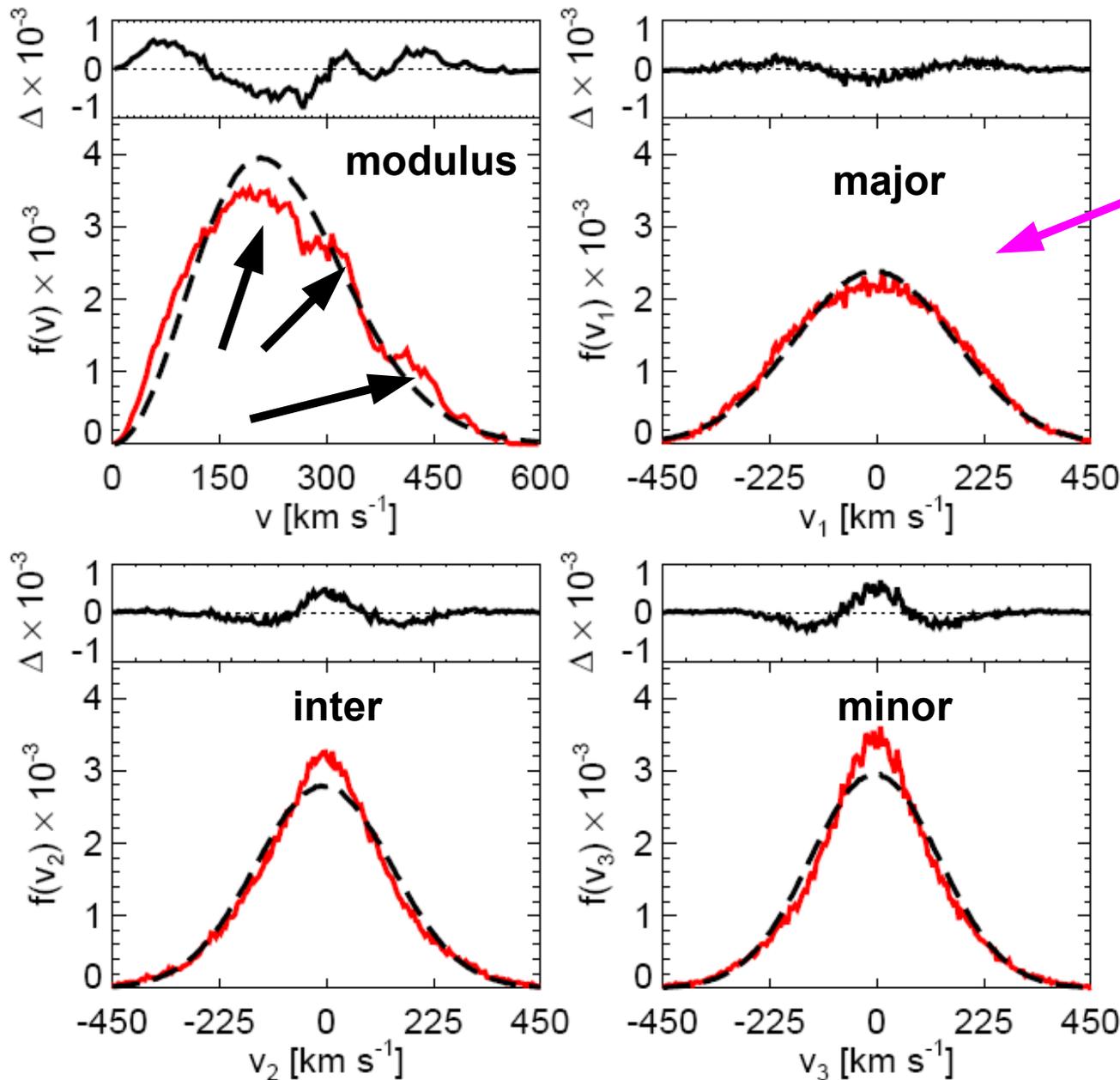
Chance of 'hitting'
a subhalo is
very small: $\sim 10^{-4}$



**Experimentalists can
use smooth models**

MV et al (2009)

Velocity Distribution near the Sun



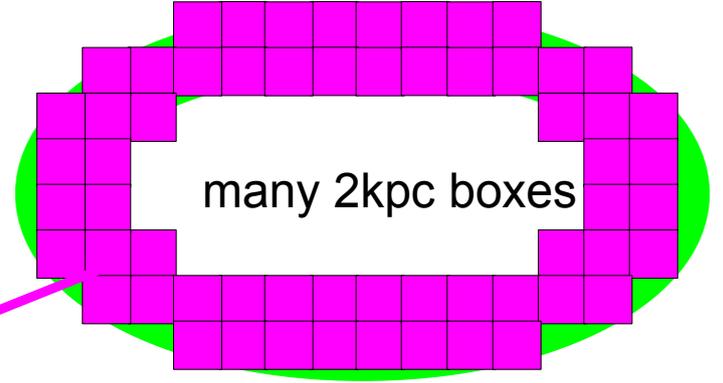
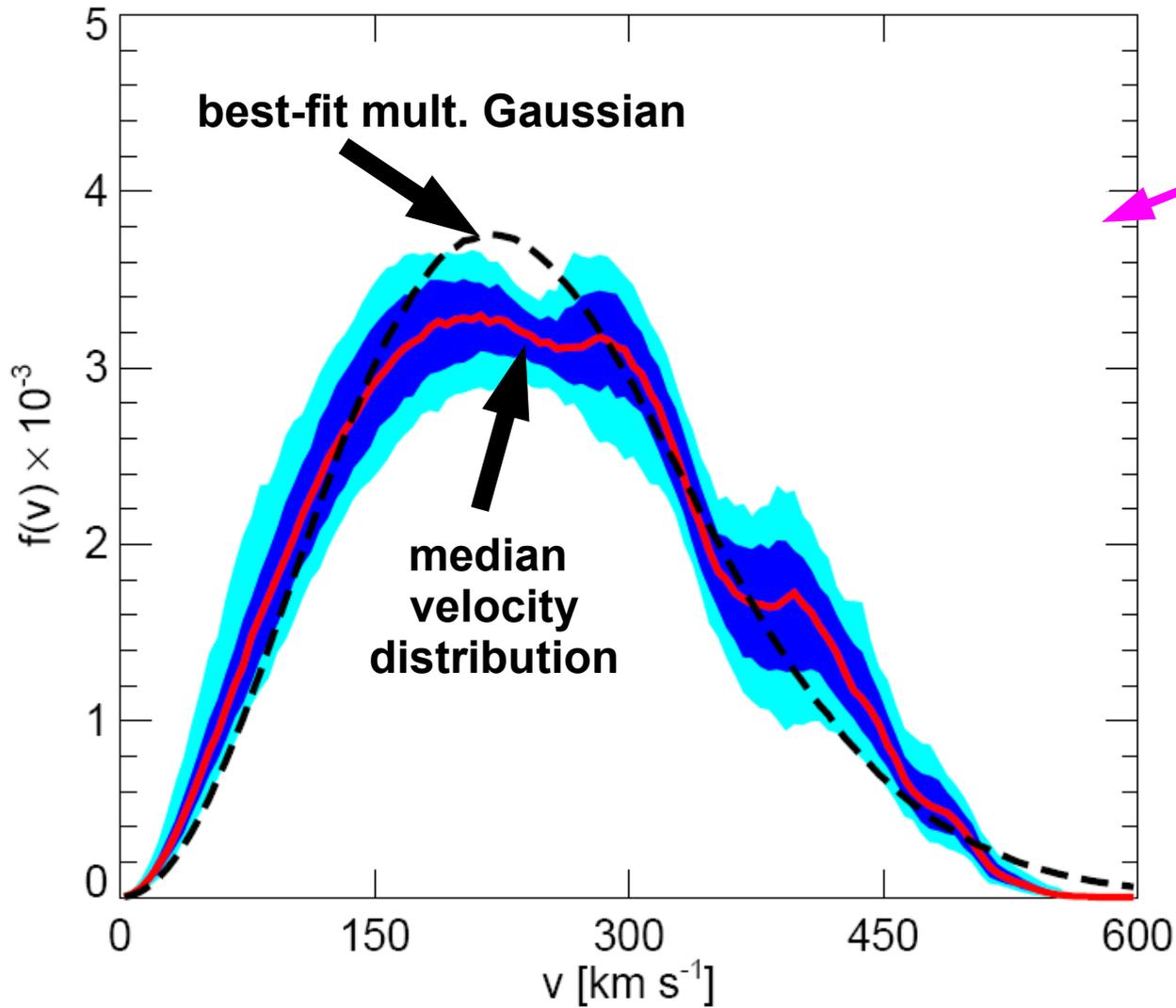
No signs for distinct/massive streams

Bumps/dips in velocity vector modulus

Not Maxwellian

— simulation
- - - Best-fit multivariate Gaussian model

... at Solar Circle

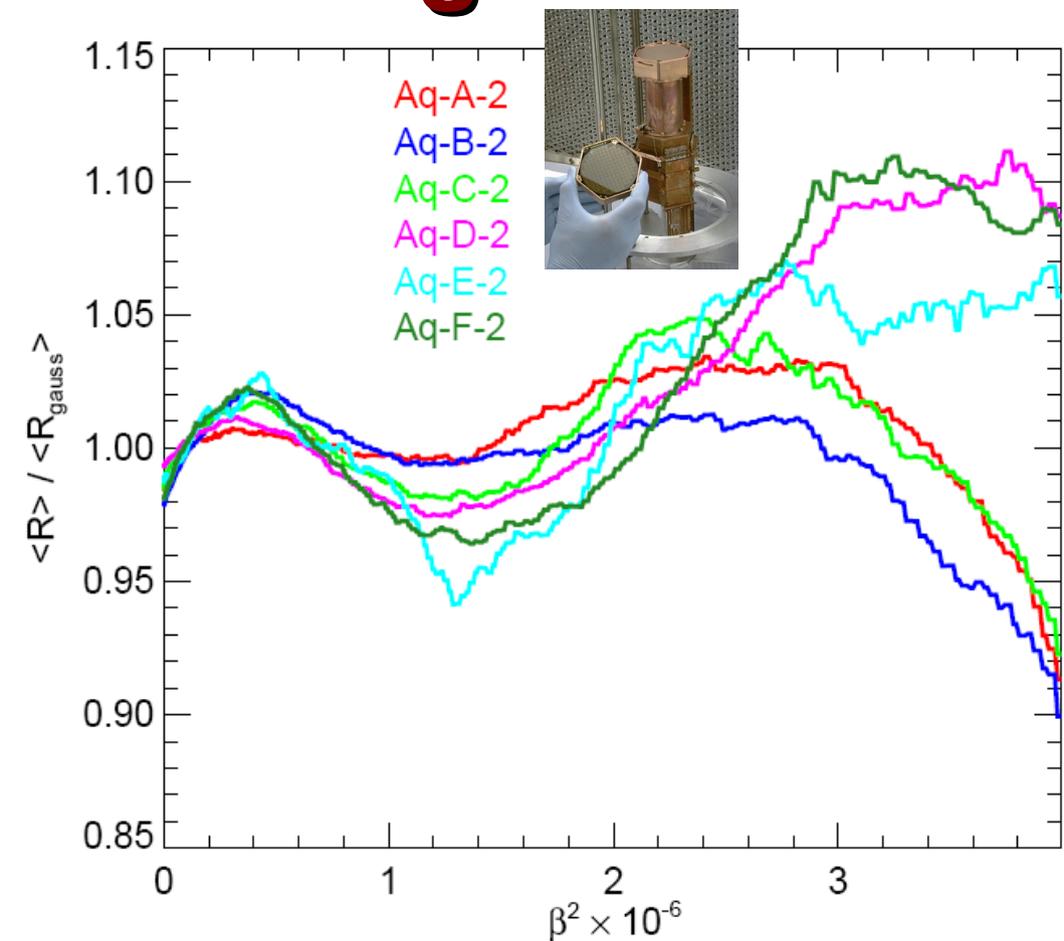


**Bumps in velocity
modulus at
the same velocity**

Not Maxwellian

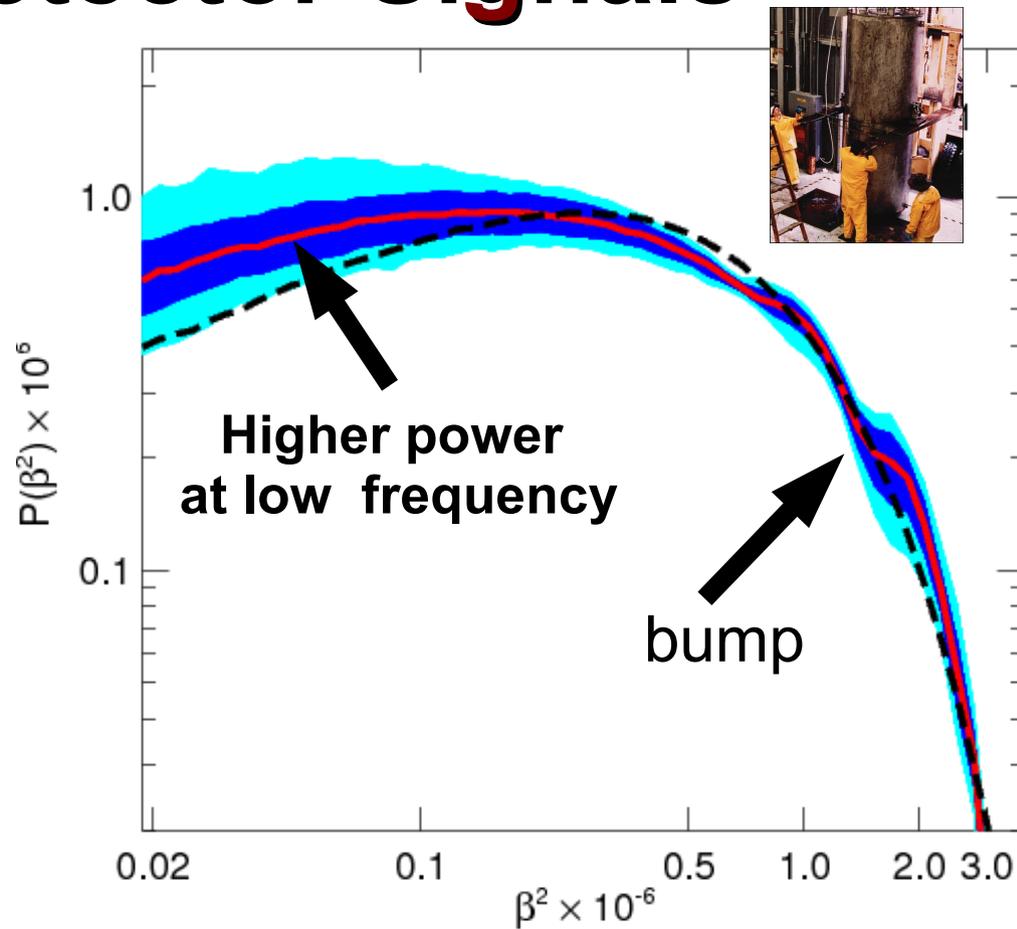
Not Gaussian

Signature in Detector Signals



WIMP recoil spectrum

$$E = \frac{2 m_\chi^2 m_A}{(m_\chi + m_A)^2} c^2 \beta^2 \times 10^6 \text{ keV}$$



Axion microwave spectrum

$$\nu_a = 241.8 \left(\frac{m_a}{1 \mu\text{eV}/c^2} \right) \left(1 + \frac{1}{2} \beta^2 \right) \text{ MHz}$$

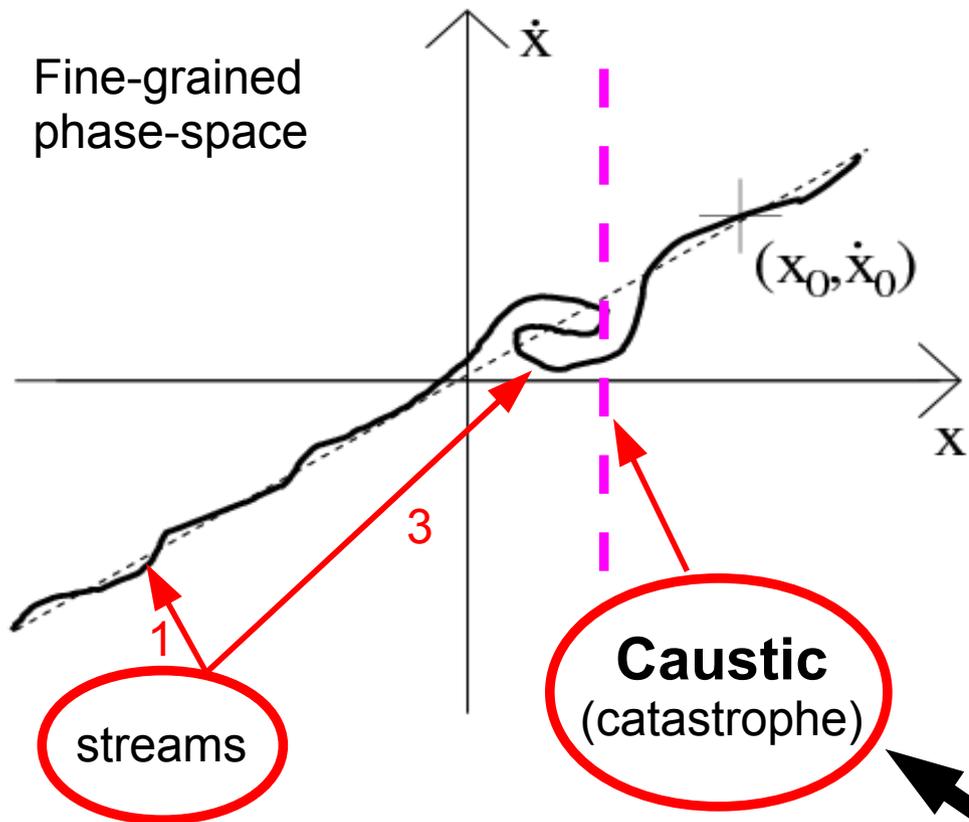
2) Towards the fine-grained structure of LCDM halos

CDM – very small scales

CDM is **cold** and **collisionless**



CDM lies on **3D hypersurface**
in **6D phase-space**



Thickness of line:
primordial velocity dispersion

Amplitude of wiggles:
velocity due to density perturbations

Wind-up:
growth of an overdensity

Phase space sheet:
 $(\vec{r}, \vec{v}) : H(t)\vec{r} + \Delta\vec{v}(\vec{r}, t)$

regions of **very high CDM** density

Estimates/Calculations so far

Self-similar halo formation:

Fillmore & Goldreich (1984), Bertschinger (1985),
Mohayaee & Salati (2008);
Mohayaee et al (2006); ...

Caustic ring model:

Duffy & Sikivie (2008);
Natarajan & Sikivie (2008);
Onemli & Sikivie (2007);
Natarajan & Sikivie (2007);
Sikivie et al (1997); ...

General arguments:

Hogan (2001),
Afshordi et al (2009), ...

Predictions

- ~100 streams at solar position
- significant annihilation boost
- strong caustic rings
- discrete velocity distribution
- distinct caustic structures

→ **Significant effects
on search experiments**

How realistic are these models?

Correct caustic structure?

Correct caustic densities?

Correct number of streams?

Correct boost?

SELF-SIMILAR GRAVITATIONAL COLLAPSE IN AN EXPANDING UNIVERSE¹

JAMES A. FILLMORE AND PETER GOLDREICH

California Institute of Technology

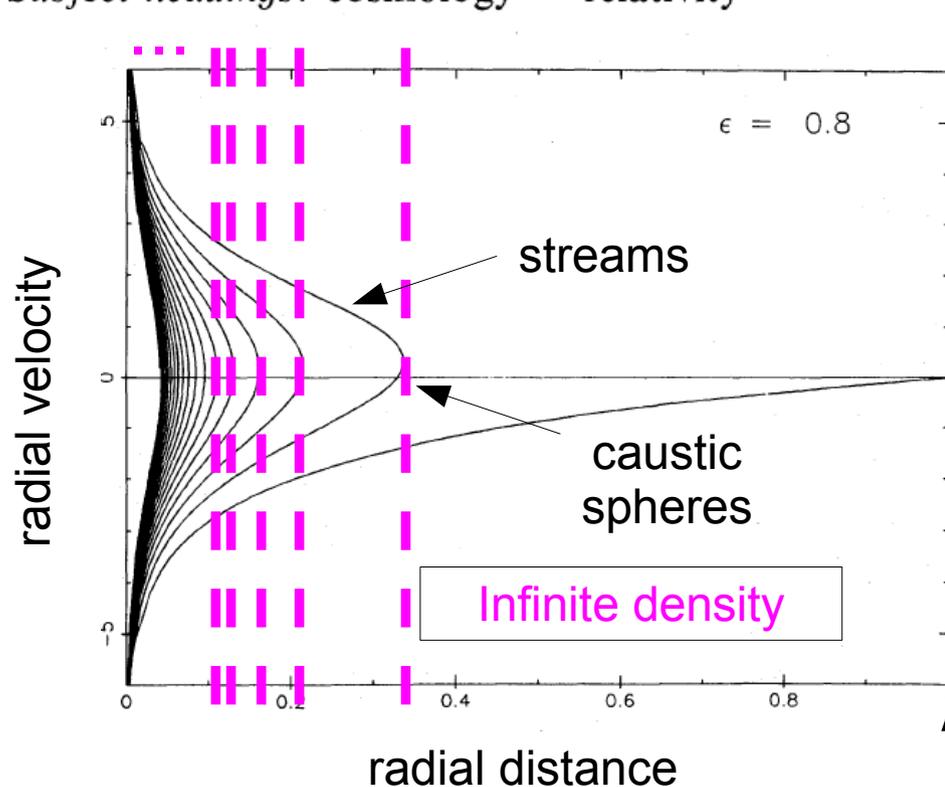
Received 1983 October 10; accepted 1983 December 5

Starting point
Analytic 1D model

ABSTRACT

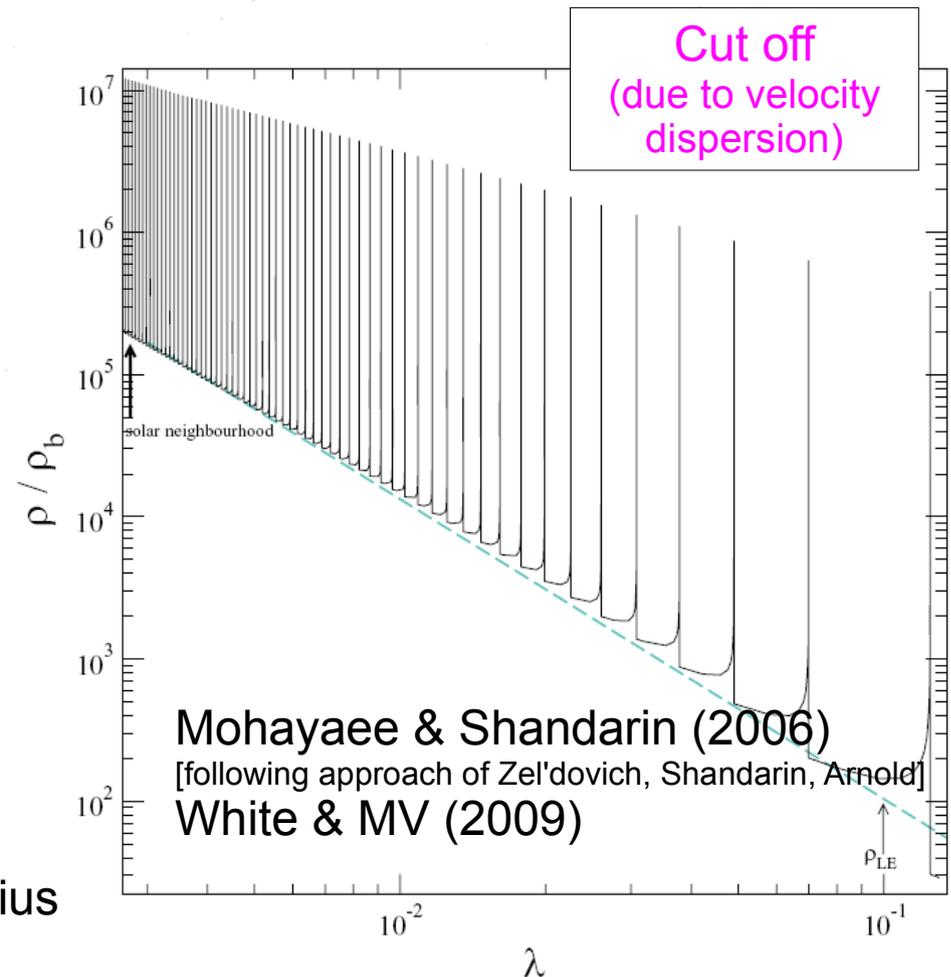
We derive similarity solutions which describe the collapse of cold, collisionless matter in a perturbed Einstein–de Sitter universe. We obtain three classes of solutions, one each with planar, cylindrical, and spherical symmetry. Our solutions can be computed to arbitrary accuracy, and they follow the development of structure in both the linear and nonlinear regimes.

Subject headings: cosmology — relativity



[also Bertschinger (1985)]

turnaround radius



Mohayaee & Shandarin (2006)

[following approach of Zel'dovich, Shandarin, Arnold]

White & MV (2009)

Resolving fine-grained caustics with N-body simulations

Problem: N-body simulations have too coarse phase-space sampling
(→ missing many orders of magnitude in mass resolution/particle number)

Solution: Follow the local phase-space evolution for each particle
(→ with a phase-space geodesic deviation equation)

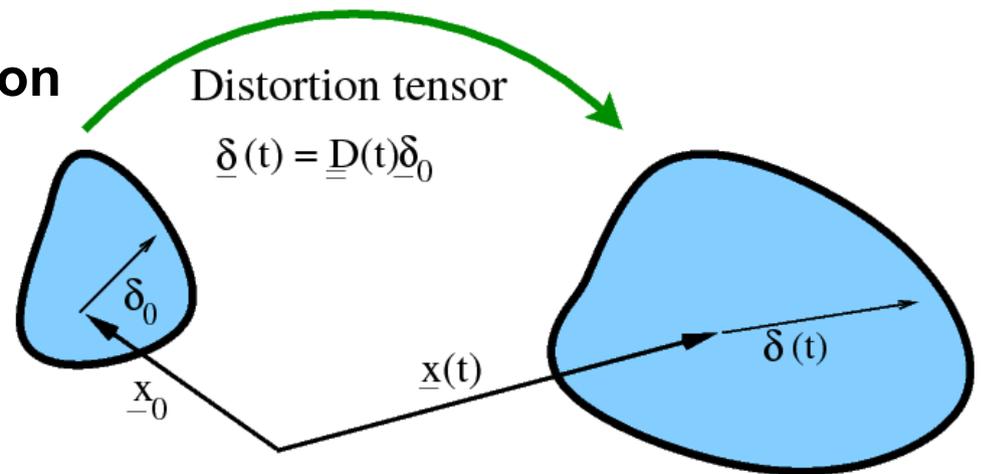
- calculation of **stream density**
- **identification of caustics**
- Monte-Carlo estimate for **intra-stream annihilation**

gaining resolution *without*
using larger computers

→ allows **caustic annihilation** calculation

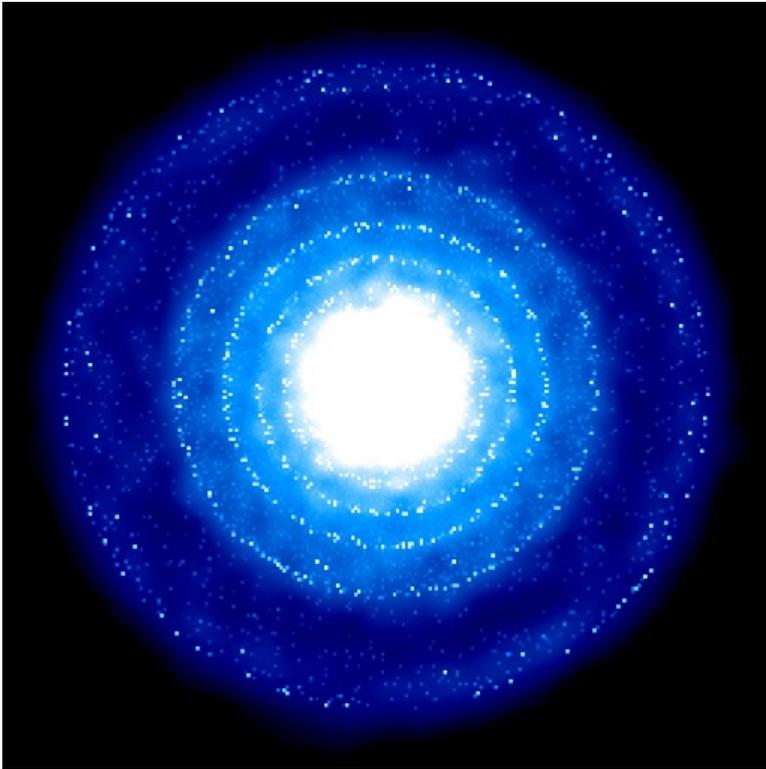
$$\frac{d\mathcal{A}_{s,i}}{dt} = \frac{\langle \sigma v \rangle_{\chi}}{m_{\chi}^2} m_i \rho_{s,i}$$

[Implementation in GADGET-3]

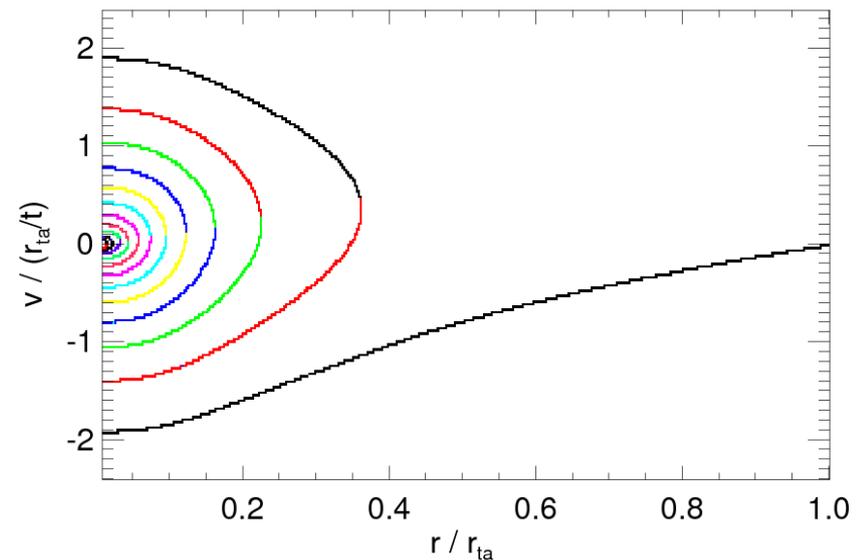
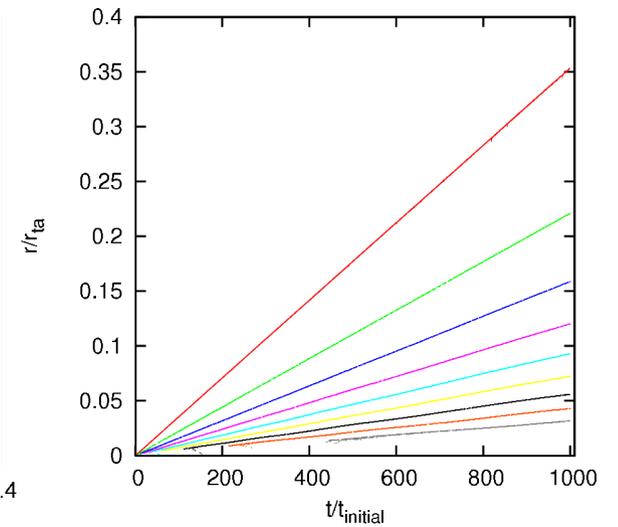
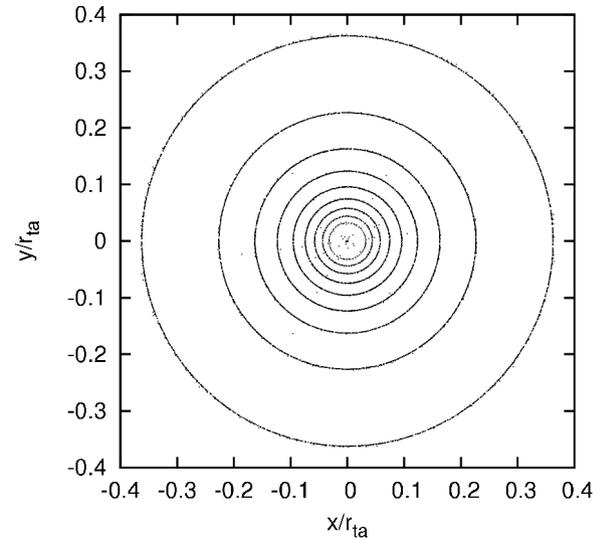


MV et al (2008)

Caustic Annihilation radiation - 1D gravity -

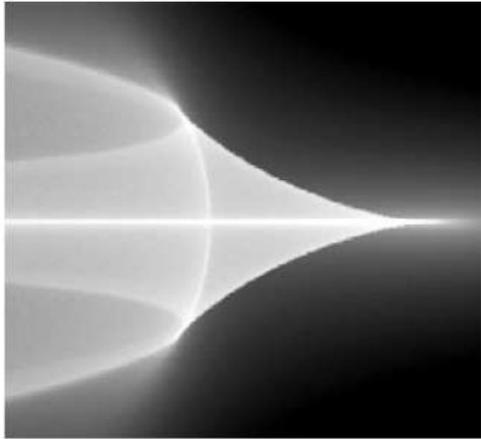


caustic spheres
on top of smooth
annihilation signal



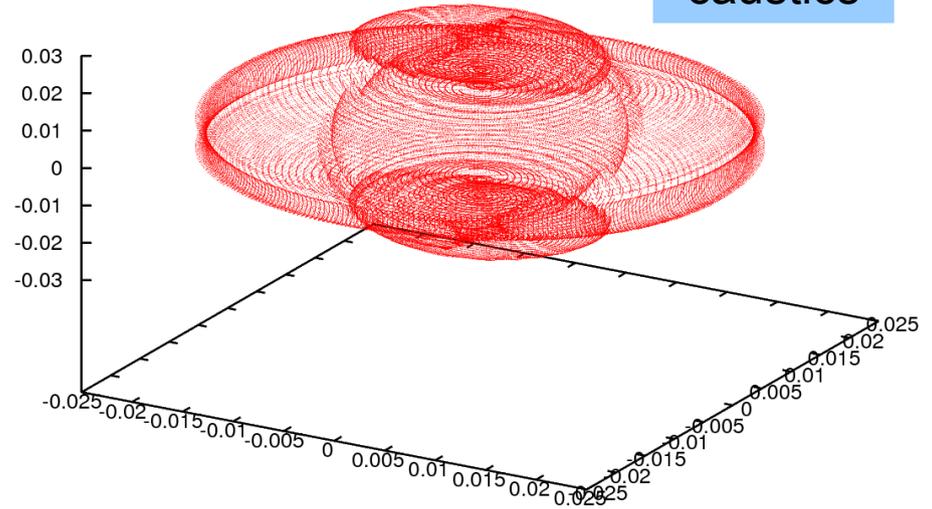
Caustic structures - non-radial halo model -

lead to interest in
impact on annihilation radiation

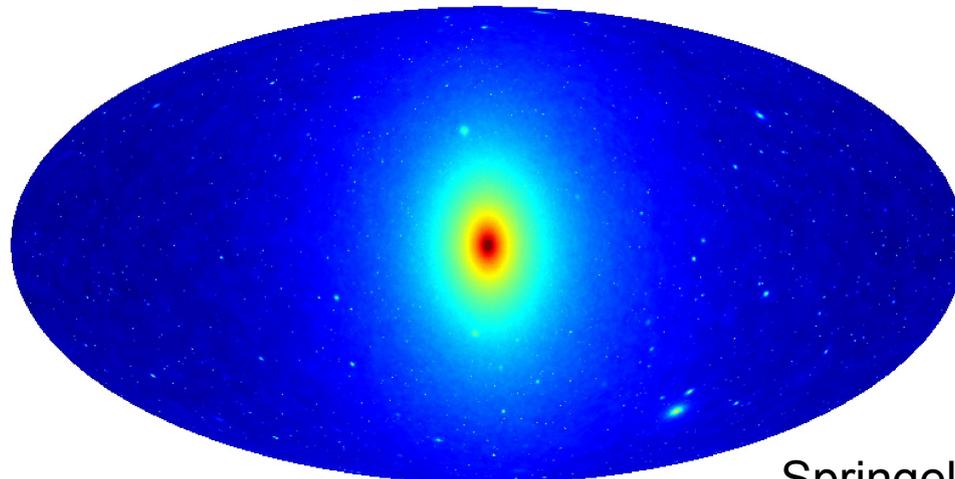
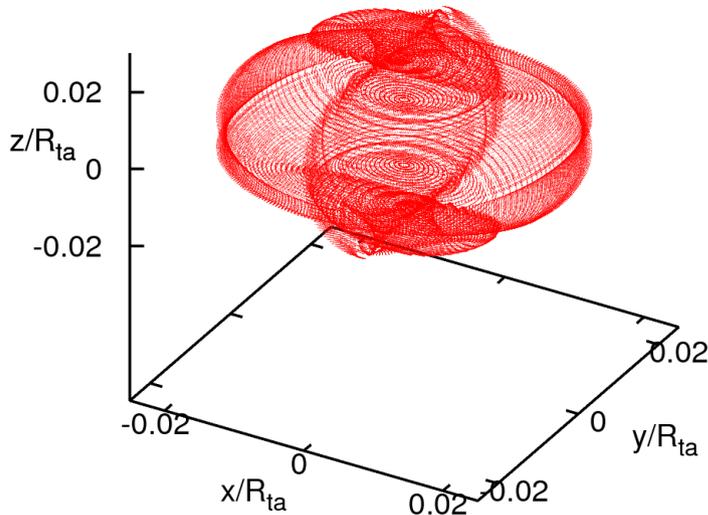


Natarajan & Sikivie (2008)

Inner
caustics



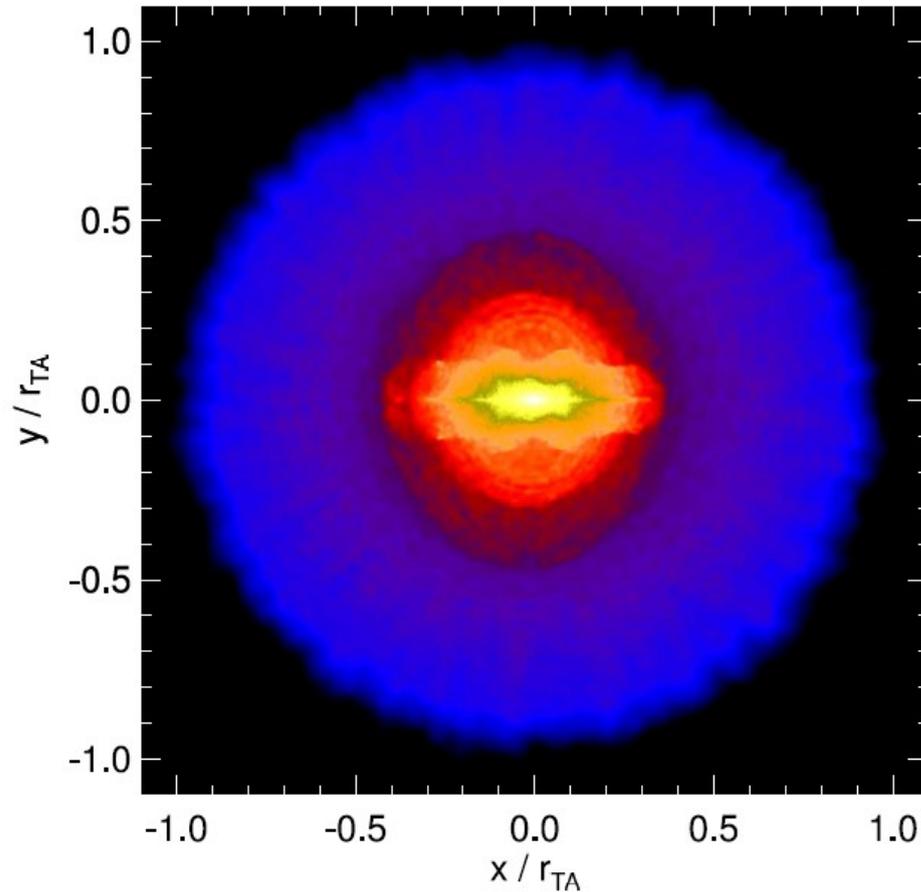
But N-body simulations predict:



14  18
 $\log S \text{ (} M_{\text{sun}}^2 \text{ kpc}^{-5} \text{ sr}^{-1} \text{)}$

Springel et al (2008)

Collapse of an isolated halo in 3D gravity

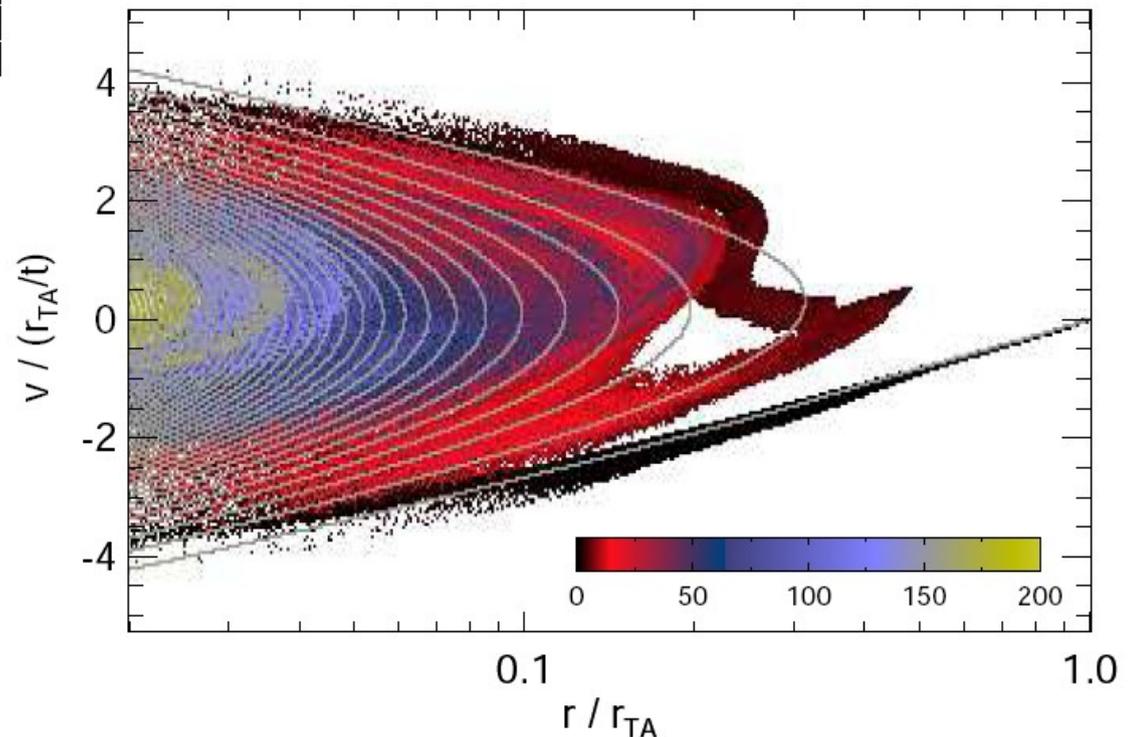


Density slice → **bar formation**

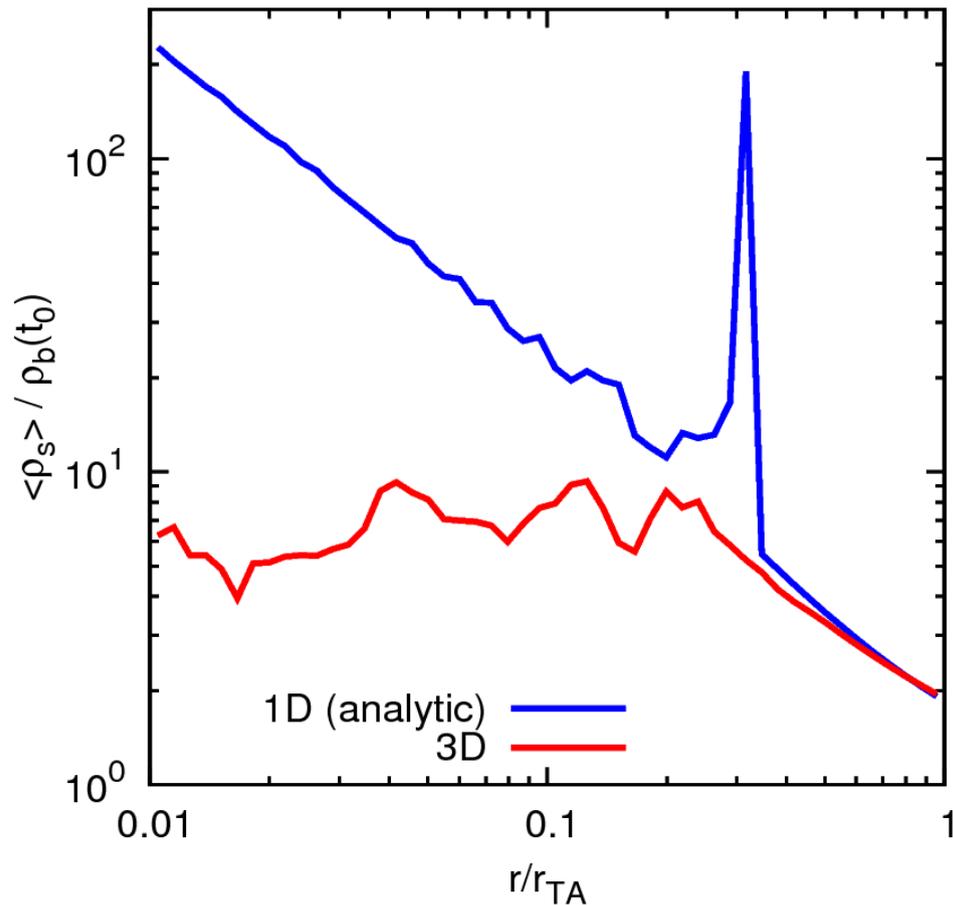
Instabilities lead to different phase-space evolution!

MV et al (2009)

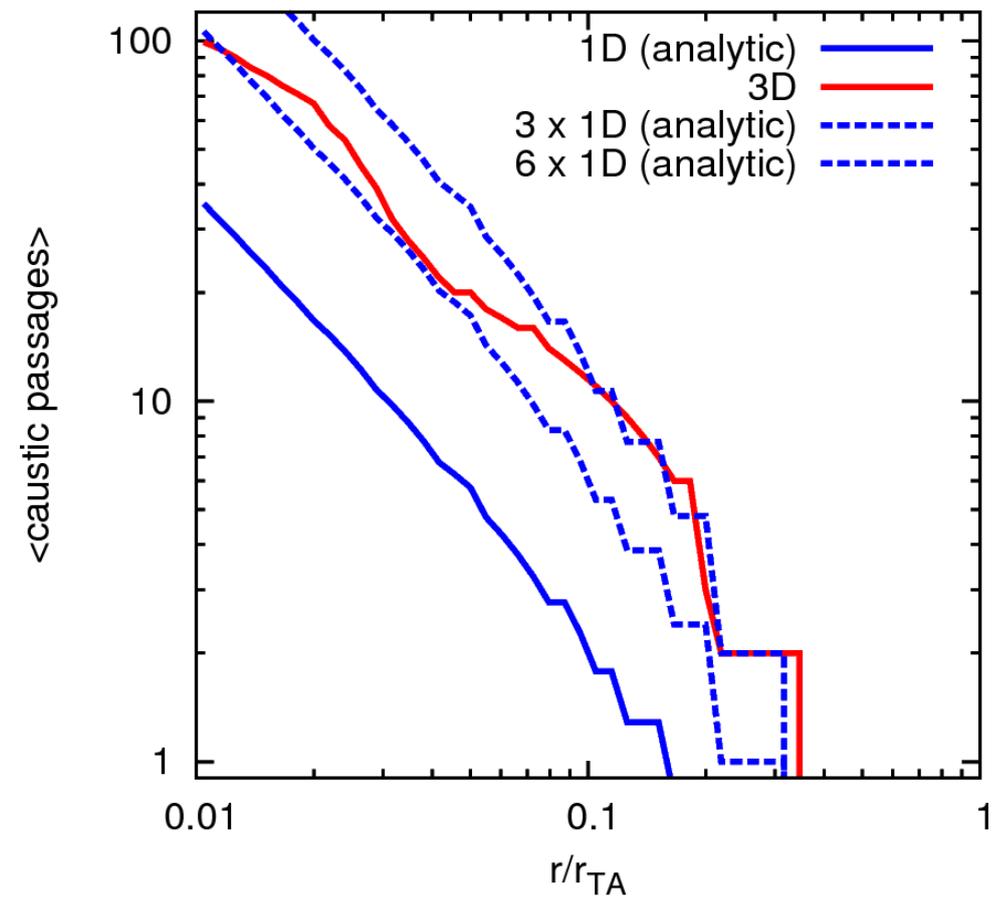
No clear phase-space pattern



Fine-grained phase-space in 3D



Stream density: **lower in 3D**

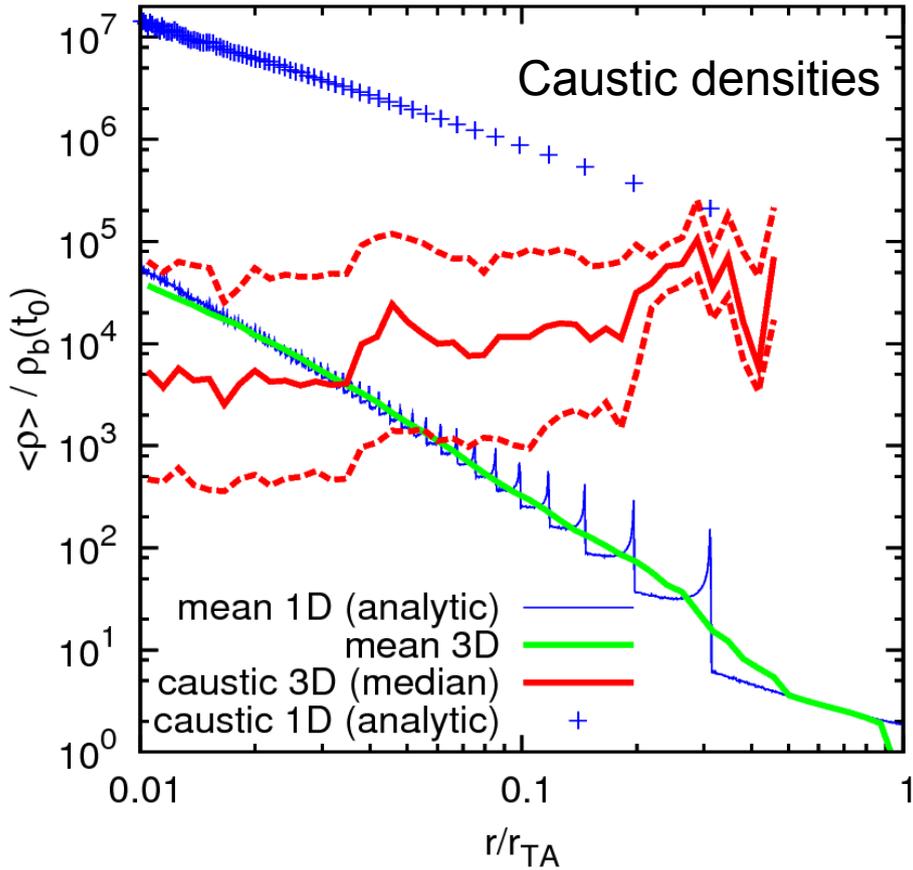


Caustic passages:
Increases in 3D
→ more turning points

More efficient mixing in higher dimensions

Caustics in 3D

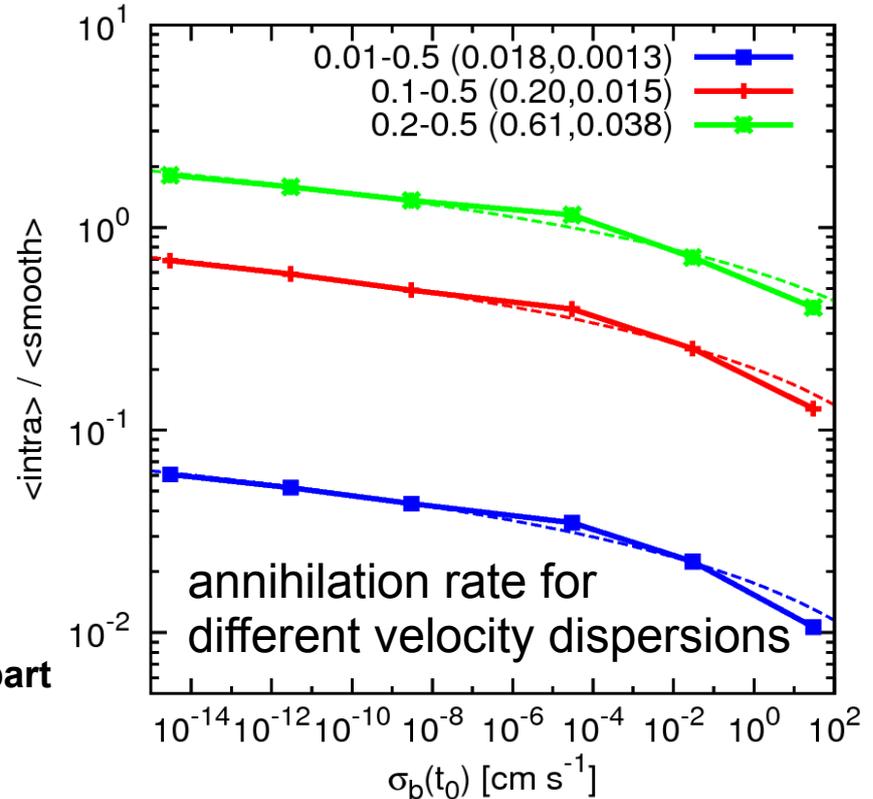
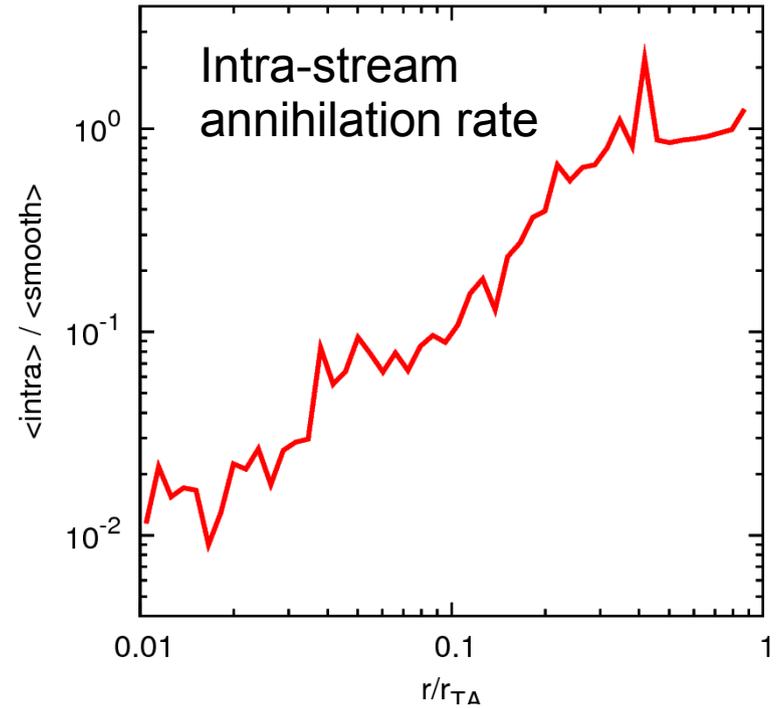
100 GeV/c² Neutralino



boost factor due to caustics:

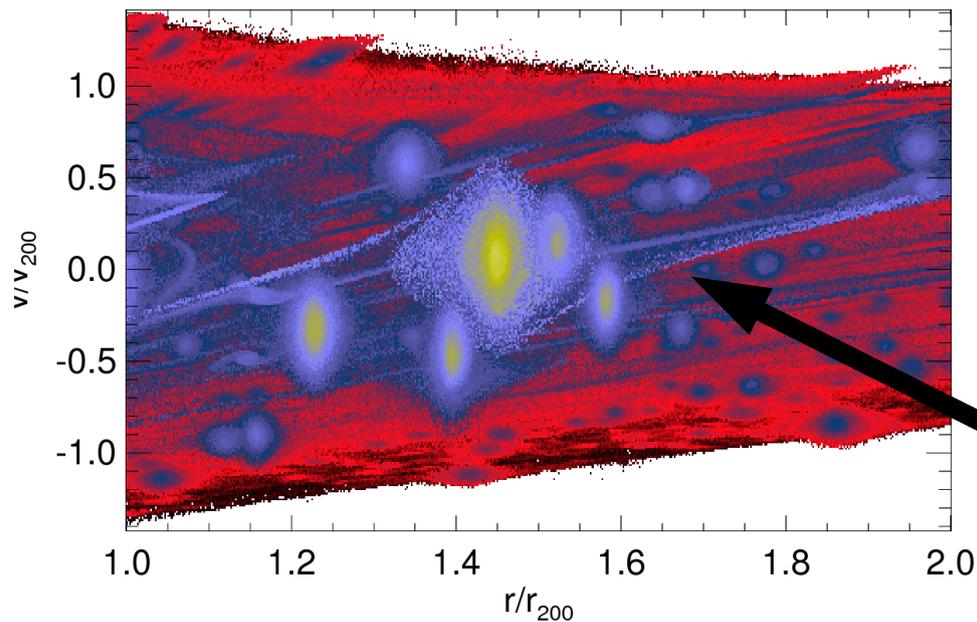
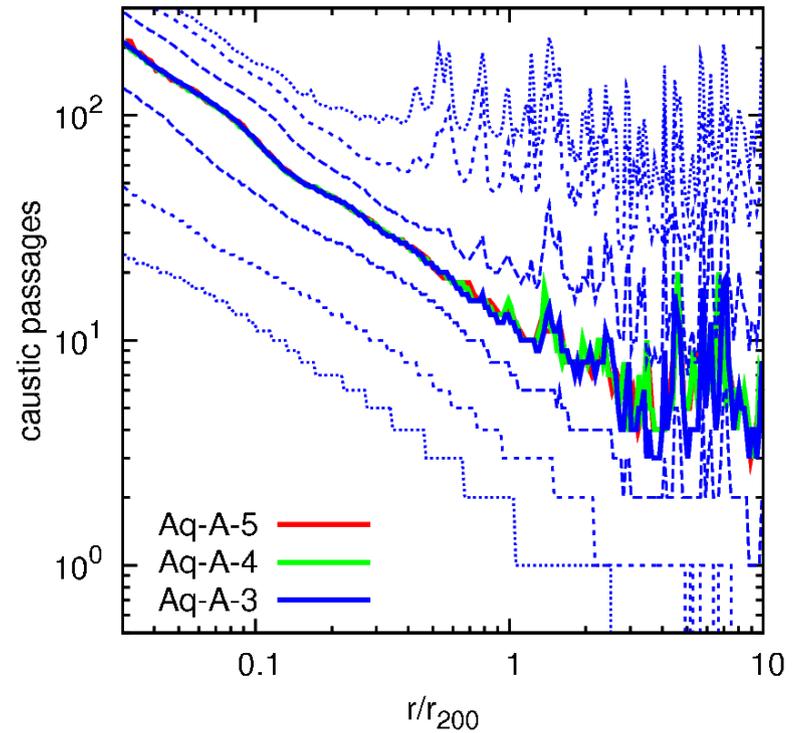
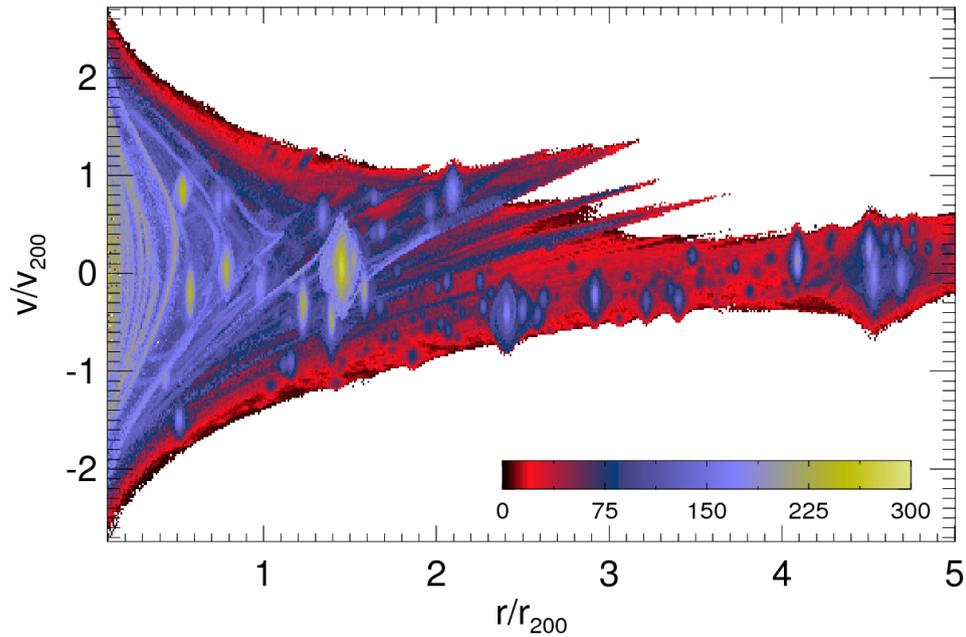
- 4% in range 0.01 to 0.5 x turnaround radius
- 24% ... 0.1 to 0.5 x ...
- 64% ... 0.2 to 0.5 x ...

→ caustic annihilation is negligible in the inner halo, but can boost the signal by more than 50% in the outer part



3) The fine-grained structure of LCDM halos

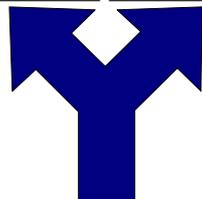
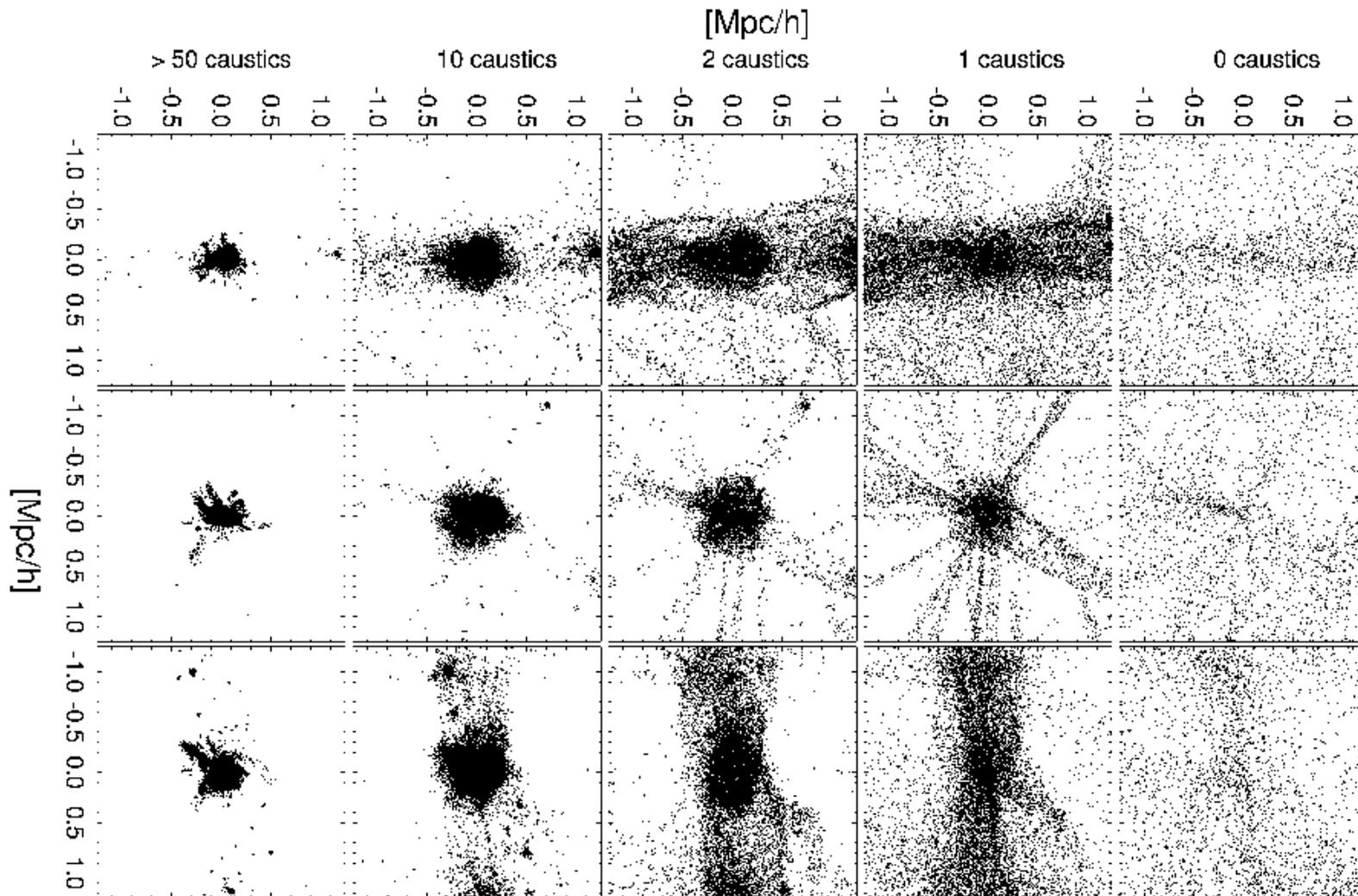
Caustics in LCDM Haloes



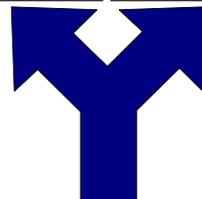
MV & White (in prep)

phase-space sheet
Wind-up around subhalo

'Filtering' the cosmic web



(sub)halo 'cores'

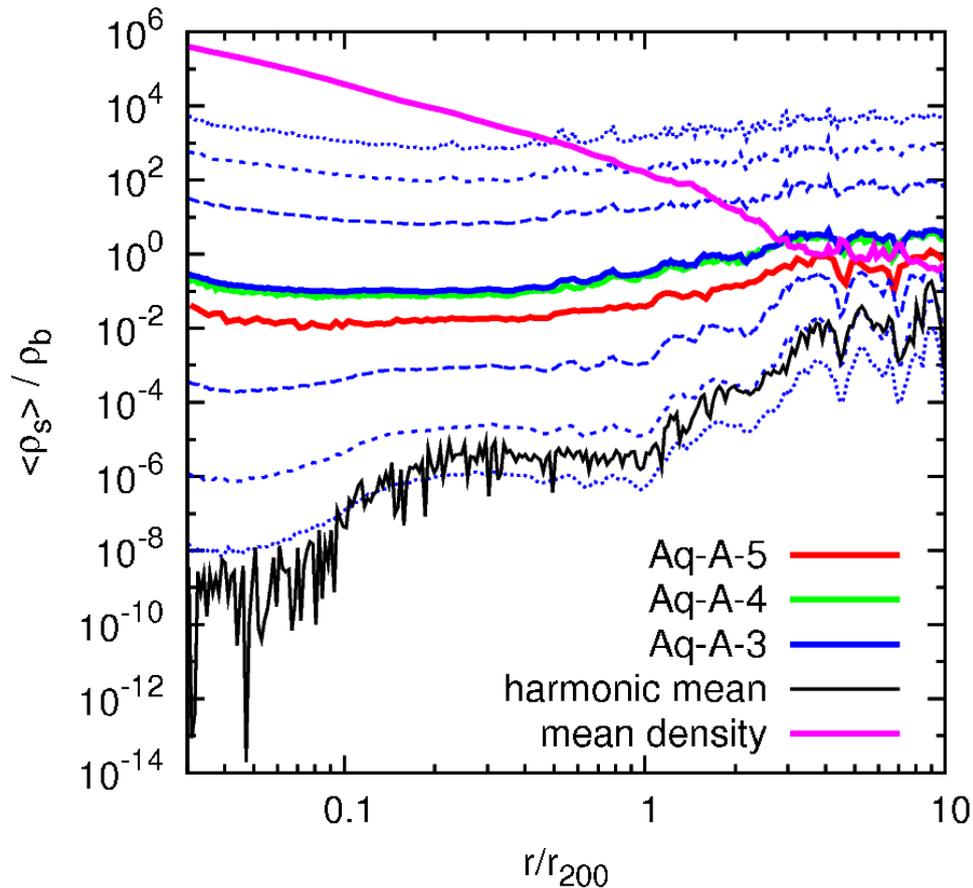


filaments

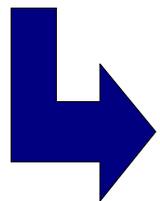
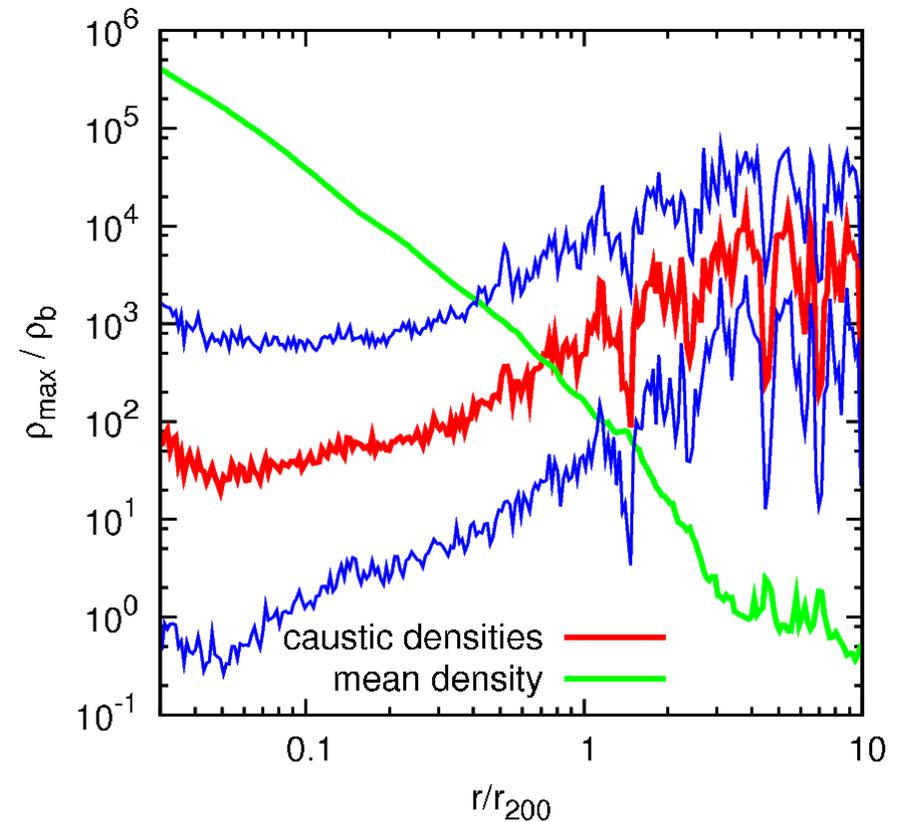


smooth

Stream and caustic densities



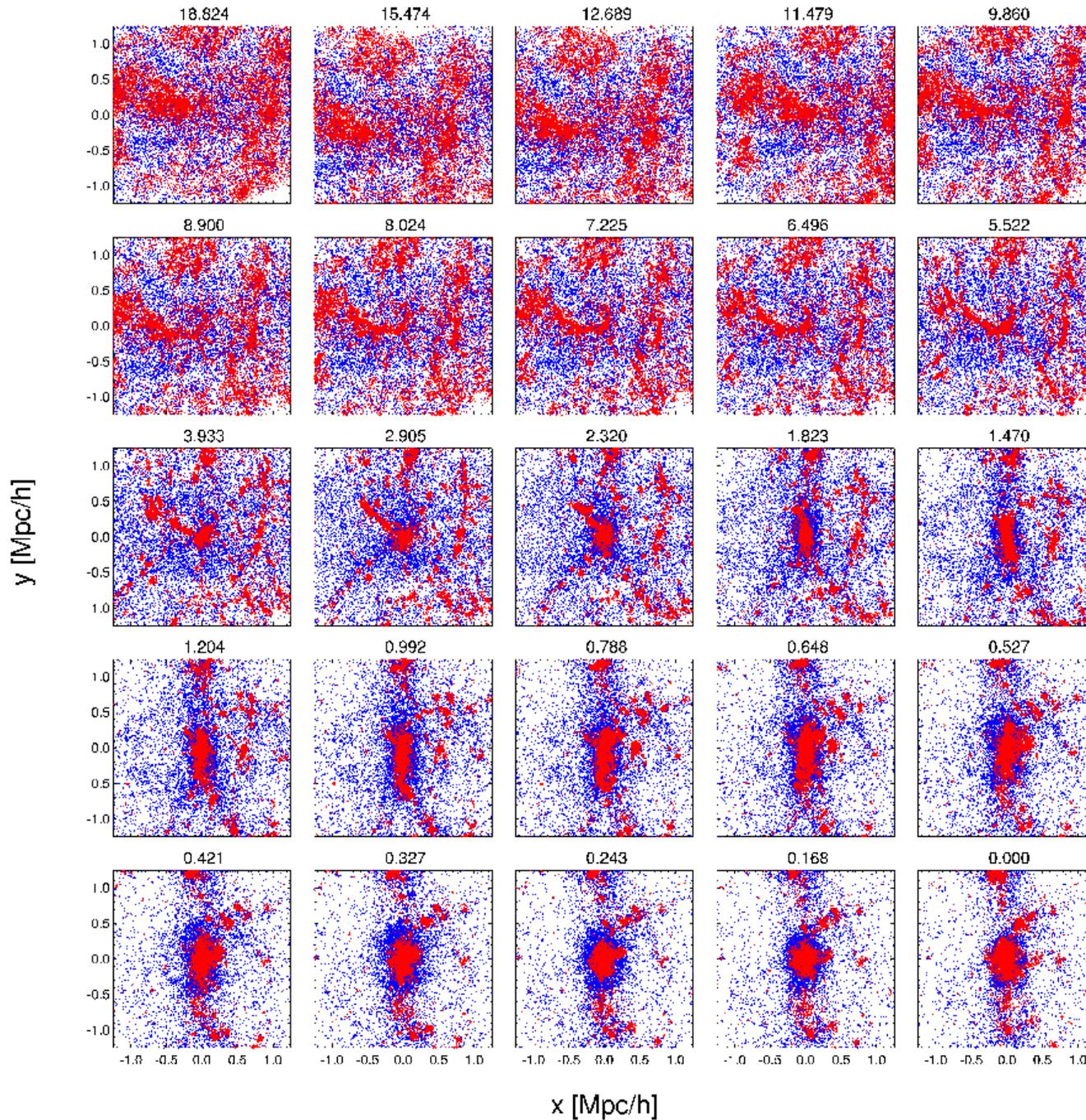
caustics subdominant
within virial radius



directly influences
annihilation radiation
due to caustics

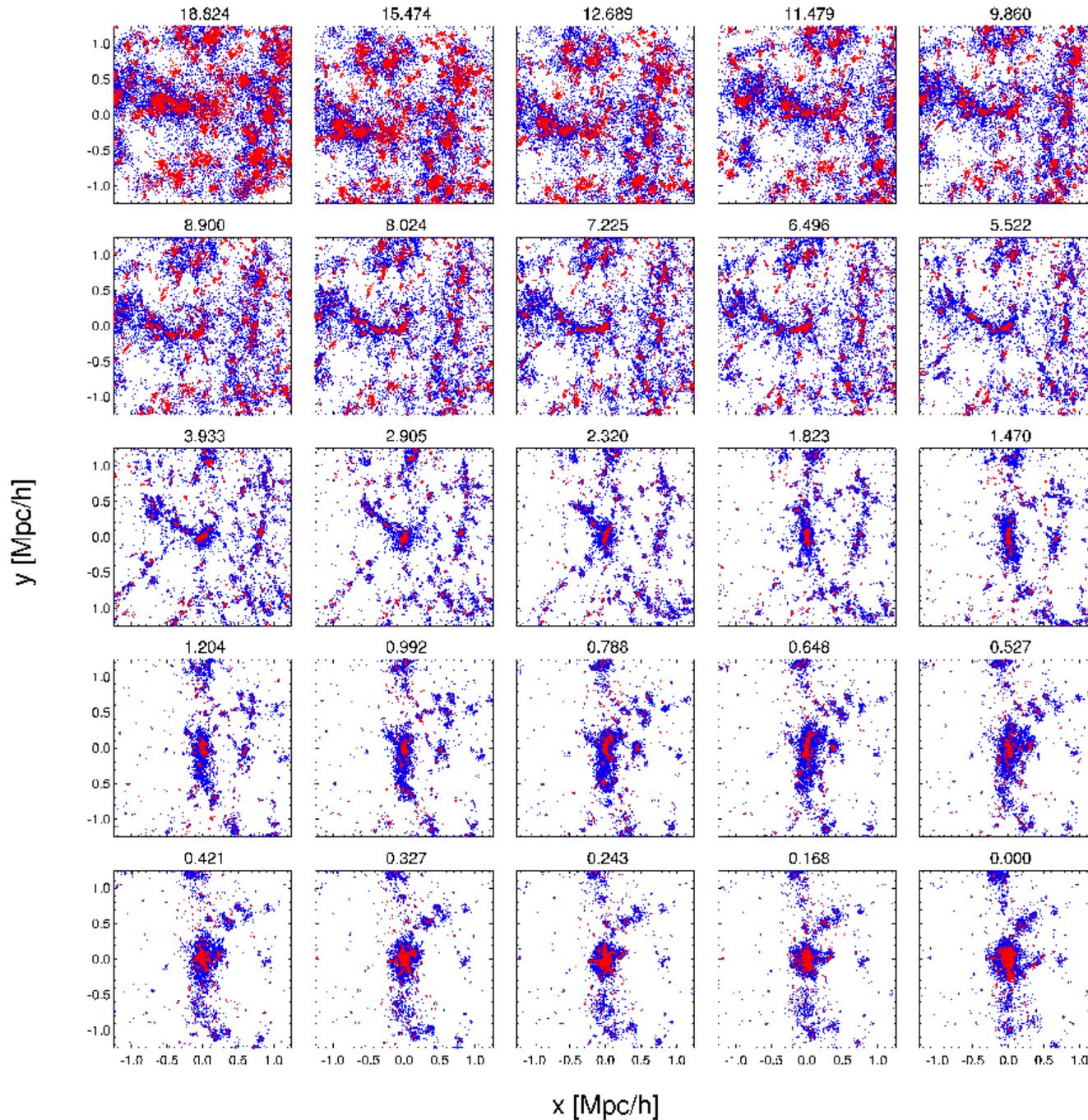
High/Low stream density particles

Where do they come from?

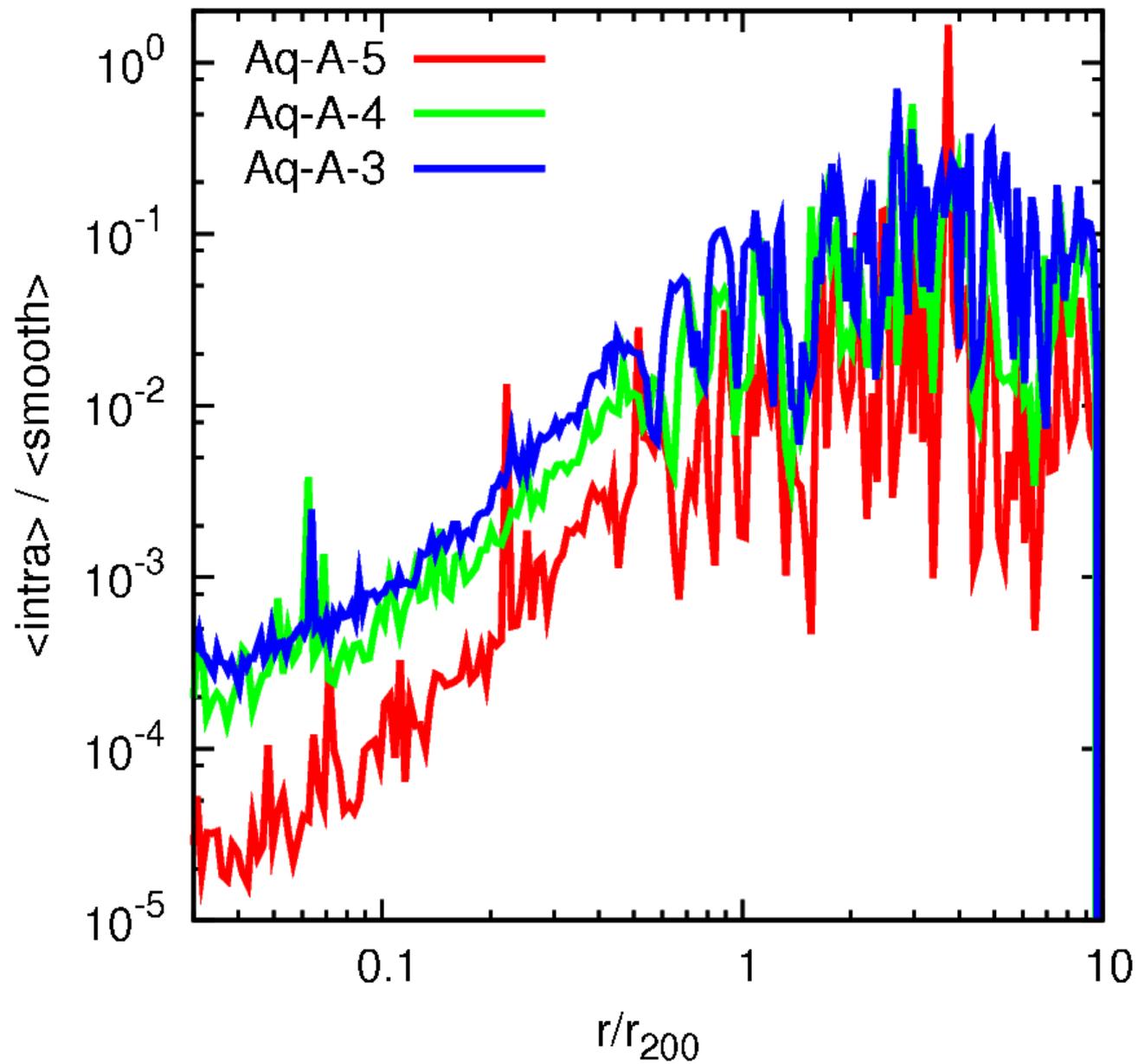


High/Low caustic counter particles

Where do they come from?



Local annihilation boost factor



4) A note: Dynamics with the Geodesic Deviation Equation

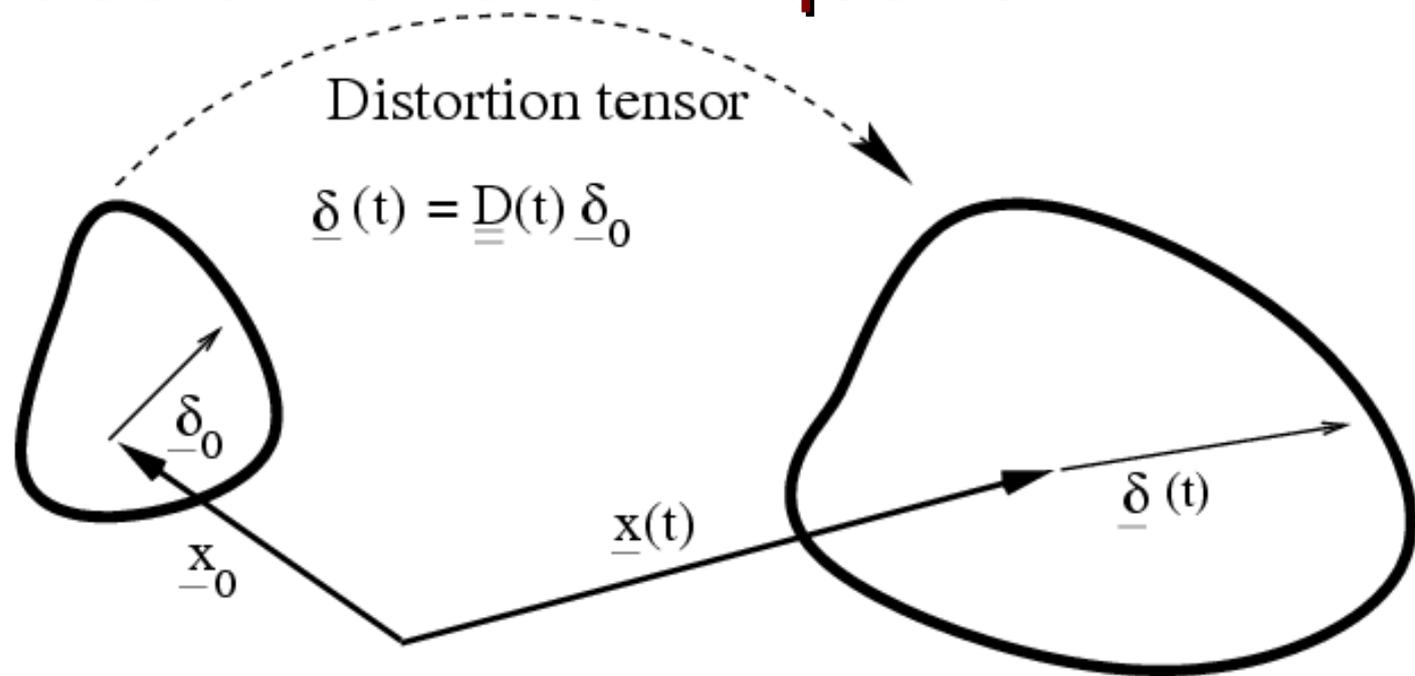
The Geodesic Deviation Equation

$$\overline{\overline{D}}(t; \overline{x}_0) = \frac{\partial \overline{x}}{\partial \overline{x}_0}(t; \overline{x}_0)$$

$$\dot{\overline{\overline{D}}}(t; \overline{x}_0) = \overline{\overline{T}}(t; \overline{x}_0) \overline{\overline{D}}(t; \overline{x}_0)$$

phase-space tidal tensor

$$\overline{\overline{T}}(t; \overline{x}_0) = \begin{pmatrix} \underline{\underline{0}} & \underline{\underline{1}} \\ \underline{\underline{T}}(t; \underline{x}_0) & \underline{\underline{0}} \end{pmatrix}$$



projection to configuration space

projection operators:
phase-space to
configuration-space

$$\underline{\underline{D}}(t; \overline{x}_0) = \begin{pmatrix} \underline{\underline{1}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{0}} \end{pmatrix} \overline{\overline{D}}(t; \overline{x}_0) \begin{pmatrix} \underline{\underline{1}} \\ \underline{\underline{V}}_x(\overline{x}_0) \end{pmatrix}$$

initial CDM

sheet orientation

zeroth order perturbation theory
=> Hubble flow

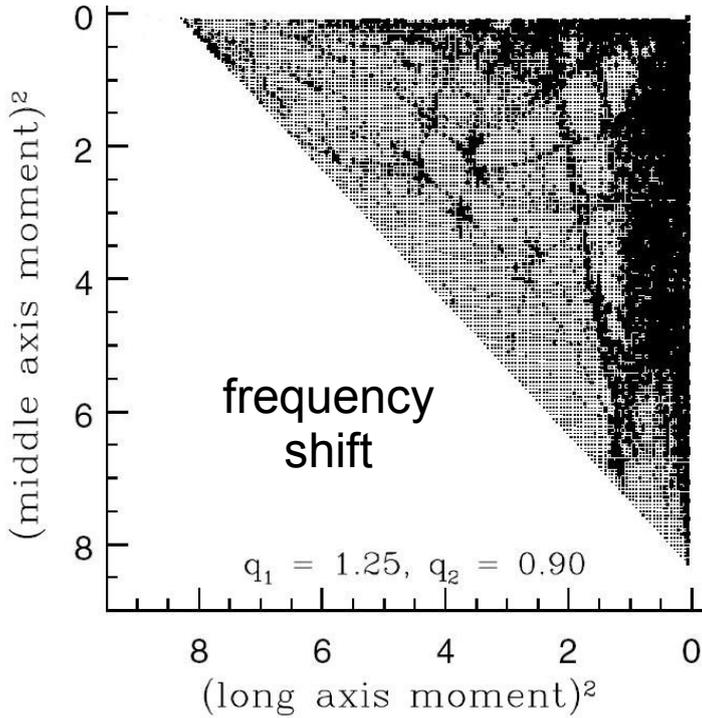
CDM stream density:

$$\rho_{\text{stream}}(t) \propto \frac{1}{\left| \det \left(\underline{\underline{D}}(t; \overline{x}_0) \right) \right|}$$

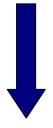
Properties:

- phase-space distortion tensor volume conserved
- configuration-space distortion tensor changes sign when passing through caustic

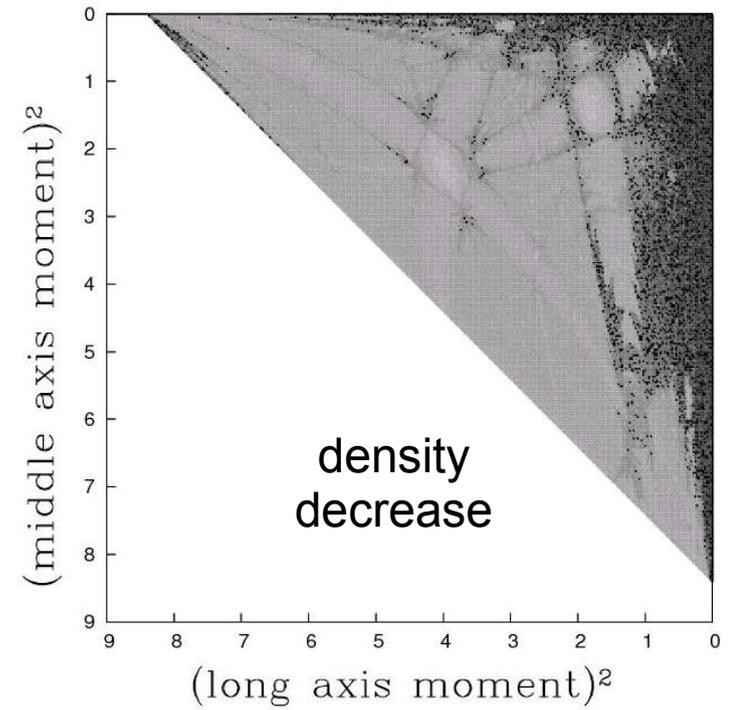
Chaos maps



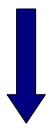
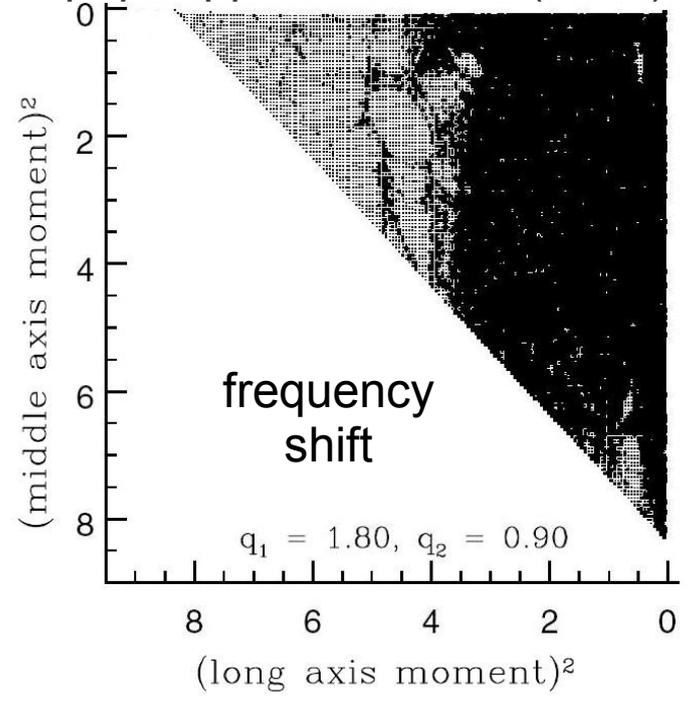
moderate triaxiality



density mostly
decaying like
power law



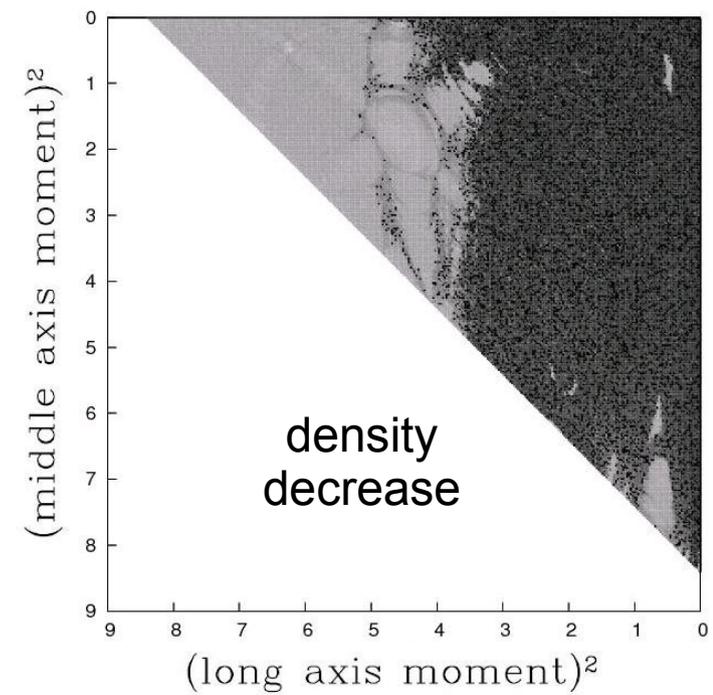
Papaphilippou & Laskar (1998)



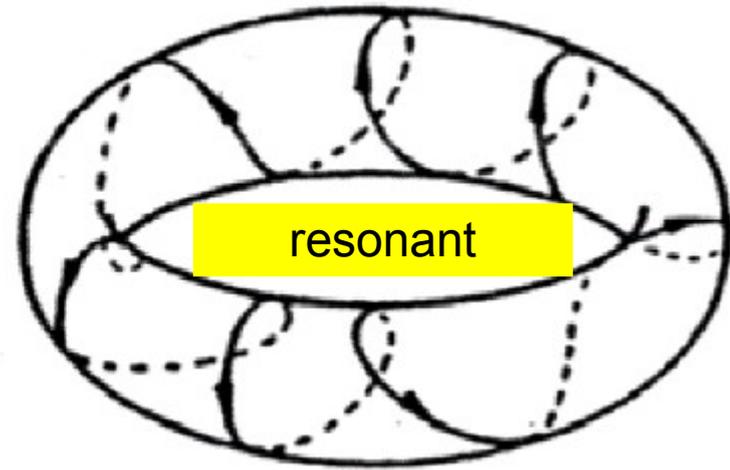
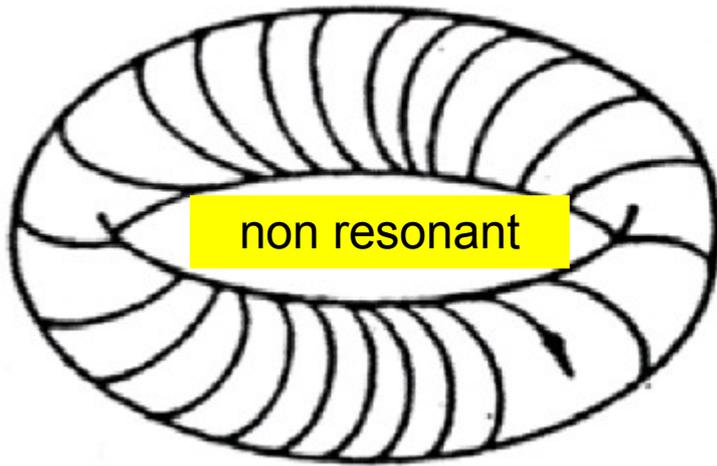
large fraction of
chaotic orbits

density mostly
decaying
lots of faster than
power law

MV et al (2008)



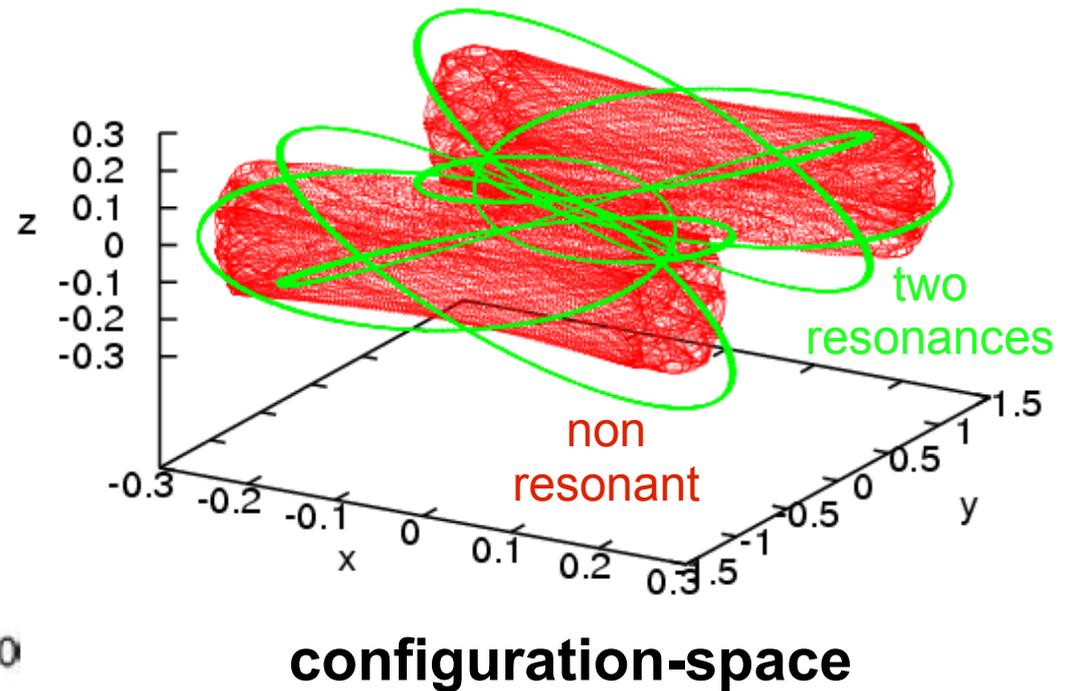
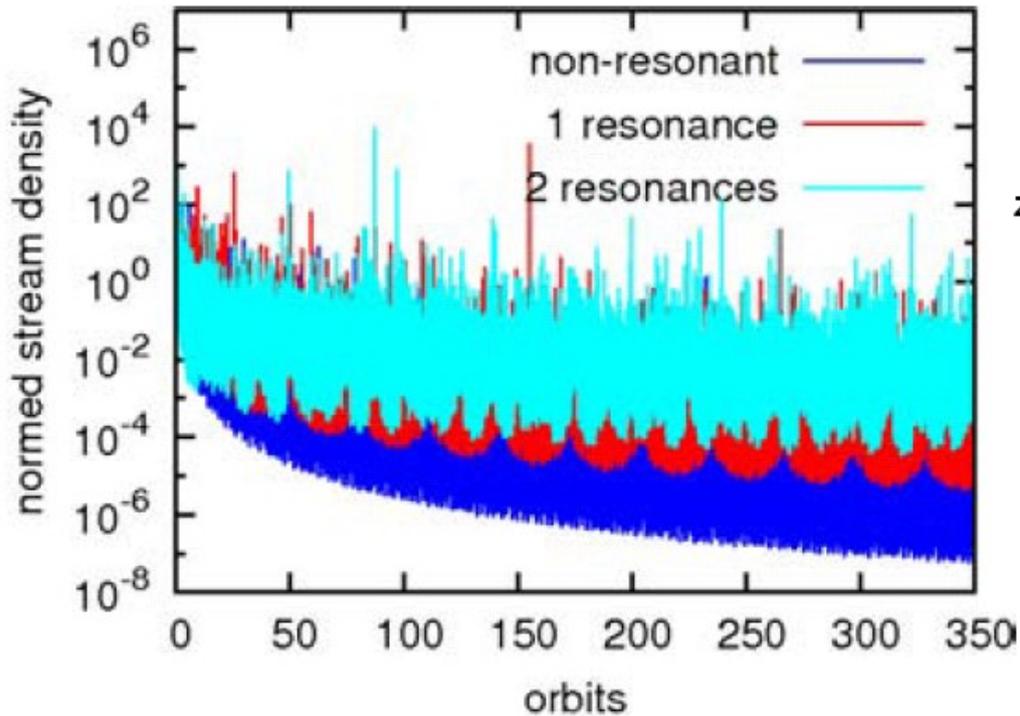
Resonances in phase-space



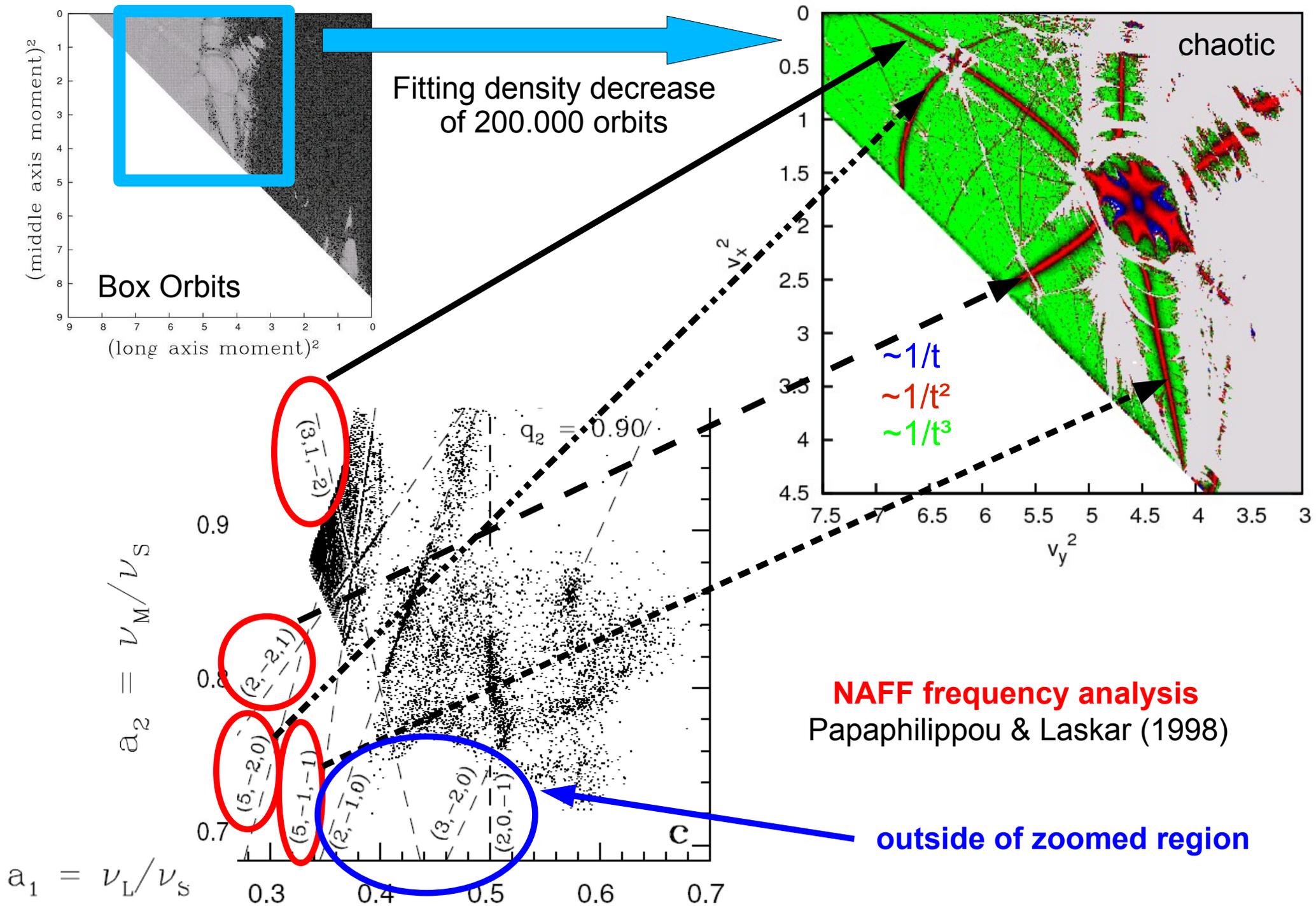
$$m_1 \omega_1 + m_2 \omega_2 + m_3 \omega_3 = 0$$



KAM torus not densely covered



Resonances: scanning phase-space



Conclusions

- ~100 streams near the Sun **[wrong]** [**~millions due to faster mixing**]
- massive caustic structures **[wrong]** [**non-regular fine-grained phase-space**]
- 1D models predict fine-grained phase-space **[wrong]** [**missing instabilities**]
- simulations miss much caustic annihilation **[wrong]** [**~10% in outskirts**]

The smooth halo model is not too bad!