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# The mass and anisotropy profiles of nearby galaxy clusters from the projected phase-space density

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Yehuda Hoffman



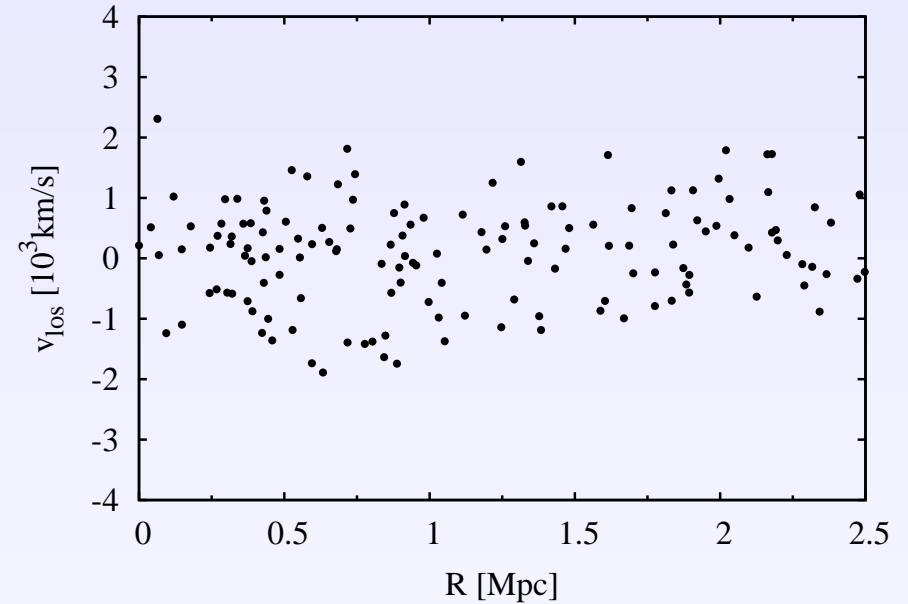
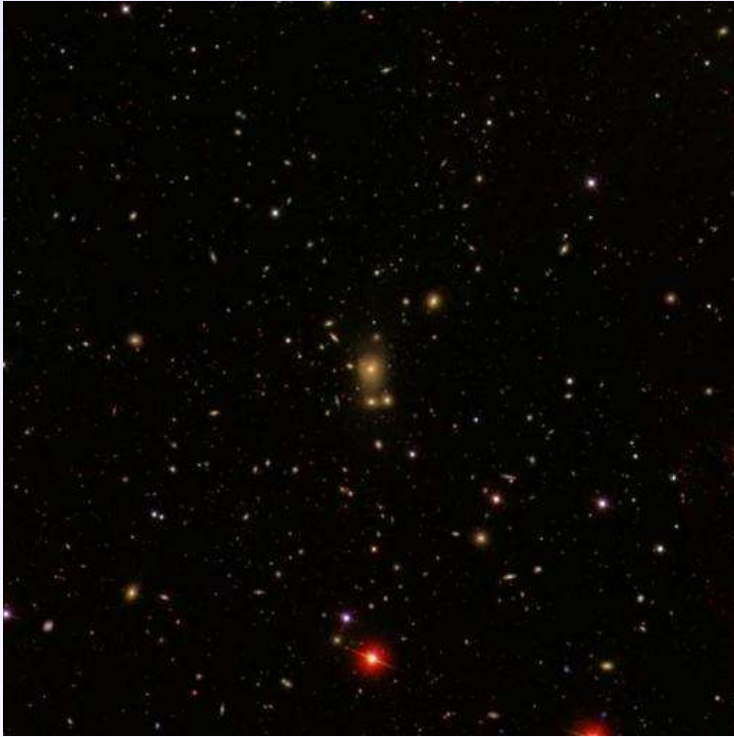
# Outline

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- Motivation
- DM density profile/anisotropy profile
- Model of the phase-space density: brief description
- Tests on mock kinematic data
- Application to nearby galaxy clusters
- Summary/Perspective for future



# Motivation

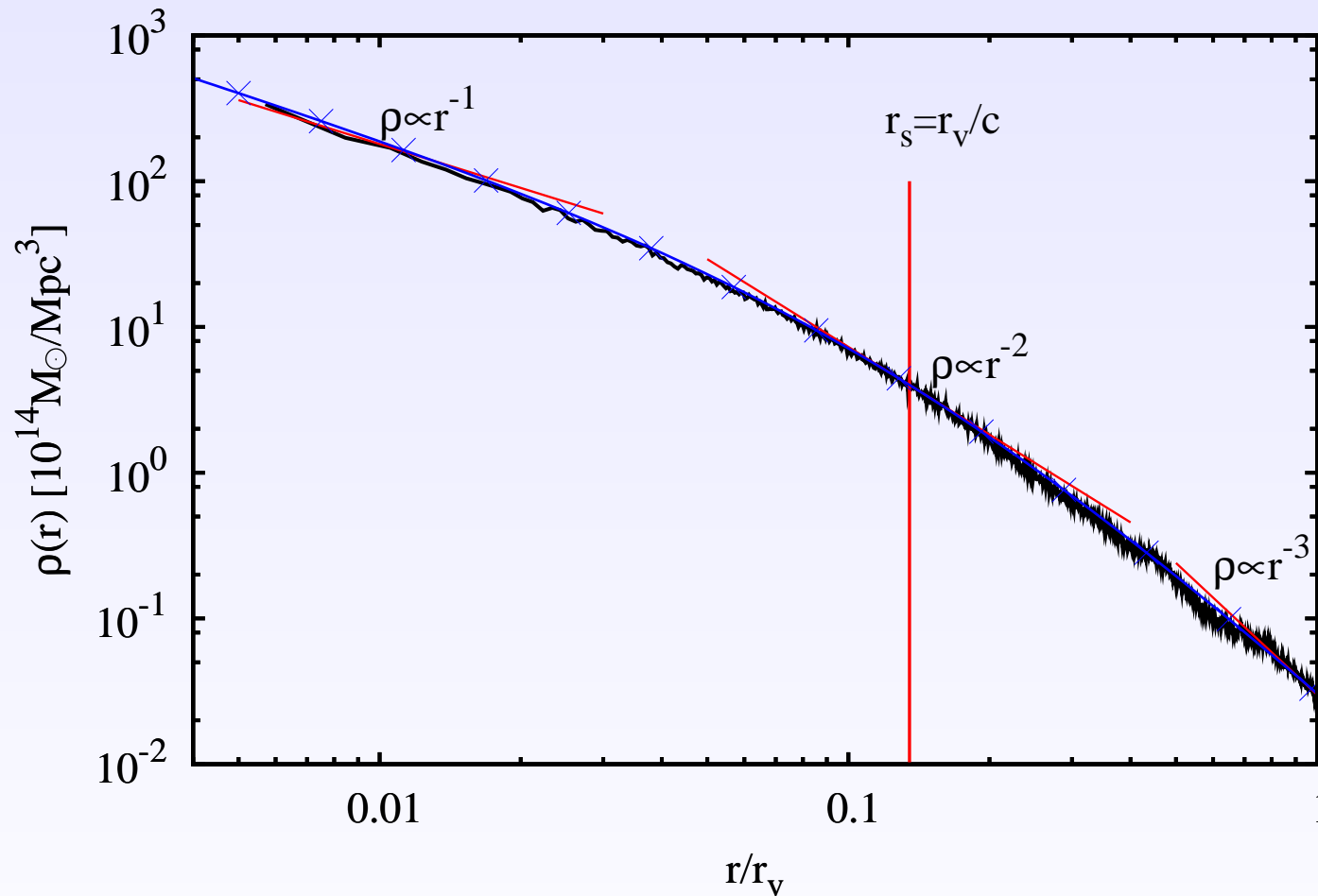


$$f_{los}(R, v_{los}) \rightarrow \rho_{DM}(r), \beta(r)$$

- no data binning
- breaking  $M - \beta$  degeneracy



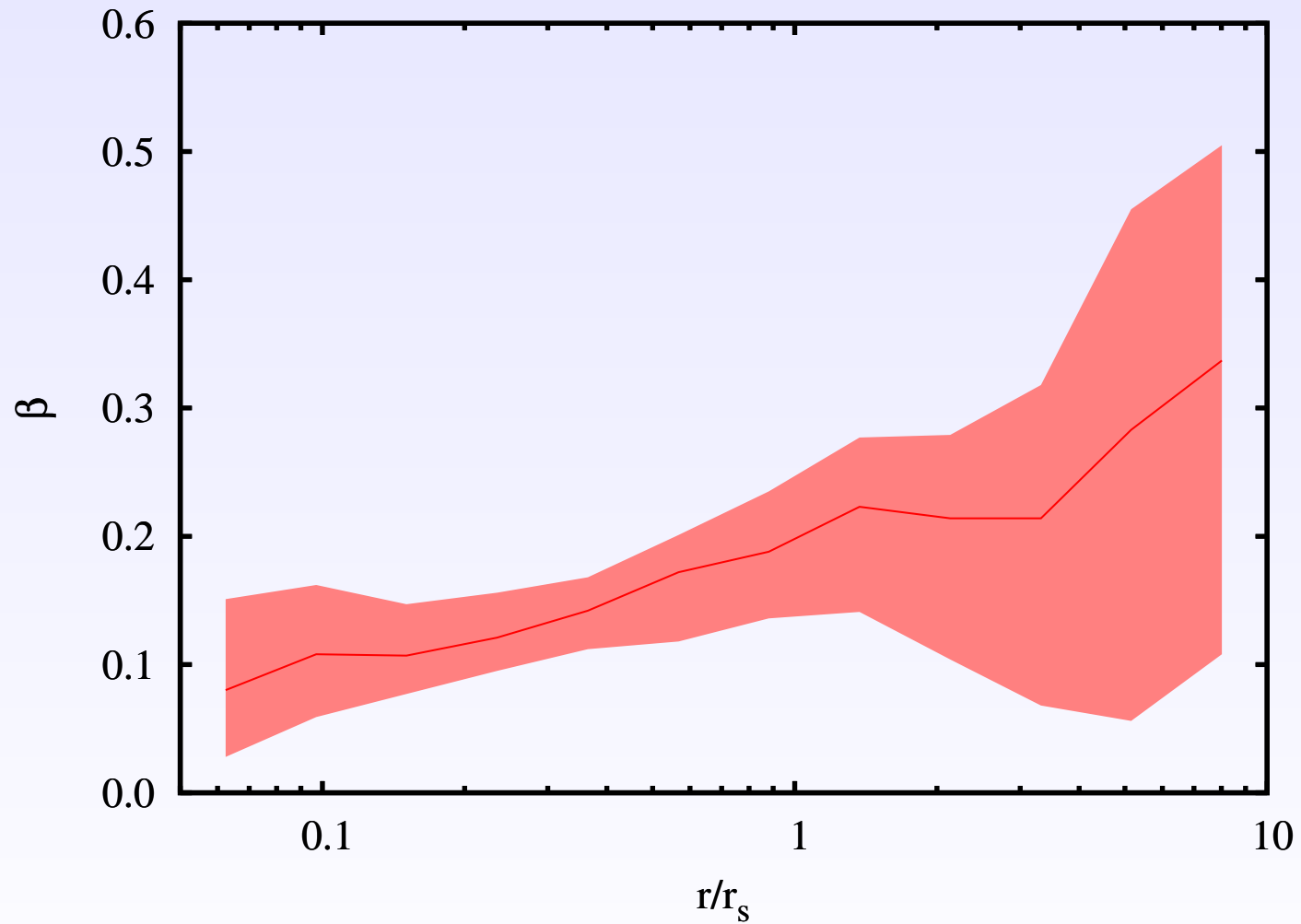
# DM density profile



$$\rho_{NFW}(r) \propto \frac{1}{(r/r_s)(1+r/r_s)^2}$$



# Anisotropy profile



$$\beta(r) = 1 - \frac{\sigma_{\theta}^2(r)}{\sigma_r^2(r)}$$



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# Model of the phase-space density



# $f(\mathbf{r}, \mathbf{v})$

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- spherical symmetry:  $f(\mathbf{r}, \mathbf{v}) = f(E, L)$
- 1st component:  $\rho(r) \rightarrow f(E, L)$
- 2nd component:  $\beta(r) \rightarrow f(E, L)$

$$\beta(r) = 1 - \frac{\sigma_{\theta}^2(r)}{\sigma_r^2(r)}$$

- 1: radially-biased model
- 0: isotropic dispersion tensor
- $-\infty$ : tangentially-biased model



# History

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- $\beta(r) = 0$

Eddington (1916)

- $\beta(r) = \text{const.}$

Hénon (1973)

- $\beta(r) = \frac{r^2}{r^2 + r_a^2}$

Osipkov (1979), Merritt (1985)

- $\beta(r) = \frac{r^2 - \alpha r_a^2}{r^2 + r_a^2}$

Cuddeford (1991)

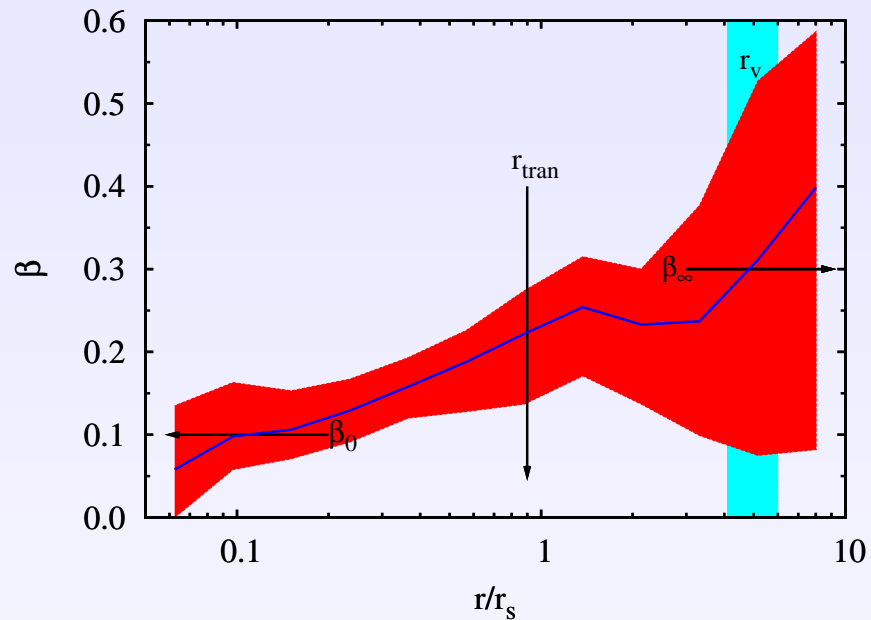
- $f(E, L) = f_E(E) \left(1 + L^2 / (2L_0^2)\right)^{-\beta_\infty}$

Cuddeford & Louis (1995)





# $f_L(L)$



Free parameters:

- $\beta_0 = \lim_{r \rightarrow 0} \beta(r)$
- $\beta_\infty = \lim_{r \rightarrow \infty} \beta(r)$
- $\beta(r_{tran}) \approx \frac{\beta_0 + \beta_\infty}{2}$

$$f_L(L) \propto \begin{cases} L^{-2\beta_0} & \text{for } L \ll L_0 \\ L^{-2\beta_\infty} & \text{for } L \gg L_0, \end{cases}$$

$$\text{ansatz: } f_L(L) = \left(1 + \frac{L^2}{2L_0^2}\right)^{-\beta_\infty + \beta_0} L^{-2\beta_0}$$



# $f_E(E)$

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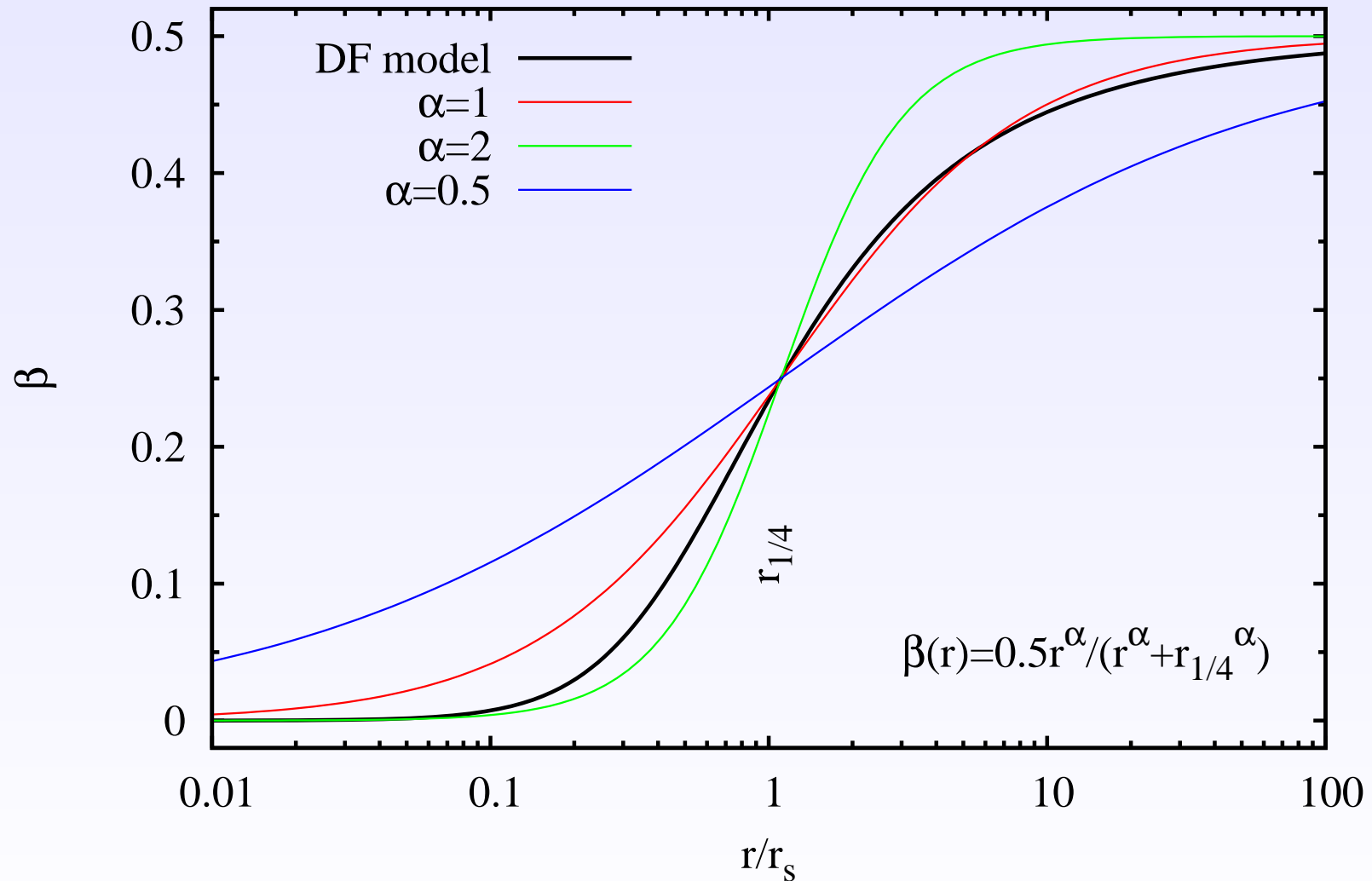
- $n_{gal}(r)$
- $\rho_{DM}(r) \rightarrow \Psi(r)$ 
  - $n_{gal}(r) \propto \rho_{DM}(r)$
  - $\rho_{DM} \propto \frac{1}{(r/r_s)(1+r/r_s)^2}$

## $f_E(E)$

- $n_{gal}(r) = \int \int \int f_E(E) f_L(L) d^3v$
- $n_{gal}(r) = \int_0^{\Psi(r)} f_E(E) K(E, \Psi(r)) dE$
- $n_{gal,i} = \sum_j K_{i,j} f_E(\Psi_j)$



# The shape of $\beta(r)$ profile

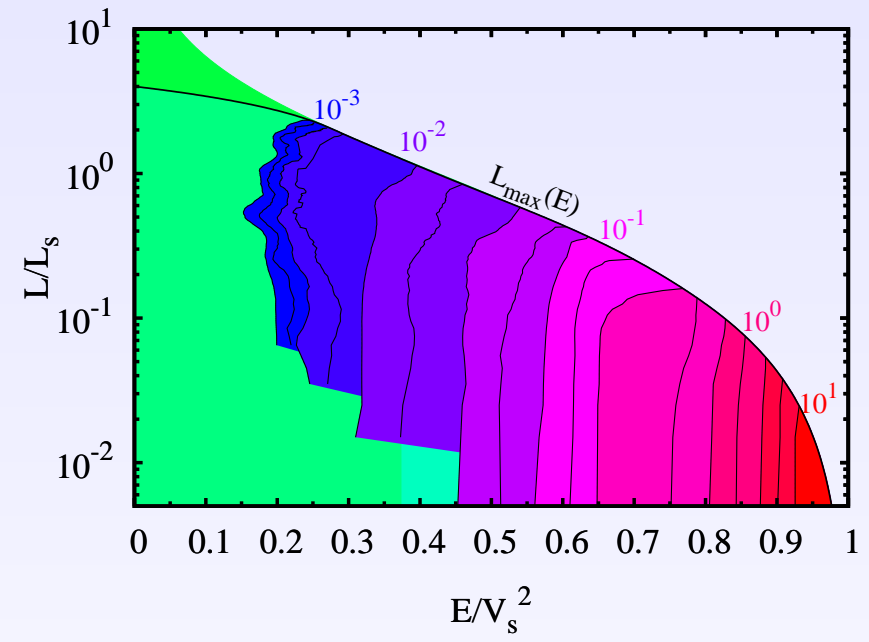
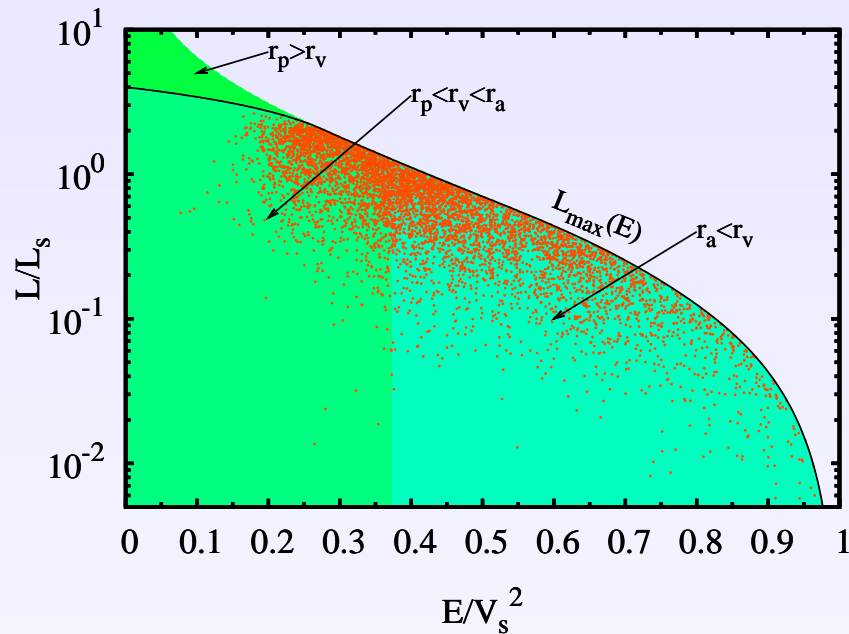


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# Comparison with the simulation



# $f(E, L)$ from the simulation



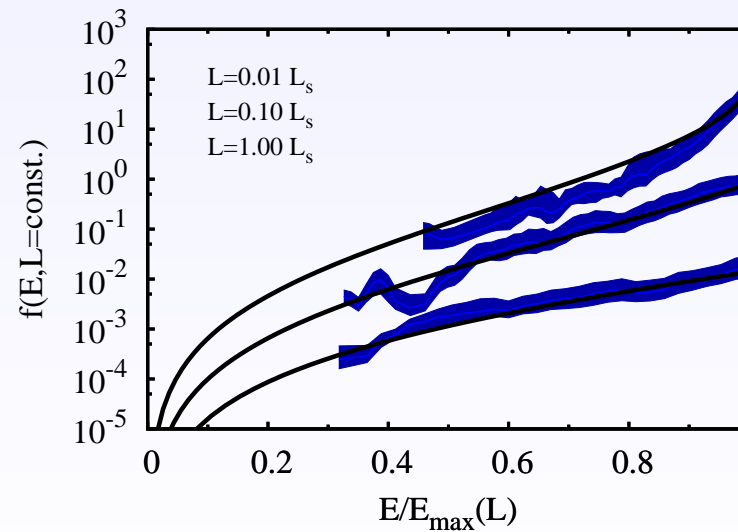
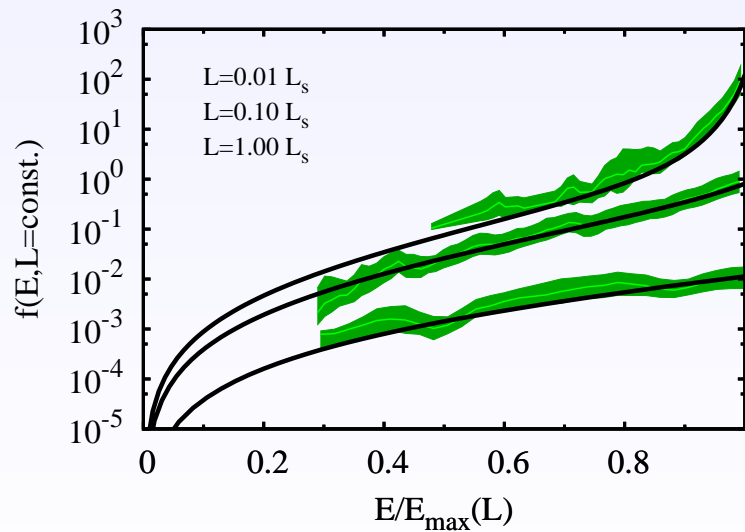
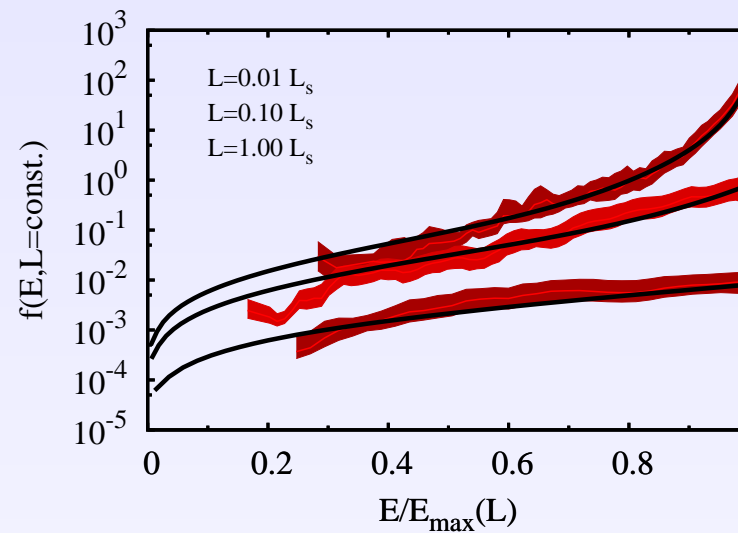
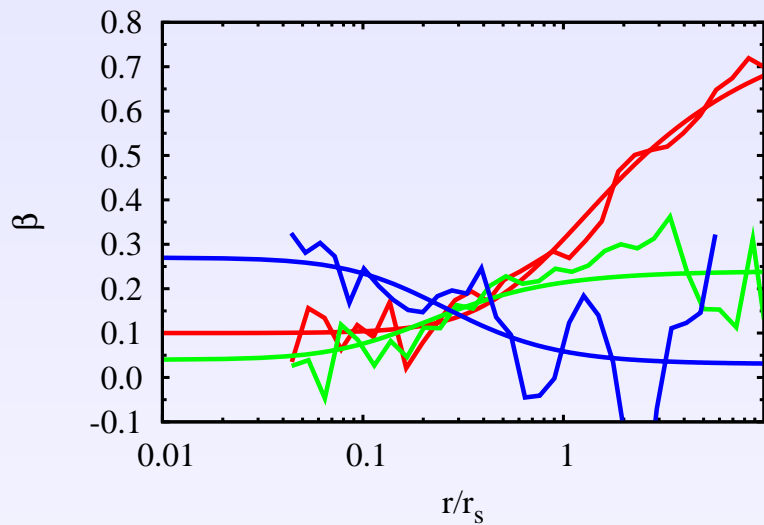
$$N(E, L) = \frac{d^2 M}{dE dL}$$

$$f(E, L) = \frac{N(E, L)}{g(E, L)}$$

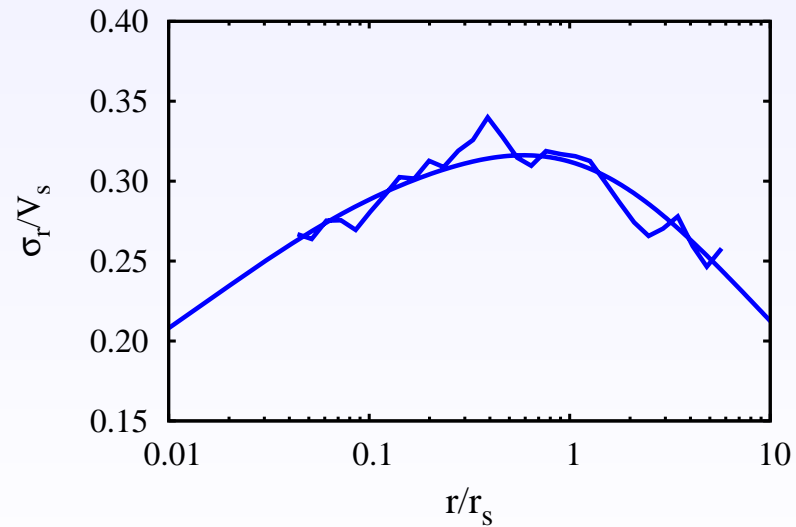
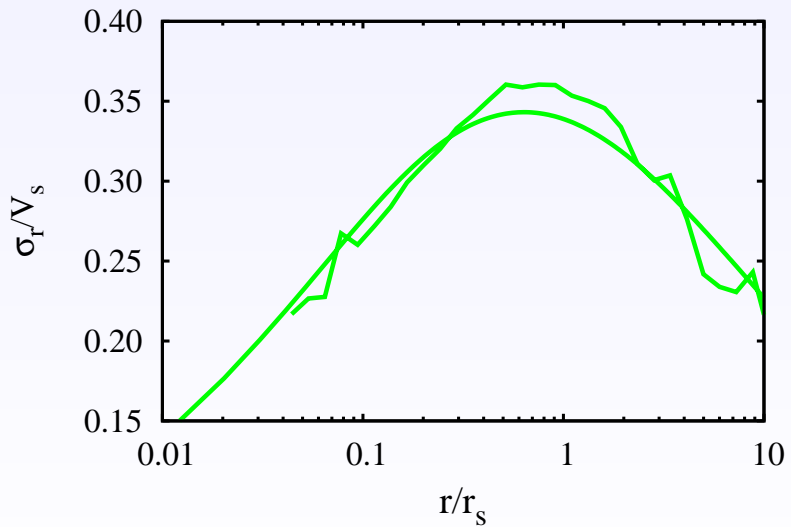
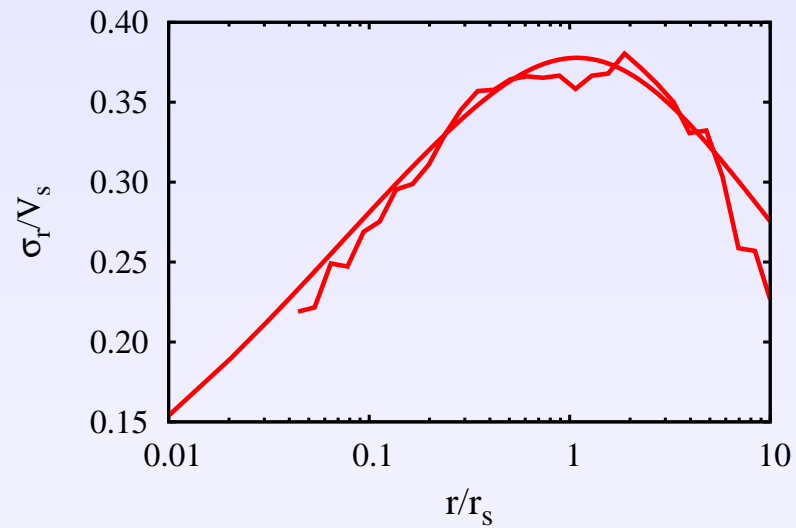
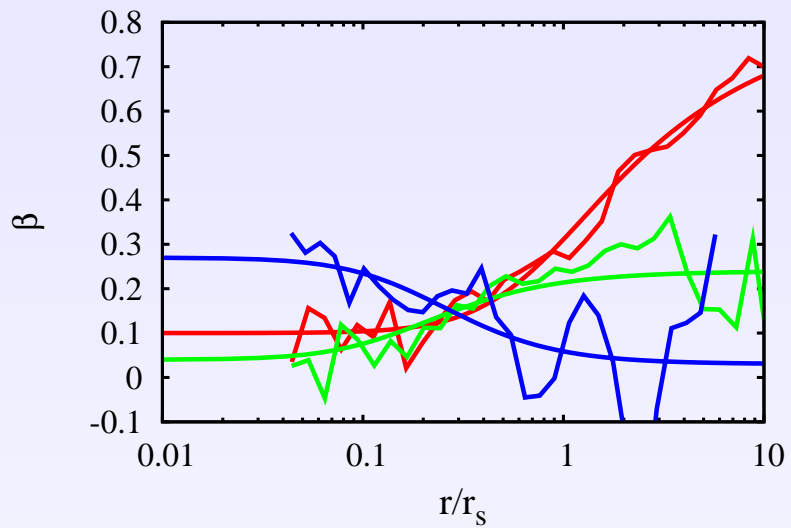
■  $g(E, L) = \int \delta\left(\frac{1}{2}v^2 - \Psi(r) - E\right) \delta(|\mathbf{r} \times \mathbf{v}| - L) d^3 r d^3 v$



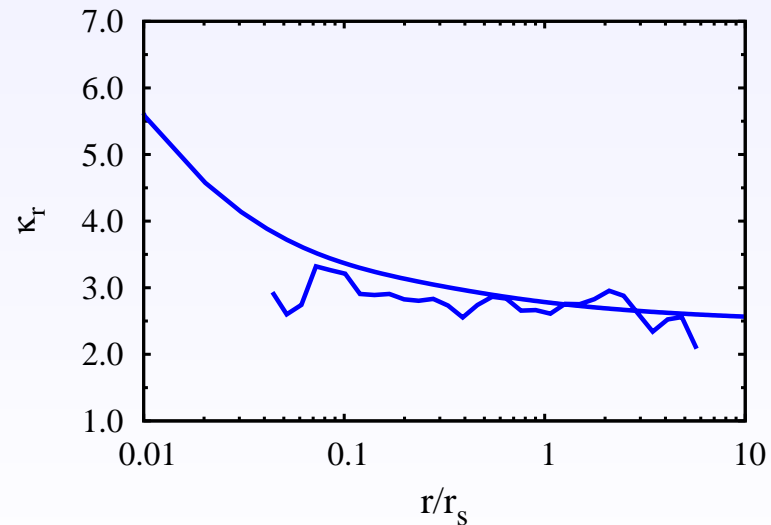
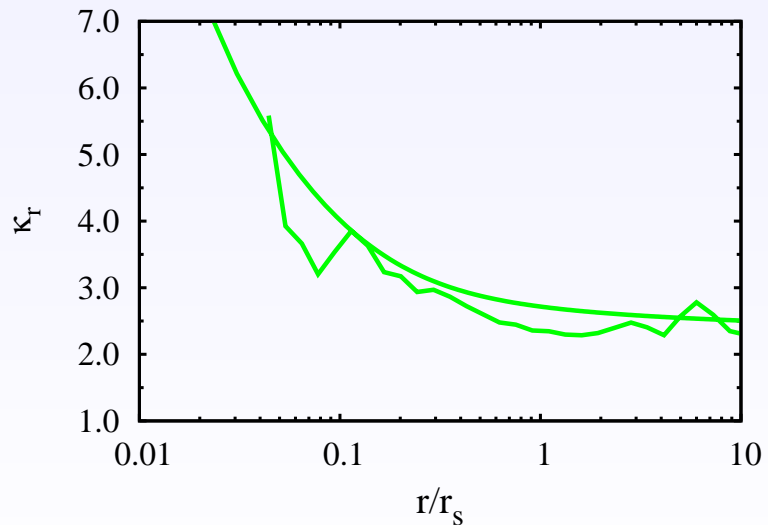
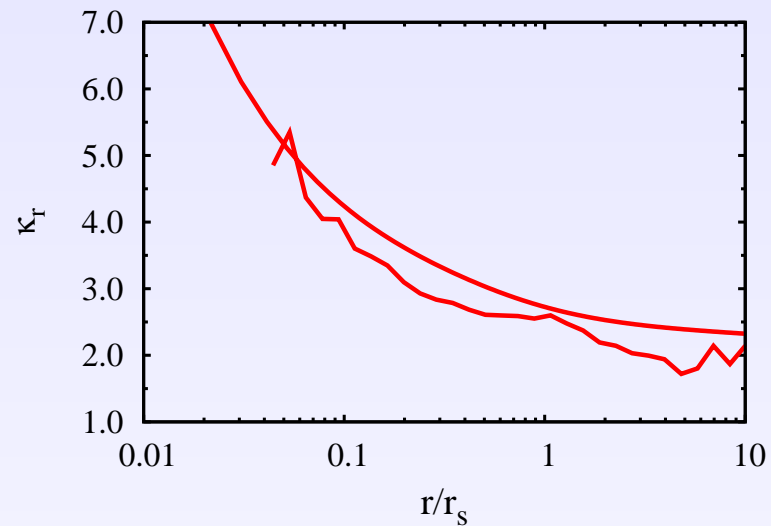
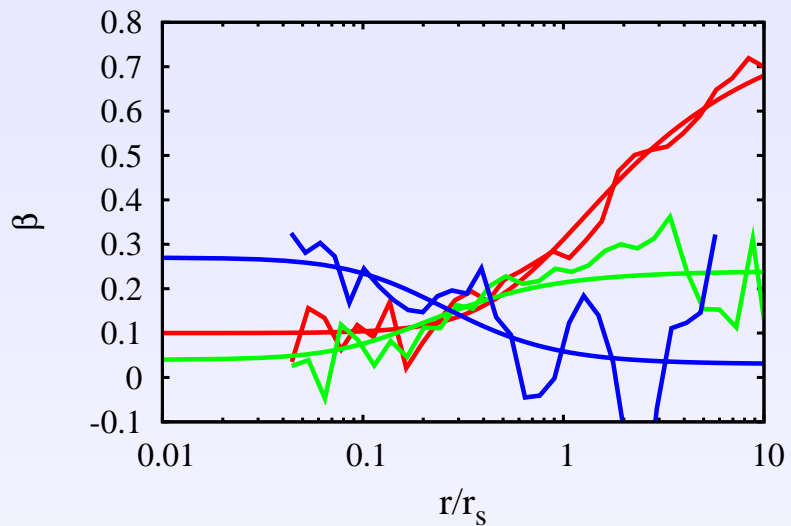
# Comparison: $f(E, L)$



# Comparison: $\sigma_r(r)$



# Comparison: $\kappa_r = \langle v_r^4 \rangle / \sigma_r^4$





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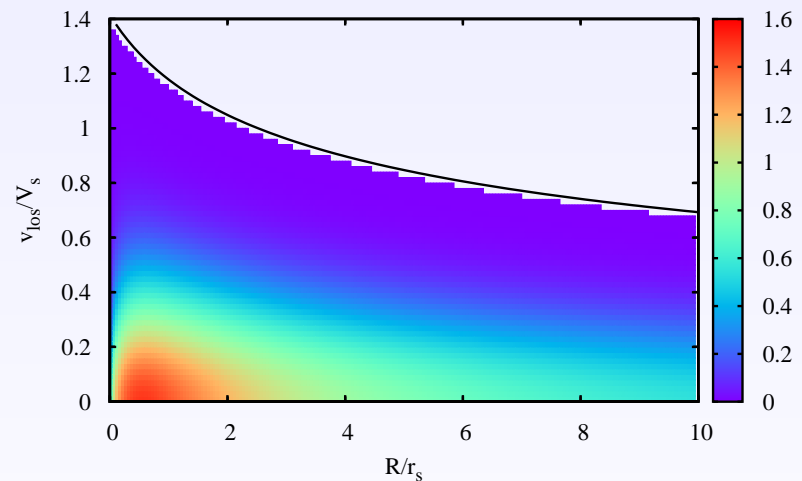
# Projected phase-space density



# Projected phase-space density

$$f_{los}(R, v_{los}) = 2\pi R \int_{-z_{max}}^{z_{max}} dz \int \int_{E>0} dv_R dv_\phi f(\mathbf{r}, \mathbf{v})$$

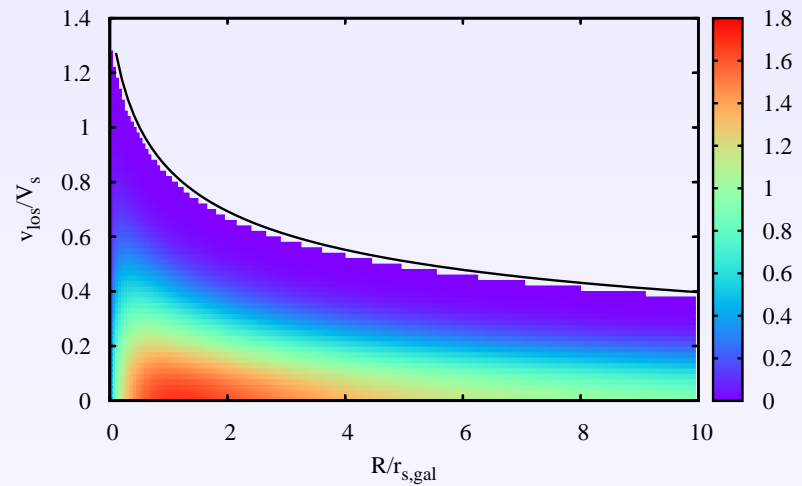
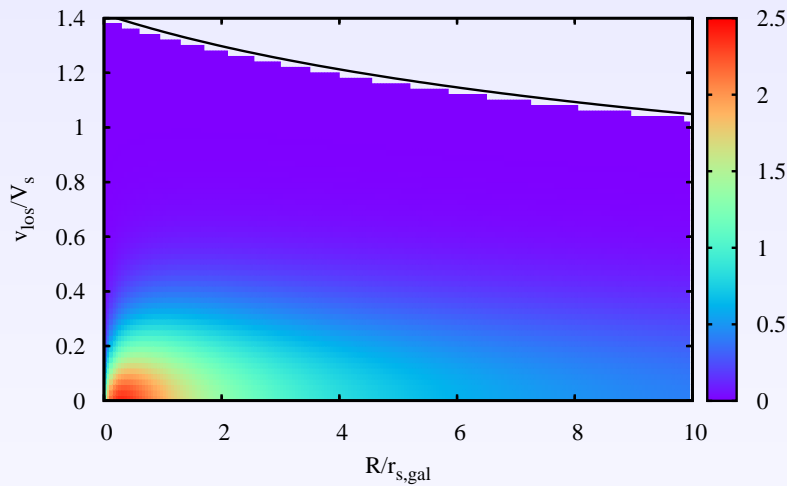
- $V_s \propto (GM_s/r_s)^{1/2}$
- $r_s$



# Different $\rho_{DM}(r)$ and $n_{gal}(r)$

■  $r_{s,DM}/r_{s,gal} = 5$

■  $r_{s,DM}/r_{s,gal} = 1/5$



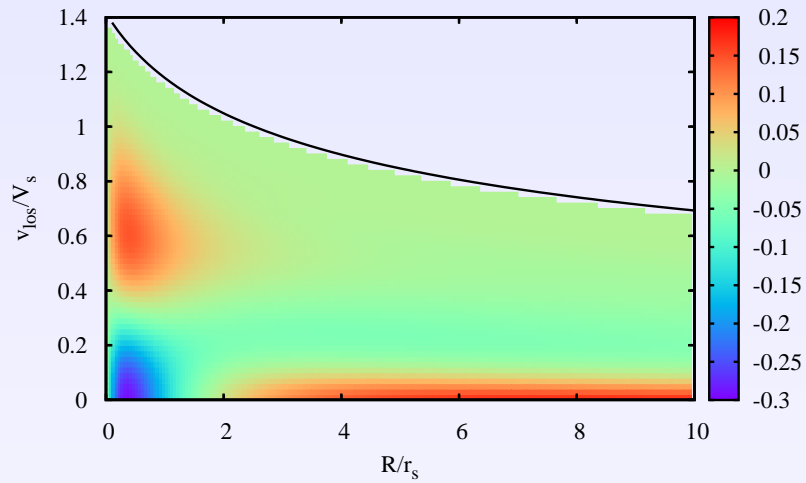
■  $V_s \propto \sqrt{GM_{s,DM}/r_{s,DM}}$

■  $r_{s,gal} \rightarrow \Sigma_{gal}(R)$

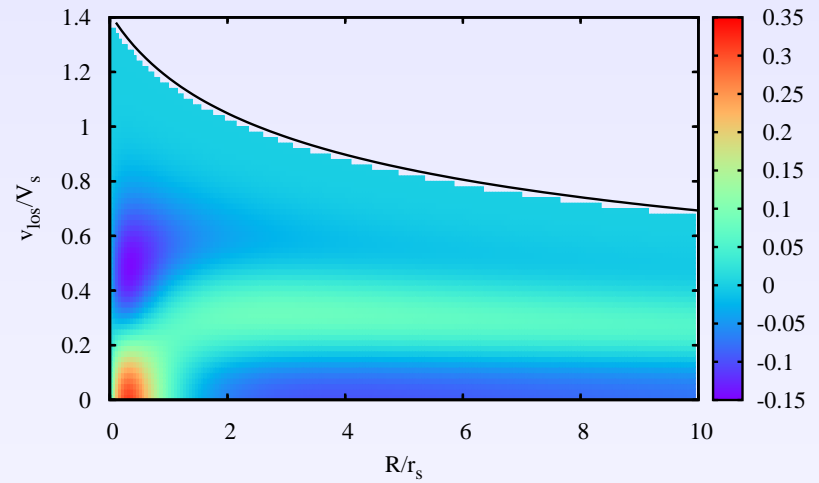
■  $r_{s,DM} \rightarrow f_{los}(R, v_{los}) = \text{const.}$



$$f_{los}(R, v_{los}) \leftrightarrow \beta_{\infty}$$



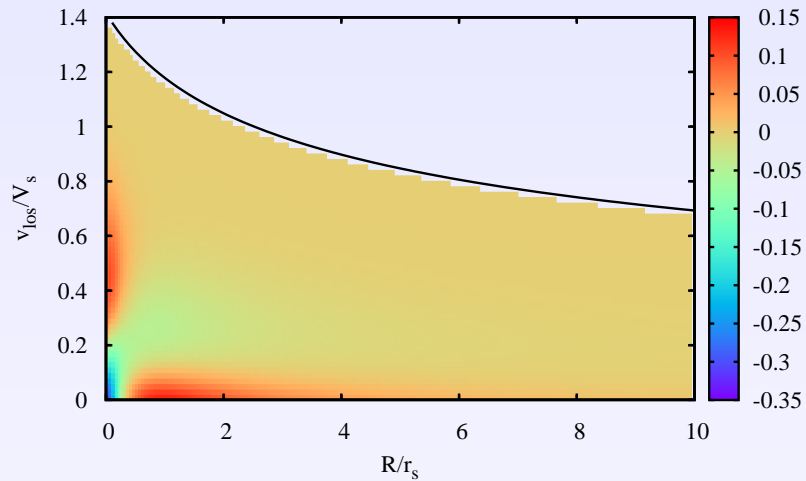
- $\beta_0 = 0$
- $\beta_{\infty} = 1$



- $\beta_0 = 0$
- $\beta_{\infty} = -2$

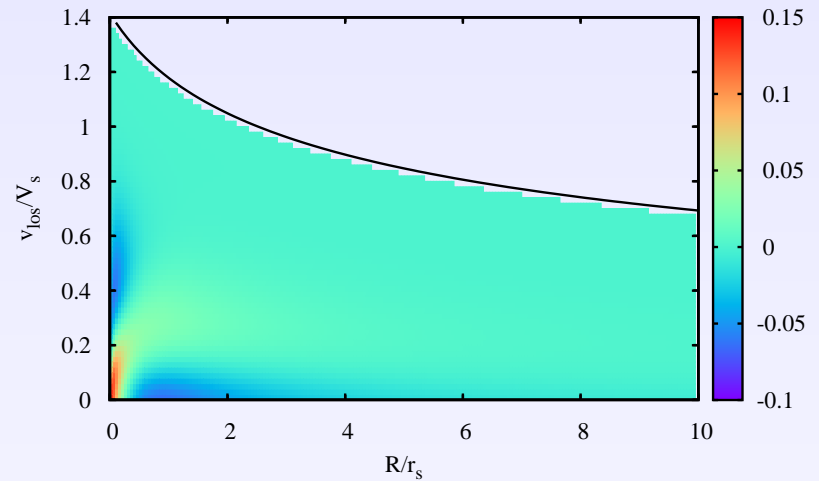


$$f_{los}(R, v_{los}) \leftrightarrow \beta_0$$



- $\beta_0 = 0.5$

- $\beta_\infty = 0$



- $\beta_0 = -0.5$

- $\beta_\infty = 0$



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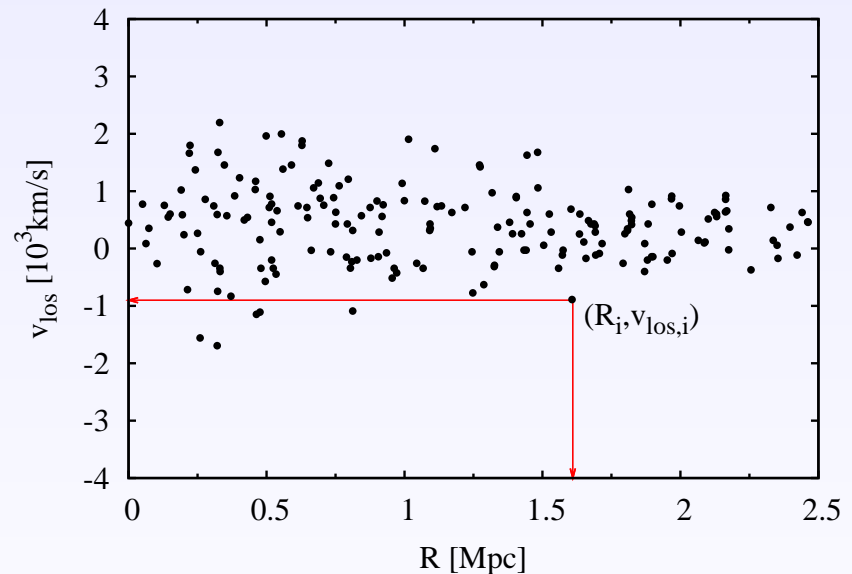
# Towards data analysis



# Bayesian data analysis

$$f_{los}(R, v_{los}) = 2\pi R \int_{-z_{max}}^{z_{max}} dz \int \int_{E>0} dv_R dv_\phi f(\mathbf{r}, \mathbf{v})$$

- $p_{post} \propto p_{prior} \mathcal{L}$



- $\mathcal{L} = \prod_{i=1}^n f_{los}(R_i, v_{los,i} | \{M_s, r_s, \dots\})$



# Parameters

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- $n_{gal}(r) \propto \rho_{DM}(r)$
- $\rho_{DM}(r) \propto 1/((r/r_s)(1 + r/r_s)^2)$

## Free parameters

- $M_s$  and  $r_s \rightarrow M_v$  and  $c$
- $\ln(1 - \beta_0)$  and  $\ln(1 - \beta_\infty)$

## Fixed parameter

- $L_0 = 0.198L_s \rightarrow r_{tran} \approx 1r_s$





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# Test on mock kinematic data



# Tests on simulated data

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## Motivation

- equilibrium
- finite size of the virial sphere
- infall zone
- substructures
- projection effects
- spherical symmetry

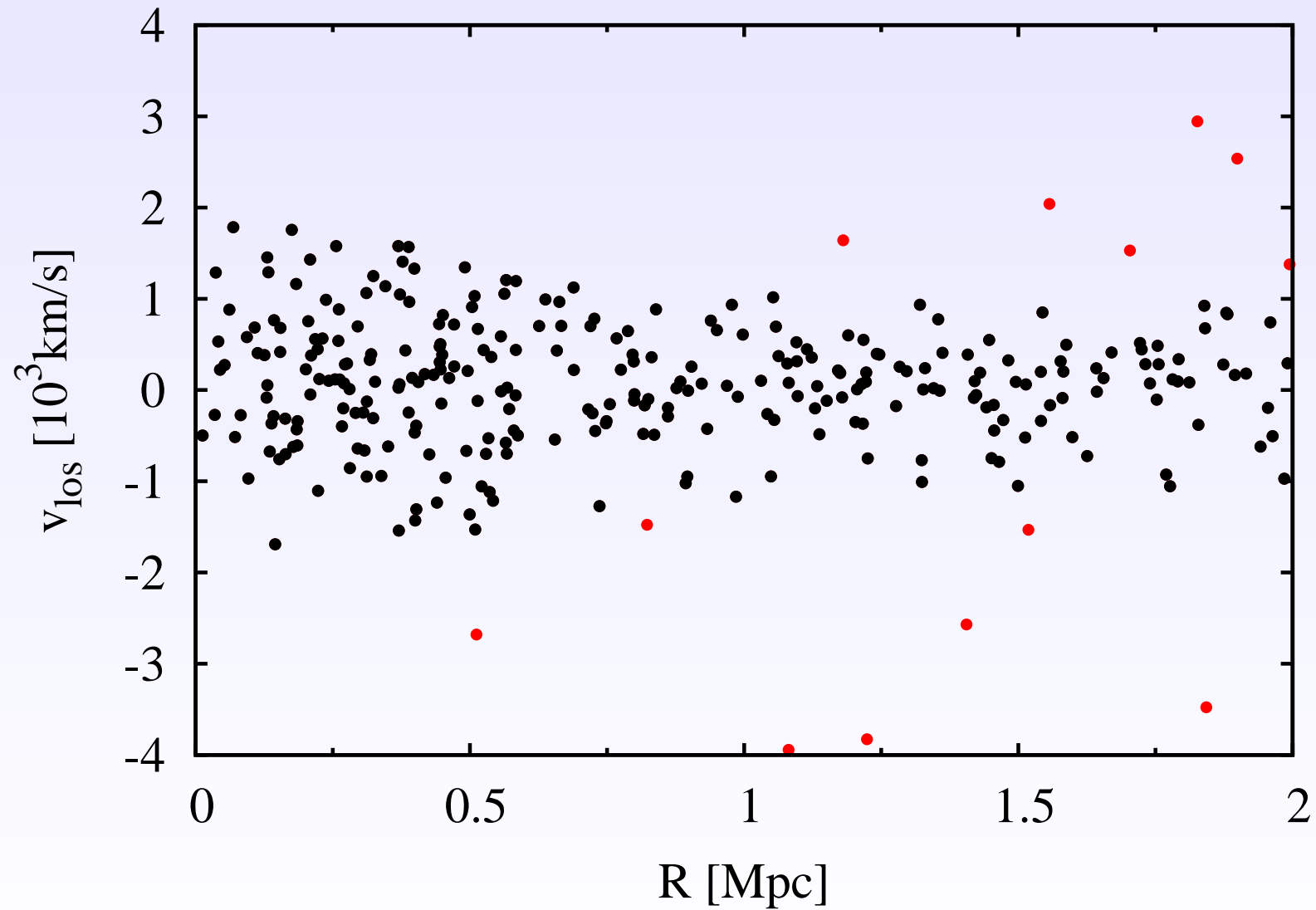
## Mock kinematic data

- relaxed haloes of the mass  $10^{14} - 10^{15} M_{\odot}$
- tracer: particles
- 300 particles per “cluster”

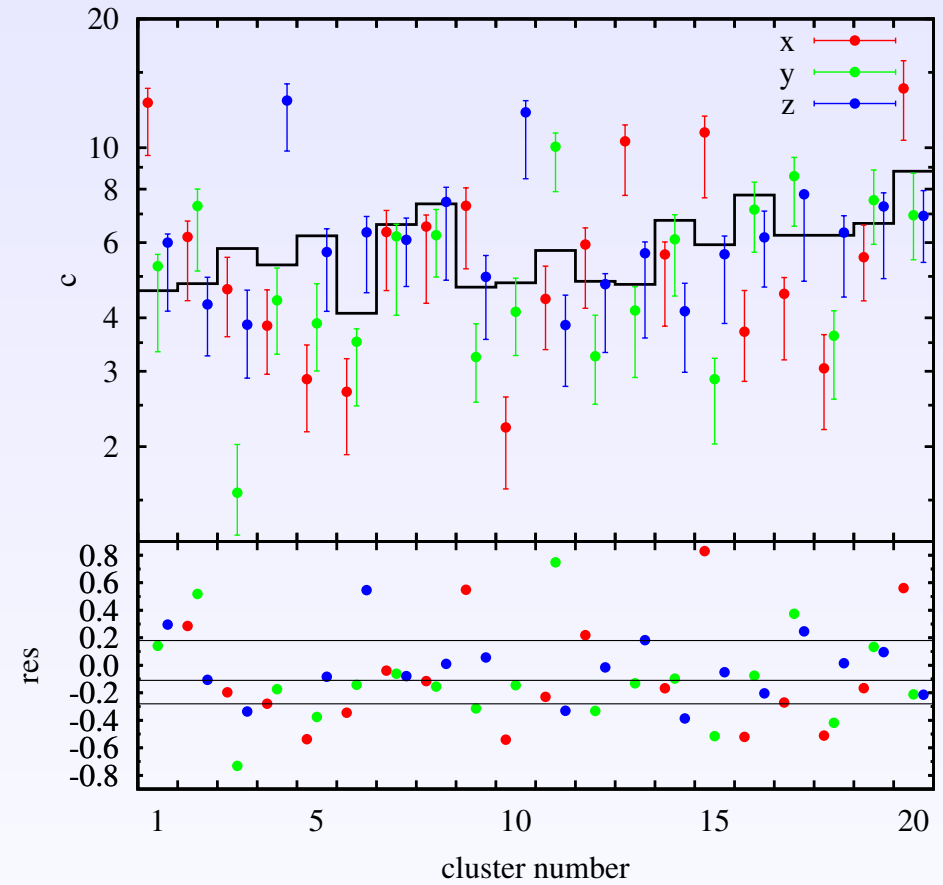
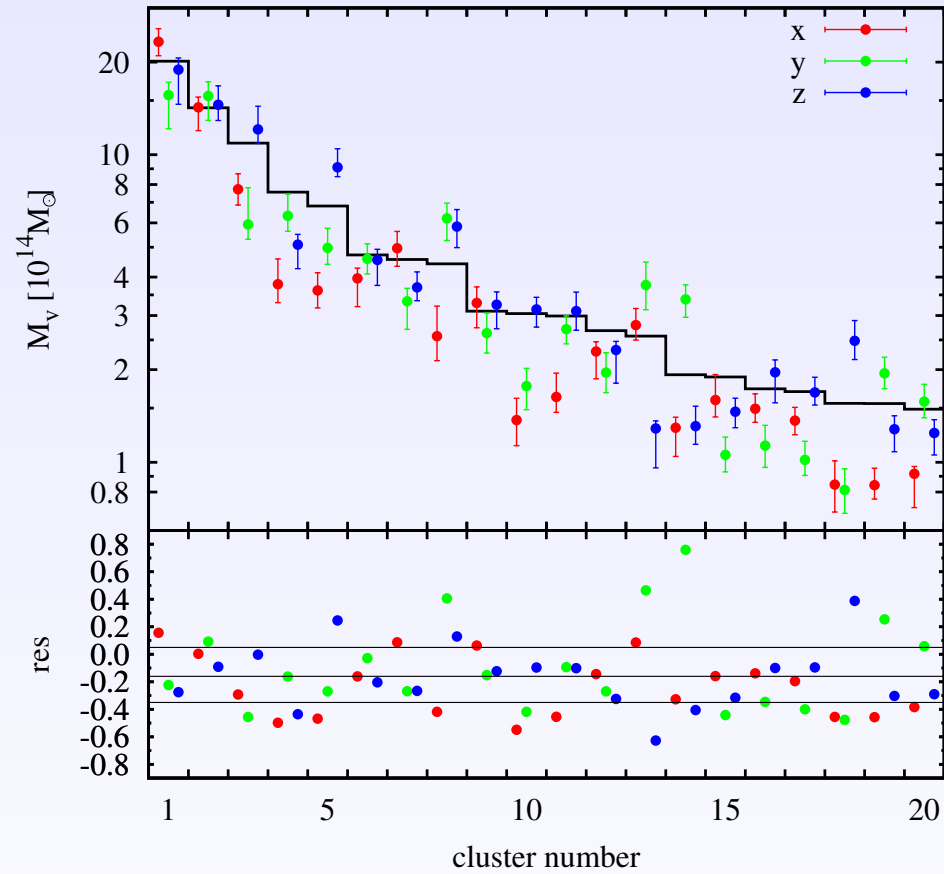


# Mock kinematic data

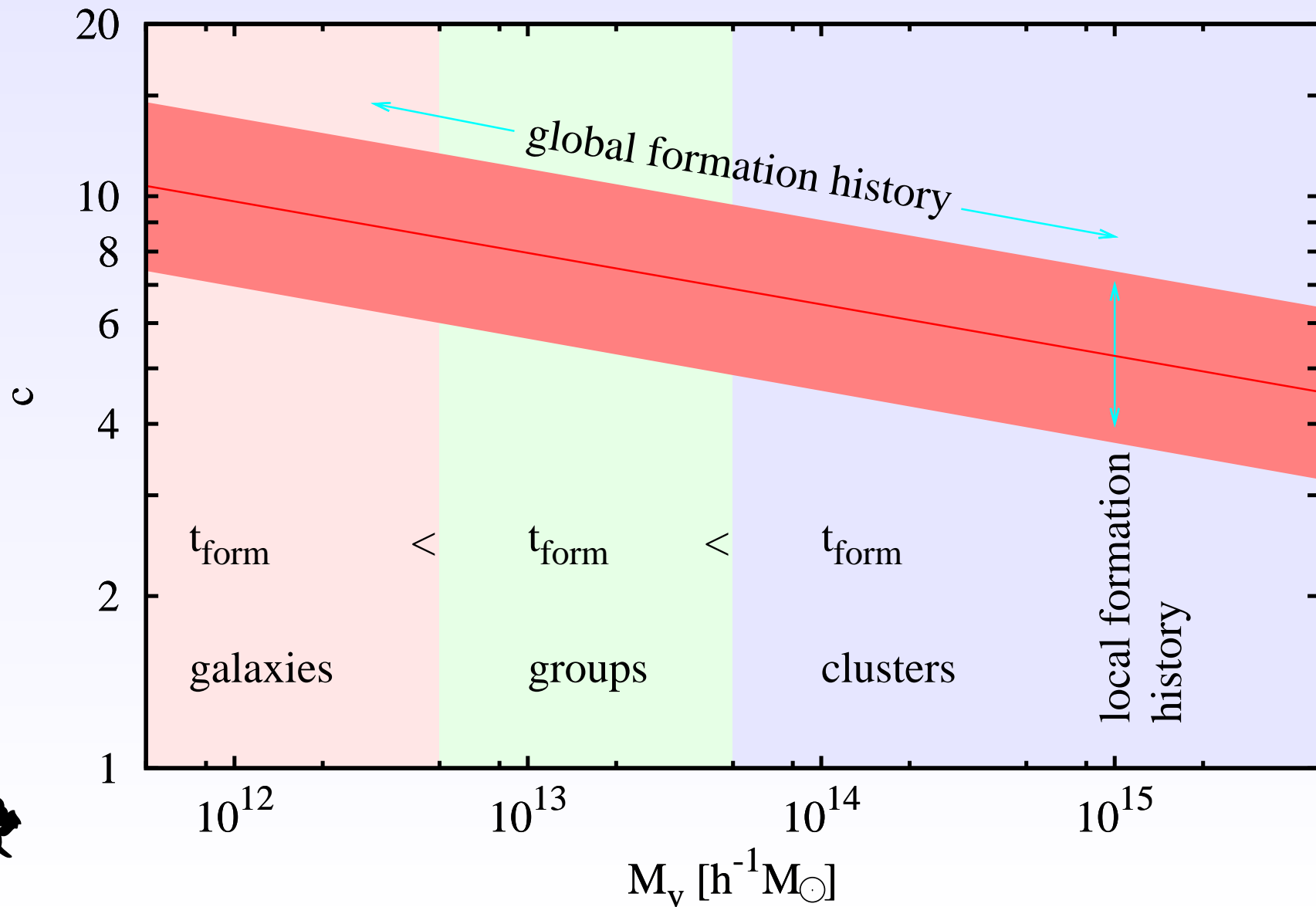
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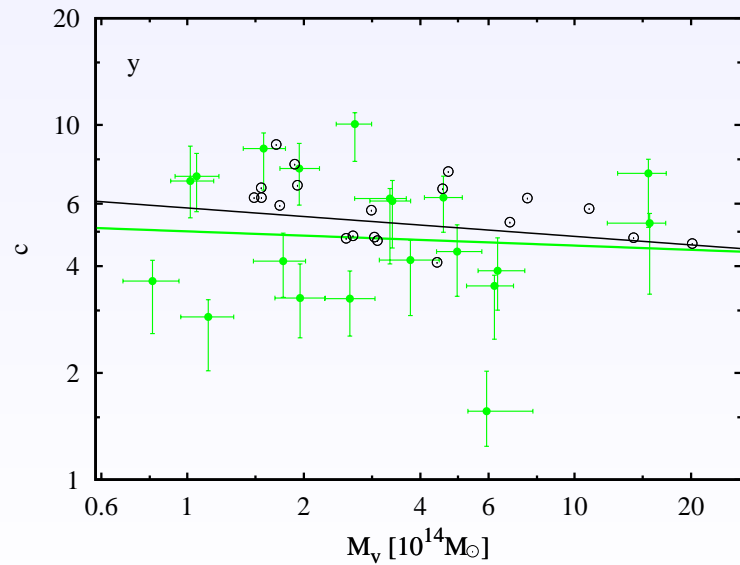
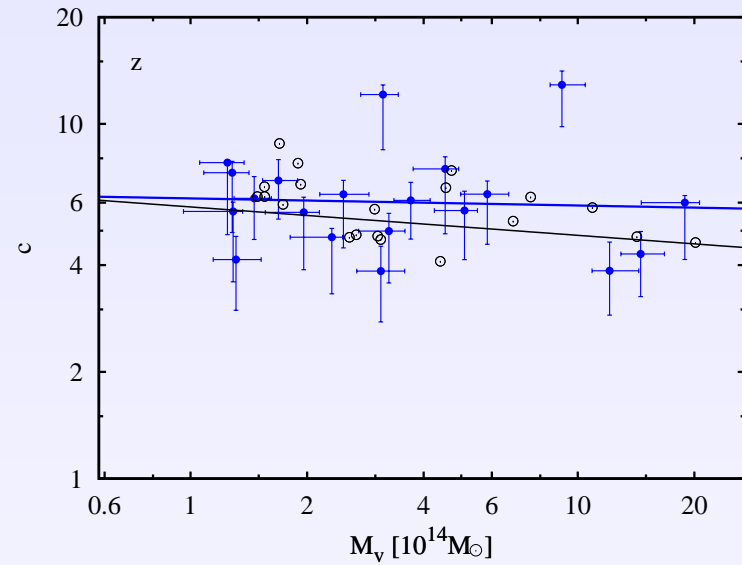
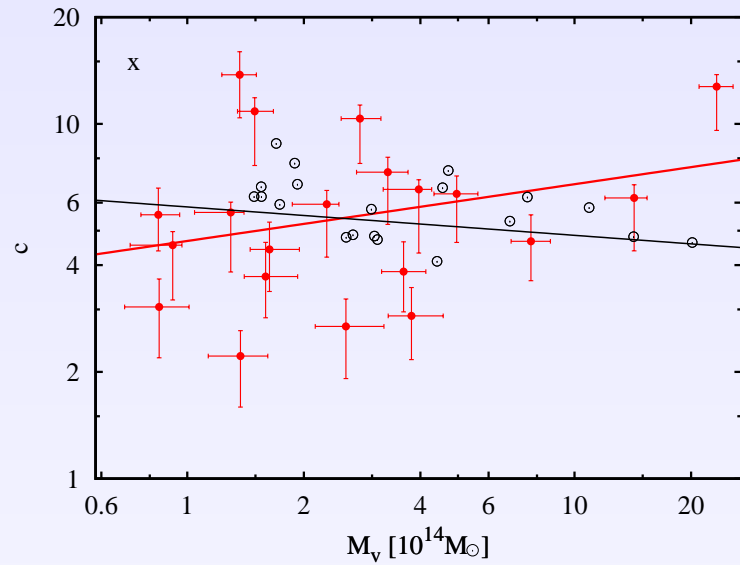
# Constraints on $M_\nu$ and $c$



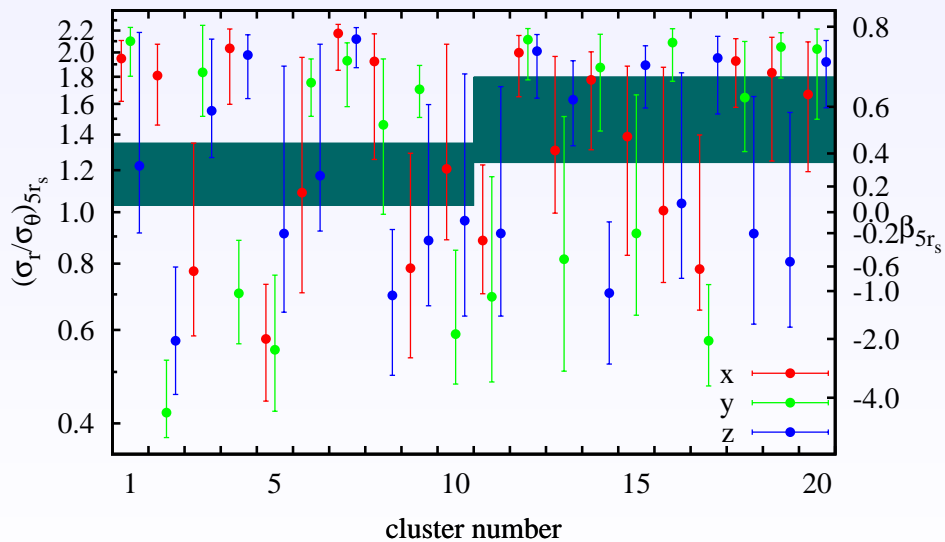
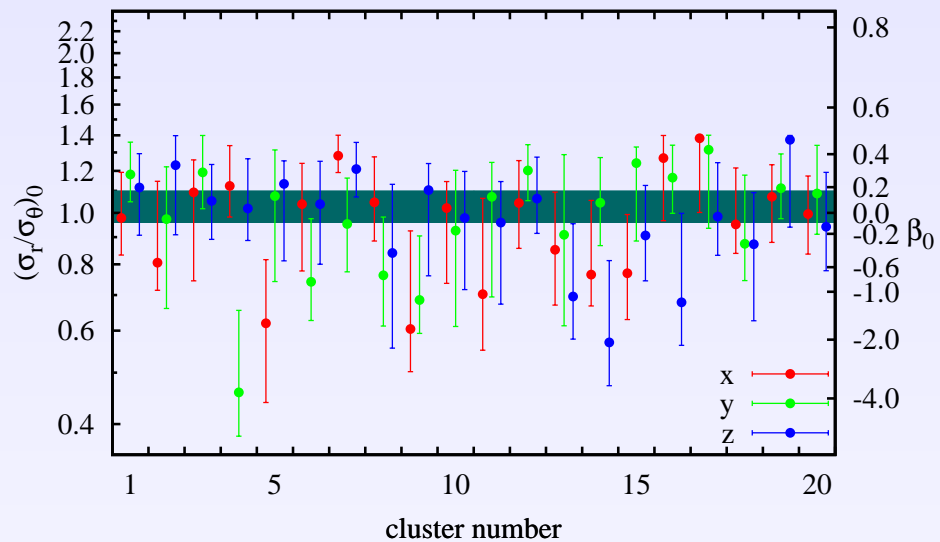
# $c - M_v$ relation



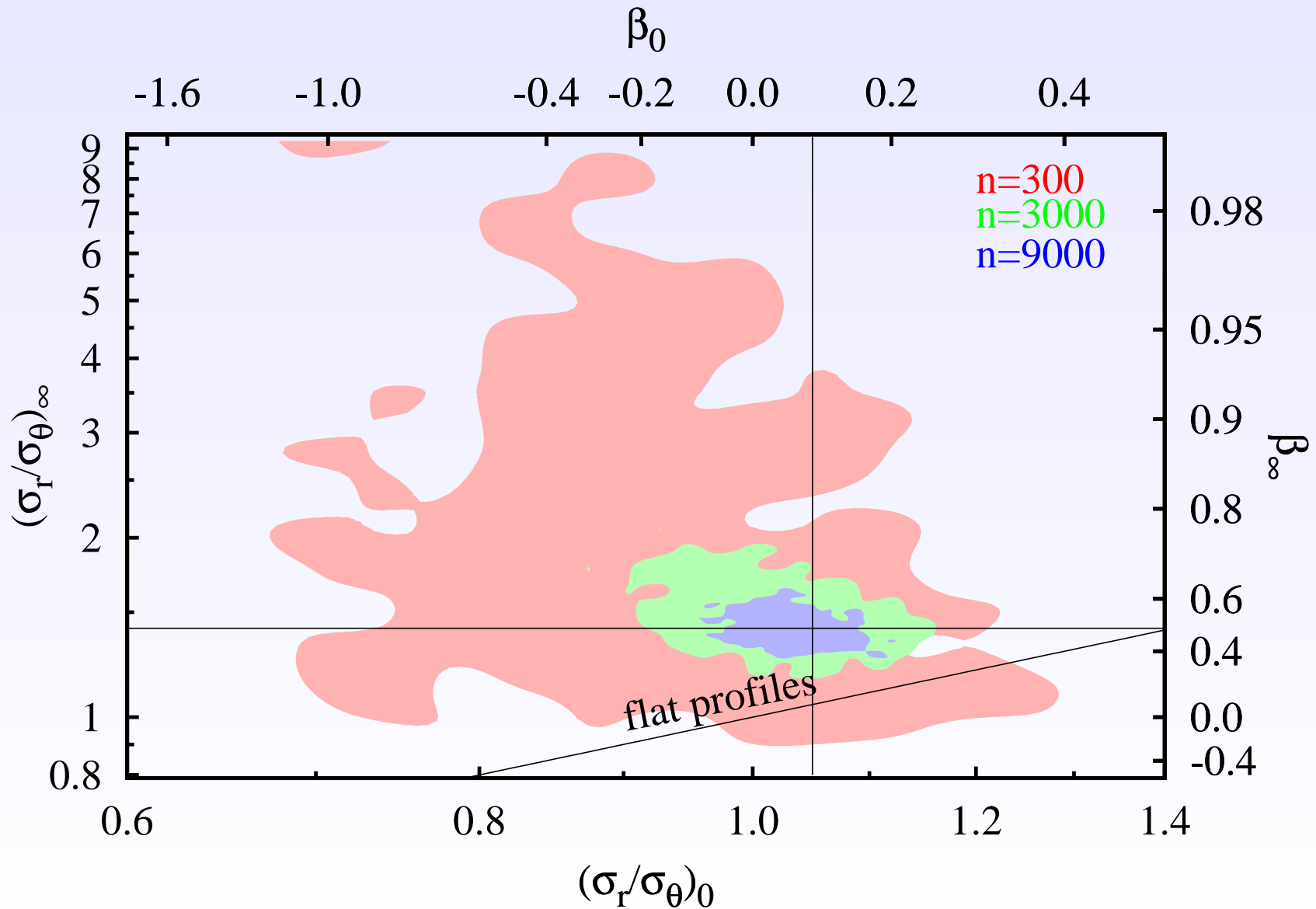
# Recovering $c - M_v$ relation



# Constraints on $\beta_0$ and $\beta_{5r_s}$



# $\beta_0$ and $\beta_\infty$ : need for more data points





# $\beta_0$ and $\beta_\infty$ : hierarchical modelling

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- cluster 1:  $p_1(r_{s,1}, M_s)$
- cluster 2:  $p_2(r_{s,2}, M_{s,2})$
- ...
- cluster  $N$ :  $p_N(r_{s,N}, M_{s,N})$

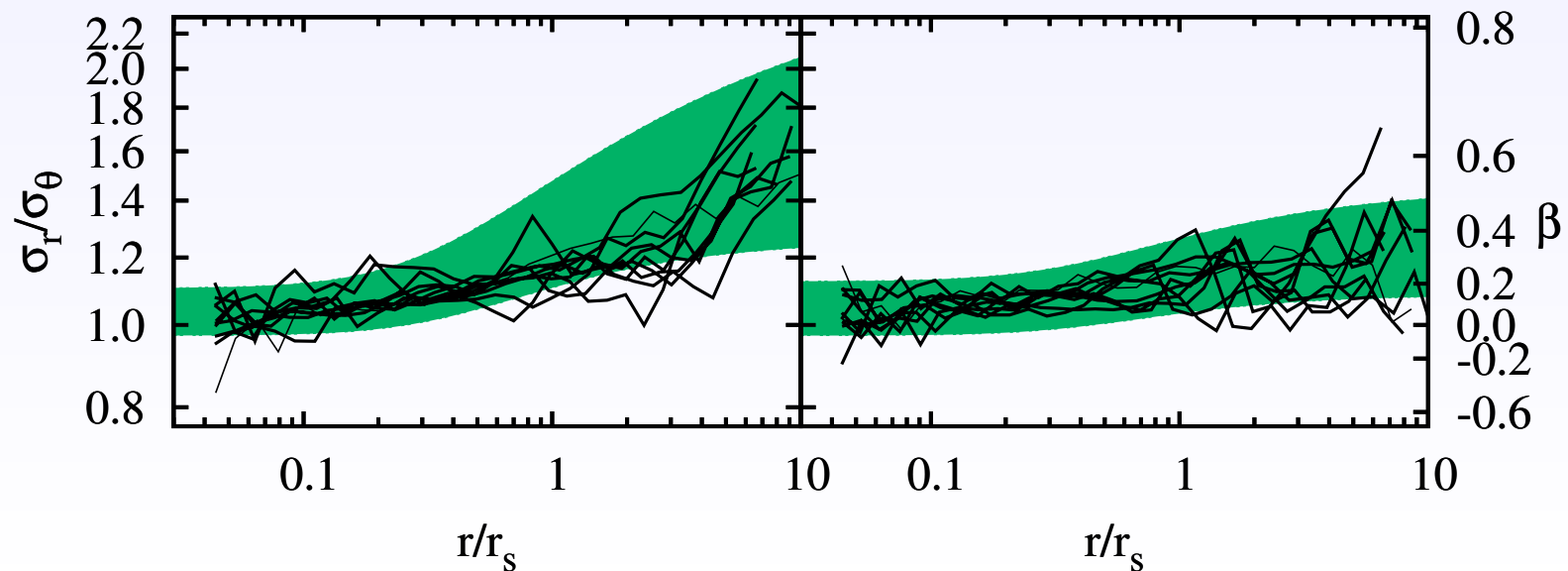
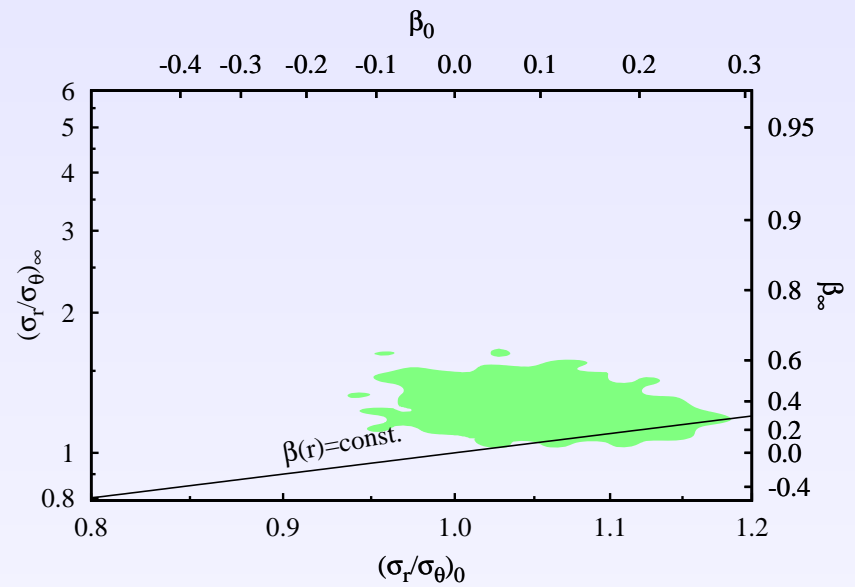
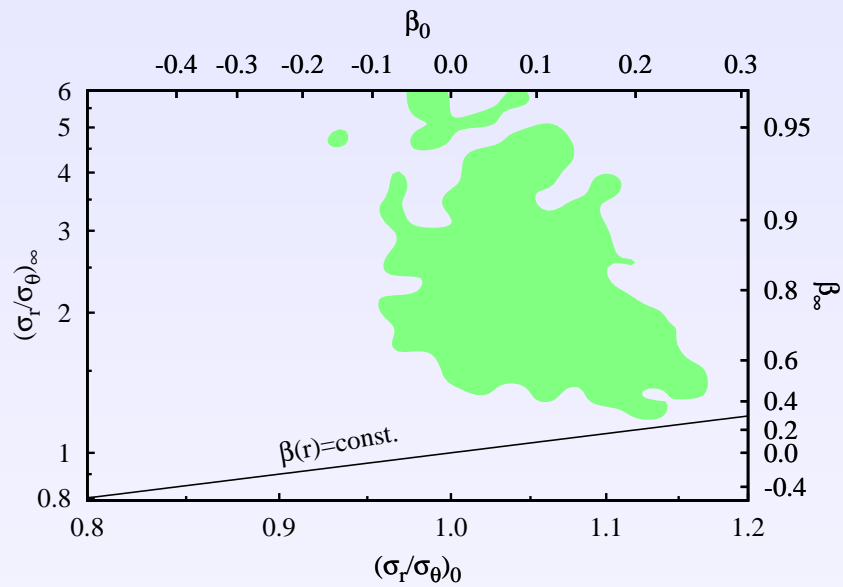
$$p_{post}(\beta_0, \beta_\infty, r_{s,1}, M_{s,1}, \dots) \propto p_{prior} \mathcal{L}$$

$$\mathcal{L} = \prod_{i=1}^N \prod_{j=1}^{n_i} f_{los}(R_{j,i}, v_{los,j,i} | \{\beta_0, \beta_\infty, r_{s,i}, M_{s,i}\})$$

$$p_{prior}(r_{s,1}, M_{s,1}, \dots) = \prod_{i=1}^N p_i(r_{s,i}, M_{s,i})$$



# Constraints on $\beta(r)$ from 10 clusters



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# Application to nearby galaxy clusters



# Nearby ( $z < 0.1$ ) galaxy clusters

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Publically available database

- NASA/IPAC Extragalactic Database
- WINGS

Cluster selection:

- symmetric X-ray image
- cool core
- regular velocity diagrams

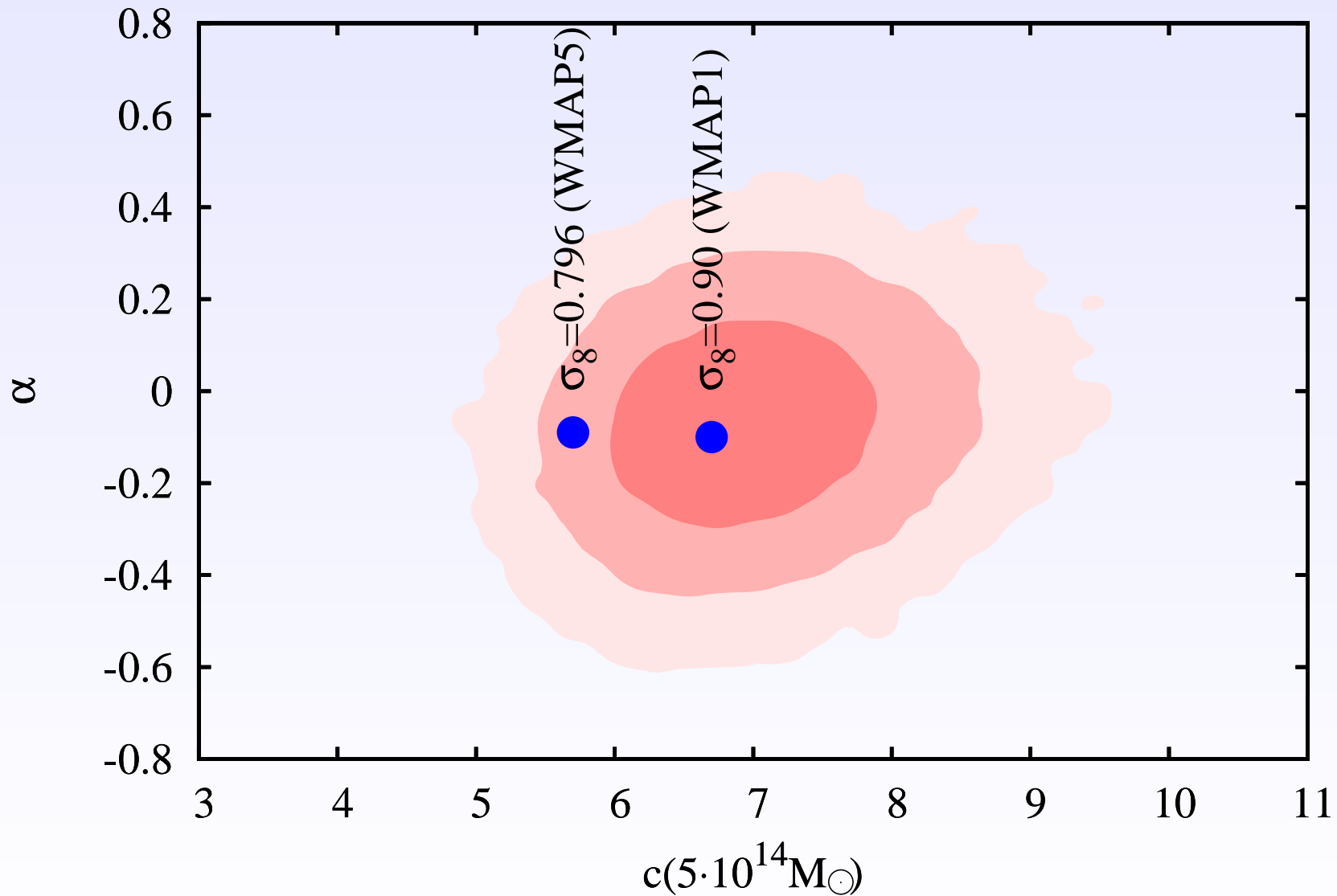
Final sample

- 41 clusters
- 100 – 200 redshifts per cluster within  $R < 2.5 Mpc$

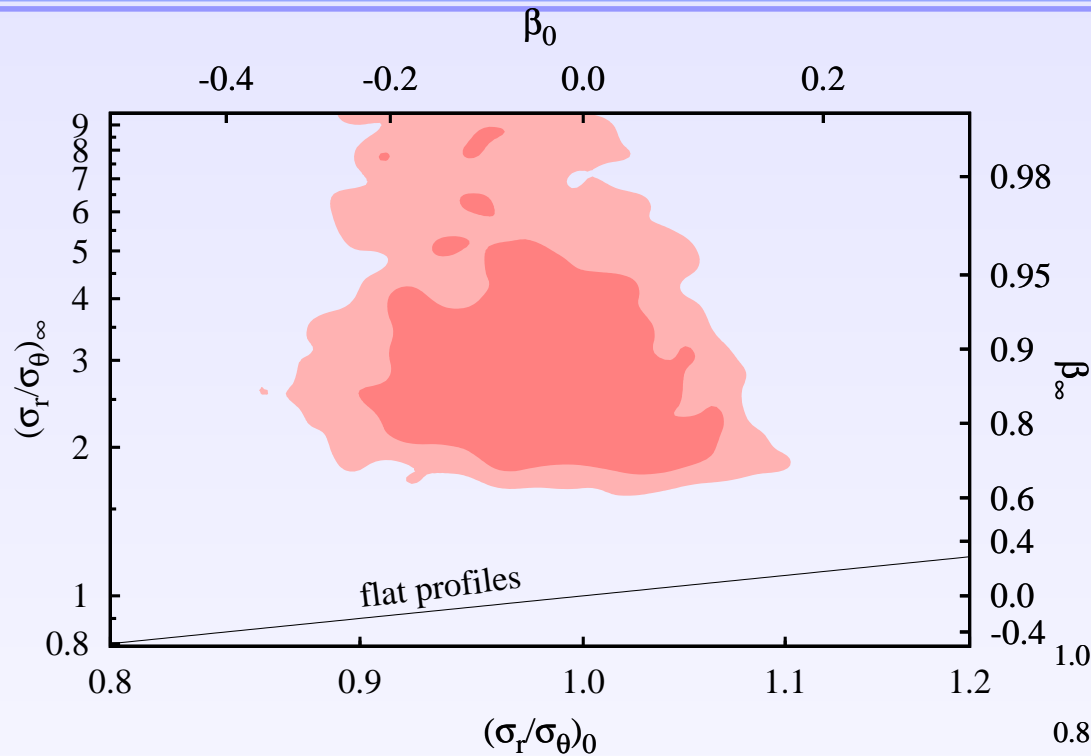




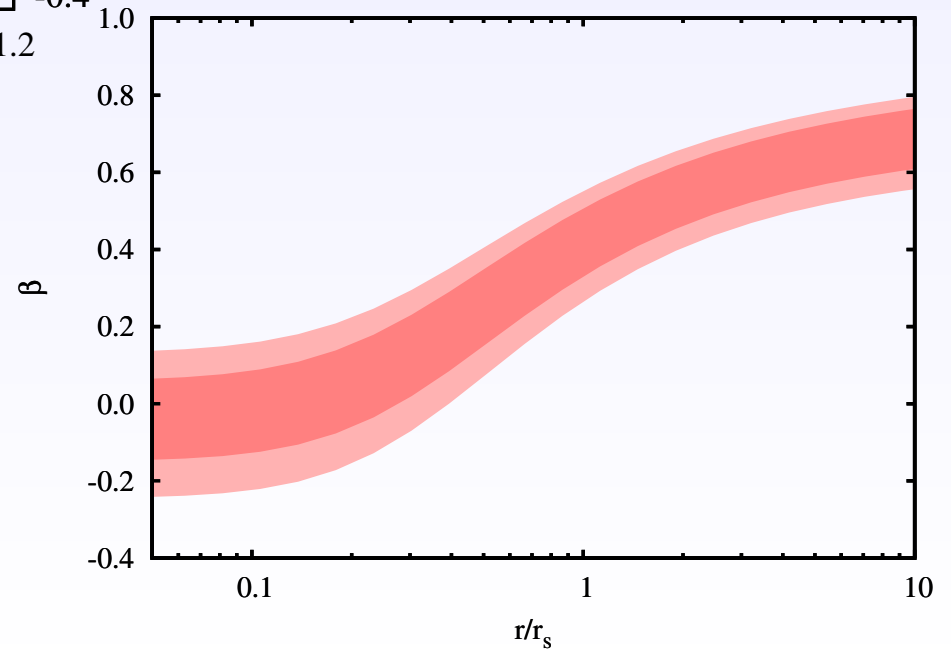
# Slope and normalization of $c \propto M^\alpha$



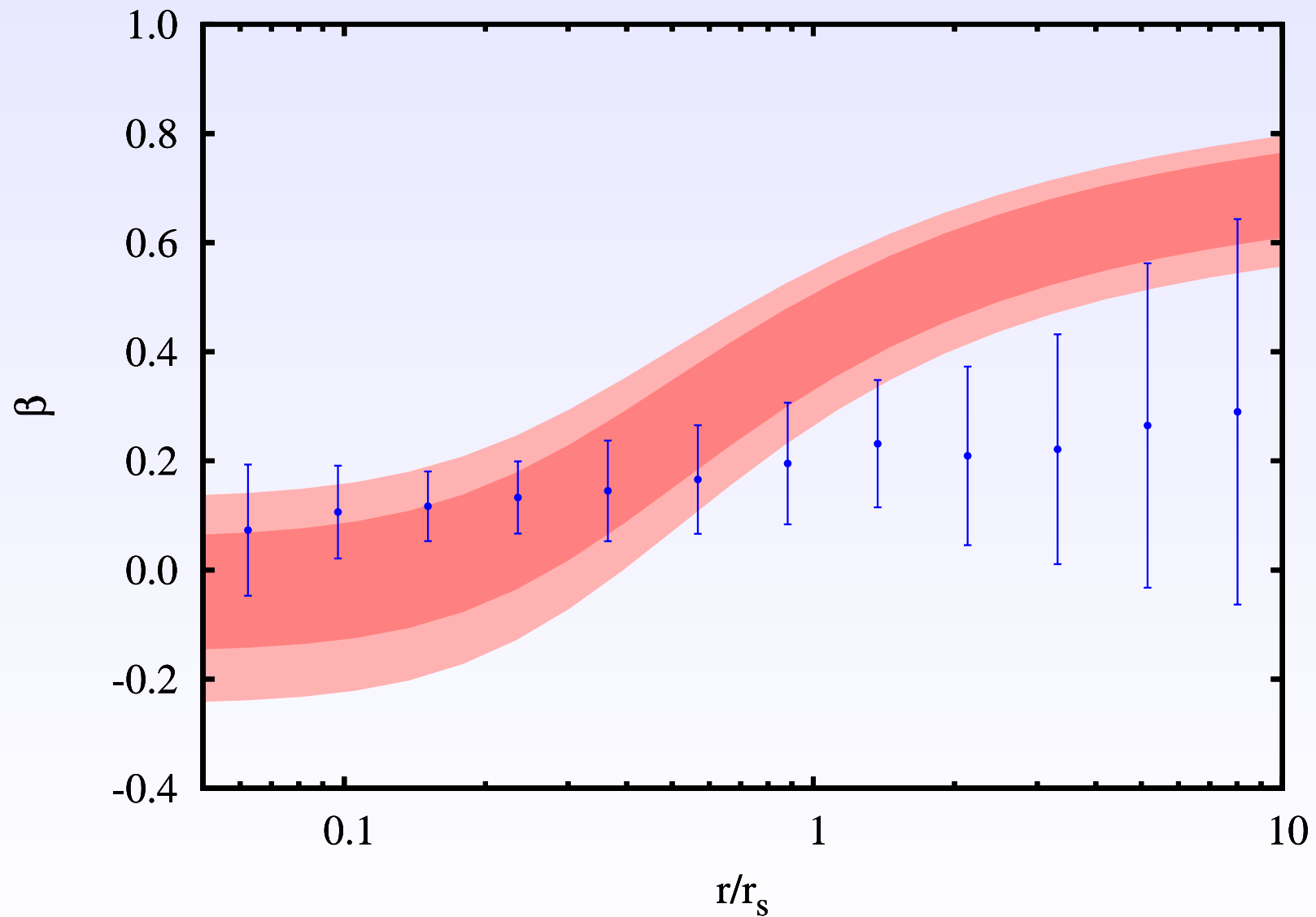
# Constraints on the anisotropy profile



- 41 clusters
- 5707 redshifts

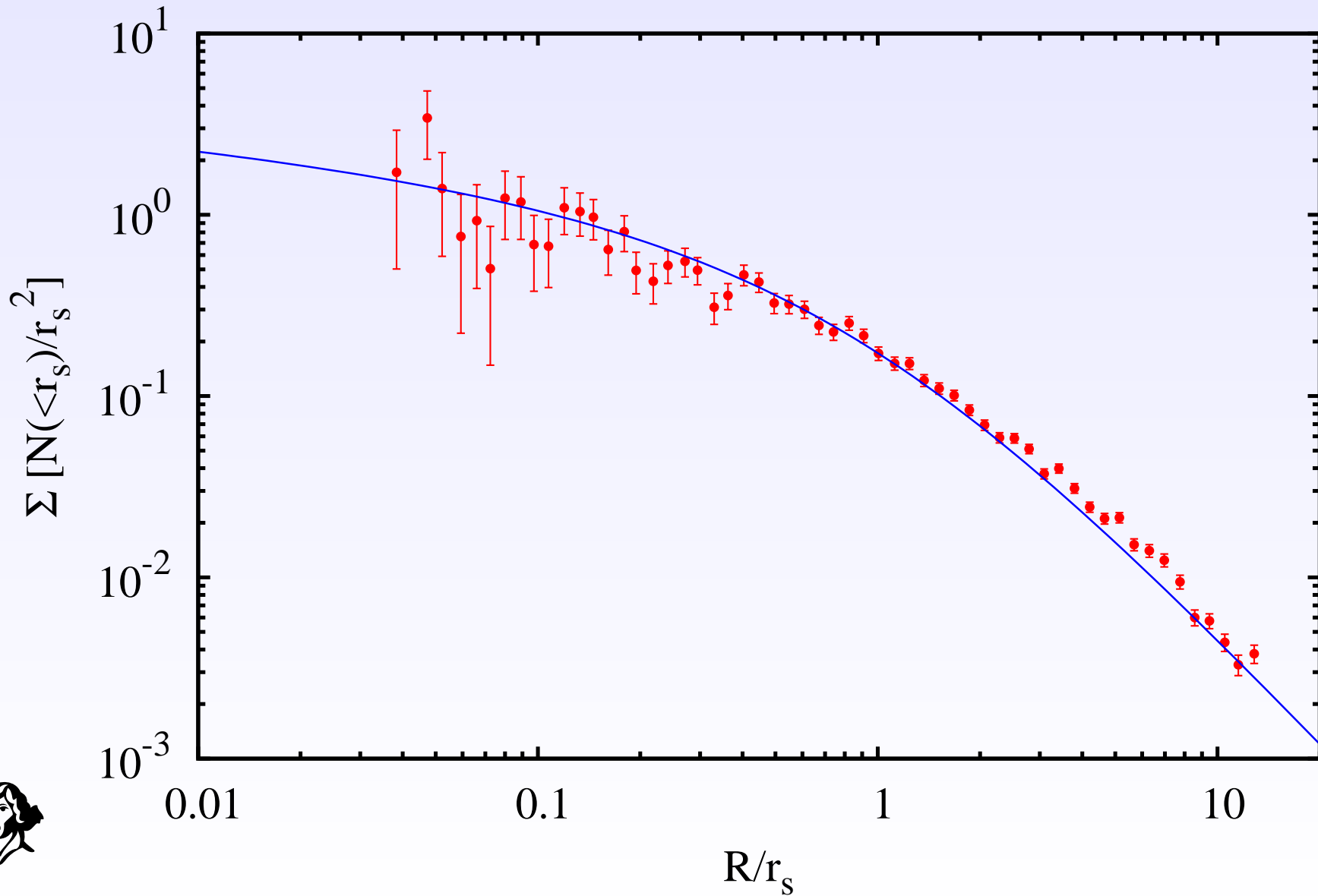


# Comparison with the simulations



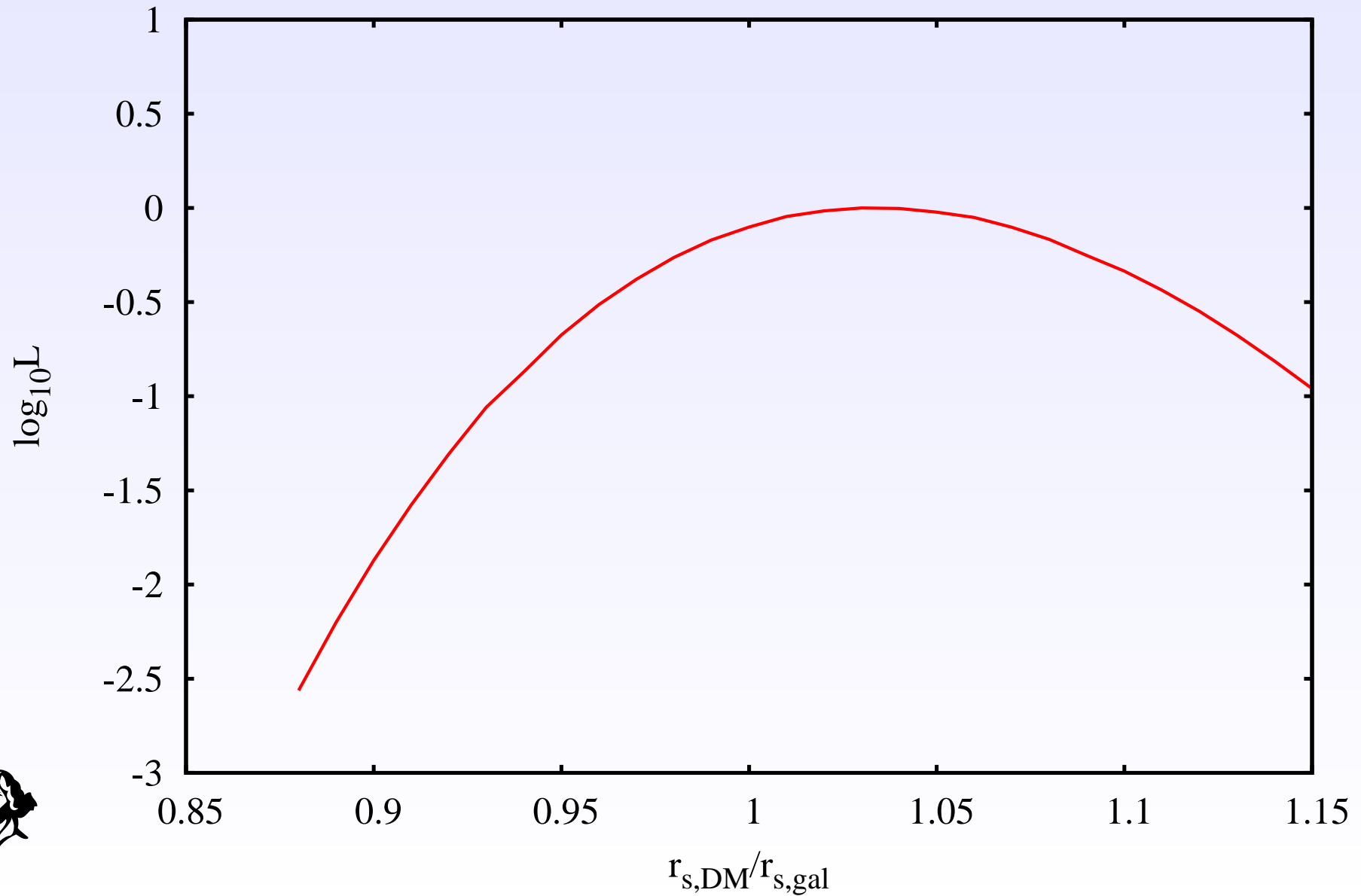


$$n_{gal}(r) \propto \rho_{DM}(r)?$$



$$n_{gal}(r) \propto \rho_{DM}(r)?$$

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# Summary

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- model of distribution function
- tests on mock data
- application to nearby clusters
  - constraints on  $c - M_v$  relation
  - $\beta_0 \approx 0, \beta(r_v) \approx 0.6$

## Perspective for future

- SDSS satellites
- ellipticals
- groups

