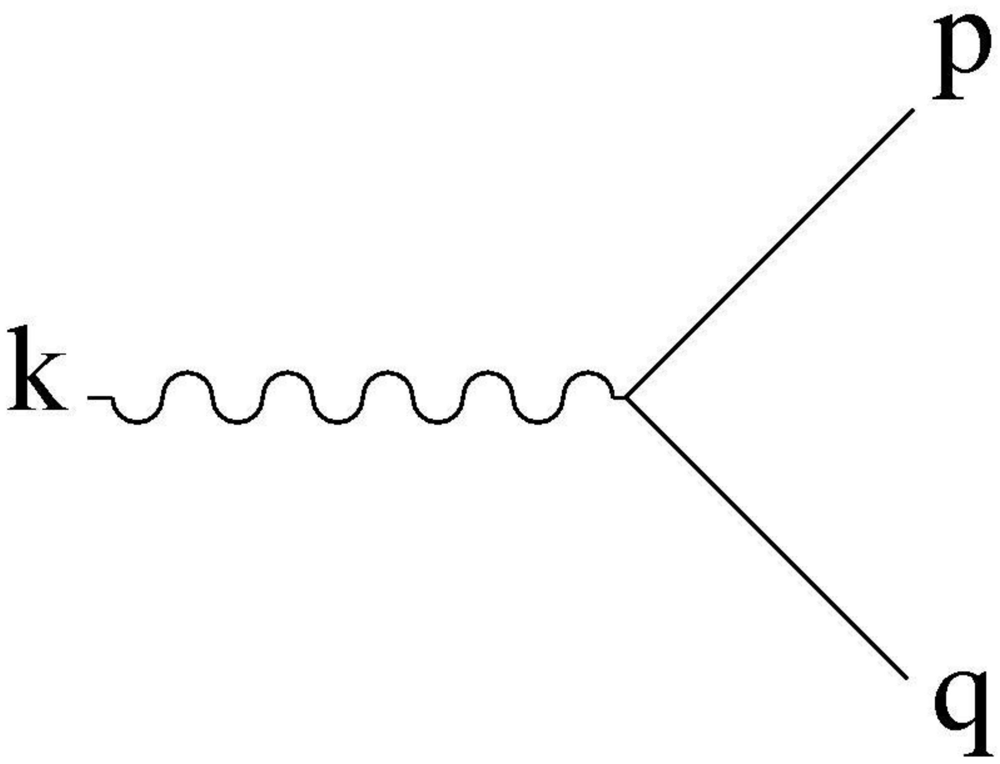
Recent Results in dS QFT

Ugo Moschella
Università dell'Insubria – Como
ugo.moschella@uninsubria.it

Workshop SW3 Cargese
April 2009

Decays of unstable particles



$$\langle pq|S|k\rangle \simeq \frac{2gi(2\pi)^4}{\sqrt{8p_0q_0k_0}}\delta_4(k-p-q)$$

Decay rate and lifetime

Sum over all the final states

$$\Gamma(1,2) \;=\; \frac{1}{T}\int dq \int dp |\langle pq|S|k\rangle|^2$$
$$\rightarrow\; \frac{g^2}{8\pi M}\sqrt{M^2-4m^2}\;\;\theta(M-2m)$$

Lifetime in the rest frame:

$$\tau_0 = \frac{1}{\Gamma(1,2)}$$

Lifetime dilation in a general frame:

$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

First task: x-space approach

Three scalar fields on a general *d*-dimensional Minkowski space (the dimension can be a complex number)

$\phi_0, \ \phi_1, \ \phi_2$ with masses $m_0, \ m_1, \ m_2$

The fields are independent (uncorrelated):

$$(\Omega, \ \phi_j(x)\,\phi_k(y)\Omega) = \delta_{jk}\,\mathcal{W}_{m_j}(x, \ y)$$

Fock space: tensor product of the individual Fock spaces

$$\mathcal{H} = \mathcal{F}^{(0)} \otimes \mathcal{F}^{(1)} \otimes \mathcal{F}^{(2)}$$

Interaction Lagrangian

Switch on an interaction term

$$\int \gamma\, g(x)\, \mathcal{L}(x)\, dx$$

$$\mathcal{L}(x) = \,:\phi_0(x)\phi_1(x)^{n_1}\phi_2(x)^{n_2}:$$

g(x) is a smooth rapidly decreasing function over the spacetime (an infrared cutoff). In the end *g(x)* $\to$ 1 (adiabatic limit).

Special case $\qquad \mathcal{L}(x) = \,:\phi^n(x):$

In the following we consider a trilinear interaction term

$$\mathcal{L}(x) = \,:\phi_0(x)\phi_1(x)\phi_2(x):$$

Perturbation theory

The transition amplitude between two normalized states ψ_0 and ψ_1 is given by the scalar product

$$(\psi_0,\ S(\gamma g)\psi_1),$$

$$S(\gamma g) = \sum_{n=0}^{\infty} \frac{i^n \gamma^n}{n!} \int_{\mathcal{X}^n} g(x_1)\, dx_1 \cdots g(x_n)\, dx_n\, T(\mathcal{L}(x_1) \ldots \mathcal{L}(x_n))$$

At first order, if the states ψ_0 and ψ_1 are orthogonal

$$(\psi_0,\ S(\gamma g)\psi_1) = (\psi_0,\ iT_1(\gamma g)\psi_1)$$

$$T_1(\gamma g) = \int \gamma g(x)\, \mathcal{L}(x)\, dx$$

Quantum states

The subspace of states containing j_k k-particles: $\mathcal{H}_{j_0,j_1,j_2}$

States of the form $\psi_0 = \int f_0(x)\,\phi_0(x)\Omega\,dx$

generate $\mathcal{H}_{1,0,0}$

$$(\psi_0, \psi_0) = \int \bar{f}_0(x)\,\mathcal{W}_{m_0}(x,y) f_0(y)\,dx\,dy$$

States of the form

$$\psi_1 = \int f(x_1, x_2) : \phi_1(x_1)\phi_2(x_2) : \Omega\,dx_1\,dx_2$$

generate $\mathcal{H}_{0,1,1}$

Transition probability

The probability of transition from ψ_0 to any state in $\mathcal{H}_{0,1,1}$

$$\Gamma = \frac{\sum_n \left|(\psi_0,\ T_1(\gamma g)\,\psi_{1(n)})\right|^2}{(\psi_0,\ \psi_0)}$$

$$= \frac{(\psi_0,\ T_1(\gamma g)\Pi_{0,1,1}T_1(\gamma g)^*\,\psi_0)}{(\psi_0,\ \psi_0)}$$

Wick's theorem gives

$$\Gamma = \frac{\gamma^2}{(\psi_0,\ \psi_0)}\int \overline{f_0(x)}\, f_0(y)\, g(u)\, g(v) \times$$

$$\times \quad \mathcal{W}_{m_0}(x,\ u)\ [\mathcal{W}_{m_1}(u,\ v)\mathcal{W}_{m_2}(u,\ v)]\, \mathcal{W}_{m_0}(v,\ y)\, dx\, du\, dv\, dy$$

Graphical interpretation

$$\Gamma = \frac{\gamma^2}{(\psi_0,\ \psi_0)} \int \overline{f_0(x)}\, f_0(y)\, g(u)\, g(v) \times$$

$$\times \quad \mathcal{W}_{m_0}(x,\ u)\ [\mathcal{W}_{m_1}(u,\ v) \mathcal{W}_{m_2}(u,\ v)]\, \mathcal{W}_{m_0}(v,\ y)\, dx\, du\, dv\, dy\ .$$

Infinite volume (adiabatic) limit

$$g(u) \to 1 \qquad g(v) \to 1.$$

Källen-Lehmann decomposition

$$\Gamma \;=\; \frac{\gamma^2}{(\psi_0,\;\psi_0)} \int \overline{f_0(x)}\, f_0(y)\, g(u)\, g(v) \times$$

$$\times \quad \mathcal{W}_{m_0}(x,\;u)\;[\mathcal{W}_{m_1}(u,\;v)\mathcal{W}_{m_2}(u,\;v)]\,\mathcal{W}_{m_0}(v,\;y)\,dx\,du\,dv\,dy$$

$$\boxed{\mathcal{W}_{m_1}(u,\;v)\mathcal{W}_{m_2}(u,\;v) = \int \rho(a^2; m_1, m_2)\,\mathcal{W}_a(u,\;v)\,da^2}$$

$$\Gamma \;=\; \frac{\gamma^2}{(\psi_0,\;\psi_0)} \int \overline{f_0(x)}\, f_0(y)\, g(u)\, g(v) \int \rho(a^2; m_1, m_2) \times$$

$$\times \quad \mathcal{W}_{m_0}(x,\;u)\,\mathcal{W}_a(u,\;v)\;\mathcal{W}_{m_0}(v,\;y)\,dx\,du\,dv\,dy\,da^2$$

Infrared limit: step 1

$$\Gamma = \frac{\gamma^2}{(\psi_0,\ \psi_0)} \int \overline{f_0(x)}\, f_0(y)\, g(u)\, g(v) \int \rho(a^2; m_1, m_2) \times$$
$$\times \quad \mathcal{W}_{m_0}(x,\ u)\, \mathcal{W}_a(u,\ v)\ \mathcal{W}_{m_0}(v,\ y)\, dx\, du\, dv\, dy\, da^2$$

Infinite volume (adiabatic) limit: take one $g(u) \to 1.$
(Cannot take both limits because of the infrared divergence)

$$\Gamma = \frac{\gamma^2}{(\psi_0,\ \psi_0)} \int \overline{f_0(x)}\, f_0(y)\, g(v) \int \rho(a^2; m_1, m_2) \times$$
$$\times \quad \mathcal{W}_{m_0}(x,\ u)\, \mathcal{W}_a(u,\ v)\ \mathcal{W}_{m_0}(v,\ y)\, dx\, du\, dv\, dy\, da^2$$

Projector identity (completeness)

$$\Gamma \;=\; \frac{\gamma^2}{(\psi_0,\ \psi_0)} \int \overline{f_0(x)}\, f_0(y)\, g(v) \int \rho(a^2; m_1, m_2) \times$$

$$\times \quad \mathcal{W}_{m_0}(x,\ u)\, \mathcal{W}_a(u,\ v)\ \mathcal{W}_{m_0}(v,\ y)\, dx\, du\, dv\, dy\, da^2$$

The convolution can be trivially computed

$$\int \mathcal{W}_{m_0}(x,\ u)\, \mathcal{W}_a(u,\ v)\, du = C(m_0,\ d)\delta(m_0^2 - a^2)\mathcal{W}_{m_0}(x,\ v).$$

$$C(m_0,\ d) = 2\pi$$

$$\Gamma \;=\; \gamma^2 C(m_0, d)\rho(m_0^2; m_1, m_2) \times$$

$$\times \quad \frac{\int \overline{f_0(x)}\, \mathcal{W}_{m_0}(x,\ v)\, g(v)\, \mathcal{W}_{m_0}(v,\ y)\ f_0(y) dx\, dv\, dy}{\int \overline{f_0(x)}\, \mathcal{W}_{m_0}(x,\ y)\ f_0(y) dx\, dy}$$

$$= \gamma^2 C(m_0, d)\ \rho(m_0^2; m_1, m_2)\ \frac{\int\ |F_0(v)|^2 g(v)\, dv}{\int \overline{f_0(x)}\, \mathcal{W}_{m_0}(x,\ y)\ f_0(y) dx\, dy}$$

The formula

$$\Gamma = \gamma^2 C(m_0, d)\ \rho(m_0^2; m_1, m_2)\ \frac{\int |F_0(v)|^2 g(v)\, dv}{\int \overline{f_0(x)}\, \mathcal{W}_{m_0}(x,\ y)\, f_0(y) dx\, dy}$$

$$C(m_0,\ d) = 2\pi \qquad\qquad F_0(v) = \int \mathcal{W}_{m_0}(v,\ x)\, f_0(x) dx$$

This formula also holds for dS processes provided the mass is greater than a critical mass

To complete the computation we need to know the two-point function

Klein-Gordon field: a crash review.

$$(\Box + m^2)\phi(x) = 0 \qquad (\Box_{x,y} + m^2)\mathcal{W}(x,y) = 0$$

$$\xi = x - x' \qquad (\Box_\xi + m^2)\mathcal{W}(\xi) = 0$$

$$(p^2 - m^2)\tilde{\mathcal{W}}(p) = 0$$

$$\tilde{\mathcal{W}}(p) = A\,\theta(p^0)\delta(p^2 - m^2) + \cancel{B\,\theta(-p^0)\delta(p^2 - m^2)}$$

Spectral Condition

$$\mathcal{W}(x - x') = \frac{1}{(2\pi)^{d-1}} \int e^{-ip\cdot(x-x')}\theta(p^0)\delta(p^2 - m^2)dp$$

Technical intermezzo 1: computing the KL weight

Fourier (momentum space) representation of the two-point function satisfying the positivity of the energy spectrum axiom:

$$\mathcal{W}_m(u,\ v) = \frac{1}{(2\pi)^{d-1}} \int e^{-ip(u-v)} \theta(p^0)\delta(p^2 - m^2)dp$$

$$p\xi = p^0\xi^0 - p^1\xi^1 - \ldots - p^{d-1}\xi^{d-1} \qquad \xi = u - v$$

$$\mathcal{W}_{m_1}(\xi) \cdot \mathcal{W}_{m_2}(\xi) = \int \rho(a^2, m_1, m_2) W_a(\xi) da^2$$
$$= \frac{1}{(2\pi)^{2d-2}} \int e^{-i(p_1+p_2)\xi} \theta(p_1^0)\delta(p_1^2 - m_1^2)\, \theta(p_2^0)\delta(p_2^2 - m_2^2)\, dp_1\, dp_2$$

Computing the KL weight

$$\rho(s;\ m_1, m_2) = (2\pi)^{1-d} \int \delta(P - p_1 - p_2)\, \delta(p_1^2 - m_1^2)\theta(p_1^0)\, \delta(p_2^2 - m_2^2)\theta(p_2^0)\, d^d p_1\, d^d p_2\ ,$$

where $P^0 = \sqrt{s}$, $\vec{P} = 0$, $s \geq 0$.

$$\begin{aligned}
\rho(s;\ m_1, m_2) &= (2\pi)^{1-d} \int \delta(p_1^2 - m_1^2)\theta(p_1^0)\, \delta\left(s - 2\sqrt{s}p_1^0 + m_1^2 - m_2^2\right)\, d^d p_1 \\
&= \frac{(2\pi)^{1-d}\Omega_{d-1}}{2\sqrt{s}} \int_0^\infty \delta\left(r^2 + m_1^2 - \frac{(s + m_1^2 - m_2^2)^2}{4s}\right) r^{d-2}\, dr \\
&= \frac{(2\pi)^{1-d}\Omega_{d-1}}{4\sqrt{s}} \left(\frac{(s - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2}{4s}\right)^{\frac{d-3}{2}} \\
&= \frac{(2\pi)^{1-d}\Omega_{d-1}}{4\sqrt{s}} \left(\frac{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)}{4s}\right)^{\frac{d-3}{2}}
\end{aligned}$$

$$\Omega_n = \frac{2\pi^{n/2}}{\Gamma(n/2)} \qquad \forall\ n \geq 1$$

Equal masses

$$\rho(m_0^2;\ m_1,\ m_1) = \frac{1}{2^{2d-3}\pi^{\frac{d-1}{2}}\Gamma\left(\frac{d-1}{2}\right) m_0}\left(m_0^2 - 4m_1^2\right)^{\frac{d-3}{2}}\theta(m_0^2 - 4m_1^2)$$

$$d = 4 \ : \quad \rho(m_0^2;\ m_1,\ m_1) \ = \frac{1}{16\pi^2 m_0}(m_0^2 - 4m_1^2)^{\frac{1}{2}}\theta(m_0^2 - 4m_1^2)$$

Infrared limit: II step

$$\Gamma(1,2,g) = \frac{2\gamma^2 C(m_0,d)\,\rho(m_0^2;\ m_1,m_1)\ \int g(t)\,|F_0(t,\vec{x})|^2\,dt d\vec{x}}{\int \overline{f_0(x)} w_{m_0}(x,\ y)\,f_0(y) dx\,dy}$$

- $\rho(m_0, m_1, m_1)$ can be computed by Fourier transforming $W_{m_1}(x,y)^2$

$$\rho(m_0,m_1,m_1) = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}\frac{(2\pi)^{1-d}}{4m_0}\left(\frac{m_0^2-4m_1^2}{4}\right)^{\frac{d-3}{2}}\theta(m_0^2-4m_1^2).$$

Infrared limit: let g be the characteristic function of a time interval T. The transition probability is proportional to T: it diverges when $T \to \infty$. Apply Fermi's golden rule.

$$\begin{aligned}\frac{1}{\tau} &= \lim_{T\to\infty}\frac{\Gamma(1,2,g(t))}{T} = \\ &= 4\pi\gamma^2\rho(m_0^2,m_1,m_1)\times\frac{1}{2}\frac{\int \frac{|\tilde{f}_0(p)|^2}{(\vec{p}^2+M^2)}\,d\vec{p}}{\int \frac{|\tilde{f}_0(p)|^2}{\sqrt{\vec{p}^2+M^2}}d\vec{p}}\end{aligned}$$

Minkowski case: results

$$\frac{1}{\tau} = 4\pi\gamma^2\rho(m_0^2, m_1, m_1) \times \frac{1}{2}\,\frac{\int \frac{|\tilde{f}_0(p)|^2}{(\vec{p}^2+M^2)}\, d\vec{p}}{\int \frac{|\tilde{f}_0(p)|^2}{\sqrt{\vec{p}^2+M^2}} d\vec{p}}$$

Let $|\tilde{f}(p^0,\vec{p})|^2 \to \delta(\vec{p})$ (particle at rest):

$$\frac{1}{\tau_0} = \frac{2(2\pi)^{2-d}\gamma^2}{8m_0^2}\,\frac{2\pi^{(d-1)/2}}{\Gamma(\frac{d-1}{2})}\left(\frac{m_0^2 - 4m_1^2}{4}\right)^{(d-3)/2} \theta(m_0^2 - 4m_1^2)$$

$$(\mathrm{d}=4) \;\to\; \frac{\gamma^2}{8\pi m_0}\sqrt{m_0^2 - 4m_1^2}\;\; \theta(m_0 - 2m_1)$$

Use $|\tilde{f}(\Lambda_v p)|^2$ (particle moving vith velocity v):

$$\tau = \tau_0 / \sqrt{1 - \frac{v^2}{c^2}}$$

de Sitter

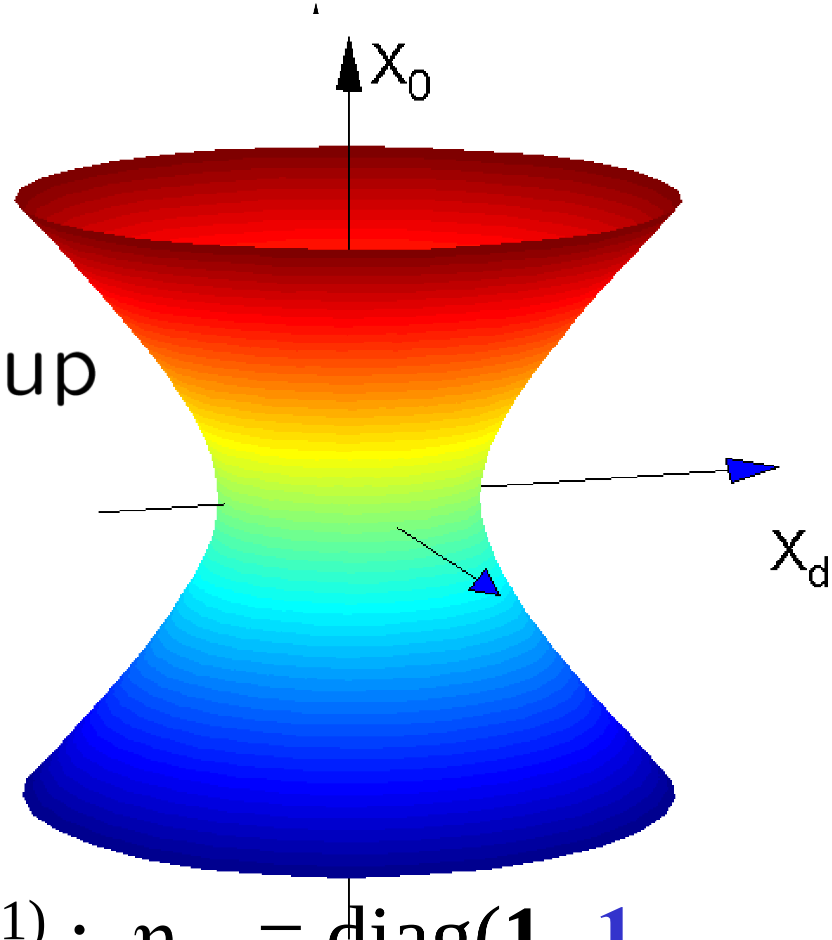
$$\Gamma = \gamma^2 C(m_0, d)\ \rho(m_0^2; m_1, m_2)\ \frac{\int |F_0(v)|^2 g(v)\, dv}{\int \overline{f_0(x)}\, \mathcal{W}_{m_0}(x,\ y)\, f_0(y) dx\, dy}$$

$$F_0(v) = \int \mathcal{W}_{m_0}(v,\ x)\, f_0(x) dx$$

$\mathcal{W}_m(x,\ y) = ?$ $\qquad C(m,\ d) = ?$ $\qquad \rho(m_0^2; m_1, m_2) = ?$

The de Sitter Universe (1917)

$$X_0^2 - X_1^2 - \ldots X_d^2 = -R^2$$



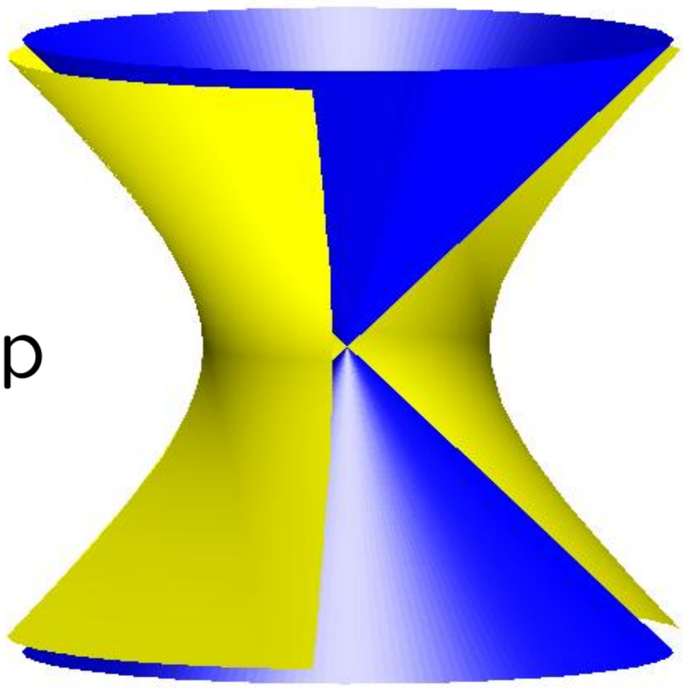
Relativity group

$SO(1, d)$

$M^{(d+1)}$: $\eta_{\mu\nu} = \mathrm{diag}(\mathbf{1}, \mathbf{-1}, \ldots\ldots, \mathbf{-1})$

The asymptotic cone

$$\{\xi_0^2 - \xi_1^2 - \ldots - \xi_d^2 = 0\}$$

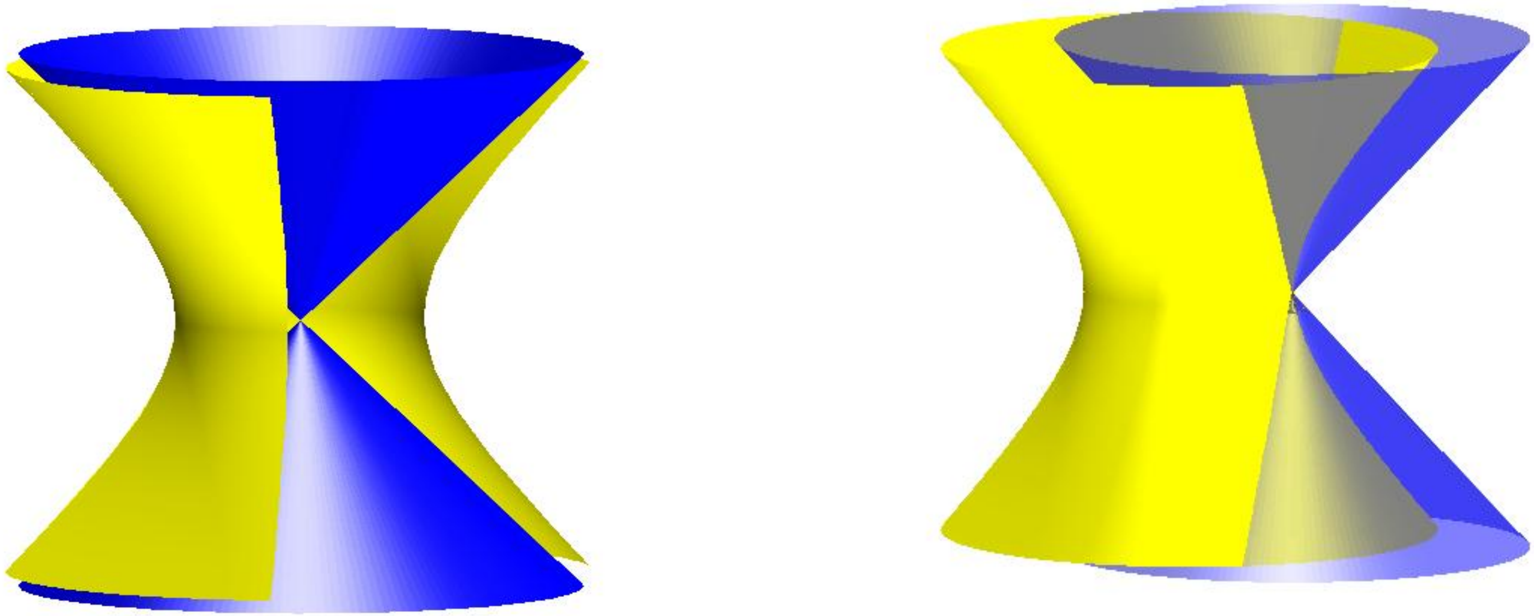


Isometry group
$SO(1,d)$

$M^{(d+1)}$: $\eta_{\mu\nu}$ = diag(**1**,**-1**,**…….**,**-1**)

The asymptotic cone provides the causal structure

$$\{\xi_0^2 - \xi_1^2 - \ldots - \xi_d^2 = 0\}$$



X , Y are spacelike separated iff (X-Y)^2 < 0
(X-Y is outside the cone)

Classical free particles

Write the action in a coordinate system

$$S = -mc \int \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} d\lambda\,;$$

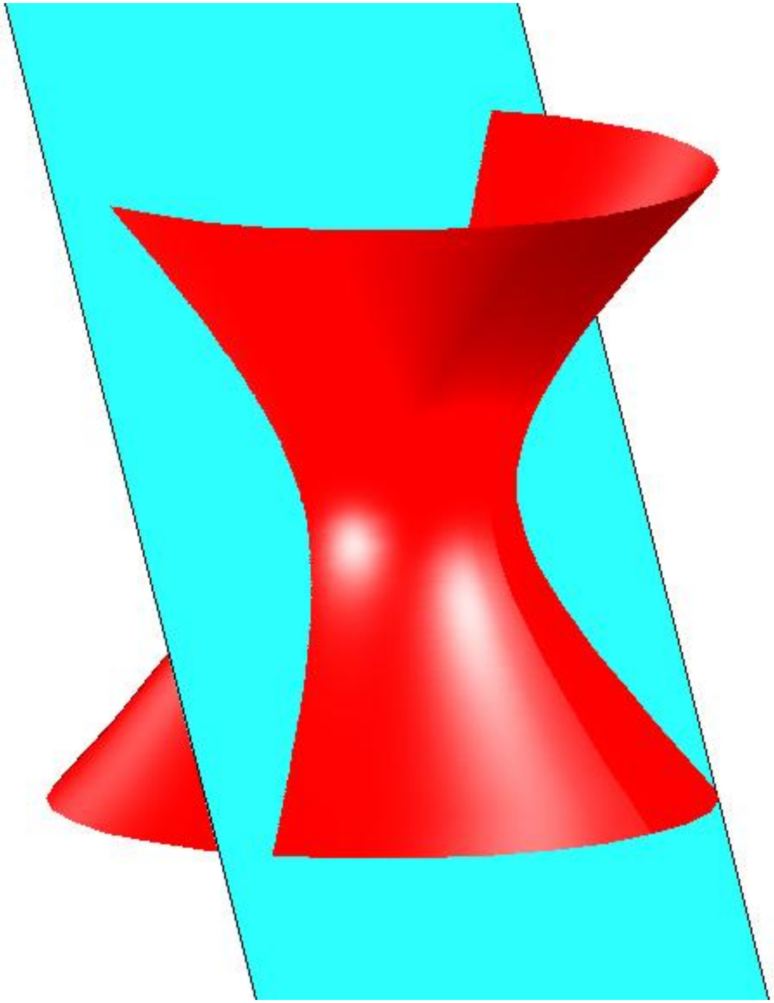
Solve the affine geodesic equations

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0$$

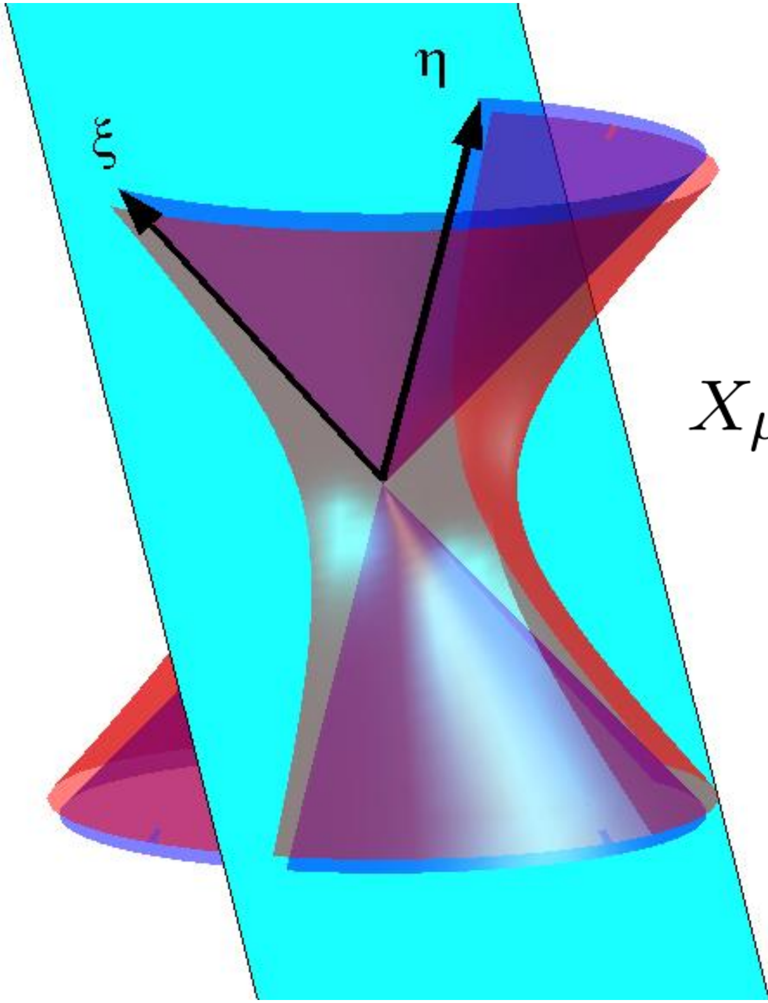
Trivial but not easy in practice!

Results bury an otherwise simple structure

Geodesics: a geometrical approach



Parametrization: the asymptotic cone as de Sitter momentum space

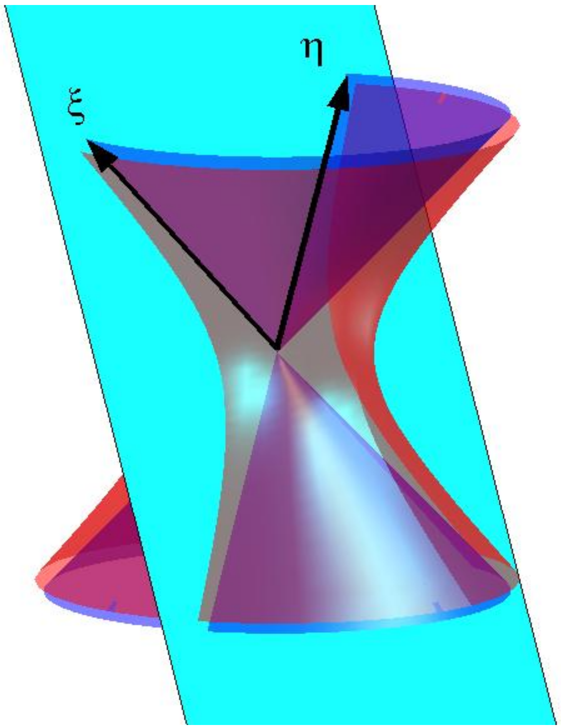


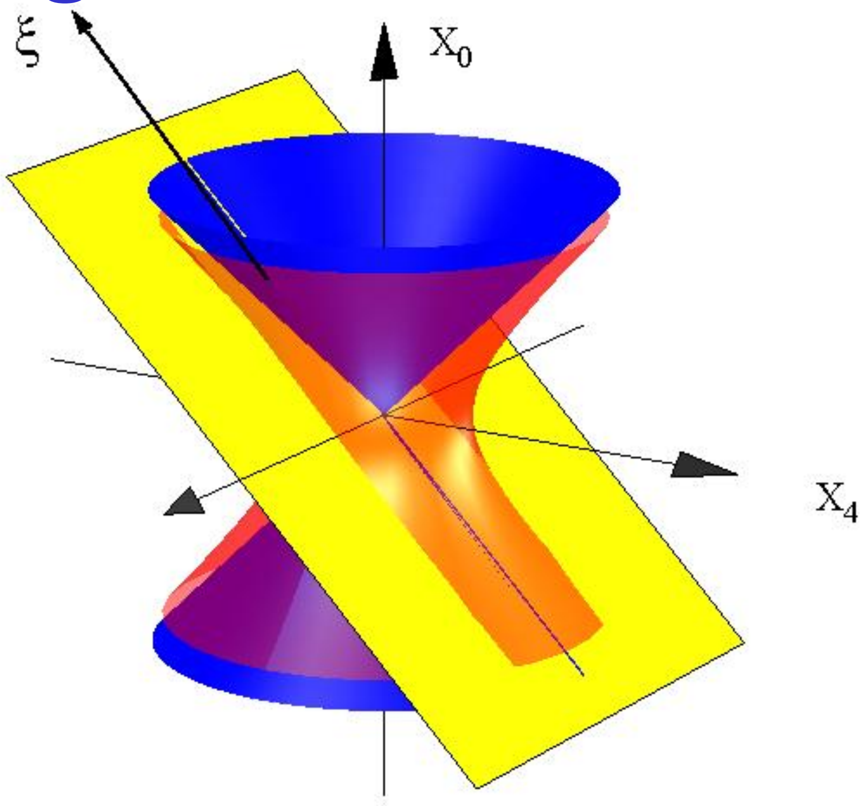
$$X_\mu(s) = \frac{R}{\sqrt{2\xi \cdot \eta}} \left(\xi_\mu e^{\frac{s}{R}} - \eta_\mu e^{-\frac{s}{R}} \right)$$

to be compared with

$$x_\mu(s) = x_\mu(0) + \left(\frac{p_\mu}{mc} \right) s$$

Lightlike geodesics





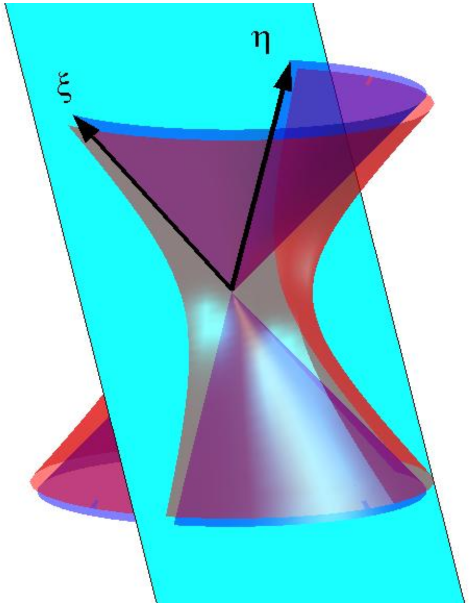
$$X_\mu(\lambda) = x_0 + \xi_\mu \lambda, \qquad \text{with } \xi \cdot x_0 = 0$$

to be compared with

$$x_\mu(\lambda) = x_\mu(0) + k_\mu \lambda$$

Conserved quantities

$$X_\mu(\tau) = \frac{R}{\sqrt{2\xi\cdot\eta}}\left(\xi_\mu e^{\frac{c\tau}{R}} - \eta_\mu e^{-\frac{c\tau}{R}}\right)$$



$$K_{\xi,\eta} = mc\frac{\xi\wedge\eta}{\xi\cdot\eta}$$

Classical scattering

$$b_1 + b_2 \longrightarrow c_1 + \ldots + c_M$$

$$X_i^\mu(\tau) = \frac{R}{\sqrt{2\chi\cdot\zeta}}\left(\chi^\mu e^{\frac{c\tau}{R}} - \zeta^\mu e^{-\frac{c\tau}{R}}\right) \qquad X_f^\mu(\tau) = \frac{R}{\sqrt{2\xi\cdot\eta}}\left(\xi^\mu e^{\frac{c\tau}{R}} - \eta^\mu e^{-\frac{c\tau}{R}}\right)$$

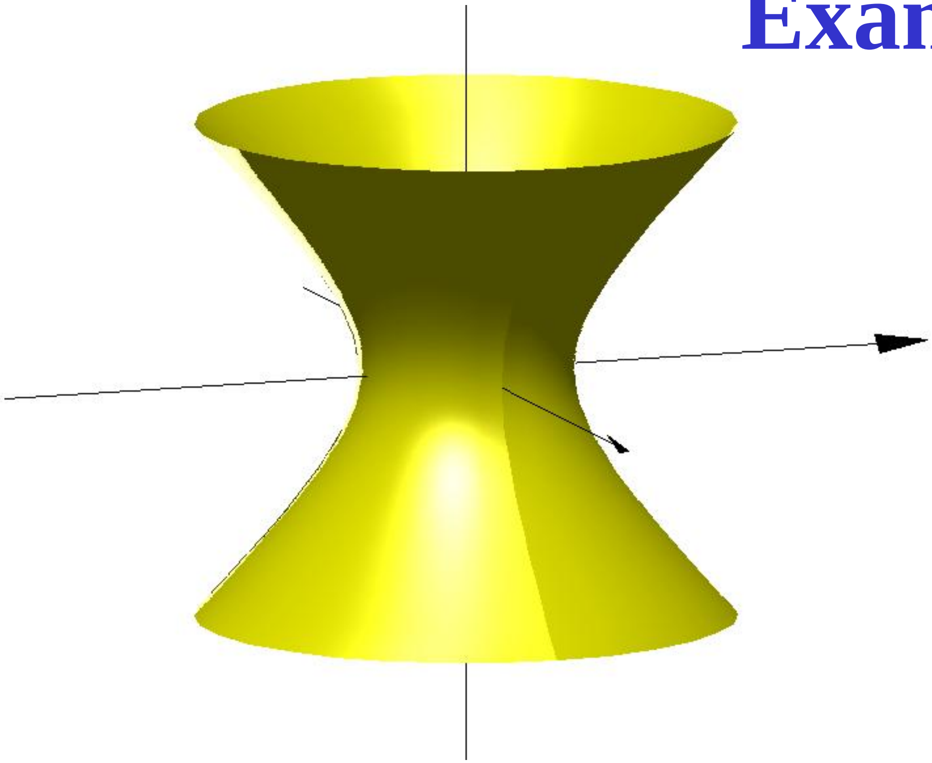
Solving the collision problem amounts to find the outgoing vectors (ξ_f, η_f) given the ingoing ones (χ_i, ζ_i).

$$K_i = m_i c \frac{\chi_i \wedge \zeta_i}{\chi_i \cdot \zeta_i}\,, \quad i = 1, 2\,; \qquad K_f = \tilde{m}_f c \frac{\xi_f \wedge \eta_f}{\xi_f \cdot \eta_f}\,, \quad f = 1, \ldots, M\,.$$

$$\sum_{i=1}^{2} K_i = \sum_{i=1}^{2} K_i\Big|_{x=\bar{x}} = \sum_{f=1}^{M} K_f\Big|_{x=\bar{x}} = \sum_{f=1}^{M} K_f.$$

$$\boxed{\frac{\chi_i - \zeta_i}{\sqrt{\chi_i\cdot\zeta_i}} = X = \frac{\xi_f - \eta_f}{\sqrt{\xi_f\cdot\eta_f}}\,,}$$

Example



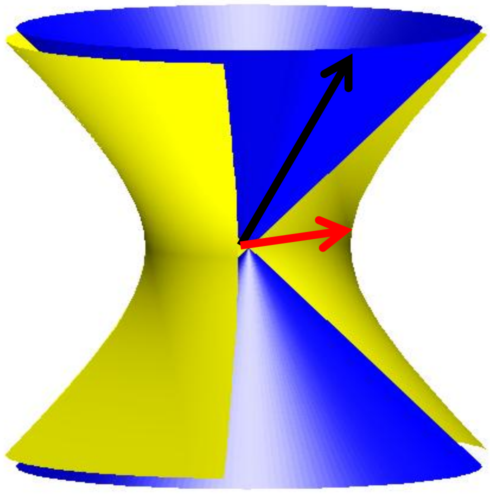
$$m_1 \longrightarrow \mu_1 + \mu_2 \ ,$$

$$\begin{aligned}
\xi &= \frac{1}{\sqrt{2}}\left(\frac{m_1^2+\mu_1^2-\mu_2^2}{2m_1\mu_1}, \mp\frac{\sqrt{(m_1^2-\mu_1^2-\mu_2^2)^2-4\mu_1^2\mu_2^2}}{2m_1\mu_1}, 1\right) \\
\eta &= \frac{1}{\sqrt{2}}\left(\frac{m_1^2-\mu_1^2+\mu_2^2}{2m_1\mu_2}, \pm\frac{\sqrt{(m_1^2-\mu_1^2-\mu_2^2)^2-4\mu_1^2\mu_2^2}}{2m_1\mu_2}, 1\right) \qquad (1)
\end{aligned}$$

with the restriction $m_1 > \mu_1 + \mu_2$. This condition no more holds true at the quantum level

What is it, a de Sitter Plane Wave?

$$\psi(x,p) = \exp(ix \cdot p)$$



$$\Psi(X,\xi) = (\xi \cdot X)^s$$

ξ in C^+ **X in dS** s complex

Physical values: $s = -\frac{d-1}{2} + i\nu \begin{cases} \nu \in \mathbf{R} \\ \nu' = i\nu \in \mathbf{R} \quad |\nu| < \frac{d-1}{2} \end{cases}$

Two types of de Sitter Waves

$\psi(X,\xi) = (X \cdot \xi)^{-\frac{d-1}{2}+i\nu}$ principal waves

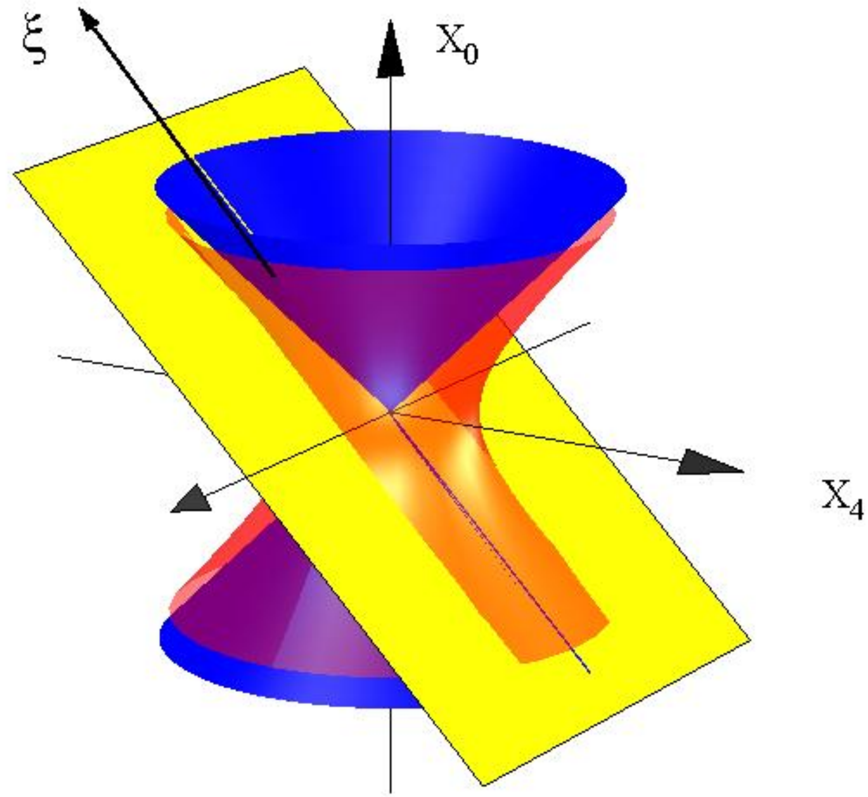
$\psi(X,\xi) = (X \cdot \xi)^{-\frac{d-1}{2}+\nu'}$ complementary waves

However, they are irregular

$$\psi(x,p) = \exp(ix \cdot p)$$

$$\Psi(X,\xi) = (\xi \cdot X)^s$$

ξ **in C$^+$** **X in dS,** s complex



How to glue them?
Spectral property!

Spectral Property

There exists a complete set of nonegative energy states

(The energy-momentum spectrum is in the closed future cone)

equivalent to

$W(x_1,\ldots\ldots, x_n)$ is the boundary value of a function $W(z_1,\ldots\ldots, z_n)$ holomorphic in a "tube" of the complex Minkowski spacetime

(tube = {Im $(z_{k+1} - z_k)$ contained in the closed future cone})

Consequences

1) **Two-point functions**

2) **Free fields**

3) **Perturbation theory**

4) **Renormalization**

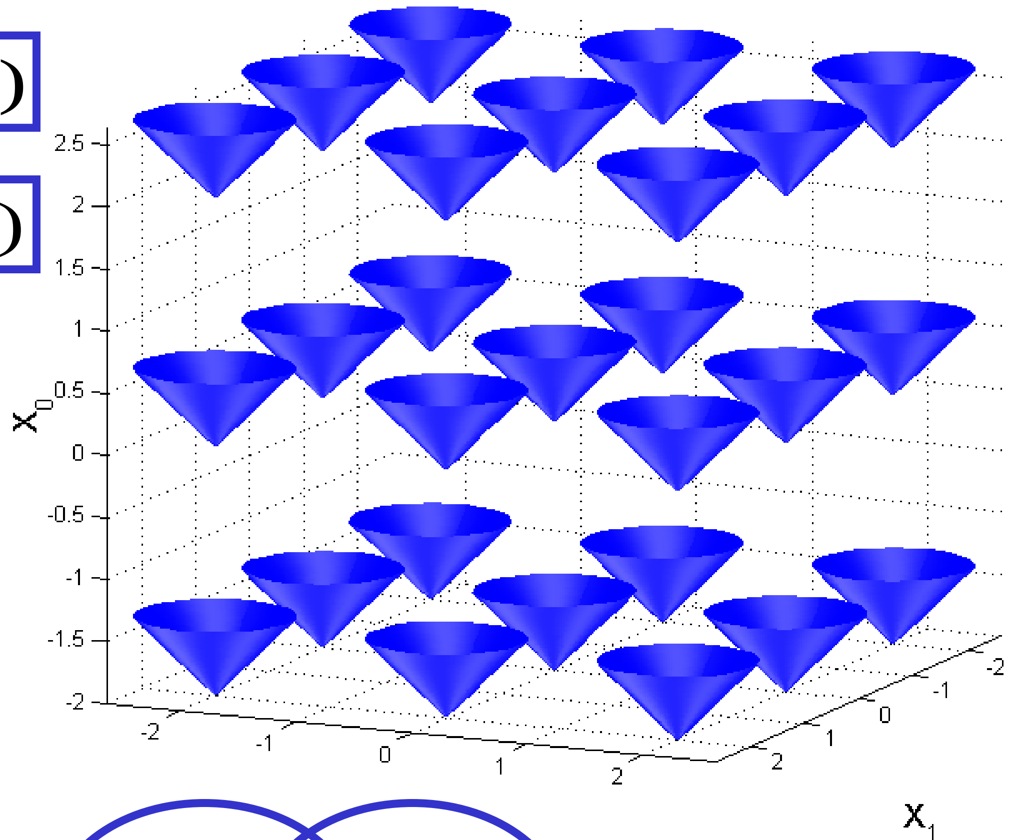
2) The spectral condition implies that the integral representation makes sense in a larger complex domain of the complex Minkowski space-time

$$\mathrm{z} \in T^{-} \ (\mathrm{Im}\, \mathrm{z} \in \mathrm{V}^{-})$$

$$\mathrm{z}' \in T^{+} \ (\mathrm{Im}\, \mathrm{z}' \in \mathrm{V}^{+})$$

$$\Downarrow$$

$$\mathrm{z}' - \mathrm{z} \in T^{+}$$
$$(\mathrm{Im}(\mathrm{z}' - \mathrm{z}) \in \mathrm{V}^{+})$$



$$W(z - z') = \int e^{-ip\cdot z} e^{ip\cdot z'} \theta(p^0)\delta(p^2 - m^2)$$

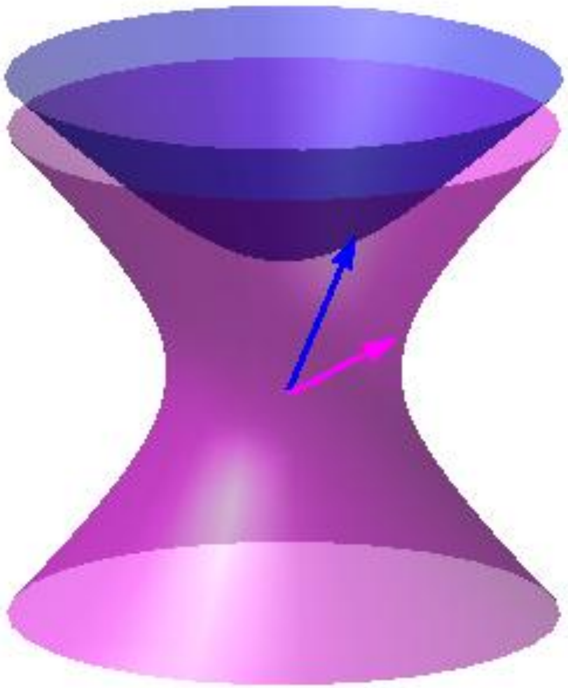
One point tubes (dS)

$$dS^c = Z_0^2 - Z_1^2 - \ldots Z_d^2 = -R^2$$

Z=X+iY

$$\boxed{\mathbf{X}^2 - \mathbf{Y}^2 = -R^2} \qquad \boxed{\mathbf{X}\cdot\mathbf{Y} = 0}$$

$$\mathcal{T}^{\pm} = T^{\pm} \cap \mathrm{dS}^{(c)} = \left\{ \begin{matrix} Z \in \mathrm{dS}^{(c)} \to Y^2 > 0, \\ Y_0 > 0 \ \ (\text{or } Y_0 < 0) \end{matrix} \right\}$$

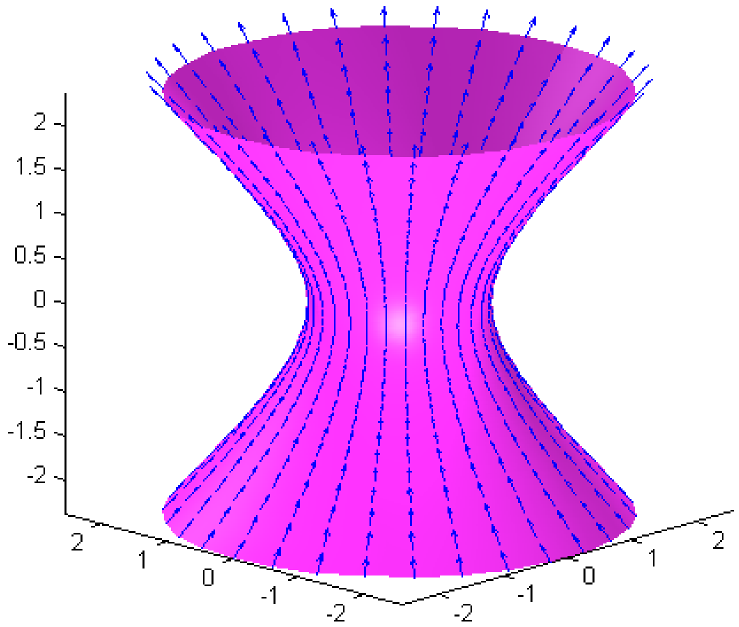


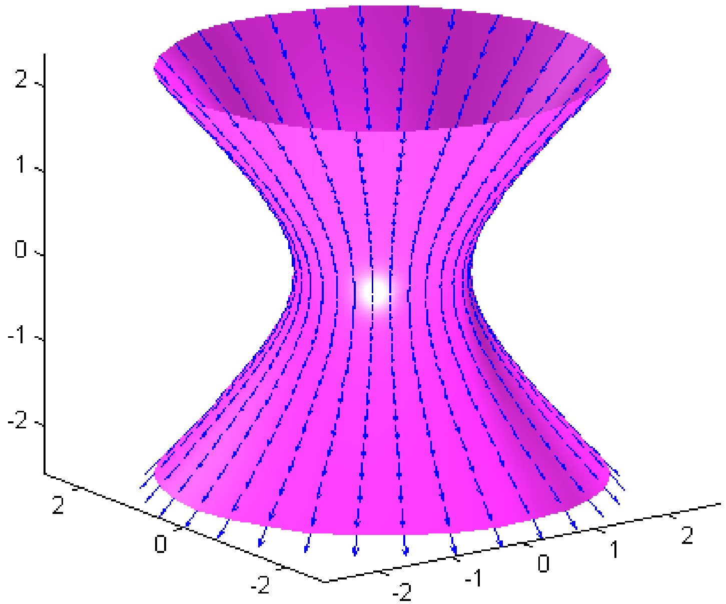
One point tubes (dS)

$\mathbf{Z} = \mathbf{X} + i\mathbf{Y}$

$$\boxed{\mathbf{X}^2 - \mathbf{Y}^2 = -R^2 \qquad \mathbf{X}\cdot\mathbf{Y} = 0}$$

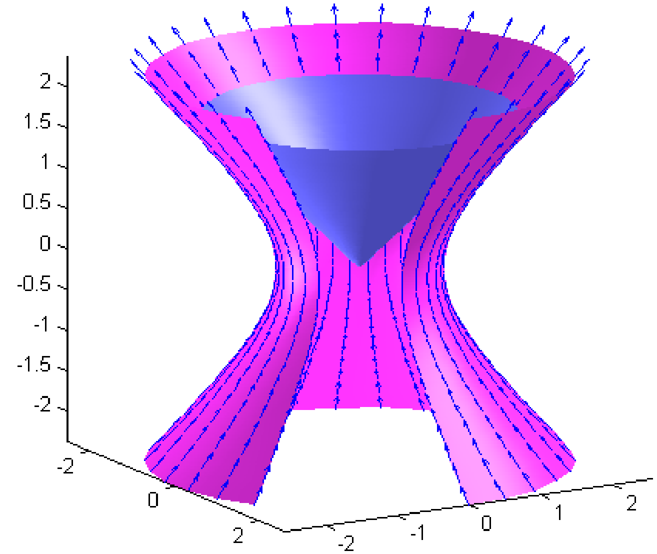
$$\boxed{\begin{array}{l}\mathcal{T}^+ = T^+ \cap dS^{(c)} = \\ \left\{ Y^2 > 0,\ Y_0 > 0 \right\}\end{array}} \qquad \boxed{\begin{array}{l}\mathcal{T}^- = T^- \cap dS^{(c)} = \\ \left\{ Y^2 > 0,\ Y_0 < 0 \right\}\end{array}}$$





$$\Psi(Z,\xi) = (\xi\cdot Z)^s$$

are globally well-defined in the tubes.

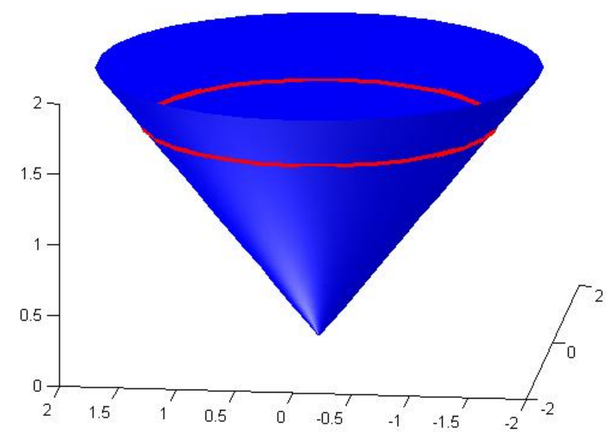


For $Z \in \mathcal{T}^-$ and $Z' \in \mathcal{T}^+$

$$W(Z,Z') = \int_\gamma (Z\cdot\xi)^{-\frac{d-1}{2}+i\nu}(\xi\cdot Z')^{-\frac{d-1}{2}-i\nu}d\mu(\xi)$$

To be compared with $(z \in T$

$$W(z-z') = \int e^{-ip\cdot z}e^{ip\cdot z'}($$



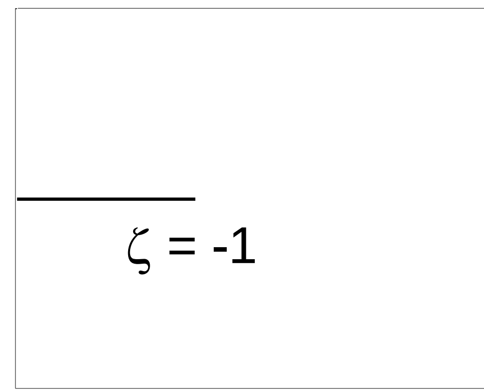
For $Z \in \mathcal{T}^-$ and $Z' \in \mathcal{T}^+$, $\quad \zeta = \frac{Z \cdot Z'}{R^2}$

$$W_\nu(Z,Z') = \int_\gamma (Z \cdot \xi)^{-\frac{d-1}{2}+i\nu} (\xi \cdot Z)^{-\frac{d-1}{2}-i\nu} d\mu(\xi)$$

$$= \frac{\Gamma\left(\frac{d-1}{2}+i\nu\right)\Gamma\left(\frac{d-1}{2}-i\nu\right)}{2(2\pi)^{\frac{d}{2}} R^{d-2}} \times (\zeta^2-1)^{-\frac{d-2}{4}} P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(\zeta);$$

a) $W(Z,Z')$ is de Sitter invariant

b) $W(Z,Z')$ is maximally analytic

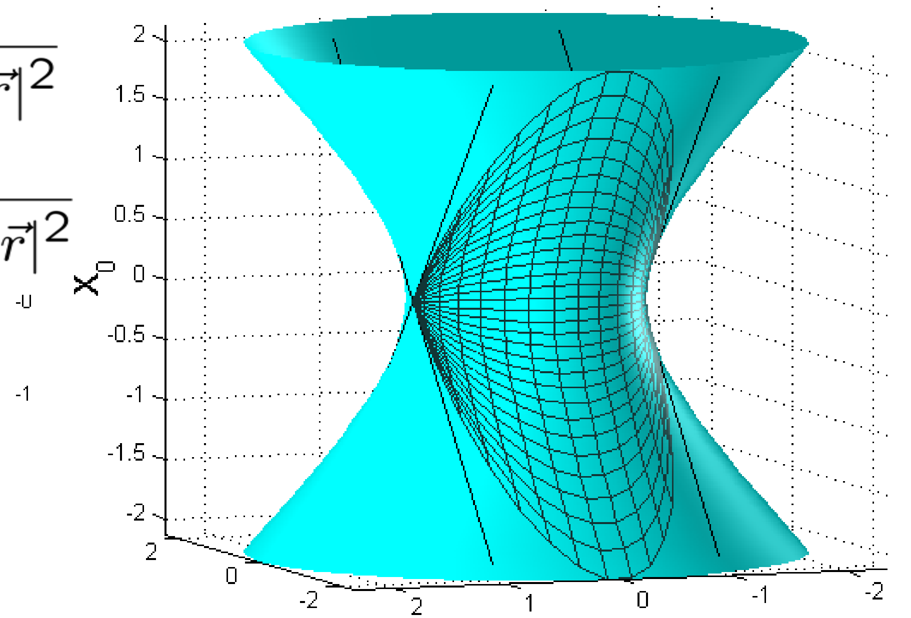


c) $W(X,X')$ is the b.v. of $W(Z,Z')$ from $\mathcal{T}^- \times \mathcal{T}^+$

d) $W(X',X)$ is the b.v. of $W(Z,Z')$ from $\mathcal{T}^+ \times \mathcal{T}^-$

Physical interpretation:
de Sitter temperature

$$X(t,\vec{r}) = \begin{cases} X_0 & = & R \sinh \frac{t}{R} \sqrt{R^2 - |\vec{r}|^2} \\ X_i & = & r_i \qquad (|\vec{r}|^2 \leq R^2) \\ X_d & = & R \cosh \frac{t}{R} \sqrt{R^2 - |\vec{r}|^2} \end{cases}$$

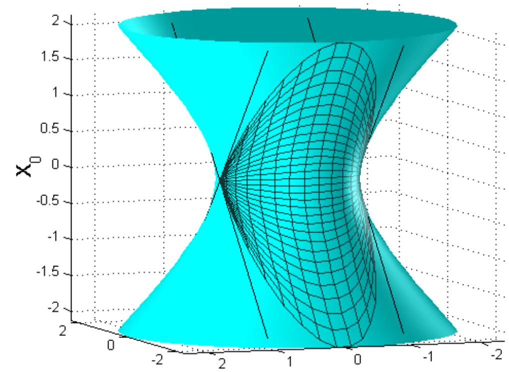


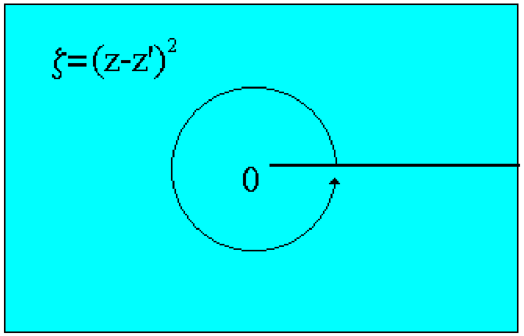
Time translations:

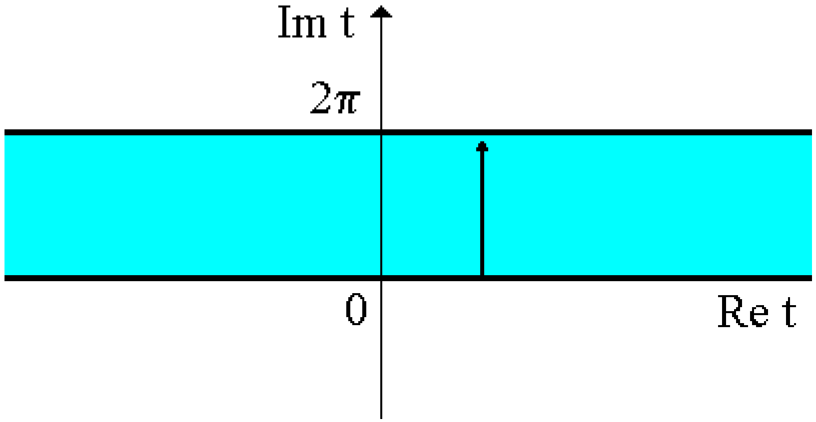
$$\alpha(s) X(t,\vec{r}) = X(t+s,\vec{r}) \equiv X(s)$$

Maximal analyticity ☛ KMS condition

$$W(X,X'(t)) = \left(\Omega,\ \varphi(X)U(t)\varphi(X')\,U(t)^{-1}\,\Omega\right)$$



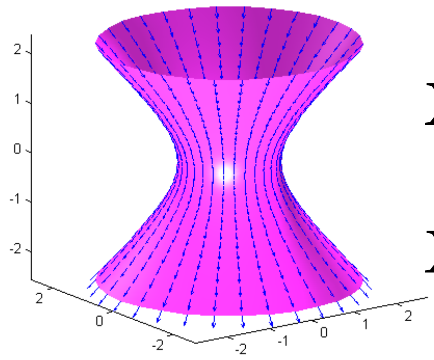




1) $W(X,X'(t))$ is analytic in the strip $0 < \mathrm{Im}\, t < 2\pi R$
2) For real t, $W(X,X'(t+2\pi i)) = W(X'(t),X)$

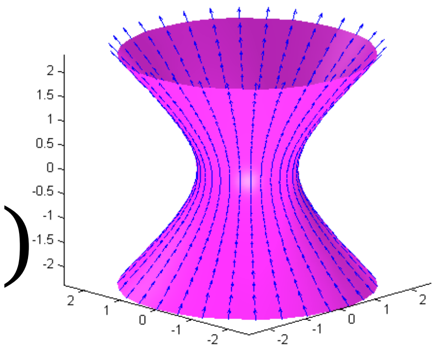
KMS condition at inverse temperature $2\pi R$

Fourier transforms



$$x - i0 \in \mathcal{T}^{-} \; (\mathrm{Im}\, Z \in V^{-})$$

$$x + i0 \in \mathcal{T}^{+} \; (\mathrm{Im}\, Z' \in V^{+})$$



$$\tilde{f}_{+}(\xi,\nu) = \int_{dS} f(x)(x_{+}\cdot\xi)^{-\frac{d-1}{2}-i\nu}dx$$

$$\tilde{f}_{-}(\xi,\nu) = \int_{dS} f(x)(x_{-}\cdot\xi)^{-\frac{d-1}{2}-i\nu}dx$$

One particle spaces

$\mathcal{D} =$ smooth complex functions on dS

$\mathcal{H}_\nu =$ completion of $\mathcal{D}$ w.r.t. the scalar product

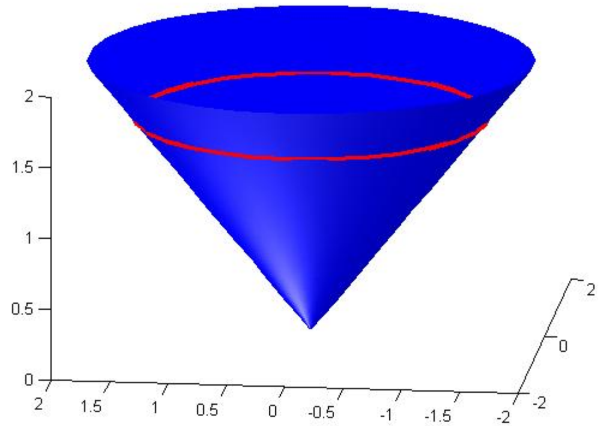
$$\langle f, g\rangle = \int \bar{f}(x) W_\nu(x,y) g(y)\, dx\, dy$$

$$= c \int_\gamma \int_{dS\times dS} \bar{f}(x)(x_-\cdot\xi)^{-\frac{d-1}{2}+i\nu}(\xi\cdot y_+)^{-\frac{d-1}{2}-i\nu} g(y)\, dx\, dy\, d\mu(\xi)$$

$$= c \int_\gamma \bar{\tilde{f}}_-(\xi,-\nu)\tilde{g}_+(\xi,\nu)\, d\mu(\xi)$$

$$\langle f, g\rangle = \int \bar{\tilde{f}}_+(\xi,\nu)\tilde{g}_+(\xi,\nu)\, d\mu(\xi)$$

$$\mathcal{H}_\nu = L^2(\gamma, d\mu(\xi)), \quad \nu \in \mathbf{R}$$



de Sitter decays

$$\kappa \longrightarrow \nu + \nu$$

$$A(1,2,g) = \frac{2\lambda^2\, C(\kappa)\rho(\kappa^2,\nu)\ \int g(x)\, |F(x)|^2\, dx}{\int \overline{f_0(x)} W_\kappa(x,\ y)\, f_0(y)\, dx\, dy}$$

- Projector identity: nontrivial; holds only for the principal series

$$\int w_\nu(z,\ x)\, w_{\nu'}(x,\ y)\, dx = 2\pi \coth \pi\nu\delta(\nu^2 - \nu'^2) w_\nu(z,\ y)$$

KL weight: nontrivial

$$A(1,2,g) = \frac{4\lambda^2 \coth(\pi\kappa)\ \rho(\kappa^2,\nu)\ \int g(x)\,|F(x)|^2\,dx}{\int \overline{f_0(x)} W_\kappa(x,\ y)\, f_0(y)\,dx\,dy}$$

$$W_\nu(z,z') = \frac{\Gamma\left(\frac{d-1}{2}+i\nu\right)\Gamma\left(\frac{d-1}{2}+i\nu\right)}{2(2\pi)^{\frac{d}{2}}R^{d-2}} \times (\zeta^2-1)^{-\frac{d-2}{4}}\, P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(\zeta)$$

Mehler-Fock transform of the 2-point function squared:

$$\rho(\kappa^2,\nu) = \frac{\left(\Gamma\left(\frac{d-1}{2}+i\nu\right)\Gamma\left(\frac{d-1}{2}-i\nu\right)\right)^2 \sinh\pi\kappa}{2(2\pi)^{1+\frac{d}{2}}R^{d-2}} \int_1^\infty P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\kappa}(x)\,[P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(x)]^2(x^2-1)^{-\frac{d-2}{4}}\,dx.$$

Formula 19, p. 535 of Prudnikov et al. Vol. III:

Three dimensions: easy

$$\int_0^\infty \frac{\sin\kappa v\,\sin^2\nu v}{\sinh v}dv = \frac{\sinh(\pi\nu)^2\,\tanh(\frac{\pi\kappa}{2})}{(\cosh(\pi\kappa)+\cosh(2\pi\nu))}$$

Sketch of proof of the formula for ρ

$$H_\nu(\kappa) \;=\; \int_1^\infty P^{-\frac{d-2}{2}}_{-\frac12+i\kappa}(x)\,[P^{-\frac{d-2}{2}}_{-\frac12+i\nu}(x)]^2(x^2-1)^{-\frac{d-2}{4}}\,dx = \int_0^\infty \frac{P^\mu_\alpha(\sqrt{1+\zeta})}{2\sqrt{1+\zeta}}\,\zeta^{\mu/2}[P^\mu_\beta(\sqrt{1+\zeta})]^2\,d\zeta = \int_0^\infty G_1(\zeta)\,G_2(\zeta)\,\frac{d\zeta}{\zeta}, \tag{1}$$

with

$$\alpha = -\tfrac12 + i\kappa, \quad \beta = -\tfrac12 + i\nu, \quad \mu = 1-\frac{d}{2}\,. \quad G_1(\zeta) = \frac{P^\mu_\alpha(\sqrt{1+\zeta})}{2\sqrt{1+\zeta}}, \quad G_2(\zeta) = \zeta^{\frac{\mu}{2}+1}[P^\mu_\beta(\sqrt{1+\zeta})]^2\,. \tag{2}$$

Compute the Mellin transforms

$$\widehat{G}_j(s) = \int_0^\infty \zeta^{s-1}\,G_j(\zeta)\,d\zeta \tag{3}$$

$$\widehat{G}_1(s) = \frac{2^{\mu-1}}{\Gamma\left(1+\frac{\alpha-\mu}{2}\right)\Gamma\left(\frac{1-\mu-\alpha}{2}\right)}\Gamma\left[\begin{array}{l} s-\frac{\mu}{2},\;\; 1+\frac{\alpha}{2}-s,\;\; \frac{1-\alpha}{2}-s \\ 1-\frac{\mu}{2}-s \end{array}\right] \tag{4}$$

$$\widehat{G}_2(s) = \frac{1}{\pi^{\frac12}\Gamma(1+\beta-\mu)\Gamma(-\beta-\mu)}\,\Gamma\left[\begin{array}{l} 1-\frac{\mu}{2}+s,\;\; \beta-\frac{\mu}{2}-s,\;\; -\beta-1-\frac{\mu}{2}-s,\;\; -\frac{1+\mu}{2}-s \\ -\frac{3\mu}{2}-s,\;\; -\frac{\mu}{2}-s \end{array}\right] \tag{5}$$

We now use the Mellin-Plancherel formula

$$\int_0^\infty G_1(\zeta)\,G_2(\zeta)\,\frac{d\zeta}{\zeta} = \frac{1}{2i\pi}\int_{\sigma-i\infty}^{\sigma+i\infty}\widehat{G}_1(s)\widehat{G}_2(-s)\,ds\,. \tag{6}$$

combined with Barnes' Second Lemma:

Lemma 1 (Barnes)

$$\frac{1}{2i\pi}\int_{-i\infty}^{i\infty}\Gamma\left[\begin{array}{l} \alpha_1+u,\; \alpha_2+u,\; \alpha_3+u,\; 1-\beta_1-u,\; -u \\ \beta_2+u \end{array}\right]du =$$
$$= \Gamma\left[\begin{array}{l} \alpha_1,\; \alpha_2,\; \alpha_3,\; 1-\beta_1+\alpha_1,\; 1-\beta_1+\alpha_2,\; 1-\beta_1+\alpha_3 \\ \beta_2-\alpha_1,\; \beta_2-\alpha_2,\; \beta_2-\alpha_3 \end{array}\right], \tag{7}$$

provided

$$\alpha_1+\alpha_2+\alpha_3+1-\beta_1-\beta_2 = 0\,, \tag{8}$$

and that the contour of integration separates the increasing and decreasing series of poles.

As a function of s, $\widehat{G}_1(s)\widehat{G}_2(-s)$, is, up to a factor, of the form of the integrand in Barnes lemma if we take

$$a_1 = \frac{d-4}{4}+i\nu,\; a_2 = \frac{d-4}{4}-i\nu,\; a_3 = \frac{d-4}{4},\; b_1 = \frac34+\frac{i\kappa}{2},\; b_2 = \frac34-\frac{i\kappa}{2},\; c = \frac{3d-6}{4}. \tag{9}$$

This choice satisfies the conditions. Therefore

$$H_\nu(\kappa) = \frac{1}{2^{\frac{d}{2}}\sqrt{\pi}\,\Gamma\left[\frac{d+1}{4}+\frac{i\kappa}{2},\; \frac{d+1}{4}-\frac{i\kappa}{2},\; \frac{d-1}{2}+i\nu,\; \frac{d-1}{2}-i\nu\right]}\times$$
$$\times\Gamma\left[\begin{array}{l} \frac{d-1}{4}+\frac{i\kappa}{2}+i\nu,\; \frac{d-1}{4}+\frac{i\kappa}{2}-i\nu,\; \frac{d-1}{4}+\frac{i\kappa}{2},\; \frac{d-1}{4}-\frac{i\kappa}{2}+i\nu,\; \frac{d-1}{4}-\frac{i\kappa}{2}-i\nu,\; \frac{d-1}{4}-\frac{i\kappa}{2} \\ \frac{d-1}{2}-i\nu,\; \frac{d-1}{2}+i\nu,\; \frac{d-1}{2} \end{array}\right]. \tag{10}$$

KL weight: properties

$$A(1,2,g) = \frac{4\lambda^2 \coth(\pi\kappa)\ \rho(\kappa^2,\nu)\ \int g(x)\, |F(x)|^2\, dx}{\int \overline{f_0(x)} W_\kappa(x,\ y)\, f_0(y)\, dx\, dy}$$

$$\rho(\kappa^2,\nu) = \frac{R^{2-d} \sinh \pi\kappa}{(4\pi)^{\frac{d+2}{2}} \sqrt{\pi}\, \Gamma(\frac{d-1}{2})} \frac{\Gamma\left(\frac{d-1}{4} + \frac{i\kappa}{2}\right) \Gamma\left(\frac{d-1}{4} - \frac{i\kappa}{2}\right)}{\Gamma\left(\frac{d+1}{4} + \frac{i\kappa}{2}\right) \Gamma\left(\frac{d+1}{4} - \frac{i\kappa}{2}\right)}$$
$$\times\, \Gamma\left(\frac{d-1}{4} + i\nu + \frac{i\kappa}{2}\right) \Gamma\left(\frac{d-1}{4} - i\nu + \frac{i\kappa}{2}\right) \Gamma\left(\frac{d-1}{4} + i\nu - \frac{i\kappa}{2}\right) \Gamma\left(\frac{d-1}{4} - i\nu - \frac{i\kappa}{2}\right)$$

The weight ρ never vanishes. For $m > m_c$ decays into heavier particles are always possible.

The Minkowskian result is recovered in the limit of zero curvature: $\kappa = MR$ and $\nu = mR$:

$$\lim_{R\to\infty} \rho(\kappa^2;\nu)\ d\kappa^2 = \rho(M^2;m)\ dM^2. \qquad (1)$$

Corrections

Lowest order corrections to the flat case give:

$$R^2\rho \sim \frac{|\Delta m|^{\frac{d-3}{2}}}{2^d \pi^{\frac{d-1}{2}} \Gamma\left(\frac{d-1}{2}\right) M} \left(\frac{M+2m}{4}\right)^{\frac{d-3}{2}} \times$$

$$\times \left(1 + \frac{A}{R^2}\right) \left[\theta(\Delta m) + e^{-|\Delta m|R} \theta(-\Delta m)\right]$$

$\Delta m = M - 2m$. The lack of particle stability ($\Delta m < 0$) is exponentially small in R. If $\Delta m > 0$ there is a correction of the order of the cosmological constant $\Lambda = \frac{(d-1)(d-2)}{2R^2}$. In the four dimensional case

$$A = \frac{17}{64\left(m + \frac{M}{2}\right)^2} - \frac{107}{24M^2} + \frac{17}{64\left(\frac{M}{2} - m\right)^2} \quad (1)$$

Unequal masses

Compute the following Mehler-Fock transform

$$h_d(\kappa,\nu,\lambda) = \int_1^\infty P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\kappa}(u)\, P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(u) P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\lambda}(u)(u^2-1)^{-\frac{d-2}{4}}\, du$$

Kallen-Lehmann weight is given by

$$\rho(\kappa^2,\nu,\lambda) \;=\; \frac{\Gamma\left(\frac{d-1}{2}+i\nu\right)\Gamma\left(\frac{d-1}{2}-i\nu\right)\Gamma\left(\frac{d-1}{2}+i\lambda\right)\Gamma\left(\frac{d-1}{2}-i\lambda\right)}{2(2\pi)^{1+\frac{d}{2}}} \sinh(\pi\kappa)\; h_d(\kappa,\nu,\lambda),$$

Previous Mellin method does not work. Need something like a vector Fourier transform

Unequal masses

Evaluate the two-point function at purely imaginary events

$$\mathcal{W}_\nu(-iy, iy') = \frac{\Gamma(\frac{d-1}{2}+i\nu)\Gamma(\frac{d-1}{2}-i\nu)}{2^{d+1}\pi^d}\int_\gamma (y\cdot\xi)^{-\frac{d-1}{2}+i\nu}\left(\xi\cdot y'\right)^{-\frac{d-1}{2}-i\nu} d\mu_\gamma(\xi) =$$

$$= \frac{\Gamma\left(\frac{d-1}{2}+i\nu\right)\Gamma\left(\frac{d-1}{2}-i\nu\right)}{2(2\pi)^{\frac{d}{2}}}\left(\left(y\cdot y'\right)^2-1\right)^{-\frac{d-2}{4}} P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}\left(y\cdot y'\right)$$

$\gamma = \gamma_0$ and $y' = (1,0,\ldots,0)$ so that $y\cdot y' = y^0 = u \geq 1,$

$$\left(u^2-1\right)^{-\frac{d-2}{4}} P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(u) = \frac{1}{(2\pi)^{\frac{d}{2}}}\int_{\gamma_0}(y\cdot\xi)^{-\frac{d-1}{2}-i\nu} d\mu_\gamma(\xi)$$

Unequal masses

$$h_d(\kappa,\nu,\lambda) = \int_1^\infty P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\kappa}(u)\, P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(u) P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\lambda}(u)(u^2-1)^{-\frac{d-2}{4}}\, du =$$

$$\frac{1}{(2\pi)^{\frac{3d}{2}}\omega_{d-1}} \int_{{\gamma_0}^3}\int_{H_d} (y\cdot\xi_1)^{-\frac{d-1}{2}-i\kappa}\,(y\cdot\xi_2)^{-\frac{d-1}{2}-i\nu}\,(y\cdot\xi_3)^{-\frac{d-1}{2}-i\lambda}\, dy\, d\mu_\gamma(\xi_1)\, d\mu_\gamma(\xi_2)\, d\mu_\gamma(\xi_3)$$

First step: a star-triangle identity

$$\begin{aligned} F_{a_1,a_2,a_3}(\xi_1,\xi_2,\xi_3) &= \int_H (y\cdot\xi_1)^{a_2+a_3}(y\cdot\xi_2)^{a_3+a_1}(y\cdot\xi_3)^{a_1+a_2} dy = \\ &= c(a_1,a_2,a_3)(\xi_1\cdot\xi_2)^{a_3}(\xi_2\cdot\xi_3)^{a_1}(\xi_3\cdot\xi_1)^{a_2} \end{aligned}$$

Second step: integrate the star-triangle identity.
Probability of random triangles on a sphere

$$\hat{J}(a_1,a_2,a_3) = \int_{{\gamma_0}^3} (\xi_1{\cdot}\xi_2)^{a_3}(\xi_2{\cdot}\xi_3)^{a_1}(\xi_3{\cdot}\xi_1)^{a_2}\, d\mu(\xi_1)\, d\mu(\xi_2)\, d\mu(\xi_3),$$

Unequal masses

$$h_d(\kappa,\nu,\lambda) = \int_1^\infty P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\kappa}(u)\, P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(u) P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\lambda}(u)(u^2-1)^{-\frac{d-2}{4}}\, du =$$

$$\frac{1}{(2\pi)^{\frac{3d}{2}}\omega_{d-1}} \int_{\gamma_0}\int_{\gamma_0}\int_{\gamma_0}\int_{H_d} (y\cdot\xi_1)^{-\frac{d-1}{2}-i\kappa}\,(y\cdot\xi_2)^{-\frac{d-1}{2}-i\nu}\,(y\cdot\xi_3)^{-\frac{d-1}{2}-i\lambda}\,dy\, d\mu_\gamma(\xi_1)\, d\mu_\gamma(\xi_2)\, d\mu_\gamma(\xi_3)$$

$$= \frac{2^{\frac{d}{2}}}{(4\pi)^{\frac{3}{2}}\Gamma\left(\frac{d-1}{2}\right)} \frac{\prod_{\epsilon,\epsilon',\epsilon''=\pm1}\Gamma\left(\frac{d-1}{4}+\frac{i\epsilon\kappa+i\epsilon'\nu+i\epsilon''\lambda}{2}\right)}{\left[\prod_{\epsilon=\pm1}\Gamma\left(\frac{d-1}{2}+i\epsilon\kappa\right)\right]\left[\prod_{\epsilon'=\pm1}\Gamma\left(\frac{d-1}{2}+i\epsilon'\nu\right)\right]\left[\prod_{\epsilon''=\pm1}\Gamma\left(\frac{d-1}{2}+i\epsilon''\lambda\right)\right]}$$

Complementary fields. Inflation

$$W_\nu^2(z,z') = \int_{-\infty}^{\infty} \kappa d\kappa \;\rho(\kappa^2,\nu) W_\kappa(z,z')$$

$$\rho(\kappa^2,\nu) = \frac{R^{2-d}\sinh\pi\kappa}{(4\pi)^{\frac{d+2}{2}}\sqrt{\pi}\Gamma(\frac{d-1}{2})} \frac{\Gamma\left(\frac{d-1}{4}+\frac{i\kappa}{2}\right)\Gamma\left(\frac{d-1}{4}-\frac{i\kappa}{2}\right)}{\Gamma\left(\frac{d+1}{4}+\frac{i\kappa}{2}\right)\Gamma\left(\frac{d+1}{4}-\frac{i\kappa}{2}\right)}$$

$$\times\,\Gamma\left(\frac{d-1}{4}+i\nu+\frac{i\kappa}{2}\right)\Gamma\left(\frac{d-1}{4}-i\nu+\frac{i\kappa}{2}\right)\Gamma\left(\frac{d-1}{4}+i\nu-\frac{i\kappa}{2}\right)\Gamma\left(\frac{d-1}{4}-i\nu-\frac{i\kappa}{2}\right)$$

$$W_\nu^2 = \int_{-\infty}^{\infty} \kappa\, d\kappa\; \rho_\nu(\kappa) W_\kappa \;+\; \sum_{n=0}^{N-1} A_n(\nu)\, W_{i(\mu+2i\nu+2n)}$$

$$A_n(\nu) \;=\; \frac{8\pi(-1)^n}{n!2^d\pi^{\frac{1+d}{2}}R^{d-2}\Gamma(\mu)} \frac{\Gamma(\mu+2i\nu+n)\Gamma(-2i\nu-n)}{\Gamma(\mu+2i\nu+2n)\Gamma(-\mu-2i\nu-2n)}$$

$$\times\frac{\Gamma(\mu+n)\Gamma(-i\nu-n)\Gamma(\mu+i\nu+n)}{\Gamma(-i\nu-n+\frac{1}{2})\Gamma(\mu+i\nu+n+\frac{1}{2})}$$

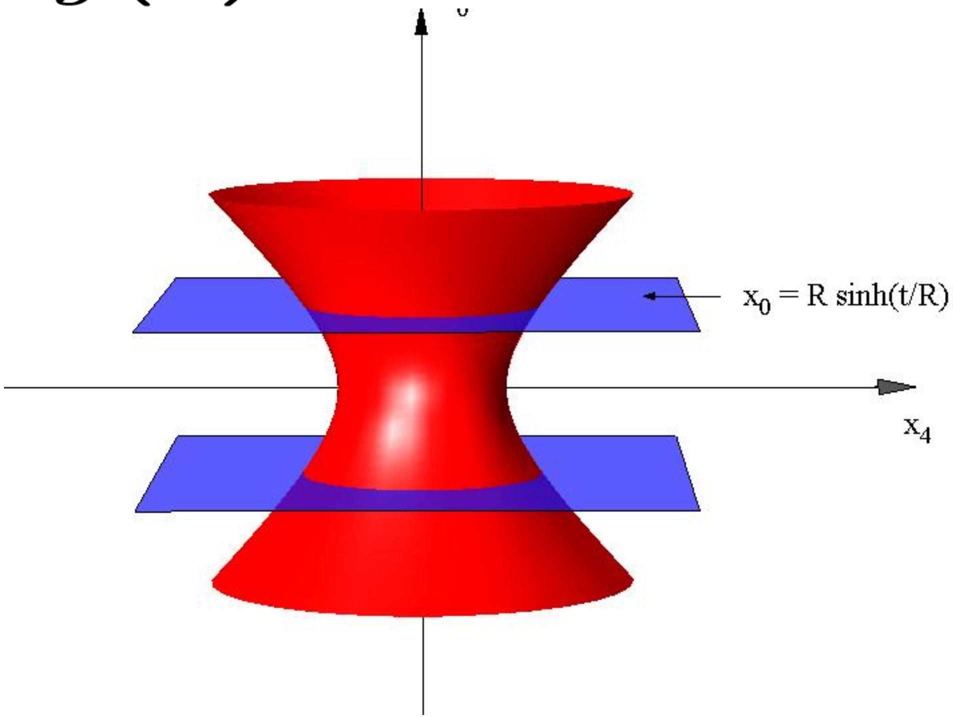
$$\mu = (d-1)/2$$

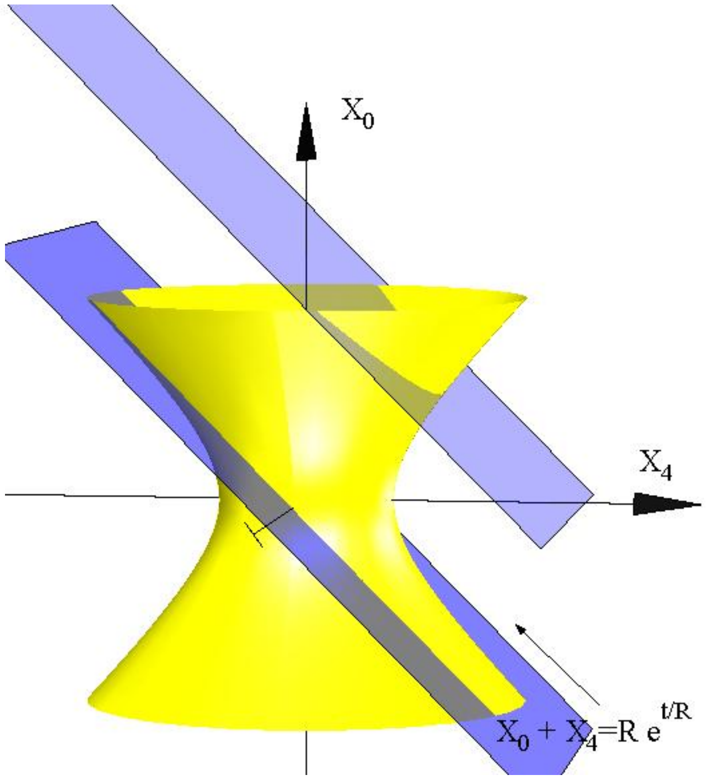
The number of discrete terms is the largest N satisfying $N < 1 + |\Im\nu| - \mu/2$, or 0 if this is negative. A particle of the complementary series with parameter $\kappa = i\beta$ can only decay into two particles with parameter $\nu = \frac{i}{2}(|\beta| + \mu + 2n)$, where n is any integer such that $0 \le 2n < \mu - |\beta|$, and the decay is instantaneous.

A particle with mass $M \ll m_c$ can only decay into two particles of mass $m \sim M/\sqrt{2}$.

de Sitter lifetime

$$g(x) \to 1$$





$$\lim_{T\to\infty} \frac{2\lambda^2 C(\kappa)\rho(\kappa^2,\nu)\ \int g(x)\,|F(x)|^2\,dx}{T\int \overline{f_0(x)}W_\kappa(x,\ y)\,f_0(y)\,dx\,dy} = \rho(\kappa^2,\nu)\frac{2\lambda^2\pi\coth(\pi\kappa)^2}{|\kappa|}$$

These works were done with

Lifetime of a de Sitter Particle

- Jacques Bros, Vincent Pasquier, Michel Gaudin (SPhT-Cea-Saclay)
- Henri Epstein (IHES-Bures sur Yvette)

Symmtries and conservation laws on the de Sitter universe

- Vittorio Gorini, Sergio Cacciatori (Insubria)
- Alexander Kamenshchik (Bologna)