Electrodynamics at non-zero temperature, chemical potential, and Bose condensate

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Introduction

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Associated to the polarization of the medium.

Introduction

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$$U \sim \frac{Q_1 Q_2}{r} \exp^{-m_D r} \qquad \mathbf{m_D} = m_D(T, \mu, m...)$$

The potential is calculated from the photon EOM:

$$\left[\mathbf{k}^\rho \mathbf{k}_\rho \mathbf{g}^{\mu\nu} - \mathbf{k}^\mu \mathbf{k}^\nu + \mathbf{\Pi}^{\mu\nu}(\mathbf{k})\right] \mathbf{A}_\nu(\mathbf{k}) \quad = \quad \mathcal{J}^\mu(\mathbf{k})$$

$$U(r) = Q_1 Q_2 \int \frac{d^3k}{(2\pi)^3} \frac{\exp(i\mathbf{kr})}{k^2 - \Pi_{00}(k)} \quad = \quad \frac{Q_1 Q_2}{2\pi^2} \int_0^\infty \frac{dk k^2}{k^2 - \Pi_{00}(k)} \frac{\sin kr}{kr}$$

Introduction - 2

Usually Π_{00} is independent of the photon momentum k and Debye screening comes from poles at purely imaginary k ($\Pi_{00} = -m_D^2$). When a Bose condensate is present the situation is quite different:

- There is k dependence of Π_{00}. As a consequence, the potential is not Yukawa-like but exponentially decreasing and oscillating at the same time. Moreover there are power-law decreasing terms.
- The dependence on the coupling constant e is not analytic anymore.
- Debye screening is stronger than in standard cases.

The model

We studied an abelian gauge theory containing massive charged fermions and bosons:

$$\mathcal{L} = -\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \mathbf{m_B^2}|\phi|^\mathbf{2} + |(\partial_\mu + \mathbf{i\,eA}_\mu)\phi|^\mathbf{2} + \bar{\psi}(\mathbf{i}\not{\partial} - \mathbf{e}\not{\mathbf{A}} - \mathbf{m_F})\psi$$

and assumed $\mu_F \neq 0, Q = 0$.

The model

Bosons will compensate the fermion net charge, hence naively $\mu_B = -\mu_F$. But this condition is realized only when $\mu_B < m_B$ because $\mu_B > m_B$ leads to senseless distribution function:

$$f_B(E,T) = \frac{1}{\exp[(E-\mu_B)/T] - 1}$$

$$f_B(E = m_B, \mu_B) \leq 0 \text{ ?!?!?}$$

Bose condensate

If the boson-antiboson asymmetry is so large that $\mu_B = m_B$ is not sufficient to ensure $Q = 0$, a Bose condensate would be formed:

$$\begin{aligned} f_B^{(C)}(E, C, T) &= \frac{1}{\exp[(E - m_B)/T] - 1} + C\,\delta(\mathbf{q}) \equiv f_B(E, m_B, T) + C\,\delta(\mathbf{q}), \\ \bar{f}_B(E, -m_B, T) &= \frac{1}{\exp[(E + m_B)/T] - 1} \end{aligned}$$

The formation of the condensate depends on the boson density and takes place below a critical temperature $T_C = \sqrt{\frac{3\mathcal{J}_0^B}{e\, m_B}}$ while its amplitude C depends on the fermion chemical potential, the temperature T, and the particle masses:

$$C = -4\pi \int dq\, q^2 \left[f_B(E_B, m_B, T) - \bar{f}_B(E_B, -m_B, T) - 2 f_F(E_F, \mu_F, T) + 2 \bar{f}_F(E_F, \mu_F, T) \right]$$

The physical problem

We included in our system:

- non zero temperature $\rightarrow$ astrophysics and cosmology.
- chemical potential $\rightarrow$ matter-antimatter asymmetry.
- Bose condensate $\rightarrow$ applications to
  - Primordial universe (BG).
  - E-W breaking.
  - Helium white dwarfs.

and calculated the photon polarization tensor and the electrostatic potential.

Our approach

In abelian theories at the lowest order many subtleties can be ignored. Hence we were able to use a physically intuitive procedure to perform our analysis, in particular:

- We are dealing with an abelian theory and considering up to the second order in the c.c. e. Hence we could use a standard perturbative approach.

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- Thermal effects are taken into account by calculating the average over the thermal bath.

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- We are dealing with an abelian theory and considering up to the second order in the c.c. e. Hence we could use a standard perturbative approach.
- Thermal effects are taken into account by calculating the average over the thermal bath.
- After performing a Fourier transform, we finally found the photon polarization tensor $\Pi^{\mu\nu}$ which is a quite complicated function of the temperature T, the fermion and boson masses (m_F, m_B) and the photon momentum k^μ.

Thermal field theory

- Observables measured in empty space-time, e.g. cross sections measured in accelerators

$\boxed{\text{Conventional QFT}}$

- If we are dealing with non-negligible matter and radiation densities, e.g. early-universe cosmology, astrophysics of compact stars, extreme situations in experiments . . .

$\boxed{\text{Thermal Field Theory}}$

$$
\begin{aligned}
\langle a^\dagger(\mathbf{q})a(\mathbf{q}')\rangle &= f_B(E_q)\delta^{(3)}(\mathbf{q}-\mathbf{q}') \\
\langle a(\mathbf{q})a^\dagger(\mathbf{q}')\rangle &= [1+f_B(E_p)]\delta^{(3)}(\mathbf{q}-\mathbf{q}') \\
\langle c^\dagger(\mathbf{q})c(\mathbf{q}')\rangle &= f_F(E_p)\delta^{(3)}(\mathbf{q}-\mathbf{q}') \\
\langle c(\mathbf{q})c^\dagger(\mathbf{q}')\rangle &= [1-f_F(E_p)]\delta^{(3)}(\mathbf{q}-\mathbf{q}')
\end{aligned}
$$

The polarization tensor

The spatial-spatial component of $\Pi_{\mu\nu}$, Π_{00}, is needed to work out the electrostatic potential and is in general a function of C, T, m_B, m_F:

$$\begin{aligned}
\mathbf{\Pi_{00}}(\omega = \mathbf{0}) &= \mathbf{e^2} \int \frac{\mathbf{d^3q}}{(\mathbf{2}\pi)^{\mathbf{3}}\mathbf{E}} \mathbf{C}\, \delta^{\mathbf{3}}(\mathbf{q}) \left(\mathbf{1} + \frac{\mathbf{4E^2}}{\mathbf{k^2}}\right) \\
&+ \quad \frac{\mathbf{e^2}}{\mathbf{2}\pi^{\mathbf{2}}} \int_{\mathbf{0}}^{\infty} \frac{\mathbf{dq\, q^2}}{\mathbf{E}} \left[\mathbf{f_B}(\mathbf{E_B}, \mathbf{m_B}, \mathbf{T}) + \bar{\mathbf{f}}_{\mathbf{B}}(\mathbf{E_B}, -\mathbf{m_B}, \mathbf{T})\right] \left[\mathbf{1} + \frac{\mathbf{E_B^2}}{\mathbf{kq}} \ln \left|\frac{\mathbf{2q + k}}{\mathbf{2q - k}}\right|\right] \\
&+ \quad \frac{\mathbf{e^2}}{\mathbf{2}\pi^{\mathbf{2}}} \int_{\mathbf{0}}^{\infty} \frac{\mathbf{dq\, q^2}}{\mathbf{E}} \left[\mathbf{f_F}(\mathbf{E_F}, \mu_{\mathbf{F}}, \mathbf{T}) + \bar{\mathbf{f}}_{\mathbf{F}}(\mathbf{E_F}, -\mu_{\mathbf{F}}, \mathbf{T})\right] \left[\mathbf{2} + \frac{(\mathbf{4E_F^2} - \mathbf{k^2})}{\mathbf{2kq}} \ln \left|\frac{\mathbf{2q + k}}{\mathbf{2q - k}}\right|\right]
\end{aligned}$$

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$$\mathbf{\Pi_{00}}(\omega = \mathbf{0}, |\mathbf{k}| \to \mathbf{0}) = \mathbf{e^2} \int \frac{\mathbf{d^3q}}{(\mathbf{2}\pi)^\mathbf{3}\mathbf{E}} \mathbf{C}\,\delta^\mathbf{3}(\mathbf{q}) \left(\mathbf{1} + \frac{\mathbf{4E^2}}{\mathbf{k^2}}\right)$$

$$+ \quad \frac{\mathbf{e^2}}{\mathbf{2}\pi^\mathbf{2}} \int_\mathbf{0}^\infty \frac{\mathbf{dq\,q^2}}{\mathbf{E}} \left[\mathbf{f_B}(\mathbf{E_B}, \mathbf{m_B}, \mathbf{T}) + \mathbf{\bar{f}_B}(\mathbf{E_B}, -\mathbf{m_B}, \mathbf{T})\right] \left[\mathbf{1} + \frac{\mathbf{E_B^2}}{\mathbf{kq}} \ln\left|\frac{\mathbf{2q+k}}{\mathbf{2q-k}}\right|\right]$$

$$+ \quad \frac{\mathbf{e^2}}{\mathbf{2}\pi^\mathbf{2}} \int_\mathbf{0}^\infty \frac{\mathbf{dq\,q^2}}{\mathbf{E}} \left[\mathbf{f_F}(\mathbf{E_F}, \mu_\mathbf{F}, \mathbf{T}) + \mathbf{\bar{f}_F}(\mathbf{E_F}, -\mu_\mathbf{F}, \mathbf{T})\right] \left[\mathbf{2} + \frac{(\mathbf{4E_F^2} - \mathbf{k^2})}{\mathbf{2kq}} \ln\left|\frac{\mathbf{2q+k}}{\mathbf{2q-k}}\right|\right]$$

Besides some "standard", k-independent terms, Π_{00} contains two singular in k terms which are strictly related to the condensate:

$$\frac{\mathbf{e^2}}{(\mathbf{2}\pi)^\mathbf{3}} \frac{\mathbf{C}}{\mathbf{m_B}} \left(\frac{\mathbf{4m_B^2}}{\mathbf{k^2}}\right)$$

The polarization tensor

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$$+\quad\frac{\mathbf{e^2}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathbf{0}}^{\infty}\frac{\mathbf{dq\,q^2}}{\mathbf{E}}\left[\mathbf{f_B(E_B,m_B,T)}+\bar{\mathbf{f}}_{\mathbf{B}}(\mathbf{E_B},-\mathbf{m_B},\mathbf{T})\right]\left[\mathbf{1}+\frac{\mathbf{E_B^2}}{\mathbf{kq}}\ln\left|\frac{\mathbf{2q+k}}{\mathbf{2q-k}}\right|\right]$$

$$+\quad\frac{\mathbf{e^2}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathbf{0}}^{\infty}\frac{\mathbf{dq\,q^2}}{\mathbf{E}}\left[\mathbf{f_F}(\mathbf{E_F},\mu_{\mathbf{F}},\mathbf{T})+\bar{\mathbf{f}}_{\mathbf{F}}(\mathbf{E_F},-\mu_{\mathbf{F}},\mathbf{T})\right]\left[\mathbf{2}+\frac{(\mathbf{4E_F^2}-\mathbf{k^2})}{\mathbf{2kq}}\ln\left|\frac{\mathbf{2q+k}}{\mathbf{2q-k}}\right|\right]$$

Besides some "standard", k-independent terms, Π_{00} contains two singular in k terms which are strictly related to the condensate:

$$\int_0^\infty = \int_0^{k/2} + \int_{k/2}^\infty \qquad\qquad \frac{e^2 m_B^2 T}{2k}$$

The polarization tensor

The spatial-spatial component of $\Pi_{\mu\nu}$, Π_{00}, is needed to work out the electrostatic potential and is in general a function of C, T, m_B, m_F:

$$-\Pi_{00} = e^2 \left[\left(m_0^2 + \frac{m_1^3}{k} + \frac{m_2^4}{k^2} \right) \right]$$

$$m_0^2 = \left(\frac{2T^2}{3} \right) + \frac{C}{(2\pi)^3 m_B} \qquad m_1^3 = \frac{m_B^2 T}{2} \qquad m_2^4 = \frac{4 C m_B}{(2\pi)^3}$$

The electrostatic potential can hence be calculated as:

$$U(r) = Q_1 Q_2 \int \frac{d^3k}{(2\pi)^3} \frac{\exp(i\mathbf{kr})}{k^2 - \Pi_{00}(k)} = \frac{q}{2\pi^2} \int_0^\infty \frac{dk k^2}{k^2 - \Pi_{00}(k)} \frac{\sin kr}{kr}$$

and it contains contributions from the poles at

$$k^2 + e^2 \left[\left(m_0^2 + \frac{m_1^3}{k} + \frac{m_2^4}{k^2} \right) \right] = 0$$

The results

- $T \to 0$: the integral can be taken using the residues; poles are at complex values of ks, hence the electrostatic potential has the standard Debye screening from the imaginary component of k plus an oscillating term coming from its real component:

$$\mathbf{U(r)} \sim \mathbf{Q_1 Q_2} \frac{\exp(-\sqrt{\mathbf{e/2}}\mathbf{m_2 r}) \cos(\sqrt{\mathbf{e/2}}\mathbf{m_2 r})}{\mathbf{r}}.$$

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$$\mathbf{U(r) \sim Q_1 Q_2 \frac{\exp(-\sqrt{e/2}m_2 r)\cos(\sqrt{e/2}m_2 r)}{r}}.$$

- Finite T: integral over the imaginary axis gives a power law decreasing term:

$$\mathbf{U \sim Q_1 Q_2 \frac{1}{e^2} \frac{m_1^3}{m_2^8 r^6}}$$

Please note: Debye screening is stronger than the standard cases ($m_D \sim e$) and non analytic function of e.

Friedel oscillations

Friedel oscillations appear in degenerate fermionic plasma at $T \to 0$ and are due to the sharp Fermi distribution function.
Mathematically they come from the $\log$ term in Π_{00}:

$$\Pi_{00}^F = \frac{\mathbf{e^2}}{\mathbf{2\pi^2}} \int_{\mathbf{0}}^{\infty} \frac{\mathbf{dq\,q^2}}{\mathbf{E}} \left[\mathbf{f_F}(\mathbf{E_F}, \mu_{\mathbf{F}}, \mathbf{T}) + \mathbf{\bar{f}_F}(\mathbf{E_F}, -\mu_{\mathbf{F}}, \mathbf{T})\right] \left[\mathbf{2} + \frac{(\mathbf{4E_F^2} - \mathbf{k^2})}{\mathbf{2kq}} \ln\left|\frac{\mathbf{2q+k}}{\mathbf{2q-k}}\right|\right].$$

Let us assume m_F. The distribution function has poles at

$$q_n = \mu_F \pm i\pi T(2n+1).$$

Hence performing the integral along branch cuts $(k = 2q_n + iy)$ we get

$$U_n(r) = \frac{Q_1 Q_2 e^2 T}{16\pi^2 r^3 q_n^3} \sin(2\mu_F r) \exp^{-2\pi(2n+1)Tr}$$

Friedel oscillations

Friedel oscillations appear in degenerate fermionic plasma at $T \to 0$ and are due to the sharp Fermi distribution function.
Performing the sum over n we get the potentials:

$$U(r,T) = \frac{Q_1 Q_2 e^2}{64\pi^3} \frac{\sin(2k_F r)}{k_F^3 r^4} \cdot 4\pi r T \cdot \frac{\exp(-2\pi r T)}{1-\exp(-4\pi r T)} \qquad (m_F = 0)$$

$$U(r,T) = \frac{Q_1 Q_2 e^2 T m_F^4}{16\pi^2} \frac{\cos(2k_F r)}{k_F^4 r^2} \cdot \frac{\exp(-2\pi r T m_F/k_F)}{1-\exp(-4\pi r T m_F/k_F)} \qquad (m_F \gg k_F)$$

which vanish for large T and have the low temperature limit:

$$U(r,T=0) = \frac{Q_1 Q_2 e^2}{64\pi^3} \frac{\sin(2k_F r)}{k_F^3 r^4} \qquad (m_F = 0)$$

$$U(r,T=0) = \frac{Q_1 Q_2 e^2 m_F}{64\pi^3} \frac{\cos(2k_F r)}{k_F^3 r^3} \qquad (m_F \gg k_F)$$

Summary and conclusion

- Gauge theories with a Bose condensate are of interest in Physics because of their applications to SM extensions (e.g. SUSY), cosmology, astrophysics etc.
- In this work we studied the simplest abelian one including both condensate and temperature effects and studied the photon propagation in the hot charged plasma.
- As a result, we found that the electrostatic potential in the considered system has very peculiar features. In particular, there is k dependence of Π_{00} which give rise to oscillating and exponentially decreasing potential and non analytic dependence on the coupling constant e.

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THE END

Standard perturbative approach

$$\phi(x) = \phi_0(x) + \int d^4y\, G_B(x-y)\mathcal{J}_\phi(y)$$

$$\psi(x) = \psi_0(x) + \int d^4y\, G_F(x-y)e\not{A}(y)\psi(y)$$

$$(\partial_\mu\partial^\mu + m_B^2)\phi_0(x) = 0, \qquad (i\not{\partial} - m_F)\psi_0(x) = 0$$

$$\phi_0(x) = \int \frac{d^3q}{\sqrt{(2\pi)^3 2E}} \left[a(\mathbf{q})\exp^{-iqx} + b^\dagger(\mathbf{q})\exp^{iqx}\right]$$

$$\psi_0(x) = \int \frac{d^3q}{\sqrt{(2\pi)^3}}\sqrt{\frac{m_F}{E}} \left[c_r(\mathbf{q})u_r(\mathbf{q})\exp^{-iqx} + d_r^\dagger(\mathbf{q})v_r(\mathbf{q})\exp^{iqx}\right]$$

$$G_B(k) = \frac{1}{k^2 - m_B^2 + i\epsilon}, \quad \text{and} \quad G_F(k) = \frac{\not{k} + m_F}{k^2 - m_F^2 + i\epsilon}$$

BACK

The photon equation of motion

The most general expression of the photon EOM in the momentum space is:

$$\left[\mathbf{k}^\rho \mathbf{k}_\rho \mathbf{g}^{\mu\nu} - \mathbf{k}^\mu \mathbf{k}^\nu + \mathbf{\Pi}^{\mu\nu}(\mathbf{k})\right] \mathbf{A}_\nu(\mathbf{k}) = \mathcal{J}^\mu(\mathbf{k})$$

$$\mathbf{\Pi}^{\mathbf{B}}_{\mu\nu}(\mathbf{k}) = \mathbf{e}^2 \int \frac{\mathbf{d}^3\mathbf{q}}{(2\pi)^3\mathbf{E}} \left[\mathbf{f}_{\mathbf{B}}(\mathbf{E}) + \bar{\mathbf{f}}_{\mathbf{B}}(\mathbf{E})\right] \times$$
$$\left[\frac{1}{2}\frac{(2\mathbf{q}-\mathbf{k})_\mu(2\mathbf{q}-\mathbf{k})_\nu}{(\mathbf{q}-\mathbf{k})^2 - \mathbf{m}_{\mathbf{B}}^2} + \frac{1}{2}\frac{(2\mathbf{q}+\mathbf{k})_\mu(2\mathbf{q}+\mathbf{k})_\nu}{(\mathbf{q}+\mathbf{k})^2 - \mathbf{m}_{\mathbf{B}}^2} - \mathbf{g}_{\mu\nu}\right]$$

$$\mathbf{\Pi}^{\mathbf{F}}_{\mu\nu}(\mathbf{k}) = 2\mathbf{e}^2 \int \frac{\mathbf{d}^3\mathbf{q}}{(2\pi)^3\mathbf{E}} \left[\mathbf{f}_{\mathbf{F}}(\mathbf{E}) + \bar{\mathbf{f}}_{\mathbf{F}}(\mathbf{E})\right] \times$$
$$\left[\frac{\mathbf{q}_\nu(\mathbf{k}+\mathbf{q})_\mu - \mathbf{q}^\rho\mathbf{k}_\rho\mathbf{g}_{\mu\nu} + \mathbf{q}_\mu(\mathbf{k}+\mathbf{q})_\nu}{(\mathbf{k}+\mathbf{q})^2 - \mathbf{m}_{\mathbf{F}}^2} + \frac{\mathbf{q}_\nu(\mathbf{q}-\mathbf{k})_\mu + \mathbf{q}^\rho\mathbf{k}_\rho\mathbf{g}_{\mu\nu} + \mathbf{q}_\mu(\mathbf{q}-\mathbf{k})_\nu}{(\mathbf{k}-\mathbf{q})^2 - \mathbf{m}_{\mathbf{F}}^2}\right]$$

$$\mathcal{J}_\mu = -\mathbf{e}\int \frac{\mathbf{d}^4\mathbf{x}}{(2\pi)^4} \exp^{-\mathbf{ikx}} \int \frac{\mathbf{d}^3\mathbf{q}}{(2\pi)^3}\frac{\mathbf{q}_\mu}{\mathbf{E}}\left[\mathbf{f}_{\mathbf{B}}(\mathbf{E}) - \bar{\mathbf{f}}_{\mathbf{B}}(\mathbf{E}) - 2\left(\mathbf{f}_{\mathbf{F}}(\mathbf{E}) - \bar{\mathbf{f}}_{\mathbf{F}}(\mathbf{E})\right)\right]$$

BACK

Π^{00} with BEC - general form

$$\Pi_{00}(\omega=0,|\mathbf{k}|\to 0) = \frac{e^2}{(2\pi)^3}\frac{C}{m_B}\left(1+\frac{4m_B^2}{k^2}\right)$$

$$+ \quad \frac{e^2}{2\pi^2}\int_0^\infty \frac{dq\,q^2}{E}\left[f_B(E_B,m_B,T)+\bar{f}_B(E_B,-m_B,T)\right]\left[1+\frac{E_B^2}{kq}\ln\left|\frac{2q+k}{2q-k}\right|\right]$$

$$+ \quad \frac{e^2}{2\pi^2}\int_0^\infty \frac{dq\,q^2}{E}\left[f_F(E_F,\mu_F,T)+\bar{f}_F(E_F,-\mu_F,T)\right]\left[2+\frac{(4E_F^2-k^2)}{2kq}\ln\left|\frac{2q+k}{2q-k}\right|\right]$$

BACK

Debye mass - standard cases

- Relativistic fermions with $m_F \ll T, \mu_F$:

$$m_D^2 = e^2 \left(\frac{T^2}{3} + \frac{\mu_F^2}{\pi^2} \right)$$

- Non relativistic fermions:

$$m_D^2 = \frac{e^2 n_F}{T}, \qquad n_F = \frac{\exp(\mu/T)}{\pi^2} \int dq q^2 e^{-q^2/2m_F T}$$

- Massless scalars without chemical potential:

$$m_D^2 = \frac{e^2 T^2}{3}$$

BACK

Plasma frequency - standard cases

- Massless QED and SQED at high T with no chemical potential:

$$\omega^2_{P\,B}(m_B = 0) = \omega^2_{P\,F}(m_F = 0) = \frac{1}{9}\, e^2 T^2$$

- Massive QED with chemical potential and $T \to 0$:

$$\omega_p^2 = \frac{e^2 n_F}{m_F}, \qquad n_F = \frac{\exp(\mu/T)}{\pi^2} \int dq q^2 e^{-q^2/2m_F T}$$

BACK

Our model...

We considered a model with charged fermions and bosons:

$$\mathcal{L} = -\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \mathbf{m_B^2}|\phi|^\mathbf{2} + |(\partial_\mu + \mathbf{i\,e}\mathbf{A}_\mu)\phi|^\mathbf{2} + \bar{\psi}(\mathbf{i}\partial\!\!\!/ - \mathbf{e}A\!\!\!/ - \mathbf{m_F})\psi$$

and studied in detail how the photon propagates in the hot plasma.

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and studied in detail how the photon propagates in the hot plasma.

The Eulero-Lagrange equations are needed:

$$\partial_\nu \mathbf{F}^{\mu\nu}(\mathbf{x}) = \mathcal{J}^\mu(\mathbf{x})$$

$$(\mathbf{i}\not{\partial} - \mathbf{m})\psi(\mathbf{x}) = \mathbf{e}\not{\mathbf{A}}\psi(\mathbf{x})$$

$$(\partial_\mu\partial^\mu + \mathbf{m^2})\phi(\mathbf{x}) = \mathcal{J}_\phi(\mathbf{x})$$

.

Our model...

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$$\mathcal{L} = -\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \mathbf{m_B^2}|\phi|^2 + |(\partial_\mu + \mathbf{i}\,\mathbf{e}\mathbf{A}_\mu)\phi|^2 + \bar{\psi}(\mathbf{i}\not{\partial} - \mathbf{e}\not{\mathbf{A}} - \mathbf{m_F})\psi$$

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$$\partial_\nu \mathbf{F}^{\mu\nu}(\mathbf{x}) = \mathcal{J}^\mu(\mathbf{x})$$

$$\mathcal{J}^\mu(\mathbf{x}) = -\mathbf{i}\,\mathbf{e}\Big[(\phi^\dagger(\mathbf{x})\partial^\mu\phi(\mathbf{x})) - (\partial^\mu\phi^\dagger(\mathbf{x}))\phi(\mathbf{x})\Big] + \mathbf{2e^2}\mathbf{A}^\mu(\mathbf{x})|\phi(\mathbf{x})|^\mathbf{2} - \mathbf{e}\bar{\psi}(\mathbf{x})\gamma^\mu\psi(\mathbf{x})$$

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We considered a model with charged fermions and bosons:

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and studied in detail how the photon propagates in the hot plasma.

The Eulero-Lagrange equations are needed:

$$\partial_\nu \mathbf{F}^{\mu\nu}(\mathbf{x}) = \mathcal{J}^\mu(\mathbf{x})$$

To study the photon propagation we need its EOM as a function of its momentum k:

$$\left[\mathbf{k}^\rho \mathbf{k}_\rho \mathbf{g}^{\mu\nu} - \mathbf{k}^\mu \mathbf{k}^\nu + \Pi^{\mu\nu}(k)\right]\mathbf{A}_\nu(\mathbf{k}) = \mathcal{J}^\mu(\mathbf{k})$$

So the key quantity to study the photon propagation is $\Pi^{\mu\nu}$, that is the **photon polarization tensor**.

Photon propagation in plasma

Generally, the electrostatic potential in a medium is given by:

$$U(r) = q \int \frac{d^3k}{(2\pi)^3} \frac{\exp(i\mathbf{kr})}{k^2 - \Pi_{00}(k)} = \frac{q}{2\pi^2} \int_0^\infty \frac{dkk^2}{k^2 - \Pi_{00}(k)} \frac{\sin kr}{kr}$$

In vacuum $\Pi_{00} = 0$ and thus

$$U(r) = \frac{q}{4\pi} \frac{1}{r}$$

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In the cases which can be found in the literature the particles obey the:

$$f_B(E,T) = \frac{1}{\exp[(E-\mu_B)/T] - 1}$$

$$f_F(E,T) = \frac{1}{\exp[(E-\mu_F)/T] + 1}$$

and it turns out that $\sqrt{-\Pi_{00}(k)} = const = m_D$ and hence

$$U(r) = \frac{q}{4\pi} \frac{\exp(-m_D r)}{r}.$$

Bose-Einstein condensate

Let us consider a charge-neutral system with a fermion asymmetry and no net electric charge:

$$\mu_F \neq 0, \qquad Q = 0.$$

Bosons will compensate the fermion net charge and hence one may naively expect that $\mu_B = -\mu_F$.
But the condition $\mu_B > m_B$ leads to senseless distribution function:

$$f_B(E,T) = \frac{1}{\exp[(E-\mu_B)/T] - 1}$$

$$f_B(E = m_B, \mu_B) \leq 0 \text{ ?!?!?}$$

Bose-Einstein condensate

If the boson-antiboson asymmetry is so large that $\mu_B = m_B$ is not sufficient to ensure $Q = 0$, a Bose condensate would be formed and the boson distribution function would take the form:

$$
\begin{aligned}
f_B^{(C)}(E, C, T) &= \frac{1}{\exp[(E - m_B)/T] - 1} + C\,\delta(\mathbf{q}) \equiv f_B(E, m_B, T) + C\,\delta(\mathbf{q}), \\
\bar{f}_B(E, -m_B, T) &= \frac{1}{\exp[(E + m_B)/T] - 1}
\end{aligned}
$$

The formation of the condensate will depend on the boson density and will take place below a critical temperature $T_C = \sqrt{\frac{3\mathcal{J}_0^B}{e\,m_B}}$ while its amplitude C will depend on the fermion chemical potential, the temperature T, and the particle masses:

$$
C = -4\pi \int dq\, q^2 \left[f_B(E_B, m_B, T) - \bar{f}_B(E_B, -m_B, T) - 2f_F(E_F, \mu_F, T) + 2\bar{f}_F(E_F, \mu_F, T) \right]
$$

Photon propagation with BEC

The spatial-spatial component of $\Pi_{\mu\nu}$, Π_{00}, is needed to work out the electrostatic potential and is in general a function of C, T, m_B, m_F:

$$\mathbf{\Pi_{00}}(\omega = \mathbf{0}, |\mathbf{k}| \to \mathbf{0}) = \mathbf{e^2} \int \frac{\mathbf{d^3q}}{(\mathbf{2}\pi)^\mathbf{3}\mathbf{E}} \mathbf{C}\, \delta^\mathbf{3}(\mathbf{q}) \left(\mathbf{1} + \frac{\mathbf{4E^2}}{\mathbf{k^2}}\right)$$

$$+ \quad \frac{\mathbf{e^2}}{\mathbf{2}\pi^\mathbf{2}} \int_\mathbf{0}^\infty \frac{\mathbf{dq\,q^2}}{\mathbf{E}} \left[\mathbf{f_B}(\mathbf{E_B}, \mathbf{m_B}, \mathbf{T}) + \mathbf{\bar{f}_B}(\mathbf{E_B}, -\mathbf{m_B}, \mathbf{T})\right] \left[\mathbf{1} + \frac{\mathbf{E_B^2}}{\mathbf{kq}} \ln\left|\frac{\mathbf{2q+k}}{\mathbf{2q-k}}\right|\right]$$

$$+ \quad \frac{\mathbf{e^2}}{\mathbf{2}\pi^\mathbf{2}} \int_\mathbf{0}^\infty \frac{\mathbf{dq\,q^2}}{\mathbf{E}} \left[\mathbf{f_F}(\mathbf{E_F}, \mu_\mathbf{F}, \mathbf{T}) + \mathbf{\bar{f}_F}(\mathbf{E_F}, -\mu_\mathbf{F}, \mathbf{T})\right] \left[\mathbf{2} + \frac{(\mathbf{4E_F^2} - \mathbf{k^2})}{\mathbf{2kq}} \ln\left|\frac{\mathbf{2q+k}}{\mathbf{2q-k}}\right|\right]$$

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$$\mathbf{\Pi_{00}(\omega = 0, |k| \to 0) = e^2 \int \frac{d^3q}{(2\pi)^3 E} C\, \delta^3(q) \left(1 + \frac{4E^2}{k^2}\right)}$$

$$\mathbf{+ \quad \frac{e^2}{2\pi^2} \int_0^\infty \frac{dq\, q^2}{E} \left[f_B(E_B, m_B, T) + \bar{f}_B(E_B, -m_B, T)\right] \left[1 + \frac{E_B^2}{kq} \ln\left|\frac{2q + k}{2q - k}\right|\right]}$$

$$\mathbf{+ \quad \frac{e^2}{2\pi^2} \int_0^\infty \frac{dq\, q^2}{E} \left[f_F(E_F, \mu_F, T) + \bar{f}_F(E_F, -\mu_F, T)\right] \left[2 + \frac{(4E_F^2 - k^2)}{2kq} \ln\left|\frac{2q + k}{2q - k}\right|\right]}$$

Besides some "standard", k-independent terms, Π_{00} contains two singular in k terms which are strictly related to the condensate:

$$\mathbf{\frac{e^2}{(2\pi)^3} \frac{C}{m_B} \left(\frac{4m_B^2}{k^2}\right)}$$

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$$+ \quad \frac{\mathbf{e^2}}{\mathbf{2}\pi^{\mathbf{2}}} \int_{\mathbf{0}}^{\infty} \frac{\mathbf{dq\,q^2}}{\mathbf{E}} \left[\mathbf{f_F(E_F, \mu_F, T)} + \mathbf{\bar{f}_F(E_F, -\mu_F, T)}\right] \left[\mathbf{2} + \frac{(\mathbf{4E_F^2 - k^2})}{\mathbf{2kq}} \ln \left|\frac{\mathbf{2q+k}}{\mathbf{2q-k}}\right|\right]$$

Besides some "standard", k-independent terms, Π_{00} contains two singular in k terms which are strictly related to the condensate:

$$\int_0^\infty = \int_0^{k/2} + \int_{k/2}^\infty \qquad\qquad \frac{e^2 m_B^2 T}{2k}$$

Photon propagation with BEC

The spatial-spatial component of $\Pi_{\mu\nu}$, Π_{00}, is needed to work out the electrostatic potential and is in general a function of C, T, m_B, m_F:

$$\Pi_{00} \quad = \quad e^2 \left[\left(m_0^2 + \frac{m_1^3}{k} + \frac{m_2^4}{k^2} \right) \right]$$

$$m_0^2 \quad = \quad \left(\frac{2T^2}{3} \right) + \frac{C}{(2\pi)^3 m_B} \qquad\qquad m_1^3 = \frac{m_B^2 T}{2} \qquad\qquad m_2^4 = \frac{4Cm_B}{(2\pi)^3}$$

The electrostatic potential can hence be calculated as:

$$U(r) = q \int \frac{d^3k}{(2\pi)^3} \frac{\exp(i\mathbf{kr})}{k^2 - \Pi_{00}(k)} = \frac{q}{2\pi^2} \int_0^\infty \frac{dkk^2}{k^2 - \Pi_{00}(k)} \frac{\sin kr}{kr}$$

The electrostatic potential

- The potential contains an oscillating exponentially suppressed term. In the special case $T \to 0$, that is, without the $1/k$ term:

$$U(r) \sim q \, \frac{\exp(-\sqrt{e/2} m_2 r) \cos(\sqrt{e/2} m_2 r)}{r}.$$

  - Generally, $T \neq 0$ moves the poles but doesn't change the shape of the potential;
  - The screening lenght $\lambda_D \sim m_D^{-1} \sim (\sqrt{e})^{-1}$ is non analytic in e;
  - A similar potential has been found at higher order for fermion system, where it is dominant at $T \to 0$. The oscillations, known as *Friedel oscillations*, have been experimentally observed .

The electrostatic potential

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- Moreover, thermal effects induces a power-law decreasing term proportional to e^{-2} and arising from integration on the imaginary axis:

$$U \sim q \, \frac{1}{e^2} \, \frac{m_1^3}{m_2^8 r^6}$$

The propagating modes

The spatial components of the photon polarization tensor can be decomposed as:

$$\Pi_{ij} = a\left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) + b\frac{k_i k_j}{\mathbf{k}^2}, \quad \Pi_{0j} = \frac{k_j}{\omega}\, b, \quad \Pi_{00} = \frac{\mathbf{k^2}}{\omega^2}\, b$$

which can be used to calculate the dispersion relation of the transversal and longitudinal part of the photon. We focused mainly on the calculation of the plasma frequency ω_p, defined as:

$$\omega_p^2 = b(\omega, |\mathbf{k} = 0|) = a(\omega, |\mathbf{k} = 0|).$$

Boson condensate contributes to ω_p with a constant additive term:

$$\delta\omega_p^2 = \frac{e^2 C}{(2\pi)^2 m_B}$$

Summary and conclusion

- Altough it has not been studied until very recently, scalar electrodynamics with a Bose condensate is of interest in Physics because its applications to SM extensions (e.g. SUSY), cosmology, astrophysics etc.
- In this work we included both condensate and temperature effects and studied the photon propagation in the hot charged plasma.
- As a result, we found that the electrostatic potential in the considered system has very peculiar features, both because of its functional form and its dependence on the coupling constant e.
- Interpretation..... Well, there is still a lot of work to do!

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THE END