Massive Gravity and the Vainshtein Mechanism

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based on arXiv:0901.0393 + work in progress

OUTLINE

- DECOUPLING LIMIT
  - solutions in different regimes
  - solutions with the source
- FULL SYSTEM: STATIC SPHERICALLY SYMMETRIC SOLUTION
  - solution far from a source
  - solution close to a source and the Vainshtein proposal
  - validity of DL solutions
  - numerical results for the full system
- CONCLUSION

✦ Action for Massive Gravity

$$S = \int d^4x \left(\frac{M_P^2}{2} \sqrt{-g}\, R[g] + \mathcal{V}_{\rm int}[f, g] + \sqrt{-g}\, \mathcal{L}_m[g] \right)$$

Decoupling Limit

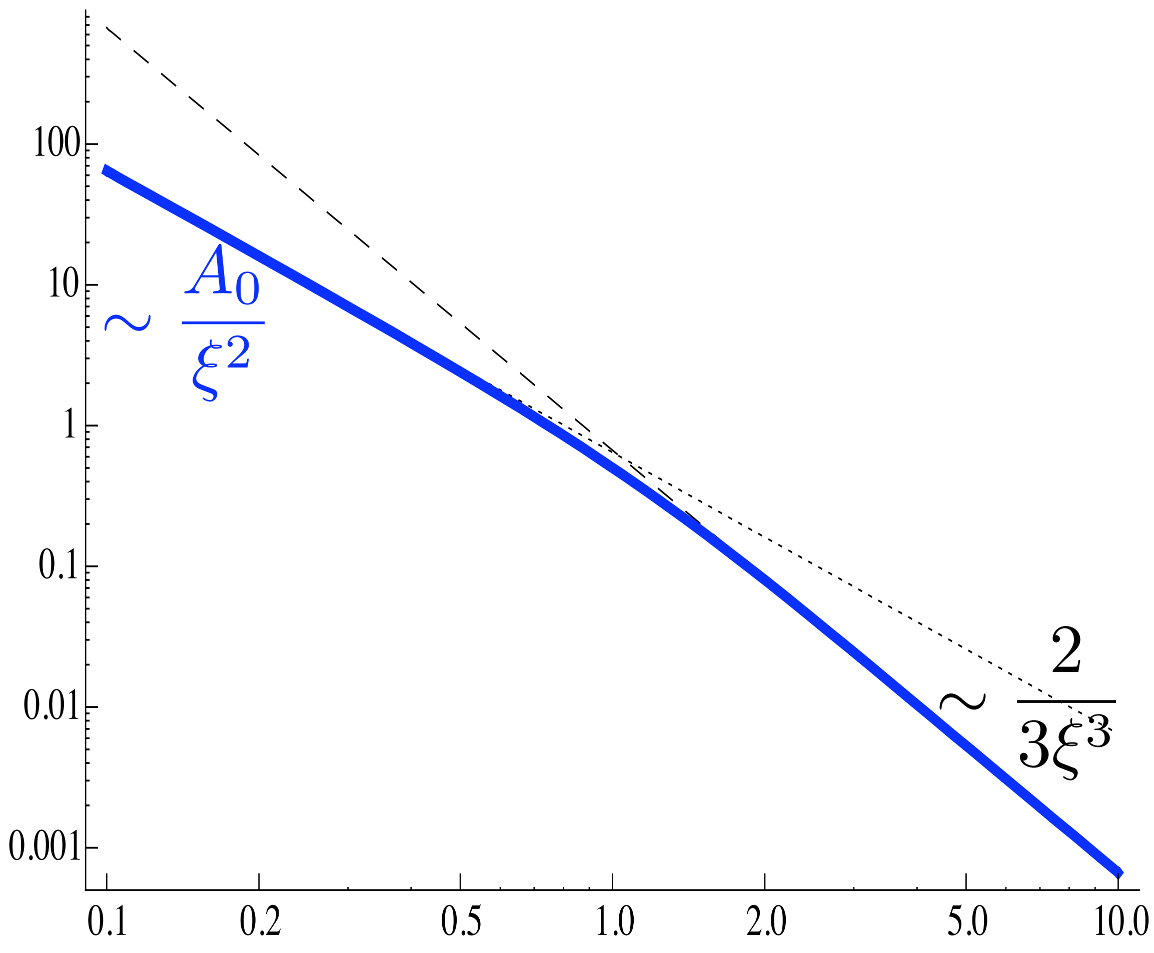
$$\boxed{\begin{array}{cc} M_P \to \infty, & m \to 0 \\ \left(M_P m^4\right)^{1/5} \sim const, & T_{\mu\nu}/M_P \sim const \end{array}}$$

$$\boxed{2\, Q(w) + \frac{3}{2} w = \frac{1}{\xi^3}}$$

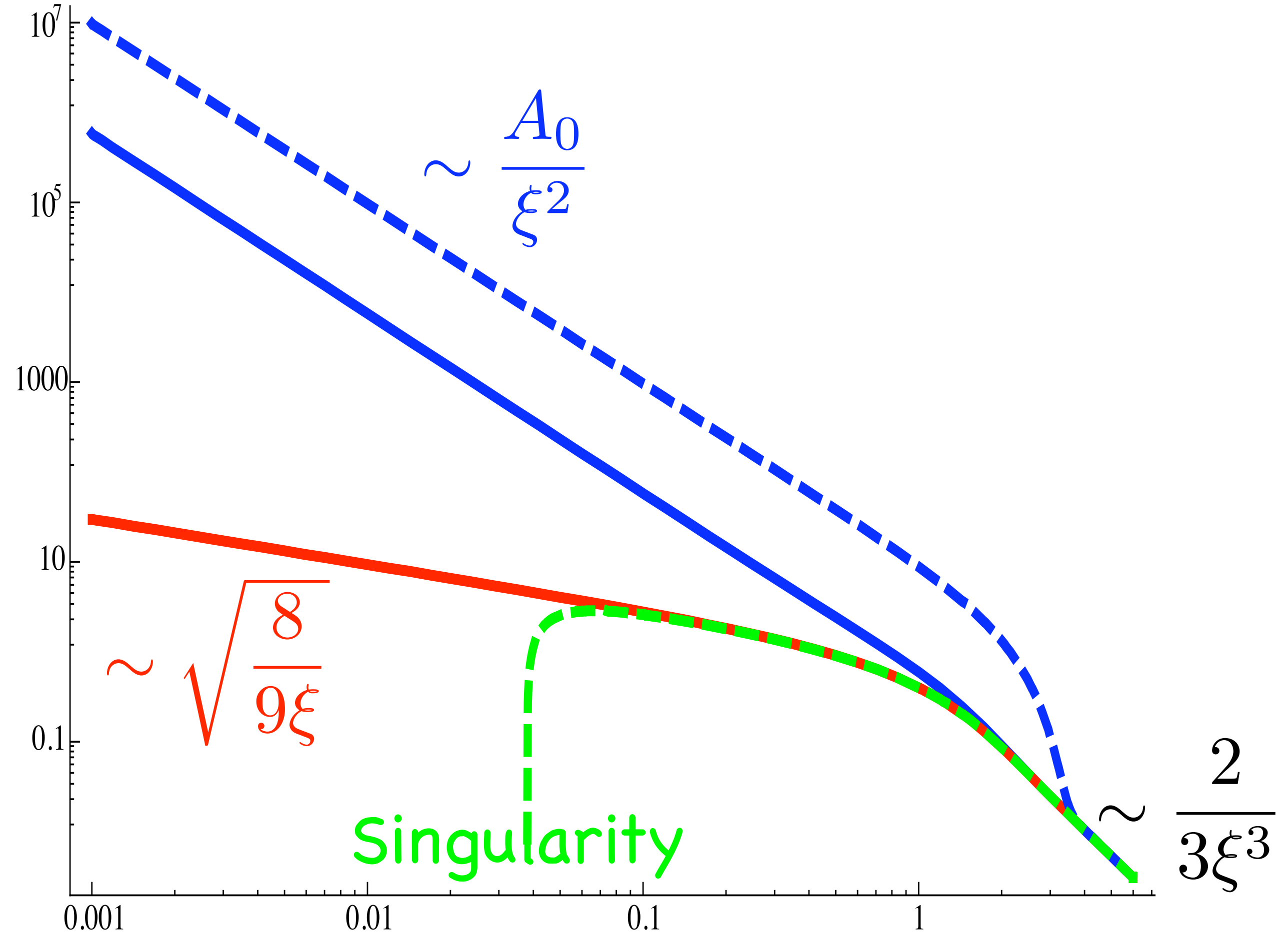
$$Q(w) = -\frac{1}{2}\left\{ 3\alpha \left(\frac{\xi}{2}\dot{w}\ddot{w} + \frac{3}{2} w\ddot{w} + 2\dot{w}^2 + \frac{6w\dot{w}}{\xi} \right) + \beta \left(\frac{3\xi}{2}\dot{w}\ddot{w} + \frac{5}{2} w\ddot{w} + 5\dot{w}^2 + \frac{10w\dot{w}}{\xi} \right) \right\}.$$

Solutions in the Decoupling Limit

BD potential



AGS potential



Unique solution for the fixed "flat" asymptotic at infinity

A family of solutions with the same "flat" asymptotic at infinity

Physical solutions?

✦ Let us include a smoothed source and ask for regularity at r=0

Q-scaling

$$w \to \frac{A}{\xi^2}$$

$$\Downarrow$$

$$\begin{cases} u \to -\frac{A}{2} \\ v \to \frac{A}{2}\ln\xi \end{cases}$$

Curvature ("conical") singularity at r=0

Vainshtein scaling

$$w \to \sqrt{\frac{8}{9\xi}}$$

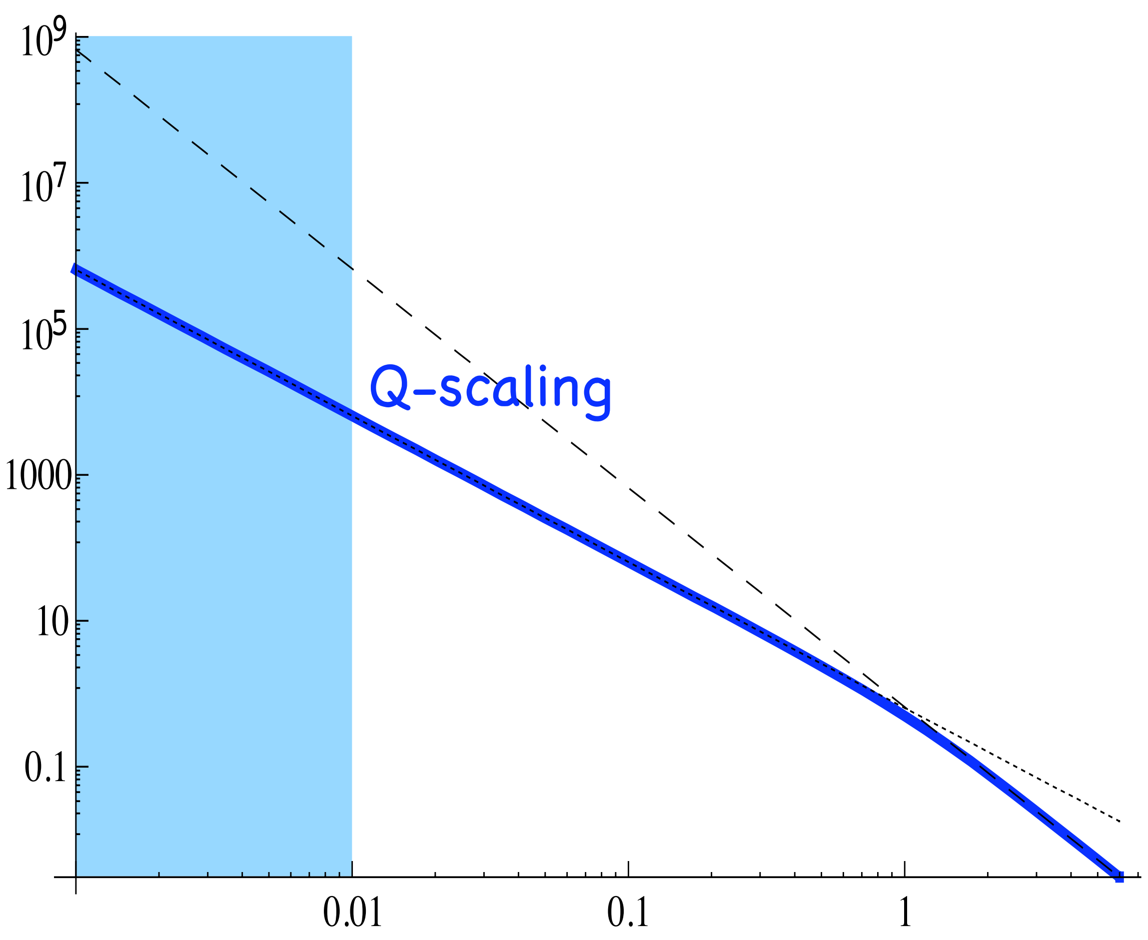
$$\Downarrow$$

$$\begin{cases} u \to 0 \\ v \to const. \end{cases}$$

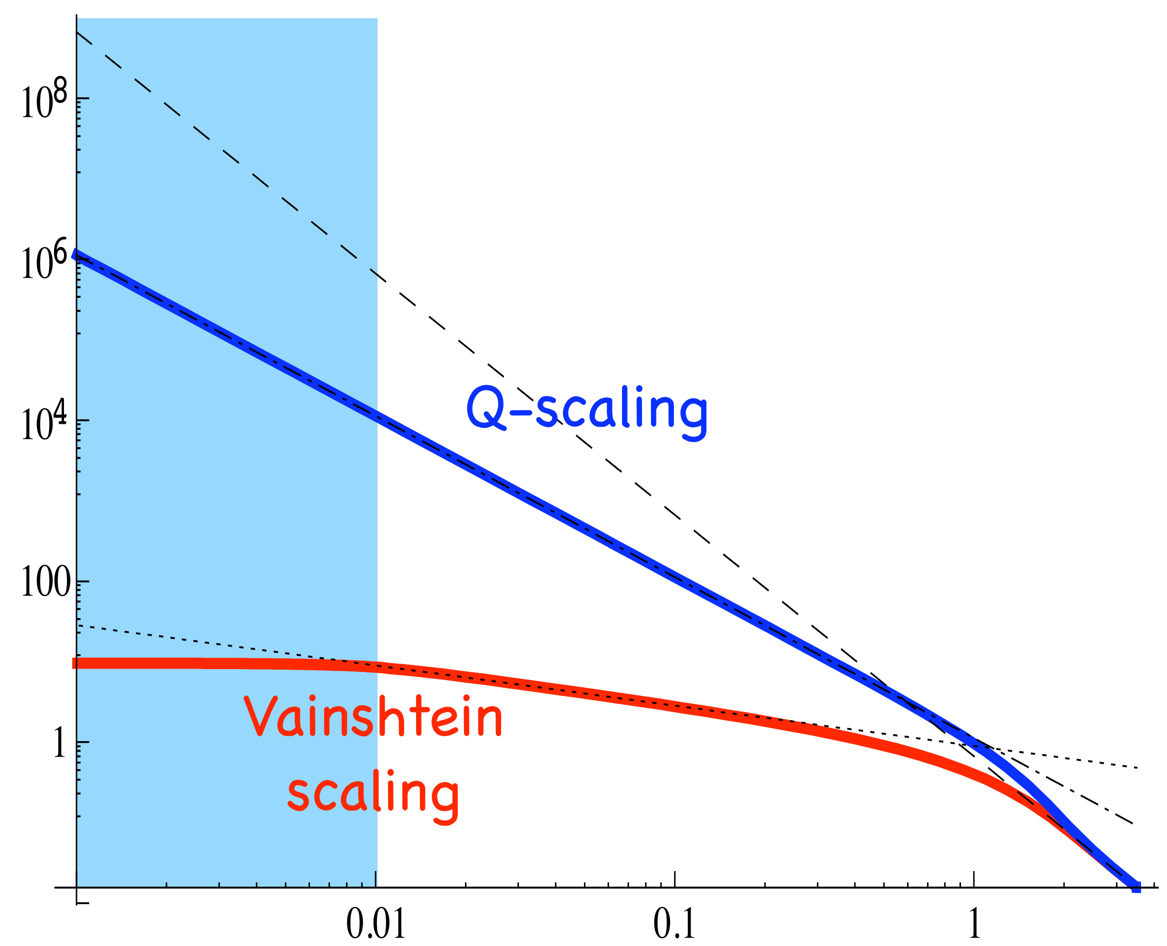
No singularity at r=0

solutions with source

BD potential



AGS potential



Full system: Metrics and Equations of Motion

- Bi-diagonal ansatz in the "Unitary" gauge:

$$\begin{aligned} g_{AB}dx^A dx^B &= -J(r)dt^2 + K(r)dr^2 + L(r)r^2 d\Omega^2 \\ f_{AB}dx^A dx^B &= -dt^2 + dr^2 + r^2 d\Omega^2 \end{aligned}$$

- "Schwarzschild" gauge:

$$\boxed{\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu &= -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2 && \text{Schwarzschild-like} \\ f_{\mu\nu}dx^\mu dx^\nu &= -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2 && \text{flat} \end{aligned}}$$

- Equations of motion:

$$\begin{aligned} e^{\nu-\lambda}\left(\frac{\lambda'}{R} + \frac{1}{R^2}(e^\lambda - 1)\right) &= 8\pi G_N\left(T^g_{tt} + \rho e^\nu\right), \\ \frac{\nu'}{R} + \frac{1}{R^2}\left(1 - e^\lambda\right) &= 8\pi G_N\left(T^g_{RR} + P e^\lambda\right), \\ \nabla^\mu T^g_{\mu R} &= 0. \end{aligned}$$

Relation between w, μ and ϕ

- μ is defined via the gauge transformation

$$
\begin{aligned}
f_{AB} dX^A dX^B &= -dt^2 + dr^2 + r^2 d\Omega^2 \\
\to f_{\mu\nu} dx^\mu dx^\nu &= -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)} dR^2 + e^{-\mu(R)} R^2 d\Omega^2
\end{aligned}
$$

- While the Stuckelberg field X is defined such that:

$$f_{\mu\nu} dx^\mu dx^\nu = \left[\partial_\mu X^A(x) \partial_\nu X^B(x) f_{AB}\left(X(x)\right)\right] dx^\mu dx^\nu$$

- This corresponds to the Stuckelberg field:

$$X^A \equiv x^A + f^{AB} \partial_B \phi = \left(t, R\, e^{-\frac{\mu(R)}{2}}, \theta, \phi\right) \Leftrightarrow \phi' \equiv \partial_R \phi = R\left(e^{-\frac{\mu(R)}{2}} - 1\right)$$

- In the Decoupling Limit:

$$\phi' = -\frac{R\mu}{2}$$

- Rescaling

$$w = (R_V m)^{-2} \mu$$

solution far from source

- Expansion in Newton's constant:

$$\lambda = \lambda_0 + \lambda_1 + \dots \quad \text{etc., with} \qquad \lambda_i, \nu_i, \mu_i \propto G_N^{i+1}$$

- Equations of motion:

$$\boxed{\begin{array}{lll} E_{tt} & \Rightarrow & \dfrac{\lambda_0'}{R} + \dfrac{\lambda_0}{R^2} = -\dfrac{m^2}{2}(\lambda_0 + 3\mu_0 + R\mu_0') \\ E_{rr} & \Rightarrow & \dfrac{\nu_0'}{R} - \dfrac{\lambda_0}{R^2} = \dfrac{m^2}{2}(\nu_0 + 2\mu_0) \\ \text{Bianchi} & \Rightarrow & \dfrac{\lambda_0}{R^2} = \dfrac{\nu_0'}{2R} \end{array}}$$

solution:

$$\boxed{\begin{aligned} \lambda_0 &= \frac{mC_1}{2}\left(1 + \frac{1}{mR}\right) e^{-mR}, \\ \nu_0 &= -\frac{C_1}{R} e^{-mR}, \\ \mu_0 &= \frac{C_1}{2R}\left(1 + \frac{1}{mR} + \frac{1}{(mR)^2}\right) e^{-mR} \end{aligned}}$$

solutions very far from source:
infinitely many solutions!

Decoupling Limit

$$\mu \sim R^{-3} + \ldots$$

$$\delta\mu \sim Ce^{-\#R}$$

Full system

$$\mu \sim e^{\#-R} + \ldots$$

$$\delta\mu \sim Ce^{-\#e^{\#\sqrt{R}}}$$

Important for numerics!!!

not very far from source $mR \ll 1$

$$\boxed{\begin{aligned} \frac{\lambda_1'}{R} + \frac{\lambda_1}{R^2} &= -\frac{m^2}{2}(3\mu_1 + R\mu_1') \\ \frac{\nu_1'}{R} - \frac{\lambda_1}{R^2} &= m^2\mu_1 \\ \frac{\lambda_1}{R^2} &= \frac{\nu_1'}{2R} + Q(\mu_0), \end{aligned}}$$

$$\begin{aligned} Q(\mu) &= -\frac{1}{2R}\left\{3\alpha\left(6\mu\mu' + 2R\mu'^2 + \frac{3}{2}R\mu\mu'' + \frac{1}{2}R^2\mu'\mu''\right)\right. \\ &\left. +\beta\left(10\mu\mu' + 5R\mu'^2 + \frac{5}{2}R\mu\mu'' + \frac{3}{2}R^2\mu'\mu''\right)\right\} \end{aligned}$$

solution:

$$\boxed{\begin{aligned} \nu &= -\frac{2}{3}\frac{R_S}{R} + \frac{R_S^2}{R^2}\frac{n_1}{(mR)^4} + \mathcal{O}(R_S^3) \\ \lambda &= \frac{1}{3}\frac{R_S}{R} + \frac{R_S^2}{R^2}\frac{l_1}{(mR)^4} + \mathcal{O}(R_S^3) \\ \mu &= \frac{1}{3(mR)^2}\frac{R_S}{R} + \frac{R_S^2}{R^2}\frac{m_1}{(mR)^6} + \mathcal{O}(R_S^3) \end{aligned}}$$

relevant at

$$\boxed{R_V = \left(\frac{R_S}{m^4}\right)^{1/5}}$$

Vainshtein scaling close to source

- Expansion in m: $$f(R) = \sum_{n=0}^{\infty} m^{2n} f_n(R)$$
- 0th order:

$$\lambda_0 = -\nu_0 = -\ln\left(1 - \frac{R_S}{R}\right)$$
$$\mu_0 = m_0 \sqrt{R_S/R} \gg \lambda_0, \nu_0$$

assume $R \gg R_S$

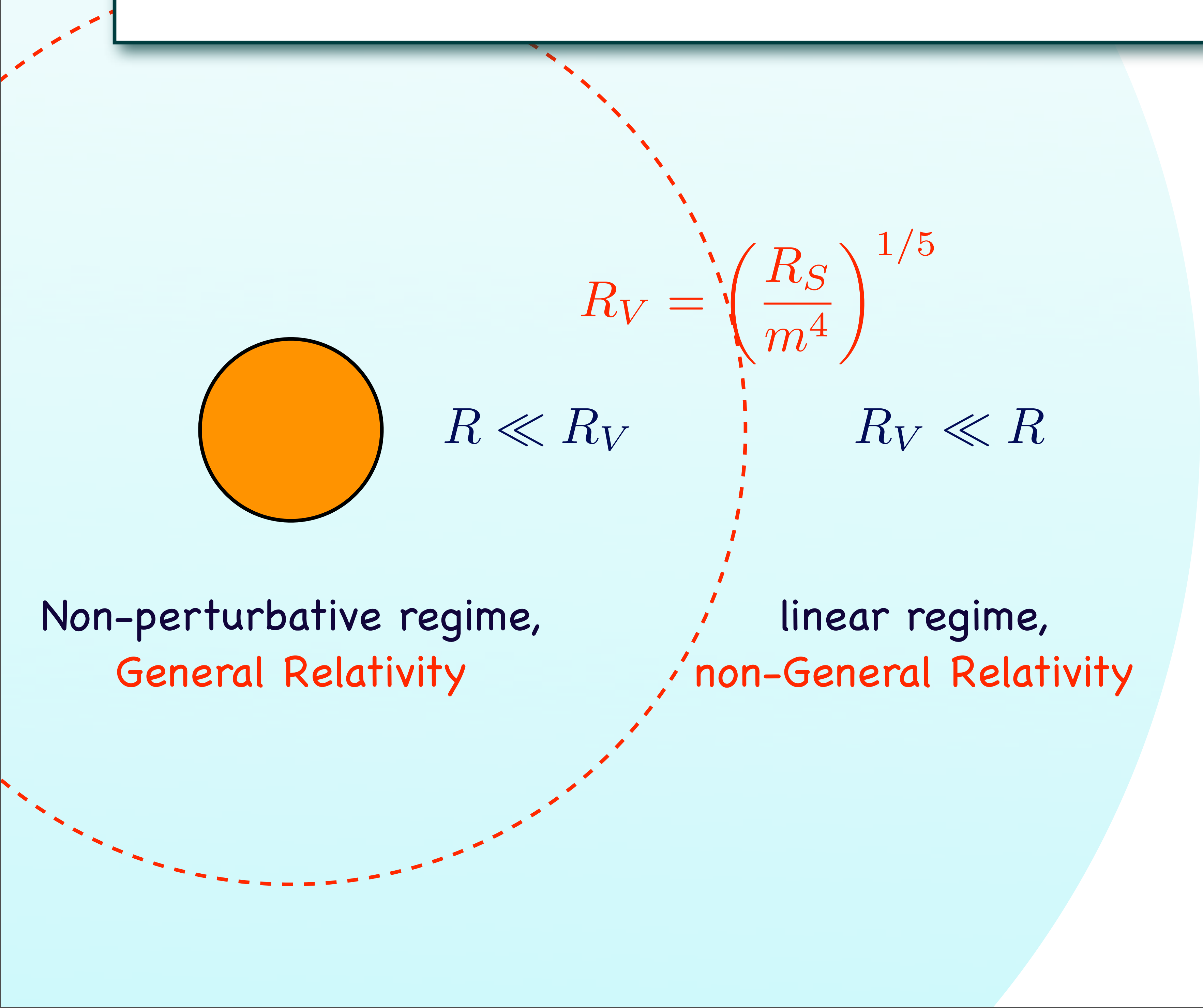
1st order solution

$$\boxed{\begin{aligned} \nu &= -\frac{R_S}{R} + n_1 (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}\left(m^4\right) \\ \lambda &= \frac{R_S}{R} + l_1 (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}\left(m^4\right) \\ \mu &= m_0 \sqrt{\frac{R_S}{R}} + m_1 (mR)^2 + \mathcal{O}\left(m^4\right) \end{aligned}}$$

$$\boxed{R_V = \left(\frac{R_S}{m^4}\right)^{1/5}}$$

General picture

$$S \propto \int d^4x \sqrt{-g}\, R + \mathcal{V}_{\rm int}[\tilde{g}, g]$$



Validity of DL solutions

All other terms <=> cubic interaction (kept in DL)

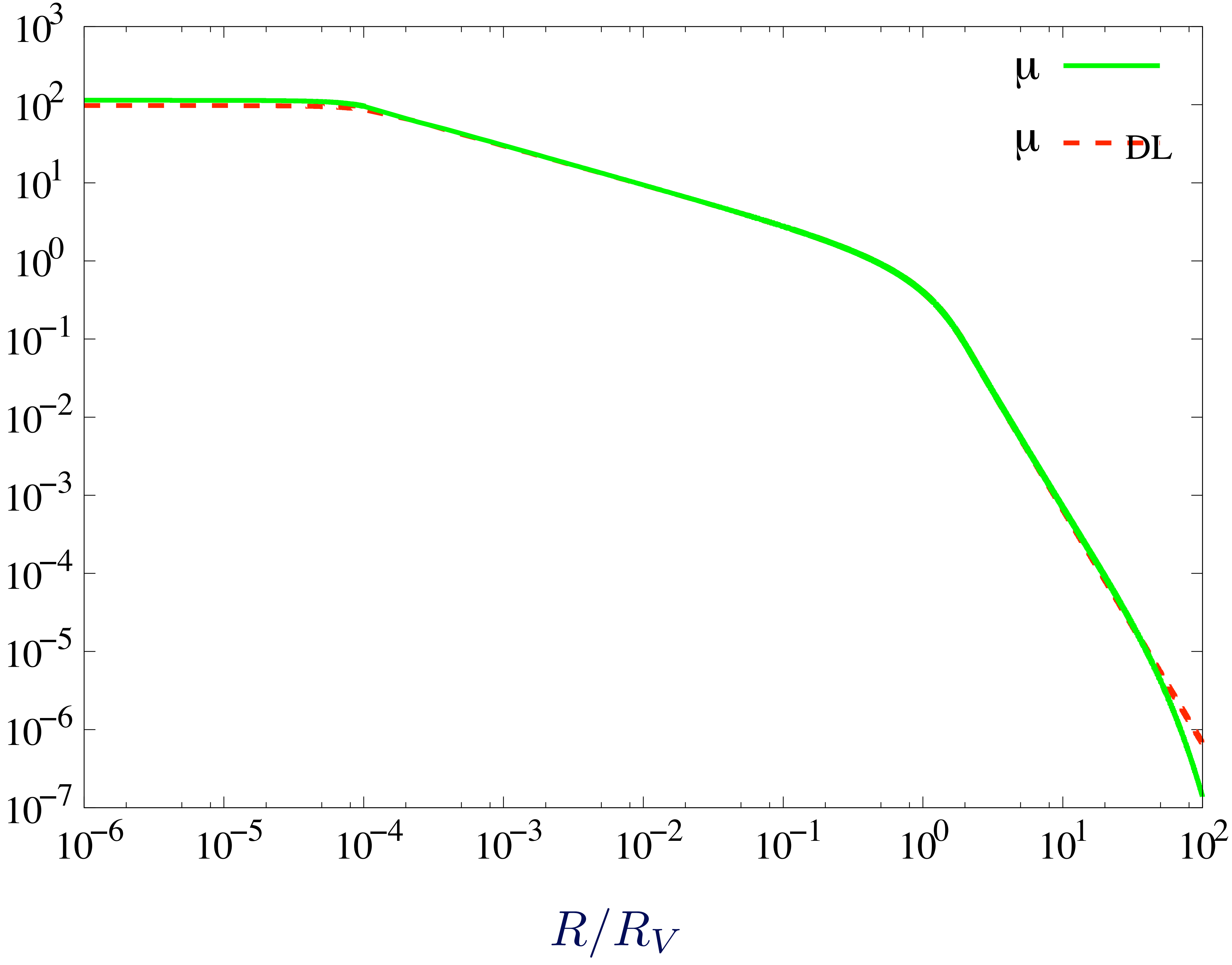
- for R>(Vainshtein radius) DL is valid up to 1/m
- for R<(Vainshtein radius),

Q-scaling	Vainshtein scaling
$h \sim R_S/R$	$h \sim R_S/R$
$\partial\partial\phi \sim \mu \sim m^2 R_V^4/R^2$	$\partial\partial\phi \sim \mu \sim \sqrt{R_S/R}$
$A \sim 0$	$A \sim 0$
$\Downarrow$	$\Downarrow$
$R \sim R_V^2 m$	$R \sim R_S$

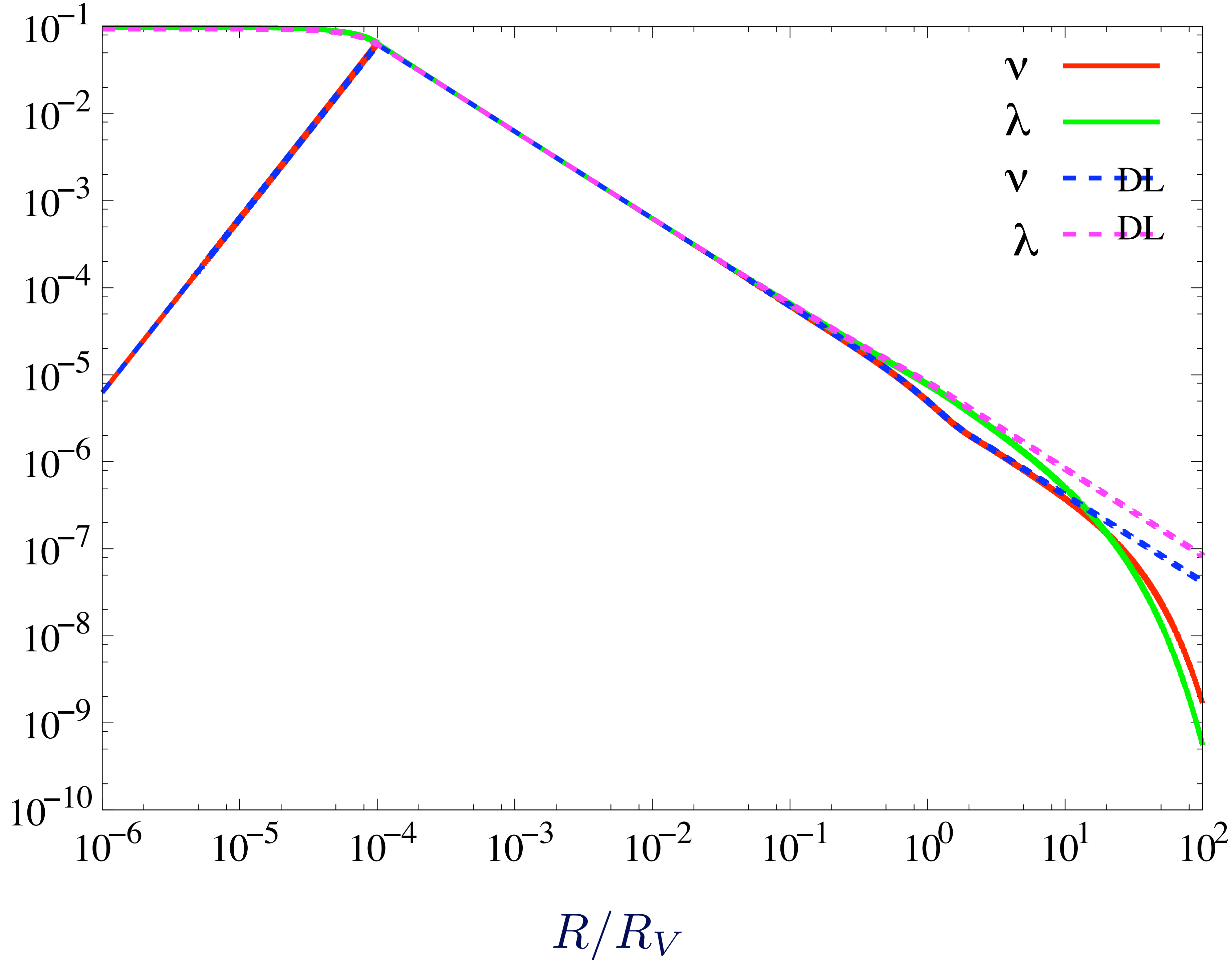
$$R_V^2 m \ll R \ll m^{-1} \qquad\qquad R_S \ll R \ll m^{-1}$$

N.B. Inside the star the solution changes,
DL is still valid.

Decoupling Limit <=> full system

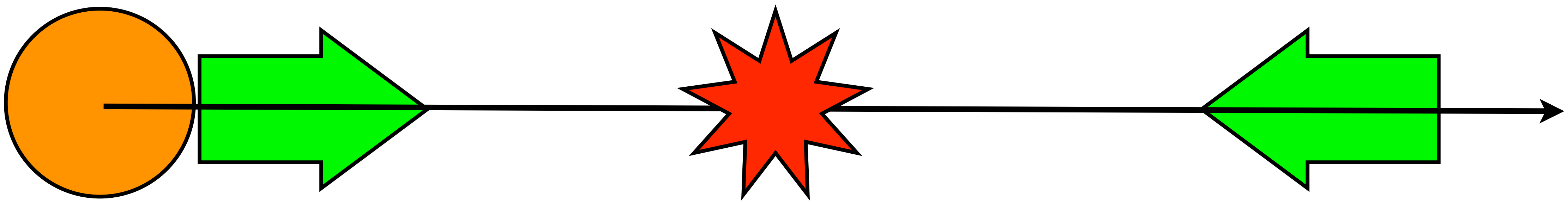


Decoupling Limit <=> full system



Numerics

RELAXATION vs SHOOTING



Damour, Kogan, Papazoglou'03

Relaxation

Impose all the boundary conditions

Might miss a singularity

Shooting

More reliable for checking singular solutions

Requires adjusting initial conditions to get required boundaries

Extremely difficult for highly non-linear systems and for several equations

Conclusion

- It is possible to obtain the DL in the case of static spherically symmetric ansatz. This decoupling limit corresponds to DL in the Goldstone picture.
- The scaling conjectured by Vainshtein at small radius is only a limiting case in an infinite family of non singular solutions each showing a Vainshtein recovery of GR solutions below the Vainshtein radius but a different common scaling at small distances.
- For AGS potential a family of solutions exists containing the new scaling solution with an Vainshtein-like solution as an asymptotic. The requirement of no-conical singularity at zero chooses uniquely the Vainshtein-like solution.
- For the full system (not DL) regular (everywhere) solution exist for AGS potential featuring a Vainshtein-like recovery of solutions of General Relativity and flat asymptotic at infinity.
- ? Compact objects: neutron stars and black holes ?

Conclusion

- For the full system of Massive Gravity (not DL) we have found numerically solution(s) for in the case of AGS potential term and extended objects.

- It is possible to show analytically that there is an infinite number of solutions at the infinity (the initial data at infinity does not fix uniquely the solution)

- Compact objects: neutron stars and black holes?