The (fine-grained) phase-space structure of Cold Dark Matter Halos - and its influence on dark matter searches -

Mak Vogelsberger, Harvard/CfA
The content of the Universe

Evidence ↔ Existence: WIMP, Axion, …?

N-body simulations:
→ calculate DM gravitational interaction
→ structure formation
Large-scale structure

Springel et al (2005)
Large-scale structure

Millennium II


a playground for galaxy formation
(semi-analytic modeling)

~6,000 Milky Way-mass halos with ~100,000 particles

use a large calculator
Do we need more?!  
Do we simulate reality?  
Is there really dark matter?  

How to be sure?

Detect it!  
- indirectly via annihilation products  
- directly via scattering in underground detectors  

And ideally:  
- produce a suitable particle at LHC  

Complete the puzzle
The Hunt for Dark Matter

Direct searches: nuclear recoil events

Usually assumed astrophysical input:
- Standard Halo Model (SHM):
  - Smooth mass distribution
  - Smooth velocity distribution
  - 'Featureless' phase-space

Density: \(~0.3\) GeV / \(c^2 / \text{cm}^3\)

Velocity: Maxwellian

Indirect searches: annihilation products

FERMI, PAMELA, ...

Accelerator searches: producing DM

CRESST, XENON, ZEPLIN, EDELWEISS, CDMS, DAMA, ...

LHC

Accelerator searches: producing DM

CRESST, XENON, ZEPLIN, EDELWEISS, CDMS, DAMA, ...

LHC
'Non-standard' Halo models

N-body simulations:
lots of phase-space substructure

Standard halo model assumption wrong?!

Dark matter parameter-space limits wrong?!

Analytic models:
→ massive streams
→ caustic ring model


Van Bibber (2008)

Natarajan & Sikivie (2008)

Outline

1) The coarse-grained structure of LCDM halos

2) Towards the fine-grained structure of LCDM halos

3) The fine-grained structure of LCDM halos

4) A note: Dynamics with the Geodesic Deviation Equation

5) Conclusions
1) The coarse-grained structure of LCDM halos
Aquarius Project
-six Milky Way-like Halos-

\[
\begin{array}{ccc}
    m_p & \epsilon & N_{hr} \\
    [M_\odot] & [pc] & \\
    1.712 \times 10^3 & 20.5 & 4,252,607,000
\end{array}
\]

Probing DM near the Sun!

DM smoothness near the Sun

Scatter standard deviation < 5%
(variation due to halo shape taken out)

Chance of 'hitting' a subhalo is very small: $\sim 10^{-4}$

Experimentalists can use smooth models

MV et al (2009)
Velocity Distribution near the Sun

No signs for distinct/massive streams

Bumps/dips in velocity vector modulus

Not Maxwellian

**Alignment**
- No alignment issues detected.
Bumps in velocity modulus at the same velocity
Not Maxwellian
Not Gaussian

**... at Solar Circle**

- best-fit mult. Gaussian
- median velocity distribution

Many 2kpc boxes
Signature in Detector Signals

WIMP recoil spectrum

\[ E = \frac{2 \, m^2 \, m_A}{(m_\chi + m_A)^2} \, c^2 \, \beta^2 \times 10^6 \, \text{keV} \]

Axion microwave spectrum

\[ \nu_a = 241.8 \left( \frac{m_\alpha}{1 \, \mu\text{eV}/c^2} \right) \left( 1 + \frac{1}{2} \beta^2 \right) \, \text{MHz} \]
2) Towards the fine-grained structure of LCDM halos
CDM – very small scales

CDM is cold and collisionless

CDM lies on 3D hypersurface in 6D phase-space

Thickness of line: primordial velocity dispersion

Amplitude of wiggles: velocity due to density perturbations

Wind-up: growth of an overdensity

Phase space sheet:

\[ (\vec{r}, \vec{v}) : H(t) \vec{r} + \Delta \vec{v}(\vec{r}, t) \]

Caustic (catastrophe)

regions of very high CDM density

Fine-grained phase-space

streams

1 3
Estimates/Calculations so far

Self-similar halo formation:
Fillmore & Goldreich (1984), Bertschinger (1985), Mohayaee & Salati (2008); Mohayaee et al (2006); ...

Caustic ring model:
Duffy & Sikivie (2008); Natarajan & Sikivie (2008); Onemli & Sikivie (2007); Natarajan & Sikivie (2007); Sikivie et al (1997); ...

Predictions
• ~100 streams at solar position
• significant annihilation boost
• strong caustic rings
• discrete velocity distribution
• distinct caustic structures
→ Significant effects on search experiments

General arguments:
Hogan (2001), Afshordi et al (2009), ...

How realistic are these models?
Correct caustic structure?
Correct caustic densities?
Correct number of streams?
Correct boost?
Infinite density

Cut off (due to velocity dispersion)

Mohayaee & Shandarin (2006)
[following approach of Zel’dovich, Shandarin, Arnold]
White & MV (2009)

ABSTRACT

We derive similarity solutions which describe the collapse of cold, collisionless matter in a perturbed Einstein–de Sitter universe. We obtain three classes of solutions, one each with planar, cylindrical, and spherical symmetry. Our solutions can be computed to arbitrary accuracy, and they follow the development of structure in both the linear and nonlinear regimes.

Subject headings: cosmology — relativity
Resolving fine-grained caustics with N-body simulations

**Problem:** N-body simulations have too coarse phase-space sampling (→ missing many orders of magnitude in mass resolution/particle number)

**Solution:** Follow the local phase-space evolution for each particle (→ with a phase-space geodesic deviation equation)

- **calculation of stream density**
- **identification of caustics**
- **Monte-Carlo estimate for intra-stream annihilation**

→ allows **caustic annihilation calculation**

\[ \frac{dA_{s,i}}{dt} = \frac{<\sigma v>_{\chi}}{m_{\chi}^2} m_i \rho_{s,i} \]

[Implementation in GADGET-3]

MV et al (2008)
Caustic Annihilation radiation - 1D gravity -

causitic spheres on top of smooth annihilation signal
Caustic structures
- non-radial halo model -

lead to interest in impact on annihilation radiation

Natarajan & Sikivie (2008)

But N-body simulations predict:

Collapse of an isolated halo in 3D gravity

No clear phase-space pattern

Density slice → bar formation

Instabilities lead to different phase-space evolution!

MV et al (2009)
Fine-grained phase-space in 3D

Stream density: lower in 3D

Caustic passages: Increases in 3D → more turning points

More efficient mixing in higher dimensions
Caustics in 3D

100 GeV/c^2 Neutralino

Caustic densities

boost factor due to caustics:
→ 4% in range 0.01 to 0.5 x turnaround radius
→ 24% ... 0.1 to 0.5 x ...
→ 64% ... 0.2 to 0.5 x ...

→ caustic annihilation is negligible in the inner halo, but can boost the signal by more than 50% in the outer part
3) The fine-grained structure of LCDM halos
Caustics in LCDM Haloes

MV & White (in prep)

phase-space sheet
Wind-up around subhalo
'Filtering' the cosmic web

(sub)halo 'cores'
filaments
smooth
Stream and caustic densities directly influences annihilation radiation due to caustics.

caustics subdominant within virial radius
High/Low stream density particles

Where do they come from?
High/Low caustic counter particles

Where do they come from?
Local annihilation boost factor

\[ \frac{\langle \text{intra} \rangle}{<\text{smooth}>} \]

\[ r/r_{200} \]

- Aq-A-5
- Aq-A-4
- Aq-A-3
4) A note: Dynamics with the Geodesic Deviation Equation
The Geodesic Deviation Equation

\[ \overline{D}(t; \overline{x}_0) = \frac{\partial \overline{x}}{\partial \overline{x}_0}(t; \overline{x}_0) \]

\[ \overline{D}(t; \overline{x}_0) = \overline{T}(t; \overline{x}_0) \overline{D}(t; \overline{x}_0) \]

phase-space tidal tensor

\[ \overline{T}(t; \overline{x}_0) = \begin{pmatrix} 0 & 1 \\ \overline{T}(t; \overline{x}_0) & 0 \end{pmatrix} \]

projection to configuration space

\[ \underline{D}(t; \underline{x}) = \left( \begin{array}{cc} 1 & 0 \\ 0 & \frac{1}{\sqrt{\overline{V}_x(\overline{x}_0)}} \end{array} \right) \overline{D}(t; \overline{x}_0) \]

projection operators: phase-space to configuration-space

properties:
- phase-space distortion tensor volume conserved
- configuration-space distortion tensor changes sign when passing through caustic

zeroth order perturbation theory => Hubble flow

CDM stream density:

\[ \rho_{\text{stream}}(t) \propto \frac{1}{\det \left( \frac{\overline{D}(t; \overline{x}_0)}{D(t; \overline{x}_0)} \right)} \]
Chaos maps

- Moderate triaxiality
  - Density mostly decaying like power law

- Large fraction of chaotic orbits
  - Density mostly decaying lots of faster than power law

Papaphilippou & Laskar (1998)

MV et al (2008)
Resonances in phase-space

\[ m_1 \omega_1 + m_2 \omega_2 + m_3 \omega_3 = 0 \]

KAM torus not densely covered

configuration-space

non resonant
resonant

non resonant
1 resonance
2 resonances

two resonances
non resonant
Resonances: scanning phase-space

Fitting density decrease of 200,000 orbits

Box Orbits

Naff frequency analysis

Papaphilippou & Laskar (1998)

Outside of zoomed region
Conclusions

- ~100 streams near the Sun [wrong] [~millions due to faster mixing]
- massive caustic structures [wrong] [non-regular fine-grained phase-space]
- 1D models predict fine-grained phase-space [wrong] [missing instabilities]
- simulations miss much caustic annihilation [wrong] [~10% in outskirts]

The smooth halo model is not too bad!