EXACT BLACK HOLE SOLUTIONS IN MASSIVE GRAVITY

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Outline

Why study modifications of gravity?

- Gravity is the most mysterious force which is quite poorly understood
- GR is a beautiful and unique classical theory. However:
  - problems at quantum level
  - problems at largest scales

  $\Longrightarrow$ better understanding of gravity is needed
- Such an understanding may come from attempts to construct a consistent modified theory of gravity
- To be of phenomenological interest, the model has to coincide with GR at scales from mm to kpc

Why black holes are of particular interest in such models

- Black holes are the most fascinating non-linear manifestations of gravity. Hence they are good theoretical laboratories for studying gravity models.
- The properties of black holes are deeply interrelated with fundamental structure of the underlying theory.
- Black holes are states where macroscopic and microscopic features of theory (including quantum properties) are mixed together.
- Finally, BH's most likely exist in nature and will be observed and perhaps studied at the quantitative level in the near future.

Massive gravity as a "deformation" of GR

- Massive gravity: scalar-tensor theory with Lorentz invariance that is spontaneously broken by space-time dependent condensates of scalar fields. The action is *[Dubovsky'2004]*

$$S = \int \sqrt{g}\left\{-R + L(\text{matter}) + \Lambda^4 F(\text{scalars})\right\}$$

  Note: no direct coupling between scalars and ordinary matter
- The model is understood as an effective theory below the scale Λ

Coupling of scalars to gravity

- Four scalars ϕ^0, ϕ^i with derivative coupling

$$F = F(X, W^{ij})$$

$$X = g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^0$$

$$W^{ij} = g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j - g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^i \cdot g^{\alpha\beta}\partial_\alpha\phi^0\partial_\beta\phi^j/X$$

- Obeys the symmetry

$$\phi^i \to \phi^i + \xi^i(\phi^0)$$

which ensures the absence of ghosts and instabilities at the perturbative level.

Vacuum and perturbations

- For a generic function F, the energy-momentum of scalars $T_{\mu\nu}$ vanishes for the following configuration

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} \\ \phi^0 &= \Lambda^2 \cdot t \\ \phi^i &= \Lambda^2 \cdot x^i \end{aligned}$$

$\Longrightarrow$ Minkowski space is a solution

- The spectrum of perturbations contains massive tensor modes and no other particles. Graviton mass is:

$$m \sim \Lambda^2 / M_{\rm Pl}$$

Cosmological solutions

- Flat cosmological ansatz:

$$\begin{aligned} ds^2 &= dt^2 - a^2(t)dx_i^2 \\ \phi^0 &= \Lambda^2 \cdot \phi(t) \\ \phi^i &= \Lambda^2 \cdot x^i \end{aligned}$$

- Cosmological evolution has an attractor where $F = F(X^\gamma W^{ij})$. In that point the scalars give two contributions to ρ, behaving like cosmological constant and like matter with the equation of state depending on γ.

Summary of the model:

- Spectrum of perturbations: massive graviton (2 polarizations) with

$$m \sim \Lambda^2 / M_{\rm Pl}$$

  Current bound $m \lesssim 10^{-20}$ eV *[Dubovsky,PT,Tkachev'2005]*
- Force between static sources: at the attractor point, standard Newton's law + strongly suppressed corrections
- Cosmology: flat Universe; extra contributions from scalar fields behaving like cosmological constant and matter with the equation of state $-1 < w < \infty$
- Cosmological perturbations: standard behavior in some range of parameters *[Bebronne,PT'2007]*
- !Rem: to be tested against BBN

Do black holes get modified in this model?

- Schwarzschild metric is a solution to the field equations with

$$\phi_0 = \Lambda^2 \left(t + 2\sqrt{r r_s} + r_s \ln \frac{\sqrt{r} - \sqrt{r_s}}{\sqrt{r} + \sqrt{r_s}} \right)$$

[Dubovsky,PT,Zaldarriaga'2007]

- Kerr metric is not a solution no matter what are scalar fields
- Generically, black holes may have hair due to presence of instantaneous interaction
- Modified black hole solutions have been found in bi-gravity models *[Berezhiani,Comelli,Nesti,Pilo'2008]*

Spherically-symmetric ansatz

6 functions to begin with:

$$
\begin{aligned}
ds^2 &= \alpha(r)dt^2 + 2\delta(r)dtdr - \beta(r)dr^2 - \kappa(r)d\Omega^2 \\
\phi^0 &= \Lambda^2(t + h(r)) \\
\phi^i &= \phi(r)\frac{\Lambda^2 x^i}{r}
\end{aligned}
$$

Use gauge freedom to set

$$
\begin{aligned}
\kappa(r) &= r^2 \\
\delta(r) &= 0
\end{aligned}
$$

$\Longrightarrow$ 4 radial functions $\alpha(r)$, $\beta(r)$, $h(r)$ and $\phi(r)$

Einstein equations

$$G^0_0 = \frac{1}{M^2_{Pl}} T^0_0 \qquad G^r_r = \frac{1}{M^2_{Pl}} T^r_r$$

$$G^\theta_\theta = \frac{1}{M^2_{Pl}} T^\theta_\theta \qquad 0 = T^r_0$$

When the last equation holds, $T^0_0 = T^r_r$, which implies that $G^0_0 = G^r_r$, which in turn implies that

$$\alpha(r)\beta(r) = 1.$$

We are left with 3 differential equations for 3 variables.

Analytical example

Main tool — choice of the function F.

From rotational invariance, F has to depend on W^{ij} through three combinations $w_n = \mathrm{Tr}(W^{ij})^n$. Take

$$F = c_0 \left(\frac{1}{X} + w_1 \right) + c_1 \left(w_1^3 - 3w_1 w_2 - 6w_1 + 2w_3 - 12 \right)$$

Constraints to be satisfied:

- Minkowski space has to be a solution
- Graviton should be non-tachyonic

$$c_0 - 6c_1 \geq 0$$

- Stability upon addition of higher-derivative terms

$$c_0 > 0$$

Solution

Due to specific choice of F, equaiton $T^r_0 = 0$ simplifies:

$$\frac{1}{X}\left[-c_0 + \phi'^2\left(6c_1\frac{\phi^4}{r^4} + c_0 - 6c_1\right)\right] = 0$$

The solution to this equation is

$$\phi = br,$$

with

$$(b^2 - 1)(6b^4 + 6b^2 + c_0/c_1) = 0$$

Solution

The remaining two equations can also be solved analytically,

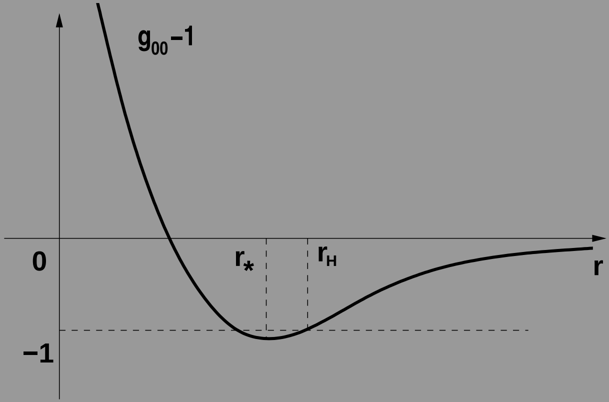
$$
\begin{aligned}
\alpha(r) &= 1 - \frac{2M}{r} - \frac{S}{r^{\lambda}} + \Lambda_c r^2 \\
\beta(r) &= 1/\alpha(r) \\
h(r) &= \pm \int \frac{dr}{\alpha} \left[1 - \alpha \left(\frac{S}{c_0 m^2} \frac{\lambda - 1}{r^{\lambda+2}} + 1 \right)^{-1} \right]^{1/2} \\
\phi(r) &= b \cdot r
\end{aligned}
$$

where

$$
\begin{aligned}
\lambda &= -12 b^6 \frac{c_1}{c_0} \\
\Lambda_c &= 2 m^2 c_1 (b^6 - 1)
\end{aligned}
$$

Behavior of solutions

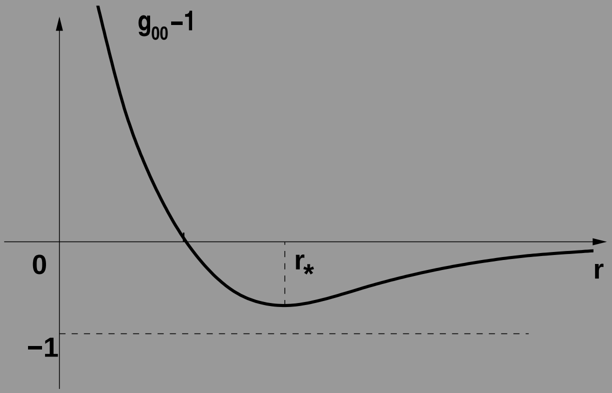
Case of interest: $c_1 = -1$, $c_0 > 0 \Longrightarrow \lambda > 0$.
Consider the branch $\Lambda_c = 0$ (no cosmological constant)
$M > 0$, $S < 0$.



Gravity is weaker close to horizon

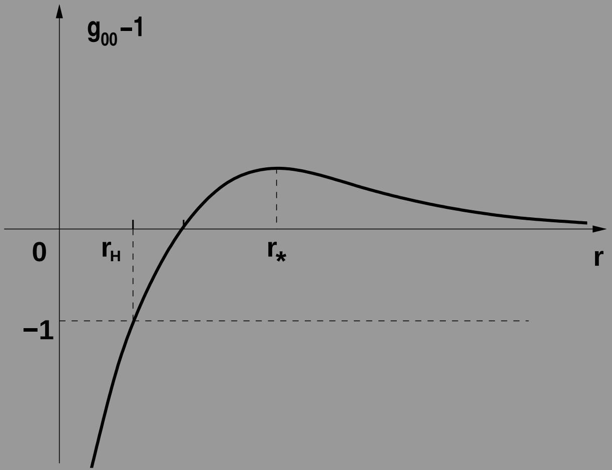
Behavior of solutions

$M > 0$, $S < 0$.



Behavior of solutions

$M < 0$, $S > 0$.



Gravity is stronger close to horizon

Summary

- Exact spherically-symmetric solutions in modified gravity are different from Schwarzschild black holes
- They show rich behavior, including anti-gravitation at large distances
- Open questions:
  - stability
  - formation of configurations with $S \neq 0$