Breaking of Lorentz symmetry by vector fields and accelerated cosmology

spontaneous workshop III

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Plan

- The CC problem and the idea of self-accelerated cosmology
- Tensor condensates and large-distance modification of gravity
- Cosmology with vector condensates
- 5d and 4d interpretations
- Problems
- Conclusions and open issues

Our Universe is expanding with acceleration

The first guess is the cosmological constant

\+ ΛCDM well fits the data

-- CC+particle physics= HUGE hierarchy problem

try alternatives (quintescence, modified gravity, ...)

Self-accelerated cosmology

It is conceivable to have a mechanism nullifying the CC: e.g. symmetry (Linde), quantum gravity (Hawking, Coleman)

... or rather a mechanism pushing the CC to the smallest possible non-negative value

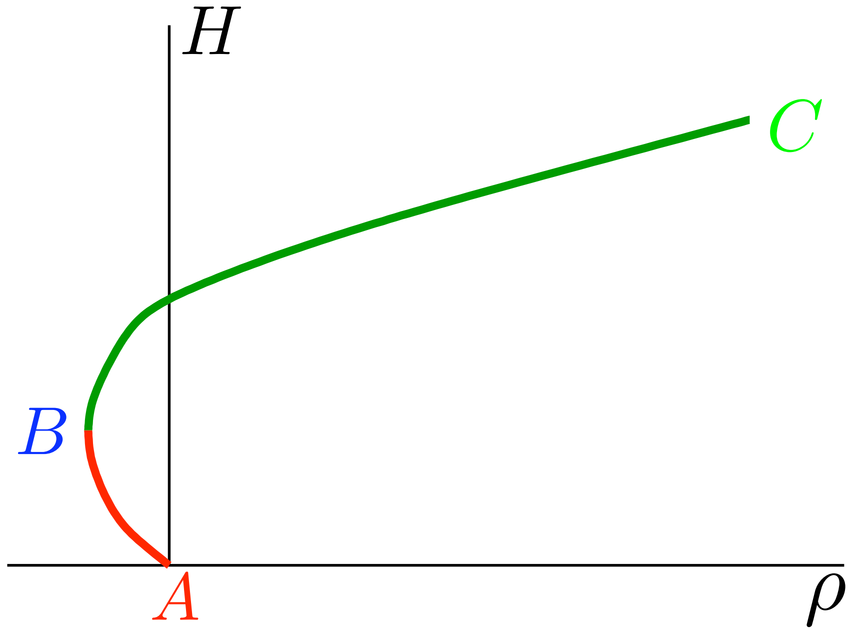
$\Rightarrow$ look for a theory where the effective CC is bounded from below

= expansion of the Universe always ends up in the accelerated phase

Looking for modified Friedman equation

$$H^2 = \frac{8\pi G}{3}\rho_{mat} + \Lambda + \delta$$

$$\delta = H_0 (H/H_0)^\alpha \ , \quad \alpha < 2$$



- Purely gravitational realization: DGP (Dvali, Gabadadze, Porrati)

  but self-accelerated solution contains ghost

- Try matter which reacts to the change of the metric

  simple choice: space-like vector with fixed norm

$$g^{\mu\nu} A_\mu A_\nu = -v^2$$

$$g^{ij} = -\frac{1}{a^2(t)}\delta^{ij} \quad \Rightarrow \quad A_i = n_i v a(t)$$

- To preserve spatial rotations
  $\Rightarrow$ 3 vectors A^a_μ

$$g^{\mu\nu} A^a_\mu A^b_\nu = -v^2 \delta^{ab} \quad \Rightarrow \quad A^a_i = v a(t) \delta^a_i$$

- Does not work in 4d (Libanov, Rubakov)

$$T_{00} \propto g^{ij} \partial_0 A_i \partial_0 A_j \propto v^2 H^2$$

- Go to higher dimensions (brane world): non-locality in extra dimensions reduces the number of time derivatives in effective EMT
- Gravity is localized by warped bulk geometry

The model

- 5d AdS bulk + positive tension brane (Randall, Sundrum)
- 3 massless vectors A_M^a , $a = 1, 2, 3$,living in the bulk

standard bulk action

$$S_{A,bulk} = -\frac{1}{4\alpha^2}\int d^5x\sqrt{g}F^a_{MN}F^{a\,MN} \;,\;\; F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M$$

boundary conditions on the brane

$$\bar{g}^{\mu\nu}A^a_\mu A^b_\nu = -v^2\delta^{ab}$$

Symmetries

- 5d diff-invariance
- global $SO(3)$ acting on indices a, b
- gauge $[U(1)]^3$ in the bulk; broken explicitly on the brane

Parameters: $\varkappa \sim 1,\;\; v < \alpha^{-2} \sim l^{-1} \lesssim M_5 \lesssim \; M_{Pl} = (lM_5^3)^{1/2}$

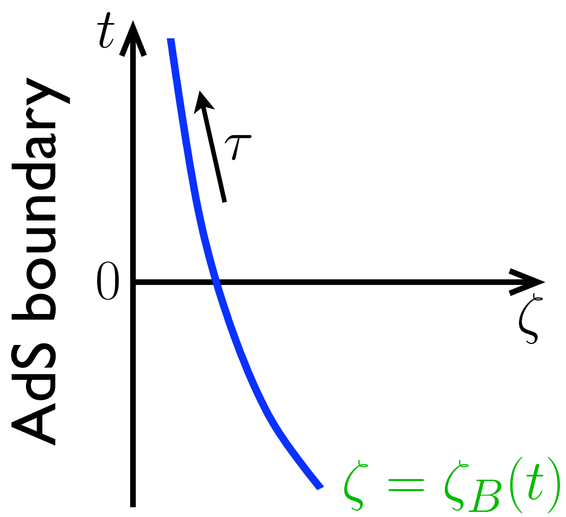
Cosmology

General Ansatz preserving spatial rotations and translations

$$ds^2 = F(\zeta, t)(dt^2 - d\zeta^2) - r^2(\zeta, t)d\mathbf{x}^2$$

$$A_i^a = v\delta_i^a A(\zeta, t) \ , \quad A_0^a = A_5^a = 0$$

Brane moves in the bulk



$$d\bar{s}^2 = d\tau^2 - a^2(\tau)d\mathbf{x}^2$$

$$d\tau = \sqrt{F(\zeta_B(t), t)}\, dt$$

$$a = r(\zeta_B(t), t)$$

Qualitative picture

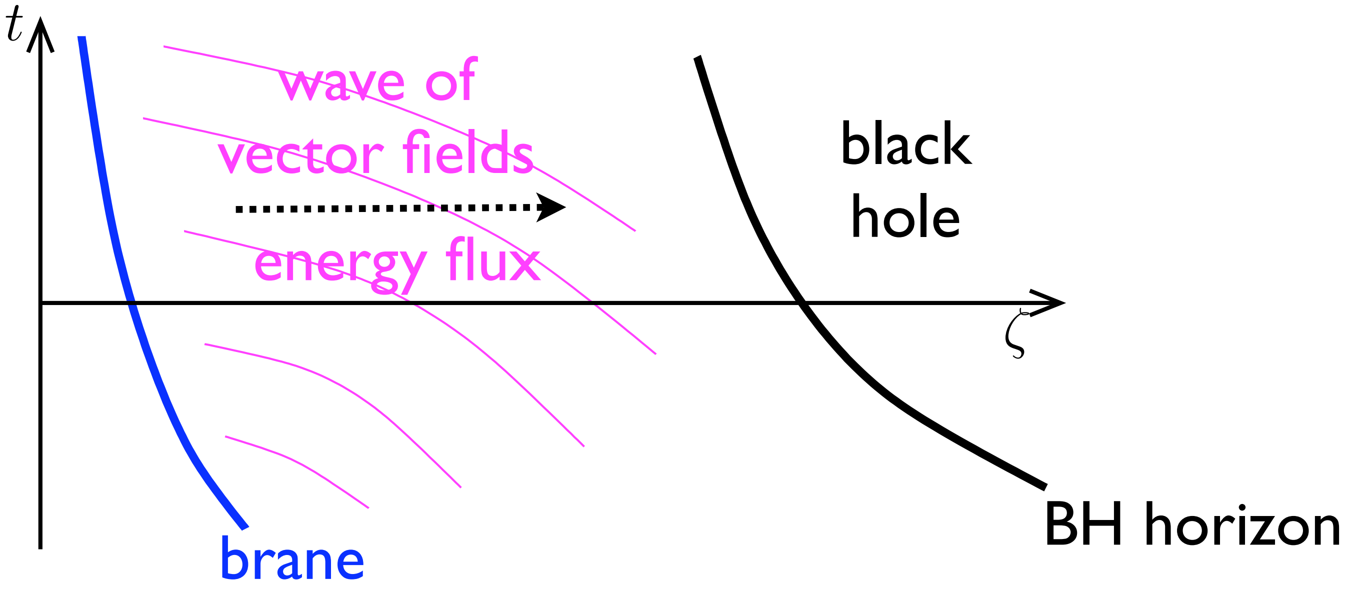
The brane sources the vector fields

Far from the brane the vector fields get blue-shifted

$$(T^{vec})^2 \propto (\zeta/l)^8$$

and form a black hole

The BH grows in time and back-reacts on the brane



A remark on boundary conditions

Equation for vectors in the vicinity of the brane

$$-\partial_\zeta^2 A + \frac{\partial_\zeta A}{\zeta} + \partial_t^2 A = 0$$

$\Rightarrow$ there is well-defined limit $A(\zeta = 0)$; $\partial_\zeta A \to 0\ ,\quad \zeta \to 0$

The brane is near the boundary,

$$\zeta_B \partial_\zeta A \ll A$$

will be confirmed by the result

$\Rightarrow$ possible to set boundary conditions for vectors on the AdS boundary:

$$A\big|_{\zeta=0} = a(t)$$

Bulk dynamics separated from the brane dynamics

Bulk solution

- Boundary conditions

$$F \to \left(\frac{l}{\zeta}\right)^2 , \quad r \to \frac{l}{\zeta} , \quad A \to a(t) \qquad \text{at} \qquad \zeta \to 0$$

- Einstein equations

$$U = l^{-1}(t - \zeta)$$

$$V = l^{-1}(t + \zeta)$$

$$-\frac{\partial_v^2 r}{r} + \frac{\partial_v F \partial_v r}{F r} = \frac{\lambda}{r^2}(\partial_v A)^2 ,$$

$$-\frac{\partial_u^2 r}{r} + \frac{\partial_u F \partial_u r}{F r} = \frac{\lambda}{r^2}(\partial_u A)^2 ,$$

$$-\partial_u \partial_v \ln F - \frac{\partial_u \partial_v r}{r} + \frac{4\partial_u r \partial_v r}{r^2} = \frac{\lambda}{r^2}\partial_u A \partial_v A ,$$

$$\frac{\partial_u \partial_v r}{r} + \frac{2\partial_u r \partial_v r}{r^2} = -F$$

$$\boxed{\lambda = \frac{8\pi G_5 v^2}{\alpha^2}}$$

- Vector equation

$$\partial_u(r\partial_v A) + \partial_v(r\partial_u A) = 0$$

Dilatation symmetry of equations

$$A(U,V) \mapsto \gamma A(\gamma U, \gamma V) \ ,$$

$$r(U,V) \mapsto \gamma r(\gamma U, \gamma V) \ ,$$

$$F(U,V) \mapsto \gamma^2 F(\gamma U, \gamma V)$$

Boundary conditions for metrics preserve this symmetry whereas BC for vector fields breaks it in general case

Important exception:

$$a(t) = -\frac{1}{Ht}$$

$$\Updownarrow$$

inflationary cosmology on the brane

Ansatz

$$A = -\frac{1}{\sqrt{\lambda}U}\alpha\left(\frac{V}{U}\right)\;,\quad r = -\frac{1}{U}\rho\left(\frac{V}{U}\right)\;,\quad F = \frac{1}{U^2}f\left(\frac{V}{U}\right)$$

reduces the problem to ordinary differential equations

$$-\frac{\rho''}{\rho} + \frac{f'\rho'}{f\rho} = \left(\frac{\alpha'}{\rho}\right)^2\;,$$

$$-\frac{\rho''}{\rho} - \frac{2\rho'}{x\rho} + \frac{f'\rho'}{f\rho} + \frac{f'}{xf} = \left(\frac{\alpha'}{\rho} + \frac{\alpha}{x\rho}\right)^2\;,$$

$$\left(\frac{f'}{f}\right)' + \frac{\rho''}{\rho} + \frac{f'}{xf} - \frac{2\rho'}{x\rho} - 4\left(\frac{\rho'}{\rho}\right)^2 = -\left(\frac{\alpha'}{\rho}\right)^2 - \frac{\alpha\alpha'}{x\rho^2}\;,$$

$$\frac{\rho''}{\rho} + \frac{4\rho'}{x\rho} + 2\left(\frac{\rho'}{\rho}\right)^2 = \frac{f}{x}\;,$$

$$\frac{\alpha''}{\alpha} + \frac{\rho'\alpha'}{\rho\alpha} + \frac{5\alpha'}{2x\alpha} + \frac{\rho'}{2x\rho} = 0$$

$$x = \frac{V}{U}$$

with boundary conditions

$$f \to \frac{4}{(1-x)^2}\;,\quad \rho \to \frac{2}{1-x}\;,\quad \alpha \to \frac{\sqrt{\lambda}}{Hl}\;,\quad x \to 1$$

Approximate solution at $\frac{\sqrt{\lambda}}{Hl} \gg 1$

$$\frac{1}{\rho_h}\left(-\frac{1}{2}\ln\frac{\rho-\rho_h}{\rho+\rho_h}+\frac{\pi}{2}-\mathrm{arctg}\,\frac{\rho}{\rho_h}\right)=1-x$$

$$f=\rho^2-\frac{\rho_h^4}{\rho^2}$$

$$\rho_h=\left(\frac{\lambda}{2H^2l^2}\right)^{1/3}$$

$$\alpha(x)\approx\frac{\sqrt{\lambda}}{Hl}$$

Metric in a more suggestive form

$$ds^2=l^2\left(\mathrm{e}^{-2y}f(\rho)\frac{d\eta^2}{\eta^2}-\frac{d\rho^2}{\mathrm{e}^{-2y}f(\rho)}-\frac{\mathrm{e}^{-2y}\rho^2}{\eta^2}d\mathbf{x}^2\right)$$

where $U=l^{-1}\eta\mathrm{e}^{y}$, $V=l^{-1}\eta\mathrm{e}^{-y}$

At $y\ll 1\,,\quad \Delta\eta\ll\eta$ $\quad AdS$-Schwartzschild

Friedman equation

$$H^2 = \frac{8\pi G}{3}\rho_{mat} + \Lambda + H^2 \left(\frac{\lambda^2}{4Hl}\right)^{2/3}$$

Fixed point at $H_c = \frac{\lambda^2}{4l} \sim \frac{v^4}{M_{Pl}^3}$

$\Rightarrow$ $\Lambda_{eff} \sim v^2/M_{Pl}$ $\Leftrightarrow$ "see-saw" for cosmological constant

H_c=present Hubble parameter for $v \sim 1\mathrm{Tev}$

This calculation is valid when the expansion is nearly de Sitter

Going beyond de Sitter

Use quasi-stationary approximation for the bulk BH

$$H^2 = \frac{8\pi G}{3}\rho_{mat} + \Lambda + \delta$$

$$\delta = \frac{1}{l^2 a^4}\left(\frac{3\lambda l}{2}\int d\tau a \dot{a}^2\right)^{4/3}$$

If expansion is dominated by matter with $w < 1$

$\rho_{mat} \propto a^{-3(1+w)}$, $a \propto \tau^{\frac{2}{3(1+w)}}$ ⟹ $\delta = H^2 \left(\frac{\lambda^2}{(1-w)^2 l H}\right)^{2/3}$

$\delta \propto a^{-2(1+w)}$ dilutes slowlier than matter density

AdS/CFT description

theory in AdS	CFT (?) on the brane with $N_c \sim \sqrt{l^3/G_5}$
A_μ	global current j_μ
$A_\mu\big\rvert_{\zeta=\zeta_B} = A_\mu^{(4)}$	$\sqrt{\frac{l}{\alpha^2}}\int d^4x\sqrt{-g}A_\mu^{(4)}j^\mu$
flux of energy into the bulk	heating of conformal plasma
bulk black hole	CFT at non-zero temperature

- $A_i^{(4)\,a} = va(\tau)\delta_i^a$ $\Rightarrow$ 3 electric fields $E_i^a = \frac{v\sqrt{l}}{\alpha} H \delta_i^a$
- Electric fields produce currents; on dimensional grounds $\mathbf{j}^a = (\gamma\omega + \beta T)\mathbf{E}^a$. For cosmological setup $\omega \sim H \ll T$ so $\mathbf{j}^a = \beta T \mathbf{E}^a$
- Energy balance of plasma

$$\dot{\mathcal{E}} + 4H\mathcal{E} = \mathbf{E}^a \mathbf{j}^a = \beta T \left(\mathbf{E}^a\right)^2$$

$$\mathcal{E} = N_c^2 T^4$$

- Contribution into the Friedman equation $\delta = \frac{8\pi G}{3}\mathcal{E}$

N.B. Possible generalization to the case of the fluid with

$$\mathbf{j}^a = c(\mathcal{E})\mathbf{E}^a$$

Problems with stability

The condition $g^{\mu\nu} A^a_\mu A^b_\nu = -v^2 \delta^{ab}$ explicitly breaks gauge invariance $\Rightarrow$ introduces new degrees of freedom with nasty properties

Example: "vector Goldstone model"

$$V(A^a_\mu) = \varkappa(g^{\mu\nu}A^a_\mu A^b_\nu + v^2\delta^{ab})^2$$

summation over $a,\ b$

dangerous modes --- longitudinal components

$$A^a_\mu \to \partial_\mu\phi^a \quad \Rightarrow \quad \mathcal{L}_\phi = -\varkappa(\partial_\mu\phi^a\partial_\nu\phi^b + v^2\delta^{ab})^2$$

$$\phi^a = v'x^a + \pi^a$$

background ($v'x^a$) perturbations (π^a)

$$\mathcal{L}_\pi \propto 2(v'^2 - v^2)(\partial_\mu\pi^a)^2 - v'^2(\partial_a\pi^b + \partial_b\pi^a)^2$$

$v' > v$ $\Rightarrow$ OK

$v' < v$ $\Rightarrow$ tachyonic ghost $\omega^2 = -\dfrac{v'^2}{v^2 - v'^2}k^2$

Preserve as much gauge symmetry as possible

$$A_\mu^a \to \bar{A}_\mu^a \equiv A_\mu^a - A_\nu^a \partial_\nu \phi \partial_\mu \phi / \sqrt{X}, \qquad X = (\partial_\mu \phi)^2$$

Remnant of gauge symmetry:

$$A_\mu^a \mapsto A_\mu^a + \partial_\mu \alpha^a(\phi)$$

Lagrangian of longitudinal modes

$$\mathcal{L}_\phi = \tilde{v}^4 \big((\partial_\mu \phi)^2 - 1\big)^2 - \varkappa(\partial_\mu \bar{\phi}^a \partial_\nu \bar{\phi}^b + v^2 \delta^{ab})^2$$

Take background $\phi = \alpha t, \quad \phi^a = v' x^a$

and consider perturbations

→ 3 modes are non-dynamical, the fourth mode with dispersion relation

$$\omega^2 = \left[(\alpha^2 - 1) + \frac{\varkappa v'^2}{\tilde{v}^4}(v^2 - v'^2)\right] k^2$$

The instability is alleviated but not removed

Ways to proceed

- Hope that the instability is cut by non-linear effects (a la Arkani-Hamed et al. (2005))
- Kill the unstable mode by constraint

$$(\partial_\mu \phi)^2 - 1 = 0$$

→ model becomes stable at linear level

gravitational field of point sources coincides with that in GR

Beware: the instability may come back at non-linear level

Conclusions and open questions

- Vector (tensor) condensates may provide self-accelerated when combined with the brane-world idea 
- 4d interpretation using AdS/CFT allows phenomenological generalizations in terms of conducting fluids 
- Hard to make the model stable / instabilities harmless. 

New ideas needed